Computer algebra independent integration tests

6-Hyperbolic-functions/6.7-Miscellaneous/6.7.1-Hyperbolic-functions

Nasser M. Abbasi

May 24, 2020 Compiled on May 24, 2020 at 3:18pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1059]. This is test number [185].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1059)	% 0. (0)
Mathematica	% 99.24 (1051)	% 0.76 (8)
Maple	% 88.39 (936)	% 11.61 (123)
Maxima	% 69.88 (740)	% 30.12 (319)
Fricas	% 91.41 (968)	% 8.59 (91)
Sympy	% 24.83 (263)	% 75.17 (796)
Giac	% 73.56 (779)	% 26.44 (280)

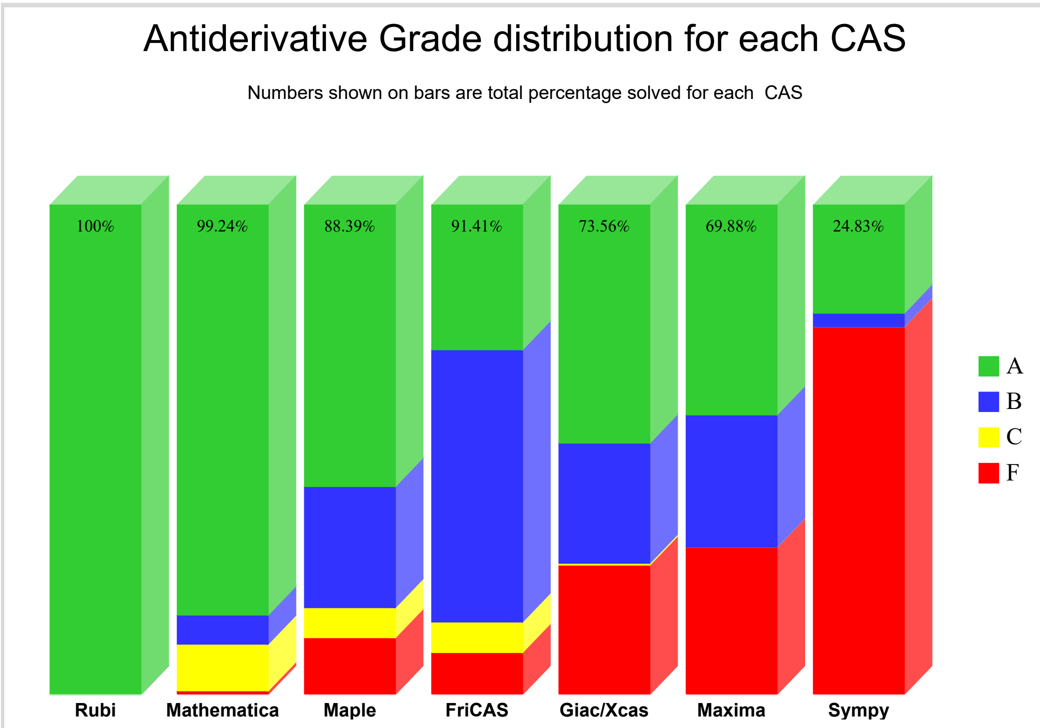
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

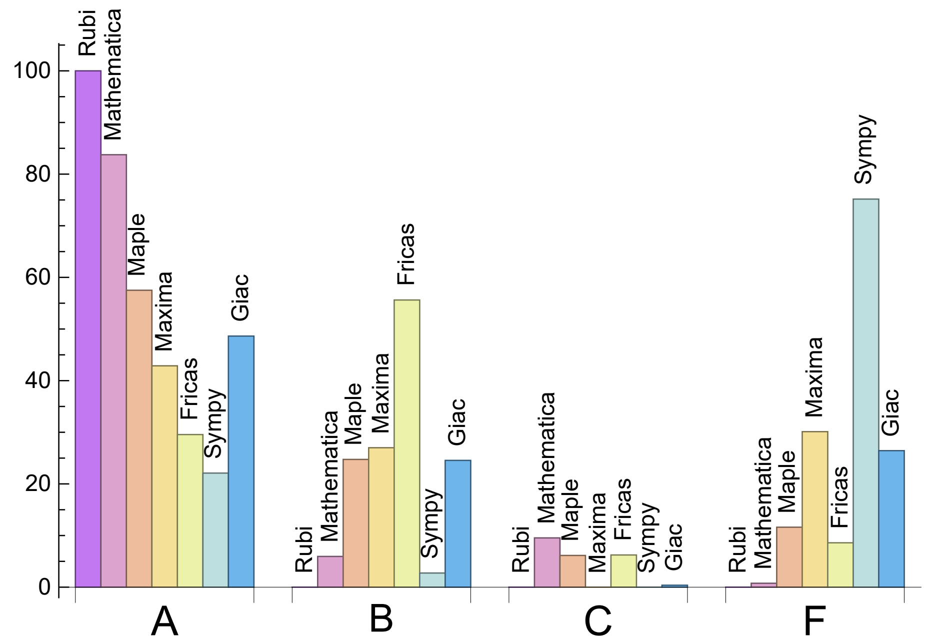
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	83.76	5.95	9.54	0.76
Maple	57.51	24.74	6.14	11.61
Maxima	42.87	27.01	0.	30.12
Fricas	29.56	55.62	6.23	8.59
Sympy	22.1	2.74	0.	75.17
Giac	48.63	24.55	0.38	26.44

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	67.91	0.92	50.	1.
Mathematica	2.48	186.14	2.23	52.	1.
Maple	0.08	202.49	2.2	64.5	1.38
Maxima	1.18	120.83	2.64	82.	1.85
Fricas	2.07	1339.59	16.39	455.	9.4
Sympy	15.07	211.46	3.79	61.	1.77
Giac	1.07	117.85	2.11	77.	1.82

1.4 list of integrals that has no closed form antiderivative

{334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 870, 1014, 1015, 1016, 1017}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {219, 337, 349, 364, 372, 393, 405, 406, 425, 433, 439, 446, 462, 474, 475, 488, 490, 503, 531, 547, 556, 590, 592, 594, 622, 761, 762, 763, 764, 765, 766, 767, 768, 769, 774, 775, 776, 777, 854, 911, 918, 930, 935, 943, 944, 966}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

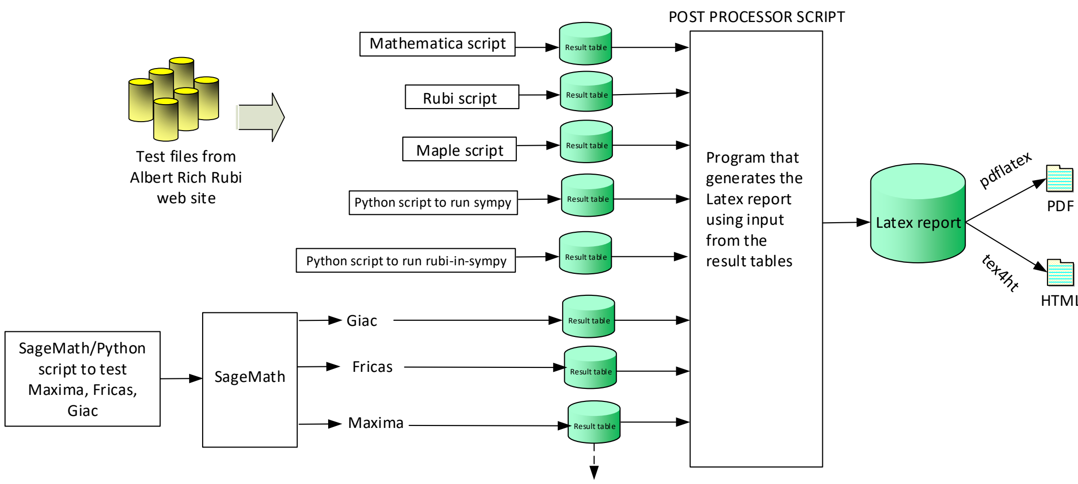
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 32, 34, 35, 36, 37, 38, 40, 42, 44, 45, 46, 47, 48, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 148, 149, 151, 152, 154, 157, 160, 161, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 214, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 237, 240, 242, 243, 244, 245, 246, 248, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 423, 424, 425, 428, 429, 430, 431, 434, 436, 437, 438, 439, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 543, 544, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 665, 667, 668, 669, 671, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 751, 753, 754, 755, 756, 757, 758, 759, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 938, 941, 942, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 982, 983, 984, 986, 987, 988, 990, 991, 992, 993, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1057, 1058, 1059 }

B grade: { 1, 2, 3, 4, 6, 8, 24, 100, 127, 143, 144, 150, 155, 156, 162, 188, 189, 205, 239, 254, 358, 426, 427, 432, 440, 460, 461, 470, 482, 495, 496, 508, 509, 516, 563, 584, 614, 616, 656, 664, 666, 677, 687, 702, 704, 745, 749, 750, 752, 760, 882, 949, 994, 998, 999, 1000, 1001, 1002, 1026, 1027, 1053, 1055, 1056 }

C grade: { 29, 31, 33, 39, 41, 43, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 146, 147, 153, 158, 159, 165, 211, 213, 215, 217, 229, 230, 231, 232, 233, 236, 238, 241, 247, 249, 337, 349, 364, 393, 420, 422, 433, 435, 446, 462, 491, 504, 529, 531, 540, 542, 545, 547, 556, 558, 590, 591, 592, 593, 594, 595, 620, 622, 670, 672, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 775, 776, 777, 852, 869, 911, 918, 930, 935, 937, 939, 940, 943, 944, 966 }

F grade: { 772, 773, 778, 779, 981, 985, 989, 997}

2.1.3 Maple

A grade: { 1, 8, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 90, 93, 94, 95, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 121, 122, 123, 127, 131, 132, 133, 134, 151, 154, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 206, 212, 216, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 245, 246, 253, 254, 255, 256, 257, 258, 263, 264, 265, 266, 267, 271, 272, 273, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 315, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 463, 464, 465, 466, 467, 470, 471, 472, 473, 476, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 491, 492, 493, 494, 498, 499, 500, 501, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 517, 518, 519, 520, 521, 522, 525, 526, 527, 572, 573, 578, 579, 580, 581, 583, 585, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 626, 627, 628, 630, 632, 633, 636, 637, 638, 640, 642, 643, 644, 645, 646, 647, 648, 650, 655, 657, 658, 660, 661, 662, 663, 667, 668, 670, 671, 672, 673, 674, 675, 676, 680, 681, 682, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 701, 702, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 730, 731, 732, 735, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 754, 755, 756, 762, 764, 771, 777, 784, 786, 787, 788, 803, 805, 806, 848, 855, 856, 857, 864, 866, 870, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 909, 910, 911, 912, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 982, 993, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1038, 1039, 1041, 1043, 1044, 1045, 1046, 1049, 1050, 1051, 1055 }

B grade: { 2, 3, 4, 5, 6, 7, 84, 85, 88, 96, 97, 100, 115, 116, 119, 124, 125, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 152, 153, 158, 159, 160, 161, 164, 165, 207, 208, 209, 210, 211, 214, 236, 237, 238, 247, 248, 249, 251, 252, 260, 261, 262, 269, 270, 281, 282, 283, 290, 291, 292, 299, 300, 308, 309, 317, 318, 326, 327, 328, 343, 358, 372, 385, 398, 399, 400, 407, 414, 426, 427, 432, 433, 446, 455, 460, 461, 462, 468, 469, 483, 490, 495, 496, 497, 503, 510, 523, 524, 531, 540, 547, 556, 560, 563, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 582, 584, 587, 589, 619, 620, 621, 622, 623, 624, 625, 629, 631, 634, 635, 639, 641, 649, 651, 652, 653, 654, 656, 659, 664, 665, 666, 669, 677, 683, 697, 700, 703, 704, 705, 724, 727, 733, 734, 736, 743, 744, 745, 753, 758, 759, 760, 761, 763, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 780, 781, 782, 783, 785, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 804, 807, 808, 809, 810, 817, 818, 819, 820, 821, 822, 830, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 851, 852, 853, 858, 859, 860, 861, 862, 863, 865, 867, 868, 869, 908, 913, 980, 981, 983, 984, 986, 987, 990, 991, 992, 994, 997, 1022, 1023, 1024, 1025, 1035, 1036, 1040, 1054, 1056, 1057 }

C grade: { 13, 89, 92, 120, 126, 143, 144, 145, 150, 155, 156, 157, 162, 200, 201, 202, 203, 204, 213, 215, 217, 218, 219, 220, 240, 241, 242, 243, 244, 344, 386, 757, 811, 812, 813, 814, 815, 816, 823, 824, 825, 826, 827, 828, 829, 831, 840, 841, 937, 938, 939, 940, 941, 942, 943, 944, 985, 988, 989, 995, 1037, 1052, 1053, 1058, 1059 }

F grade: { 10, 11, 14, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 91, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 342, 356, 357, 370, 371, 384, 474, 475, 488, 489, 502, 515, 516, 528, 529, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 561, 562, 564, 565, 678, 679, 684, 685, 849, 850, 854, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 998, 999, 1000, 1001, 1002, 1042, 1047, 1048 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 29, 30, 42, 70, 75, 77, 82, 84, 85, 106, 113, 115, 116, 139, 140, 143, 149, 161, 193, 194, 197, 198, 206, 216, 222, 223, 226, 227, 235, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 344, 346, 347, 348, 349, 352, 353, 354, 355, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 374, 375, 376, 377, 378, 379, 381, 382, 383, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 423, 424, 425, 426, 429, 430, 431, 432, 433, 435, 436, 437, 438, 439, 440, 443, 444, 445, 446, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 464, 465, 466, 467, 468, 469, 471, 472, 473, 476, 478, 479, 480, 481, 482, 483, 485, 486, 487, 491, 492, 493, 494, 496, 498, 499, 500, 501, 505, 506, 507, 508, 509, 510, 512, 513, 514, 517, 519, 520, 521, 522, 523, 524, 526, 527, 575, 576, 580, 581, 583, 586, 596, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 617, 618, 627, 628, 629, 630, 631, 632, 633, 637, 638, 640, 641, 642, 647, 648, 658, 660, 662, 668, 669, 670, 671, 672, 674, 675, 680, 681, 688, 690, 691, 693, 697, 700, 707, 709, 711, 713, 715, 717, 719, 721, 723, 730, 731, 732, 733, 739, 740, 741, 746, 747, 748, 753, 754, 755, 756, 783, 786, 802, 803, 804, 805, 806, 807, 809, 811, 812, 813, 814, 815, 816, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 851, 852, 853, 855, 856, 857, 870, 872, 873, 874, 875, 876, 877, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 967, 968, 969, 973, 974, 975, 979, 981, 986, 993, 994, 1013, 1014, 1015, 1016, 1017, 1019, 1026, 1027, 1028, 1031, 1037, 1038, 1043, 1044, 1045, 1046, 1050, 1051, 1055, 1056

B grade: { 5, 7, 10, 11, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 67, 68, 71, 72, 73, 74, 76, 78, 79, 80, 81, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 192, 196, 200, 201, 207, 208, 211, 212, 217, 218, 221, 225, 229, 230, 234, 236, 240, 241, 245, 246, 247, 262, 277, 283, 302, 320, 329, 345, 350, 351, 365, 373, 380, 387, 394, 401, 408, 415, 421, 422, 427, 428, 434, 441, 442, 447, 448, 456, 461, 462, 463, 470, 477, 484, 495, 497, 504, 511, 518, 525, 560, 561, 562, 563, 564, 565, 574, 577, 582, 584, 588, 597, 598, 606, 607, 614, 615, 616, 619, 621, 623, 624, 625, 626, 634, 635, 636, 639, 643, 644, 645, 646, 649, 651, 653, 654, 655, 656, 657, 659, 661, 663, 664, 665, 666, 667, 673, 676, 677, 678, 679, 682, 683, 684, 685, 686, 687, 702, 703, 704, 705, 735, 749, 750, 751, 752, 768, 769, 770, 774, 775, 776, 808, 810, 817, 818, 819, 820, 821, 822, 840, 841, 980, 983, 984, 987, 990, 991, 992, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1018, 1021, 1022, 1023, 1024, 1025, 1030, 1032, 1040, 1048, 1049, 1053, 1054, 1057, 1058 }

C grade: { }

F grade: { 9, 12, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 86, 117, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 195, 199, 202, 203, 204, 205, 209, 210, 213, 214, 215, 219, 220, 224, 228, 231, 232, 233, 237, 238, 239, 242, 243, 244, 248, 249, 289, 342, 343, 356, 357, 358, 370, 371, 372, 384, 385, 418, 419, 420, 474, 475, 488, 489, 490, 502, 503, 515, 516, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 566, 567, 568, 569, 570, 571, 572, 573, 578, 579, 585, 587, 589, 590, 591, 592, 593, 594, 595, 620, 622, 650, 652, 689, 692, 694, 695, 696, 698, 699, 701, 706, 708, 710, 712, 714, 716, 718, 720, 722, 724, 725, 726, 727, 728, 729, 734, 736, 737, 738, 742, 743, 744, 745, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 780, 781, 782, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 849, 850, 854, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 895, 896, 899, 900, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 982, 985, 988, 989, 997, 998, 999, 1000, 1001, 1002, 1020, 1029, 1033, 1034, 1035, 1036, 1039, 1041, 1042, 1047, 1052, 1059 }

2.1.5 FriCAS

A grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 71, 75, 78, 102, 106, 109, 167, 173, 179, 182, 193, 195, 196, 199, 216, 224, 225, 226, 228, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 293, 294, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 312, 313, 315, 316, 317, 318, 319, 321, 322, 324, 325, 326, 330, 334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 387, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 442, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 572, 580, 581, 585, 596, 597, 599, 600, 603, 604, 605, 606, 608, 609, 610, 612, 613, 617, 625, 627, 628, 629, 630, 632, 635, 637, 638, 639, 640, 642, 647, 657, 658, 660, 667, 668, 669, 670, 675, 681, 688, 691, 694, 695, 724, 727, 730, 732, 733, 736, 739, 740, 741, 742, 746, 747, 748, 780, 781, 782, 783, 784, 785, 786, 787, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 811, 812, 813, 814, 815, 816, 823, 824, 825, 838, 839, 842, 855, 856, 857, 870, 872, 879, 880, 886, 887, 893, 894, 901, 903, 910, 915, 920, 922, 927, 948, 955, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 989, 1000, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1035, 1036, 1043, 1044, 1055 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 183, 184, 185, 186, 187, 192, 194, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 227, 229, 230, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 263, 272, 277, 284, 290, 291, 292, 295, 296, 297, 302, 304, 305, 311, 314, 320, 323, 327, 328, 329, 331, 332, 333, 338, 343, 344, 345, 350, 351, 352, 358, 359, 365, 366, 372, 373, 380, 385, 386, 394, 400, 401, 407, 408, 414, 415, 418, 419, 421, 426, 427, 428, 433, 434, 435, 440, 441, 446, 447, 448, 449, 455, 456, 462, 463, 470, 476, 477, 484, 490, 491, 497, 498, 503, 504, 511, 517, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 586, 587, 588, 589, 598, 601, 602, 607, 611, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 626, 631, 633, 634, 636, 641, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 665, 666, 671, 672, 673, 674, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 731, 734, 735, 737, 738, 743, 744, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 774, 775, 776, 788, 790, 791, 793, 794, 796, 797, 799, 800, 801, 808, 809, 810, 817, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 845, 848, 858, 859, 860, 869, 873, 874, 875, 876, 877, 878, 885, 897, 898, 902, 904, 905, 906, 907, 908, 909, 911, 912, 913, 914, 916, 917, 918, 919, 921, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 959, 960, 961, 962, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1001, 1002, 1003, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1039, 1040, 1041, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1056, 1057, 1058, 1059 }

C grade: { 335, 336, 337, 342, 349, 356, 357, 363, 364, 370, 371, 377, 378, 379, 384, 391, 392, 393, 398, 399, 405, 406, 412, 413, 420, 422, 423, 425, 432, 439, 453, 454, 460, 461, 467, 468, 469, 474, 475, 481, 482, 483, 488, 489, 495, 496, 502, 508, 509, 510, 515, 516, 522, 523, 524, 843, 844, 846, 847, 849, 850, 851, 852, 853, 867, 868 }

F grade: { 188, 189, 190, 191, 205, 231, 239, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 590, 591, 592, 593, 594, 595, 745, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 854, 861, 862, 863, 864, 865, 866, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 1042 }

2.1.6 Sympy

A grade: { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 81, 82, 83, 85, 87, 88, 96, 97, 113, 131, 132, 133, 134, 167, 168, 169, 170, 173, 174, 175, 176, 179, 180, 181, 182, 183, 185, 193, 194, 195, 196, 197, 198, 199, 222, 223, 224, 225, 226, 227, 228, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 290, 291, 292, 293, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 326, 327, 328, 329, 339, 340, 346, 347, 353, 354, 360, 361, 367, 368, 374, 381, 382, 388, 402, 403, 409, 429, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 566, 567, 580, 585, 586, 596, 597, 598, 599, 600, 601, 602, 605, 606, 607, 608, 609, 610, 611, 619, 623, 688, 691, 697, 700, 715, 724, 727, 730, 731, 733, 736, 739, 740, 741, 746, 747, 748, 749, 755, 756, 838, 839, 855, 856, 857, 870, 872, 873, 874, 879, 880, 886, 887, 893, 894, 897, 898, 901, 902, 903, 908, 909, 915, 919, 920, 921, 926, 927, 933, 946, 947, 948, 953, 954, 955, 962, 967, 969, 973, 975, 1003, 1014, 1015, 1016, 1017, 1018, 1028, 1031, 1032, 1043, 1045, 1050, 1051, 1055 }

B grade: { 2, 192, 221, 575, 576, 577, 581, 582, 583, 584, 629, 631, 639, 641, 753, 754, 783, 808, 809, 811, 812, 813, 840, 841, 983, 984, 1026, 1027, 1044 }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 171, 172, 177, 178, 184, 186, 187, 188, 189, 190, 191, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 334, 335, 336, 337, 338, 341, 342, 343, 344, 345, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 474, 475, 476, 477, 481, 482, 483, 484, 488, 489, 490, 491, 495, 496, 497, 498, 502, 503, 504, 508, 509, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 578, 579, 587, 588, 589, 590, 591, 592, 593, 594, 595, 603, 604, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 624, 625, 626, 627, 628, 630, 632, 633, 634, 635, 636, 637, 638, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 694, 695, 696, 698, 699, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 732, 734, 735, 737, 738, 742, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 810, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 875, 876, 877, 878, 881, 882, 883, 884, 885, 888, 889, 890, 891, 892, 895, 896, 899, 900, 904, 905, 906, 907, 910, 911, 912, 913, 914, 916, 917, 918, 922, 923, 924, 925, 928, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 949, 950, 951, 952, 956, 957, 958, 959, 960, 961, 963, 964, 965, 966, 968, 970, 971, 972, 974, 976, 977, 978, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1029, 1030, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1056, 1057, 1058, 1059 }

2.1.7 Giac

A grade: { 1, 2, 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 32, 40, 42, 43, 48, 70, 72, 75, 77, 79, 82, 87, 88, 90, 93, 94, 95, 97, 101, 103, 106, 110, 113, 118, 119, 121, 122, 123, 143, 144, 145, 149, 150, 151, 152, 154, 155, 156, 157, 161, 162, 163, 164, 166, 171, 177, 182, 184, 193, 194, 197, 198, 202, 203, 204, 206, 209, 216, 217, 219, 222, 223, 226, 227, 235, 237, 241, 243, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 276, 281, 282, 283, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 303, 304, 305, 306, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 346, 347, 348, 352, 353, 354, 355, 359, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 449, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 498, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 566, 567, 568, 569, 572, 573, 575, 576, 578, 579, 585, 586, 587, 588, 597, 598, 600, 601, 602, 603, 604, 606, 607, 609, 610, 611, 612, 613, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 680, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 740, 742, 743, 746, 747, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 795, 796, 798, 799, 801, 802, 803, 804, 805, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 819, 822, 823, 824, 825, 828, 830, 834, 835, 838, 839, 840, 841, 855, 856, 857, 858, 859, 860, 870, 872, 873, 874, 876, 877, 878, 879, 880, 885, 886, 887, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 973, 980, 981, 985, 986, 987, 989, 995, 1014, 1015, 1016, 1017, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1046, 1053, 1054, 1055, 1056, 1057, 1058 }

B grade: { 24, 25, 26, 27, 28, 29, 31, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 71, 73, 74, 76, 78, 80, 81, 84, 85, 96, 98, 99, 100, 102, 104, 105, 107, 108, 109, 111, 112, 115, 116, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 153, 158, 159, 160, 165, 167, 168, 169, 170, 172, 173, 174, 175, 176, 178, 179, 180, 181, 183, 185, 186, 187, 192, 195, 196, 199, 200, 201, 207, 208, 210, 211, 212, 213, 214, 215, 218, 220, 221, 224, 225, 228, 229, 230, 231, 232, 233, 234, 236, 238, 240, 242, 244, 245, 246, 247, 248, 249, 263, 272, 277, 278, 279, 284, 302, 311, 320, 338, 344, 345, 350, 351, 365, 366, 373, 380, 386, 387, 394, 401, 408, 415, 418, 427, 428, 434, 435, 441, 442, 447, 448, 456, 463, 470, 477, 484, 491, 497, 504, 511, 518, 525, 570, 571, 574, 577, 580, 581, 582, 583, 584, 589, 596, 605, 614, 616, 644, 646, 648, 658, 659, 668, 674, 675, 676, 681, 682, 686, 687, 702, 703, 704, 705, 739, 741, 744, 745, 748, 749, 750, 751, 752, 753, 768, 769, 770, 774, 775, 776, 791, 794, 797, 800, 810, 817, 818, 820, 821, 836, 875, 893, 894, 979, 983, 984, 990, 991, 992, 993, 994, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1006, 1007, 1010, 1011, 1018, 1019, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1040, 1045, 1047 }

C grade: { 897, 898, 1050, 1051 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 83, 86, 89, 91, 92, 114, 117, 120, 126, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 474, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 516, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 590, 591, 592, 593, 594, 595, 599, 608, 677, 678, 679, 683, 684, 685, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 826, 827, 829, 831, 832, 833, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 968, 969, 970, 971, 972, 974, 975, 976, 977, 978, 982, 988, 1004, 1005, 1008, 1009, 1012, 1013, 1020, 1028, 1029, 1039, 1041, 1042, 1048, 1049, 1052, 1059 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	17	28	120	19	28
normalized size	1	1.	2.14	0.77	1.27	5.45	0.86	1.27
time (sec)	N/A	0.016	0.07	0.014	1.484	2.04	0.323	1.186

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	28	147	280	28
normalized size	1	1.	2.14	7.09	1.27	6.68	12.73	1.27
time (sec)	N/A	0.021	0.067	0.078	1.522	2.089	9.607	1.279

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	28	147	0	0
normalized size	1	1.	2.14	7.09	1.27	6.68	0.	0.
time (sec)	N/A	0.04	0.078	0.066	1.554	2.108	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	28	147	0	0
normalized size	1	1.	2.14	7.09	1.27	6.68	0.	0.
time (sec)	N/A	0.039	0.073	0.06	1.707	2.163	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	44	93	263	0	0
normalized size	1	1.	1.91	2.	4.23	11.95	0.	0.
time (sec)	N/A	0.039	0.101	0.052	1.603	2.112	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	132	28	147	0	0
normalized size	1	1.	2.14	6.	1.27	6.68	0.	0.
time (sec)	N/A	0.04	0.105	0.093	1.647	2.102	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	102	51	263	0	0
normalized size	1	1.	1.91	4.64	2.32	11.95	0.	0.
time (sec)	N/A	0.039	0.105	0.054	1.675	2.093	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	18	58	19	32
normalized size	1	1.	2.47	0.93	1.2	3.87	1.27	2.13
time (sec)	N/A	0.012	0.013	0.006	0.999	2.007	0.215	1.321

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	193	49	0
normalized size	1	1.	1.	1.05	0.	10.16	2.58	0.
time (sec)	N/A	0.023	0.01	0.014	0.	2.203	1.669	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	504	482	668	0
normalized size	1	1.	1.	0.	12.92	12.36	17.13	0.
time (sec)	N/A	0.045	0.061	0.5	1.837	2.269	14.346	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	926	1015	2574	0
normalized size	1	1.	0.83	0.	15.69	17.2	43.63	0.
time (sec)	N/A	0.052	0.195	0.478	1.79	2.337	80.103	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	193	49	0
normalized size	1	1.	1.	1.05	0.	10.16	2.58	0.
time (sec)	N/A	0.027	0.009	0.012	0.	2.113	1.785	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	923	396	520	678	0
normalized size	1	1.	1.1	23.08	9.9	13.	16.95	0.
time (sec)	N/A	0.05	0.126	0.372	1.668	2.185	14.716	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	753	1094	2351	0
normalized size	1	1.	1.31	0.	12.76	18.54	39.85	0.
time (sec)	N/A	0.061	0.218	0.345	1.698	2.208	78.712	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	53	104	92	65
normalized size	1	1.	0.5	0.93	1.15	2.26	2.	1.41
time (sec)	N/A	0.044	0.03	0.008	1.012	2.059	1.118	1.167

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	61	119	243	136	122
normalized size	1	1.	0.58	0.88	1.72	3.52	1.97	1.77
time (sec)	N/A	0.074	0.078	0.01	0.979	2.037	3.871	1.22

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	52	79	149	382	189	154
normalized size	1	1.	0.57	0.86	1.62	4.15	2.05	1.67
time (sec)	N/A	0.104	0.123	0.01	1.006	2.044	12.828	1.206

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	40	56	119	243	136	124
normalized size	1	1.	0.6	0.84	1.78	3.63	2.03	1.85
time (sec)	N/A	0.052	0.057	0.01	1.024	2.087	3.792	1.26

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	33	74	89	263	189	92
normalized size	1	1.	0.37	0.82	0.99	2.92	2.1	1.02
time (sec)	N/A	0.084	0.043	0.011	1.212	2.064	12.088	1.212

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	62	92	178	544	231	184
normalized size	1	1.	0.55	0.81	1.58	4.81	2.04	1.63
time (sec)	N/A	0.118	0.173	0.013	1.133	2.071	31.062	1.229

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	66	149	382	189	154
normalized size	1	1.	0.59	0.75	1.69	4.34	2.15	1.75
time (sec)	N/A	0.064	0.094	0.012	1.008	2.057	12.183	1.226

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	84	178	544	231	184
normalized size	1	1.	0.56	0.76	1.6	4.9	2.08	1.66
time (sec)	N/A	0.099	0.128	0.011	1.02	2.068	32.458	1.251

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	43	102	116	498	277	124
normalized size	1	1.	0.32	0.76	0.87	3.72	2.07	0.93
time (sec)	N/A	0.132	0.082	0.013	1.058	2.095	79.081	1.214

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	68	154	0	47
normalized size	1	1.	2.82	1.09	6.18	14.	0.	4.27
time (sec)	N/A	0.012	0.02	0.013	1.561	2.165	0.	1.163

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	23	82	462	0	95
normalized size	1	1.	1.13	1.	3.57	20.09	0.	4.13
time (sec)	N/A	0.026	0.031	0.013	1.019	2.087	0.	1.163

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	119	1035	0	135
normalized size	1	1.	1.33	0.96	4.41	38.33	0.	5.
time (sec)	N/A	0.024	0.036	0.018	1.602	2.216	0.	1.218

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	41	33	146	1958	0	124
normalized size	1	1.	1.08	0.87	3.84	51.53	0.	3.26
time (sec)	N/A	0.029	0.027	0.016	1.042	2.087	0.	1.189

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	39	177	2952	0	171
normalized size	1	1.	1.15	0.98	4.42	73.8	0.	4.28
time (sec)	N/A	0.033	0.094	0.02	1.636	2.104	0.	1.226

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	58	305	0	78
normalized size	1	1.	1.21	1.12	2.42	12.71	0.	3.25
time (sec)	N/A	0.022	0.019	0.015	1.508	2.134	0.	1.212

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	24	213	0	24
normalized size	1	1.	0.57	1.39	1.04	9.26	0.	1.04
time (sec)	N/A	0.031	0.015	0.013	1.13	1.949	0.	1.172

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	52	122	1401	0	140
normalized size	1	1.	0.59	1.06	2.49	28.59	0.	2.86
time (sec)	N/A	0.041	0.015	0.019	1.529	2.08	0.	1.229

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	44	127	622	0	82
normalized size	1	1.	1.21	1.16	3.34	16.37	0.	2.16
time (sec)	N/A	0.035	0.038	0.019	1.155	2.068	0.	1.21

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	29	71	184	3312	0	174
normalized size	1	1.	0.41	1.01	2.63	47.31	0.	2.49
time (sec)	N/A	0.043	0.015	0.019	1.779	2.205	0.	1.17

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	123	1037	0	135
normalized size	1	1.	1.21	0.96	4.39	37.04	0.	4.82
time (sec)	N/A	0.025	0.043	0.018	1.615	2.4	0.	1.18

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	68	43	143	1935	0	154
normalized size	1	1.	1.39	0.88	2.92	39.49	0.	3.14
time (sec)	N/A	0.046	0.033	0.02	1.084	2.466	0.	1.174

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	47	48	138	2102	0	136
normalized size	1	1.	1.09	1.12	3.21	48.88	0.	3.16
time (sec)	N/A	0.044	0.016	0.023	1.513	2.569	0.	1.216

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	83	53	201	4433	0	182
normalized size	1	1.	1.26	0.8	3.05	67.17	0.	2.76
time (sec)	N/A	0.05	0.032	0.025	1.086	2.562	0.	1.203

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	61	244	5837	0	240
normalized size	1	1.	0.93	1.05	4.21	100.64	0.	4.14
time (sec)	N/A	0.049	0.461	0.024	1.593	2.784	0.	1.221

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	39	122	1423	0	111
normalized size	1	1.	0.89	1.05	3.3	38.46	0.	3.
time (sec)	N/A	0.027	0.017	0.017	1.785	2.336	0.	1.255

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	50	122	622	0	82
normalized size	1	1.	1.22	1.35	3.3	16.81	0.	2.22
time (sec)	N/A	0.037	0.039	0.02	1.117	2.168	0.	1.194

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	73	178	3283	0	173
normalized size	1	1.	0.5	1.11	2.7	49.74	0.	2.62
time (sec)	N/A	0.043	0.016	0.025	1.783	2.547	0.	1.165

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	62	122	900	0	42
normalized size	1	1.	0.81	1.17	2.3	16.98	0.	0.79
time (sec)	N/A	0.04	0.019	0.025	1.174	2.268	0.	1.161

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	33	92	240	6041	0	207
normalized size	1	1.	0.37	1.03	2.7	67.88	0.	2.33
time (sec)	N/A	0.048	0.017	0.024	1.56	2.625	0.	1.191

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	46	39	180	2952	0	171
normalized size	1	1.	1.18	1.	4.62	75.69	0.	4.38
time (sec)	N/A	0.032	0.123	0.019	1.554	2.416	0.	1.209

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	105	61	209	4477	0	185
normalized size	1	1.	1.5	0.87	2.99	63.96	0.	2.64
time (sec)	N/A	0.05	0.035	0.019	1.033	2.505	0.	1.161

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	242	5836	0	240
normalized size	1	1.	0.97	1.19	4.17	100.62	0.	4.14
time (sec)	N/A	0.045	0.194	0.022	1.561	2.587	0.	1.192

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	121	71	263	8095	0	215
normalized size	1	1.	1.36	0.8	2.96	90.96	0.	2.42
time (sec)	N/A	0.055	0.037	0.024	1.012	2.742	0.	1.178

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	77	81	203	6263	0	174
normalized size	1	1.	1.12	1.17	2.94	90.77	0.	2.52
time (sec)	N/A	0.054	0.025	0.024	1.656	2.744	0.	1.223

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	2819	0	0
normalized size	1	1.	0.56	0.	0.	26.59	0.	0.
time (sec)	N/A	0.116	0.058	0.079	0.	2.713	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	1705	0	0
normalized size	1	1.	0.73	0.	0.	21.05	0.	0.
time (sec)	N/A	0.081	0.048	0.063	0.	2.687	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	923	0	0
normalized size	1	1.	0.75	0.	0.	11.68	0.	0.
time (sec)	N/A	0.073	0.051	0.063	0.	2.552	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	0	0	423	0	0
normalized size	1	1.	1.09	0.	0.	7.83	0.	0.
time (sec)	N/A	0.037	0.034	0.104	0.	2.454	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	421	0	0
normalized size	1	1.	1.06	0.	0.	7.8	0.	0.
time (sec)	N/A	0.038	0.023	0.09	0.	2.478	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	0	0	925	0	0
normalized size	1	1.	0.72	0.	0.	11.71	0.	0.
time (sec)	N/A	0.071	0.033	0.062	0.	2.322	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	1705	0	0
normalized size	1	1.	0.73	0.	0.	21.05	0.	0.
time (sec)	N/A	0.073	0.03	0.063	0.	2.282	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	2819	0	0
normalized size	1	1.	0.56	0.	0.	26.59	0.	0.
time (sec)	N/A	0.105	0.038	0.061	0.	2.387	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	3047	0	0
normalized size	1	1.	0.38	0.	0.	19.66	0.	0.
time (sec)	N/A	0.182	0.054	0.042	0.	2.331	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	2222	0	0
normalized size	1	1.	0.38	0.	0.	14.34	0.	0.
time (sec)	N/A	0.182	0.056	0.037	0.	2.137	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	59	0	0	3081	0	0
normalized size	1	1.	0.24	0.	0.	12.68	0.	0.
time (sec)	N/A	0.244	0.051	0.038	0.	2.282	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	59	0	0	2232	0	0
normalized size	1	1.	0.27	0.	0.	10.24	0.	0.
time (sec)	N/A	0.223	0.04	0.037	0.	2.218	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	1694	0	0
normalized size	1	1.	0.46	0.	0.	13.23	0.	0.
time (sec)	N/A	0.092	0.034	0.059	0.	1.933	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	1694	0	0
normalized size	1	1.	0.46	0.	0.	13.23	0.	0.
time (sec)	N/A	0.089	0.024	0.039	0.	1.951	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	57	0	0	2232	0	0
normalized size	1	1.	0.26	0.	0.	10.24	0.	0.
time (sec)	N/A	0.201	0.026	0.054	0.	2.045	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	57	0	0	3081	0	0
normalized size	1	1.	0.23	0.	0.	12.68	0.	0.
time (sec)	N/A	0.236	0.028	0.038	0.	2.133	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	2222	0	0
normalized size	1	1.	0.38	0.	0.	14.34	0.	0.
time (sec)	N/A	0.125	0.035	0.038	0.	1.925	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	3047	0	0
normalized size	1	1.	0.38	0.	0.	19.66	0.	0.
time (sec)	N/A	0.121	0.031	0.038	0.	2.072	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	82	335	0	0
normalized size	1	1.	1.	0.	5.12	20.94	0.	0.
time (sec)	N/A	0.03	0.011	0.027	1.706	1.814	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	82	333	0	0
normalized size	1	1.	1.	0.	5.12	20.81	0.	0.
time (sec)	N/A	0.03	0.016	0.029	1.63	1.849	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	0	213	0	0
normalized size	1	1.	1.	0.7	0.	21.3	0.	0.
time (sec)	N/A	0.026	0.008	0.006	0.	1.829	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	55	254	0	43
normalized size	1	1.	1.	1.04	2.39	11.04	0.	1.87
time (sec)	N/A	0.017	0.012	0.016	1.581	1.936	0.	1.209

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	32	73	85	0	62
normalized size	1	1.	1.	1.52	3.48	4.05	0.	2.95
time (sec)	N/A	0.026	0.031	0.013	1.022	1.751	0.	1.188

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	70	123	1289	0	88
normalized size	1	1.	0.98	1.43	2.51	26.31	0.	1.8
time (sec)	N/A	0.03	0.054	0.019	1.599	1.967	0.	1.205

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	67	132	257	0	96
normalized size	1	1.	1.	1.81	3.57	6.95	0.	2.59
time (sec)	N/A	0.033	0.035	0.017	1.029	1.759	0.	1.277

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	76	551	0	77
normalized size	1	1.	0.89	0.96	2.71	19.68	0.	2.75
time (sec)	N/A	0.025	0.019	0.017	1.547	1.974	0.	1.233

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	86	150	0	92
normalized size	1	1.	0.78	0.98	2.15	3.75	0.	2.3
time (sec)	N/A	0.04	0.11	0.015	1.013	1.825	0.	1.301

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	47	139	2049	0	130
normalized size	1	1.	0.81	1.09	3.23	47.65	0.	3.02
time (sec)	N/A	0.044	0.051	0.017	1.589	1.895	0.	1.292

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	38	96	836	0	85
normalized size	1	1.	1.	1.	2.53	22.	0.	2.24
time (sec)	N/A	0.026	0.014	0.019	1.565	1.872	0.	1.182

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	50	107	177	0	103
normalized size	1	1.	1.05	1.32	2.82	4.66	0.	2.71
time (sec)	N/A	0.041	0.033	0.019	1.064	1.829	0.	1.264

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	92	157	2390	0	130
normalized size	1	1.	0.98	1.39	2.38	36.21	0.	1.97
time (sec)	N/A	0.042	0.103	0.02	1.631	1.941	0.	1.377

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	39	109	1270	0	116
normalized size	1	1.	0.85	0.98	2.72	31.75	0.	2.9
time (sec)	N/A	0.032	0.03	0.017	1.585	1.819	0.	1.191

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	31	154	17	32
normalized size	1	1.	1.	1.09	2.82	14.	1.55	2.91
time (sec)	N/A	0.013	0.007	0.007	1.134	1.69	0.321	1.164

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	235	22	36
normalized size	1	1.	1.	0.93	1.2	15.67	1.47	2.4
time (sec)	N/A	0.02	0.011	0.005	1.009	1.759	0.678	1.194

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	49	338	39	0
normalized size	1	1.	1.	1.06	3.06	21.12	2.44	0.
time (sec)	N/A	0.03	0.038	0.007	1.772	1.851	0.678	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	18	378	0	42
normalized size	1	1.	1.	2.8	1.2	25.2	0.	2.8
time (sec)	N/A	0.029	0.006	0.018	1.037	1.861	0.	1.218

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	18	572	44	50
normalized size	1	1.	1.	2.8	1.2	38.13	2.93	3.33
time (sec)	N/A	0.03	0.005	0.018	1.079	1.808	2.389	1.257

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	190	0	0
normalized size	1	1.	1.	1.05	0.	10.	0.	0.
time (sec)	N/A	0.035	0.019	0.014	0.	2.071	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	50	200	467	41	66
normalized size	1	1.	1.	1.85	7.41	17.3	1.52	2.44
time (sec)	N/A	0.024	0.031	0.011	1.02	1.748	1.281	1.249

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	289	946	46	70
normalized size	1	1.	1.	2.19	9.32	30.52	1.48	2.26
time (sec)	N/A	0.035	0.056	0.023	1.166	1.823	4.517	1.199

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	275	466	605	0	0
normalized size	1	1.	0.89	7.64	12.94	16.81	0.	0.
time (sec)	N/A	0.049	0.122	0.134	2.017	1.86	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	52	373	836	0	72
normalized size	1	1.	1.81	1.68	12.03	26.97	0.	2.32
time (sec)	N/A	0.033	0.041	0.019	1.129	1.853	0.	1.254

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	0	475	1521	0	0
normalized size	1	1.	0.83	0.	13.57	43.46	0.	0.
time (sec)	N/A	0.037	0.105	0.194	1.804	1.899	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	73	535	680	477	0	0
normalized size	1	1.	1.82	13.38	17.	11.92	0.	0.
time (sec)	N/A	0.044	0.904	0.204	1.648	1.92	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	49	89	759	0	63
normalized size	1	1.	1.	1.44	2.62	22.32	0.	1.85
time (sec)	N/A	0.024	0.015	0.013	1.638	1.826	0.	1.187

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	59	90	151	2257	0	96
normalized size	1	1.	1.07	1.64	2.75	41.04	0.	1.75
time (sec)	N/A	0.044	0.166	0.013	1.746	1.866	0.	1.235

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	69	149	2236	0	90
normalized size	1	1.	1.	1.25	2.71	40.65	0.	1.64
time (sec)	N/A	0.045	0.02	0.017	1.849	1.874	0.	1.213

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	46	258	620	29	47
normalized size	1	1.	1.	2.19	12.29	29.52	1.38	2.24
time (sec)	N/A	0.019	0.012	0.007	1.283	1.839	2.502	1.217

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	76	501	2333	34	47
normalized size	1	1.	1.	3.04	20.04	93.32	1.36	1.88
time (sec)	N/A	0.029	0.015	0.014	1.256	1.816	41.691	1.214

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	48	46	115	3186	0	99
normalized size	1	1.	1.26	1.21	3.03	83.84	0.	2.61
time (sec)	N/A	0.053	0.009	0.013	1.728	1.809	0.	1.181

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	115	3189	0	99
normalized size	1	1.	1.	1.	3.19	88.58	0.	2.75
time (sec)	N/A	0.037	0.009	0.012	1.568	1.832	0.	1.203

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	72	1157	3032	0	89
normalized size	1	1.	2.03	2.18	35.06	91.88	0.	2.7
time (sec)	N/A	0.03	0.025	0.084	1.065	1.863	0.	1.199

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	21	80	355	0	59
normalized size	1	1.	1.13	0.91	3.48	15.43	0.	2.57
time (sec)	N/A	0.018	0.026	0.016	1.207	1.849	0.	1.187

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	32	76	85	0	68
normalized size	1	1.	1.	1.45	3.45	3.86	0.	3.09
time (sec)	N/A	0.022	0.014	0.015	1.195	1.797	0.	1.197

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	67	62	146	1710	0	107
normalized size	1	1.	1.37	1.27	2.98	34.9	0.	2.18
time (sec)	N/A	0.036	0.032	0.02	1.211	2.026	0.	1.285

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	67	135	238	0	96
normalized size	1	1.	1.	1.81	3.65	6.43	0.	2.59
time (sec)	N/A	0.025	0.021	0.019	1.153	1.822	0.	1.256

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	95	552	0	81
normalized size	1	1.	0.93	0.96	3.52	20.44	0.	3.
time (sec)	N/A	0.024	0.021	0.017	1.081	1.793	0.	1.163

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	89	166	0	90
normalized size	1	1.	0.78	0.98	2.22	4.15	0.	2.25
time (sec)	N/A	0.041	0.107	0.018	1.122	1.729	0.	1.256

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	48	162	2049	0	134
normalized size	1	1.	0.81	1.12	3.77	47.65	0.	3.12
time (sec)	N/A	0.042	0.048	0.02	1.078	1.866	0.	1.268

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	31	117	1037	0	104
normalized size	1	1.	1.16	0.82	3.08	27.29	0.	2.74
time (sec)	N/A	0.03	0.029	0.017	1.047	1.897	0.	1.198

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	50	107	177	0	103
normalized size	1	1.	1.	1.32	2.82	4.66	0.	2.71
time (sec)	N/A	0.035	0.018	0.017	1.082	1.857	0.	1.247

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	85	81	180	3027	0	146
normalized size	1	1.	1.29	1.23	2.73	45.86	0.	2.21
time (sec)	N/A	0.048	0.04	0.021	1.045	1.979	0.	1.329

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	39	128	1270	0	117
normalized size	1	1.	0.9	1.	3.28	32.56	0.	3.
time (sec)	N/A	0.031	0.032	0.017	1.099	1.902	0.	1.234

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	34	154	0	32
normalized size	1	1.	1.	1.09	3.09	14.	0.	2.91
time (sec)	N/A	0.011	0.009	0.007	1.046	1.736	0.	1.203

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	232	22	36
normalized size	1	1.	1.	0.93	1.2	15.47	1.47	2.4
time (sec)	N/A	0.02	0.013	0.009	1.032	1.815	4.925	1.17

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	72	338	0	0
normalized size	1	1.	1.	1.06	4.5	21.12	0.	0.
time (sec)	N/A	0.028	0.02	0.006	1.91	1.895	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	18	378	0	42
normalized size	1	1.	1.	2.8	1.2	25.2	0.	2.8
time (sec)	N/A	0.027	0.004	0.017	1.105	1.734	0.	1.238

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	18	568	0	50
normalized size	1	1.	1.	2.8	1.2	37.87	0.	3.33
time (sec)	N/A	0.029	0.005	0.022	1.042	1.85	0.	1.257

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	192	0	0
normalized size	1	1.	1.	1.05	0.	9.6	0.	0.
time (sec)	N/A	0.034	0.023	0.013	0.	2.012	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	50	200	464	0	66
normalized size	1	1.	1.	1.85	7.41	17.19	0.	2.44
time (sec)	N/A	0.021	0.014	0.014	1.053	1.761	0.	1.216

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	289	946	0	70
normalized size	1	1.	1.	2.19	9.32	30.52	0.	2.26
time (sec)	N/A	0.032	0.026	0.02	1.081	1.843	0.	1.219

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	499	559	603	0	0
normalized size	1	1.	0.92	13.49	15.11	16.3	0.	0.
time (sec)	N/A	0.042	0.066	0.152	1.914	1.983	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	45	113	1088	0	78
normalized size	1	1.	1.68	1.32	3.32	32.	0.	2.29
time (sec)	N/A	0.03	0.032	0.014	1.073	1.845	0.	1.234

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	95	58	174	3066	0	108
normalized size	1	1.	1.73	1.05	3.16	55.75	0.	1.96
time (sec)	N/A	0.06	0.044	0.022	1.092	2.027	0.	1.223

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	95	74	180	3081	0	116
normalized size	1	1.	1.73	1.35	3.27	56.02	0.	2.11
time (sec)	N/A	0.057	0.039	0.015	1.07	1.941	0.	1.311

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	28	201	555	0	41
normalized size	1	1.	1.59	1.65	11.82	32.65	0.	2.41
time (sec)	N/A	0.026	0.023	0.012	1.018	1.831	0.	1.183

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	32	188	737	0	39
normalized size	1	1.	1.	1.88	11.06	43.35	0.	2.29
time (sec)	N/A	0.028	0.009	0.013	1.023	1.709	0.	1.183

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	371	497	347	0	0
normalized size	1	1.	1.15	14.27	19.12	13.35	0.	0.
time (sec)	N/A	0.035	0.086	0.208	1.664	2.008	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	84	46	132	4327	0	96
normalized size	1	1.	2.21	1.21	3.47	113.87	0.	2.53
time (sec)	N/A	0.065	0.02	0.016	1.042	1.873	0.	1.15

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	47	50	582	1516	0	65
normalized size	1	1.	1.88	2.	23.28	60.64	0.	2.6
time (sec)	N/A	0.029	0.027	0.014	1.253	1.908	0.	1.176

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	64	258	705	0	63
normalized size	1	1.	1.	2.21	8.9	24.31	0.	2.17
time (sec)	N/A	0.019	0.017	0.017	1.096	1.688	0.	1.143

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	76	587	1571	0	73
normalized size	1	1.	1.	2.3	17.79	47.61	0.	2.21
time (sec)	N/A	0.034	0.012	0.015	1.066	1.779	0.	1.175

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	78	232	58	96
normalized size	1	1.	0.96	0.89	2.89	8.59	2.15	3.56
time (sec)	N/A	0.027	0.033	0.014	1.178	1.921	1.093	1.186

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	80	223	61	105
normalized size	1	1.	1.	0.89	2.96	8.26	2.26	3.89
time (sec)	N/A	0.027	0.042	0.01	1.048	1.905	1.075	1.172

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	78	231	58	93
normalized size	1	1.	0.96	0.89	2.89	8.56	2.15	3.44
time (sec)	N/A	0.019	0.023	0.009	1.136	1.824	1.091	1.131

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	80	223	58	100
normalized size	1	1.	0.96	0.89	2.96	8.26	2.15	3.7
time (sec)	N/A	0.019	0.023	0.009	1.196	1.793	1.073	1.188

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	151	112	701	0	149
normalized size	1	1.	0.78	4.08	3.03	18.95	0.	4.03
time (sec)	N/A	0.07	0.472	0.044	1.711	1.981	0.	1.476

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	149	117	672	0	116
normalized size	1	1.	0.83	4.14	3.25	18.67	0.	3.22
time (sec)	N/A	0.069	0.46	0.042	1.712	1.973	0.	1.196

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	155	212	701	0	159
normalized size	1	1.	0.78	4.19	5.73	18.95	0.	4.3
time (sec)	N/A	0.035	0.462	0.047	1.199	1.964	0.	1.21

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	153	216	672	0	122
normalized size	1	1.	0.89	4.25	6.	18.67	0.	3.39
time (sec)	N/A	0.036	0.444	0.047	1.143	1.917	0.	1.256

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	77	92	490	0	135
normalized size	1	1.	0.75	2.14	2.56	13.61	0.	3.75
time (sec)	N/A	0.025	0.206	0.033	1.696	1.873	0.	1.23

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	75	90	471	0	95
normalized size	1	1.	0.82	2.27	2.73	14.27	0.	2.88
time (sec)	N/A	0.023	0.199	0.033	1.619	2.148	0.	1.171

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	79	180	491	0	147
normalized size	1	1.	0.78	2.19	5.	13.64	0.	4.08
time (sec)	N/A	0.024	0.221	0.035	1.157	2.148	0.	1.2

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	77	174	471	0	100
normalized size	1	1.	0.88	2.33	5.27	14.27	0.	3.03
time (sec)	N/A	0.024	0.202	0.033	1.116	2.159	0.	1.2

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	77	914	0	66
normalized size	1	1.	2.97	5.76	2.66	31.52	0.	2.28
time (sec)	N/A	0.023	0.058	0.078	1.651	2.191	0.	1.202

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	205	142	2491	0	119
normalized size	1	1.	2.27	4.56	3.16	55.36	0.	2.64
time (sec)	N/A	0.055	0.1	0.089	1.736	2.244	0.	1.241

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	70	240	201	4772	0	151
normalized size	1	1.	0.97	3.33	2.79	66.28	0.	2.1
time (sec)	N/A	0.083	0.334	0.093	1.783	2.311	0.	1.224

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	127	1233	0	117
normalized size	1	1.	3.21	5.34	4.38	42.52	0.	4.03
time (sec)	N/A	0.02	0.053	0.041	1.253	2.332	0.	1.166

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	197	189	3363	0	163
normalized size	1	1.	2.39	4.28	4.11	73.11	0.	3.54
time (sec)	N/A	0.046	0.094	0.046	1.179	2.401	0.	1.187

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	230	251	6496	0	207
normalized size	1	1.	0.96	3.15	3.44	88.99	0.	2.84
time (sec)	N/A	0.087	0.328	0.053	1.228	2.356	0.	1.218

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	148	66	231	0	66
normalized size	1	1.	1.	5.69	2.54	8.88	0.	2.54
time (sec)	N/A	0.016	0.119	0.036	1.263	2.154	0.	1.147

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	181	95	1129	0	92
normalized size	1	1.	2.37	5.17	2.71	32.26	0.	2.63
time (sec)	N/A	0.033	0.087	0.08	1.881	2.078	0.	1.187

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	58	162	644	0	69
normalized size	1	1.	0.92	1.53	4.26	16.95	0.	1.82
time (sec)	N/A	0.044	0.162	0.03	1.258	1.993	0.	1.146

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	150	113	230	0	69
normalized size	1	1.	1.	5.77	4.35	8.85	0.	2.65
time (sec)	N/A	0.015	0.111	0.036	1.263	2.128	0.	1.207

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	172	139	1709	0	140
normalized size	1	1.	2.5	4.78	3.86	47.47	0.	3.89
time (sec)	N/A	0.032	0.08	0.039	1.134	2.196	0.	1.188

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	57	177	620	0	72
normalized size	1	1.	0.9	1.46	4.54	15.9	0.	1.85
time (sec)	N/A	0.044	0.172	0.028	1.081	2.115	0.	1.177

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	80	914	0	72
normalized size	1	1.	2.97	5.76	2.76	31.52	0.	2.48
time (sec)	N/A	0.019	0.057	0.08	1.646	2.155	0.	1.184

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	207	139	2491	0	116
normalized size	1	1.	2.27	4.6	3.09	55.36	0.	2.58
time (sec)	N/A	0.042	0.098	0.089	1.755	2.215	0.	1.19

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	115	238	201	4772	0	154
normalized size	1	1.	1.6	3.31	2.79	66.28	0.	2.14
time (sec)	N/A	0.079	0.309	0.091	1.754	2.005	0.	1.251

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	122	1233	0	115
normalized size	1	1.	3.21	5.34	4.21	42.52	0.	3.97
time (sec)	N/A	0.02	0.054	0.042	1.075	1.952	0.	1.183

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	195	194	3363	0	169
normalized size	1	1.	2.39	4.24	4.22	73.11	0.	3.67
time (sec)	N/A	0.043	0.096	0.047	1.155	2.065	0.	1.182

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	228	248	6496	0	204
normalized size	1	1.	0.96	3.12	3.4	88.99	0.	2.79
time (sec)	N/A	0.086	0.331	0.056	1.109	2.157	0.	1.244

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	146	69	230	0	68
normalized size	1	1.	1.	5.62	2.65	8.85	0.	2.62
time (sec)	N/A	0.015	0.109	0.038	1.117	1.866	0.	1.163

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	183	95	1129	0	92
normalized size	1	1.	2.37	5.23	2.71	32.26	0.	2.63
time (sec)	N/A	0.031	0.082	0.08	1.634	1.832	0.	1.149

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	56	161	644	0	66
normalized size	1	1.	0.92	1.47	4.24	16.95	0.	1.74
time (sec)	N/A	0.042	0.168	0.03	1.101	1.823	0.	1.189

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	152	108	231	0	68
normalized size	1	1.	1.	5.85	4.15	8.88	0.	2.62
time (sec)	N/A	0.015	0.107	0.036	1.118	1.857	0.	1.197

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	170	142	1709	0	143
normalized size	1	1.	2.5	4.72	3.94	47.47	0.	3.97
time (sec)	N/A	0.031	0.068	0.039	1.088	1.824	0.	1.191

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	59	178	620	0	69
normalized size	1	1.	0.9	1.51	4.56	15.9	0.	1.77
time (sec)	N/A	0.043	0.169	0.032	1.069	1.832	0.	1.199

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	170	153	115
normalized size	1	1.	1.	0.93	0.	3.95	3.56	2.67
time (sec)	N/A	0.042	0.219	0.018	0.	1.866	2.028	1.166

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	304	401	162
normalized size	1	1.	1.11	0.92	0.	4.9	6.47	2.61
time (sec)	N/A	0.063	0.769	0.013	0.	1.952	9.547	1.176

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	0	516	932	242
normalized size	1	1.	0.95	0.92	0.	5.67	10.24	2.66
time (sec)	N/A	0.075	0.465	0.015	0.	1.909	50.452	1.275

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	83	0	455	1027	211
normalized size	1	1.	1.2	0.94	0.	5.17	11.67	2.4
time (sec)	N/A	0.069	0.724	0.023	0.	1.881	37.424	1.231

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	961	0	351
normalized size	1	1.	1.1	0.92	0.	6.67	0.	2.44
time (sec)	N/A	0.122	1.624	0.017	0.	1.893	0.	1.209

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	177	184	0	1710	0	504
normalized size	1	1.	0.91	0.94	0.	8.77	0.	2.58
time (sec)	N/A	0.144	1.572	0.023	0.	2.089	0.	1.246

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	169	153	115
normalized size	1	1.	1.	0.93	0.	3.93	3.56	2.67
time (sec)	N/A	0.04	0.173	0.01	0.	1.921	2.043	1.172

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	286	398	162
normalized size	1	1.	1.11	0.92	0.	4.61	6.42	2.61
time (sec)	N/A	0.055	0.688	0.019	0.	1.902	8.783	1.219

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	520	926	242
normalized size	1	1.	0.93	0.92	0.	5.71	10.18	2.66
time (sec)	N/A	0.069	0.427	0.011	0.	1.908	46.364	1.2

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	83	0	455	1027	211
normalized size	1	1.	1.19	0.94	0.	5.17	11.67	2.4
time (sec)	N/A	0.067	0.683	0.012	0.	1.842	34.197	1.18

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	926	0	351
normalized size	1	1.	1.1	0.92	0.	6.43	0.	2.44
time (sec)	N/A	0.1	1.612	0.03	0.	1.922	0.	1.218

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	0	1716	0	504
normalized size	1	1.	0.9	0.94	0.	8.8	0.	2.58
time (sec)	N/A	0.133	1.509	0.013	0.	1.983	0.	1.193

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	169	153	115
normalized size	1	1.	1.	0.93	0.	3.93	3.56	2.67
time (sec)	N/A	0.045	0.196	0.01	0.	1.666	2.	1.205

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	304	403	162
normalized size	1	1.	1.11	0.92	0.	4.9	6.5	2.61
time (sec)	N/A	0.057	0.678	0.007	0.	1.86	8.541	1.235

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	514	940	242
normalized size	1	1.	0.93	0.92	0.	5.65	10.33	2.66
time (sec)	N/A	0.082	0.447	0.012	0.	1.853	46.386	1.192

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	74	63	0	266	405	167
normalized size	1	1.	1.09	0.93	0.	3.91	5.96	2.46
time (sec)	N/A	0.054	0.721	0.01	0.	1.834	9.197	1.168

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	107	83	0	455	1027	211
normalized size	1	1.	1.22	0.94	0.	5.17	11.67	2.4
time (sec)	N/A	0.066	0.702	0.01	0.	1.868	34.496	1.185

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	926	0	351
normalized size	1	1.	1.1	0.92	0.	6.43	0.	2.44
time (sec)	N/A	0.093	1.569	0.01	0.	1.925	0.	1.214

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	0	562	942	247
normalized size	1	1.	0.93	0.93	0.	5.79	9.71	2.55
time (sec)	N/A	0.081	0.496	0.013	0.	2.147	46.825	1.18

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	0	1054	0	346
normalized size	1	1.	1.11	0.92	0.	7.64	0.	2.51
time (sec)	N/A	0.114	1.641	0.017	0.	2.26	0.	1.186

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	0	1715	0	504
normalized size	1	1.	0.9	0.94	0.	8.79	0.	2.58
time (sec)	N/A	0.153	1.673	0.013	0.	2.361	0.	1.191

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	278	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	14.191	0.069	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	240	0	0	0	0	0
normalized size	1	1.	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	6.732	0.083	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	99	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	9.164	0.08	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	103	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	11.315	0.063	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	15	15	7	36	61	20	34
normalized size	1	1.88	1.88	0.88	4.5	7.62	2.5	4.25
time (sec)	N/A	0.009	0.005	0.007	1.049	2.038	0.643	1.251

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	80	20	36
normalized size	1	1.	1.	0.82	2.12	4.71	1.18	2.12
time (sec)	N/A	0.009	0.006	0.019	1.052	2.115	0.633	1.211

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	119	20	36
normalized size	1	1.	1.	0.82	2.12	7.	1.18	2.12
time (sec)	N/A	0.01	0.007	0.02	1.052	2.038	0.608	1.156

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	117	78	80
normalized size	1	1.	0.71	0.8	0.	3.34	2.23	2.29
time (sec)	N/A	0.032	0.041	0.022	0.	2.191	1.091	1.151

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	36	72	20	34
normalized size	1	1.	1.	0.8	2.4	4.8	1.33	2.27
time (sec)	N/A	0.01	0.005	0.01	1.091	2.076	0.608	1.175

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	109	20	35
normalized size	1	1.	1.	0.82	2.12	6.41	1.18	2.06
time (sec)	N/A	0.011	0.007	0.019	1.025	1.984	0.587	1.175

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	130	20	36
normalized size	1	1.	1.	0.82	2.12	7.65	1.18	2.12
time (sec)	N/A	0.011	0.006	0.027	1.043	2.055	0.597	1.163

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	117	37	80
normalized size	1	1.	0.71	0.8	0.	3.34	1.06	2.29
time (sec)	N/A	0.035	0.039	0.009	0.	2.136	1.351	1.139

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	54	72	420	0	49
normalized size	1	1.	1.	2.84	3.79	22.11	0.	2.58
time (sec)	N/A	0.026	0.015	0.048	1.542	2.135	0.	1.194

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	60	62	302	0	58
normalized size	1	1.	1.	3.16	3.26	15.89	0.	3.05
time (sec)	N/A	0.043	0.028	0.066	1.614	2.014	0.	1.198

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	42	0	504	0	96
normalized size	1	1.	1.	0.61	0.	7.3	0.	1.39
time (sec)	N/A	0.096	0.135	0.083	0.	2.351	0.	1.252

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	60	0	724	0	109
normalized size	1	1.	0.93	0.69	0.	8.32	0.	1.25
time (sec)	N/A	0.291	0.187	0.089	0.	2.428	0.	1.241

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	84	0	633	0	135
normalized size	1	1.	1.	0.97	0.	7.28	0.	1.55
time (sec)	N/A	0.259	0.124	0.105	0.	2.375	0.	1.192

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	164	0	0	0	0	0
normalized size	1	1.	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.171	0.06	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	22	167	0	22
normalized size	1	1.	1.	0.9	2.2	16.7	0.	2.2
time (sec)	N/A	0.023	0.008	0.019	1.517	2.053	0.	1.158

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	51	66	433	0	49
normalized size	1	1.	1.	2.55	3.3	21.65	0.	2.45
time (sec)	N/A	0.027	0.016	0.049	1.575	2.217	0.	1.178

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	143	81	493	0	73
normalized size	1	1.	1.	5.11	2.89	17.61	0.	2.61
time (sec)	N/A	0.053	0.03	0.092	1.562	2.314	0.	1.213

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	246	0	699	0	101
normalized size	1	1.	0.93	3.	0.	8.52	0.	1.23
time (sec)	N/A	0.187	0.224	0.155	0.	2.382	0.	1.252

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	172	0	624	0	92
normalized size	1	1.	1.	4.53	0.	16.42	0.	2.42
time (sec)	N/A	0.077	0.058	0.112	0.	2.234	0.	1.204

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	155	39	57	122	0	51
normalized size	1	1.	9.69	2.44	3.56	7.62	0.	3.19
time (sec)	N/A	0.019	0.281	0.033	1.524	1.986	0.	1.198

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	26	61	171	0	55
normalized size	1	1.	0.81	1.24	2.9	8.14	0.	2.62
time (sec)	N/A	0.03	0.009	0.034	1.566	2.054	0.	1.158

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	110	40	0	776	0	155
normalized size	1	1.	1.55	0.56	0.	10.93	0.	2.18
time (sec)	N/A	0.074	0.024	0.046	0.	2.165	0.	1.221

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	583	0	159
normalized size	1	1.	0.92	1.63	0.	9.4	0.	2.56
time (sec)	N/A	0.083	0.083	0.058	0.	2.193	0.	1.202

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	385	78	0	906	0	208
normalized size	1	1.	4.53	0.92	0.	10.66	0.	2.45
time (sec)	N/A	0.079	0.267	0.068	0.	2.334	0.	1.241

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	4	9	36	0	4
normalized size	1	1.	1.	0.57	1.29	5.14	0.	0.57
time (sec)	N/A	0.013	0.003	0.013	1.574	2.071	0.	1.205

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	44	40	53	115	0	26
normalized size	1	1.	2.93	2.67	3.53	7.67	0.	1.73
time (sec)	N/A	0.036	0.024	0.033	1.539	2.129	0.	1.203

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	62	68	297	0	59
normalized size	1	1.	1.	2.38	2.62	11.42	0.	2.27
time (sec)	N/A	0.027	0.021	0.04	1.521	2.188	0.	1.21

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	558	0	92
normalized size	1	1.	1.12	0.55	0.	7.44	0.	1.23
time (sec)	N/A	0.108	0.103	0.038	0.	2.355	0.	1.186

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	92	0	408	0	78
normalized size	1	1.	0.83	2.56	0.	11.33	0.	2.17
time (sec)	N/A	0.045	0.033	0.054	0.	2.16	0.	1.191

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	15	15	7	36	72	20	34
normalized size	1	1.88	1.88	0.88	4.5	9.	2.5	4.25
time (sec)	N/A	0.011	0.005	0.009	1.008	1.987	0.647	1.223

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	36	109	20	35
normalized size	1	1.	1.	0.71	2.12	6.41	1.18	2.06
time (sec)	N/A	0.011	0.007	0.014	1.047	2.112	0.558	1.194

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	130	20	36
normalized size	1	1.	1.	0.82	2.12	7.65	1.18	2.12
time (sec)	N/A	0.011	0.006	0.011	1.036	2.07	0.587	1.156

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	117	42	80
normalized size	1	1.	0.71	0.8	0.	3.34	1.2	2.29
time (sec)	N/A	0.033	0.038	0.006	0.	2.103	1.042	1.149

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	36	61	20	34
normalized size	1	1.	1.	0.8	2.4	4.07	1.33	2.27
time (sec)	N/A	0.009	0.005	0.013	1.035	1.964	0.618	1.17

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	80	20	36
normalized size	1	1.	1.	0.82	2.12	4.71	1.18	2.12
time (sec)	N/A	0.009	0.006	0.018	1.054	2.14	0.571	1.176

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	119	20	36
normalized size	1	1.	1.	0.82	2.12	7.	1.18	2.12
time (sec)	N/A	0.009	0.004	0.037	1.042	2.021	0.558	1.189

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	117	56	80
normalized size	1	1.	0.71	0.8	0.	3.34	1.6	2.29
time (sec)	N/A	0.029	0.034	0.009	0.	2.066	1.263	1.168

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	164	16	70	259	0	61
normalized size	1	1.	8.63	0.84	3.68	13.63	0.	3.21
time (sec)	N/A	0.032	0.17	0.013	1.708	2.112	0.	1.145

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	55	17	207	275	0	61
normalized size	1	1.	2.75	0.85	10.35	13.75	0.	3.05
time (sec)	N/A	0.03	0.058	0.017	1.698	2.056	0.	1.151

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	0	0	161
normalized size	1	1.	1.64	0.96	0.	0.	0.	2.33
time (sec)	N/A	0.087	0.023	0.054	0.	0.	0.	1.235

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	249	70	0	1019	0	171
normalized size	1	1.	3.04	0.85	0.	12.43	0.	2.09
time (sec)	N/A	0.117	0.028	0.061	0.	2.159	0.	1.237

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	395	102	0	976	0	212
normalized size	1	1.	4.54	1.17	0.	11.22	0.	2.44
time (sec)	N/A	0.251	0.283	0.069	0.	2.195	0.	1.22

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	9	39	231	0	35
normalized size	1	1.	1.4	0.9	3.9	23.1	0.	3.5
time (sec)	N/A	0.026	0.014	0.023	1.114	2.205	0.	1.108

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	50	77	412	0	74
normalized size	1	1.	1.04	1.11	1.71	9.16	0.	1.64
time (sec)	N/A	0.063	0.018	0.053	1.694	2.152	0.	1.177

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	192	63	95	397	0	90
normalized size	1	1.	6.86	2.25	3.39	14.18	0.	3.21
time (sec)	N/A	0.057	0.224	0.073	1.669	2.205	0.	1.16

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	133	190	0	980	0	212
normalized size	1	1.	1.21	1.73	0.	8.91	0.	1.93
time (sec)	N/A	0.176	0.115	0.083	0.	2.238	0.	1.189

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	95	87	0	586	0	120
normalized size	1	1.	2.5	2.29	0.	15.42	0.	3.16
time (sec)	N/A	0.086	0.071	0.097	0.	2.127	0.	1.136

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	156	0	0	0	0	0
normalized size	1	1.	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.169	0.063	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	44	58	254	0	53
normalized size	1	1.	1.	2.93	3.87	16.93	0.	3.53
time (sec)	N/A	0.016	0.008	0.033	1.769	2.026	0.	1.122

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	48	40	154	115	0	26
normalized size	1	1.	3.2	2.67	10.27	7.67	0.	1.73
time (sec)	N/A	0.032	0.022	0.035	1.769	2.066	0.	1.14

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	40	0	427	0	182
normalized size	1	1.	0.94	0.56	0.	6.01	0.	2.56
time (sec)	N/A	0.039	0.097	0.056	0.	2.236	0.	1.216

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	558	0	92
normalized size	1	1.	1.12	0.55	0.	7.44	0.	1.23
time (sec)	N/A	0.125	0.1	0.037	0.	2.307	0.	1.177

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	83	0	489	0	239
normalized size	1	1.	0.95	0.98	0.	5.75	0.	2.81
time (sec)	N/A	0.058	0.084	0.083	0.	2.388	0.	1.208

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	11	6	26	89	0	22
normalized size	1	1.	1.57	0.86	3.71	12.71	0.	3.14
time (sec)	N/A	0.015	0.002	0.013	1.076	1.986	0.	1.141

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	24	63	173	0	54
normalized size	1	1.	1.	1.14	3.	8.24	0.	2.57
time (sec)	N/A	0.031	0.009	0.038	1.635	2.043	0.	1.152

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	183	53	81	211	0	77
normalized size	1	1.	7.04	2.04	3.12	8.12	0.	2.96
time (sec)	N/A	0.033	0.294	0.039	1.684	2.102	0.	1.145

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	583	0	146
normalized size	1	1.	0.92	1.63	0.	9.4	0.	2.35
time (sec)	N/A	0.078	0.059	0.063	0.	2.161	0.	1.189

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	91	77	0	358	0	107
normalized size	1	1.	2.53	2.14	0.	9.94	0.	2.97
time (sec)	N/A	0.052	0.062	0.046	0.	2.172	0.	1.124

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	80	258	0	0
normalized size	1	1.	0.94	0.	1.14	3.69	0.	0.
time (sec)	N/A	0.118	0.046	0.041	1.265	2.206	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	50	203	116	180	119	99
normalized size	1	1.	0.53	2.16	1.23	1.91	1.27	1.05
time (sec)	N/A	0.069	0.104	0.004	1.108	2.057	2.389	1.161

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	39	114	86	154	75	77
normalized size	1	1.	0.61	1.78	1.34	2.41	1.17	1.2
time (sec)	N/A	0.043	0.067	0.004	1.086	2.123	1.213	1.148

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	28	53	62	112	56	55
normalized size	1	1.	0.64	1.2	1.41	2.55	1.27	1.25
time (sec)	N/A	0.023	0.055	0.003	1.198	1.99	0.61	1.163

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	18	58	19	32
normalized size	1	1.	2.47	0.93	1.2	3.87	1.27	2.13
time (sec)	N/A	0.014	0.01	0.	1.091	2.	0.222	1.127

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	31	109	0	31
normalized size	1	1.	0.93	0.96	1.15	4.04	0.	1.15
time (sec)	N/A	0.075	0.025	0.021	1.329	2.024	0.	1.142

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	42	56	36	174	0	70
normalized size	1	1.	1.08	1.44	0.92	4.46	0.	1.79
time (sec)	N/A	0.095	0.071	0.026	1.348	1.892	0.	1.139

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	61	90	41	257	0	116
normalized size	1	1.	1.02	1.5	0.68	4.28	0.	1.93
time (sec)	N/A	0.123	0.175	0.032	1.233	1.945	0.	1.136

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	124	42	286	0	162
normalized size	1	1.	0.91	1.46	0.49	3.36	0.	1.91
time (sec)	N/A	0.151	0.156	0.034	1.391	1.967	0.	1.119

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	153	486	0	0
normalized size	1	1.	0.85	0.	1.14	3.63	0.	0.
time (sec)	N/A	0.196	0.173	0.061	1.353	2.213	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	334	216	332	146	189
normalized size	1	1.	0.74	2.85	1.85	2.84	1.25	1.62
time (sec)	N/A	0.121	0.381	0.007	1.167	1.998	4.191	1.16

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	193	165	273	105	146
normalized size	1	1.	0.78	2.33	1.99	3.29	1.27	1.76
time (sec)	N/A	0.071	0.2	0.006	1.112	1.978	2.25	1.15

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	92	113	205	61	103
normalized size	1	1.	1.02	2.04	2.51	4.56	1.36	2.29
time (sec)	N/A	0.031	0.137	0.007	1.197	2.309	1.134	1.163

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	105	20	62
normalized size	1	1.	1.	0.93	1.2	7.	1.33	4.13
time (sec)	N/A	0.021	0.003	0.003	1.172	2.208	0.517	1.135

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	47	57	204	0	57
normalized size	1	1.	0.83	1.	1.21	4.34	0.	1.21
time (sec)	N/A	0.141	0.07	0.057	1.369	2.381	0.	1.183

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	70	104	68	327	0	122
normalized size	1	1.	0.88	1.3	0.85	4.09	0.	1.52
time (sec)	N/A	0.186	0.148	0.066	1.382	1.891	0.	1.138

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	105	169	78	489	0	211
normalized size	1	1.	0.88	1.42	0.66	4.11	0.	1.77
time (sec)	N/A	0.249	0.283	0.069	1.256	1.836	0.	1.157

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	234	78	548	0	301
normalized size	1	1.	0.9	1.52	0.51	3.56	0.	1.95
time (sec)	N/A	0.289	0.349	0.076	1.263	1.756	0.	1.17

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	158	518	0	0
normalized size	1	1.	0.79	0.	1.14	3.73	0.	0.
time (sec)	N/A	0.237	0.126	0.057	1.269	1.909	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	91	414	231	468	226	196
normalized size	1	1.	0.59	2.67	1.49	3.02	1.46	1.26
time (sec)	N/A	0.141	0.599	0.009	1.082	1.777	8.534	1.151

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	237	171	394	150	153
normalized size	1	1.	0.69	2.35	1.69	3.9	1.49	1.51
time (sec)	N/A	0.076	0.231	0.006	1.206	1.757	4.305	1.17

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	113	123	289	110	109
normalized size	1	1.	0.77	1.74	1.89	4.45	1.69	1.68
time (sec)	N/A	0.043	0.154	0.007	1.185	1.81	2.236	1.16

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	146	20	66
normalized size	1	1.	1.	0.93	1.2	9.73	1.33	4.4
time (sec)	N/A	0.02	0.002	0.005	1.061	1.663	1.05	1.144

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	61	223	0	61
normalized size	1	1.	0.89	0.94	1.15	4.21	0.	1.15
time (sec)	N/A	0.141	0.082	0.06	1.271	1.718	0.	1.149

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	110	72	370	0	135
normalized size	1	1.	0.9	1.24	0.81	4.16	0.	1.52
time (sec)	N/A	0.189	0.204	0.069	1.335	1.787	0.	1.201

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	112	178	81	570	0	227
normalized size	1	1.	0.9	1.42	0.65	4.56	0.	1.82
time (sec)	N/A	0.248	0.589	0.069	1.388	1.877	0.	1.183

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	80	647	0	319
normalized size	1	1.	0.89	1.46	0.47	3.83	0.	1.89
time (sec)	N/A	0.297	0.535	0.072	1.381	1.851	0.	1.151

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	18	38	0	18
normalized size	1	1.	1.	0.88	2.25	4.75	0.	2.25
time (sec)	N/A	0.03	0.006	0.017	1.285	1.978	0.	1.147

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	70	0	41
normalized size	1	1.	1.	0.94	1.25	4.38	0.	2.56
time (sec)	N/A	0.047	0.006	0.007	1.237	2.027	0.	1.129

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	18	113	0	65
normalized size	1	1.	1.	0.89	0.67	4.19	0.	2.41
time (sec)	N/A	0.063	0.007	0.007	1.206	2.009	0.	1.18

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	153	487	0	0
normalized size	1	1.	0.85	0.	1.14	3.63	0.	0.
time (sec)	N/A	0.183	0.227	0.089	1.364	1.951	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	84	334	216	332	146	189
normalized size	1	1.	0.72	2.85	1.85	2.84	1.25	1.62
time (sec)	N/A	0.145	0.399	0.005	1.131	1.764	4.191	1.162

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	66	193	165	271	105	146
normalized size	1	1.	0.8	2.33	1.99	3.27	1.27	1.76
time (sec)	N/A	0.079	0.425	0.005	1.158	1.826	2.216	1.201

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	92	113	203	61	103
normalized size	1	1.	0.84	2.04	2.51	4.51	1.36	2.29
time (sec)	N/A	0.035	0.137	0.006	1.236	1.806	1.112	1.174

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	89	20	62
normalized size	1	1.	1.	0.93	1.2	5.93	1.33	4.13
time (sec)	N/A	0.018	0.003	0.002	1.127	1.722	0.509	1.157

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	57	204	0	57
normalized size	1	1.	0.87	1.	1.21	4.34	0.	1.21
time (sec)	N/A	0.123	0.07	0.068	1.298	1.744	0.	1.124

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	104	68	340	0	123
normalized size	1	1.	0.85	1.3	0.85	4.25	0.	1.54
time (sec)	N/A	0.167	0.207	0.072	1.278	1.8	0.	1.133

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	107	169	78	485	0	211
normalized size	1	1.	0.9	1.42	0.66	4.08	0.	1.77
time (sec)	N/A	0.218	0.255	0.075	1.333	1.77	0.	1.18

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	234	78	549	0	300
normalized size	1	1.	0.9	1.52	0.51	3.56	0.	1.95
time (sec)	N/A	0.281	0.305	0.076	1.341	1.638	0.	1.194

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	377	0	0
normalized size	1	1.	0.89	0.	0.	4.44	0.	0.
time (sec)	N/A	0.128	0.206	0.053	0.	1.888	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	384	123	352	250	105
normalized size	1	1.	0.73	4.86	1.56	4.46	3.16	1.33
time (sec)	N/A	0.114	0.198	0.007	1.178	1.852	8.077	1.172

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	232	93	288	204	84
normalized size	1	1.	0.8	3.87	1.55	4.8	3.4	1.4
time (sec)	N/A	0.101	0.165	0.007	1.164	1.855	4.664	1.293

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	114	69	234	131	62
normalized size	1	1.	1.	2.78	1.68	5.71	3.2	1.51
time (sec)	N/A	0.05	0.135	0.005	1.11	1.833	2.435	1.224

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	53	104	92	65
normalized size	1	1.	0.5	0.93	1.15	2.26	2.	1.41
time (sec)	N/A	0.039	0.022	0.	1.067	1.728	1.197	1.128

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	36	130	0	36
normalized size	1	1.	0.97	0.91	1.09	3.94	0.	1.09
time (sec)	N/A	0.085	0.097	0.049	1.266	1.836	0.	1.15

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	61	43	240	0	74
normalized size	1	1.	0.87	1.17	0.83	4.62	0.	1.42
time (sec)	N/A	0.112	0.093	0.049	1.394	1.879	0.	1.167

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	95	49	371	0	120
normalized size	1	1.	0.97	1.42	0.73	5.54	0.	1.79
time (sec)	N/A	0.136	0.104	0.055	1.206	1.762	0.	1.18

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	79	129	49	436	0	166
normalized size	1	1.	0.86	1.4	0.53	4.74	0.	1.8
time (sec)	N/A	0.17	0.191	0.054	1.336	1.812	0.	1.161

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	175	0	231	764	0	0
normalized size	1	1.	0.84	0.	1.11	3.66	0.	0.
time (sec)	N/A	0.287	0.295	0.078	1.361	2.004	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	125	534	331	716	253	286
normalized size	1	1.	0.62	2.64	1.64	3.54	1.25	1.42
time (sec)	N/A	0.267	1.089	0.012	1.084	1.755	13.531	1.185

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	105	306	252	579	182	221
normalized size	1	1.	0.71	2.07	1.7	3.91	1.23	1.49
time (sec)	N/A	0.18	0.308	0.008	1.068	1.754	7.98	1.184

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	143	174	425	112	157
normalized size	1	1.	0.74	1.52	1.85	4.52	1.19	1.67
time (sec)	N/A	0.096	0.193	0.007	1.066	1.834	4.413	1.162

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	42	105	178	44	95
normalized size	1	1.	0.87	1.35	3.39	5.74	1.42	3.06
time (sec)	N/A	0.033	0.059	0.009	1.013	1.799	2.111	1.157

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	71	86	323	0	86
normalized size	1	1.	0.84	0.97	1.18	4.42	0.	1.18
time (sec)	N/A	0.19	0.105	0.105	1.304	1.747	0.	1.16

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	158	103	582	0	194
normalized size	1	1.	0.84	1.27	0.83	4.69	0.	1.56
time (sec)	N/A	0.263	0.354	0.109	1.262	1.765	0.	1.214

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	162	257	119	859	0	328
normalized size	1	1.	0.88	1.4	0.65	4.67	0.	1.78
time (sec)	N/A	0.344	0.558	0.115	1.319	1.805	0.	1.243

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	356	119	996	0	462
normalized size	1	1.	0.89	1.5	0.5	4.18	0.	1.94
time (sec)	N/A	0.444	0.57	0.122	1.301	1.79	0.	1.17

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	158	518	0	0
normalized size	1	1.	0.79	0.	1.12	3.67	0.	0.
time (sec)	N/A	0.21	0.137	0.053	1.228	1.867	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	95	414	231	468	226	196
normalized size	1	1.	0.61	2.67	1.49	3.02	1.46	1.26
time (sec)	N/A	0.139	0.635	0.006	1.072	1.692	7.942	1.215

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	72	237	171	394	150	153
normalized size	1	1.	0.71	2.35	1.69	3.9	1.49	1.51
time (sec)	N/A	0.077	0.233	0.006	1.064	1.743	4.446	1.191

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	113	123	289	110	109
normalized size	1	1.	0.77	1.74	1.89	4.45	1.69	1.68
time (sec)	N/A	0.043	0.157	0.005	1.12	1.68	2.321	1.173

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	146	20	66
normalized size	1	1.	1.	0.93	1.2	9.73	1.33	4.4
time (sec)	N/A	0.021	0.002	0.003	1.014	1.838	1.123	1.202

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	61	223	0	61
normalized size	1	1.	0.89	0.94	1.15	4.21	0.	1.15
time (sec)	N/A	0.133	0.08	0.067	1.212	1.799	0.	1.213

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	110	72	370	0	135
normalized size	1	1.	0.88	1.24	0.81	4.16	0.	1.52
time (sec)	N/A	0.176	0.25	0.066	1.249	1.805	0.	1.177

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	178	81	570	0	227
normalized size	1	1.	0.9	1.42	0.65	4.56	0.	1.82
time (sec)	N/A	0.23	0.545	0.069	1.226	1.852	0.	1.193

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	82	647	0	319
normalized size	1	1.	0.89	1.46	0.49	3.83	0.	1.89
time (sec)	N/A	0.287	0.531	0.081	1.211	1.767	0.	1.217

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	174	0	231	763	0	0
normalized size	1	1.	0.83	0.	1.11	3.65	0.	0.
time (sec)	N/A	0.28	0.261	0.062	1.344	1.951	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	136	542	331	722	253	286
normalized size	1	1.	0.67	2.68	1.64	3.57	1.25	1.42
time (sec)	N/A	0.262	0.491	0.01	1.091	1.774	13.88	1.171

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	98	314	252	581	182	221
normalized size	1	1.	0.66	2.12	1.7	3.93	1.23	1.49
time (sec)	N/A	0.182	0.445	0.007	1.061	1.789	8.046	1.225

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	149	174	433	112	157
normalized size	1	1.	0.74	1.59	1.85	4.61	1.19	1.67
time (sec)	N/A	0.09	0.189	0.007	1.202	1.692	4.184	1.177

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	48	105	216	44	95
normalized size	1	1.	0.87	1.55	3.39	6.97	1.42	3.06
time (sec)	N/A	0.035	0.07	0.009	1.021	1.884	2.124	1.155

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	71	86	323	0	86
normalized size	1	1.	0.86	0.97	1.18	4.42	0.	1.18
time (sec)	N/A	0.182	0.109	0.091	1.309	1.808	0.	1.179

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	106	158	103	548	0	189
normalized size	1	1.	0.85	1.27	0.83	4.42	0.	1.52
time (sec)	N/A	0.25	0.264	0.095	1.292	1.838	0.	1.19

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	164	257	119	857	0	323
normalized size	1	1.	0.89	1.4	0.65	4.66	0.	1.76
time (sec)	N/A	0.339	0.439	0.097	1.323	1.813	0.	1.185

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	356	119	987	0	462
normalized size	1	1.	0.89	1.5	0.5	4.15	0.	1.94
time (sec)	N/A	0.414	0.521	0.102	1.302	1.874	0.	1.178

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	119	0	158	520	0	0
normalized size	1	1.	0.77	0.	1.02	3.35	0.	0.
time (sec)	N/A	0.243	0.153	0.076	1.197	1.85	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	90	622	231	617	314	196
normalized size	1	1.	0.63	4.35	1.62	4.31	2.2	1.37
time (sec)	N/A	0.197	0.885	0.01	1.085	1.768	23.173	1.305

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	72	358	171	531	223	153
normalized size	1	1.	0.69	3.41	1.63	5.06	2.12	1.46
time (sec)	N/A	0.14	0.219	0.009	1.067	1.757	13.46	1.228

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	170	123	408	148	109
normalized size	1	1.	0.75	2.54	1.84	6.09	2.21	1.63
time (sec)	N/A	0.073	0.144	0.009	1.098	1.804	7.63	1.159

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	35	52	76	197	42	66
normalized size	1	1.	1.13	1.68	2.45	6.35	1.35	2.13
time (sec)	N/A	0.037	0.012	0.007	1.012	1.76	3.746	1.166

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	61	225	0	61
normalized size	1	1.	0.89	0.94	1.15	4.25	0.	1.15
time (sec)	N/A	0.157	0.18	0.088	1.228	1.806	0.	1.186

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	110	72	432	0	135
normalized size	1	1.	0.88	1.24	0.81	4.85	0.	1.52
time (sec)	N/A	0.196	0.243	0.095	1.245	1.721	0.	1.188

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	118	178	82	699	0	227
normalized size	1	1.	0.9	1.36	0.63	5.34	0.	1.73
time (sec)	N/A	0.257	0.247	0.092	1.264	1.896	0.	1.168

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	82	792	0	319
normalized size	1	1.	0.89	1.46	0.49	4.69	0.	1.89
time (sec)	N/A	0.315	0.331	0.097	1.378	1.796	0.	1.191

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.459	0.046	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	116	113	761	0	0
normalized size	1	1.	0.97	1.27	1.24	8.36	0.	0.
time (sec)	N/A	0.16	2.232	0.098	1.203	2.082	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	66	94	85	595	0	0
normalized size	1	1.	1.02	1.45	1.31	9.15	0.	0.
time (sec)	N/A	0.135	2.058	0.026	1.207	1.965	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	149	70	54	427	0	0
normalized size	1	1.	3.31	1.56	1.2	9.49	0.	0.
time (sec)	N/A	0.082	3.526	0.026	1.197	1.917	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	28	88	0	36
normalized size	1	1.	1.	1.18	2.55	8.	0.	3.27
time (sec)	N/A	0.006	0.008	0.005	1.004	1.868	0.	1.267

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	11.788	0.076	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	18.557	0.056	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.394	3.169	0.033	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	130	0	0	1890	0	0
normalized size	1	1.	1.15	0.	0.	16.73	0.	0.
time (sec)	N/A	0.105	2.192	0.102	0.	2.029	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	125	154	0	1395	0	0
normalized size	1	1.	1.81	2.23	0.	20.22	0.	0.
time (sec)	N/A	0.061	0.435	0.054	0.	2.066	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	59	50	327	0	95
normalized size	1	1.	1.33	2.46	2.08	13.62	0.	3.96
time (sec)	N/A	0.019	0.046	0.039	1.787	1.774	0.	1.312

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	31	154	0	31
normalized size	1	1.	1.	1.09	2.82	14.	0.	2.82
time (sec)	N/A	0.015	0.005	0.005	0.996	1.697	0.	1.251

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	6.741	0.063	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	7.772	0.063	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.47	6.253	0.036	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	228	121	149	2966	0	0
normalized size	1	1.	2.75	1.46	1.8	35.73	0.	0.
time (sec)	N/A	0.183	6.123	0.028	1.512	2.059	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	73	127	977	0	192
normalized size	1	1.	1.31	1.74	3.02	23.26	0.	4.57
time (sec)	N/A	0.061	0.118	0.029	1.348	1.825	0.	1.225

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	177	270	0	248
normalized size	1	1.	1.	1.43	5.9	9.	0.	8.27
time (sec)	N/A	0.03	0.072	0.028	1.062	1.751	0.	1.252

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	31	235	0	36
normalized size	1	1.	1.	0.93	2.07	15.67	0.	2.4
time (sec)	N/A	0.021	0.009	0.007	1.012	1.752	0.	1.169

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	23.712	0.063	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	20.619	0.065	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	13.577	0.096	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	211	0	0	1704	0	0
normalized size	1	1.	1.08	0.	0.	8.74	0.	0.
time (sec)	N/A	0.198	1.437	0.125	0.	2.384	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	153	0	0	1345	0	0
normalized size	1	1.	1.13	0.	0.	9.96	0.	0.
time (sec)	N/A	0.13	1.429	0.057	0.	2.355	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	212	162	0	973	0	0
normalized size	1	1.	2.75	2.1	0.	12.64	0.	0.
time (sec)	N/A	0.064	0.109	0.057	0.	2.24	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	55	254	0	51
normalized size	1	1.	1.	1.04	2.39	11.04	0.	2.22
time (sec)	N/A	0.015	0.012	0.013	1.606	2.086	0.	1.167

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	8.738	0.085	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	8.736	0.066	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.585	0.059	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	104	125	146	1890	0	0
normalized size	1	1.	1.17	1.4	1.64	21.24	0.	0.
time (sec)	N/A	0.178	4.533	0.056	1.348	2.319	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	168	99	113	1424	0	0
normalized size	1	1.	2.58	1.52	1.74	21.91	0.	0.
time (sec)	N/A	0.118	3.204	0.055	1.379	2.254	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	128	475	0	128
normalized size	1	1.	1.48	1.74	4.13	15.32	0.	4.13
time (sec)	N/A	0.027	0.141	0.055	1.182	2.048	0.	1.196

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	18	34	82	0	38
normalized size	1	1.	1.77	1.38	2.62	6.31	0.	2.92
time (sec)	N/A	0.009	0.007	0.013	1.043	1.972	0.	1.205

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	18.244	0.067	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	11.661	0.066	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	15.33	0.066	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	245	0	0	5955	0	0
normalized size	1	1.	1.02	0.	0.	24.81	0.	0.
time (sec)	N/A	0.298	3.293	0.224	0.	2.733	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	180	0	0	4242	0	0
normalized size	1	1.	1.26	0.	0.	29.66	0.	0.
time (sec)	N/A	0.199	1.605	0.182	0.	2.527	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	93	178	0	2963	0	0
normalized size	1	1.	1.02	1.96	0.	32.56	0.	0.
time (sec)	N/A	0.102	0.753	0.068	0.	2.354	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	49	89	759	0	108
normalized size	1	1.	1.	1.44	2.62	22.32	0.	3.18
time (sec)	N/A	0.024	0.016	0.014	1.618	1.998	0.	1.163

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	16.04	0.145	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	12.831	0.195	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	21.897	0.069	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	191	189	244	2568	0	0
normalized size	1	1.	1.03	1.02	1.32	13.88	0.	0.
time (sec)	N/A	0.247	2.862	0.071	1.326	2.47	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	154	152	186	2090	0	0
normalized size	1	1.	1.18	1.17	1.43	16.08	0.	0.
time (sec)	N/A	0.19	2.639	0.069	1.39	2.314	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	102	110	128	1561	0	0
normalized size	1	1.	1.15	1.24	1.44	17.54	0.	0.
time (sec)	N/A	0.116	0.221	0.065	1.362	2.285	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	76	551	0	92
normalized size	1	1.	0.89	0.96	2.71	19.68	0.	3.29
time (sec)	N/A	0.025	0.017	0.016	1.566	2.077	0.	1.175

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	18.774	0.102	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	53	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	19.689	0.081	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	16.835	0.08	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	196	0	0	3351	0	0
normalized size	1	1.	1.21	0.	0.	20.69	0.	0.
time (sec)	N/A	0.183	3.255	0.103	0.	2.503	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	172	205	0	2500	0	0
normalized size	1	1.	1.65	1.97	0.	24.04	0.	0.
time (sec)	N/A	0.118	0.492	0.074	0.	2.475	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	50	94	109	779	0	138
normalized size	1	1.	1.09	2.04	2.37	16.93	0.	3.
time (sec)	N/A	0.051	0.115	0.082	1.853	2.132	0.	1.236

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	32	73	85	0	61
normalized size	1	1.	1.	1.52	3.48	4.05	0.	2.9
time (sec)	N/A	0.024	0.029	0.016	1.039	1.949	0.	1.209

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	11.504	0.082	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	9.81	0.089	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	5.265	0.087	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	230	234	319	5662	0	0
normalized size	1	1.	1.26	1.28	1.74	30.94	0.	0.
time (sec)	N/A	0.326	2.821	0.108	1.256	2.669	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	185	164	247	4211	0	0
normalized size	1	1.	1.59	1.41	2.13	36.3	0.	0.
time (sec)	N/A	0.202	2.242	0.082	1.288	2.516	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	232	111	177	2992	0	0
normalized size	1	1.	2.83	1.35	2.16	36.49	0.	0.
time (sec)	N/A	0.117	6.123	0.079	1.425	2.404	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	82	930	0	93
normalized size	1	1.	1.	0.96	3.04	34.44	0.	3.44
time (sec)	N/A	0.018	0.012	0.017	1.729	2.166	0.	1.136

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	13.394	0.218	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	7.957	0.221	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	7.181	0.058	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	200	176	644	0	0
normalized size	1	1.	1.05	2.3	2.02	7.4	0.	0.
time (sec)	N/A	0.153	0.01	0.026	1.301	2.078	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	166	130	489	0	0
normalized size	1	1.	1.05	2.63	2.06	7.76	0.	0.
time (sec)	N/A	0.133	0.01	0.023	1.294	2.105	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	122	78	336	0	0
normalized size	1	1.	1.04	2.71	1.73	7.47	0.	0.
time (sec)	N/A	0.081	0.006	0.023	1.298	2.071	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	13	31	88	0	38
normalized size	1	1.	1.73	1.18	2.82	8.	0.	3.45
time (sec)	N/A	0.006	0.013	0.007	1.1	2.073	0.	1.215

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.435	0.061	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.833	0.059	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	17.877	0.112	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	202	246	278	1409	0	0
normalized size	1	1.	1.22	1.49	1.68	8.54	0.	0.
time (sec)	N/A	0.178	3.909	0.092	1.472	2.18	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	138	196	205	1084	0	0
normalized size	1	1.	1.2	1.7	1.78	9.43	0.	0.
time (sec)	N/A	0.123	3.82	0.059	1.349	2.207	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	131	139	127	753	0	0
normalized size	1	1.	1.98	2.11	1.92	11.41	0.	0.
time (sec)	N/A	0.061	0.133	0.061	1.295	2.163	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	21	80	355	0	73
normalized size	1	1.	1.13	0.91	3.48	15.43	0.	3.17
time (sec)	N/A	0.017	0.02	0.013	1.049	2.121	0.	1.149

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	25.979	0.084	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	36.98	0.089	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	22.368	0.074	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	236	272	304	2325	0	0
normalized size	1	1.	1.31	1.51	1.69	12.92	0.	0.
time (sec)	N/A	0.237	2.527	0.087	1.45	2.178	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	178	222	231	1858	0	0
normalized size	1	1.	1.41	1.76	1.83	14.75	0.	0.
time (sec)	N/A	0.197	2.515	0.072	1.411	2.174	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	82	162	153	1355	0	0
normalized size	1	1.	0.93	1.84	1.74	15.4	0.	0.
time (sec)	N/A	0.121	0.234	0.07	1.364	2.161	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	95	552	0	93
normalized size	1	1.	0.93	0.96	3.52	20.44	0.	3.44
time (sec)	N/A	0.023	0.02	0.015	1.035	2.105	0.	1.151

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	10.907	0.073	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	16.409	0.073	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	48	0	1022	0	136
normalized size	1	1.	1.	1.45	0.	30.97	0.	4.12
time (sec)	N/A	0.054	0.029	0.04	0.	2.152	0.	1.191

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	87	0	1840	0	0
normalized size	1	1.	0.88	1.19	0.	25.21	0.	0.
time (sec)	N/A	0.158	0.093	0.045	0.	2.174	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	94	117	0	2541	0	0
normalized size	1	1.	0.92	1.15	0.	24.91	0.	0.
time (sec)	N/A	0.183	0.147	0.046	0.	2.322	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	82	197	2881	0	0
normalized size	1	1.	0.89	1.3	3.13	45.73	0.	0.
time (sec)	N/A	0.163	0.077	0.047	1.362	2.258	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	98	127	235	4618	0	0
normalized size	1	1.	1.02	1.32	2.45	48.1	0.	0.
time (sec)	N/A	0.28	0.277	0.061	1.335	2.319	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	133	184	321	5913	0	0
normalized size	1	1.	0.84	1.16	2.03	37.42	0.	0.
time (sec)	N/A	0.386	0.334	0.061	1.29	2.38	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	3.825	0.033	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	167	174	163	1461	0	0
normalized size	1	1.	1.8	1.87	1.75	15.71	0.	0.
time (sec)	N/A	0.105	6.869	0.028	1.381	2.115	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	133	134	112	1022	0	0
normalized size	1	1.	2.25	2.27	1.9	17.32	0.	0.
time (sec)	N/A	0.061	0.83	0.027	1.369	2.135	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	114	54	86	483	0	126
normalized size	1	1.	4.56	2.16	3.44	19.32	0.	5.04
time (sec)	N/A	0.019	0.048	0.029	1.317	2.055	0.	1.274

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	34	154	0	34
normalized size	1	1.	1.	1.09	3.09	14.	0.	3.09
time (sec)	N/A	0.011	0.007	0.006	1.034	2.02	0.	1.281

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	31.313	0.035	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	37.301	0.063	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	9.639	0.056	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	204	198	197	1643	0	0
normalized size	1	1.	2.34	2.28	2.26	18.89	0.	0.
time (sec)	N/A	0.19	0.601	0.056	1.385	2.162	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	163	156	146	1211	0	0
normalized size	1	1.	2.51	2.4	2.25	18.63	0.	0.
time (sec)	N/A	0.125	4.882	0.056	1.4	2.173	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	155	477	0	132
normalized size	1	1.	1.48	1.74	5.	15.39	0.	4.26
time (sec)	N/A	0.028	0.152	0.054	1.194	2.167	0.	1.153

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	27	18	34	82	0	38
normalized size	1	1.	2.08	1.38	2.62	6.31	0.	2.92
time (sec)	N/A	0.009	0.013	0.013	1.095	2.208	0.	1.16

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.378	0.059	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.89	0.095	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	15.924	0.082	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	225	241	292	2685	0	0
normalized size	1	1.	1.57	1.69	2.04	18.78	0.	0.
time (sec)	N/A	0.184	0.325	0.079	1.463	2.577	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	230	185	212	1952	0	0
normalized size	1	1.	2.42	1.95	2.23	20.55	0.	0.
time (sec)	N/A	0.118	0.439	0.075	1.442	2.415	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	66	89	147	1008	0	194
normalized size	1	1.	1.4	1.89	3.13	21.45	0.	4.13
time (sec)	N/A	0.054	0.219	0.077	1.368	2.295	0.	1.236

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	32	76	85	0	66
normalized size	1	1.	1.	1.45	3.45	3.86	0.	3.
time (sec)	N/A	0.022	0.013	0.013	1.11	2.198	0.	1.161

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	23.097	0.086	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	20.455	0.086	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	8.15	0.036	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	228	177	176	2545	0	0
normalized size	1	1.	2.75	2.13	2.12	30.66	0.	0.
time (sec)	N/A	0.163	6.129	0.032	1.537	2.351	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	72	144	979	0	188
normalized size	1	1.	1.31	1.71	3.43	23.31	0.	4.48
time (sec)	N/A	0.058	0.115	0.03	1.529	2.461	0.	1.175

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	176	267	0	248
normalized size	1	1.	1.	1.43	5.87	8.9	0.	8.27
time (sec)	N/A	0.03	0.073	0.028	1.19	2.213	0.	1.169

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	232	0	36
normalized size	1	1.	1.	0.93	2.27	15.47	0.	2.4
time (sec)	N/A	0.021	0.011	0.007	1.13	2.293	0.	1.227

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	15.019	0.039	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.236	19.786	0.069	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	23.88	0.069	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	280	340	354	4602	0	0
normalized size	1	1.	1.39	1.69	1.76	22.9	0.	0.
time (sec)	N/A	0.36	6.85	0.078	1.628	2.481	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	222	210	266	3370	0	0
normalized size	1	1.	1.8	1.71	2.16	27.4	0.	0.
time (sec)	N/A	0.236	4.162	0.072	1.591	2.578	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	144	156	167	2294	0	0
normalized size	1	1.	1.76	1.9	2.04	27.98	0.	0.
time (sec)	N/A	0.125	2.071	0.064	1.599	2.37	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	45	113	1088	0	122
normalized size	1	1.	1.68	1.32	3.32	32.	0.	3.59
time (sec)	N/A	0.033	0.033	0.017	1.232	2.346	0.	1.165

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	52.279	0.155	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	44.293	0.239	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	13.982	0.091	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	390	375	408	5023	0	0
normalized size	1	1.	2.18	2.09	2.28	28.06	0.	0.
time (sec)	N/A	0.337	3.335	0.091	1.43	2.707	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	295	246	305	3729	0	0
normalized size	1	1.	2.59	2.16	2.68	32.71	0.	0.
time (sec)	N/A	0.212	2.225	0.087	1.376	2.823	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	232	164	201	2569	0	0
normalized size	1	1.	2.83	2.	2.45	31.33	0.	0.
time (sec)	N/A	0.122	6.123	0.083	1.36	2.527	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	107	930	0	95
normalized size	1	1.	1.26	0.96	3.96	34.44	0.	3.52
time (sec)	N/A	0.018	0.071	0.019	1.079	2.537	0.	1.167

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.727	0.228	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.422	0.234	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	9.32	0.024	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	150	241	274	1323	0	0
normalized size	1	1.	1.01	1.63	1.85	8.94	0.	0.
time (sec)	N/A	0.15	4.009	0.082	1.163	2.637	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	108	186	200	1018	0	0
normalized size	1	1.	1.11	1.92	2.06	10.49	0.	0.
time (sec)	N/A	0.106	4.119	0.043	1.119	2.527	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	110	125	117	686	0	0
normalized size	1	1.	1.9	2.16	2.02	11.83	0.	0.
time (sec)	N/A	0.056	0.079	0.039	1.143	2.458	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	68	154	0	47
normalized size	1	1.	2.82	1.09	6.18	14.	0.	4.27
time (sec)	N/A	0.013	0.013	0.	1.64	2.29	0.	1.199

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	17.162	0.073	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	15.66	0.064	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	21.087	0.035	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	282	0	0	3553	0	0
normalized size	1	1.	1.25	0.	0.	15.72	0.	0.
time (sec)	N/A	0.337	0.378	0.521	0.	2.896	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	225	0	0	2657	0	0
normalized size	1	1.	1.54	0.	0.	18.2	0.	0.
time (sec)	N/A	0.227	0.642	0.432	0.	2.721	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	106	95	122	1166	0	0
normalized size	1	1.	1.58	1.42	1.82	17.4	0.	0.
time (sec)	N/A	0.114	0.283	0.085	1.725	2.506	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	23	82	462	0	95
normalized size	1	1.	1.13	1.	3.57	20.09	0.	4.13
time (sec)	N/A	0.026	0.029	0.013	1.113	2.319	0.	1.191

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	33.609	0.256	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	24.701	0.314	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.475	18.337	0.041	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	462	359	444	8806	0	0
normalized size	1	1.	1.92	1.5	1.85	36.69	0.	0.
time (sec)	N/A	0.431	7.317	0.088	1.202	3.187	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	362	256	309	6546	0	0
normalized size	1	1.	2.45	1.73	2.09	44.23	0.	0.
time (sec)	N/A	0.251	3.877	0.051	1.192	2.891	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	139	166	192	4325	0	0
normalized size	1	1.	1.46	1.75	2.02	45.53	0.	0.
time (sec)	N/A	0.12	0.414	0.04	1.282	2.708	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	119	1035	0	135
normalized size	1	1.	1.33	0.96	4.41	38.33	0.	5.
time (sec)	N/A	0.026	0.031	0.	1.608	2.323	0.	1.173

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	54.798	0.41	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	28.096	0.53	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	21.035	0.033	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	333	0	0	3559	0	0
normalized size	1	1.	1.41	0.	0.	15.02	0.	0.
time (sec)	N/A	0.344	1.768	0.454	0.	2.802	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	312	0	0	2685	0	0
normalized size	1	1.	1.99	0.	0.	17.1	0.	0.
time (sec)	N/A	0.234	1.85	0.371	0.	2.765	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	112	179	0	1624	0	0
normalized size	1	1.	1.42	2.27	0.	20.56	0.	0.
time (sec)	N/A	0.112	0.805	0.041	0.	2.678	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	58	305	0	78
normalized size	1	1.	1.21	1.12	2.42	12.71	0.	3.25
time (sec)	N/A	0.025	0.016	0.015	1.717	2.347	0.	1.192

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	21.954	0.25	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	24.505	0.322	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.489	7.172	0.032	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	284	263	243	5019	0	0
normalized size	1	1.	3.34	3.09	2.86	59.05	0.	0.
time (sec)	N/A	0.238	5.738	0.052	1.174	2.829	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	216	199	159	3605	0	0
normalized size	1	1.	3.38	3.11	2.48	56.33	0.	0.
time (sec)	N/A	0.167	4.067	0.041	1.223	2.822	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	62	117	788	0	97
normalized size	1	1.	0.87	2.07	3.9	26.27	0.	3.23
time (sec)	N/A	0.057	0.145	0.037	1.119	2.276	0.	1.162

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	24	213	0	24
normalized size	1	1.	0.57	1.39	1.04	9.26	0.	1.04
time (sec)	N/A	0.033	0.01	0.	1.077	2.251	0.	1.172

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	23.573	0.093	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	20.432	0.057	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.578	23.053	0.043	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	397	0	0	10452	0	0
normalized size	1	1.	1.93	0.	0.	50.74	0.	0.
time (sec)	N/A	0.434	7.508	0.441	0.	3.08	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	133	232	0	6375	0	0
normalized size	1	1.	1.11	1.93	0.	53.12	0.	0.
time (sec)	N/A	0.166	1.191	0.082	0.	2.74	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	52	122	1401	0	140
normalized size	1	1.	0.59	1.06	2.49	28.59	0.	2.86
time (sec)	N/A	0.042	0.016	0.	1.674	2.122	0.	1.158

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	40.461	0.385	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	33.509	0.507	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.453	19.443	0.04	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	490	417	475	8660	0	0
normalized size	1	1.	2.04	1.74	1.98	36.08	0.	0.
time (sec)	N/A	0.424	7.49	0.056	1.227	3.008	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	369	266	328	6546	0	0
normalized size	1	1.	2.49	1.8	2.22	44.23	0.	0.
time (sec)	N/A	0.232	4.508	0.05	1.178	2.624	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	137	170	196	4327	0	0
normalized size	1	1.	1.44	1.79	2.06	45.55	0.	0.
time (sec)	N/A	0.121	0.657	0.043	1.128	2.346	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	123	1037	0	135
normalized size	1	1.	1.21	0.96	4.39	37.04	0.	4.82
time (sec)	N/A	0.027	0.043	0.018	1.59	1.937	0.	1.169

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	54.769	0.408	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	26.442	0.5	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.601	21.15	0.043	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	433	0	0	14033	0	0
normalized size	1	1.	1.37	0.	0.	44.27	0.	0.
time (sec)	N/A	1.177	8.268	0.441	0.	3.409	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	441	0	0	10167	0	0
normalized size	1	1.	2.24	0.	0.	51.61	0.	0.
time (sec)	N/A	0.489	7.95	0.291	0.	2.565	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	168	148	224	4531	0	0
normalized size	1	1.	1.54	1.36	2.06	41.57	0.	0.
time (sec)	N/A	0.165	3.053	0.05	1.715	2.042	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	68	43	143	1935	0	154
normalized size	1	1.	1.39	0.88	2.92	39.49	0.	3.14
time (sec)	N/A	0.048	0.035	0.019	1.144	1.877	0.	1.154

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	64.212	0.39	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.333	43.584	0.498	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.613	24.146	0.043	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	274	445	514	17071	0	0
normalized size	1	1.	1.14	1.85	2.14	71.13	0.	0.
time (sec)	N/A	0.304	7.511	0.064	1.168	2.989	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	192	299	369	12081	0	0
normalized size	1	1.	1.29	2.01	2.48	81.08	0.	0.
time (sec)	N/A	0.198	6.369	0.057	1.187	2.63	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	148	197	221	8173	0	0
normalized size	1	1.	1.63	2.16	2.43	89.81	0.	0.
time (sec)	N/A	0.11	1.387	0.056	1.159	2.318	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	47	48	138	2102	0	136
normalized size	1	1.	1.09	1.12	3.21	48.88	0.	3.16
time (sec)	N/A	0.044	0.011	0.023	1.57	1.85	0.	1.137

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	58.834	0.42	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	42.161	0.521	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.344	0.028	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	142	0	0	0	0	0
normalized size	1	1.	2.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	1.844	0.026	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.159	0.03	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	190	250	0	0	0	0
normalized size	1	1.	5.14	6.76	0.	0.	0.	0.
time (sec)	N/A	0.03	1.468	0.066	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.163	0.026	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.191	0.026	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.234	0.026	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	69	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.279	0.026	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	69	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.326	0.033	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.24	0.033	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	57	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.196	0.034	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.138	0.033	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	100	250	0	0	0	0
normalized size	1	1.	1.75	4.39	0.	0.	0.	0.
time (sec)	N/A	0.04	1.075	0.053	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.159	0.033	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	125	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.134	0.033	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.273	0.032	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.31	0.027	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	143	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	2.164	0.028	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	77	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.2	0.03	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	182	229	0	0	0	0
normalized size	1	1.	2.56	3.23	0.	0.	0.	0.
time (sec)	N/A	0.039	1.69	0.062	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.165	0.026	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.204	0.026	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.271	0.026	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.273	0.027	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.515	0.037	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	75	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.298	0.036	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	70	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.207	0.034	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.145	0.036	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	183	229	0	0	0	0
normalized size	1	1.	2.58	3.23	0.	0.	0.	0.
time (sec)	N/A	0.041	1.015	0.062	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.414	0.033	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	111	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	1.913	0.03	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.397	0.031	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	42	47	201	0	0
normalized size	1	1.	1.	3.23	3.62	15.46	0.	0.
time (sec)	N/A	0.05	0.062	0.145	1.653	2.227	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	0	93	348	0	0
normalized size	1	1.	0.74	0.	3.	11.23	0.	0.
time (sec)	N/A	0.088	0.067	0.105	1.63	2.303	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	29	0	139	883	0	0
normalized size	1	1.	0.58	0.	2.78	17.66	0.	0.
time (sec)	N/A	0.118	0.113	0.103	1.752	2.396	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	73	201	0	0
normalized size	1	1.	2.69	3.23	5.62	15.46	0.	0.
time (sec)	N/A	0.053	0.081	0.142	1.634	2.417	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	147	348	0	0
normalized size	1	1.	0.68	0.	4.74	11.23	0.	0.
time (sec)	N/A	0.097	0.049	0.103	1.664	2.327	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	220	883	0	0
normalized size	1	1.	0.88	0.	4.4	17.66	0.	0.
time (sec)	N/A	0.132	0.312	0.106	1.691	2.462	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	60	120	0	458	770	117
normalized size	1	1.	1.15	2.31	0.	8.81	14.81	2.25
time (sec)	N/A	0.132	0.106	0.025	0.	2.626	94.037	1.152

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	119	0	458	772	119
normalized size	1	1.	1.17	2.25	0.	8.64	14.57	2.25
time (sec)	N/A	0.133	0.106	0.023	0.	2.569	94.661	1.166

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	127	0	717	0	81
normalized size	1	1.	0.98	2.23	0.	12.58	0.	1.42
time (sec)	N/A	0.133	0.094	0.018	0.	2.526	0.	1.172

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	154	0	716	0	84
normalized size	1	1.	0.95	2.61	0.	12.14	0.	1.42
time (sec)	N/A	0.14	0.095	0.021	0.	2.497	0.	1.15

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	0	396	0	130
normalized size	1	1.	1.	2.32	0.	15.84	0.	5.2
time (sec)	N/A	0.058	0.169	0.175	0.	2.405	0.	1.194

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	55	0	394	0	135
normalized size	1	1.	1.	2.2	0.	15.76	0.	5.4
time (sec)	N/A	0.058	0.123	0.062	0.	2.431	0.	1.214

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	89	0	643	0	72
normalized size	1	1.	1.02	1.44	0.	10.37	0.	1.16
time (sec)	N/A	0.157	0.124	0.04	0.	5.789	0.	1.151

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	86	0	482	0	122
normalized size	1	1.	1.16	1.48	0.	8.31	0.	2.1
time (sec)	N/A	0.168	0.136	0.032	0.	5.794	0.	1.171

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	86	220	0	55
normalized size	1	1.	1.26	2.84	4.53	11.58	0.	2.89
time (sec)	N/A	0.042	0.031	0.02	1.612	2.423	0.	1.154

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	34	19	54	41	19
normalized size	1	1.	1.	4.25	2.38	6.75	5.12	2.38
time (sec)	N/A	0.042	0.015	0.024	1.064	2.175	1.713	1.168

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	28	19	54	12	19
normalized size	1	1.	1.	3.5	2.38	6.75	1.5	2.38
time (sec)	N/A	0.043	0.006	0.022	1.08	2.375	1.589	1.165

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	102	138	220	61	51
normalized size	1	1.	1.26	5.37	7.26	11.58	3.21	2.68
time (sec)	N/A	0.039	0.03	0.014	1.612	2.28	5.584	1.157

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	127	112	0	1480	0	96
normalized size	1	1.	1.72	1.51	0.	20.	0.	1.3
time (sec)	N/A	0.245	0.218	0.04	0.	6.237	0.	1.137

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	125	112	0	1037	0	147
normalized size	1	1.	1.81	1.62	0.	15.03	0.	2.13
time (sec)	N/A	0.263	0.41	0.036	0.	6.54	0.	1.18

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	31	8	31
normalized size	1	1.	1.	1.11	1.33	3.44	0.89	3.44
time (sec)	N/A	0.009	0.004	0.001	1.02	2.435	0.129	1.124

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	37	62	126	78	100
normalized size	1	1.	0.97	1.	1.68	3.41	2.11	2.7
time (sec)	N/A	0.018	0.054	0.02	1.048	2.184	0.244	1.152

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	63	68	93	261	66	181
normalized size	1	1.	1.8	1.94	2.66	7.46	1.89	5.17
time (sec)	N/A	0.025	0.127	0.025	1.017	2.381	0.438	1.14

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	87	118	139	425	265	281
normalized size	1	1.	1.21	1.64	1.93	5.9	3.68	3.9
time (sec)	N/A	0.037	0.145	0.027	1.078	2.35	0.946	1.148

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	133	160	258	747	172	464
normalized size	1	1.	2.18	2.62	4.23	12.25	2.82	7.61
time (sec)	N/A	0.045	0.225	0.056	1.161	2.462	1.655	1.161

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	423	119	47
normalized size	1	1.	1.21	1.03	0.	11.13	3.13	1.24
time (sec)	N/A	0.029	0.044	0.036	0.	2.383	6.542	1.19

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	29	39	162	206	35
normalized size	1	1.	1.	1.71	2.29	9.53	12.12	2.06
time (sec)	N/A	0.015	0.024	0.056	1.235	2.363	151.74	1.174

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	96	167	0	3501	0	119
normalized size	1	1.	1.25	2.17	0.	45.47	0.	1.55
time (sec)	N/A	0.047	0.478	0.066	0.	2.648	0.	1.147

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	87	672	1234	0	72
normalized size	1	1.	0.96	1.3	10.03	18.42	0.	1.07
time (sec)	N/A	0.032	0.133	0.081	1.125	2.305	0.	1.144

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	147	462	0	15709	0	319
normalized size	1	1.	1.31	4.12	0.	140.26	0.	2.85
time (sec)	N/A	0.069	0.985	0.135	0.	3.008	0.	1.164

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	206	33	0	0	0	0
normalized size	1	1.	3.17	0.51	0.	0.	0.	0.
time (sec)	N/A	0.027	0.703	0.129	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	92	171	0	0	0	0
normalized size	1	1.	0.89	1.66	0.	0.	0.	0.
time (sec)	N/A	0.053	0.574	0.238	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	193	51	0	0	0	0
normalized size	1	1.	1.87	0.5	0.	0.	0.	0.
time (sec)	N/A	0.05	0.814	0.152	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	97	0	0	0	0
normalized size	1	1.	1.25	1.49	0.	0.	0.	0.
time (sec)	N/A	0.027	0.093	0.19	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	148	33	0	0	0	0
normalized size	1	1.	1.32	0.29	0.	0.	0.	0.
time (sec)	N/A	0.051	0.447	0.12	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	37	0	0	0	0
normalized size	1	1.	1.15	0.32	0.	0.	0.	0.
time (sec)	N/A	0.048	0.558	0.144	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	45	19	31	53	29	76
normalized size	1	1.	1.96	0.83	1.35	2.3	1.26	3.3
time (sec)	N/A	0.015	0.017	0.009	1.068	1.978	0.254	1.142

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	24	119	109	44	23
normalized size	1	1.	0.96	0.92	4.58	4.19	1.69	0.88
time (sec)	N/A	0.016	0.051	0.003	1.061	1.99	0.485	1.132

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	24	197	163	83	23
normalized size	1	1.	0.96	0.92	7.58	6.27	3.19	0.88
time (sec)	N/A	0.015	0.076	0.003	1.065	1.952	0.877	1.154

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	24	93	36	0
normalized size	1	1.	0.92	1.04	0.92	3.58	1.38	0.
time (sec)	N/A	0.015	0.074	0.009	1.024	2.029	0.284	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	23	59	34	23
normalized size	1	1.	1.	1.	0.96	2.46	1.42	0.96
time (sec)	N/A	0.016	0.038	0.003	1.088	1.982	0.625	1.122

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	23	124	66	23
normalized size	1	1.	1.	0.92	0.88	4.77	2.54	0.88
time (sec)	N/A	0.016	0.043	0.003	1.153	1.985	1.218	1.135

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	23	181	90	23
normalized size	1	1.	1.	0.92	0.88	6.96	3.46	0.88
time (sec)	N/A	0.016	0.046	0.	1.07	1.942	2.208	1.171

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	23	61	0	23
normalized size	1	1.	0.92	0.96	0.88	2.35	0.	0.88
time (sec)	N/A	0.016	0.022	0.002	1.016	2.003	0.	1.16

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	23	113	0	23
normalized size	1	1.	0.92	0.96	0.88	4.35	0.	0.88
time (sec)	N/A	0.017	0.034	0.003	1.073	1.955	0.	1.149

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	21	32	54	29	76
normalized size	1	1.	1.96	0.88	1.33	2.25	1.21	3.17
time (sec)	N/A	0.015	0.019	0.002	0.988	1.841	0.198	1.134

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	120	113	44	23
normalized size	1	1.	1.	0.96	4.44	4.19	1.63	0.85
time (sec)	N/A	0.015	0.028	0.	1.073	1.955	0.267	1.127

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	198	165	83	23
normalized size	1	1.	1.	0.96	7.33	6.11	3.07	0.85
time (sec)	N/A	0.015	0.023	0.	1.065	2.019	0.676	1.152

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	27	97	37	0
normalized size	1	1.	0.96	1.04	0.96	3.46	1.32	0.
time (sec)	N/A	0.016	0.045	0.002	1.054	2.168	0.238	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	25	18	53	32	18
normalized size	1	1.	0.92	1.04	0.75	2.21	1.33	0.75
time (sec)	N/A	0.016	0.007	0.	1.037	2.248	0.476	1.123

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	23	109	65	23
normalized size	1	1.	1.	0.96	0.85	4.04	2.41	0.85
time (sec)	N/A	0.015	0.044	0.001	1.069	2.179	0.923	1.113

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	23	158	88	23
normalized size	1	1.	1.	0.96	0.85	5.85	3.26	0.85
time (sec)	N/A	0.016	0.068	0.001	1.084	2.366	1.74	1.139

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	23	62	0	23
normalized size	1	1.	0.96	0.96	0.85	2.3	0.	0.85
time (sec)	N/A	0.016	0.022	0.	1.038	2.265	0.	1.155

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	23	109	0	23
normalized size	1	1.	0.96	0.96	0.85	4.04	0.	0.85
time (sec)	N/A	0.017	0.029	0.001	1.026	2.35	0.	1.16

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	355	223	377	5216	0	324
normalized size	1	1.	2.86	1.8	3.04	42.06	0.	2.61
time (sec)	N/A	0.185	1.875	0.046	1.689	2.811	0.	1.137

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	123	284	505	0	149
normalized size	1	1.	0.79	1.23	2.84	5.05	0.	1.49
time (sec)	N/A	0.209	0.176	0.032	1.06	2.31	0.	1.137

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	194	79	162	1359	0	158
normalized size	1	1.	3.34	1.36	2.79	23.43	0.	2.72
time (sec)	N/A	0.107	1.869	0.03	1.574	2.477	0.	1.145

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	36	58	95	0	42
normalized size	1	1.	0.9	1.24	2.	3.28	0.	1.45
time (sec)	N/A	0.063	0.048	0.014	1.057	2.265	0.	1.137

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	16	12	15	104	0	28
normalized size	1	1.	1.45	1.09	1.36	9.45	0.	2.55
time (sec)	N/A	0.01	0.004	0.003	1.044	2.413	0.	1.113

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	50	38	72	32	30
normalized size	1	1.	1.	4.55	3.45	6.55	2.91	2.73
time (sec)	N/A	0.039	0.006	0.046	1.022	2.288	0.654	1.156

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	659	119	0	933	0	131
normalized size	1	1.	10.63	1.92	0.	15.05	0.	2.11
time (sec)	N/A	0.127	3.684	0.072	0.	2.401	0.	1.195

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	241	158	1362	0	101
normalized size	1	1.	0.88	5.02	3.29	28.38	0.	2.1
time (sec)	N/A	0.08	0.133	0.088	1.088	2.568	0.	1.159

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	3458	972	0	6724	0	360
normalized size	1	1.	23.68	6.66	0.	46.05	0.	2.47
time (sec)	N/A	0.365	6.456	0.109	0.	3.082	0.	1.188

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	721	401	6251	2166	205
normalized size	1	1.	0.87	7.59	4.22	65.8	22.8	2.16
time (sec)	N/A	0.116	0.229	0.128	1.204	2.91	28.256	1.189

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	62	82	317	273	0	49
normalized size	1	1.	1.55	2.05	7.92	6.82	0.	1.22
time (sec)	N/A	0.061	0.111	0.046	1.6	2.442	0.	1.156

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	89	244	153	0	30
normalized size	1	1.	1.95	2.34	6.42	4.03	0.	0.79
time (sec)	N/A	0.113	0.127	0.054	1.056	2.326	0.	1.146

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	39	45	99	142	0	28
normalized size	1	1.	1.5	1.73	3.81	5.46	0.	1.08
time (sec)	N/A	0.054	0.027	0.029	1.605	2.475	0.	1.12

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	14	26	34	43	0	16
normalized size	1	1.	0.7	1.3	1.7	2.15	0.	0.8
time (sec)	N/A	0.075	0.005	0.015	1.05	2.295	0.	1.113

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	11	12	34	0	24
normalized size	1	1.	1.55	1.	1.09	3.09	0.	2.18
time (sec)	N/A	0.008	0.004	0.	1.014	2.403	0.	1.156

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	33	20	32	22	18
normalized size	1	1.	1.31	2.54	1.54	2.46	1.69	1.38
time (sec)	N/A	0.029	0.017	0.044	1.051	2.48	0.667	1.135

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	29	19	43	0	16
normalized size	1	1.	1.55	1.45	0.95	2.15	0.	0.8
time (sec)	N/A	0.044	0.027	0.051	1.065	2.589	0.	1.146

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	40	56	45	142	513	36
normalized size	1	1.	1.43	2.	1.61	5.07	18.32	1.29
time (sec)	N/A	0.052	0.038	0.072	1.054	2.285	30.051	1.156

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	41	54	154	0	30
normalized size	1	1.	1.97	1.08	1.42	4.05	0.	0.79
time (sec)	N/A	0.077	0.075	0.116	1.092	2.054	0.	1.139

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	45	68	81	274	0	54
normalized size	1	1.	1.07	1.62	1.93	6.52	0.	1.29
time (sec)	N/A	0.055	0.099	0.106	1.091	2.141	0.	1.162

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	82	317	274	0	49
normalized size	1	1.	1.48	1.95	7.55	6.52	0.	1.17
time (sec)	N/A	0.059	0.058	0.048	1.648	2.092	0.	1.138

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	89	244	154	0	30
normalized size	1	1.	1.97	2.34	6.42	4.05	0.	0.79
time (sec)	N/A	0.109	0.054	0.054	1.067	2.098	0.	1.132

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	45	99	142	0	28
normalized size	1	1.	1.39	1.61	3.54	5.07	0.	1.
time (sec)	N/A	0.06	0.028	0.029	1.58	2.091	0.	1.151

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	14	26	34	43	0	16
normalized size	1	1.	0.7	1.3	1.7	2.15	0.	0.8
time (sec)	N/A	0.074	0.005	0.017	0.997	2.015	0.	1.123

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	11	12	32	0	24
normalized size	1	1.	1.55	1.	1.09	2.91	0.	2.18
time (sec)	N/A	0.008	0.004	0.002	0.991	2.089	0.	1.138

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	33	20	34	22	18
normalized size	1	1.	1.55	3.	1.82	3.09	2.	1.64
time (sec)	N/A	0.03	0.017	0.045	1.05	2.053	0.663	1.175

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	29	19	43	0	16
normalized size	1	1.	1.55	1.45	0.95	2.15	0.	0.8
time (sec)	N/A	0.047	0.029	0.054	1.04	2.028	0.	1.132

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	56	45	142	513	36
normalized size	1	1.	1.04	2.15	1.73	5.46	19.73	1.38
time (sec)	N/A	0.052	0.036	0.078	1.034	2.075	29.503	1.145

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	41	54	153	0	30
normalized size	1	1.	1.95	1.08	1.42	4.03	0.	0.79
time (sec)	N/A	0.077	0.064	0.118	1.079	2.105	0.	1.15

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	68	81	273	0	54
normalized size	1	1.	1.12	1.7	2.02	6.82	0.	1.35
time (sec)	N/A	0.055	0.098	0.098	1.082	2.167	0.	1.154

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	244	223	446	6710	0	316
normalized size	1	1.	1.97	1.8	3.6	54.11	0.	2.55
time (sec)	N/A	0.243	0.469	0.028	1.073	2.472	0.	1.18

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	95	123	289	497	0	151
normalized size	1	1.	0.94	1.22	2.86	4.92	0.	1.5
time (sec)	N/A	0.239	0.266	0.02	1.042	2.053	0.	1.224

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	99	79	205	1712	0	155
normalized size	1	1.	1.68	1.34	3.47	29.02	0.	2.63
time (sec)	N/A	0.123	0.241	0.02	1.113	2.243	0.	1.201

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	36	61	95	0	39
normalized size	1	1.	0.85	1.33	2.26	3.52	0.	1.44
time (sec)	N/A	0.068	0.121	0.01	1.044	1.982	0.	1.133

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	15	14	18	108	0	45
normalized size	1	1.	1.25	1.17	1.5	9.	0.	3.75
time (sec)	N/A	0.01	0.004	0.002	1.054	2.103	0.	1.141

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	35	72	0	26
normalized size	1	1.	1.	4.64	3.18	6.55	0.	2.36
time (sec)	N/A	0.046	0.018	0.034	1.037	2.069	0.	1.129

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	95	0	1647	0	92
normalized size	1	1.	0.91	1.42	0.	24.58	0.	1.37
time (sec)	N/A	0.132	0.333	0.05	0.	2.192	0.	1.171

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	77	144	150	1362	0	89
normalized size	1	1.	1.54	2.88	3.	27.24	0.	1.78
time (sec)	N/A	0.107	0.108	0.055	1.068	2.171	0.	1.163

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	507	0	13226	0	327
normalized size	1	1.	0.94	3.19	0.	83.18	0.	2.06
time (sec)	N/A	0.373	0.434	0.066	0.	2.958	0.	1.154

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	138	309	385	6251	0	182
normalized size	1	1.	1.41	3.15	3.93	63.79	0.	1.86
time (sec)	N/A	0.156	0.305	0.07	1.247	2.57	0.	1.203

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	75	319	925	0	45
normalized size	1	1.	1.89	2.68	11.39	33.04	0.	1.61
time (sec)	N/A	0.067	0.087	0.021	1.276	2.082	0.	1.127

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	57	247	234	0	30
normalized size	1	1.	1.	1.9	8.23	7.8	0.	1.
time (sec)	N/A	0.114	0.048	0.033	1.185	1.992	0.	1.112

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	41	39	89	336	0	30
normalized size	1	1.	2.28	2.17	4.94	18.67	0.	1.67
time (sec)	N/A	0.056	0.051	0.018	1.056	2.023	0.	1.134

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	10	21	34	77	0	14
normalized size	1	1.	0.71	1.5	2.43	5.5	0.	1.
time (sec)	N/A	0.082	0.026	0.009	1.05	1.969	0.	1.109

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	11	10	12	47	0	34
normalized size	1	1.	1.22	1.11	1.33	5.22	0.	3.78
time (sec)	N/A	0.008	0.003	0.001	1.043	2.054	0.	1.139

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	9	20	15	47	0	15
normalized size	1	1.	1.8	4.	3.	9.4	0.	3.
time (sec)	N/A	0.032	0.019	0.027	1.058	2.031	0.	1.121

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	24	16	77	0	14
normalized size	1	1.	0.83	2.	1.33	6.42	0.	1.17
time (sec)	N/A	0.049	0.02	0.033	1.118	2.012	0.	1.125

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	28	42	336	0	28
normalized size	1	1.	1.29	2.	3.	24.	0.	2.
time (sec)	N/A	0.057	0.02	0.034	1.178	2.103	0.	1.127

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	51	234	0	30
normalized size	1	1.	1.15	1.23	1.96	9.	0.	1.15
time (sec)	N/A	0.085	0.021	0.06	1.283	1.975	0.	1.119

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	32	36	70	925	0	41
normalized size	1	1.	1.45	1.64	3.18	42.05	0.	1.86
time (sec)	N/A	0.063	0.022	0.041	1.103	2.003	0.	1.112

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	55	77	321	923	0	38
normalized size	1	1.	2.29	3.21	13.38	38.46	0.	1.58
time (sec)	N/A	0.063	0.085	0.022	1.231	2.05	0.	1.128

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	57	247	234	0	30
normalized size	1	1.	1.15	2.19	9.5	9.	0.	1.15
time (sec)	N/A	0.117	0.007	0.031	1.19	1.896	0.	1.159

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	43	41	92	335	0	26
normalized size	1	1.	2.69	2.56	5.75	20.94	0.	1.62
time (sec)	N/A	0.058	0.052	0.02	1.192	2.077	0.	1.12

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	21	34	77	0	14
normalized size	1	1.	1.5	1.75	2.83	6.42	0.	1.17
time (sec)	N/A	0.084	0.005	0.01	1.221	1.937	0.	1.122

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	13	12	15	46	0	34
normalized size	1	1.	1.18	1.09	1.36	4.18	0.	3.09
time (sec)	N/A	0.008	0.005	0.003	1.151	2.135	0.	1.127

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	23	18	46	0	14
normalized size	1	1.	1.	2.56	2.	5.11	0.	1.56
time (sec)	N/A	0.034	0.021	0.03	1.224	2.128	0.	1.138

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	26	16	77	0	14
normalized size	1	1.	1.71	1.86	1.14	5.5	0.	1.
time (sec)	N/A	0.051	0.009	0.036	1.24	2.212	0.	1.131

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	29	47	335	0	27
normalized size	1	1.	0.9	1.45	2.35	16.75	0.	1.35
time (sec)	N/A	0.059	0.023	0.04	1.153	2.242	0.	1.115

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	34	51	234	0	30
normalized size	1	1.	0.93	1.13	1.7	7.8	0.	1.
time (sec)	N/A	0.083	0.008	0.068	1.306	2.385	0.	1.168

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	32	37	78	923	0	42
normalized size	1	1.	1.07	1.23	2.6	30.77	0.	1.4
time (sec)	N/A	0.062	0.024	0.046	1.134	1.939	0.	1.142

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	11	236	0	32
normalized size	1	1.	1.25	1.12	1.38	29.5	0.	4.
time (sec)	N/A	0.008	0.004	0.001	1.181	1.672	0.	1.136

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	35	109	0	53
normalized size	1	1.	0.82	0.68	1.59	4.95	0.	2.41
time (sec)	N/A	0.025	0.004	0.011	1.176	1.765	0.	1.147

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	45	28	90	2090	0	84
normalized size	1	1.	1.32	0.82	2.65	61.47	0.	2.47
time (sec)	N/A	0.052	0.068	0.017	1.136	1.896	0.	1.125

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	73	201	0	0
normalized size	1	1.	2.69	3.23	5.62	15.46	0.	0.
time (sec)	N/A	0.07	0.069	0.121	1.811	1.711	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	147	348	0	0
normalized size	1	1.	0.68	0.	4.74	11.23	0.	0.
time (sec)	N/A	0.122	0.05	0.079	1.848	1.907	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	220	883	0	0
normalized size	1	1.	0.88	0.	4.4	17.66	0.	0.
time (sec)	N/A	0.157	0.313	0.079	1.713	1.87	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	11	166	0	22
normalized size	1	1.	1.75	1.12	1.38	20.75	0.	2.75
time (sec)	N/A	0.006	0.004	0.002	1.18	1.781	0.	1.164

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	35	101	0	50
normalized size	1	1.	0.73	0.59	1.59	4.59	0.	2.27
time (sec)	N/A	0.025	0.027	0.015	1.198	1.911	0.	1.145

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	29	76	1648	0	89
normalized size	1	1.	1.09	0.85	2.24	48.47	0.	2.62
time (sec)	N/A	0.048	0.028	0.018	1.751	2.073	0.	1.152

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	53	208	0	0
normalized size	1	1.	1.	3.07	3.79	14.86	0.	0.
time (sec)	N/A	0.057	0.052	0.185	1.79	1.826	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	24	0	104	354	0	0
normalized size	1	1.	0.73	0.	3.15	10.73	0.	0.
time (sec)	N/A	0.097	0.083	0.129	1.958	1.847	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	30	0	155	887	0	0
normalized size	1	1.	0.57	0.	2.92	16.74	0.	0.
time (sec)	N/A	0.126	0.129	0.134	1.763	1.88	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	24	35	17	53	386	0	58
normalized size	1	1.33	1.94	0.94	2.94	21.44	0.	3.22
time (sec)	N/A	0.075	0.026	0.03	1.136	1.819	0.	1.153

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	24	50	17	54	386	0	58
normalized size	1	1.2	2.5	0.85	2.7	19.3	0.	2.9
time (sec)	N/A	0.073	0.043	0.032	1.221	1.697	0.	1.113

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	70	54	109	146	58
normalized size	1	1.	0.74	1.79	1.38	2.79	3.74	1.49
time (sec)	N/A	0.067	0.061	0.037	1.183	1.776	0.723	1.141

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	89	93	0	1106	0	82
normalized size	1	1.	1.2	1.26	0.	14.95	0.	1.11
time (sec)	N/A	0.083	0.193	0.041	0.	1.88	0.	1.143

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	175	117	818	0	154
normalized size	1	1.	0.74	1.73	1.16	8.1	0.	1.52
time (sec)	N/A	0.133	0.145	0.05	1.142	1.884	0.	1.154

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	71	55	108	146	58
normalized size	1	1.	0.74	1.82	1.41	2.77	3.74	1.49
time (sec)	N/A	0.062	0.042	0.038	1.151	1.832	0.716	1.143

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	80	93	0	1106	0	82
normalized size	1	1.	1.08	1.26	0.	14.95	0.	1.11
time (sec)	N/A	0.079	0.156	0.043	0.	1.833	0.	1.165

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	175	116	818	0	150
normalized size	1	1.	0.74	1.73	1.15	8.1	0.	1.49
time (sec)	N/A	0.116	0.103	0.049	1.202	1.881	0.	1.122

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	564	0	65
normalized size	1	1.	1.2	1.08	0.	11.28	0.	1.3
time (sec)	N/A	0.096	0.122	0.049	0.	2.025	0.	1.151

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	53	0	675	0	81
normalized size	1	1.	1.16	1.04	0.	13.24	0.	1.59
time (sec)	N/A	0.1	0.08	0.05	0.	2.044	0.	1.126

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	125	99	0	1473	0	97
normalized size	1	1.	1.89	1.5	0.	22.32	0.	1.47
time (sec)	N/A	0.063	0.189	0.052	0.	1.9	0.	1.13

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	146	140	826	2574	153
normalized size	1	1.	0.9	2.15	2.06	12.15	37.85	2.25
time (sec)	N/A	0.138	0.23	0.074	1.258	1.829	143.99	1.131

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	301	205	164	0	3664	0	235
normalized size	1	1.54	1.05	0.84	0.	18.79	0.	1.21
time (sec)	N/A	1.225	0.439	0.072	0.	2.096	0.	1.152

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	124	98	0	1474	0	97
normalized size	1	1.	1.94	1.53	0.	23.03	0.	1.52
time (sec)	N/A	0.055	0.136	0.056	0.	1.927	0.	1.153

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	149	140	826	2533	154
normalized size	1	1.	0.99	2.22	2.09	12.33	37.81	2.3
time (sec)	N/A	0.126	0.233	0.067	1.255	1.876	141.206	1.156

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	204	167	0	3667	0	235
normalized size	1	1.45	1.53	1.26	0.	27.57	0.	1.77
time (sec)	N/A	0.782	0.349	0.072	0.	2.022	0.	1.15

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	54	31	225	518	0	68
normalized size	1	1.	2.84	1.63	11.84	27.26	0.	3.58
time (sec)	N/A	0.032	0.115	0.068	1.267	1.786	0.	1.168

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	117	404	390	2854	0	339
normalized size	1	1.	1.12	3.88	3.75	27.44	0.	3.26
time (sec)	N/A	0.238	0.646	0.092	1.339	2.03	0.	1.145

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	40	55	225	518	0	65
normalized size	1	1.	2.11	2.89	11.84	27.26	0.	3.42
time (sec)	N/A	0.031	0.058	0.065	1.264	1.724	0.	1.148

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	119	494	394	2853	0	339
normalized size	1	1.	1.14	4.75	3.79	27.43	0.	3.26
time (sec)	N/A	0.201	0.856	0.085	1.34	2.005	0.	1.179

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	1103	0	81
normalized size	1	1.	1.1	1.28	0.	15.32	0.	1.12
time (sec)	N/A	0.089	0.202	0.033	0.	1.942	0.	1.141

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	145	112	826	0	136
normalized size	1	1.	0.72	1.42	1.1	8.1	0.	1.33
time (sec)	N/A	0.16	0.229	0.044	1.214	1.85	0.	1.12

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	166	0	4208	0	220
normalized size	1	1.	1.31	1.21	0.	30.72	0.	1.61
time (sec)	N/A	0.203	1.105	0.049	0.	2.136	0.	1.157

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	146	113	826	0	138
normalized size	1	1.	0.72	1.43	1.11	8.1	0.	1.35
time (sec)	N/A	0.163	0.223	0.04	1.178	1.837	0.	1.148

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	168	0	4230	0	215
normalized size	1	1.	1.47	1.38	0.	34.67	0.	1.76
time (sec)	N/A	0.229	0.979	0.046	0.	2.151	0.	1.152

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	128	321	207	2612	0	269
normalized size	1	1.	0.66	1.65	1.07	13.46	0.	1.39
time (sec)	N/A	0.342	0.602	0.058	1.273	1.941	0.	1.146

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	167	200	0	4208	0	220
normalized size	1	1.	1.22	1.46	0.	30.72	0.	1.61
time (sec)	N/A	0.195	1.134	0.045	0.	2.146	0.	1.157

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	126	322	203	2612	0	273
normalized size	1	1.	0.65	1.66	1.05	13.46	0.	1.41
time (sec)	N/A	0.338	0.57	0.051	1.327	2.231	0.	1.159

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	325	344	0	10982	0	439
normalized size	1	1.	1.53	1.62	0.	51.8	0.	2.07
time (sec)	N/A	0.432	2.218	0.056	0.	3.25	0.	1.181

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	181	144	892	1986	173
normalized size	1	1.	0.65	1.95	1.55	9.59	21.35	1.86
time (sec)	N/A	0.204	0.184	0.063	1.234	2.41	146.537	1.15

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	222	219	0	4070	0	242
normalized size	1	1.	1.35	1.33	0.	24.67	0.	1.47
time (sec)	N/A	0.307	1.039	0.07	0.	2.612	0.	1.169

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	176	253	325	3650	0	321
normalized size	1	1.	0.82	1.18	1.51	16.98	0.	1.49
time (sec)	N/A	0.536	0.951	0.081	1.275	2.69	0.	1.177

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	264	217	0	4068	0	242
normalized size	1	1.	1.62	1.33	0.	24.96	0.	1.48
time (sec)	N/A	0.316	0.872	0.067	0.	2.697	0.	1.151

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	174	286	329	3779	0	313
normalized size	1	1.	0.85	1.4	1.6	18.43	0.	1.53
time (sec)	N/A	0.659	1.864	0.078	1.219	2.695	0.	1.173

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	474	326	0	11158	0	419
normalized size	1	1.	1.82	1.25	0.	42.75	0.	1.61
time (sec)	N/A	1.034	3.772	0.081	0.	3.162	0.	1.192

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	183	253	324	3650	0	324
normalized size	1	1.	0.85	1.18	1.51	16.98	0.	1.51
time (sec)	N/A	0.558	1.2	0.079	1.222	2.639	0.	1.15

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	481	289	0	11158	0	419
normalized size	1	1.	1.86	1.12	0.	43.08	0.	1.62
time (sec)	N/A	0.908	2.056	0.08	0.	3.407	0.	1.155

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	366	398	518	8631	0	518
normalized size	1	1.	1.17	1.27	1.65	27.49	0.	1.65
time (sec)	N/A	1.732	1.379	0.089	1.151	3.164	0.	1.183

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	181	0	644	367	108
normalized size	1	1.	0.98	2.26	0.	8.05	4.59	1.35
time (sec)	N/A	0.082	0.224	0.057	0.	2.497	47.269	1.16

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	155	115	0	1648	0	112
normalized size	1	1.	1.89	1.4	0.	20.1	0.	1.37
time (sec)	N/A	0.079	0.439	0.067	0.	2.425	0.	1.147

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	134	187	0	4243	0	205
normalized size	1	1.	1.09	1.52	0.	34.5	0.	1.67
time (sec)	N/A	0.127	1.129	0.082	0.	2.713	0.	1.149

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	182	0	647	408	108
normalized size	1	1.	0.98	2.28	0.	8.09	5.1	1.35
time (sec)	N/A	0.061	0.171	0.055	0.	2.462	46.995	1.13

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	151	116	0	1648	0	112
normalized size	1	1.	1.94	1.49	0.	21.13	0.	1.44
time (sec)	N/A	0.061	0.281	0.062	0.	2.416	0.	1.143

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	134	214	0	4246	0	205
normalized size	1	1.	1.12	1.78	0.	35.38	0.	1.71
time (sec)	N/A	0.119	1.037	0.094	0.	2.719	0.	1.166

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	17	8	61	8	8
normalized size	1	1.	1.55	1.55	0.73	5.55	0.73	0.73
time (sec)	N/A	0.039	0.003	0.	1.022	2.209	0.376	1.148

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	17	8	68	10	8
normalized size	1	1.	1.55	1.55	0.73	6.18	0.91	0.73
time (sec)	N/A	0.034	0.005	0.002	1.082	2.286	0.332	1.12

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	13	14	35	0	18
normalized size	1	1.	1.07	0.93	1.	2.5	0.	1.29
time (sec)	N/A	0.027	0.034	0.027	1.049	2.323	0.	1.115

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	145	117	143	326	73
normalized size	1	1.	0.81	2.74	2.21	2.7	6.15	1.38
time (sec)	N/A	0.046	0.115	0.057	1.115	2.245	0.883	1.138

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	152	0	1789	0	119
normalized size	1	1.	1.12	1.95	0.	22.94	0.	1.53
time (sec)	N/A	0.072	0.228	0.073	0.	2.218	0.	1.136

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	63	455	556	0	95
normalized size	1	1.	0.99	0.89	6.41	7.83	0.	1.34
time (sec)	N/A	0.069	0.158	0.083	1.112	1.985	0.	1.203

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	253	0	701	643	120
normalized size	1	1.	0.98	2.75	0.	7.62	6.99	1.3
time (sec)	N/A	0.07	0.256	0.058	0.	2.222	80.001	1.135

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	167	0	1897	0	128
normalized size	1	1.	1.2	1.9	0.	21.56	0.	1.45
time (sec)	N/A	0.069	0.261	0.073	0.	2.201	0.	1.203

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	146	228	0	4510	0	247
normalized size	1	1.	1.08	1.69	0.	33.41	0.	1.83
time (sec)	N/A	0.14	0.828	0.087	0.	2.397	0.	1.162

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	116	130	185	421	196	296
normalized size	1	1.	0.97	1.09	1.55	3.54	1.65	2.49
time (sec)	N/A	0.132	0.187	0.028	1.026	1.986	0.723	1.122

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	54	85	171	100	112
normalized size	1	1.	0.92	0.92	1.44	2.9	1.69	1.9
time (sec)	N/A	0.035	0.081	0.023	1.008	1.957	0.352	1.136

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	39	12	35
normalized size	1	1.	1.	1.08	1.33	3.25	1.	2.92
time (sec)	N/A	0.009	0.002	0.002	1.046	1.961	0.165	1.158

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	53	0	660	0	62
normalized size	1	1.	1.06	1.04	0.	12.94	0.	1.22
time (sec)	N/A	0.069	0.073	0.041	0.	2.091	0.	1.167

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	105	191	0	2912	0	150
normalized size	1	1.	1.17	2.12	0.	32.36	0.	1.67
time (sec)	N/A	0.088	0.267	0.061	0.	2.199	0.	1.133

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	183	747	0	15389	0	410
normalized size	1	1.	1.25	5.12	0.	105.4	0.	2.81
time (sec)	N/A	0.165	0.507	0.083	0.	3.186	0.	1.181

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	488	1842	0	0	0	968
normalized size	1	1.	2.22	8.37	0.	0.	0.	4.4
time (sec)	N/A	0.302	0.997	0.118	0.	0.	0.	1.206

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	112	129	185	381	189	251
normalized size	1	1.	1.07	1.23	1.76	3.63	1.8	2.39
time (sec)	N/A	0.116	0.163	0.035	1.029	2.412	1.062	1.118

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	55	85	163	100	109
normalized size	1	1.	0.96	0.96	1.49	2.86	1.75	1.91
time (sec)	N/A	0.035	0.07	0.02	0.981	2.322	0.433	1.146

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	39	12	35
normalized size	1	1.	1.	1.08	1.33	3.25	1.	2.92
time (sec)	N/A	0.009	0.002	0.	0.998	2.329	0.276	1.14

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	14	49	109	17	53
normalized size	1	1.	2.33	0.93	3.27	7.27	1.13	3.53
time (sec)	N/A	0.019	0.037	0.036	1.025	2.296	1.66	1.152

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	58	116	657	0	113
normalized size	1	1.	2.02	1.35	2.7	15.28	0.	2.63
time (sec)	N/A	0.041	0.292	0.063	1.047	2.416	0.	1.137

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	148	138	335	3359	0	277
normalized size	1	1.	1.66	1.55	3.76	37.74	0.	3.11
time (sec)	N/A	0.094	0.495	0.08	1.122	2.828	0.	1.172

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	300	250	657	9150	0	509
normalized size	1	1.	2.14	1.79	4.69	65.36	0.	3.64
time (sec)	N/A	0.213	0.574	0.098	1.215	3.329	0.	1.165

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	208	409	374	3148	626	527
normalized size	1	1.	1.11	2.18	1.99	16.74	3.33	2.8
time (sec)	N/A	0.148	0.493	0.082	1.027	2.831	2.632	1.175

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	134	202	217	1665	298	262
normalized size	1	1.	0.99	1.49	1.6	12.24	2.19	1.93
time (sec)	N/A	0.09	0.257	0.046	1.034	2.646	2.257	1.159

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	72	80	107	656	122	130
normalized size	1	1.	0.8	0.89	1.19	7.29	1.36	1.44
time (sec)	N/A	0.048	0.113	0.033	1.178	2.595	0.497	1.168

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	196	20	49
normalized size	1	1.	1.	0.96	1.25	8.17	0.83	2.04
time (sec)	N/A	0.011	0.008	0.002	1.	2.724	0.269	1.149

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	596	0	235	0	0
normalized size	1	1.	1.06	17.53	0.	6.91	0.	0.
time (sec)	N/A	0.037	0.073	0.254	0.	2.987	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	217	0	1639	0	0
normalized size	1	1.	0.68	2.17	0.	16.39	0.	0.
time (sec)	N/A	0.081	0.144	0.086	0.	3.05	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	184	488	0	6947	0	0
normalized size	1	1.	1.26	3.34	0.	47.58	0.	0.
time (sec)	N/A	0.12	0.379	0.132	0.	3.122	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	425	828	0	15194	0	0
normalized size	1	1.	2.15	4.18	0.	76.74	0.	0.
time (sec)	N/A	0.178	0.794	0.229	0.	4.055	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	3775	1036	0	0	0	0
normalized size	1	1.	12.84	3.52	0.	0.	0.	0.
time (sec)	N/A	0.483	6.335	0.879	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	2292	318	0	0	0	0
normalized size	1	1.	9.2	1.28	0.	0.	0.	0.
time (sec)	N/A	0.27	6.141	0.362	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	1401	314	0	0	0	0
normalized size	1	1.	13.74	3.08	0.	0.	0.	0.
time (sec)	N/A	0.071	6.103	0.373	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	237	248	0	0	0	0
normalized size	1	1.	2.32	2.43	0.	0.	0.	0.
time (sec)	N/A	0.071	0.483	0.244	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	1522	1430	0	0	0	0
normalized size	1	1.	9.76	9.17	0.	0.	0.	0.
time (sec)	N/A	0.098	6.197	0.958	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	2492	6019	0	0	0	0
normalized size	1	1.	7.74	18.69	0.	0.	0.	0.
time (sec)	N/A	0.34	6.234	2.066	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	4093	58437	0	0	0	0
normalized size	1	1.	9.96	142.18	0.	0.	0.	0.
time (sec)	N/A	0.544	6.531	10.031	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	10223	518	2407	2079	0	887
normalized size	1	1.	73.02	3.7	17.19	14.85	0.	6.34
time (sec)	N/A	0.123	74.012	0.638	15.586	2.613	0.	1.455

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	4392	190	864	967	0	409
normalized size	1	1.	47.74	2.07	9.39	10.51	0.	4.45
time (sec)	N/A	0.077	70.947	0.513	3.217	2.428	0.	1.264

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	455	201	207	468	0	140
normalized size	1	1.	12.3	5.43	5.59	12.65	0.	3.78
time (sec)	N/A	0.039	68.272	0.442	1.882	2.394	0.	1.195

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	211	129	0	0	0	0
normalized size	1	1.	2.13	1.3	0.	0.	0.	0.
time (sec)	N/A	0.113	29.113	0.279	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F(-2)	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	0	417	0	0	0	0
normalized size	1	1.	0.	2.69	0.	0.	0.	0.
time (sec)	N/A	0.132	180.002	0.785	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	0	954	0	0	0	0
normalized size	1	1.	0.	4.65	0.	0.	0.	0.
time (sec)	N/A	0.176	180.009	1.046	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	9943	288	2415	2079	0	887
normalized size	1	1.	68.1	1.97	16.54	14.24	0.	6.08
time (sec)	N/A	0.122	75.477	0.574	16.286	2.483	0.	1.469

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	9861	190	869	967	0	408
normalized size	1	1.	102.72	1.98	9.05	10.07	0.	4.25
time (sec)	N/A	0.079	73.353	0.436	3.211	2.539	0.	1.29

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	9771	202	211	468	0	139
normalized size	1	1.	250.54	5.18	5.41	12.	0.	3.56
time (sec)	N/A	0.037	73.33	0.444	2.122	2.611	0.	1.205

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	52609	129	0	0	0	0
normalized size	1	1.	515.77	1.26	0.	0.	0.	0.
time (sec)	N/A	0.094	30.645	0.323	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F(-2)	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	0	415	0	0	0	0
normalized size	1	1.	0.	2.61	0.	0.	0.	0.
time (sec)	N/A	0.121	180.002	0.783	0.	0.	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	0	984	0	0	0	0
normalized size	1	1.	0.	4.66	0.	0.	0.	0.
time (sec)	N/A	0.167	180.009	1.115	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	86	422	0	1077	0	143
normalized size	1	1.	0.8	3.94	0.	10.07	0.	1.34
time (sec)	N/A	0.137	0.178	0.062	0.	2.579	0.	1.185

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	421	0	1103	0	143
normalized size	1	1.	0.76	3.73	0.	9.76	0.	1.27
time (sec)	N/A	0.156	0.197	0.047	0.	2.595	0.	1.17

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	86	429	0	1110	0	143
normalized size	1	1.	0.83	4.12	0.	10.67	0.	1.38
time (sec)	N/A	0.115	0.187	0.036	0.	2.644	0.	1.147

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	14	72	34	14
normalized size	1	1.	1.	1.56	0.78	4.	1.89	0.78
time (sec)	N/A	0.025	0.04	0.024	1.162	2.438	0.702	1.223

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	0	637	0	62
normalized size	1	1.	1.	0.98	0.	11.8	0.	1.15
time (sec)	N/A	0.082	0.041	0.05	0.	2.465	0.	1.184

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	96	406	0	1254	0	170
normalized size	1	1.	0.66	2.78	0.	8.59	0.	1.16
time (sec)	N/A	0.481	0.236	0.049	0.	14.231	0.	1.141

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	20	15	59	0	18
normalized size	1	1.	1.47	1.05	0.79	3.11	0.	0.95
time (sec)	N/A	0.05	0.069	0.037	1.093	2.318	0.	1.153

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	54	53	0	657	0	62
normalized size	1	1.	1.08	1.06	0.	13.14	0.	1.24
time (sec)	N/A	0.09	0.04	0.039	0.	2.432	0.	1.171

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	97	180	0	1372	0	165
normalized size	1	1.	0.82	1.53	0.	11.63	0.	1.4
time (sec)	N/A	0.584	0.193	0.038	0.	14.582	0.	1.156

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	573	0	1212	0	165
normalized size	1	1.	0.87	4.78	0.	10.1	0.	1.38
time (sec)	N/A	0.118	0.201	0.053	0.	2.387	0.	1.156

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	130	287	0	4805	0	242
normalized size	1	1.	1.2	2.66	0.	44.49	0.	2.24
time (sec)	N/A	0.116	0.307	0.074	0.	2.52	0.	1.151

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	373	1091	0	25911	0	844
normalized size	1	1.	1.88	5.51	0.	130.86	0.	4.26
time (sec)	N/A	0.291	0.769	0.099	0.	4.26	0.	1.227

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	574	0	1214	0	165
normalized size	1	1.	0.86	4.74	0.	10.03	0.	1.36
time (sec)	N/A	0.118	0.179	0.051	0.	2.39	0.	1.144

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	125	287	0	4775	0	239
normalized size	1	1.	1.16	2.66	0.	44.21	0.	2.21
time (sec)	N/A	0.128	0.241	0.072	0.	2.592	0.	1.153

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	336	1112	0	26118	0	844
normalized size	1	1.	1.73	5.73	0.	134.63	0.	4.35
time (sec)	N/A	0.276	0.614	0.096	0.	4.225	0.	1.199

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	107	873	0	1364	0	169
normalized size	1	1.	0.86	6.98	0.	10.91	0.	1.35
time (sec)	N/A	0.143	0.263	0.056	0.	2.304	0.	1.185

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	123	287	0	4648	0	242
normalized size	1	1.	1.14	2.66	0.	43.04	0.	2.24
time (sec)	N/A	0.126	0.302	0.077	0.	2.361	0.	1.145

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	319	885	0	21315	0	779
normalized size	1	1.	1.64	4.56	0.	109.87	0.	4.02
time (sec)	N/A	0.26	0.64	0.105	0.	4.329	0.	1.184

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	119	1009	0	1407	0	184
normalized size	1	1.	0.87	7.36	0.	10.27	0.	1.34
time (sec)	N/A	0.219	0.27	0.056	0.	2.731	0.	1.16

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	143	376	0	5477	0	279
normalized size	1	1.	1.18	3.11	0.	45.26	0.	2.31
time (sec)	N/A	0.15	0.373	0.081	0.	2.827	0.	1.193

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	465	1425	0	29412	0	1106
normalized size	1	1.	2.	6.12	0.	126.23	0.	4.75
time (sec)	N/A	0.511	0.934	0.109	0.	5.184	0.	1.319

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	34	73	0	178	0	47
normalized size	1	1.	1.55	3.32	0.	8.09	0.	2.14
time (sec)	N/A	0.083	0.083	0.082	0.	2.325	0.	1.184

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	136	78	284	0	78
normalized size	1	1.	1.21	1.92	1.1	4.	0.	1.1
time (sec)	N/A	0.056	0.211	0.053	1.123	2.297	0.	1.134

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	84	137	77	285	0	78
normalized size	1	1.	1.09	1.78	1.	3.7	0.	1.01
time (sec)	N/A	0.05	0.169	0.052	1.137	2.253	0.	1.131

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	103	232	134	342	0	107
normalized size	1	1.	1.2	2.7	1.56	3.98	0.	1.24
time (sec)	N/A	0.083	0.284	0.055	1.262	2.462	0.	1.114

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	125	88	155	0	66
normalized size	1	1.	1.12	1.62	1.14	2.01	0.	0.86
time (sec)	N/A	0.051	0.234	0.048	1.126	2.33	0.	1.131

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	125	84	154	0	65
normalized size	1	1.	1.1	1.6	1.08	1.97	0.	0.83
time (sec)	N/A	0.046	0.157	0.044	1.072	2.3	0.	1.157

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	102	213	142	194	0	93
normalized size	1	1.	1.26	2.63	1.75	2.4	0.	1.15
time (sec)	N/A	0.08	0.281	0.05	1.184	2.452	0.	1.182

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	116	47	69	117	7
normalized size	1	1.	1.	38.67	15.67	23.	39.	2.33
time (sec)	N/A	0.017	0.005	0.039	1.619	2.298	11.341	1.108

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	8	36	11	135	48	14
normalized size	1	1.	0.73	3.27	1.	12.27	4.36	1.27
time (sec)	N/A	0.025	0.003	0.023	1.101	2.193	5.284	1.098

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	166	86	1022	0	62
normalized size	1	1.	0.85	6.38	3.31	39.31	0.	2.38
time (sec)	N/A	0.029	0.006	0.042	1.756	2.244	0.	1.169

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	10	1
normalized size	1	1.	1.	8.	1.	4.	10.	1.
time (sec)	N/A	0.015	0.	0.01	1.102	2.064	0.504	1.155

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	22	1
normalized size	1	1.	1.	8.	1.	4.	22.	1.
time (sec)	N/A	0.013	0.	0.011	1.061	2.178	1.358	1.191

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	34	1
normalized size	1	1.	1.	8.	1.	4.	34.	1.
time (sec)	N/A	0.014	0.	0.013	1.155	2.227	4.735	1.119

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.013	0.	0.019	1.05	2.09	0.	1.183

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.012	0.	0.024	1.06	2.121	0.	1.154

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.013	0.	0.024	1.106	2.293	0.	1.145

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	54	86	220	0	55
normalized size	1	1.	1.	2.84	4.53	11.58	0.	2.89
time (sec)	N/A	0.028	0.089	0.041	1.67	2.41	0.	1.156

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	42	108	119	883	0	85
normalized size	1	1.	1.35	3.48	3.84	28.48	0.	2.74
time (sec)	N/A	0.054	0.156	0.046	1.767	2.284	0.	1.143

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	66	140	154	2365	0	104
normalized size	1	1.	1.22	2.59	2.85	43.8	0.	1.93
time (sec)	N/A	0.064	0.192	0.058	1.69	2.13	0.	1.172

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	102	49	219	0	49
normalized size	1	1.	1.	5.67	2.72	12.17	0.	2.72
time (sec)	N/A	0.03	0.081	0.034	1.691	2.277	0.	1.131

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	64	129	81	883	0	81
normalized size	1	1.	2.	4.03	2.53	27.59	0.	2.53
time (sec)	N/A	0.047	0.131	0.048	1.697	2.357	0.	1.158

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	66	145	113	2363	0	97
normalized size	1	1.	1.22	2.69	2.09	43.76	0.	1.8
time (sec)	N/A	0.088	0.187	0.049	1.772	2.415	0.	1.136

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.016	0.	0.02	1.111	1.972	0.	1.148

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.015	0.	0.023	1.137	1.79	0.	1.137

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	4	0	1
normalized size	1	1.	1.	8.	1.	4.	0.	1.
time (sec)	N/A	0.016	0.	0.027	1.107	1.89	0.	1.121

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	217	74	0	7035	0	0
normalized size	1	1.	0.8	0.27	0.	25.96	0.	0.
time (sec)	N/A	0.915	0.712	0.037	0.	3.689	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	70	0	7015	0	0
normalized size	1	1.	0.87	0.25	0.	25.05	0.	0.
time (sec)	N/A	0.724	0.481	0.035	0.	4.152	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	283	108	0	9954	0	7
normalized size	1	1.	0.92	0.35	0.	32.21	0.	0.02
time (sec)	N/A	1.081	0.512	0.046	0.	6.568	0.	9.316

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	326	144	0	13673	0	0
normalized size	1	1.	0.9	0.4	0.	37.67	0.	0.
time (sec)	N/A	4.677	0.805	0.049	0.	9.062	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	36	0	159	0	38
normalized size	1	1.	1.17	3.	0.	13.25	0.	3.17
time (sec)	N/A	0.091	0.034	0.046	0.	1.747	0.	1.193

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	258	79	0	14202	0	0
normalized size	1	1.	0.86	0.26	0.	47.34	0.	0.
time (sec)	N/A	0.758	0.512	0.056	0.	32.526	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	198	1264	0	7035	0	0
normalized size	1	1.	0.89	5.67	0.	31.55	0.	0.
time (sec)	N/A	0.629	0.554	0.058	0.	2.728	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	227	1262	0	7015	0	0
normalized size	1	1.	0.99	5.49	0.	30.5	0.	0.
time (sec)	N/A	0.583	0.466	0.03	0.	2.798	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	264	1957	0	9950	0	7
normalized size	1	1.	1.04	7.67	0.	39.02	0.	0.03
time (sec)	N/A	1.291	0.564	0.043	0.	4.402	0.	5.946

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	309	2530	0	13678	0	32
normalized size	1	1.	1.03	8.46	0.	45.75	0.	0.11
time (sec)	N/A	6.389	0.766	0.053	0.	7.91	0.	5.149

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	0	159	0	35
normalized size	1	1.	1.	2.64	0.	14.45	0.	3.18
time (sec)	N/A	0.087	0.058	0.026	0.	1.537	0.	1.135

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	241	2556	0	14195	0	0
normalized size	1	1.	0.98	10.39	0.	57.7	0.	0.
time (sec)	N/A	0.682	0.445	0.039	0.	33.021	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	414	0	1007	258	62
normalized size	1	1.	0.87	10.62	0.	25.82	6.62	1.59
time (sec)	N/A	0.151	0.095	0.045	0.	1.915	2.83	1.158

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	389	0	1006	250	61
normalized size	1	1.	0.87	10.24	0.	26.47	6.58	1.61
time (sec)	N/A	0.112	0.054	0.036	0.	1.888	2.635	1.16

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	96	99	340	136	45
normalized size	1	1.	1.05	2.53	2.61	8.95	3.58	1.18
time (sec)	N/A	0.154	0.131	0.071	1.632	1.752	2.425	1.166

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	96	99	340	136	45
normalized size	1	1.	1.05	2.53	2.61	8.95	3.58	1.18
time (sec)	N/A	0.092	0.096	0.062	1.63	2.031	2.501	1.114

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	136	49	285	0	0
normalized size	1	1.	0.93	2.31	0.83	4.83	0.	0.
time (sec)	N/A	0.701	0.054	0.079	1.82	2.098	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	83	209	81	559	0	0
normalized size	1	1.	0.8	2.01	0.78	5.38	0.	0.
time (sec)	N/A	0.813	0.066	0.069	1.658	2.099	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	113	281	108	807	0	0
normalized size	1	1.	0.75	1.87	0.72	5.38	0.	0.
time (sec)	N/A	0.833	0.08	0.072	1.817	2.101	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	175	58	454	0	0
normalized size	1	1.	0.6	2.4	0.79	6.22	0.	0.
time (sec)	N/A	0.556	0.048	0.077	1.938	2.088	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	253	90	848	0	0
normalized size	1	1.	0.58	2.58	0.92	8.65	0.	0.
time (sec)	N/A	0.608	0.049	0.072	1.981	2.114	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	68	329	117	1223	0	0
normalized size	1	1.	0.53	2.55	0.91	9.48	0.	0.
time (sec)	N/A	0.554	0.054	0.072	1.918	2.178	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	150	81	1173	0	0
normalized size	1	1.	0.84	1.7	0.92	13.33	0.	0.
time (sec)	N/A	0.36	0.056	0.092	1.784	2.122	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	154	0	0	2506	0	0
normalized size	1	1.	0.82	0.	0.	13.4	0.	0.
time (sec)	N/A	0.512	0.187	0.154	0.	2.467	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	249	0	0	3722	0	0
normalized size	1	1.	0.87	0.	0.	12.97	0.	0.
time (sec)	N/A	0.673	0.632	0.133	0.	2.51	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	71	252	124	5553	0	0
normalized size	1	1.	0.54	1.91	0.94	42.07	0.	0.
time (sec)	N/A	0.407	0.22	0.071	1.705	2.539	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	120	441	208	10301	0	0
normalized size	1	1.	0.59	2.16	1.02	50.5	0.	0.
time (sec)	N/A	0.638	0.62	0.083	1.873	3.075	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	157	602	279	13428	0	0
normalized size	1	1.	0.48	1.85	0.86	41.19	0.	0.
time (sec)	N/A	0.683	0.994	0.085	1.811	3.398	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	162	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.646	0.239	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	77	124	170	409	190	232
normalized size	1	1.	0.71	1.14	1.56	3.75	1.74	2.13
time (sec)	N/A	0.099	0.259	0.047	1.246	2.014	5.902	1.143

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	68	85	198	129	143
normalized size	1	1.	0.79	1.08	1.35	3.14	2.05	2.27
time (sec)	N/A	0.035	0.113	0.039	1.168	2.179	1.461	1.185

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	24	77	24	39
normalized size	1	1.	1.9	0.95	1.2	3.85	1.2	1.95
time (sec)	N/A	0.019	0.007	0.001	1.231	2.481	0.288	1.134

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	48	207	0	772	0	107
normalized size	1	1.	1.09	4.7	0.	17.55	0.	2.43
time (sec)	N/A	0.123	0.08	0.087	0.	2.338	0.	1.182

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	469	0	1895	0	208
normalized size	1	1.	1.01	5.27	0.	21.29	0.	2.34
time (sec)	N/A	0.104	0.337	0.119	0.	2.163	0.	1.191

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	121	2082	0	5688	0	351
normalized size	1	1.	0.85	14.56	0.	39.78	0.	2.45
time (sec)	N/A	0.172	0.64	0.158	0.	2.078	0.	1.57

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	239	1260	0	0	0	0
normalized size	1	1.	0.79	4.19	0.	0.	0.	0.
time (sec)	N/A	0.392	1.375	0.542	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	202	935	0	0	0	0
normalized size	1	1.	0.81	3.77	0.	0.	0.	0.
time (sec)	N/A	0.253	0.804	0.49	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	94	351	0	0	0	0
normalized size	1	1.	0.98	3.66	0.	0.	0.	0.
time (sec)	N/A	0.073	0.109	0.396	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	181	0	0	0	0
normalized size	1	1.	0.94	1.89	0.	0.	0.	0.
time (sec)	N/A	0.078	0.135	0.362	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	119	630	0	0	0	0
normalized size	1	1.	0.75	3.99	0.	0.	0.	0.
time (sec)	N/A	0.113	0.474	0.452	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	237	641	0	0	0	0
normalized size	1	1.	0.73	1.97	0.	0.	0.	0.
time (sec)	N/A	0.381	1.6	0.98	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	279	687	0	3650	0	0
normalized size	1	1.	0.72	1.78	0.	9.46	0.	0.
time (sec)	N/A	0.604	0.367	0.059	0.	2.008	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	210	530	0	2738	0	0
normalized size	1	1.	0.75	1.89	0.	9.74	0.	0.
time (sec)	N/A	0.515	0.251	0.053	0.	1.982	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	956	376	0	1814	0	0
normalized size	1	1.	5.14	2.02	0.	9.75	0.	0.
time (sec)	N/A	0.303	1.474	0.05	0.	1.875	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.965	0.05	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.069	0.044	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	80	72	289	124	72
normalized size	1	1.	0.76	1.21	1.09	4.38	1.88	1.09
time (sec)	N/A	0.036	0.034	0.022	1.24	1.752	38.216	1.134

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	32	61	50	254	139	41
normalized size	1	1.	0.8	1.52	1.25	6.35	3.48	1.02
time (sec)	N/A	0.035	0.019	0.016	1.255	1.806	7.29	1.148

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	25	52	35	153	54	31
normalized size	1	1.	0.78	1.62	1.09	4.78	1.69	0.97
time (sec)	N/A	0.017	0.013	0.005	1.287	1.696	2.514	1.137

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	40	61	163	0	68
normalized size	1	1.	0.88	1.54	2.35	6.27	0.	2.62
time (sec)	N/A	0.018	0.017	0.033	1.288	1.829	0.	1.124

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	43	84	286	0	65
normalized size	1	1.	0.88	1.02	2.	6.81	0.	1.55
time (sec)	N/A	0.043	0.049	0.032	1.45	1.775	0.	1.157

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	67	119	1098	0	105
normalized size	1	1.	0.84	0.92	1.63	15.04	0.	1.44
time (sec)	N/A	0.044	0.058	0.037	1.055	1.726	0.	1.151

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	108	166	0	752	0	113
normalized size	1	1.	0.78	1.19	0.	5.41	0.	0.81
time (sec)	N/A	0.063	0.489	0.028	0.	1.798	0.	1.136

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	112	0	382	476	76
normalized size	1	1.	0.66	1.27	0.	4.34	5.41	0.86
time (sec)	N/A	0.036	0.152	0.016	0.	1.529	14.812	1.156

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	176	167	54
normalized size	1	1.	0.7	1.44	0.	3.26	3.09	1.
time (sec)	N/A	0.019	0.069	0.004	0.	1.518	3.034	1.221

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	59	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.133	0.03	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	131	0	0	0	0	0
normalized size	1	1.	2.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	3.229	0.031	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	2.354	0.038	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.062	0.024	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	106	166	0	756	0	113
normalized size	1	1.	0.76	1.19	0.	5.44	0.	0.81
time (sec)	N/A	0.05	0.497	0.007	0.	1.401	0.	1.128

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	112	0	382	476	76
normalized size	1	1.	0.64	1.27	0.	4.34	5.41	0.86
time (sec)	N/A	0.031	0.153	0.008	0.	1.302	13.15	1.138

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	174	201	54
normalized size	1	1.	0.7	1.44	0.	3.22	3.72	1.
time (sec)	N/A	0.018	0.069	0.004	0.	1.25	3.517	1.163

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.018	0.024	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.017	0.043	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	80	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.147	0.049	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	89	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.074	0.053	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.094	0.046	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	196	434	255	999	1731	2125
normalized size	1	1.	0.77	1.71	1.	3.93	6.81	8.37
time (sec)	N/A	0.407	1.709	0.128	1.089	1.378	128.612	1.358

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	141	119	317	400	1214
normalized size	1	1.	0.88	1.33	1.12	2.99	3.77	11.45
time (sec)	N/A	0.179	0.612	0.046	1.038	1.338	17.498	1.255

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	3.37	0.026	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	255	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	3.011	0.06	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	230	426	252	5389	1719	2128
normalized size	1	1.	0.92	1.7	1.	21.47	6.85	8.48
time (sec)	N/A	0.313	0.573	0.073	1.07	1.731	112.801	1.296

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	135	117	1080	391	1215
normalized size	1	1.	0.87	1.34	1.16	10.69	3.87	12.03
time (sec)	N/A	0.146	0.196	0.036	1.036	1.544	11.166	1.242

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.046	0.033	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	127	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.332	0.059	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	90	76	302	139	70
normalized size	1	1.	0.74	1.3	1.1	4.38	2.01	1.01
time (sec)	N/A	0.051	0.062	0.01	0.994	1.561	92.698	1.149

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	71	68	261	177	77
normalized size	1	1.	0.79	1.25	1.19	4.58	3.11	1.35
time (sec)	N/A	0.053	0.066	0.01	1.038	1.534	26.828	1.13

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	54	39	144	76	35
normalized size	1	1.	0.8	1.54	1.11	4.11	2.17	1.
time (sec)	N/A	0.027	0.01	0.005	1.011	1.46	8.25	1.127

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	51	158	0	53
normalized size	1	1.	0.88	1.08	2.04	6.32	0.	2.12
time (sec)	N/A	0.018	0.016	0.014	1.084	1.57	0.	1.144

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	30	61	284	0	63
normalized size	1	1.	0.83	0.73	1.49	6.93	0.	1.54
time (sec)	N/A	0.045	0.047	0.017	1.025	1.472	0.	1.255

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	69	105	1089	0	92
normalized size	1	1.	0.84	0.99	1.5	15.56	0.	1.31
time (sec)	N/A	0.06	0.08	0.017	1.011	1.549	0.	1.37

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	107	104	456	0	109
normalized size	1	1.	0.74	1.18	1.14	5.01	0.	1.2
time (sec)	N/A	0.074	0.09	0.012	1.008	1.49	0.	1.254

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	84	51	258	144	49
normalized size	1	1.	0.82	1.71	1.04	5.27	2.94	1.
time (sec)	N/A	0.053	0.026	0.01	0.992	1.55	137.372	1.148

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	71	68	259	175	77
normalized size	1	1.	0.75	1.25	1.19	4.54	3.07	1.35
time (sec)	N/A	0.047	0.044	0.007	1.019	1.558	35.022	1.129

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	39	52	68	190	0	57
normalized size	1	1.	0.93	1.24	1.62	4.52	0.	1.36
time (sec)	N/A	0.043	0.029	0.018	1.024	1.652	0.	1.17

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	179	47	84	585	0	85
normalized size	1	1.	3.38	0.89	1.58	11.04	0.	1.6
time (sec)	N/A	0.038	2.543	0.019	0.999	1.598	0.	1.146

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	43	93	711	0	81
normalized size	1	1.	0.74	0.69	1.5	11.47	0.	1.31
time (sec)	N/A	0.065	0.081	0.024	1.019	1.629	0.	1.141

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	120	73	420	0	70
normalized size	1	1.	0.74	1.74	1.06	6.09	0.	1.01
time (sec)	N/A	0.059	0.038	0.013	1.043	1.533	0.	1.181

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	102	104	458	0	109
normalized size	1	1.	0.74	1.12	1.14	5.03	0.	1.2
time (sec)	N/A	0.079	0.103	0.01	1.033	1.48	0.	1.179

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	84	76	302	139	70
normalized size	1	1.	0.74	1.22	1.1	4.38	2.01	1.01
time (sec)	N/A	0.047	0.051	0.008	1.012	1.651	107.192	1.143

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	68	50	88	520	0	99
normalized size	1	1.	1.15	0.85	1.49	8.81	0.	1.68
time (sec)	N/A	0.054	0.347	0.02	0.985	1.835	0.	1.176

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	67	92	594	0	96
normalized size	1	1.	0.83	1.06	1.46	9.43	0.	1.52
time (sec)	N/A	0.065	0.082	0.02	1.023	1.922	0.	1.152

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	286	88	119	1291	0	105
normalized size	1	1.	3.53	1.09	1.47	15.94	0.	1.3
time (sec)	N/A	0.052	4.43	0.025	1.	1.866	0.	1.14

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	89	70	413	241	81
normalized size	1	1.	0.75	1.56	1.23	7.25	4.23	1.42
time (sec)	N/A	0.054	0.073	0.016	1.03	1.886	90.015	1.168

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	69	72	292	128	74
normalized size	1	1.	0.77	1.05	1.09	4.42	1.94	1.12
time (sec)	N/A	0.05	0.08	0.01	1.025	1.878	30.294	1.124

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	33	32	250	117	24
normalized size	1	1.	1.09	1.43	1.39	10.87	5.09	1.04
time (sec)	N/A	0.023	0.015	0.006	1.002	1.746	9.813	1.132

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	38	77	178	0	41
normalized size	1	1.	0.95	1.03	2.08	4.81	0.	1.11
time (sec)	N/A	0.032	0.025	0.034	1.019	1.851	0.	1.164

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	62	65	103	587	0	74
normalized size	1	1.	1.15	1.2	1.91	10.87	0.	1.37
time (sec)	N/A	0.043	0.116	0.039	1.054	1.851	0.	1.16

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	47	56	116	714	0	65
normalized size	1	1.	0.75	0.89	1.84	11.33	0.	1.03
time (sec)	N/A	0.072	0.08	0.04	1.035	1.855	0.	1.187

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	105	501	0	108
normalized size	1	1.	0.73	1.08	1.05	5.01	0.	1.08
time (sec)	N/A	0.078	0.103	0.016	1.004	1.764	0.	1.21

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	38	58	57	304	70	58
normalized size	1	1.	0.73	1.12	1.1	5.85	1.35	1.12
time (sec)	N/A	0.059	0.038	0.008	1.014	1.756	111.514	1.145

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	69	72	290	128	74
normalized size	1	1.	0.82	1.05	1.09	4.39	1.94	1.12
time (sec)	N/A	0.047	0.056	0.007	1.013	1.779	65.197	1.154

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	58	54	82	298	0	73
normalized size	1	1.	1.29	1.2	1.82	6.62	0.	1.62
time (sec)	N/A	0.034	0.263	0.073	1.008	1.84	0.	1.15

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	57	116	540	0	76
normalized size	1	1.	0.81	0.97	1.97	9.15	0.	1.29
time (sec)	N/A	0.05	0.062	0.069	1.017	1.878	0.	1.209

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	247	78	130	1295	0	97
normalized size	1	1.	2.91	0.92	1.53	15.24	0.	1.14
time (sec)	N/A	0.068	4.029	0.08	1.232	1.849	0.	1.186

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	61	70	513	0	81
normalized size	1	1.	0.79	1.07	1.23	9.	0.	1.42
time (sec)	N/A	0.062	0.044	0.014	1.146	1.74	0.	1.204

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	103	501	0	108
normalized size	1	1.	0.73	1.08	1.03	5.01	0.	1.08
time (sec)	N/A	0.074	0.095	0.01	1.05	1.759	0.	1.219

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	89	70	413	240	82
normalized size	1	1.	0.75	1.56	1.23	7.25	4.21	1.44
time (sec)	N/A	0.052	0.058	0.007	1.069	1.881	103.65	1.179

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	55	95	354	0	70
normalized size	1	1.	0.81	0.93	1.61	6.	0.	1.19
time (sec)	N/A	0.06	0.042	0.115	1.034	1.946	0.	1.212

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	220	79	117	797	0	101
normalized size	1	1.	3.01	1.08	1.6	10.92	0.	1.38
time (sec)	N/A	0.054	2.072	0.119	1.049	1.868	0.	1.192

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	60	70	143	1099	0	95
normalized size	1	1.	0.75	0.88	1.79	13.74	0.	1.19
time (sec)	N/A	0.063	0.11	0.127	1.016	1.867	0.	1.231

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	42	40	122	500	0	122
normalized size	1	1.	0.37	0.35	1.08	4.42	0.	1.08
time (sec)	N/A	0.082	0.028	0.052	1.495	1.999	0.	1.163

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	120	44	136	637	0	128
normalized size	1	1.	0.93	0.34	1.05	4.94	0.	0.99
time (sec)	N/A	0.096	0.118	0.05	1.691	1.934	0.	1.182

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	58	48	142	651	0	134
normalized size	1	1.	0.45	0.37	1.09	5.01	0.	1.03
time (sec)	N/A	0.106	0.06	0.061	1.623	2.108	0.	1.152

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	64	50	155	798	0	139
normalized size	1	1.	0.43	0.34	1.04	5.36	0.	0.93
time (sec)	N/A	0.128	0.062	0.067	1.634	2.241	0.	1.135

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	48	46	745	0	47
normalized size	1	1.	0.91	1.41	1.35	21.91	0.	1.38
time (sec)	N/A	0.028	0.056	0.061	1.558	1.844	0.	1.163

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	54	63	1750	0	57
normalized size	1	1.	1.02	1.02	1.19	33.02	0.	1.08
time (sec)	N/A	0.042	0.091	0.065	1.506	1.891	0.	1.191

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	161	56	63	1868	0	57
normalized size	1	1.	2.93	1.02	1.15	33.96	0.	1.04
time (sec)	N/A	0.048	3.155	0.097	1.534	1.894	0.	1.156

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	310	60	80	3366	0	65
normalized size	1	1.	4.13	0.8	1.07	44.88	0.	0.87
time (sec)	N/A	0.067	5.271	0.105	1.515	2.002	0.	1.175

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	1161	0	126
normalized size	1	1.	0.63	1.47	0.	8.47	0.	0.92
time (sec)	N/A	0.101	1.12	0.03	0.	1.954	0.	1.164

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	887	1059	116
normalized size	1	1.	0.63	1.4	0.	6.98	8.34	0.91
time (sec)	N/A	0.091	0.953	0.024	0.	1.825	102.679	1.208

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	47	102	0	360	347	63
normalized size	1	1.	0.71	1.55	0.	5.45	5.26	0.95
time (sec)	N/A	0.047	0.053	0.006	0.	1.762	16.925	1.182

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	240	201	54
normalized size	1	1.	0.7	1.44	0.	4.44	3.72	1.
time (sec)	N/A	0.017	0.031	0.005	0.	1.842	4.725	1.279

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	120	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	1.929	0.056	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	92	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.657	0.07	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	159	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	1.54	0.1	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	117	278	0	2201	0	178
normalized size	1	1.	0.6	1.43	0.	11.29	0.	0.91
time (sec)	N/A	0.136	1.19	0.021	0.	1.881	0.	1.207

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	124	0	770	831	78
normalized size	1	1.	0.7	1.49	0.	9.28	10.01	0.94
time (sec)	N/A	0.079	0.355	0.012	0.	1.982	150.693	1.186

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	886	994	116
normalized size	1	1.	0.63	1.4	0.	6.98	7.83	0.91
time (sec)	N/A	0.09	0.524	0.007	0.	2.138	86.463	1.177

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	124	0	433	456	78
normalized size	1	1.	0.58	1.31	0.	4.56	4.8	0.82
time (sec)	N/A	0.036	0.142	0.007	0.	2.08	16.198	1.176

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.555	0.119	0.	0.	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	145	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.874	0.101	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	111	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.343	1.321	0.136	0.	0.	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	113	202	0	1611	0	126
normalized size	1	1.	0.82	1.47	0.	11.76	0.	0.92
time (sec)	N/A	0.113	0.985	0.023	0.	2.024	0.	1.205

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	118	278	0	2202	0	178
normalized size	1	1.	0.61	1.43	0.	11.29	0.	0.91
time (sec)	N/A	0.133	1.21	0.009	0.	2.113	0.	1.172

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	1161	0	126
normalized size	1	1.	0.63	1.47	0.	8.47	0.	0.92
time (sec)	N/A	0.094	0.83	0.009	0.	2.077	0.	1.16

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	106	178	0	898	1114	116
normalized size	1	1.	0.74	1.24	0.	6.24	7.74	0.81
time (sec)	N/A	0.057	0.444	0.006	0.	2.127	82.623	1.171

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	172	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	1.037	0.15	0.	0.	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	145	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	1.208	0.163	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	3.724	0.176	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.614	1.321	0.214	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	74	36	36
normalized size	1	1.	1.	1.	1.29	4.35	2.12	2.12
time (sec)	N/A	0.015	0.041	0.007	1.026	2.345	0.59	1.158

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	30	88	0	0
normalized size	1	1.	1.	1.05	1.36	4.	0.	0.
time (sec)	N/A	0.017	0.209	0.088	0.984	2.336	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	30	88	48	0
normalized size	1	1.	0.96	1.	1.3	3.83	2.09	0.
time (sec)	N/A	0.015	0.044	0.009	1.004	2.267	3.989	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	0	31	0	0
normalized size	1	1.	1.	1.31	0.	2.38	0.	0.
time (sec)	N/A	0.024	0.06	0.01	0.	2.31	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	0	42	0	0
normalized size	1	1.	1.	1.28	0.	2.33	0.	0.
time (sec)	N/A	0.023	0.16	0.046	0.	2.158	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	0	42	0	0
normalized size	1	1.	0.95	1.21	0.	2.21	0.	0.
time (sec)	N/A	0.024	0.061	0.039	0.	2.048	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	74	36	39
normalized size	1	1.	1.	1.	1.29	4.35	2.12	2.29
time (sec)	N/A	0.014	0.016	0.006	1.008	2.032	1.148	1.151

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	23	30	88	0	0
normalized size	1	1.	1.05	1.05	1.36	4.	0.	0.
time (sec)	N/A	0.015	0.128	0.064	0.99	2.046	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	30	88	48	0
normalized size	1	1.	1.	1.	1.3	3.83	2.09	0.
time (sec)	N/A	0.014	0.042	0.009	1.054	2.011	3.696	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	0	31	0	0
normalized size	1	1.	1.	1.31	0.	2.38	0.	0.
time (sec)	N/A	0.023	0.033	0.01	0.	1.973	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	0	42	0	0
normalized size	1	1.	1.	1.28	0.	2.33	0.	0.
time (sec)	N/A	0.023	0.062	0.051	0.	2.137	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	0	42	0	0
normalized size	1	1.	0.95	1.21	0.	2.21	0.	0.
time (sec)	N/A	0.023	0.058	0.042	0.	2.024	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	15	126	0	61
normalized size	1	1.	1.82	1.09	1.36	11.45	0.	5.55
time (sec)	N/A	0.04	0.055	0.023	1.044	2.065	0.	1.17

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	116	47	69	0	7
normalized size	1	1.	1.	38.67	15.67	23.	0.	2.33
time (sec)	N/A	0.032	0.003	0.038	1.515	1.995	0.	1.135

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	A	B	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	11	11	0	116	15	82	0	15
normalized size	1	1.	0.	10.55	1.36	7.45	0.	1.36
time (sec)	N/A	0.034	0.02	0.071	1.574	2.33	0.	1.159

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	0	220	0	0
normalized size	1	1.	0.95	1.05	0.	11.58	0.	0.
time (sec)	N/A	0.043	0.156	0.018	0.	2.448	0.	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	34	16	50	29	16
normalized size	1	1.	1.	8.5	4.	12.5	7.25	4.
time (sec)	N/A	0.052	0.006	0.032	1.024	2.314	1.484	1.156

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	34	16	50	29	16
normalized size	1	1.	1.	8.5	4.	12.5	7.25	4.
time (sec)	N/A	0.074	0.003	0.033	1.065	2.11	1.425	1.14

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	B	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	5	5	0	42	0	74	0	12
normalized size	1	1.	0.	8.4	0.	14.8	0.	2.4
time (sec)	N/A	0.049	0.039	0.059	0.	2.032	0.	1.168

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	32	39	267	0	39
normalized size	1	1.	0.73	2.13	2.6	17.8	0.	2.6
time (sec)	N/A	0.053	0.03	0.049	1.005	2.066	0.	1.159

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	32	43	188	0	35
normalized size	1	1.	0.73	2.13	2.87	12.53	0.	2.33
time (sec)	N/A	0.058	0.024	0.05	1.039	2.002	0.	1.204

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	74	34	0	1177	0	0
normalized size	1	1.	0.73	0.33	0.	11.54	0.	0.
time (sec)	N/A	0.112	0.106	0.052	0.	2.14	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	A	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	0	62	0	132	0	26
normalized size	1	1.	0.	2.82	0.	6.	0.	1.18
time (sec)	N/A	0.068	0.031	0.069	0.	2.092	0.	1.155

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	54	100	89	467	0	153
normalized size	1	1.	1.93	3.57	3.18	16.68	0.	5.46
time (sec)	N/A	0.099	0.329	0.042	1.566	2.262	0.	1.153

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	61	251	204	1667	0	356
normalized size	1	1.	1.15	4.74	3.85	31.45	0.	6.72
time (sec)	N/A	0.158	0.566	0.063	1.583	2.372	0.	1.155

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	134	542	373	4383	0	733
normalized size	1	1.	1.72	6.95	4.78	56.19	0.	9.4
time (sec)	N/A	0.163	0.785	0.082	1.636	2.709	0.	1.182

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	250	0	43
normalized size	1	1.	1.	0.92	1.17	20.83	0.	3.58
time (sec)	N/A	0.086	0.042	0.041	0.991	1.993	0.	1.512

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	306	34	3032	0	89
normalized size	1	1.	2.03	9.27	1.03	91.88	0.	2.7
time (sec)	N/A	0.108	0.028	0.088	1.021	2.034	0.	1.211

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	78	165	170	0	38
normalized size	1	1.	1.04	3.	6.35	6.54	0.	1.46
time (sec)	N/A	0.091	0.219	0.075	1.628	2.441	0.	1.22

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	16	50	0	16
normalized size	1	1.	1.	1.25	4.	12.5	0.	4.
time (sec)	N/A	0.02	0.002	0.014	1.053	2.172	0.	1.159

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	0	63	0	244	0	47
normalized size	1	1.	0.	3.15	0.	12.2	0.	2.35
time (sec)	N/A	0.124	0.029	0.069	0.	2.353	0.	1.156

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	43	0	0	367	0	59
normalized size	1	1.	4.78	0.	0.	40.78	0.	6.56
time (sec)	N/A	0.049	0.046	0.112	0.	2.402	0.	1.167

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	47	0	0	398	0	59
normalized size	1	1.	5.22	0.	0.	44.22	0.	6.56
time (sec)	N/A	0.05	0.053	0.17	0.	2.56	0.	1.219

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	46	0	0	4	0	132
normalized size	1	1.	3.29	0.	0.	0.29	0.	9.43
time (sec)	N/A	0.051	0.045	0.165	0.	2.214	0.	1.189

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	51	0	0	786	0	162
normalized size	1	1.	2.68	0.	0.	41.37	0.	8.53
time (sec)	N/A	0.049	0.209	0.232	0.	2.069	0.	1.176

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	55	0	0	1181	0	196
normalized size	1	1.	2.29	0.	0.	49.21	0.	8.17
time (sec)	N/A	0.043	0.092	0.142	0.	2.084	0.	1.183

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	46	1157	19	55
normalized size	1	1.	1.47	1.18	2.71	68.06	1.12	3.24
time (sec)	N/A	0.078	0.015	0.013	1.027	1.994	18.369	1.134

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	140	193	0	0
normalized size	1	1.	0.65	1.42	3.26	4.49	0.	0.
time (sec)	N/A	0.039	0.056	0.071	1.328	2.031	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	140	193	0	0
normalized size	1	1.	0.65	1.42	3.26	4.49	0.	0.
time (sec)	N/A	0.04	0.026	0.002	1.316	2.116	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	158	258	0	344
normalized size	1	1.	0.56	1.02	2.47	4.03	0.	5.38
time (sec)	N/A	0.039	0.06	0.07	1.412	2.121	0.	1.235

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	158	258	0	344
normalized size	1	1.	0.56	1.02	2.47	4.03	0.	5.38
time (sec)	N/A	0.044	0.029	0.	1.372	2.202	0.	1.176

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	139	193	0	0
normalized size	1	1.	0.65	1.37	3.23	4.49	0.	0.
time (sec)	N/A	0.042	0.116	0.086	1.316	2.023	0.	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	139	193	0	0
normalized size	1	1.	0.65	1.37	3.23	4.49	0.	0.
time (sec)	N/A	0.039	0.031	0.	1.344	2.065	0.	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	158	258	0	343
normalized size	1	1.	0.56	0.98	2.47	4.03	0.	5.36
time (sec)	N/A	0.049	0.158	0.091	1.34	2.024	0.	1.189

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	158	258	0	343
normalized size	1	1.	0.56	0.98	2.47	4.03	0.	5.36
time (sec)	N/A	0.042	0.032	0.	1.309	2.078	0.	1.173

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	128	38	0	0
normalized size	1	1.	1.	0.89	14.22	4.22	0.	0.
time (sec)	N/A	0.026	0.006	0.017	4.902	1.987	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	38	0	0
normalized size	1	1.	1.	0.89	1.	4.22	0.	0.
time (sec)	N/A	0.024	0.009	0.015	1.013	2.111	0.	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.033	0.029	0.	0.	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.037	0.009	0.	0.	0.	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.076	0.026	0.	0.	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.07	0.026	0.	0.	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	31	309	12	32
normalized size	1	1.	1.	0.92	2.38	23.77	0.92	2.46
time (sec)	N/A	0.036	0.007	0.013	1.036	2.026	0.741	1.112

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	16	127	0	62
normalized size	1	1.	1.67	1.08	1.33	10.58	0.	5.17
time (sec)	N/A	0.048	0.054	0.019	1.001	2.161	0.	1.099

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	0	221	0	0
normalized size	1	1.	0.95	1.05	0.	11.05	0.	0.
time (sec)	N/A	0.048	0.196	0.021	0.	2.155	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	16	50	0	16
normalized size	1	1.	1.	1.25	4.	12.5	0.	4.
time (sec)	N/A	0.02	0.003	0.013	1.028	1.997	0.	1.112

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	32	16	50	0	16
normalized size	1	1.	1.	8.	4.	12.5	0.	4.
time (sec)	N/A	0.062	0.006	0.03	1.062	1.972	0.	1.113

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	56	94	104	467	0	153
normalized size	1	1.	2.	3.36	3.71	16.68	0.	5.46
time (sec)	N/A	0.097	0.301	0.036	1.199	2.21	0.	1.121

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	203	239	1669	0	358
normalized size	1	1.	1.17	3.83	4.51	31.49	0.	6.75
time (sec)	N/A	0.145	0.466	0.06	1.222	2.3	0.	1.159

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	136	378	427	4383	0	734
normalized size	1	1.	1.74	4.85	5.47	56.19	0.	9.41
time (sec)	N/A	0.153	1.146	0.073	1.232	2.788	0.	1.209

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	136	40	53	1041	100	302
normalized size	1	1.	3.78	1.11	1.47	28.92	2.78	8.39
time (sec)	N/A	0.095	0.248	0.007	1.116	2.207	24.082	1.164

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	114	40	53	1041	82	302
normalized size	1	1.	3.17	1.11	1.47	28.92	2.28	8.39
time (sec)	N/A	0.094	0.584	0.01	1.021	2.109	20.601	1.233

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	464	46	0
normalized size	1	1.	1.	0.84	1.05	24.42	2.42	0.
time (sec)	N/A	0.062	0.012	0.009	1.068	2.2	1.909	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	0	171	0	0
normalized size	1	1.	1.	0.81	0.	6.33	0.	0.
time (sec)	N/A	0.17	0.034	0.074	0.	2.23	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	23	146	0	23
normalized size	1	1.	1.	0.88	2.88	18.25	0.	2.88
time (sec)	N/A	0.191	0.017	0.013	1.079	2.082	0.	1.131

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	8	58	7	23
normalized size	1	1.	1.5	0.88	1.	7.25	0.88	2.88
time (sec)	N/A	0.014	0.004	0.006	1.178	1.95	0.363	1.119

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	20	146	8	20
normalized size	1	1.	1.	0.88	2.5	18.25	1.	2.5
time (sec)	N/A	0.189	0.016	0.012	1.041	1.766	0.432	1.12

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	68	59	75	0	730	0	124
normalized size	1	1.31	1.13	1.44	0.	14.04	0.	2.38
time (sec)	N/A	0.162	0.084	0.053	0.	1.899	0.	1.214

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	68	59	75	0	730	0	124
normalized size	1	1.31	1.13	1.44	0.	14.04	0.	2.38
time (sec)	N/A	0.13	0.061	0.055	0.	1.933	0.	1.152

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	92	0	821	0	63
normalized size	1	1.	0.92	1.77	0.	15.79	0.	1.21
time (sec)	N/A	0.134	0.077	0.034	0.	1.964	0.	1.148

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	92	0	821	0	66
normalized size	1	1.	0.96	1.77	0.	15.79	0.	1.27
time (sec)	N/A	0.106	0.061	0.033	0.	1.963	0.	1.148

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	39	24	842	0	49
normalized size	1	1.	1.03	1.3	0.8	28.07	0.	1.63
time (sec)	N/A	0.039	0.037	0.114	1.63	1.884	0.	1.226

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	54	455	0	46
normalized size	1	1.	1.58	1.	1.74	14.68	0.	1.48
time (sec)	N/A	0.039	0.066	0.077	1.712	1.856	0.	1.206

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	68	0	450	0	0
normalized size	1	1.	1.65	1.58	0.	10.47	0.	0.
time (sec)	N/A	0.073	0.032	0.038	0.	1.835	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	45	297	0	63
normalized size	1	1.	1.	2.17	3.75	24.75	0.	5.25
time (sec)	N/A	0.036	0.02	0.032	1.629	1.829	0.	1.15

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	78	75	0	1442	0	0
normalized size	1	1.	1.47	1.42	0.	27.21	0.	0.
time (sec)	N/A	0.132	0.064	0.037	0.	1.937	0.	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	1.101	0.158	0.	0.	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	30	89	31	14
normalized size	1	1.	1.	0.77	1.36	4.05	1.41	0.64
time (sec)	N/A	0.034	0.004	0.007	1.02	2.031	0.216	1.14

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	37	48	39	346	382	36
normalized size	1	1.	0.54	0.7	0.57	5.01	5.54	0.52
time (sec)	N/A	0.201	0.034	0.039	1.052	2.036	1.411	1.122

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	120	194	288	976	241	516
normalized size	1	1.	0.93	1.5	2.23	7.57	1.87	4.
time (sec)	N/A	0.14	0.498	0.069	1.105	2.24	137.366	1.163

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	48	36	346	0	36
normalized size	1	1.	0.85	1.23	0.92	8.87	0.	0.92
time (sec)	N/A	0.049	0.032	0.011	1.036	2.094	0.	1.138

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	30	0	0	1465	0	62
normalized size	1	1.	1.2	0.	0.	58.6	0.	2.48
time (sec)	N/A	0.1	0.201	0.102	0.	2.072	0.	1.192

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	0	525	953	0	0
normalized size	1	1.	0.92	0.	14.19	25.76	0.	0.
time (sec)	N/A	0.106	0.059	0.113	1.447	2.096	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	28	61	0	0
normalized size	1	1.	1.	1.25	7.	15.25	0.	0.
time (sec)	N/A	0.14	0.071	0.02	2.233	2.073	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	34	38	200	94	370
normalized size	1	1.	1.22	1.26	1.41	7.41	3.48	13.7
time (sec)	N/A	0.091	0.086	0.012	1.053	2.075	8.383	1.202

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	37	37	49	201	92	381
normalized size	1	1.	1.16	1.16	1.53	6.28	2.88	11.91
time (sec)	N/A	0.095	0.072	0.006	1.157	2.114	8.527	1.167

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	25	138	0	559	0	0
normalized size	1	1.	0.49	2.71	0.	10.96	0.	0.
time (sec)	N/A	0.172	0.036	0.097	0.	2.069	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	115	120	126	370	0	51
normalized size	1	1.	2.45	2.55	2.68	7.87	0.	1.09
time (sec)	N/A	0.328	1.337	0.142	1.576	2.174	0.	1.178

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	148	66	104	0	19
normalized size	1	1.	1.55	13.45	6.	9.45	0.	1.73
time (sec)	N/A	0.062	0.004	0.063	1.753	2.013	0.	1.16

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	65	36	19	108	39	19
normalized size	1	1.	2.95	1.64	0.86	4.91	1.77	0.86
time (sec)	N/A	0.05	0.024	0.	1.025	2.01	0.649	1.137

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	65	36	19	107	0	19
normalized size	1	1.	4.64	2.57	1.36	7.64	0.	1.36
time (sec)	N/A	0.207	0.025	0.129	1.11	2.087	0.	1.133

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	148	68	103	0	20
normalized size	1	1.	1.42	12.33	5.67	8.58	0.	1.67
time (sec)	N/A	0.263	0.008	0.101	1.581	1.947	0.	1.166

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	120	126	369	0	51
normalized size	1	1.	1.11	2.55	2.68	7.85	0.	1.09
time (sec)	N/A	0.41	0.319	0.247	1.775	2.04	0.	1.164

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	26	138	0	558	0	0
normalized size	1	1.	0.51	2.71	0.	10.94	0.	0.
time (sec)	N/A	1.418	0.026	0.134	0.	1.71	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [423] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.	14	0.214
2	A	2	1	1.	21	0.048
3	A	2	2	1.	23	0.087
4	A	2	2	1.	21	0.095
5	A	2	2	1.	23	0.087
6	A	2	2	1.	23	0.087
7	A	2	2	1.	23	0.087
8	A	2	2	1.	13	0.154
9	A	2	2	1.	15	0.133
10	A	3	2	1.	17	0.118
11	A	3	2	1.	17	0.118
12	A	2	2	1.	15	0.133
13	A	3	2	1.	17	0.118
14	A	3	2	1.	17	0.118
15	A	3	3	1.	17	0.176
16	A	4	3	1.	17	0.176
17	A	5	3	1.	17	0.176
18	A	4	3	1.	17	0.176
19	A	5	3	1.	17	0.176
20	A	6	3	1.	17	0.176
21	A	5	3	1.	17	0.176
22	A	6	3	1.	17	0.176
23	A	7	3	1.	17	0.176
24	A	2	2	1.	13	0.154
25	A	3	3	1.	15	0.2
26	A	3	2	1.	15	0.133
27	A	4	3	1.	15	0.2
28	A	4	3	1.	15	0.2
29	A	3	3	1.	15	0.2
30	A	3	2	1.	17	0.118
31	A	4	4	1.	17	0.235

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	3	2	1.	17	0.118
33	A	5	4	1.	17	0.235
34	A	3	2	1.	15	0.133
35	A	4	4	1.	17	0.235
36	A	4	3	1.	17	0.176
37	A	5	4	1.	17	0.235
38	A	4	3	1.	17	0.176
39	A	4	3	1.	15	0.2
40	A	3	2	1.	17	0.118
41	A	5	4	1.	17	0.235
42	A	3	2	1.	17	0.118
43	A	6	4	1.	17	0.235
44	A	4	3	1.	15	0.2
45	A	5	4	1.	17	0.235
46	A	4	3	1.	17	0.176
47	A	6	4	1.	17	0.235
48	A	4	3	1.	17	0.176
49	A	6	5	1.	21	0.238
50	A	5	5	1.	21	0.238
51	A	5	5	1.	21	0.238
52	A	4	4	1.	21	0.19
53	A	4	4	1.	21	0.19
54	A	5	5	1.	21	0.238
55	A	5	5	1.	21	0.238
56	A	6	5	1.	21	0.238
57	A	9	9	1.	21	0.429
58	A	9	9	1.	21	0.429
59	A	12	8	1.	21	0.381
60	A	11	7	1.	21	0.333
61	A	8	8	1.	21	0.381
62	A	8	8	1.	21	0.381
63	A	11	7	1.	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	12	8	1.	21	0.381
65	A	9	9	1.	21	0.429
66	A	9	9	1.	21	0.429
67	A	1	1	1.	13	0.077
68	A	1	1	1.	13	0.077
69	A	2	2	1.	9	0.222
70	A	3	3	1.	13	0.231
71	A	3	2	1.	15	0.133
72	A	4	4	1.	15	0.267
73	A	3	2	1.	15	0.133
74	A	3	2	1.	15	0.133
75	A	4	4	1.	17	0.235
76	A	4	3	1.	17	0.176
77	A	4	3	1.	15	0.2
78	A	3	2	1.	17	0.118
79	A	5	4	1.	17	0.235
80	A	4	3	1.	15	0.2
81	A	2	2	1.	13	0.154
82	A	2	2	1.	15	0.133
83	A	2	2	1.	17	0.118
84	A	2	2	1.	17	0.118
85	A	2	2	1.	17	0.118
86	A	2	2	1.	17	0.118
87	A	2	1	1.	15	0.067
88	A	3	2	1.	17	0.118
89	A	3	2	1.	19	0.105
90	A	3	2	1.	17	0.118
91	A	3	2	1.	19	0.105
92	A	3	2	1.	17	0.118
93	A	2	2	1.	15	0.133
94	A	3	2	1.	15	0.133
95	A	3	3	1.	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.	7	0.286
97	A	3	2	1.	9	0.222
98	A	4	3	1.	9	0.333
99	A	4	3	1.	9	0.333
100	A	3	2	1.	9	0.222
101	A	3	3	1.	13	0.231
102	A	3	2	1.	15	0.133
103	A	4	4	1.	15	0.267
104	A	3	2	1.	15	0.133
105	A	3	2	1.	15	0.133
106	A	4	4	1.	17	0.235
107	A	4	3	1.	17	0.176
108	A	4	3	1.	15	0.2
109	A	3	2	1.	17	0.118
110	A	5	4	1.	17	0.235
111	A	4	3	1.	15	0.2
112	A	2	2	1.	13	0.154
113	A	2	2	1.	15	0.133
114	A	2	2	1.	17	0.118
115	A	2	2	1.	17	0.118
116	A	2	2	1.	17	0.118
117	A	2	2	1.	17	0.118
118	A	2	1	1.	15	0.067
119	A	3	2	1.	17	0.118
120	A	3	2	1.	19	0.105
121	A	2	2	1.	15	0.133
122	A	3	3	1.	17	0.176
123	A	3	2	1.	15	0.133
124	A	3	2	1.	9	0.222
125	A	3	2	1.	9	0.222
126	A	3	2	1.	9	0.222
127	A	4	3	1.	9	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	3	2	1.	9	0.222
129	A	3	2	1.	11	0.182
130	A	3	2	1.	9	0.222
131	A	3	2	1.	13	0.154
132	A	3	2	1.	14	0.143
133	A	3	2	1.	13	0.154
134	A	3	2	1.	14	0.143
135	A	4	3	1.	13	0.231
136	A	4	3	1.	14	0.214
137	A	4	3	1.	13	0.231
138	A	4	3	1.	14	0.214
139	A	3	2	1.	13	0.154
140	A	3	2	1.	14	0.143
141	A	3	2	1.	13	0.154
142	A	3	2	1.	14	0.143
143	A	3	3	1.	13	0.231
144	A	6	6	1.	15	0.4
145	A	9	7	1.	15	0.467
146	A	3	3	1.	13	0.231
147	A	6	6	1.	15	0.4
148	A	9	7	1.	15	0.467
149	A	3	3	1.	13	0.231
150	A	4	4	1.	15	0.267
151	A	5	5	1.	15	0.333
152	A	3	3	1.	13	0.231
153	A	4	4	1.	15	0.267
154	A	5	5	1.	15	0.333
155	A	3	3	1.	13	0.231
156	A	6	6	1.	15	0.4
157	A	9	7	1.	15	0.467
158	A	3	3	1.	13	0.231
159	A	6	6	1.	15	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	9	7	1.	15	0.467
161	A	3	3	1.	13	0.231
162	A	4	4	1.	15	0.267
163	A	5	5	1.	15	0.333
164	A	3	3	1.	13	0.231
165	A	4	4	1.	15	0.267
166	A	5	5	1.	15	0.333
167	A	4	2	1.	13	0.154
168	A	5	2	1.	15	0.133
169	A	6	2	1.	15	0.133
170	A	6	2	1.	17	0.118
171	A	8	2	1.	17	0.118
172	A	10	2	1.	17	0.118
173	A	4	2	1.	13	0.154
174	A	5	2	1.	15	0.133
175	A	6	2	1.	15	0.133
176	A	6	2	1.	17	0.118
177	A	8	2	1.	17	0.118
178	A	10	2	1.	17	0.118
179	A	4	2	1.	13	0.154
180	A	5	2	1.	15	0.133
181	A	6	2	1.	15	0.133
182	A	5	2	1.	15	0.133
183	A	6	2	1.	17	0.118
184	A	8	2	1.	17	0.118
185	A	6	2	1.	15	0.133
186	A	8	2	1.	17	0.118
187	A	10	2	1.	17	0.118
188	A	6	3	1.	13	0.231
189	A	6	3	1.	13	0.231
190	A	6	3	1.	13	0.231
191	A	6	3	1.	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	1	1	1.88	7	0.143
193	A	1	1	1.	7	0.143
194	A	1	1	1.	7	0.143
195	A	4	2	1.	7	0.286
196	A	1	1	1.	7	0.143
197	A	1	1	1.	7	0.143
198	A	1	1	1.	7	0.143
199	A	4	2	1.	7	0.286
200	A	4	3	1.	7	0.429
201	A	5	3	1.	7	0.429
202	A	6	4	1.	7	0.571
203	A	9	4	1.	7	0.571
204	A	10	5	1.	7	0.714
205	A	6	3	1.	7	0.429
206	A	3	2	1.	7	0.286
207	A	3	2	1.	7	0.286
208	A	6	3	1.	7	0.429
209	A	6	3	1.	7	0.429
210	A	7	3	1.	7	0.429
211	A	2	2	1.	7	0.286
212	A	5	5	1.	7	0.714
213	A	4	3	1.	7	0.429
214	A	7	6	1.	7	0.857
215	A	7	4	1.	7	0.571
216	A	2	2	1.	7	0.286
217	A	2	1	1.	7	0.143
218	A	4	2	1.	7	0.286
219	A	4	2	1.	7	0.286
220	A	7	3	1.	7	0.429
221	A	1	1	1.88	7	0.143
222	A	1	1	1.	7	0.143
223	A	1	1	1.	7	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	4	2	1.	7	0.286
225	A	1	1	1.	7	0.143
226	A	1	1	1.	7	0.143
227	A	1	1	1.	7	0.143
228	A	4	2	1.	7	0.286
229	A	4	3	1.	7	0.429
230	A	3	2	1.	7	0.286
231	A	6	4	1.	7	0.571
232	A	6	3	1.	7	0.429
233	A	10	5	1.	7	0.714
234	A	4	3	1.	7	0.429
235	A	9	4	1.	7	0.571
236	A	6	3	1.	7	0.429
237	A	10	4	1.	7	0.571
238	A	7	3	1.	7	0.429
239	A	6	3	1.	7	0.429
240	A	2	2	1.	7	0.286
241	A	2	1	1.	7	0.143
242	A	4	3	1.	7	0.429
243	A	4	2	1.	7	0.286
244	A	7	4	1.	7	0.571
245	A	2	2	1.	7	0.286
246	A	5	5	1.	7	0.714
247	A	4	2	1.	7	0.286
248	A	7	6	1.	7	0.857
249	A	7	3	1.	7	0.429
250	A	5	4	1.	16	0.25
251	A	5	5	1.	16	0.312
252	A	3	3	1.	16	0.188
253	A	3	3	1.	14	0.214
254	A	2	2	1.	13	0.154
255	A	5	5	1.	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	6	6	1.	16	0.375
257	A	7	6	1.	16	0.375
258	A	8	6	1.	16	0.375
259	A	8	3	1.	18	0.167
260	A	7	5	1.	18	0.278
261	A	4	4	1.	18	0.222
262	A	3	2	1.	16	0.125
263	A	2	2	1.	15	0.133
264	A	8	4	1.	18	0.222
265	A	10	5	1.	18	0.278
266	A	12	5	1.	18	0.278
267	A	14	5	1.	18	0.278
268	A	8	3	1.	18	0.167
269	A	9	5	1.	18	0.278
270	A	4	3	1.	18	0.167
271	A	4	3	1.	16	0.188
272	A	2	2	1.	15	0.133
273	A	8	4	1.	18	0.222
274	A	10	5	1.	18	0.278
275	A	12	5	1.	18	0.278
276	A	14	5	1.	18	0.278
277	A	3	3	1.	8	0.375
278	A	4	4	1.	8	0.5
279	A	5	4	1.	8	0.5
280	A	8	3	1.	18	0.167
281	A	7	5	1.	18	0.278
282	A	4	4	1.	18	0.222
283	A	3	2	1.	16	0.125
284	A	2	2	1.	15	0.133
285	A	8	4	1.	18	0.222
286	A	10	5	1.	18	0.278
287	A	12	5	1.	18	0.278

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
288	A	14	5	1.	18	0.278
289	A	5	3	1.	20	0.15
290	A	6	3	1.	20	0.15
291	A	5	3	1.	20	0.15
292	A	4	3	1.	18	0.167
293	A	3	3	1.	17	0.176
294	A	5	4	1.	20	0.2
295	A	6	5	1.	20	0.25
296	A	7	5	1.	20	0.25
297	A	8	5	1.	20	0.25
298	A	11	3	1.	20	0.15
299	A	14	3	1.	20	0.15
300	A	11	3	1.	20	0.15
301	A	8	3	1.	18	0.167
302	A	3	2	1.	17	0.118
303	A	11	4	1.	20	0.2
304	A	14	5	1.	20	0.25
305	A	17	5	1.	20	0.25
306	A	20	5	1.	20	0.25
307	A	8	3	1.	18	0.167
308	A	9	5	1.	18	0.278
309	A	4	3	1.	18	0.167
310	A	4	3	1.	16	0.188
311	A	2	2	1.	15	0.133
312	A	8	4	1.	18	0.222
313	A	10	5	1.	18	0.278
314	A	12	5	1.	18	0.278
315	A	14	5	1.	18	0.278
316	A	11	3	1.	20	0.15
317	A	14	3	1.	20	0.15
318	A	11	3	1.	20	0.15
319	A	8	3	1.	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
320	A	3	2	1.	17	0.118
321	A	11	4	1.	20	0.2
322	A	14	5	1.	20	0.25
323	A	17	5	1.	20	0.25
324	A	20	5	1.	20	0.25
325	A	8	3	1.	20	0.15
326	A	10	3	1.	20	0.15
327	A	8	3	1.	20	0.15
328	A	6	3	1.	18	0.167
329	A	3	2	1.	17	0.118
330	A	8	4	1.	20	0.2
331	A	10	5	1.	20	0.25
332	A	12	5	1.	20	0.25
333	A	14	5	1.	20	0.25
334	A	0	0	0.	0	0.
335	A	6	6	1.	10	0.6
336	A	5	5	1.	10	0.5
337	A	4	4	1.	8	0.5
338	A	1	1	1.	6	0.167
339	A	0	0	0.	0	0.
340	A	0	0	0.	0	0.
341	A	0	0	0.	0	0.
342	A	8	5	1.	16	0.312
343	A	6	4	1.	16	0.25
344	A	2	2	1.	14	0.143
345	A	2	2	1.	13	0.154
346	A	0	0	0.	0	0.
347	A	0	0	0.	0	0.
348	A	0	0	0.	0	0.
349	A	6	6	1.	18	0.333
350	A	3	3	1.	18	0.167
351	A	3	3	1.	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	2	2	1.	15	0.133
353	A	0	0	0.	0	0.
354	A	0	0	0.	0	0.
355	A	0	0	0.	0	0.
356	A	14	8	1.	16	0.5
357	A	11	7	1.	16	0.438
358	A	8	6	1.	14	0.429
359	A	3	3	1.	13	0.231
360	A	0	0	0.	0	0.
361	A	0	0	0.	0	0.
362	A	0	0	0.	0	0.
363	A	7	7	1.	12	0.583
364	A	6	6	1.	12	0.5
365	A	3	3	1.	10	0.3
366	A	2	2	1.	8	0.25
367	A	0	0	0.	0	0.
368	A	0	0	0.	0	0.
369	A	0	0	0.	0	0.
370	A	25	9	1.	18	0.5
371	A	17	7	1.	18	0.389
372	A	12	5	1.	16	0.312
373	A	2	2	1.	15	0.133
374	A	0	0	0.	0	0.
375	A	0	0	0.	0	0.
376	A	0	0	0.	0	0.
377	A	12	12	1.	18	0.667
378	A	9	9	1.	18	0.5
379	A	8	8	1.	16	0.5
380	A	3	2	1.	15	0.133
381	A	0	0	0.	0	0.
382	A	0	0	0.	0	0.
383	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
384	A	13	8	1.	18	0.444
385	A	10	7	1.	18	0.389
386	A	5	5	1.	16	0.312
387	A	3	2	1.	15	0.133
388	A	0	0	0.	0	0.
389	A	0	0	0.	0	0.
390	A	0	0	0.	0	0.
391	A	13	10	1.	12	0.833
392	A	9	8	1.	12	0.667
393	A	7	7	1.	10	0.7
394	A	2	2	1.	8	0.25
395	A	0	0	0.	0	0.
396	A	0	0	0.	0	0.
397	A	0	0	0.	0	0.
398	A	6	6	1.	10	0.6
399	A	5	5	1.	10	0.5
400	A	4	4	1.	8	0.5
401	A	1	1	1.	6	0.167
402	A	0	0	0.	0	0.
403	A	0	0	0.	0	0.
404	A	0	0	0.	0	0.
405	A	14	8	1.	16	0.5
406	A	11	7	1.	16	0.438
407	A	8	6	1.	14	0.429
408	A	3	3	1.	13	0.231
409	A	0	0	0.	0	0.
410	A	0	0	0.	0	0.
411	A	0	0	0.	0	0.
412	A	12	12	1.	18	0.667
413	A	9	9	1.	18	0.5
414	A	8	8	1.	16	0.5
415	A	3	2	1.	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
416	A	0	0	0.	0	0.
417	A	0	0	0.	0	0.
418	A	6	5	1.	10	0.5
419	A	11	10	1.	12	0.833
420	A	12	10	1.	12	0.833
421	A	16	10	1.	10	1.
422	A	19	11	1.	12	0.917
423	A	26	15	1.	12	1.25
424	A	0	0	0.	0	0.
425	A	8	5	1.	16	0.312
426	A	6	4	1.	16	0.25
427	A	2	2	1.	14	0.143
428	A	2	2	1.	13	0.154
429	A	0	0	0.	0	0.
430	A	0	0	0.	0	0.
431	A	0	0	0.	0	0.
432	A	7	7	1.	12	0.583
433	A	6	6	1.	12	0.5
434	A	3	3	1.	10	0.3
435	A	2	2	1.	8	0.25
436	A	0	0	0.	0	0.
437	A	0	0	0.	0	0.
438	A	0	0	0.	0	0.
439	A	13	8	1.	18	0.444
440	A	10	7	1.	18	0.389
441	A	5	5	1.	16	0.312
442	A	3	2	1.	15	0.133
443	A	0	0	0.	0	0.
444	A	0	0	0.	0	0.
445	A	0	0	0.	0	0.
446	A	6	6	1.	18	0.333
447	A	3	3	1.	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	3	3	1.	16	0.188
449	A	2	2	1.	15	0.133
450	A	0	0	0.	0	0.
451	A	0	0	0.	0	0.
452	A	0	0	0.	0	0.
453	A	25	9	1.	18	0.5
454	A	17	7	1.	18	0.389
455	A	12	5	1.	16	0.312
456	A	2	2	1.	15	0.133
457	A	0	0	0.	0	0.
458	A	0	0	0.	0	0.
459	A	0	0	0.	0	0.
460	A	13	10	1.	12	0.833
461	A	9	8	1.	12	0.667
462	A	7	7	1.	10	0.7
463	A	2	2	1.	8	0.25
464	A	0	0	0.	0	0.
465	A	0	0	0.	0	0.
466	A	0	0	0.	0	0.
467	A	10	6	1.	16	0.375
468	A	8	5	1.	16	0.312
469	A	6	4	1.	14	0.286
470	A	2	2	1.	13	0.154
471	A	0	0	0.	0	0.
472	A	0	0	0.	0	0.
473	A	0	0	0.	0	0.
474	A	21	13	1.	18	0.722
475	A	17	14	1.	18	0.778
476	A	10	10	1.	16	0.625
477	A	3	3	1.	15	0.2
478	A	0	0	0.	0	0.
479	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
480	A	0	0	0.	0	0.
481	A	20	16	1.	18	0.889
482	A	15	12	1.	18	0.667
483	A	11	10	1.	16	0.625
484	A	3	2	1.	15	0.133
485	A	0	0	0.	0	0.
486	A	0	0	0.	0	0.
487	A	0	0	0.	0	0.
488	A	21	13	1.	18	0.722
489	A	17	14	1.	18	0.778
490	A	10	10	1.	16	0.625
491	A	3	3	1.	15	0.2
492	A	0	0	0.	0	0.
493	A	0	0	0.	0	0.
494	A	0	0	0.	0	0.
495	A	7	7	1.	20	0.35
496	A	6	6	1.	20	0.3
497	A	3	3	1.	18	0.167
498	A	3	2	1.	17	0.118
499	A	0	0	0.	0	0.
500	A	0	0	0.	0	0.
501	A	0	0	0.	0	0.
502	A	29	18	1.	20	0.9
503	A	13	12	1.	18	0.667
504	A	4	4	1.	17	0.235
505	A	0	0	0.	0	0.
506	A	0	0	0.	0	0.
507	A	0	0	0.	0	0.
508	A	20	16	1.	18	0.889
509	A	15	12	1.	18	0.667
510	A	11	10	1.	16	0.625
511	A	3	2	1.	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
512	A	0	0	0.	0	0.
513	A	0	0	0.	0	0.
514	A	0	0	0.	0	0.
515	A	40	19	1.	20	0.95
516	A	29	19	1.	20	0.95
517	A	13	12	1.	18	0.667
518	A	4	4	1.	17	0.235
519	A	0	0	0.	0	0.
520	A	0	0	0.	0	0.
521	A	0	0	0.	0	0.
522	A	16	9	1.	20	0.45
523	A	10	7	1.	20	0.35
524	A	7	5	1.	18	0.278
525	A	4	3	1.	17	0.176
526	A	0	0	0.	0	0.
527	A	0	0	0.	0	0.
528	A	4	3	1.	18	0.167
529	A	3	3	1.	18	0.167
530	A	3	3	1.	18	0.167
531	A	2	2	1.	18	0.111
532	A	2	2	1.	18	0.111
533	A	3	3	1.	18	0.167
534	A	3	3	1.	18	0.167
535	A	4	3	1.	18	0.167
536	A	5	4	1.	18	0.222
537	A	4	4	1.	18	0.222
538	A	4	4	1.	18	0.222
539	A	3	3	1.	18	0.167
540	A	3	3	1.	18	0.167
541	A	4	4	1.	18	0.222
542	A	4	4	1.	18	0.222
543	A	5	4	1.	18	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
544	A	5	4	1.	18	0.222
545	A	4	4	1.	18	0.222
546	A	4	4	1.	18	0.222
547	A	3	3	1.	18	0.167
548	A	3	3	1.	18	0.167
549	A	4	4	1.	18	0.222
550	A	4	4	1.	18	0.222
551	A	5	4	1.	18	0.222
552	A	5	4	1.	18	0.222
553	A	4	4	1.	18	0.222
554	A	4	4	1.	18	0.222
555	A	3	3	1.	18	0.167
556	A	3	3	1.	18	0.167
557	A	4	4	1.	18	0.222
558	A	4	4	1.	18	0.222
559	A	5	4	1.	18	0.222
560	A	3	3	1.	9	0.333
561	A	4	4	1.	9	0.444
562	A	5	5	1.	9	0.556
563	A	3	3	1.	9	0.333
564	A	4	4	1.	9	0.444
565	A	5	5	1.	9	0.556
566	A	7	6	1.	14	0.429
567	A	7	6	1.	15	0.4
568	A	6	5	1.	14	0.357
569	A	6	5	1.	15	0.333
570	A	3	3	1.	17	0.176
571	A	3	3	1.	16	0.188
572	A	5	5	1.	15	0.333
573	A	6	6	1.	15	0.4
574	A	3	3	1.	17	0.176
575	A	4	4	1.	17	0.235

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
576	A	4	4	1.	17	0.235
577	A	3	3	1.	17	0.176
578	A	6	6	1.	17	0.353
579	A	7	7	1.	17	0.412
580	A	3	2	1.	9	0.222
581	A	2	2	1.	11	0.182
582	A	2	1	1.	11	0.091
583	A	3	2	1.	11	0.182
584	A	3	2	1.	11	0.182
585	A	2	2	1.	11	0.182
586	A	1	1	1.	11	0.091
587	A	3	3	1.	11	0.273
588	A	2	2	1.	11	0.182
589	A	4	3	1.	11	0.273
590	A	2	2	1.	13	0.154
591	A	3	3	1.	13	0.231
592	A	3	3	1.	13	0.231
593	A	2	2	1.	13	0.154
594	A	3	3	1.	13	0.231
595	A	3	3	1.	13	0.231
596	A	3	2	1.	17	0.118
597	A	1	1	1.	19	0.053
598	A	1	1	1.	19	0.053
599	A	1	1	1.	19	0.053
600	A	1	1	1.	19	0.053
601	A	1	1	1.	19	0.053
602	A	1	1	1.	19	0.053
603	A	1	1	1.	21	0.048
604	A	1	1	1.	21	0.048
605	A	3	2	1.	18	0.111
606	A	1	1	1.	20	0.05
607	A	1	1	1.	20	0.05

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
608	A	1	1	1.	20	0.05
609	A	1	1	1.	20	0.05
610	A	1	1	1.	20	0.05
611	A	1	1	1.	20	0.05
612	A	1	1	1.	22	0.045
613	A	1	1	1.	22	0.045
614	A	8	8	1.	11	0.727
615	A	4	4	1.	11	0.364
616	A	7	7	1.	11	0.636
617	A	4	3	1.	11	0.273
618	A	3	2	1.	9	0.222
619	A	3	3	1.	11	0.273
620	A	6	6	1.	11	0.546
621	A	4	3	1.	11	0.273
622	A	8	8	1.	11	0.727
623	A	4	3	1.	11	0.273
624	A	4	3	1.	11	0.273
625	A	5	4	1.	11	0.364
626	A	4	3	1.	11	0.273
627	A	4	4	1.	11	0.364
628	A	3	2	1.	9	0.222
629	A	3	3	1.	11	0.273
630	A	3	3	1.	11	0.273
631	A	4	3	1.	11	0.273
632	A	4	3	1.	11	0.273
633	A	4	3	1.	11	0.273
634	A	4	3	1.	11	0.273
635	A	5	4	1.	11	0.364
636	A	4	3	1.	11	0.273
637	A	4	4	1.	11	0.364
638	A	3	2	1.	9	0.222
639	A	3	3	1.	11	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
640	A	3	3	1.	11	0.273
641	A	4	3	1.	11	0.273
642	A	4	3	1.	11	0.273
643	A	4	3	1.	11	0.273
644	A	8	8	1.	11	0.727
645	A	4	4	1.	11	0.364
646	A	7	7	1.	11	0.636
647	A	4	3	1.	11	0.273
648	A	3	2	1.	9	0.222
649	A	3	3	1.	11	0.273
650	A	5	5	1.	11	0.454
651	A	4	3	1.	11	0.273
652	A	7	7	1.	11	0.636
653	A	4	3	1.	11	0.273
654	A	4	3	1.	7	0.429
655	A	5	4	1.	7	0.571
656	A	4	3	1.	7	0.429
657	A	4	4	1.	7	0.571
658	A	3	2	1.	5	0.4
659	A	3	3	1.	7	0.429
660	A	3	3	1.	7	0.429
661	A	4	3	1.	7	0.429
662	A	4	3	1.	7	0.429
663	A	4	3	1.	7	0.429
664	A	4	3	1.	9	0.333
665	A	5	4	1.	9	0.444
666	A	4	3	1.	9	0.333
667	A	4	4	1.	9	0.444
668	A	3	2	1.	7	0.286
669	A	3	3	1.	9	0.333
670	A	3	3	1.	9	0.333
671	A	4	3	1.	9	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
672	A	4	3	1.	9	0.333
673	A	4	3	1.	9	0.333
674	A	3	2	1.	5	0.4
675	A	4	3	1.	7	0.429
676	A	6	5	1.	7	0.714
677	A	4	4	1.	9	0.444
678	A	5	5	1.	9	0.556
679	A	6	6	1.	9	0.667
680	A	3	2	1.	7	0.286
681	A	4	3	1.	9	0.333
682	A	6	5	1.	9	0.556
683	A	3	3	1.	11	0.273
684	A	4	4	1.	11	0.364
685	A	5	5	1.	11	0.454
686	A	6	6	1.33	7	0.857
687	A	6	6	1.2	9	0.667
688	A	2	2	1.	14	0.143
689	A	4	4	1.	16	0.25
690	A	5	5	1.	16	0.312
691	A	2	2	1.	14	0.143
692	A	4	4	1.	16	0.25
693	A	5	5	1.	16	0.312
694	A	5	4	1.	14	0.286
695	A	5	4	1.	14	0.286
696	A	3	3	1.	14	0.214
697	A	4	4	1.	16	0.25
698	A	16	10	1.54	16	0.625
699	A	3	3	1.	14	0.214
700	A	4	4	1.	16	0.25
701	A	8	4	1.45	16	0.25
702	A	2	2	1.	14	0.143
703	A	5	5	1.	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
704	A	2	2	1.	14	0.143
705	A	5	5	1.	16	0.312
706	A	5	5	1.	16	0.312
707	A	7	7	1.	18	0.389
708	A	9	8	1.	18	0.444
709	A	7	7	1.	18	0.389
710	A	10	8	1.	20	0.4
711	A	13	8	1.	20	0.4
712	A	9	8	1.	18	0.444
713	A	13	9	1.	20	0.45
714	A	17	9	1.	20	0.45
715	A	6	5	1.	16	0.312
716	A	13	8	1.	18	0.444
717	A	17	13	1.	18	0.722
718	A	13	8	1.	18	0.444
719	A	21	10	1.	20	0.5
720	A	33	12	1.	20	0.6
721	A	17	13	1.	18	0.722
722	A	33	12	1.	20	0.6
723	A	48	12	1.	20	0.6
724	A	3	3	1.	18	0.167
725	A	3	3	1.	18	0.167
726	A	4	4	1.	18	0.222
727	A	3	3	1.	18	0.167
728	A	3	3	1.	18	0.167
729	A	4	4	1.	18	0.222
730	A	1	1	1.	15	0.067
731	A	1	1	1.	15	0.067
732	A	1	1	1.	21	0.048
733	A	1	1	1.	21	0.048
734	A	3	3	1.	21	0.143
735	A	3	3	1.	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
736	A	3	3	1.	22	0.136
737	A	3	3	1.	22	0.136
738	A	4	4	1.	22	0.182
739	A	5	4	1.	12	0.333
740	A	4	3	1.	12	0.25
741	A	3	2	1.	10	0.2
742	A	3	3	1.	12	0.25
743	A	5	5	1.	12	0.417
744	A	5	5	1.	12	0.417
745	A	6	6	1.	12	0.5
746	A	5	4	1.	12	0.333
747	A	4	3	1.	12	0.25
748	A	3	2	1.	10	0.2
749	A	2	2	1.	12	0.167
750	A	4	4	1.	12	0.333
751	A	4	4	1.	12	0.333
752	A	5	5	1.	12	0.417
753	A	6	3	1.	24	0.125
754	A	5	3	1.	24	0.125
755	A	4	3	1.	24	0.125
756	A	3	2	1.	22	0.091
757	A	1	1	1.	24	0.042
758	A	2	2	1.	24	0.083
759	A	3	2	1.	24	0.083
760	A	4	2	1.	24	0.083
761	A	7	7	1.	14	0.5
762	A	6	6	1.	14	0.429
763	A	2	2	1.	14	0.143
764	A	2	2	1.	14	0.143
765	A	3	3	1.	14	0.214
766	A	7	7	1.	14	0.5
767	A	8	7	1.	14	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
768	A	3	2	1.	26	0.077
769	A	2	2	1.	26	0.077
770	A	1	1	1.	26	0.038
771	A	3	3	1.	26	0.115
772	A	4	4	1.	26	0.154
773	A	5	4	1.	26	0.154
774	A	3	2	1.	28	0.071
775	A	2	2	1.	28	0.071
776	A	1	1	1.	28	0.036
777	A	3	3	1.	28	0.107
778	A	4	4	1.	28	0.143
779	A	5	4	1.	28	0.143
780	A	5	5	1.	12	0.417
781	A	5	5	1.	12	0.417
782	A	4	4	1.	15	0.267
783	A	1	1	1.	11	0.091
784	A	4	4	1.	15	0.267
785	A	10	9	1.	17	0.529
786	A	4	4	1.	15	0.267
787	A	4	4	1.	15	0.267
788	A	9	7	1.	17	0.412
789	A	4	4	1.	19	0.21
790	A	4	4	1.	19	0.21
791	A	5	5	1.	19	0.263
792	A	4	4	1.	19	0.21
793	A	4	4	1.	19	0.21
794	A	5	5	1.	19	0.263
795	A	4	4	1.	22	0.182
796	A	4	4	1.	22	0.182
797	A	5	5	1.	22	0.227
798	A	4	4	1.	23	0.174
799	A	4	4	1.	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
800	A	5	5	1.	23	0.217
801	A	1	1	1.	32	0.031
802	A	1	1	1.	19	0.053
803	A	1	1	1.	19	0.053
804	A	1	1	1.	23	0.043
805	A	1	1	1.	20	0.05
806	A	1	1	1.	20	0.05
807	A	1	1	1.	24	0.042
808	A	2	1	1.	11	0.091
809	A	2	1	1.	11	0.091
810	A	4	3	1.	11	0.273
811	A	2	2	1.	13	0.154
812	A	2	2	1.	13	0.154
813	A	2	2	1.	13	0.154
814	A	2	2	1.	11	0.182
815	A	2	2	1.	11	0.182
816	A	2	2	1.	11	0.182
817	A	4	2	1.	13	0.154
818	A	6	4	1.	13	0.308
819	A	6	4	1.	13	0.308
820	A	4	2	1.	11	0.182
821	A	6	4	1.	11	0.364
822	A	6	4	1.	11	0.364
823	A	2	2	1.	13	0.154
824	A	2	2	1.	13	0.154
825	A	2	2	1.	13	0.154
826	A	7	4	1.	14	0.286
827	A	8	4	1.	17	0.235
828	A	9	5	1.	19	0.263
829	A	10	6	1.	19	0.316
830	A	3	3	1.	27	0.111
831	A	7	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
832	A	5	3	1.	14	0.214
833	A	6	3	1.	17	0.176
834	A	7	4	1.	19	0.21
835	A	8	5	1.	19	0.263
836	A	3	3	1.	27	0.111
837	A	5	3	1.	21	0.143
838	A	4	3	1.	20	0.15
839	A	4	3	1.	20	0.15
840	A	6	4	1.	16	0.25
841	A	6	4	1.	16	0.25
842	A	6	4	1.	16	0.25
843	A	8	5	1.	18	0.278
844	A	10	6	1.	18	0.333
845	A	5	5	1.	16	0.312
846	A	6	6	1.	18	0.333
847	A	7	7	1.	18	0.389
848	A	10	10	1.	16	0.625
849	A	17	14	1.	18	0.778
850	A	21	13	1.	18	0.722
851	A	12	11	1.	16	0.688
852	A	16	13	1.	18	0.722
853	A	21	17	1.	18	0.944
854	A	4	4	1.	18	0.222
855	A	3	3	1.	18	0.167
856	A	2	2	1.	18	0.111
857	A	3	2	1.	16	0.125
858	A	4	4	1.	18	0.222
859	A	6	6	1.	18	0.333
860	A	7	7	1.	18	0.389
861	A	8	8	1.	20	0.4
862	A	7	7	1.	20	0.35
863	A	3	3	1.	20	0.15

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
864	A	3	3	1.	20	0.15
865	A	5	5	1.	20	0.25
866	A	8	8	1.	20	0.4
867	A	13	8	1.	14	0.571
868	A	11	7	1.	14	0.5
869	A	9	6	1.	12	0.5
870	A	0	0	0.	0	0.
871	A	2	2	1.	18	0.111
872	A	4	3	1.	18	0.167
873	A	5	4	1.	18	0.222
874	A	3	2	1.	16	0.125
875	A	4	4	1.	16	0.25
876	A	5	4	1.	18	0.222
877	A	5	4	1.	18	0.222
878	A	2	2	1.	16	0.125
879	A	2	2	1.	16	0.125
880	A	1	1	1.	14	0.071
881	A	1	1	1.	14	0.071
882	A	1	1	1.	16	0.062
883	A	2	2	1.	16	0.125
884	A	2	2	1.	18	0.111
885	A	2	2	1.	16	0.125
886	A	2	2	1.	16	0.125
887	A	1	1	1.	14	0.071
888	A	1	1	1.	14	0.071
889	A	1	1	1.	16	0.062
890	A	2	2	1.	16	0.125
891	A	2	2	1.	18	0.111
892	A	2	2	1.	18	0.111
893	A	8	6	1.	25	0.24
894	A	6	5	1.	23	0.217
895	A	2	2	1.	25	0.08

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
896	A	3	3	1.	25	0.12
897	A	8	6	1.	22	0.273
898	A	6	5	1.	20	0.25
899	A	2	2	1.	22	0.091
900	A	3	3	1.	22	0.136
901	A	4	3	1.	22	0.136
902	A	5	4	1.	22	0.182
903	A	4	3	1.	20	0.15
904	A	3	3	1.	14	0.214
905	A	5	4	1.	20	0.2
906	A	5	5	1.	22	0.227
907	A	5	4	1.	24	0.167
908	A	4	3	1.	24	0.125
909	A	5	4	1.	22	0.182
910	A	5	4	1.	20	0.2
911	A	5	4	1.	16	0.25
912	A	5	4	1.	22	0.182
913	A	4	3	1.	24	0.125
914	A	5	4	1.	24	0.167
915	A	4	3	1.	22	0.136
916	A	5	4	1.	22	0.182
917	A	5	4	1.	22	0.182
918	A	7	6	1.	16	0.375
919	A	5	4	1.	24	0.167
920	A	4	3	1.	24	0.125
921	A	4	3	1.	22	0.136
922	A	4	3	1.	16	0.188
923	A	5	5	1.	22	0.227
924	A	5	4	1.	24	0.167
925	A	4	3	1.	26	0.115
926	A	4	3	1.	26	0.115
927	A	4	3	1.	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
928	A	5	4	1.	22	0.182
929	A	4	3	1.	18	0.167
930	A	6	6	1.	24	0.25
931	A	5	4	1.	26	0.154
932	A	4	3	1.	26	0.115
933	A	5	4	1.	24	0.167
934	A	5	4	1.	24	0.167
935	A	6	5	1.	24	0.208
936	A	4	3	1.	18	0.167
937	A	12	9	1.	12	0.75
938	A	13	10	1.	14	0.714
939	A	13	10	1.	14	0.714
940	A	14	11	1.	16	0.688
941	A	6	6	1.	12	0.5
942	A	7	7	1.	14	0.5
943	A	7	7	1.	14	0.5
944	A	8	8	1.	16	0.5
945	A	4	2	1.	22	0.091
946	A	4	2	1.	22	0.091
947	A	3	3	1.	20	0.15
948	A	1	1	1.	14	0.071
949	A	4	3	1.	14	0.214
950	A	4	2	1.	20	0.1
951	A	4	2	1.	22	0.091
952	A	5	2	1.	24	0.083
953	A	4	3	1.	24	0.125
954	A	4	2	1.	22	0.091
955	A	2	2	1.	16	0.125
956	A	6	4	1.	20	0.2
957	A	5	3	1.	16	0.188
958	A	5	2	1.	22	0.091
959	A	4	2	1.	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
960	A	5	2	1.	24	0.083
961	A	4	2	1.	22	0.091
962	A	2	2	1.	16	0.125
963	A	8	4	1.	22	0.182
964	A	7	4	1.	22	0.182
965	A	6	3	1.	16	0.188
966	A	10	5	1.	56	0.089
967	A	2	2	1.	17	0.118
968	A	2	2	1.	22	0.091
969	A	2	2	1.	22	0.091
970	A	2	2	1.	17	0.118
971	A	2	2	1.	22	0.091
972	A	2	2	1.	22	0.091
973	A	2	2	1.	17	0.118
974	A	2	2	1.	22	0.091
975	A	2	2	1.	22	0.091
976	A	2	2	1.	17	0.118
977	A	2	2	1.	22	0.091
978	A	2	2	1.	22	0.091
979	A	2	2	1.	13	0.154
980	A	2	2	1.	13	0.154
981	A	2	2	1.	13	0.154
982	A	2	2	1.	13	0.154
983	A	3	1	1.	17	0.059
984	A	4	3	1.	23	0.13
985	A	3	3	1.	17	0.176
986	A	3	2	1.	16	0.125
987	A	3	2	1.	18	0.111
988	A	7	7	1.	15	0.467
989	A	3	3	1.	19	0.158
990	A	3	2	1.	19	0.105
991	A	3	2	1.	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
992	A	3	2	1.	21	0.095
993	A	2	2	1.	17	0.118
994	A	4	3	1.	19	0.158
995	A	5	5	1.	19	0.263
996	A	2	2	1.	11	0.182
997	A	3	2	1.	17	0.118
998	A	2	2	1.	17	0.118
999	A	2	2	1.	17	0.118
1000	A	3	3	1.	15	0.2
1001	A	3	3	1.	15	0.2
1002	A	3	3	1.	15	0.2
1003	A	4	3	1.	15	0.2
1004	A	4	3	1.	20	0.15
1005	A	4	3	1.	19	0.158
1006	A	4	3	1.	24	0.125
1007	A	4	3	1.	21	0.143
1008	A	4	3	1.	20	0.15
1009	A	4	3	1.	19	0.158
1010	A	4	3	1.	24	0.125
1011	A	4	3	1.	21	0.143
1012	A	1	3	1.	8	0.375
1013	A	1	2	1.	8	0.25
1014	A	0	0	0.	0	0.
1015	A	0	0	0.	0	0.
1016	A	0	0	0.	0	0.
1017	A	0	0	0.	0	0.
1018	A	3	2	1.	13	0.154
1019	A	2	2	1.	13	0.154
1020	A	2	2	1.	13	0.154
1021	A	2	2	1.	11	0.182
1022	A	3	2	1.	19	0.105
1023	A	3	2	1.	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1024	A	3	2	1.	21	0.095
1025	A	3	2	1.	21	0.095
1026	A	4	3	1.	17	0.176
1027	A	4	3	1.	17	0.176
1028	A	2	2	1.	17	0.118
1029	A	3	3	1.	16	0.188
1030	A	3	3	1.	18	0.167
1031	A	1	1	1.	18	0.056
1032	A	3	3	1.	18	0.167
1033	A	9	7	1.31	15	0.467
1034	A	8	7	1.31	15	0.467
1035	A	4	2	1.	15	0.133
1036	A	4	2	1.	15	0.133
1037	A	3	3	1.	21	0.143
1038	A	3	3	1.	21	0.143
1039	A	12	7	1.	8	0.875
1040	A	5	4	1.	10	0.4
1041	A	19	6	1.	10	0.6
1042	A	8	5	1.	18	0.278
1043	A	6	5	1.	8	0.625
1044	A	5	3	1.	15	0.2
1045	A	7	4	1.	22	0.182
1046	A	4	3	1.	12	0.25
1047	A	6	5	1.	15	0.333
1048	A	5	4	1.	15	0.267
1049	A	3	2	1.	13	0.154
1050	A	4	4	1.	23	0.174
1051	A	4	4	1.	25	0.16
1052	A	6	3	1.	39	0.077
1053	A	5	3	1.	39	0.077
1054	A	3	2	1.	39	0.051
1055	A	1	1	1.	31	0.032

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1056	A	2	1	1.	31	0.032
1057	A	2	1	1.	39	0.026
1058	A	5	3	1.	39	0.077
1059	A	6	3	1.	39	0.077

Chapter 3

Listing of integrals

3.1 $\int \frac{2}{-1+3\cosh(4+6x)}\, dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0155441, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 206}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2}{-1+3\cosh(4+6x)}\,dx &= 2\int \frac{1}{-1+3\cosh(4+6x)}\,dx\\ &= -\left(\frac{2}{3}i\,\text{Subst}\left(\int \frac{1}{2-4x^2}\,dx, x, \tan\left(\frac{1}{2}(4i+6ix)\right)\right)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [B] time = 0.0704974, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{\left(3+2e^4+3e^8\right)\tanh(3x)+3\left(e^8-1\right)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

Maple [A] time = 0.014, size = 17, normalized size = 0.8

$$\frac{\arctan\left(\sqrt{2}\tanh\left(2+3\,x\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(-1+3*cosh(4+6*x)),x)
```

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Maxima [A] time = 1.48378, size = 28, normalized size = 1.27

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Fricas [B] time = 2.04042, size = 120, normalized size = 5.45

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}\cosh\left(6x+4\right)+\frac{3}{4}\sqrt{2}\sinh\left(6x+4\right)-\frac{1}{4}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*arctan(3/4*sqrt(2)*cosh(6*x + 4) + 3/4*sqrt(2)*sinh(6*x + 4) -
1/4*sqrt(2))
```

Sympy [A] time = 0.322538, size = 19, normalized size = 0.86

$$\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tanh\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*tanh(3*x + 2))/6
```

<u>Giac</u> [A] time = 1.186, size = 28, normalized size = 1.27

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.2 $\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0212245, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tanh(2+3x)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [B] time = 0.0665493, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{(3+2e^4+3e^8)\tanh(3x)+3(e^8-1)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

Maple [B] time = 0.078, size = 156, normalized size = 7.1

$$-\frac{\sqrt{6}}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{2\sqrt{3}+2\sqrt{2}}\right)-\frac{2}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{2\sqrt{3}+2\sqrt{2}}\right)+\frac{\sqrt{6}}{6\sqrt{3}-6\sqrt{2}}\arctan\left(2\frac{\tanh(1}{2\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x)
```

```
[Out] -1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1
/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2)
))+1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^
(1/2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))
```

Maxima [A] time = 1.52181, size = 28, normalized size = 1.27

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Fricas [B] time = 2.08918, size = 147, normalized size = 6.68

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3x+2\right)+2\sqrt{2}\sinh\left(3x+2\right)}{2\left(\cosh\left(3x+2\right)-\sinh\left(3x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [B] time = 9.60681, size = 280, normalized size = 12.73

$$\frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{66\sqrt{5-2\sqrt{6}}+27\sqrt{6}\sqrt{5-2\sqrt{6}}}+\frac{5\,\mathrm{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{66\sqrt{5-2\sqrt{6}}+27\sqrt{6}\sqrt{5-2\sqrt{6}}}-\frac{5\sqrt{5-2\sqrt{6}}\sqrt{2\sqrt{6}+5}\,\mathrm{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{6}+5}}\right)}{66\sqrt{5-2\sqrt{6}}+27\sqrt{6}\sqrt{5-2\sqrt{6}}}-2\sqrt{6}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)**2+2*sinh(2+3*x)**2),x)
```

```
[Out] 2*sqrt(6)*atan(tanh(3*x/2 + 1)/sqrt(5 - 2*sqrt(6)))/(66*sqrt(5 - 2*sqrt(6))
 + 27*sqrt(6)*sqrt(5 - 2*sqrt(6))) + 5*atan(tanh(3*x/2 + 1)/sqrt(5 - 2*sqrt
(6)))/(66*sqrt(5 - 2*sqrt(6)) + 27*sqrt(6)*sqrt(5 - 2*sqrt(6))) - 5*sqrt(5
- 2*sqrt(6))*sqrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6) + 5))/
(66*sqrt(5 - 2*sqrt(6)) + 27*sqrt(6)*sqrt(5 - 2*sqrt(6))) - 2*sqrt(6)*sqrt(
5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6) + 5)
)/(66*sqrt(5 - 2*sqrt(6)) + 27*sqrt(6)*sqrt(5 - 2*sqrt(6)))
```

Giac [A] time = 1.27923, size = 28, normalized size = 1.27

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.3 $\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0398909, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tanh(2+3x)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.0781197, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{\left(3+2e^4+3e^8\right)\tanh(3x)+3\left(e^8-1\right)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

Maple [B] time = 0.066, size = 156, normalized size = 7.1

$$-\frac{\sqrt{6}}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{2\sqrt{3}+2\sqrt{2}}\right)-\frac{2}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{2\sqrt{3}+2\sqrt{2}}\right)+\frac{\sqrt{6}}{6\sqrt{3}-6\sqrt{2}}\arctan\left(2\frac{\tanh(1}{2\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x)
```

```
[Out] -1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1
/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2)
))+1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^
(1/2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))
```

Maxima [A] time = 1.55416, size = 28, normalized size = 1.27

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Fricas [B] time = 2.10787, size = 147, normalized size = 6.68

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3\,x+2\right)+2\,\sqrt{2}\sinh\left(3\,x+2\right)}{2\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^2\left(3x+2\right)}{2\tanh^2\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)**2/(1+2*tanh(2+3*x)**2),x)
```

```
[Out] Integral(sech(3*x + 2)**2/(2*tanh(3*x + 2)**2 + 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.4 $\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0392334, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.0728385, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{(3+2e^4+3e^8)\tanh(3x)+3(e^8-1)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

Maple [B] time = 0.06, size = 156, normalized size = 7.1

$$-\frac{\sqrt{6}}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2\,x)}{2\sqrt{3}+2\sqrt{2}}\right)-\frac{2}{6\sqrt{3}+6\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2\,x)}{2\sqrt{3}+2\sqrt{2}}\right)+\frac{\sqrt{6}}{6\sqrt{3}-6\sqrt{2}}\arctan\left(2\frac{\tanh(1}{2\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(2+coth(2+3*x)^2),x)
```

```
[Out] -1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1
/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2)
))+1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^
(1/2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))
```

Maxima [A] time = 1.70741, size = 28, normalized size = 1.27

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Fricas [B] time = 2.16255, size = 147, normalized size = 6.68

$$-\frac{1}{6}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3\,x+2\right)+2\,\sqrt{2}\sinh\left(3\,x+2\right)}{2\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^2\left(3x+2\right)}{\coth^2\left(3x+2\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(2+coth(2+3*x)**2),x)
```

```
[Out] Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 + 2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.5 $\int \frac{\mathbf{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] -ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0386269, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.100644, size = 42, normalized size = 1.91

$$-\frac{\tanh^{-1}\left(\frac{(1+6e^4+e^8)\tanh(3x)+e^8-1}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[(-1 + E^8 + (1 + 6*E^4 + E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*Sqrt[
2])
```

Maple [B] time = 0.052, size = 44, normalized size = 2.

$$-\frac{\sqrt{2}}{6}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(1+3/2\,x)-2)\right)-\frac{\sqrt{2}}{6}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(1+3/2\,x)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(2-coth(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/4*(2*tanh(1+3/2*x)-2)*2^(1/2))-1/6*2^(1/2)*arctanh(1
/4*(2*tanh(1+3/2*x)+2)*2^(1/2))
```

Maxima [B] time = 1.60329, size = 93, normalized size = 4.23

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}+1}{\sqrt{2}+e^{(-3x-2)}-1}\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}-1}{\sqrt{2}+e^{(-3x-2)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) + 1)/(sqrt(2) + e^(-3*x - 2) - 1
)) + 1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) - 1)/(sqrt(2) + e^(-3*x - 2)
 + 1))
```

Fricas [B] time = 2.11168, size = 263, normalized size = 11.95

$$\frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^{2}-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh\left(3x+2\right)^{2}-2\sqrt{}}{\cosh\left(3x+2\right)^{2}+\sinh\left(3x+2\right)^{2}-3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 - 2*sqrt(2) -
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{csch}^{2}(3x+2)}{\coth^{2}(3x+2)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(2-coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 - 2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.6 $\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0401457, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.105043, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{\left(3-2e^4+3e^8\right)\tanh(3x)+3\left(e^8-1\right)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 - 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

Maple [B] time = 0.093, size = 132, normalized size = 6.

$$\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{\tanh(1+3/2x)}{\sqrt{6}-\sqrt{2}}\right)-\frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\arctan\left(2\frac{\tanh(1-}{\sqrt{6}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)-2^(1/2)))-1/3
/(6^(1/2)-2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6
^(1/2)+2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)+2^(1/2)))-1/3/(6^(1/2)+2^(1
/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)+2^(1/2)))
```

Maxima [A] time = 1.64696, size = 28, normalized size = 1.27

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) + 1))
```

Fricas [B] time = 2.10221, size = 147, normalized size = 6.68

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{2\sqrt{2}\cosh(3x+2)+\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(2*sqrt(2)*cosh(3*x + 2) + sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(1+2*coth(2+3*x)**2),x)
```

```
[Out] Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 + 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.7 $\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0394867, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$-\frac{\tanh^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.104643, size = 42, normalized size = 1.91

$$\frac{\tanh^{-1}\left(\frac{(1-6e^4+e^8)\tanh(3x)+e^8-1}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTanh[(-1 + E^8 + (1 - 6*E^4 + E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*Sqrt[2
])
```

Maple [B] time = 0.054, size = 102, normalized size = 4.6

$$-\frac{\sqrt{2}}{24}\ln\left(\left(\left(\tanh\left(1+\frac{3x}{2}\right)\right)^2+\sqrt{2}\tanh\left(1+\frac{3x}{2}\right)+1\right)\left(\left(\tanh\left(1+\frac{3x}{2}\right)\right)^2-\sqrt{2}\tanh\left(1+\frac{3x}{2}\right)+1\right)^{-1}\right)+\frac{\sqrt{2}}{24}\ln\left(\left(\left(\tan\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x)
```

```
[Out] -1/24*2^(1/2)*ln((tanh(1+3/2*x)^2+2^(1/2)*tanh(1+3/2*x)+1)/(tanh(1+3/2*x)^2
-2^(1/2)*tanh(1+3/2*x)+1))+1/24*2^(1/2)*ln((tanh(1+3/2*x)^2-2^(1/2)*tanh(1+
3/2*x)+1)/(tanh(1+3/2*x)^2+2^(1/2)*tanh(1+3/2*x)+1))
```

Maxima [B] time = 1.67463, size = 51, normalized size = 2.32

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-6x-4)}-3}{2\sqrt{2}+e^{(-6x-4)}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-6*x - 4) - 3)/(2*sqrt(2) + e^(-6*x - 4)
+ 3))
```

Fricas [B] time = 2.09315, size = 263, normalized size = 11.95

$$\frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^{2}-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh\left(3x+2\right)^{2}+2}{\cosh\left(3x+2\right)^{2}+\sinh\left(3x+2\right)^{2}+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 + 2*sqrt(2) +
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{csch}^{2}(3x+2)}{2\coth^{2}(3x+2)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(1-2*coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 - 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.8 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0124951, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sinh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0130464, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\frac{(\cosh(bx+a))^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b
```

Maxima [A] time = 0.998941, size = 18, normalized size = 1.2

$$\frac{\cosh(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(b*x + a)^2/b
```

Fricas [A] time = 2.00678, size = 58, normalized size = 3.87

$$\frac{\cosh(bx+a)^2+\sinh(bx+a)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

Sympy [A] time = 0.21501, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

Giac [A] time = 1.32101, size = 32, normalized size = 2.13

$$\frac{e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/b
```

3.9 $\int \cosh(a + bx)\sinh^n(a + bx)\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0225391, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^n(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0096959, size = 19, normalized size = 1.

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.014, size = 20, normalized size = 1.1

$$\frac{(\sinh(bx+a))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^n,x)
```

```
[Out] sinh(b*x+a)^(n+1)/b/(n+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20277, size = 193, normalized size = 10.16

$$\frac{\cosh(n\log(\sinh(bx+a)))\sinh(bx+a)+\sinh(bx+a)\sinh(n\log(\sinh(bx+a)))}{(bn+b)\cosh(bx+a)^2-(bn+b)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (cosh(n*log(sinh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sinh(n*log(sinh(b
*x + a))))/((b*n + b)*cosh(b*x + a)^2 - (b*n + b)*sinh(b*x + a)^2)
```

Sympy [A] time = 1.66918, size = 49, normalized size = 2.58

$$\begin{cases} \frac{x\cosh(a)}{\sinh(a)} & \text{for}\ b = 0 \wedge n = -1 \\ x\sinh^n(a)\cosh(a) & \text{for}\ b = 0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{for}\ n = -1 \\ \frac{\sinh(a+bx)\sinh^n(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*cosh(a)/sinh(a), Eq(b, 0) & Eq(n, -1)), (x*sinh(a)**n*cosh(a),
 Eq(b, 0)), (log(sinh(a + b*x))/b, Eq(n, -1)), (sinh(a + b*x)*sinh(a + b*x)
**n/(b*n + b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^n \cosh(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^n*cosh(b*x + a), x)
```

3.10 $\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rubi [A] time = 0.0448459, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{\text{Subst}\left(\int x^n\left(1+x^2\right)\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^n+x^{2+n}\right)\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{\sinh^{3+n}(a+bx)}{b(3+n)}$$

Mathematica [A] time = 0.0608808, size = 39, normalized size = 1.

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)} + \frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^3 (\sinh(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^n,x)
```

```
[Out] int(cosh(b*x+a)^3*sinh(b*x+a)^n,x)
```

Maxima [B] time = 1.83658, size = 504, normalized size = 12.92

$$\frac{ne^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\left(2^n n^2+2^{n+2}n+3\cdot 2^n\right)b} + \frac{(n+9)e^{\left((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{8\left(2^n n^2+2^{n+2}n+3\cdot 2^n\right)b} - \frac{(n+9)e^{\left((bx+a)n-\right.}}{8\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/8*n*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a
) + 1) + 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*(n + 9)*e^((b*x + a
)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + a)/((2^n*n
^2 + 2^(n + 2)*n + 3*2^n)*b) - 1/8*(n + 9)*e^((b*x + a)*n - b*x + n*log(e^(
-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) - a)/((2^n*n^2 + 2^(n + 2)*n + 3*
2^n)*b) - 1/8*(n + 1)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*
log(-e^(-b*x - a) + 1) - 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*e^(
(b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) +
3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b)
```

Fricas [B] time = 2.26868, size = 482, normalized size = 12.36

$$\frac{\left((n+1)\sinh\left(bx+a\right)^3+\left(3\left(n+1\right)\cosh\left(bx+a\right)^2+n+9\right)\sinh\left(bx+a\right)\right)\cosh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)+\left((n+1)\sinh\left(b\right.\right.}{4\left(\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)^4-2\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)^2\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/4*(((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*cosh(b*x + a)^2 + n + 9)*sinh(b*
x + a))*cosh(n*log(sinh(b*x + a))) + ((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*
cosh(b*x + a)^2 + n + 9)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^2
 + 4*b*n + 3*b)*cosh(b*x + a)^4 - 2*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)^2*s
inh(b*x + a)^2 + (b*n^2 + 4*b*n + 3*b)*sinh(b*x + a)^4)
```

Sympy [A] time = 14.3459, size = 668, normalized size = 17.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**3, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**4/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*b*x*tanh(a/2 +
```

```
b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*
tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x
/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/
2)**2 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
+ b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*
x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)*
*2 + b) - 2*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2
)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 +
 b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + tanh(a/2 + b*x/2)**4/(b*tanh(a
/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 1/(b*tanh(a/2 + b*x/2)**4
- 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n*sinh(a + b*x)*sinh(a + b*x)
**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b) - 2*sinh(a + b*x)**3*sinh(a + b
*x)**n/(b*n**2 + 4*b*n + 3*b) + 3*sinh(a + b*x)*sinh(a + b*x)**n*cosh(a + b
*x)**2/(b*n**2 + 4*b*n + 3*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^n \cosh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^n*cosh(b*x + a)^3, x)
```

3.11 $\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{2\sinh^{n+3}(a+bx)}{b(n+3)}+\frac{\sinh^{n+5}(a+bx)}{b(n+5)}$$

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + (2*Sinh[a + b*x]^(3 + n))/(b*(3 + n)) +
 Sinh[a + b*x]^(5 + n)/(b*(5 + n))
```

Rubi [A] time = 0.0518165, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{2\sinh^{n+3}(a+bx)}{b(n+3)}+\frac{\sinh^{n+5}(a+bx)}{b(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + (2*Sinh[a + b*x]^(3 + n))/(b*(3 + n)) +
 Sinh[a + b*x]^(5 + n)/(b*(5 + n))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \frac{\text{Subst}\left(\int x^n\left(1+x^2\right)^2\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^n+2x^{2+n}+x^{4+n}\right)dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{2\sinh^{3+n}(a+bx)}{b(3+n)} + \frac{\sinh^{5+n}(a+bx)}{b(5+n)}$$

Mathematica [A] time = 0.195388, size = 49, normalized size = 0.83

$$\frac{\sinh^{n+1}(a+bx)\left(\frac{\sinh^4(a+bx)}{n+5}+\frac{2\sinh^2(a+bx)}{n+3}+\frac{1}{n+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

```
[Out] (Sinh[a + b*x]^(1 + n)*((1 + n)^(-1) + (2*Sinh[a + b*x]^2)/(3 + n) + Sinh[a
 + b*x]^4/(5 + n)))/b
```

Maple [F] time = 0.478, size = 0, normalized size = 0.

$$\int \left(\cosh\left(bx+a\right)\right)^5\left(\sinh\left(bx+a\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
```

```
[Out] int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
```

Maxima [B] time = 1.79, size = 926, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/32*n^2*e^((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
- a) + 1) + 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/8*n*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/32*(3*n^2 + 28*n + 25
)*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) +
1) + 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
+ 1) + a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n - b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
+ 1) - a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(3*n^2 + 28*
n + 25)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x -
 a) + 1) - 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(n^2 +
 4*n + 3)*e^((b*x + a)*n - 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
 - a) + 1) - 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 3/32*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b)
```

Fricas [B] time = 2.33701, size = 1015, normalized size = 17.2

$$\left(\left(n^2+4\,n+3\right)\sinh\left(bx+a\right)^5+\left(10\left(n^2+4\,n+3\right)\cosh\left(bx+a\right)^2+3\,n^2+28\,n+25\right)\sinh\left(bx+a\right)^3+\left(5\left(n^2+4\,n+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/16*(((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2 + 4*n + 3)*cosh(b*x + a)^
2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2 + 4*n + 3)*cosh(b*x + a)^4
 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2 + 24*n + 150)*sinh(b*x + a
))*cosh(n*log(sinh(b*x + a))) + ((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2
 + 4*n + 3)*cosh(b*x + a)^2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2
+ 4*n + 3)*cosh(b*x + a)^4 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2
+ 24*n + 150)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^3 + 9*b*n^2
+ 23*b*n + 15*b)*cosh(b*x + a)^6 - 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh
(b*x + a)^4*sinh(b*x + a)^2 + 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh(b*x
+ a)^2*sinh(b*x + a)^4 - (b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*sinh(b*x + a)^6)
```

Sympy [A] time = 80.1033, size = 2574, normalized size = 43.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**5*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**5, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2) - cosh(a + b*x)**4/(4*b*sinh(a + b*x)**
4), Eq(n, -5)), (16*b*x*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16
*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*b*x*tanh(a/2 + b*x
/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2
 + b*x/2)**2) + 16*b*x*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*
b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/
2) + 1)*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*
x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 64*log(tanh(a/2 + b*x/2) + 1)*tanh(a/
2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*t
anh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(
8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2
)**2) + 16*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/
2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh
(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a
/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 16*log(tanh(a/2 + b*x/2))*tanh
(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*
b*tanh(a/2 + b*x/2)**2) - tanh(a/2 + b*x/2)**8/(8*b*tanh(a/2 + b*x/2)**6 -
16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 18*tanh(a/2 + b*x/2
)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 +
 b*x/2)**2) - 1/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b
*tanh(a/2 + b*x/2)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 +
 b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh
(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8
- 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/
2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(
a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
- 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2
)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*ta
nh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 -
4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x
/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 +
b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*ta
nh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b
*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 12*log(tanh(a/2 + b
*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
```

```
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(
tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*t
anh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 +
 b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
 b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + log
(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh
(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
 - 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 -
4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)
**2 + b) + 6*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/
2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
+ b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/
2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*t
anh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 + b*x/2)**8 -
 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2
)**2 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
 b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*t
anh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*
b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 4*tanh(a/2 + b*x/2
)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n**2*sinh(a + b*x)*si
nh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*n*s
inh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**3 + 9*b*n**2 + 23*b
*n + 15*b) + 8*n*sinh(a + b*x)*sinh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 +
9*b*n**2 + 23*b*n + 15*b) + 8*sinh(a + b*x)**5*sinh(a + b*x)**n/(b*n**3 + 9
*b*n**2 + 23*b*n + 15*b) - 20*sinh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*
x)**2/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) + 15*sinh(a + b*x)*sinh(a + b*x)*
*n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^n \cosh(bx+a)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^n*cosh(b*x + a)^5, x)
```

3.12 $\int \cosh^m(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Rubi [A] time = 0.0267734, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh^m(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^m\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^{1+m}(a+bx)}{b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0087856, size = 19, normalized size = 1.

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Maple [A] time = 0.012, size = 20, normalized size = 1.1

$$\frac{(\cosh(bx+a))^{1+m}}{b(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a),x)
```

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1134, size = 193, normalized size = 10.16

$$\frac{\cosh(bx+a)\cosh(m\log(\cosh(bx+a)))+\cosh(bx+a)\sinh(m\log(\cosh(bx+a)))}{(bm+b)\cosh(bx+a)^2-(bm+b)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)*cosh(m*log(cosh(b*x + a))) + cosh(b*x + a)*sinh(m*log(cosh(b
*x + a))))/((b*m + b)*cosh(b*x + a)^2 - (b*m + b)*sinh(b*x + a)^2)
```

Sympy [A] time = 1.78465, size = 49, normalized size = 2.58

$$\begin{cases} \frac{x \sinh(a)}{\cosh(a)} & \text{for}\: b = 0 \wedge m = -1 \\ x \sinh(a) \cosh^{m}(a) & \text{for}\: b = 0 \\ \frac{\log(\cosh(a+bx))}{b} & \text{for}\: m = -1 \\ \frac{\cosh(a+bx)\cosh^{m}(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)/cosh(a), Eq(b, 0) & Eq(m, -1)), (x*sinh(a)*cosh(a)**m,
 Eq(b, 0)), (log(cosh(a + b*x))/b, Eq(m, -1)), (cosh(a + b*x)*cosh(a + b*x)
**m/(b*m + b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^{m} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^m*sinh(b*x + a), x)
```

3.13 $\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\cosh^{m+3}(a+bx)}{b(m+3)}-\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[Out] -(Cosh[a + b*x]^(1 + m)/(b*(1 + m))) + Cosh[a + b*x]^(3 + m)/(b*(3 + m))
```

Rubi [A] time = 0.0502823, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cosh^{m+3}(a+bx)}{b(m+3)}-\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]^(1 + m)/(b*(1 + m))) + Cosh[a + b*x]^(3 + m)/(b*(3 + m))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^m\left(1-x^2\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(x^m-x^{2+m}\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\cosh^{1+m}(a+bx)}{b(1+m)} + \frac{\cosh^{3+m}(a+bx)}{b(3+m)}$$

Mathematica [A] time = 0.125609, size = 44, normalized size = 1.1

$$\frac{\cosh^{m+1}(a+bx)((m+1)\cosh(2(a+bx))-m-5)}{2b(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

```
[Out] (Cosh[a + b*x]^(1 + m)*(-5 - m + (1 + m)*Cosh[2*(a + b*x)]))/(2*b*(1 + m)*(
3 + m))
```

Maple [C] time = 0.372, size = 923, normalized size = 23.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^3,x)
```

```
[Out] 1/8/b/(3+m)*(1+exp(2*b*x+2*a))^m*(1/2)^m*exp(b*x+a)^(-m)*exp(-3*b*x-3*a)*ex
p(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*
x-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*
exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(
I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+
exp(2*b*x+2*a)))^3)+1/8/b/(3+m)*(1+exp(2*b*x+2*a))^m*(1/2)^m*exp(b*x+a)^(-m
)*exp(3*b*x+3*a)*exp(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x
-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2
*b*x+2*a)))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I
*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csg
n(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^3)-1/8*(m+9)/b/(m^2+4*m+3)*(1+exp(2*b*x
```

```
+2*a))^m*(1/2)^m*exp(b*x+a)^(-m)*exp(-b*x-a)*exp(-1/2*I*Pi*m*csgn(I*(1+exp(
2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a))))*ex
p(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a
)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2
*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^3)-1/8*(m+9
)/b/(m^2+4*m+3)*(1+exp(2*b*x+2*a))^m*(1/2)^m*exp(b*x+a)^(-m)*exp(b*x+a)*exp
(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x
-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*e
xp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(I
*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+e
xp(2*b*x+2*a)))^3)
```

Maxima [B] time = 1.66797, size = 396, normalized size = 9.9

$$\frac{me^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}-\frac{(m+9)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}-\frac{(m+9)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*m*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2
 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^((b*x + a)*m + b*x + m*log(e^(-2
*b*x - 2*a) + 1) + a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^(
(b*x + a)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^2 + 2^(m + 2)*
m + 3*2^m)*b) + 1/8*(m + 1)*e^((b*x + a)*m - 3*b*x + m*log(e^(-2*b*x - 2*a)
 + 1) - 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) + 1/8*e^((b*x + a)*m + 3*b
*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b)
```

Fricas [B] time = 2.18545, size = 520, normalized size = 13.

$$\frac{\left((m+1)\cosh\left(bx+a\right)^3+3\,(m+1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-(m+9)\cosh\left(bx+a\right)\right)\cosh\left(m\log\left(\cosh\left(bx+a\right)\right)\right)+}{4\left(\left(bm^2+4\,bm+3\,b\right)\cosh\left(bx+a\right)^4-2\left(bm^2+4\,bm+3\,b\right)\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(((m + 1)*cosh(b*x + a)^3 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (
m + 9)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m + 1)*cosh(b*x + a)^3
 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (m + 9)*cosh(b*x + a))*sinh(m*
log(cosh(b*x + a))))/((b*m^2 + 4*b*m + 3*b)*cosh(b*x + a)^4 - 2*(b*m^2 + 4*
b*m + 3*b)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*m^2 + 4*b*m + 3*b)*sinh(b*x
 + a)^4)
```

Sympy [A] time = 14.7158, size = 678, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**2/(2*b*cosh(a + b*x)**2), Eq(m, -3)), (-b*x*tanh(a/2 + b*x/2)**4
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*b*x*tanh(a/2 +
 b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - b*x/(b
*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*
x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x
/2)**2 + b) - 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2) + 1)
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b
*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 +
 b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b*x/2)
**2 + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + tanh(a/2
 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 1/(b
*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(m, -1)), (m*sinh(
a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m + 3*b) + 3*sinh(
a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m + 3*b) - 2*cosh(
a + b*x)**3*cosh(a + b*x)**m/(b*m**2 + 4*b*m + 3*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^m \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^m*sinh(b*x + a)^3, x)
```

3.14 $\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}-\frac{2\cosh^{m+3}(a+bx)}{b(m+3)}+\frac{\cosh^{m+5}(a+bx)}{b(m+5)}$$

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m)) - (2*Cosh[a + b*x]^(3 + m))/(b*(3 + m)) +
 Cosh[a + b*x]^(5 + m)/(b*(5 + m))
```

Rubi [A] time = 0.0607817, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}-\frac{2\cosh^{m+3}(a+bx)}{b(m+3)}+\frac{\cosh^{m+5}(a+bx)}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m)) - (2*Cosh[a + b*x]^(3 + m))/(b*(3 + m)) +
 Cosh[a + b*x]^(5 + m)/(b*(5 + m))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \frac{\text{Subst}\left(\int x^m\left(1-x^2\right)^2\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\left(x^m-2x^{2+m}+x^{4+m}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^{1+m}(a+bx)}{b(1+m)} - \frac{2\cosh^{3+m}(a+bx)}{b(3+m)} + \frac{\cosh^{5+m}(a+bx)}{b(5+m)}$$

Mathematica [A] time = 0.217858, size = 77, normalized size = 1.31

$$\frac{\cosh^{m+1}(a+bx)\left(-4\left(m^2+8m+7\right)\cosh(2(a+bx))+\left(m^2+4m+3\right)\cosh(4(a+bx))+3m^2+28m+89\right)}{8b(m+1)(m+3)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

```
[Out] (Cosh[a + b*x]^(1 + m)*(89 + 28*m + 3*m^2 - 4*(7 + 8*m + m^2)*Cosh[2*(a + b
*x)] + (3 + 4*m + m^2)*Cosh[4*(a + b*x)]))/(8*b*(1 + m)*(3 + m)*(5 + m))
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^m (\sinh(bx+a))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
```

```
[Out] int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
```

Maxima [B] time = 1.69795, size = 753, normalized size = 12.76

$$\frac{m^2 e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\left(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m\right)b} + \frac{m e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{8\left(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m\right)b} - \frac{\left(3\,m^2+28\,m+25\right)e^{\left((bx+\right.}}{32\left(2^m m^3+9\cdot 2^m m^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/32*m^2*e^((b*x + a)*m + 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*
m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/8*m*e^((b*x + a)*m + 5*b*x + m*
log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)
*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2
*a) + 1) + 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 +
 12*m + 75)*e^((b*x + a)*m + b*x + m*log(e^(-2*b*x - 2*a) + 1) + a)/((2^m*m
^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 + 12*m + 75)*e^((b*x + a
)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m - 3*b*x + m*log(e
^(-2*b*x - 2*a) + 1) - 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) +
 1/32*(m^2 + 4*m + 3)*e^((b*x + a)*m - 5*b*x + m*log(e^(-2*b*x - 2*a) + 1)
- 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 3/32*e^((b*x + a)*m
+ 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b)
```

Fricas [B] time = 2.20771, size = 1094, normalized size = 18.54

$$\frac{\left(\left(m^2 + 4\,m + 3\right)\cosh\left(bx + a\right)^5 + 5\left(m^2 + 4\,m + 3\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right)^4 - \left(3\,m^2 + 28\,m + 25\right)\cosh\left(bx + a\right)^3 + \left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(((m^2 + 4*m + 3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*si
nh(b*x + a)^4 - (3*m^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*c
osh(b*x + a)^3 - 3*(3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(
m^2 + 12*m + 75)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m^2 + 4*m +
3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*sinh(b*x + a)^4 - (3*m
^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*cosh(b*x + a)^3 - 3*(
3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(m^2 + 12*m + 75)*cos
h(b*x + a))*sinh(m*log(cosh(b*x + a))))/((b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*
cosh(b*x + a)^6 - 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^4*sinh(
b*x + a)^2 + 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^2*sinh(b*x +
 a)^4 - (b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*sinh(b*x + a)^6)
```

Sympy [A] time = 78.7123, size = 2351, normalized size = 39.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((x*sinh(a)**5*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**4/(4*b*cosh(a + b*x)**4) - sinh(a + b*x)**2/(2*b*cosh(a + b*x)**
2), Eq(m, -5)), (-2*b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*
tanh(a/2 + b*x/2)**4 + b) + 4*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)
**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*b*x/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**
8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 8*log(tanh(a/2
+ b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 +
 b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 2*
b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*
x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*log(tan
h(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*
x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/
2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b), Eq(m, -3)), (b*x*tanh(
a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*ta
nh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2
)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x
/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
 + b*x/2)**2 + b) + b*x/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2
+ b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*
log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4
*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)*
*2 + b) - 12*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 +
b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(
a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b
*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4
 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2
 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*ta
nh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**
```

```
8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2
)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2)**2 + 1)*tanh
(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*t
anh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*log(tanh(a/2 + b*x/
2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(
tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*
b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**
2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a
/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) -
 2*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*tanh(a/2 + b
*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2
+ b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*tanh(a/2 + b*x/2)**2/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b), Eq(m, -1)), (m**2*sinh(a + b*x)**4*cosh(a + b
*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*m*sinh(a + b*x
)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) - 4
*m*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 +
23*b*m + 15*b) + 15*sinh(a + b*x)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3
 + 9*b*m**2 + 23*b*m + 15*b) - 20*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a
+ b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*cosh(a + b*x)**5*cosh(a +
 b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^m \sinh(bx+a)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^m*sinh(b*x + a)^5, x)
```

3.15 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rubi [A] time = 0.0435783, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{1}{4}\int \cosh^2(a+bx)\,dx\\ &= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{\int 1\,dx}{8}\\ &= -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0295206, size = 23, normalized size = 0.5

$$\frac{\sinh(4(a+bx)) - 4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.008, size = 43, normalized size = 0.9

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3\sinh(bx+a)}{4} - \frac{\cosh(bx+a)\sinh(bx+a)}{8} - \frac{bx}{8} - \frac{a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a)
```

Maxima [A] time = 1.01234, size = 53, normalized size = 1.15

$$-\frac{bx+a}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A] time = 2.05852, size = 104, normalized size = 2.26

$$\frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

Sympy [A] time = 1.118, size = 92, normalized size = 2.

$$\begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b\neq 0\\ x\sinh^2(a)\cosh^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

Giac [A] time = 1.16746, size = 65, normalized size = 1.41

$$-\frac{8\,bx-\left(2\,e^{(4\,bx+4\,a)}-1\right)e^{(-4\,bx-4\,a)}+8\,a-e^{(4\,bx+4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*b*x - (2*e^(4*b*x + 4*a) - 1)*e^(-4*b*x - 4*a) + 8*a - e^(4*b*x +
4*a))/b
```

3.16 $\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] x/16 + (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*x
])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b)
```

Rubi [A] time = 0.0744825, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

```
[Out] x/16 + (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*x
])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} - \frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx\\ &= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} + \frac{1}{8}\int \cosh^2(a+bx)\,d\\ &= \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+}{6b}\\ &= \frac{x}{16} + \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh}{6b}\end{aligned}$$

Mathematica [A] time = 0.0783301, size = 40, normalized size = 0.58

$$\frac{-3\sinh(2(a+bx)) - 3\sinh(4(a+bx)) + \sinh(6(a+bx)) + 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

```
[Out] (12*b*x - 3*Sinh[2*(a + b*x)] - 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(1
92*b)
```

Maple [A] time = 0.01, size = 61, normalized size = 0.9

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3(\sinh(bx+a))^3}{6} - \frac{(\cosh(bx+a))^3\sinh(bx+a)}{8} + \frac{\cosh(bx+a)\sinh(bx+a)}{16} + \frac{bx}{16} + \frac{a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a)
```

Maxima [A] time = 0.979105, size = 119, normalized size = 1.72

$$-\frac{\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b}+\frac{bx+a}{16\,b}+\frac{3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b + 1/
16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - e^(-6*b*x
 - 6*a))/b
```

Fricas [A] time = 2.03687, size = 243, normalized size = 3.52

$$\frac{3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+2\left(5\,\cosh\left(bx+a\right)^{3}-3\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+6\,bx+3\left(\cosh\left(bx+a\right)^{5}-2\,\text{co}\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 6*b*x + 3*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 3.87096, size = 136, normalized size = 1.97

$$\begin{cases}-\frac{x\sinh^{6}(a+bx)}{16}+\frac{3x\sinh^{4}(a+bx)\cosh^{2}(a+bx)}{16}-\frac{3x\sinh^{2}(a+bx)\cosh^{4}(a+bx)}{16}+\frac{x\cosh^{6}(a+bx)}{16}+\frac{\sinh^{5}(a+bx)\cosh(a+bx)}{16b}+\frac{\sinh^{3}(a+bx)\cosh}{6b} \\ x\sinh^{4}(a)\cosh^{2}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/16 + 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/1
6 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + x*cosh(a + b*x)**6/16 + sinh
(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
```

```
 - sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)*
*2, True))
```

Giac [A] time = 1.22006, size = 122, normalized size = 1.77

$$\frac{24\,bx-\left(22\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-3\,e^{(2\,bx+2\,a)}+1\right)e^{(-6\,bx-6\,a)}+24\,a+e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-3\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/384*(24*b*x - (22*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 3*e^(2*b*x + 2*a)
 + 1)*e^(-6*b*x - 6*a) + 24*a + e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 3*e^(
2*b*x + 2*a))/b
```

3.17 $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{48b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{64b}-\frac{5\sinh(a+bx)\cosh(a+}{128b}$$

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(64*b) - (5*Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(48*b) + (Cosh[a
 + b*x]^3*Sinh[a + b*x]^5)/(8*b)
```

Rubi [A] time = 0.103597, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{48b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{64b}-\frac{5\sinh(a+bx)\cosh(a+}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```



```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(64*b) - (5*Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(48*b) + (Cosh[a
 + b*x]^3*Sinh[a + b*x]^5)/(8*b)
```


Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} - \frac{5}{8}\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= -\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} + \frac{5}{16}\int \cosh^2(a+b$$
$$= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh$$
$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a+bx)\sin}{48b}$$
$$= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a+$$

Mathematica [A] time = 0.123181, size = 52, normalized size = 0.57

$$\frac{48\sinh(2(a+bx)) + 24\sinh(4(a+bx)) - 16\sinh(6(a+bx)) + 3\sinh(8(a+bx)) - 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] - 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Maple [A] time = 0.01, size = 79, normalized size = 0.9

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^5(\cosh(bx+a))^3}{8} - \frac{5(\cosh(bx+a))^3(\sinh(bx+a))^3}{48} + \frac{5(\cosh(bx+a))^3\sinh(bx+a)}{64} - \frac{5\cosh(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)^5*cosh(b*x+a)^3-5/48*cosh(b*x+a)^3*sinh(b*x+a)^3+5/64*
cosh(b*x+a)^3*sinh(b*x+a)-5/128*cosh(b*x+a)*sinh(b*x+a)-5/128*b*x-5/128*a)
```

Maxima [A] time = 1.00577, size = 149, normalized size = 1.62

$$-\frac{\left(16\,e^{(-2\,bx-2\,a)} - 24\,e^{(-4\,bx-4\,a)} - 48\,e^{(-6\,bx-6\,a)} - 3\right)e^{(8\,bx+8\,a)}}{6144\,b} - \frac{5\,(bx+a)}{128\,b} - \frac{48\,e^{(-2\,bx-2\,a)} + 24\,e^{(-4\,bx-4\,a)} - 16\,e^{(-6\,bx-6\,a)} +}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/6144*(16*e^(-2*b*x - 2*a) - 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) -
3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b - 1/6144*(48*e^(-2*b*x - 2*a) + 24
*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a))/b
```

Fricas [A] time = 2.04398, size = 382, normalized size = 4.15

$$\frac{3\,\cosh(bx+a)\sinh(bx+a)^7 + 3\left(7\,\cosh(bx+a)^3 - 4\,\cosh(bx+a)\right)\sinh(bx+a)^5 + \left(21\,\cosh(bx+a)^5 - 40\,\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 - 40*cosh(b*x + a)^3 + 12*cosh(
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 - 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 12.8281, size = 189, normalized size = 2.05

$$\begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} + \frac{5\sinh^7(a+}{} \\ x\sinh^6(a)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b
*x)/(128*b) + 73*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) - 55*sinh(a + b*
x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b),
Ne(b, 0)), (x*sinh(a)**6*cosh(a)**2, True))
```

Giac [A] time = 1.20593, size = 154, normalized size = 1.67

$$-\frac{240\,bx - \left(250\,e^{(8\,bx+8\,a)} - 48\,e^{(6\,bx+6\,a)} - 24\,e^{(4\,bx+4\,a)} + 16\,e^{(2\,bx+2\,a)} - 3\right)e^{(-8\,bx-8\,a)} + 240\,a - 3\,e^{(8\,bx+8\,a)} + 16\,e^{(6\,bx+6\,a)} - 24}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/6144*(240*b*x - (250*e^(8*b*x + 8*a) - 48*e^(6*b*x + 6*a) - 24*e^(4*b*x
+ 4*a) + 16*e^(2*b*x + 2*a) - 3)*e^(-8*b*x - 8*a) + 240*a - 3*e^(8*b*x + 8*
a) + 16*e^(6*b*x + 6*a) - 24*e^(4*b*x + 4*a) - 48*e^(2*b*x + 2*a))/b
```

3.18 $\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{24b}-\frac{\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{x}{16}$$

```
[Out] -x/16 - (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*
x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rubi [A] time = 0.0524301, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{24b}-\frac{\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

```
[Out] -x/16 - (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*
x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{6}\int \cosh^4(a+bx)\,dx$$
$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{8}\int \cosh^2(a+bx)\,dx$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$
$$= -\frac{x}{16} - \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

Mathematica [A] time = 0.0567505, size = 40, normalized size = 0.6

$$\frac{-3\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + \sinh(6(a+bx)) - 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-12*b*x - 3*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(
192*b)
```

Maple [A] time = 0.01, size = 56, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\sinh(bx+a)(\cosh(bx+a))^5}{6} - \frac{\sinh(bx+a)}{6}\left(\frac{(\cosh(bx+a))^3}{4} + \frac{3\cosh(bx+a)}{8}\right) - \frac{bx}{16} - \frac{a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*
sinh(b*x+a)-1/16*b*x-1/16*a)
```

Maxima [A] time = 1.02397, size = 119, normalized size = 1.78

$$\frac{\left(3 e^{(-2 bx-2 a)} - 3 e^{(-4 bx-4 a)} + 1\right)e^{(6 bx+6 a)}}{384 b} - \frac{bx + a}{16 b} + \frac{3 e^{(-2 bx-2 a)} - 3 e^{(-4 bx-4 a)} - e^{(-6 bx-6 a)}}{384 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b - 1/1
6*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - e^(-6*b*x
- 6*a))/b
```

Fricas [A] time = 2.08663, size = 243, normalized size = 3.63

$$\frac{3 \cosh(bx+a)\sinh(bx+a)^5 + 2\left(5 \cosh(bx+a)^3 + 3 \cosh(bx+a)\right)\sinh(bx+a)^3 - 6 bx + 3\left(\cosh(bx+a)^5 + 2 \cosh(bx+a)^3 - \cosh(bx+a)\right)\sinh(bx+a)}{96 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 - 6*b*x + 3*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 3.79201, size = 136, normalized size = 2.03

$$\begin{cases} \frac{x \sinh^6(a+bx)}{16} - \frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16} + \frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16} - \frac{x\cosh^6(a+bx)}{16} - \frac{\sinh^5(a+bx)\cosh(a+bx)}{16b} + \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b} \\ x\sinh^2(a)\cosh^4(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**6/16 - 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/16
 + 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - x*cosh(a + b*x)**6/16 - sinh(
a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
```

```
+ sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**
4, True))
```

Giac [A] time = 1.2603, size = 124, normalized size = 1.85

$$-\frac{24\,bx - \left(22\,e^{(6\,bx+6\,a)} + 3\,e^{(4\,bx+4\,a)} - 3\,e^{(2\,bx+2\,a)} - 1\right)e^{(-6\,bx-6\,a)} + 24\,a - e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*b*x - (22*e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) - 3*e^(2*b*x + 2*a
) - 1)*e^(-6*b*x - 6*a) + 24*a - e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) + 3*e^
(2*b*x + 2*a))/b
```

3.19 $\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{64b}+\frac{3\sinh(a+bx)\cosh(a+bx}{128b}$$

```
[Out] (3*x)/128 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(64*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(16*b) + (Cosh[a + b*x]
^5*Sinh[a + b*x]^3)/(8*b)
```

Rubi [A] time = 0.0839527, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{64b}+\frac{3\sinh(a+bx)\cosh(a+bx}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(64*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(16*b) + (Cosh[a + b*x]
^5*Sinh[a + b*x]^3)/(8*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} - \frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} + \frac{1}{16}\int \cosh^4(a+bx)$$
$$= \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a}{8b}$$
$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx)\sinh(a}{16b}$$
$$= \frac{3x}{128} + \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx)\,s}{16b}$$

Mathematica [A] time = 0.0428134, size = 33, normalized size = 0.37

$$\frac{24(a+bx) - 8\sinh(4(a+bx)) + \sinh(8(a+bx))}{1024b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

```
[Out] (24*(a + b*x) - 8*Sinh[4*(a + b*x)] + Sinh[8*(a + b*x)])/(1024*b)
```

Maple [A] time = 0.011, size = 74, normalized size = 0.8

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^3(\cosh(bx+a))^5}{8} - \frac{\sinh(bx+a)(\cosh(bx+a))^5}{16} + \frac{\sinh(bx+a)}{16}\left(\frac{(\cosh(bx+a))^3}{4} + \frac{3\cosh(bx+a}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)^3*cosh(b*x+a)^5-1/16*sinh(b*x+a)*cosh(b*x+a)^5+1/16*(1
/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/128*b*x+3/128*a)
```

Maxima [A] time = 1.21238, size = 89, normalized size = 0.99

$$-\frac{\left(8\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{2048\,b}+\frac{3\,(bx+a)}{128\,b}+\frac{8\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2048*(8*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/128*(b*x + a)/b + 1/
2048*(8*e^(-4*b*x - 4*a) - e^(-8*b*x - 8*a))/b
```

Fricas [A] time = 2.06411, size = 263, normalized size = 2.92

$$\frac{7\cosh(bx+a)^3\sinh(bx+a)^5+\cosh(bx+a)\sinh(bx+a)^7+\left(7\cosh(bx+a)^5-4\cosh(bx+a)\right)\sinh(bx+a)^3+3}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(7*cosh(b*x + a)^3*sinh(b*x + a)^5 + cosh(b*x + a)*sinh(b*x + a)^7 +
(7*cosh(b*x + a)^5 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x +
 a)^7 - 4*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [A] time = 12.0885, size = 189, normalized size = 2.1

$$\begin{cases}\frac{3x\sinh^8(a+bx)}{128}-\frac{3x\sinh^6(a+bx)\cosh^2(a+bx)}{32}+\frac{9x\sinh^4(a+bx)\cosh^4(a+bx)}{64}-\frac{3x\sinh^2(a+bx)\cosh^6(a+bx)}{32}+\frac{3x\cosh^8(a+bx)}{128}-\frac{3\sinh^7(a+bx)}{128} \\ x\sinh^4(a)\cosh^4(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**8/128 - 3*x*sinh(a + b*x)**6*cosh(a + b*x)**2
/32 + 9*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 - 3*x*sinh(a + b*x)**2*cosh(
a + b*x)**6/32 + 3*x*cosh(a + b*x)**8/128 - 3*sinh(a + b*x)**7*cosh(a + b*x
```

```
)/(128*b) + 11*sinh(a + b*x)**5*cosh(a + b*x)**3/(128*b) + 11*sinh(a + b*x)
**3*cosh(a + b*x)**5/(128*b) - 3*sinh(a + b*x)*cosh(a + b*x)**7/(128*b), Ne
(b, 0)), (x*sinh(a)**4*cosh(a)**4, True))
```

Giac [A] time = 1.2121, size = 92, normalized size = 1.02

$$\frac{48\,bx - \left(18\,e^{(8\,bx+8\,a)} - 8\,e^{(4\,bx+4\,a)} + 1\right)e^{(-8\,bx-8\,a)} + 48\,a + e^{(8\,bx+8\,a)} - 8\,e^{(4\,bx+4\,a)}}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/2048*(48*b*x - (18*e^(8*b*x + 8*a) - 8*e^(4*b*x + 4*a) + 1)*e^(-8*b*x - 8
*a) + 48*a + e^(8*b*x + 8*a) - 8*e^(4*b*x + 4*a))/b
```

3.20 $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{32b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{128b}$$

```
[Out] (-3*x)/256 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) - (Cosh[a + b*x]^3*Sin
h[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(32*b) - (Cosh[a + b*
x]^5*Sinh[a + b*x]^3)/(16*b) + (Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b)
```

Rubi [A] time = 0.118413, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{32b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-3*x)/256 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) - (Cosh[a + b*x]^3*Sin
h[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(32*b) - (Cosh[a + b*
x]^5*Sinh[a + b*x]^3)/(16*b) + (Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{1}{2}\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} + \frac{3}{16}\int \cosh^4(a+bx$$
$$= \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a}{10b}$$
$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a}{16b}$$
$$= -\frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a}{32b}$$
$$= -\frac{3x}{256} - \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)}{32}$$

Mathematica [A] time = 0.172735, size = 62, normalized size = 0.55

$$\frac{20\sinh(2(a+bx)) + 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) - 5\sinh(8(a+bx)) + 2\sinh(10(a+bx)) - 120bx}{10240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 20*Sinh[2*(a + b*x)] + 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*
x)] - 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Maple [A] time = 0.013, size = 92, normalized size = 0.8

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^5(\cosh(bx+a))^5}{10} - \frac{(\sinh(bx+a))^3(\cosh(bx+a))^5}{16} + \frac{\sinh(bx+a)(\cosh(bx+a))^5}{32} - \frac{\sinh(bx+a)}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/10*sinh(b*x+a)^5*cosh(b*x+a)^5-1/16*sinh(b*x+a)^3*cosh(b*x+a)^5+1/32
*sinh(b*x+a)*cosh(b*x+a)^5-1/32*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*
x+a)-3/256*b*x-3/256*a)
```

Maxima [A] time = 1.13314, size = 178, normalized size = 1.58

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}-20\,e^{(-8\,bx-8\,a)}-2\right)e^{(10\,bx+10\,a)}}{20480\,b}-\frac{3\,(bx+a)}{256\,b}-\frac{20\,e^{(-2\,bx-2\,a)}+40\,e^{(-4\,bx-4\,a)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20480*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) -
20*e^(-8*b*x - 8*a) - 2)*e^(10*b*x + 10*a)/b - 3/256*(b*x + a)/b - 1/20480*
(20*e^(-2*b*x - 2*a) + 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) - 5*e^(-8*
b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Fricas [A] time = 2.07082, size = 544, normalized size = 4.81

$$\frac{5\,\cosh(bx+a)\sinh(bx+a)^9+10\left(6\,\cosh(bx+a)^3-\cosh(bx+a)\right)\sinh(bx+a)^7+\left(126\,\cosh(bx+a)^5-70\,\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*
cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 5*(cosh(b*x +
 a)^9 - 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + 2*cosh(
b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 31.0621, size = 231, normalized size = 2.04

$$\begin{cases}\frac{3x\sinh^{10}(a+bx)}{256}-\frac{15x\sinh^{8}(a+bx)\cosh^{2}(a+bx)}{256}+\frac{15x\sinh^{6}(a+bx)\cosh^{4}(a+bx)}{128}-\frac{15x\sinh^{4}(a+bx)\cosh^{6}(a+bx)}{128}+\frac{15x\sinh^{2}(a+bx)\cosh^{8}(a+bx)}{256} \\ x\sinh^{6}(a)\cosh^{4}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**10/256 - 15*x*sinh(a + b*x)**8*cosh(a + b*x)*
*2/256 + 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 - 15*x*sinh(a + b*x)**4
*cosh(a + b*x)**6/128 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 - 3*x*co
sh(a + b*x)**10/256 - 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) + 7*sinh(a +
 b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*b
) - 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) + 3*sinh(a + b*x)*cosh(a +
b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**4, True))
```

Giac [A] time = 1.22946, size = 184, normalized size = 1.63

$$-\frac{240\,bx-\left(274\,e^{(10\,bx+10\,a)}-20\,e^{(8\,bx+8\,a)}-40\,e^{(6\,bx+6\,a)}+10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}-2\right)e^{(-10\,bx-10\,a)}+240\,a-2\,e^{(10\,bx+10\,a)}}{20480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/20480*(240*b*x - (274*e^(10*b*x + 10*a) - 20*e^(8*b*x + 8*a) - 40*e^(6*b
*x + 6*a) + 10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) - 2)*e^(-10*b*x - 10*a)
+ 240*a - 2*e^(10*b*x + 10*a) + 5*e^(8*b*x + 8*a) + 10*e^(6*b*x + 6*a) - 40
*e^(4*b*x + 4*a) - 20*e^(2*b*x + 2*a))/b
```

3.21 $\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{48b}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{192b}-\frac{5\sinh(a+bx)\cosh(a+bx)}{128b}$$

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) - (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(192*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(48*b) + (Cosh[a +
b*x]^7*Sinh[a + b*x])/(8*b)
```

Rubi [A] time = 0.0643681, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{48b}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{192b}-\frac{5\sinh(a+bx)\cosh(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) - (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(192*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(48*b) + (Cosh[a +
b*x]^7*Sinh[a + b*x])/(8*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{1}{8}\int \cosh^6(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{5}{48}\int \cosh^4(a+bx)\,d$$
$$= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(}{8b}$$
$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx)\sinh}{48b}$$
$$= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx}{4}$$

Mathematica [A] time = 0.0942673, size = 52, normalized size = 0.59

$$\frac{-48\sinh(2(a+bx)) + 24\sinh(4(a+bx)) + 16\sinh(6(a+bx)) + 3\sinh(8(a+bx)) - 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

```
[Out] (-120*b*x - 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] + 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Maple [A] time = 0.012, size = 66, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\sinh(bx+a)(\cosh(bx+a))^7}{8} - \frac{\sinh(bx+a)}{8}\left(\frac{(\cosh(bx+a))^5}{6} + \frac{5(\cosh(bx+a))^3}{24} + \frac{5\cosh(bx+a)}{16}\right) - \frac{5bx}{128} - \frac{5}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)*cosh(b*x+a)^7-1/8*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^
3+5/16*cosh(b*x+a))*sinh(b*x+a)-5/128*b*x-5/128*a)
```

Maxima [A] time = 1.00818, size = 149, normalized size = 1.69

$$\frac{\left(16\,e^{(-2\,bx-2\,a)}+24\,e^{(-4\,bx-4\,a)}-48\,e^{(-6\,bx-6\,a)}+3\right)e^{(8\,bx+8\,a)}}{6144\,b}-\frac{5\,(bx+a)}{128\,b}+\frac{48\,e^{(-2\,bx-2\,a)}-24\,e^{(-4\,bx-4\,a)}-16\,e^{(-6\,bx-6\,a)}-3}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6144*(16*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) + 3
)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b + 1/6144*(48*e^(-2*b*x - 2*a) - 24*
e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a))/b
```

Fricas [A] time = 2.05741, size = 382, normalized size = 4.34

$$3\,\cosh(bx+a)\sinh(bx+a)^7+3\left(7\,\cosh(bx+a)^3+4\,\cosh(bx+a)\right)\sinh(bx+a)^5+\left(21\,\cosh(bx+a)^5+40\,\cosh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 + 40*cosh(b*x + a)^3 + 12*cosh(
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 + 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 12.1828, size = 189, normalized size = 2.15

$$\begin{cases} -\frac{5x\sinh^8(a+bx)}{128}+\frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32}-\frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64}+\frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32}-\frac{5x\cosh^8(a+bx)}{128}+\frac{5\sinh^7(a+\dots}{\dots} \\ x\sinh^2(a)\cosh^6(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b
*x)/(128*b) - 55*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) + 73*sinh(a + b*
x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b),
Ne(b, 0)), (x*sinh(a)**2*cosh(a)**6, True))
```

Giac [A] time = 1.22557, size = 154, normalized size = 1.75

$$-\frac{240\,bx-\left(250\,e^{(8\,bx+8\,a)}+48\,e^{(6\,bx+6\,a)}-24\,e^{(4\,bx+4\,a)}-16\,e^{(2\,bx+2\,a)}-3\right)e^{(-8\,bx-8\,a)}+240\,a-3\,e^{(8\,bx+8\,a)}-16\,e^{(6\,bx+6\,a)}-24}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/6144*(240*b*x - (250*e^(8*b*x + 8*a) + 48*e^(6*b*x + 6*a) - 24*e^(4*b*x
+ 4*a) - 16*e^(2*b*x + 2*a) - 3)*e^(-8*b*x - 8*a) + 240*a - 3*e^(8*b*x + 8*
a) - 16*e^(6*b*x + 6*a) - 24*e^(4*b*x + 4*a) + 48*e^(2*b*x + 2*a))/b
```

3.22 $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=111

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3\sinh(a+bx)\cosh^7(a+bx)}{80b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{160b}+\frac{\sinh(a+bx)\cosh^3(a+b$$

```
[Out] (3*x)/256 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(160*b) - (3*Cosh[a +
b*x]^7*Sinh[a + b*x])/(80*b) + (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b)
```

Rubi [A] time = 0.0989776, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3\sinh(a+bx)\cosh^7(a+bx)}{80b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{160b}+\frac{\sinh(a+bx)\cosh^3(a+b$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(160*b) - (3*Cosh[a +
b*x]^7*Sinh[a + b*x])/(80*b) + (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx &= \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} - \frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx \\ &= -\frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{3}{80}\int \cosh^6(a+b \\ &= \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(}{10b} \\ &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a}{80b} \\ &= \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a}{160b} \\ &= \frac{3x}{256} + \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\,\mathrm{s}}{160}\end{aligned}$$

Mathematica [A] time = 0.128019, size = 62, normalized size = 0.56

$$\frac{20\sinh(2(a+bx)) - 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) + 5\sinh(8(a+bx)) + 2\sinh(10(a+bx)) + 120bx}{10240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sinh[2*(a + b*x)] - 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*x
)] + 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Maple [A] time = 0.011, size = 84, normalized size = 0.8

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^3(\cosh(bx+a))^7}{10} - \frac{3\sinh(bx+a)(\cosh(bx+a))^7}{80} + \frac{3\sinh(bx+a)}{80}\left(\frac{(\cosh(bx+a))^5}{6} + \frac{5(\cosh(b}{24}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/10*sinh(b*x+a)^3*cosh(b*x+a)^7-3/80*sinh(b*x+a)*cosh(b*x+a)^7+3/80*(
1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+3/256*b*
x+3/256*a)
```

Maxima [A] time = 1.02041, size = 178, normalized size = 1.6

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}+20\,e^{(-8\,bx-8\,a)}+2\right)e^{(10\,bx+10\,a)}}{20480\,b}+\frac{3\,(bx+a)}{256\,b}-\frac{20\,e^{(-2\,bx-2\,a)}-40\,e^{(-4\,bx-4\,a)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20480*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) + 2
0*e^(-8*b*x - 8*a) + 2)*e^(10*b*x + 10*a)/b + 3/256*(b*x + a)/b - 1/20480*(
20*e^(-2*b*x - 2*a) - 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) + 5*e^(-8*b
*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Fricas [A] time = 2.06793, size = 544, normalized size = 4.9

$$\frac{5\,\cosh(bx+a)\sinh(bx+a)^{9}+10\left(6\,\cosh(bx+a)^{3}+\cosh(bx+a)\right)\sinh(bx+a)^{7}+\left(126\,\cosh(bx+a)^{5}+70\,\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 + cosh(b*x
+ a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*
cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 30*b*x + 5*(cosh(b*x +
 a)^9 + 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + 2*cosh(
b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 32.4585, size = 231, normalized size = 2.08

$$\begin{cases} -\frac{3x\sinh^{10}(a+bx)}{256}+\frac{15x\sinh^{8}(a+bx)\cosh^{2}(a+bx)}{256}-\frac{15x\sinh^{6}(a+bx)\cosh^{4}(a+bx)}{128}+\frac{15x\sinh^{4}(a+bx)\cosh^{6}(a+bx)}{128}-\frac{15x\sinh^{2}(a+bx)\cosh^{8}(a+b}{256} \\ x\sinh^{4}(a)\cosh^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((-3*x*sinh(a + b*x)**10/256 + 15*x*sinh(a + b*x)**8*cosh(a + b*x)
**2/256 - 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 + 15*x*sinh(a + b*x)**
4*cosh(a + b*x)**6/128 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 + 3*x*c
osh(a + b*x)**10/256 + 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) - 7*sinh(a
+ b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*
b) + 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) - 3*sinh(a + b*x)*cosh(a +
 b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**6, True))
```

Giac [A] time = 1.25138, size = 184, normalized size = 1.66

$$\frac{240\,bx - \left(274\,e^{(10\,bx+10\,a)} + 20\,e^{(8\,bx+8\,a)} - 40\,e^{(6\,bx+6\,a)} - 10\,e^{(4\,bx+4\,a)} + 5\,e^{(2\,bx+2\,a)} + 2\right)e^{(-10\,bx-10\,a)} + 240\,a + 2\,e^{(10\,bx+10\,a)} +}{20480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/20480*(240*b*x - (274*e^(10*b*x + 10*a) + 20*e^(8*b*x + 8*a) - 40*e^(6*b*
x + 6*a) - 10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 2)*e^(-10*b*x - 10*a) +
 240*a + 2*e^(10*b*x + 10*a) + 5*e^(8*b*x + 8*a) - 10*e^(6*b*x + 6*a) - 40*
e^(4*b*x + 4*a) + 20*e^(2*b*x + 2*a))/b
```

3.23 $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{24b}+\frac{\sinh(a+bx)\cosh^7(a+bx)}{64b}-\frac{\sinh(a+bx)\cosh^5(a+bx}{384b}$$

```
[Out] (-5*x)/1024 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(1024*b) - (5*Cosh[a + b*x]^3
*Sinh[a + b*x])/(1536*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(384*b) + (Cosh[
a + b*x]^7*Sinh[a + b*x])/(64*b) - (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(24*b)
 + (Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b)
```

Rubi [A] time = 0.132118, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{24b}+\frac{\sinh(a+bx)\cosh^7(a+bx)}{64b}-\frac{\sinh(a+bx)\cosh^5(a+bx}{384b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/1024 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(1024*b) - (5*Cosh[a + b*x]^3
*Sinh[a + b*x])/(1536*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(384*b) + (Cosh[
a + b*x]^7*Sinh[a + b*x])/(64*b) - (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(24*b)
 + (Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx &= \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}-\frac{5}{12}\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx\\ &= -\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}+\frac{1}{8}\int \cosh^6(a+bx)\\ &= \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh^7(a+bx)\sinh^5(a}{12b}\\ &= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{384b}+\frac{\cosh^7(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^7(a+bx)\sinh^3(}{24b}\\ &= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{384b}+\frac{\cosh^7(a+bx)\sinh(}{64b}\\ &= -\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b}-\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b}-\frac{\cosh^5(a+bx)\sinh}{384b}\\ &= -\frac{5x}{1024}-\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b}-\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b}-\frac{\cosh^5(a+}{}\end{aligned}$$

Mathematica [A] time = 0.0821472, size = 43, normalized size = 0.32

$$\frac{45\sinh(4(a+bx))-9\sinh(8(a+bx))+\sinh(12(a+bx))-120a-120bx}{24576b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*a - 120*b*x + 45*Sinh[4*(a + b*x)] - 9*Sinh[8*(a + b*x)] + Sinh[12*(a
 + b*x)])/(24576*b)
```

Maple [A] time = 0.013, size = 102, normalized size = 0.8

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^5(\cosh(bx+a))^7}{12}-\frac{(\sinh(bx+a))^3(\cosh(bx+a))^7}{24}+\frac{\sinh(bx+a)(\cosh(bx+a))^7}{64}-\frac{\sinh(bx+a)}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/12*sinh(b*x+a)^5*cosh(b*x+a)^7-1/24*sinh(b*x+a)^3*cosh(b*x+a)^7+1/64
*sinh(b*x+a)*cosh(b*x+a)^7-1/64*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*
cosh(b*x+a))*sinh(b*x+a)-5/1024*b*x-5/1024*a)
```

Maxima [A] time = 1.0582, size = 116, normalized size = 0.87

$$-\frac{\left(9\,e^{\left(-4\,bx-4\,a\right)}-45\,e^{\left(-8\,bx-8\,a\right)}-1\right)e^{\left(12\,bx+12\,a\right)}}{49152\,b}-\frac{5\,(bx+a)}{1024\,b}-\frac{45\,e^{\left(-4\,bx-4\,a\right)}-9\,e^{\left(-8\,bx-8\,a\right)}+e^{\left(-12\,bx-12\,a\right)}}{49152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/49152*(9*e^(-4*b*x - 4*a) - 45*e^(-8*b*x - 8*a) - 1)*e^(12*b*x + 12*a)/b
 - 5/1024*(b*x + a)/b - 1/49152*(45*e^(-4*b*x - 4*a) - 9*e^(-8*b*x - 8*a) +
 e^(-12*b*x - 12*a))/b
```

Fricas [A] time = 2.0948, size = 498, normalized size = 3.72

$$\frac{55\,\cosh(bx+a)^3\sinh(bx+a)^9+3\,\cosh(bx+a)\sinh(bx+a)^{11}+18\left(11\,\cosh(bx+a)^5-\cosh(bx+a)\right)\sinh(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/6144*(55*cosh(b*x + a)^3*sinh(b*x + a)^9 + 3*cosh(b*x + a)*sinh(b*x + a)^
11 + 18*(11*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^7 + 18*(11*cosh(
b*x + a)^7 - 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + (55*cosh(b*x + a)^9 - 126
*cosh(b*x + a)^5 + 45*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x
 + a)^11 - 6*cosh(b*x + a)^7 + 15*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [A] time = 79.0808, size = 277, normalized size = 2.07

$$\begin{cases} -\frac{5x\sinh^{12}(a+bx)}{1024} + \frac{15x\sinh^{10}(a+bx)\cosh^2(a+bx)}{512} - \frac{75x\sinh^8(a+bx)\cosh^4(a+bx)}{1024} + \frac{25x\sinh^6(a+bx)\cosh^6(a+bx)}{256} - \frac{75x\sinh^4(a+bx)\cosh^8(a+bx)}{1024} \\ x\sinh^6(a)\cosh^6(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**12/1024 + 15*x*sinh(a + b*x)**10*cosh(a + b*
x)**2/512 - 75*x*sinh(a + b*x)**8*cosh(a + b*x)**4/1024 + 25*x*sinh(a + b*x
)**6*cosh(a + b*x)**6/256 - 75*x*sinh(a + b*x)**4*cosh(a + b*x)**8/1024 + 1
5*x*sinh(a + b*x)**2*cosh(a + b*x)**10/512 - 5*x*cosh(a + b*x)**12/1024 + 5
*sinh(a + b*x)**11*cosh(a + b*x)/(1024*b) - 85*sinh(a + b*x)**9*cosh(a + b*
x)**3/(3072*b) + 33*sinh(a + b*x)**7*cosh(a + b*x)**5/(512*b) + 33*sinh(a +
 b*x)**5*cosh(a + b*x)**7/(512*b) - 85*sinh(a + b*x)**3*cosh(a + b*x)**9/(3
072*b) + 5*sinh(a + b*x)*cosh(a + b*x)**11/(1024*b), Ne(b, 0)), (x*sinh(a)*
*6*cosh(a)**6, True))
```

Giac [A] time = 1.21426, size = 124, normalized size = 0.93

$$-\frac{240\,bx - \left(110\,e^{(12\,bx+12\,a)} - 45\,e^{(8\,bx+8\,a)} + 9\,e^{(4\,bx+4\,a)} - 1\right)e^{(-12\,bx-12\,a)} + 240\,a - e^{(12\,bx+12\,a)} + 9\,e^{(8\,bx+8\,a)} - 45\,e^{(4\,bx+4\,a)}}{49152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/49152*(240*b*x - (110*e^(12*b*x + 12*a) - 45*e^(8*b*x + 8*a) + 9*e^(4*b*
x + 4*a) - 1)*e^(-12*b*x - 12*a) + 240*a - e^(12*b*x + 12*a) + 9*e^(8*b*x +
 8*a) - 45*e^(4*b*x + 4*a))/b
```

3.24 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] Log[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0122369, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.0202292, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sinh(a+bx))}{2b}-\frac{\log(\cosh(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] 2*(-Log[Cosh[a + b*x]]/(2*b) + Log[Sinh[a + b*x]]/(2*b))
```

Maple [A] time = 0.013, size = 12, normalized size = 1.1

$$\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.56052, size = 68, normalized size = 6.18

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

Fricas [B] time = 2.16458, size = 154, normalized size = 14.

$$-\frac{\log\left(\frac{2\,\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

Giac [B] time = 1.16251, size = 47, normalized size = 4.27

$$-\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -log(e^(2*b*x + 2*a) + 1)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.25 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rubi [A] time = 0.0257552, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 321, 207}

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.0307134, size = 26, normalized size = 1.13

$$\frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

Maple [A] time = 0.013, size = 23, normalized size = 1.

$$\frac{(\cosh(bx+a))^{-1} - 2\,\operatorname{Artanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.01946, size = 82, normalized size = 3.57

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

Fricas [B] time = 2.08739, size = 462, normalized size = 20.09

$$-\frac{\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\left(\cosh\left(b\right.\right.}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [B] time = 1.16259, size = 95, normalized size = 4.13

$$-\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}+\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 1/2*log(e^(b*x + a) + e^(-b*x
- a) - 2)/b + 2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.26 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0235788, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0362133, size = 36, normalized size = 1.33

$$-\frac{-\operatorname{sech}^2(a+bx) - 2\log(\sinh(a+bx)) + 2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.018, size = 26, normalized size = 1.

$$\frac{1}{2\,b\,(\cosh(bx+a))^2} + \frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.60227, size = 119, normalized size = 4.41

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 2.21561, size = 1035, normalized size = 38.33

$$\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2+1\right)\sinh(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

Giac [B] time = 1.21783, size = 135, normalized size = 5.

$$-\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}{2b}+\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}+\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}+6}{2b\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 1/2*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2)/b + 1/2*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/
(b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2))
```

3.27 $\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0292979, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 302, 207}

$$\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0272027, size = 41, normalized size = 1.08

$$\frac{\operatorname{sech}^3(a+bx)}{3b} + \frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

```
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

Maple [A] time = 0.016, size = 33, normalized size = 0.9

$$\frac{1}{b}\left(\frac{1}{3\left(\cosh(bx+a)\right)^3} + \left(\cosh(bx+a)\right)^{-1} - 2\,\text{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3/cosh(b*x+a)^3+1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.0422, size = 146, normalized size = 3.84

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(3\,e^{(-bx-a)}+10\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}\right)}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2/3*(3*e^(-b*x - a) +
10*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*
b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Fricas [B] time = 2.0867, size = 1958, normalized size = 51.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a)
^5 + 20*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 20*cosh(b*x + a)^3 + 60*(
cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(c
osh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a
) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5
 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*si
nh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3
 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x +
 a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**4, x)
```

Giac [B] time = 1.18892, size = 124, normalized size = 3.26

$$-\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}+\frac{2\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{3\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 1/2*log(e^(b*x + a) + e^(-b*x
- a) - 2)/b + 2/3*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 4)/(b*(e^(b*x + a) +
e^(-b*x - a))^3)
```

3.28 $\int \text{csch}(a+bx)\text{sech}^5(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/b + Tanh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0327929, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2620, 266, 43}

$$\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/b + Tanh[a + b*x]^4/(4*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(2+\frac{1}{x}+x\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.0935774, size = 46, normalized size = 1.15

$$-\frac{-\operatorname{sech}^4(a+bx) - 2\operatorname{sech}^2(a+bx) - 4\log(\sinh(a+bx)) + 4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

```
[Out] -(4*Log[Cosh[a + b*x]] - 4*Log[Sinh[a + b*x]] - 2*Sech[a + b*x]^2 - Sech[a
+ b*x]^4)/(4*b)
```

Maple [A] time = 0.02, size = 39, normalized size = 1.

$$\frac{1}{4\,b\left(\cosh\left(bx+a\right)\right)^4} + \frac{1}{2\,b\left(\cosh\left(bx+a\right)\right)^2} + \frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^5,x)
```

```
[Out] 1/4/b/cosh(b*x+a)^4+1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.63616, size = 177, normalized size = 4.42

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\left(e^{(-2\,bx-2\,a)}+4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*(e^(-2*b*x - 2*a) + 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a
) + 1))
```

Fricas [B] time = 2.10436, size = 2952, normalized size = 73.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 + 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*s
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 +
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^
6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
```

```
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**5, x)
```

Giac [B] time = 1.22604, size = 171, normalized size = 4.28

$$-\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}{2b}+\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}+\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+20e^{(2bx+2a)}+20e^{(-2b}}{4b\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 1/2*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2)/b + 1/4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2
 + 20*e^(2*b*x + 2*a) + 20*e^(-2*b*x - 2*a) + 44)/(b*(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) + 2)^2)
```

3.29 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=24

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rubi [A] time = 0.0222952, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2621, 321, 207}

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{i\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} - \frac{i\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}$$

Mathematica [C] time = 0.0185736, size = 29, normalized size = 1.21

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] time = 0.015, size = 27, normalized size = 1.1

$$-\frac{1}{b\sinh(bx+a)} - 2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -1/b/sinh(b*x+a)-2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.50841, size = 58, normalized size = 2.42

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 2.13367, size = 305, normalized size = 12.71

$$-\frac{2\left(\left(\cosh(bx+a)^2 + 2\,\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\arctan(\cosh(bx+a)+\sinh(bx+a)) + \cosh(}{b\cosh(bx+a)^2 + 2\,b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [B] time = 1.21202, size = 78, normalized size = 3.25

$$-\frac{\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{2b} - \frac{2}{b\left(e^{(bx+a)} - e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - 2/(b*(e^(b
*x + a) - e^(-b*x - a)))
```

3.30 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rubi [A] time = 0.0310056, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}$$

Mathematica [A] time = 0.0149412, size = 13, normalized size = 0.57

$$-\frac{2\coth(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*Coth[2*(a + b*x)])/b
```

Maple [A] time = 0.013, size = 32, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{1}{\cosh(bx+a)\sinh(bx+a)} - 2\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

Maxima [A] time = 1.12956, size = 24, normalized size = 1.04

$$\frac{4}{b\left(e^{(-4\,bx-4\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.94948, size = 213, normalized size = 9.26

$$-\frac{4}{b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+6\,b\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [A] time = 1.17244, size = 24, normalized size = 1.04

$$-\frac{4}{b\left(e^{(4\,bx+4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

3.31 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0406723, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$

$$= -\frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a\right.}{2b}$$

$$= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Mathematica [C] time = 0.0151644, size = 29, normalized size = 0.59

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] time = 0.019, size = 52, normalized size = 1.1

$$-\frac{1}{b\sinh(bx+a)(\cosh(bx+a))^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2b}-3\,\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)/b-3*arctan(exp(b
*x+a))/b
```

Maxima [B] time = 1.52873, size = 122, normalized size = 2.49

$$\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{3\,e^{(-bx-a)}+2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

Fricas [B] time = 2.08012, size = 1401, normalized size = 28.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
```

```
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [B] time = 1.22911, size = 140, normalized size = 2.86

$$-\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{4b}-\frac{3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4e^{(bx+a)}-4e^{(-bx-a)}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - (3*(e^(b*x
 + a) - e^(-b*x - a))^2 + 8)/(((e^(b*x + a) - e^(-b*x - a))^3 + 4*e^(b*x +
a) - 4*e^(-b*x - a))*b)
```

3.32 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^3(a+bx)}{3b}-\frac{2\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -(Coth[a + b*x]/b) - (2*Tanh[a + b*x])/b + Tanh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0352183, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tanh^3(a+bx)}{3b}-\frac{2\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]/b) - (2*Tanh[a + b*x])/b + Tanh[a + b*x]^3/(3*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth(a+bx)}{b} - \frac{2\tanh(a+bx)}{b} + \frac{\tanh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0383215, size = 46, normalized size = 1.21

$$-\frac{5\tanh(a+bx)}{3b} - \frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]/b) - (5*Tanh[a + b*x])/(3*b) - (Sech[a + b*x]^2*Tanh[a + b*
x])/(3*b)
```

Maple [A] time = 0.019, size = 44, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{1}{(\cosh(bx+a))^3\sinh(bx+a)} - 4\left(2/3+1/3\,(\operatorname{sech}(bx+a))^2\right)\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^3-4*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

Maxima [B] time = 1.1555, size = 127, normalized size = 3.34

$$-\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1\right)} - \frac{16}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8*
b*x - 8*a) + 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8
*b*x - 8*a) + 1))
```

Fricas [B] time = 2.06823, size = 622, normalized size = 16.37

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}+2\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+a\right)^{2}+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -16/3*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 -
 3*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 - b)*sinh(
b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**4, x)
```

Giac [A] time = 1.2102, size = 82, normalized size = 2.16

$$-\frac{2}{b\left(e^{(2\,bx+2\,a)}-1\right)}+\frac{2\left(3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+5\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) - 1)) + 2/3*(3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a)
+ 5)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

3.33 $\int \mathrm{csch}^2(a+bx)\mathrm{sech}^5(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\operatorname{csch}(a+bx)}{8b}-\frac{15\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

```
[Out] (-15*ArcTan[Sinh[a + b*x]])/(8*b) - (15*Csch[a + b*x])/(8*b) + (5*Csch[a +
b*x]*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]*Sech[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0427769, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{15\operatorname{csch}(a+bx)}{8b}-\frac{15\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

```
[Out] (-15*ArcTan[Sinh[a + b*x]])/(8*b) - (15*Csch[a + b*x])/(8*b) + (5*Csch[a +
b*x]*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]*Sech[a + b*x]^4)/(4*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, -i\text{csch}(a+bx)\right)}{b}$$

$$= \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b} - \frac{(5i)\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{4b}$$

$$= \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b} - \frac{(15i)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,d\right.}{8b}$$

$$= -\frac{15\text{csch}(a+bx)}{8b} + \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b} - \frac{(15}{}$$

$$= -\frac{15\tan^{-1}(\sinh(a+bx))}{8b} - \frac{15\text{csch}(a+bx)}{8b} + \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(}{}$$

Mathematica [C] time = 0.0151623, size = 29, normalized size = 0.41

$$-\frac{\text{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] time = 0.019, size = 71, normalized size = 1.

$$-\frac{1}{b\sinh\left(bx+a\right)\left(\cosh\left(bx+a\right)\right)^{4}}-\frac{5\left(\mathrm{sech}\left(bx+a\right)\right)^{3}\tanh\left(bx+a\right)}{4b}-\frac{15\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{8b}-\frac{15\arctan\left(\mathrm{e}^{bx}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^5,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^4-5/4*sech(b*x+a)^3*tanh(b*x+a)/b-15/8*sech(b*
x+a)*tanh(b*x+a)/b-15/4*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.77856, size = 184, normalized size = 2.63

$$\frac{15\arctan\left(e^{(-bx-a)}\right)}{4b}-\frac{15\,e^{(-bx-a)}+40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}+40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b\left(3\,e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 15/4*arctan(e^(-b*x - a))/b - 1/4*(15*e^(-b*x - a) + 40*e^(-3*b*x - 3*a) +
18*e^(-5*b*x - 5*a) + 40*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(3*e^(-
2*b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a)
 - e^(-10*b*x - 10*a) + 1))
```

Fricas [B] time = 2.20505, size = 3312, normalized size = 47.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
+ a)^9 + 20*(27*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
 140*(9*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^6 + 6*(315*cosh(b*
```

```
x + a)^4 + 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 18*cosh(b*x + a)^5 +
10*(189*cosh(b*x + a)^5 + 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 20*(63*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 2
)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 60*(9*cosh(b*x + a)^7 + 14*cosh(b*
x + a)^5 + 3*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(
b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 42*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
+ 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a
)^6 + 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cos
h(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 + 5*cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 + 28*cosh(b*x +
 a)^6 + 10*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cos
h(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)^
5 - 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x
 + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 + 56*cosh(b*x + a)^6 + 18*co
sh(b*x + a)^4 + 24*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(
b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^1
0 + 3*b*cosh(b*x + a)^8 + 3*(15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24
*(5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a
)^6 + 2*(105*b*cosh(b*x + a)^4 + 42*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6
+ 12*(21*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 + 105*b*cosh(b*x
 + a)^4 + 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)
^7 + 21*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15*b*cosh(b*x + a)^8 + 28*b*cosh(b*x + a
)^6 + 10*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(
5*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - 4*b*cosh
(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(a + bx) \operatorname{sech}^5(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**5, x)
```

Giac [B] time = 1.17037, size = 174, normalized size = 2.49

$$-\frac{15\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{16b}-\frac{7\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+36e^{(bx+a)}-36e^{(-bx-a)}}{4\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2b}-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -15/16*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - 1/4*(7*(
e^(b*x + a) - e^(-b*x - a))^3 + 36*e^(b*x + a) - 36*e^(-b*x - a))/(((e^(b*x
 + a) - e^(-b*x - a))^2 + 4)^2*b) - 2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.34 $\int \mathbf{csch}^3(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0247705, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.0427629, size = 34, normalized size = 1.21

$$-\frac{\operatorname{csch}^2(a+bx) + 2\log(\sinh(a+bx)) - 2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/(2*b)
```

Maple [A] time = 0.018, size = 27, normalized size = 1.

$$-\frac{1}{2\,b\,(\sinh(bx+a))^2} - \frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2-ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.6152, size = 123, normalized size = 4.39

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 2.39966, size = 1037, normalized size = 37.04

$$-\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 - 1\right)\sinh(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [B] time = 1.17974, size = 135, normalized size = 4.82

$$\frac{\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} + 2\right)}{2\,b} - \frac{\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} - 2\right)}{2\,b} + \frac{e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} - 6}{2\,b\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b - 1/2*log(e^(2*b*x + 2*a)
 + e^(-2*b*x - 2*a) - 2)/b + 1/2*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(
b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2))
```

3.35 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rubi [A] time = 0.0456363, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= -\frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= \frac{3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0333699, size = 68, normalized size = 1.39

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b} - \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) - (3*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

Maple [A] time = 0.02, size = 43, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{1}{2\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{2}}-\frac{3}{2\cosh\left(bx+a\right)}+3\,\mathrm{Artanh}\left(\mathrm{e}^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.08435, size = 143, normalized size = 2.92

$$\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}-\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{3\,e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

Fricas [B] time = 2.46611, size = 1935, normalized size = 39.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
```

```
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [B] time = 1.17436, size = 154, normalized size = 3.14

$$\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{4\,b}-\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}-\frac{3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}-4\,e^{(-bx-a)}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/4*log(e^(b*x + a) + e^(-b*x - a) + 2)/b - 3/4*log(e^(b*x + a) + e^(-b*x -
 a) - 2)/b - (3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/(((e^(b*x + a) + e^(-b*
x - a))^3 - 4*e^(b*x + a) - 4*e^(-b*x - a))*b)
```

3.36 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.044109, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0157642, size = 47, normalized size = 1.09

$$8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} - \frac{\operatorname{sech}^2(a+bx)}{16b} - \frac{\log(\tanh(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] 8*(-Csch[a + b*x]^2/(16*b) - Log[Tanh[a + b*x]]/(4*b) - Sech[a + b*x]^2/(16
*b))
```

Maple [A] time = 0.023, size = 48, normalized size = 1.1

$$-\frac{1}{2\,b\,(\sinh(bx+a))^2\,(\cosh(bx+a))^2} - \frac{1}{b\,(\cosh(bx+a))^2} - 2\,\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^2-1/b/cosh(b*x+a)^2-2*ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.51274, size = 138, normalized size = 3.21

$$-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

Fricas [B] time = 2.56934, size = 2102, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
```

```
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [B] time = 1.21559, size = 136, normalized size = 3.16

$$\frac{\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)}{b}-\frac{\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}-\frac{4\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b - log(e^(2*b*x + 2*a) + e^(-2
*b*x - 2*a) - 2)/b - 4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/(((e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a))^2 - 4)*b)
```

3.37 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\operatorname{sech}^3(a+bx)}{6b}-\frac{5\operatorname{sech}(a+bx)}{2b}+\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}$$

```
[Out] (5*ArcTanh[Cosh[a + b*x]])/(2*b) - (5*Sech[a + b*x])/(2*b) - (5*Sech[a + b*
x]^3)/(6*b) - (Csch[a + b*x]^2*Sech[a + b*x]^3)/(2*b)
```

Rubi [A] time = 0.0495726, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$-\frac{5\operatorname{sech}^3(a+bx)}{6b}-\frac{5\operatorname{sech}(a+bx)}{2b}+\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] (5*ArcTanh[Cosh[a + b*x]])/(2*b) - (5*Sech[a + b*x])/(2*b) - (5*Sech[a + b*
x]^3)/(6*b) - (Csch[a + b*x]^2*Sech[a + b*x]^3)/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$

$$= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}$$

$$= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \operatorname{sech}(a+bx)\right)}{2b}$$

$$= -\frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= \frac{5\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3}{2b}$$

Mathematica [A] time = 0.0317842, size = 83, normalized size = 1.26

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}^3(a+bx)}{3b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{2\operatorname{sech}(a+bx)}{b} - \frac{5\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) - (5*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Maple [A] time = 0.025, size = 53, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{1}{2\left(\cosh\left(bx+a\right)\right)^{3}\left(\sinh\left(bx+a\right)\right)^{2}}-\frac{5}{6\left(\cosh\left(bx+a\right)\right)^{3}}-\frac{5}{2\cosh\left(bx+a\right)}+5\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^3-5/6/cosh(b*x+a)^3-5/2/cosh(b*x+a)+5*a
rctanh(exp(b*x+a)))
```

Maxima [B] time = 1.08624, size = 201, normalized size = 3.05

$$\frac{5\log\left(e^{(-bx-a)}+1\right)}{2b}-\frac{5\log\left(e^{(-bx-a)}-1\right)}{2b}-\frac{15\,e^{(-bx-a)}+20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}+20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{3\,b\left(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 5/2*log(e^(-b*x - a) + 1)/b - 5/2*log(e^(-b*x - a) - 1)/b - 1/3*(15*e^(-b*x
 - a) + 20*e^(-3*b*x - 3*a) - 22*e^(-5*b*x - 5*a) + 20*e^(-7*b*x - 7*a) + 1
5*e^(-9*b*x - 9*a))/(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x
 - 6*a) + e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Fricas [B] time = 2.56246, size = 4433, normalized size = 67.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
+ a)^9 + 40*(27*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
 280*(9*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^6 + 4*(945*cosh(b*x
+ a)^4 + 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 44*cosh(b*x + a)^5 + 2
```

```
0*(189*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x +
a)^4 + 40*(63*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 40*(27*cosh(b*x + a)^7 + 21*cosh(b
*x + a)^5 - 11*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cos
h(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 14*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 + 1
4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)
^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cosh(
b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28*cosh(b*x + a)^6
 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x
+ a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - 4*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 +
sinh(b*x + a)^10 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)
^8 + 8*(15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b
*x + a)^4 + 14*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4
*(63*cosh(b*x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^
5 + 2*(105*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a
)^5 - 5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a
)^8 + 28*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 -
 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*
log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b*x + a)^8 + 28*cosh(b
*x + a)^6 - 22*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 30
*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 +
b*sinh(b*x + a)^10 + b*cosh(b*x + a)^8 + (45*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a)^8 + 8*(15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b
*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 + 14*b*cosh(b*x + a)^2 - b)*sin
h(b*x + a)^6 + 4*(63*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*
x + a))*sinh(b*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 +
35*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*
cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^5 - 5*b*cosh(b*x + a)^3 - b*cosh(b*x +
a))*sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (45*b*cosh(b*x + a)^8 + 28*b*cosh
(b*x + a)^6 - 30*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a
)^2 + 2*(5*b*cosh(b*x + a)^9 + 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 -
4*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**4, x)
```

Giac [B] time = 1.20296, size = 182, normalized size = 2.76

$$\frac{5\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{4\,b}-\frac{5\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}-\frac{e^{(bx+a)}+e^{(-bx-a)}}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)b}-\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+2\right)}{3\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 5/4*log(e^(b*x + a) + e^(-b*x - a) + 2)/b - 5/4*log(e^(b*x + a) + e^(-b*x -
 a) - 2)/b - (e^(b*x + a) + e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2
- 4)*b) - 4/3*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 2)/(b*(e^(b*x + a) + e^(-
b*x - a))^3)
```

3.38 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^4(a+bx)}{4b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - (3*Log[Tanh[a + b*x]])/b + (3*Tanh[a + b*x]^2)/(2*
b) - Tanh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0494925, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$-\frac{\tanh^4(a+bx)}{4b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (3*Log[Tanh[a + b*x]])/b + (3*Tanh[a + b*x]^2)/(2*
b) - Tanh[a + b*x]^4/(4*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{3\log(\tanh(a+bx))}{b} + \frac{3\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.46096, size = 54, normalized size = 0.93

$$-\frac{2\operatorname{csch}^2(a+bx)+\operatorname{sech}^4(a+bx)+4\operatorname{sech}^2(a+bx)+12\log(\sinh(a+bx))-12\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

```
[Out] -(2*Csch[a + b*x]^2 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[a + b*x]] + 4*Sec
h[a + b*x]^2 + Sech[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.024, size = 61, normalized size = 1.1

$$-\frac{1}{2\,b\,(\sinh(bx+a))^2\,(\cosh(bx+a))^4} - \frac{3}{4\,b\,(\cosh(bx+a))^4} - \frac{3}{2\,b\,(\cosh(bx+a))^2} - 3\,\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^5,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^4-3/4/b/cosh(b*x+a)^4-3/2/b/cosh(b*x+a)^2-
3*ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.59329, size = 244, normalized size = 4.21

$$-\frac{3\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{3\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b}-\frac{2\left(3e^{(-2bx-2a)}+6e^{(-4bx-4a)}-2e^{(-6bx-6a)}\right.}{b\left(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-4e^{(-6bx-6a)}-e^{(-8bx-8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3*log(e^(-b*x - a) + 1)/b - 3*log(e^(-b*x - a) - 1)/b + 3*log(e^(-2*b*x -
2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*
a) + 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) - e^
(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*
a) + e^(-12*b*x - 12*a) + 1))
```

Fricas [B] time = 2.78425, size = 5837, normalized size = 100.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^1
0 + 6*(45*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^8 + 12*cosh(b*x + a)^8 + 48*(1
5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)
^4 + 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*c
osh(b*x + a)^5 + 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(
105*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^4 + 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)
^8 + 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*si
nh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 -
 cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 - 7*cosh
(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^
7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)
```

```
^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 60*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)
^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 + 45*cosh(b*x + a)^8 - 14*
cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 + 5*cosh(b*x + a)^9 - 2*cosh
(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
 + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b
*x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x
 + a)^3 + cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 +
30*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^
6 + 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b
*x + a)^6 + 8*(99*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x +
a)^6 - 70*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(
b*x + a)^4 + 4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^
5 - 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x +
a)^10 + 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)
^11 + 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x
+ a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x +
a) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 + 8*cosh(b*x + a)^7 - 2*cosh(b
*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x +
 a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 + 2*b*cos
h(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^10 + 20*(11*b*co
sh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495
*b*cosh(b*x + a)^4 + 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*co
sh(b*x + a)^5 + 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 4
*b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 + 105*b*cosh(b*x + a)^4 - 7*b
*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 + 63*b*cosh
(b*x + a)^5 - 7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*
cosh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 + 420*b*cosh(b*x + a)^6 - 70*b*cos
h(b*x + a)^4 - 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x
 + a)^9 + 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^
3 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b
*x + a)^10 + 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 - 30*b*cosh(b*x +
a)^4 - 3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 +
 5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 - b*cosh(b
*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**5, x)
```

Giac [B] time = 1.22073, size = 240, normalized size = 4.14

$$\frac{3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}{2b}-\frac{3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}+\frac{3e^{(2bx+2a)}+3e^{(-2bx-2a)}-10}{2b\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}-\frac{9\left(e^{(2bx+2a)}+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b - 3/2*log(e^(2*b*x + 2*a)
 + e^(-2*b*x - 2*a) - 2)/b + 1/2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) -
10)/(b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)) - 1/4*(9*(e^(2*b*x + 2*a)
+ e^(-2*b*x - 2*a))^2 + 52*e^(2*b*x + 2*a) + 52*e^(-2*b*x - 2*a) + 84)/(b*(
e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)^2)
```

3.39 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\operatorname{csch}(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/b + Csch[a + b*x]/b - Csch[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0270247, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2621, 302, 207}

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\operatorname{csch}(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b + Csch[a + b*x]/b - Csch[a + b*x]^3/(3*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{csch}(a+bx)}{b} - \frac{\operatorname{csch}^3(a+bx)}{3b} + \frac{i\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\tan^{-1}(\sinh(a+bx))}{b} + \frac{\operatorname{csch}(a+bx)}{b} - \frac{\operatorname{csch}^3(a+bx)}{3b}$$

Mathematica [C] time = 0.0169181, size = 33, normalized size = 0.89

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x],x]

[Out] -(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Sinh[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.017, size = 39, normalized size = 1.1

$$-\frac{1}{3\,b\,(\sinh(bx+a))^3} + \frac{1}{b\sinh(bx+a)} + 2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a),x)

[Out] -1/3/b/sinh(b*x+a)^3+1/b/sinh(b*x+a)+2*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.78521, size = 122, normalized size = 3.3

$$-\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{2\left(3e^{(-bx-a)}-10e^{(-3bx-3a)}+3e^{(-5bx-5a)}\right)}{3b\left(3e^{(-2bx-2a)}-3e^{(-4bx-4a)}+e^{(-6bx-6a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-b*x - a))/b - 2/3*(3*e^(-b*x - a) - 10*e^(-3*b*x - 3*a) + 3*e
^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) - 1))
```

Fricas [B] time = 2.33626, size = 1423, normalized size = 38.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 10*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 30*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*
cosh(b*x + a)^4 - 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/
(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
- 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*
b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^
2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(
b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) -
b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x), x)
```

Giac [B] time = 1.25549, size = 111, normalized size = 3.

$$\frac{\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{2b} + \frac{2\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 - 4\right)}{3b\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + 2/3*(3*(e^(
b*x + a) - e^(-b*x - a))^2 - 4)/(b*(e^(b*x + a) - e^(-b*x - a))^3)
```

3.40 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{2\coth(a+bx)}{b}$$

```
[Out] (2*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + Tanh[a + b*x]/b
```

Rubi [A] time = 0.0367932, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{2\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

```
[Out] (2*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + Tanh[a + b*x]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx,x,i\tanh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,i\tanh(a+bx)\right)}{b}$$
$$= \frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Mathematica [A] time = 0.0391928, size = 45, normalized size = 1.22

$$\frac{\tanh(a+bx)}{b}+\frac{5\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

```
[Out] (5*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b) + Tanh[a +
b*x]/b
```

Maple [A] time = 0.02, size = 50, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{1}{3\cosh(bx+a)(\sinh(bx+a))^3}+\frac{4}{3\cosh(bx+a)\sinh(bx+a)}+\frac{8\tanh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)+4/3/sinh(b*x+a)/cosh(b*x+a)+8/3*tanh(b*
x+a))
```

Maxima [B] time = 1.11659, size = 122, normalized size = 3.3

$$\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1\right)}-\frac{16}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*b
*x - 8*a) - 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*
b*x - 8*a) - 1))
```

Fricas [B] time = 2.16805, size = 622, normalized size = 16.81

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}-2\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+a\right)^{2}-2\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -16/3*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 - 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 +
 b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 10*b*cosh(b*x + a)^4 + 3*b)*sinh(
b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{4}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**2, x)
```

Giac [A] time = 1.19365, size = 82, normalized size = 2.22

$$-\frac{2}{b\left(e^{(2\,bx+2\,a)}+1\right)}+\frac{2\left(3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+5\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) + 1)) + 2/3*(3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a)
+ 5)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

3.41 $\int \mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\mathrm{csch}^3(a+bx)}{6b}+\frac{5\mathrm{csch}(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)}{2b}$$

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) + (5*Csch[a + b*x])/(2*b) - (5*Csch[a + b*x
]^3)/(6*b) + (Csch[a + b*x]^3*Sech[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0432956, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{5\mathrm{csch}^3(a+bx)}{6b}+\frac{5\mathrm{csch}(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) + (5*Csch[a + b*x])/(2*b) - (5*Csch[a + b*x
]^3)/(6*b) + (Csch[a + b*x]^3*Sech[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, -i\operatorname{csch}(a+bx\right.}{2b}$$

$$= \frac{5\operatorname{csch}(a+bx)}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{5\tan^{-1}(\sinh(a+bx))}{2b} + \frac{5\operatorname{csch}(a+bx)}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(}{2b}$$

Mathematica [C] time = 0.0158004, size = 33, normalized size = 0.5

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

```
[Out] -(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, -Sinh[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.025, size = 73, normalized size = 1.1

$$-\frac{1}{3b\left(\sinh\left(bx+a\right)\right)^{3}\left(\cosh\left(bx+a\right)\right)^{2}}+\frac{5}{3b\sinh\left(bx+a\right)\left(\cosh\left(bx+a\right)\right)^{2}}+\frac{5\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b}+5\,\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^3,x)
```

```
[Out] -1/3/b/sinh(b*x+a)^3/cosh(b*x+a)^2+5/3/b/sinh(b*x+a)/cosh(b*x+a)^2+5/2*sech
(b*x+a)*tanh(b*x+a)/b+5*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.78328, size = 178, normalized size = 2.7

$$-\frac{5\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{15\,e^{(-bx-a)}-20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}-20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{3\,b\left(e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -5*arctan(e^(-b*x - a))/b - 1/3*(15*e^(-b*x - a) - 20*e^(-3*b*x - 3*a) - 22
*e^(-5*b*x - 5*a) - 20*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(e^(-2*b*
x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + e^(
-10*b*x - 10*a) - 1))
```

Fricas [B] time = 2.54743, size = 3283, normalized size = 49.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x +
 a)^9 + 20*(27*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^7 - 20*cosh(b*x + a)^7 +
140*(9*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^6 + 2*(945*cosh(b*x +
 a)^4 - 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10
```

```
*(189*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a
)^4 + 20*(63*cosh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^3 - 20*cosh(b*x + a)^3 + 20*(27*cosh(b*x + a)^7 - 21*cosh(b*
x + a)^5 - 11*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)^
3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 14*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 - 14
*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)^
6 - 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b
*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6
- 30*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x +
 a)^2 + 2*(5*cosh(b*x + a)^9 - 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 + 4*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 5*(27*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 - 22*cosh(b*x + a
)^4 - 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(b*cosh(b*x
 + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - b*cosh
(b*x + a)^8 + (45*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(15*b*cosh(b*x
 + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^6 + 2*(105*b
*cosh(b*x + a)^4 - 14*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 4*(63*b*cosh
(b*x + a)^5 - 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 - 35*b*cosh(b*x + a)^4 - 15*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 7*b*cosh(
b*x + a)^5 - 5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 + b*cos
h(b*x + a)^2 + (45*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 - 30*b*cosh(b*x
 + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^
9 - 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cos
h(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**3, x)
```

Giac [B] time = 1.16499, size = 173, normalized size = 2.62

$$\frac{5\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{4b}+\frac{e^{(bx+a)}-e^{(-bx-a)}}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)b}+\frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-2\right)}{3b\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 5/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + (e^(b*x + a
) - e^(-b*x - a))/(((e^(b*x + a) - e^(-b*x - a))^2 + 4)*b) + 4/3*(3*(e^(b*x
 + a) - e^(-b*x - a))^2 - 2)/(b*(e^(b*x + a) - e^(-b*x - a))^3)
```

3.42 $\int \mathbf{csch}^4(a+bx)\mathbf{sech}^4(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\coth(a+bx)}{b}$$

```
[Out] (3*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + (3*Tanh[a + b*x])/b - Tanh[a
+ b*x]^3/(3*b)
```

Rubi [A] time = 0.0395316, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$-\frac{\tanh^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

```
[Out] (3*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + (3*Tanh[a + b*x])/b - Tanh[a
+ b*x]^3/(3*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{3\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \frac{3\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0192102, size = 43, normalized size = 0.81

$$16\left(\frac{\coth(2(a+bx))}{3b} - \frac{\coth(2(a+bx))\operatorname{csch}^2(2(a+bx))}{6b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

```
[Out] 16*(Coth[2*(a + b*x)]/(3*b) - (Coth[2*(a + b*x)]*Csch[2*(a + b*x)]^2)/(6*b)
)
```

Maple [A] time = 0.025, size = 62, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{1}{3\,(\cosh(bx+a))^3(\sinh(bx+a))^3} + 2\,\frac{1}{(\cosh(bx+a))^3\sinh(bx+a)} + 8\,\left(2/3+1/3\,(\operatorname{sech}(bx+a))^2\right)\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^3+2/sinh(b*x+a)/cosh(b*x+a)^3+8*(2/3+1/
3*sech(b*x+a)^2)*tanh(b*x+a))
```

Maxima [A] time = 1.17381, size = 122, normalized size = 2.3

$$\frac{32\,e^{(-4\,bx-4\,a)}}{b\left(3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-8\,bx-8\,a)} + e^{(-12\,bx-12\,a)} - 1\right)} - \frac{32}{3\,b\left(3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-8\,bx-8\,a)} + e^{(-12\,bx-12\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 32*e^(-4*b*x - 4*a)/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12*b*
x - 12*a) - 1)) - 32/3/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12
*b*x - 12*a) - 1))
```

Fricas [B] time = 2.26766, size = 900, normalized size = 16.98

$$-\frac{}{3\left(b\cosh(bx+a)^{10}+120\,b\cosh(bx+a)^{3}\sinh(bx+a)^{7}+45\,b\cosh(bx+a)^{2}\sinh(bx+a)^{8}+10\,b\cosh(bx+a)\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -64/3*(cosh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(
b*cosh(b*x + a)^10 + 120*b*cosh(b*x + a)^3*sinh(b*x + a)^7 + 45*b*cosh(b*x
+ a)^2*sinh(b*x + a)^8 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x +
a)^10 - 3*b*cosh(b*x + a)^6 + 3*(70*b*cosh(b*x + a)^4 - b)*sinh(b*x + a)^6
+ 18*(14*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(14*b*co
sh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 60*(2*b*cosh(b*x + a
)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + (45*b*cosh
(b*x + a)^8 - 45*b*cosh(b*x + a)^4 + 2*b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x
 + a)^9 - 9*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**4, x)
```

Giac [A] time = 1.1614, size = 42, normalized size = 0.79

$$-\frac{32\left(3e^{(4bx+4a)}-1\right)}{3b\left(e^{(4bx+4a)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -32/3*(3*e^(4*b*x + 4*a) - 1)/(b*(e^(4*b*x + 4*a) - 1)^3)
```

3.43 $\int \textbf{csch}^4(a+bx)\textbf{sech}^5(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\text{csch}^3(a+bx)}{24b}+\frac{35\text{csch}(a+bx)}{8b}+\frac{35\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b}+\frac{7\text{csch}^3(a+bx)\text{sech}^2}{8b}$$

```
[Out] (35*ArcTan[Sinh[a + b*x]])/(8*b) + (35*Csch[a + b*x])/(8*b) - (35*Csch[a +
b*x]^3)/(24*b) + (7*Csch[a + b*x]^3*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]
^3*Sech[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0478115, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{35\text{csch}^3(a+bx)}{24b}+\frac{35\text{csch}(a+bx)}{8b}+\frac{35\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b}+\frac{7\text{csch}^3(a+bx)\text{sech}^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```


```
[Out] (35*ArcTan[Sinh[a + b*x]])/(8*b) + (35*Csch[a + b*x])/(8*b) - (35*Csch[a +
b*x]^3)/(24*b) + (7*Csch[a + b*x]^3*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]
^3*Sech[a + b*x]^4)/(4*b)
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(7i)\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{4b}$$

$$= \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(35i)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\right.}{}$$

$$= \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(35i)\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$

$$= \frac{35\operatorname{csch}(a+bx)}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx}{}$$

$$= \frac{35\tan^{-1}(\sinh(a+bx))}{8b} + \frac{35\operatorname{csch}(a+bx)}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^3(a+bx)\operatorname{sec}}{8b}$$

Mathematica [C] time = 0.017379, size = 33, normalized size = 0.37

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

```
[Out] -(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, -Sinh[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.024, size = 92, normalized size = 1.

$$-\frac{1}{3\,b\,(\sinh\,(bx+a))^{3}\,(\cosh\,(bx+a))^{4}}+\frac{7}{3\,b\sinh\,(bx+a)\,(\cosh\,(bx+a))^{4}}+\frac{35\,(\mathrm{sech}\,(bx+a))^{3}\tanh\,(bx+a)}{12\,b}+\frac{35\,\mathrm{sec}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^5,x)
```

```
[Out] -1/3/b/sinh(b*x+a)^3/cosh(b*x+a)^4+7/3/b/sinh(b*x+a)/cosh(b*x+a)^4+35/12*se
ch(b*x+a)^3*tanh(b*x+a)/b+35/8*sech(b*x+a)*tanh(b*x+a)/b+35/4*arctan(exp(b*
x+a))/b
```

Maxima [B] time = 1.56036, size = 240, normalized size = 2.7

$$-\frac{35\,\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{105\,e^{(-bx-a)}+70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}-204\,e^{(-7\,bx-7\,a)}-329\,e^{(-9\,bx-9\,a)}+70\,e^{(-11\,bx-11\,a)}+}{12\,b\left(e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}+3\,e^{(-10\,bx-10\,a)}-e^{(-12\,bx-12\,a)}-e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -35/4*arctan(e^(-b*x - a))/b + 1/12*(105*e^(-b*x - a) + 70*e^(-3*b*x - 3*a)
 - 329*e^(-5*b*x - 5*a) - 204*e^(-7*b*x - 7*a) - 329*e^(-9*b*x - 9*a) + 70*
e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a))/(b*(e^(-2*b*x - 2*a) - 3*e^(-4
*b*x - 4*a) - 3*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a) + 1))
```

Fricas [B] time = 2.62474, size = 6041, normalized size = 67.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="fricas")
```

[Out] 1/12*(105*cosh(b*x + a)^13 + 1365*cosh(b*x + a)*sinh(b*x + a)^12 + 105*sinh
(b*x + a)^13 + 70*(117*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^11 + 70*cosh(b*x
+ a)^11 + 770*(39*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^10 + 7*(10
725*cosh(b*x + a)^4 + 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 329*cosh(
b*x + a)^9 + 21*(6435*cosh(b*x + a)^5 + 550*cosh(b*x + a)^3 - 141*cosh(b*x
+ a))*sinh(b*x + a)^8 + 12*(15015*cosh(b*x + a)^6 + 1925*cosh(b*x + a)^4 -
987*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^7 - 204*cosh(b*x + a)^7 + 84*(2145*
cosh(b*x + a)^7 + 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 - 17*cosh(b*x +
a))*sinh(b*x + a)^6 + 7*(19305*cosh(b*x + a)^8 + 4620*cosh(b*x + a)^6 - 59
22*cosh(b*x + a)^4 - 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 329*cosh(b
x + a)^5 + 7(10725*cosh(b*x + a)^9 + 3300*cosh(b*x + a)^7 - 5922*cosh(b*x
+ a)^5 - 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4 + 14*(2
145*cosh(b*x + a)^10 + 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 - 510*cos
h(b*x + a)^4 - 235*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 70*cosh(b*x + a)^
3 + 14*(585*cosh(b*x + a)^11 + 275*cosh(b*x + a)^9 - 846*cosh(b*x + a)^7 -
306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 + 15*cosh(b*x + a))*sinh(b*x + a)
^2 + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + sinh(b*x +
a)^14 + (91*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^12 + cosh(b*x + a)^12 + 4*(
91*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh(b*x + a
)^4 + 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10 + 2*(10
01*cosh(b*x + a)^5 + 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b*x + a)^
9 + 3*(1001*cosh(b*x + a)^6 + 165*cosh(b*x + a)^4 - 45*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 + 33*cosh(b*
x + a)^5 - 15*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 3*(1001*co
sh(b*x + a)^8 + 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 - 28*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a)^9 + 39
6*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 - 84*cosh(b*x + a)^3 + 9*cosh(b*x +
a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 + 495*cosh(b*x + a)^8 - 630*c
osh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 + 3)*sinh(b*x + a
)^4 + 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 + 55*cosh(b*x + a)^9 - 90*
cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 + 3*cosh(b*x + a)
)*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 + 66*cosh(b*x + a)^10 - 135*cosh(b
*x + a)^8 - 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 + 6*cosh(b*x +
a)^11 - 15*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^5 + 6*co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 7*(195*cosh(b*x + a)^12 + 110*cosh(b*x + a)^10 - 423*cosh(b*
x + a)^8 - 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 +
15)*sinh(b*x + a) + 105*cosh(b*x + a))/(b*cosh(b*x + a)^14 + 14*b*cosh(b*x
+ a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 + b*cosh(b*x + a)^12 + (91*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x + a)^3 + 3*b*cosh(b*
x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (1001*b*cosh(b*x + a)^4 +
66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(1001*b*cosh(b*x + a)^5 +
110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^9 - 3*b*cosh(b*x
+ a)^8 + 3*(1001*b*cosh(b*x + a)^6 + 165*b*cosh(b*x + a)^4 - 45*b*cosh(b*x

```
 + a)^2 - b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)^7 + 33*b*cosh(b*x +
a)^5 - 15*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 3*b*cosh(b
*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 + 308*b*cosh(b*x + a)^6 - 210*b*cosh(
b*x + a)^4 - 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 2*(1001*b*cosh(b*x
 + a)^9 + 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^5 - 84*b*cosh(b*x + a
)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^4 + (1001*b*co
sh(b*x + a)^10 + 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*x + a)^6 - 210*b*cosh
(b*x + a)^4 + 45*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 4*(91*b*cosh(b*
x + a)^11 + 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^7 - 42*b*cosh(b*x + a
)^5 + 15*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*
x + a)^2 + (91*b*cosh(b*x + a)^12 + 66*b*cosh(b*x + a)^10 - 135*b*cosh(b*x
+ a)^8 - 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 + 18*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 + 6*b*cosh(b*x + a)^11 - 15
*b*cosh(b*x + a)^9 - 12*b*cosh(b*x + a)^7 + 9*b*cosh(b*x + a)^5 + 6*b*cosh(
b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**5, x)
```

Giac [A] time = 1.19093, size = 207, normalized size = 2.33

$$\frac{35\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{16b}+\frac{11\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+52e^{(bx+a)}-52e^{(-bx-a)}}{4\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2b}+\frac{2\left(9\left(e^{(bx+a)}-e^{(-bx-a)}\right)\cdots}{3b\left(e^{(bx+a)}-e^{(-bx-a)}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 35/16*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + 1/4*(11*(
e^(b*x + a) - e^(-b*x - a))^3 + 52*e^(b*x + a) - 52*e^(-b*x - a))/(((e^(b*x
```

```
+ a) - e^(-b*x - a))^2 + 4)^2*b) + 2/3*(9*(e^(b*x + a) - e^(-b*x - a))^2 -
4)/(b*(e^(b*x + a) - e^(-b*x - a))^3)
```

3.44 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\coth^4(a+bx)}{4b}+\frac{\coth^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] Coth[a + b*x]^2/b - Coth[a + b*x]^4/(4*b) + Log[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0320298, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2620, 266, 43}

$$-\frac{\coth^4(a+bx)}{4b}+\frac{\coth^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x],x]
```

```
[Out] Coth[a + b*x]^2/b - Coth[a + b*x]^4/(4*b) + Log[Tanh[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.123431, size = 46, normalized size = 1.18

$$\frac{-\operatorname{csch}^4(a+bx)+2\operatorname{csch}^2(a+bx)+4\log(\sinh(a+bx))-4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x],x]
```

```
[Out] (2*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 4*Log[Cosh[a + b*x]] + 4*Log[Sinh[a
+ b*x]])/(4*b)
```

Maple [A] time = 0.019, size = 39, normalized size = 1.

$$-\frac{1}{4\,b\left(\sinh\left(bx+a\right)\right)^4}+\frac{1}{2\,b\left(\sinh\left(bx+a\right)\right)^2}+\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a),x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4+1/2/b/sinh(b*x+a)^2+ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.55373, size = 180, normalized size = 4.62

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}-\frac{2\left(e^{(-2\,bx-2\,a)}-4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b - 2*(e^(-2*b*x - 2*a) - 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a
) - 1))
```

Fricas [B] time = 2.41554, size = 2952, normalized size = 75.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 - 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
```

```
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x), x)
```

Giac [B] time = 1.20856, size = 171, normalized size = 4.38

$$-\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}{2b}+\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}-\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-20e^{(2bx+2a)}-20e^{(-2bx-\cdots}}{4b\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 1/2*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2)/b - 1/4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2
 - 20*e^(2*b*x + 2*a) - 20*e^(-2*b*x - 2*a) + 44)/(b*(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) - 2)^2)
```

3.45 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\operatorname{sech}(a+bx)}{8b}-\frac{15\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[Out] (-15*ArcTanh[Cosh[a + b*x]])/(8*b) + (15*Sech[a + b*x])/(8*b) + (5*Csch[a +
 b*x]^2*Sech[a + b*x])/(8*b) - (Csch[a + b*x]^4*Sech[a + b*x])/(4*b)
```

Rubi [A] time = 0.0501103, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{15\operatorname{sech}(a+bx)}{8b}-\frac{15\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cosh[a + b*x]])/(8*b) + (15*Sech[a + b*x])/(8*b) + (5*Csch[a +
 b*x]^2*Sech[a + b*x])/(8*b) - (Csch[a + b*x]^4*Sech[a + b*x])/(4*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$

$$= -\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b} + \frac{5\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{4b}$$

$$= \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b} + \frac{15\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{8b}$$

$$= \frac{15\operatorname{sech}(a+bx)}{8b} + \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b} + \frac{15\,\mathrm{S}}{}$$

$$= -\frac{15\tanh^{-1}(\cosh(a+bx))}{8b} + \frac{15\operatorname{sech}(a+bx)}{8b} + \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b} - \operatorname{csch}$$

Mathematica [A] time = 0.0348478, size = 105, normalized size = 1.5

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{7\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{7\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}(a+bx)}{b} + \frac{15\log\left(\tanh\left(\frac{1}{2}\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (7*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (15*Log[Tanh[
(a + b*x)/2]])/(8*b) + (7*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4
```

```
/(64*b) + Sech[a + b*x]/b
```

Maple [A] time = 0.019, size = 61, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{1}{4\left(\sinh\left(bx+a\right)\right)^{4}\cosh\left(bx+a\right)}+\frac{5}{8\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{2}}+\frac{15}{8\cosh\left(bx+a\right)}-\frac{15\,\mathrm{Artanh}\left(\mathrm{e}^{bx+a}\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)+5/8/sinh(b*x+a)^2/cosh(b*x+a)+15/8/cosh
(b*x+a)-15/4*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.03293, size = 209, normalized size = 2.99

$$-\frac{15\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{15\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{15\,e^{(-bx-a)}-40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}-40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b\left(3\,e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(-b*x - a) + 1)/b + 15/8*log(e^(-b*x - a) - 1)/b - 1/4*(15*e^(-
b*x - a) - 40*e^(-3*b*x - 3*a) + 18*e^(-5*b*x - 5*a) - 40*e^(-7*b*x - 7*a)
+ 15*e^(-9*b*x - 9*a))/(b*(3*e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-
6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) - e^(-10*b*x - 10*a) - 1))
```

Fricas [B] time = 2.5051, size = 4477, normalized size = 63.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x +
 a)^9 + 40*(27*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^7 - 80*cosh(b*x + a)^7 +
280*(9*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^6 + 12*(315*cosh(b*
x + a)^4 - 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 36*cosh(b*x + a)^5 +
20*(189*cosh(b*x + a)^5 - 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 40*(63*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 2
)*sinh(b*x + a)^3 - 80*cosh(b*x + a)^3 + 120*(9*cosh(b*x + a)^7 - 14*cosh(b
*x + a)^5 + 3*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 42*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x +
a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*co
sh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 21*cosh(b*x + a)^5 + 5*cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 - 28*cosh(b*x
+ a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*co
sh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)
^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x
 + a)^9 + sinh(b*x + a)^10 + 3*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3
*cosh(b*x + a)^8 + 24*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 +
 2*(105*cosh(b*x + a)^4 - 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(
b*x + a)^6 + 12*(21*cosh(b*x + a)^5 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*s
inh(b*x + a)^5 + 2*(105*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 -
21*cosh(b*x + a)^5 + 5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3
*(15*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 1
2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a)
)*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b
*x + a)^8 - 56*cosh(b*x + a)^6 + 18*cosh(b*x + a)^4 - 24*cosh(b*x + a)^2 +
3)*sinh(b*x + a) + 30*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x +
a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - 3*b*cosh(b*x + a)^8 + 3*(15*b*cos
h(b*x + a)^2 - b)*sinh(b*x + a)^8 + 24*(5*b*cosh(b*x + a)^3 - b*cosh(b*x +
a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 - 42*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 12*(21*b*cosh(b*x + a)^5 - 14*b*cos
h(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^5 + 2*b*cosh(b*x + a)^4 + 2*(
105*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + b)*s
inh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 21*b*cosh(b*x + a)^5 + 5*b*cosh(
b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15
*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^9 - 12*b*cosh(b*x +
 a)^7 + 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh
(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**2, x)
```

Giac [B] time = 1.16122, size = 185, normalized size = 2.64

$$-\frac{15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{16\,b}+\frac{15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}+\frac{7\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-36\,e^{(bx+a)}-36\,e^{(-bx-a)}}{4\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2 b}+\frac{}{b\left(e^{(b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -15/16*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 15/16*log(e^(b*x + a) + e^(-
b*x - a) - 2)/b + 1/4*(7*(e^(b*x + a) + e^(-b*x - a))^3 - 36*e^(b*x + a) -
36*e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2*b) + 2/(b*(e^(b*x
+ a) + e^(-b*x - a)))
```

3.46 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\coth^2(a+bx)}{2b}+\frac{3\log(\tanh(a+bx))}{b}$$

```
[Out] (3*Coth[a + b*x]^2)/(2*b) - Coth[a + b*x]^4/(4*b) + (3*Log[Tanh[a + b*x]])/
b - Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0451177, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$-\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\coth^2(a+bx)}{2b}+\frac{3\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

```
[Out] (3*Coth[a + b*x]^2)/(2*b) - Coth[a + b*x]^4/(4*b) + (3*Log[Tanh[a + b*x]])/
b - Tanh[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{3\coth^2(a+bx)}{2b} - \frac{\coth^4(a+bx)}{4b} + \frac{3\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.194266, size = 56, normalized size = 0.97

$$\frac{-\operatorname{csch}^4(a+bx)+4\operatorname{csch}^2(a+bx)+2\operatorname{sech}^2(a+bx)+12\log(\sinh(a+bx))-12\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

```
[Out] (4*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[
a + b*x]] + 2*Sech[a + b*x]^2)/(4*b)
```

Maple [A] time = 0.022, size = 69, normalized size = 1.2

$$-\frac{1}{4b(\sinh(bx+a))^4(\cosh(bx+a))^2}+\frac{3}{4b(\sinh(bx+a))^2(\cosh(bx+a))^2}+\frac{3}{2b(\cosh(bx+a))^2}+3\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^3,x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4/cosh(b*x+a)^2+3/4/b/sinh(b*x+a)^2/cosh(b*x+a)^2+3/2/b/
cosh(b*x+a)^2+3*ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.56098, size = 242, normalized size = 4.17

$$\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{3\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}-\frac{2\left(3\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-6\right.}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}-4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 3*log(e^(-2*b*x - 2
*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a
) - 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) + e^(
-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - 1))
```

Fricas [B] time = 2.58681, size = 5836, normalized size = 100.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^10
 + 6*(45*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^8 - 12*cosh(b*x + a)^8 + 48*(15
*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)^
4 - 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*co
sh(b*x + a)^5 - 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(1
05*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^4 - 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 -
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)^
8 - 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*sin
h(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
 + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 -
cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 30*cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 - 7*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^7
 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^
```

```
5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 60*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)^
9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 - 45*cosh(b*x + a)^8 - 14*c
osh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^
2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 - 5*cosh(b*x + a)^9 - 2*cosh(
b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
+ a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*
x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 3
0*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6
 - 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*
x + a)^6 + 8*(99*cosh(b*x + a)^7 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 +
 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a
)^6 - 70*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(55*cosh(b*x + a)^9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5
 + 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a
)^10 - 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^
11 - 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 - 8*cosh(b*x + a)^7 - 2*cosh(b*
x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x +
a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 - 2*b*cosh
(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^10 + 20*(11*b*cos
h(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495*
b*cosh(b*x + a)^4 - 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*cos
h(b*x + a)^5 - 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 4*
b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 - 7*b*
cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 - 63*b*cosh(
b*x + a)^5 - 7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*c
osh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 - 420*b*cosh(b*x + a)^6 - 70*b*cosh
(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x
+ a)^9 - 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3
 - b*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b*
x + a)^10 - 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 + 30*b*cosh(b*x + a
)^4 - 3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 -
5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - b*cosh(b*
x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**3, x)
```

Giac [B] time = 1.19195, size = 240, normalized size = 4.14

$$-\frac{3\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)}{2\,b}+\frac{3\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}+\frac{3\,e^{(2\,bx+2\,a)}+3\,e^{(-2\,bx-2\,a)}+10}{2\,b\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)}-\frac{9\left(e^{(2\,bx+2\,a)}+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 3/2*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2)/b + 1/2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) +
 10)/(b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)) - 1/4*(9*(e^(2*b*x + 2*a)
 + e^(-2*b*x - 2*a))^2 - 52*e^(2*b*x + 2*a) - 52*e^(-2*b*x - 2*a) + 84)/(b*
(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)^2)
```

3.47 $\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{35\mathrm{sech}(a+bx)}{8b}-\frac{35\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3}{8b}$$

```
[Out] (-35*ArcTanh[Cosh[a + b*x]])/(8*b) + (35*Sech[a + b*x])/(8*b) + (35*Sech[a
+ b*x]^3)/(24*b) + (7*Csch[a + b*x]^2*Sech[a + b*x]^3)/(8*b) - (Csch[a + b*
x]^4*Sech[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.055344, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{35\mathrm{sech}(a+bx)}{8b}-\frac{35\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```



```
[Out] (-35*ArcTanh[Cosh[a + b*x]])/(8*b) + (35*Sech[a + b*x])/(8*b) + (35*Sech[a
+ b*x]^3)/(24*b) + (7*Csch[a + b*x]^2*Sech[a + b*x]^3)/(8*b) - (Csch[a + b*
x]^4*Sech[a + b*x]^3)/(4*b)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{7\operatorname{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{4b}\\
&= \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{35\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,d\right.}{8b}\\
&= \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{35\operatorname{Subst}\left(\int \left(1+x^2\right.\right.}{}\\
&= \frac{35\operatorname{sech}(a+bx)}{8b} + \frac{35\operatorname{sech}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx}{}\\
&= -\frac{35\tanh^{-1}(\cosh(a+bx))}{8b} + \frac{35\operatorname{sech}(a+bx)}{8b} + \frac{35\operatorname{sech}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^2(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.037301, size = 121, normalized size = 1.36

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{11\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{\operatorname{sech}^3(a+bx)}{3b} + \frac{11\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{3\operatorname{sech}(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

```
[Out] (11*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (35*Log[Tanh
[(a + b*x)/2]])/(8*b) + (11*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]
```

```
^4/(64*b) + (3*Sech[a + b*x])/b + Sech[a + b*x]^3/(3*b)
```

Maple [A] time = 0.024, size = 71, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{1}{4\,(\sinh(bx+a))^4\,(\cosh(bx+a))^3}+\frac{7}{8\,(\cosh(bx+a))^3\,(\sinh(bx+a))^2}+\frac{35}{24\,(\cosh(bx+a))^3}+\frac{35}{8\,\cosh(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^3+7/8/sinh(b*x+a)^2/cosh(b*x+a)^3+35/24
/cosh(b*x+a)^3+35/8/cosh(b*x+a)-35/4*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.01249, size = 263, normalized size = 2.96

$$-\frac{35\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{35\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{105\,e^{(-bx-a)}-70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}+204\,e^{(-7\,bx-7\,a)}-329\,e^{(-9\,bx}}{12\,b\left(e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}+3\,e^{(-10\,bx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -35/8*log(e^(-b*x - a) + 1)/b + 35/8*log(e^(-b*x - a) - 1)/b - 1/12*(105*e^
(-b*x - a) - 70*e^(-3*b*x - 3*a) - 329*e^(-5*b*x - 5*a) + 204*e^(-7*b*x - 7
*a) - 329*e^(-9*b*x - 9*a) - 70*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a)
)/(b*(e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) - 3*e^(-8*
b*x - 8*a) + 3*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a)
 - 1))
```

Fricas [B] time = 2.74153, size = 8095, normalized size = 90.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="fricas")

[Out] 1/24*(210*cosh(b*x + a)^13 + 2730*cosh(b*x + a)*sinh(b*x + a)^12 + 210*sinh
(b*x + a)^13 + 140*(117*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^11 - 140*cosh(b*
x + a)^11 + 1540*(39*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^10 + 14
*(10725*cosh(b*x + a)^4 - 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 658*c
osh(b*x + a)^9 + 42*(6435*cosh(b*x + a)^5 - 550*cosh(b*x + a)^3 - 141*cosh(
b*x + a))*sinh(b*x + a)^8 + 24*(15015*cosh(b*x + a)^6 - 1925*cosh(b*x + a)^
4 - 987*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^7 + 408*cosh(b*x + a)^7 + 168*(
2145*cosh(b*x + a)^7 - 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 + 17*cosh(
b*x + a))*sinh(b*x + a)^6 + 14*(19305*cosh(b*x + a)^8 - 4620*cosh(b*x + a)^
6 - 5922*cosh(b*x + a)^4 + 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 658*
cosh(b*x + a)^5 + 14*(10725*cosh(b*x + a)^9 - 3300*cosh(b*x + a)^7 - 5922*c
osh(b*x + a)^5 + 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4
+ 28*(2145*cosh(b*x + a)^10 - 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 +
510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^3 - 140*cosh(b
*x + a)^3 + 28*(585*cosh(b*x + a)^11 - 275*cosh(b*x + a)^9 - 846*cosh(b*x +
 a)^7 + 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(
b*x + a)^2 - 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + si
nh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cosh(b*x + a)^
12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh
(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10
 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b
*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 - 33
*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 3*
(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 28*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a
)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x + a)^3 + 9*co
sh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh(b*x + a)^8
 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 - 3)*sinh
(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*cosh(b*x + a)
^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x + a)^10 - 13
5*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 - 6*co
sh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^
5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x +
 a)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cos
h(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 +
 (1001*cosh(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(
b*x + a)^10 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x +
 a))*sinh(b*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*c
```

```
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x
+ a)^7 - 33*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
+ a)^7 + 3*(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^
4 + 28*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*c
osh(b*x + a)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x +
a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh
(b*x + a)^8 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^
2 - 3)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*co
sh(b*x + a)^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^
3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x +
 a)^10 - 135*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a
)^13 - 6*cosh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cos
h(b*x + a)^5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(co
sh(b*x + a) + sinh(b*x + a) - 1) + 14*(195*cosh(b*x + a)^12 - 110*cosh(b*x
+ a)^10 - 423*cosh(b*x + a)^8 + 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 -
 30*cosh(b*x + a)^2 + 15)*sinh(b*x + a) + 210*cosh(b*x + a))/(b*cosh(b*x +
a)^14 + 14*b*cosh(b*x + a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 - b*cosh(b
*x + a)^12 + (91*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x
 + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (100
1*b*cosh(b*x + a)^4 - 66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(100
1*b*cosh(b*x + a)^5 - 110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 3*b*cosh(b*x + a)^8 + 3*(1001*b*cosh(b*x + a)^6 - 165*b*cosh(b*x +
 a)^4 - 45*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)
^7 - 33*b*cosh(b*x + a)^5 - 15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*
x + a)^7 + 3*b*cosh(b*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 - 308*b*cosh(b*x
 + a)^6 - 210*b*cosh(b*x + a)^4 + 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6
 + 2*(1001*b*cosh(b*x + a)^9 - 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^
5 + 84*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 - 3*b*cosh(b*
x + a)^4 + (1001*b*cosh(b*x + a)^10 - 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*
x + a)^6 + 210*b*cosh(b*x + a)^4 + 45*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a
)^4 + 4*(91*b*cosh(b*x + a)^11 - 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^
7 + 42*b*cosh(b*x + a)^5 + 15*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b
*x + a)^3 - b*cosh(b*x + a)^2 + (91*b*cosh(b*x + a)^12 - 66*b*cosh(b*x + a)
^10 - 135*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 -
 18*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 - 6*b*
cosh(b*x + a)^11 - 15*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 9*b*cosh(b
*x + a)^5 - 6*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**4, x)
```

Giac [B] time = 1.17845, size = 215, normalized size = 2.42

$$-\frac{35 \log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right)}{16\, b} + \frac{35 \log\left(e^{(bx+a)} + e^{(-bx-a)} - 2\right)}{16\, b} + \frac{11 \left(e^{(bx+a)} + e^{(-bx-a)}\right)^3 - 52\, e^{(bx+a)} - 52\, e^{(-bx-a)}}{4 \left(\left(e^{(bx+a)} + e^{(-bx-a)}\right)^2 - 4\right)^2 b} + \frac{2 \left(9 \left(\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -35/16*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 35/16*log(e^(b*x + a) + e^(-
b*x - a) - 2)/b + 1/4*(11*(e^(b*x + a) + e^(-b*x - a))^3 - 52*e^(b*x + a) -
 52*e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2*b) + 2/3*(9*(e^(b
*x + a) + e^(-b*x - a))^2 + 4)/(b*(e^(b*x + a) + e^(-b*x - a))^3)
```

3.48 $\int \mathbf{csch}^5(a+bx)\mathbf{sech}^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^4(a+bx)}{4b}-\frac{2\tanh^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{2\coth^2(a+bx)}{b}+\frac{6\log(\tanh(a+bx))}{b}$$

```
[Out] (2*Coth[a + b*x]^2)/b - Coth[a + b*x]^4/(4*b) + (6*Log[Tanh[a + b*x]])/b -
(2*Tanh[a + b*x]^2)/b + Tanh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0539318, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^4(a+bx)}{4b}-\frac{2\tanh^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{2\coth^2(a+bx)}{b}+\frac{6\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```


```
[Out] (2*Coth[a + b*x]^2)/b - Coth[a + b*x]^4/(4*b) + (6*Log[Tanh[a + b*x]])/b -
(2*Tanh[a + b*x]^2)/b + Tanh[a + b*x]^4/(4*b)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{2\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{6\log(\tanh(a+bx))}{b} - \frac{2\tanh^2(a+bx)}{b} + \frac{\tanh}{}$$

Mathematica [A] time = 0.0248018, size = 77, normalized size = 1.12

$$32\left(-\frac{\operatorname{csch}^4(a+bx)}{128b}+\frac{3\operatorname{csch}^2(a+bx)}{64b}+\frac{\operatorname{sech}^4(a+bx)}{128b}+\frac{3\operatorname{sech}^2(a+bx)}{64b}+\frac{3\log(\tanh(a+bx))}{16b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

```
[Out] 32*((3*Csch[a + b*x]^2)/(64*b) - Csch[a + b*x]^4/(128*b) + (3*Log[Tanh[a +
b*x]])/(16*b) + (3*Sech[a + b*x]^2)/(64*b) + Sech[a + b*x]^4/(128*b))
```

Maple [A] time = 0.024, size = 81, normalized size = 1.2

$$-\frac{1}{4b(\sinh(bx+a))^4(\cosh(bx+a))^4}+\frac{1}{b(\sinh(bx+a))^2(\cosh(bx+a))^4}+\frac{3}{2b(\cosh(bx+a))^4}+3\frac{1}{b(\cosh(bx+a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^5,x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4/cosh(b*x+a)^4+1/b/sinh(b*x+a)^2/cosh(b*x+a)^4+3/2/b/co
sh(b*x+a)^4+3/b/cosh(b*x+a)^2+6*ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.65642, size = 203, normalized size = 2.94

$$\frac{6\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{6\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{6\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{4\left(3\,e^{(-2\,bx-2\,a)} - 11\,e^{(-6\,bx-6\,a)} - 11\,e^{(-10\,bx-10\,a)} + 3\,e\right.}{b\left(4\,e^{(-4\,bx-4\,a)} - 6\,e^{(-8\,bx-8\,a)} + 4\,e^{(-12\,bx-12\,a)} - e^{(-16}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 6*log(e^(-b*x - a) + 1)/b + 6*log(e^(-b*x - a) - 1)/b - 6*log(e^(-2*b*x - 2
*a) + 1)/b - 4*(3*e^(-2*b*x - 2*a) - 11*e^(-6*b*x - 6*a) - 11*e^(-10*b*x -
10*a) + 3*e^(-14*b*x - 14*a))/(b*(4*e^(-4*b*x - 4*a) - 6*e^(-8*b*x - 8*a) +
 4*e^(-12*b*x - 12*a) - e^(-16*b*x - 16*a) - 1))
```

Fricas [B] time = 2.74439, size = 6263, normalized size = 90.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 2*(6*cosh(b*x + a)^14 + 2184*cosh(b*x + a)^3*sinh(b*x + a)^11 + 546*cosh(b*
x + a)^2*sinh(b*x + a)^12 + 84*cosh(b*x + a)*sinh(b*x + a)^13 + 6*sinh(b*x
+ a)^14 + 22*(273*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^10 - 22*cosh(b*x + a)^
10 + 44*(273*cosh(b*x + a)^5 - 5*cosh(b*x + a))*sinh(b*x + a)^9 + 198*(91*c
osh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^8 + 528*(39*cosh(b*x + a)
^7 - 5*cosh(b*x + a)^3)*sinh(b*x + a)^7 + 22*(819*cosh(b*x + a)^8 - 210*cos
h(b*x + a)^4 - 1)*sinh(b*x + a)^6 - 22*cosh(b*x + a)^6 + 132*(91*cosh(b*x +
 a)^9 - 42*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^5 + 66*(91*cosh(b
*x + a)^10 - 70*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(2
73*cosh(b*x + a)^11 - 330*cosh(b*x + a)^7 - 55*cosh(b*x + a)^3)*sinh(b*x +
a)^3 + 6*(91*cosh(b*x + a)^12 - 165*cosh(b*x + a)^8 - 55*cosh(b*x + a)^4 +
1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^16 + 560*cosh(b*x
 + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*cosh(b
*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b*x + a)^4 - 1)*s
inh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x
 + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a)^9 + 6*(
2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^8 + 6*cosh(b*
```

x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 + 3*cosh(b*x + a))
*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x + a)^6 + 3*cosh(b
*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*cosh(b*x + a)^7 +
 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^12 - 495*cosh(b*
x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cosh(b*x + a)^4 + 1
6*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x + a)^5 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b*x + a)^10 + 7*c
osh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cosh(b*x + a)^15 -
3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1
)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*x + a)^1
6 + 560*cosh(b*x + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a
)^14 + 16*cosh(b*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)
^5 - cosh(b*x + a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(
b*x + a)^9 + 6*(2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x +
a)^8 + 6*cosh(b*x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 +
3*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x +
 a)^6 + 3*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*c
osh(b*x + a)^7 + 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^
12 - 495*cosh(b*x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cos
h(b*x + a)^4 + 16*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x +
 a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b
*x + a)^10 + 7*cosh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cos
h(b*x + a)^15 - 3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*s
inh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*
(21*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 - 33*cosh(b*x + a)^5 + 3*cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^16 + 560*b*cosh(b*x + a)^3*sinh(b*x
+ a)^13 + 120*b*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*b*cosh(b*x + a)*sinh(
b*x + a)^15 + b*sinh(b*x + a)^16 - 4*b*cosh(b*x + a)^12 + 4*(455*b*cosh(b*x
 + a)^4 - b)*sinh(b*x + a)^12 + 48*(91*b*cosh(b*x + a)^5 - b*cosh(b*x + a))
*sinh(b*x + a)^11 + 88*(91*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*
x + a)^10 + 880*(13*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^9
+ 6*b*cosh(b*x + a)^8 + 6*(2145*b*cosh(b*x + a)^8 - 330*b*cosh(b*x + a)^4 +
 b)*sinh(b*x + a)^8 + 16*(715*b*cosh(b*x + a)^9 - 198*b*cosh(b*x + a)^5 + 3
*b*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*b*cosh(b*x + a)^10 - 66*b*cosh(
b*x + a)^6 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*b*cosh(b*x + a)^
11 - 66*b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^3)*sinh(b*x + a)^5 - 4*b*cosh
(b*x + a)^4 + 4*(455*b*cosh(b*x + a)^12 - 495*b*cosh(b*x + a)^8 + 105*b*cos
h(b*x + a)^4 - b)*sinh(b*x + a)^4 + 16*(35*b*cosh(b*x + a)^13 - 55*b*cosh(b
*x + a)^9 + 21*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5
*b*cosh(b*x + a)^14 - 11*b*cosh(b*x + a)^10 + 7*b*cosh(b*x + a)^6 - b*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 16*(b*cosh(b*x + a)^15 - 3*b*cosh(b*x + a)^11
 + 3*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**5, x)
```

Giac [A] time = 1.22274, size = 174, normalized size = 2.52

$$-\frac{3\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)}{b}+\frac{3\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}+\frac{4\left(3\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^3-20\,e^{(2\,bx+2\,a)}-2\right.}{\left(\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -3*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 3*log(e^(2*b*x + 2*a) +
e^(-2*b*x - 2*a) - 2)/b + 4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 20*
e^(2*b*x + 2*a) - 20*e^(-2*b*x - 2*a))/(((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a
))^2 - 4)^2*b)
```

3.49 $\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} \, dx$

Optimal. Leaf size=106

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

Rubi [A] time = 0.116135, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2575, 298, 203, 206}

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] time = 0.0575567, size = 59, normalized size = 0.56

$$\frac{2\sinh^{\frac{9}{2}}(a+bx)\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\sinh^2(a+bx)\right)}{9b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(9/2))/(9*b*Sqrt[Cosh[a + b*x]])
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^{\frac{7}{2}}(\cosh(bx+a))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{7}{2}}}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

Fricas [B] time = 2.71279, size = 2819, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 - 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 + 2)*sinh(b*x +
a)^3 + 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/2)/cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx+a)^{\frac{7}{2}}}{\cosh (bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

3.50 $\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]
^(3/2))
```

Rubi [A] time = 0.0811908, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2574, 298, 203, 206}

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]
^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx\\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] time = 0.0479155, size = 59, normalized size = 0.73

$$\frac{2\sinh^{\frac{7}{2}}(a+bx)\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\sinh^2(a+bx)\right)}{7b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/2))/(7*b*Cosh[a + b*x]^(3/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^{\frac{5}{2}}(\cosh(bx+a))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

Fricas [B] time = 2.68691, size = 1705, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
 sinh(b*x + a)^2) + 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/2)/cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx+a\right)^{\frac{5}{2}}}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

3.51 $\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]])
```

Rubi [A] time = 0.0733108, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2575, 298, 203, 206}

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\frac{2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.0512258, size = 59, normalized size = 0.75

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\sinh^2(a+bx)\right)}{5b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(5/2))/(5*b*Sqrt[Cosh[a + b*x]])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (\sinh (bx+a))^{\frac{3}{2}} (\cosh (bx+a))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx+a)^{\frac{3}{2}}}{\cosh (bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

Fricas [B] time = 2.55228, size = 923, normalized size = 11.68

$$\frac{2\left(\cosh (bx+a)^2 + 2\,\cosh (bx+a)\sinh (bx+a) + \sinh (bx+a)^2 + 1\right)\arctan\left(-\cosh (bx+a)^2 + 2\,(\cosh (bx+a) + \sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) - 4*cosh(b*x + a)^2 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
+ a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
a) - sinh(b*x + a)^2) - 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a
))*sqrt(sinh(b*x + a)) - 8*cosh(b*x + a)*sinh(b*x + a) - 4*sinh(b*x + a)^2
- 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(3/2)/cosh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(3/2)/cosh(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

3.52 $\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b
```

Rubi [A] time = 0.0374936, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2574, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\end{aligned}$$

Mathematica [C] time = 0.0340016, size = 59, normalized size = 1.09

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\sinh^2(a+bx)\right)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]^(3/2))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \sqrt{\sinh(bx+a)}\frac{1}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

Fricas [B] time = 2.45379, size = 423, normalized size = 7.83

$$-\frac{2\arctan\left(-\cosh(bx+a)^2 + 2\left(\cosh(bx+a)+\sinh(bx+a)\right)\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)} - 2\cosh(bx+a)\sinh(bx\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
 + a)^2) + log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
 + a)^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/2)/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + b*x))/sqrt(cosh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

3.53 $\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b
```

Rubi [A] time = 0.0381403, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2575, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\end{aligned}$$

Mathematica [C] time = 0.0231303, size = 57, normalized size = 1.06

$$\frac{2\sqrt{\sinh(a+bx)}\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\sinh^2(a+bx)\right)}{b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -Sinh[a + b*x]^
2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b*x]])
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \sqrt{\cosh(bx+a)}\frac{1}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh\left(bx+a\right)}}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

Fricas [B] time = 2.47819, size = 421, normalized size = 7.8

$$2\arctan\left(-\cosh\left(bx+a\right)^{2}+2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\sqrt{\cosh\left(bx+a\right)}\sqrt{\sinh\left(bx+a\right)}-2\cosh\left(bx+a\right)\sinh\left(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2) - log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh\left(a+bx\right)}}{\sqrt{\sinh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/2)/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(a + b*x))/sqrt(sinh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

3.54 $\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b
*x]])
```

Rubi [A] time = 0.0713767, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2574, 298, 203, 206}

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b
*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx\\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.0325796, size = 57, normalized size = 0.72

$$
-\frac{2\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\sinh^2(a+bx)\right)}{b\sqrt{\sinh(a+bx)}\cosh^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(3/2)*Sqrt[Sinh[a + b*x]])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{3}{2}} (\sinh(bx+a))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

Fricas [B] time = 2.32161, size = 925, normalized size = 11.71

$$-\frac{2\left(\cosh(bx+a)^2 + 2\,\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\arctan\left(-\cosh(bx+a)^2 + 2\left(\cosh(bx+a) + \mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(
b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2) + 4*cosh(b*x + a)^2 + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
 a) - sinh(b*x + a)^2) + 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x +
a))*sqrt(sinh(b*x + a)) + 8*cosh(b*x + a)*sinh(b*x + a) + 4*sinh(b*x + a)^2
 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^{\frac{3}{2}}(a + bx)}{\sinh^{\frac{3}{2}}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(3/2)/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(3/2)/sinh(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx + a)^{\frac{3}{2}}}{\sinh(bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

3.55 $\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sinh[a + b*x]
^(3/2))
```

Rubi [A] time = 0.0729338, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2575, 298, 203, 206}

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sinh[a + b*x]
^(3/2))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx\\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] time = 0.0301987, size = 59, normalized size = 0.73

$$-\frac{2\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\sinh^2(a+bx)\right)}{3b\sinh^{\frac{3}{2}}(a+bx)\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -Sinh[a + b*
x]^2])/(3*b*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x]^(3/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{5}{2}}(\sinh(bx+a))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

Fricas [B] time = 2.28247, size = 1705, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
 sinh(b*x + a)^2) - 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/2)/sinh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(bx+a\right)^{\frac{5}{2}}}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

3.56 $\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)} dx$

Optimal. Leaf size=106

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[a + b*x]
^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rubi [A] time = 0.105444, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2574, 298, 203, 206}

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[a + b*x]
^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} + \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx \\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.}{b} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.0383257, size = 59, normalized size = 0.56

$$-\frac{2\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};-\sinh^2(a+bx)\right)}{5b\sinh^{\frac{5}{2}}(a+bx)\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -Sinh[a + b
*x]^2])/(5*b*Cosh[a + b*x]^(3/2)*Sinh[a + b*x]^(5/2))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{7}{2}}(\sinh(bx+a))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{7}{2}}}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

Fricas [B] time = 2.38686, size = 2819, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 - 2)*sinh(b*x +
a)^3 - 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/2)/sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{7}{2}}}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

3.57 $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Rubi [A] time = 0.181987, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
```

```
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{}{-3}\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh}{4b\cos}$$

Mathematica [C] time = 0.0544496, size = 59, normalized size = 0.38

$$\frac{3\sinh^{\frac{10}{3}}(a+bx)\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{5}{3},\frac{5}{3};\frac{8}{3};-\sinh^2(a+bx)\right)}{10b\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(10/3))/(10*b*Cosh[a + b*x]^(4/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^{\frac{7}{3}}(\cosh(bx+a))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

```
[Out] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{7}{3}}}{\cosh(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

Fricas [B] time = 2.33123, size = 3047, normalized size = 19.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
+ sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 + sqrt(3))*sinh(b*
x + a)^2 + 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 + sqrt(3)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^
2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sq
rt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
 a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*co
sh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(co
sh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh
(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a
)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a
)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^
(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/3)/cosh(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx + a)^{\frac{7}{3}}}{\cosh (bx + a)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

3.58 $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Rubi [A] time = 0.182096, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
```

```
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - $$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + $$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - $$

Mathematica [C] time = 0.0561897, size = 59, normalized size = 0.38

$$\frac{3\sinh^{\frac{8}{3}}(a+bx)\sqrt[3]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\sinh^2(a+bx)\right)}{8b\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(8/3))/(8*b*Cosh[a + b*x]^(2/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^{\frac{5}{3}}(\cosh(bx+a))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

```
[Out] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

Fricas [B] time = 2.13734, size = 2222, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*
sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*
cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1
/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 +
 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*s
inh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*c
osh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(-(cosh(b
*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 12*(cosh(
b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(
b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/3)/cosh(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx + a)^{\frac{5}{3}}}{\cosh (bx + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

3.59 $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)} dx$

Optimal. Leaf size=243

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b) - (3*Sinh[a + b*x]^(1/3))/(b*Cosh
[a + b*x]^(1/3))
```

Rubi [A] time = 0.244266, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 296, 634, 618, 204, 628, 206}

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b) - (3*Sinh[a + b*x]^(1/3))/(b*Cosh
[a + b*x]^(1/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{3\,\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \text{Su}\ldots$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x\ldots}\right.}{}\ldots$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} \ldots$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+\ldots}{\sinh^{\frac{2}{3}}(a+\ldots}\right.}{4\ldots}$$

Mathematica [C] time = 0.0510834, size = 59, normalized size = 0.24

$$\frac{3\sinh^{\frac{7}{3}}(a+bx)\sqrt[6]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};-\sinh^2(a+bx)\right)}{7b\sqrt[3]{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/3))/(7*b*Cosh[a + b*x]^(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (\sinh (bx+a))^{\frac{4}{3}} (\cosh (bx+a))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

```
[Out] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx+a)^{\frac{4}{3}}}{\cosh (bx+a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

Fricas [B] time = 2.28238, size = 3081, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
```

```
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 + 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x
 + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x +
 a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
 + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(4/3)/cosh(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx+a)^{\frac{4}{3}}}{\cosh (bx+a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

3.60 $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\sinh \cdots}}{\sqrt[3]{\cosh \cdots}}}{}\cdots\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b)
```

Rubi [A] time = 0.223059, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2574, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\sinh \cdots}}{\sqrt[3]{\cosh \cdots}}}{}\cdots\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b)
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x,\right.}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+}}{\sqrt[3]{\cosh(a+}}\right.}{4}$$

Mathematica [C] time = 0.0399421, size = 59, normalized size = 0.27

$$\frac{3\sinh^{\frac{5}{3}}(a+bx)\cosh^2(a+bx)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};-\sinh^2(a+bx)\right)}{5b\cosh^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(5/3))/(5*b*Cosh[a + b*x]^(5/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^{\frac{2}{3}}(\cosh(bx+a))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

```
[Out] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

Fricas [B] time = 2.21773, size = 2232, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a
)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt
(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*sqr
t(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*
x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + log((cosh(b*x
+ a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log((cosh(b
*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x
```

```
 + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)
*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(2/3)/cosh(b*x+a)**(2/3),x)
```

```
[Out] Integral(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

3.61 $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b)
```

Rubi [A] time = 0.0924688, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b)
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
```

```
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a-}{\cosh^{\frac{2}{3}}(a-}\right.}{4b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sinh}{\cosh}\right.}{2b}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.0339276, size = 59, normalized size = 0.46

$$\frac{3\sinh^{\frac{4}{3}}(a+bx)\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\sinh^2(a+bx)\right)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sinh (bx+a)}\frac{1}{\sqrt[3]{\cosh (bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

```
[Out] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx+a)^{\frac{1}{3}}}{\cosh (bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

Fricas [B] time = 1.93259, size = 1694, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqr
t(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a
)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
```

```
3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3
 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/
3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2
*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sinh{(a + bx)}}}{\sqrt[3]{\cosh{(a + bx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/3)/cosh(b*x+a)**(1/3),x)
```

```
[Out] Integral(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh{(bx + a)}^{\frac{1}{3}}}{\cosh{(bx + a)}^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

3.62 $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b)
```

Rubi [A] time = 0.0887767, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b)
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
```

```
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+b}{\sinh^{\frac{2}{3}}(a+b}\right.}{4b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cosh^{\frac{2}{3}}}{\sinh^{\frac{2}{3}}(}\right.}{2b}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.0243457, size = 59, normalized size = 0.46

$$\frac{3\sinh^{\frac{2}{3}}(a+bx)\sqrt[3]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\sinh^2(a+bx)\right)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt[3]{\cosh (bx+a)}\frac{1}{\sqrt[3]{\sinh (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

```
[Out] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^{\frac{1}{3}}}{\sinh (bx+a)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

Fricas [B] time = 1.95119, size = 1694, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqr
t(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a
)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
```

```
3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3
 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/
3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
+ 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2
*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/3)/sinh(b*x+a)**(1/3),x)
```

```
[Out] Integral(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

3.63 $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{c}}{\sqrt[3]{si}}\right.}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b)
```

Rubi [A] time = 0.200716, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{c}}{\sqrt[3]{si}}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b)
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x\right.}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right.}{4b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right.}{4b}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(}{\sinh^{\frac{2}{3}}(}\right.}{}$$

Mathematica [C] time = 0.0264577, size = 57, normalized size = 0.26

$$\frac{3\sqrt[3]{\sinh(a+bx)}\sqrt[6]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};-\sinh^2(a+bx)\right)}{b\sqrt[3]{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{2}{3}} (\sinh(bx+a))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)

[Out] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="maxima")

[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)

Fricas [B] time = 2.04492, size = 2232, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="fricas")

[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b

```
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log((co
sh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(
b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)
*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(2/3)/sinh(b*x+a)**(2/3),x)
```

```
[Out] Integral(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

3.64 $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)} dx$

Optimal. Leaf size=243

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(1/3))/(b*Sinh
[a + b*x]^(1/3))
```

Rubi [A] time = 0.235853, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 296, 634, 618, 204, 628, 206}

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(1/3))/(b*Sinh
[a + b*x]^(1/3))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx &= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}+\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx\\
&= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx,x,\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}+\frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx,x,\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}+\frac{\mathrm{S}\cdots}{\cdots}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b}+\frac{\mathrm{Subst}\left(\int \frac{1+2x}{1+x+x\cdots}\cdots\right)}{\cdots}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}-\frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}+\frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}\cdots\\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}-\frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+\cdots}}{\sqrt[3]{\cosh(a+\cdots}}\cdots\right.}{4\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.0283752, size = 57, normalized size = 0.23

$$
-\frac{3\cosh^2(a+bx)^{5/6}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};-\sinh^2(a+bx)\right)}{b\sqrt[3]{\sinh(a+bx)}\cosh^{\frac{5}{3}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(5/3)*Sinh[a + b*x]^(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (\cosh (bx+a))^{\frac{4}{3}} (\sinh (bx+a))^{-\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

```
[Out] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^{\frac{4}{3}}}{\sinh (bx+a)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

Fricas [B] time = 2.13265, size = 3081, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
```

```
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/
3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 - 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 - 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x
 + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x +
 a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
 + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(4/3)/sinh(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

3.65 $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Cosh[a + b*x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```

Rubi [A] time = 0.124612, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```


```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Cosh[a + b*x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```


Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\,\mathrm{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\,\mathrm{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{\mathrm{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\mathrm{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{}{-3}\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh}{2b\sinh}$$

Mathematica [C] time = 0.0349135, size = 59, normalized size = 0.38

$$-\frac{3\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\sinh^2(a+bx)\right)}{2b\sinh^{\frac{2}{3}}(a+bx)\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -Sinh[a + b*
x]^2])/(2*b*Cosh[a + b*x]^(4/3)*Sinh[a + b*x]^(2/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{5}{3}}(\sinh(bx+a))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

```
[Out] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

Fricas [B] time = 1.92521, size = 2222, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*
sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*
cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2
/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 +
 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*s
inh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*c
osh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(-(cosh(b
*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x
+ a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 12*(cosh(
b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(
b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/3)/sinh(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^{\frac{5}{3}}}{\sinh (bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

3.66 $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)} \, dx$

Optimal. Leaf size=155

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```

Rubi [A] time = 0.120756, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```


```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```


Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\Big(\int \frac{}{1+}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\Big(\int \frac{}{-3-}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh}{4b\sin}$$

Mathematica [C] time = 0.0314037, size = 59, normalized size = 0.38

$$-\frac{3\sqrt[3]{\cosh^2(a+bx)}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\sinh^2(a+bx)\right)}{4b\sinh^{\frac{4}{3}}(a+bx)\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -Sinh[a + b*
x]^2])/(4*b*Cosh[a + b*x]^(2/3)*Sinh[a + b*x]^(4/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^{\frac{7}{3}} (\sinh(bx+a))^{-\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

```
[Out] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

Fricas [B] time = 2.07201, size = 3047, normalized size = 19.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
+ sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 - sqrt(3))*sinh(b*
x + a)^2 - 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 - sqrt(3)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^
2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sq
rt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x +
 a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*co
sh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(co
sh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh
(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh
(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a
)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a
)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^
(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 -
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/3)/sinh(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

3.67 $\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} \, dx$

Optimal. Leaf size=16

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

```
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Rubi [A] time = 0.029653, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

```
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} \, dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.0105569, size = 16, normalized size = 1.

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

```
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (\cosh(x))^{\frac{2}{3}} (\sinh(x))^{-\frac{8}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

```
[Out] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

Maxima [B] time = 1.70559, size = 82, normalized size = 5.12

$$\frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}-\frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="maxima")
```

```
[Out] 3/5*(e^(-2*x) + 1)^(2/3)*e^(-4*x)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
- 3/5*(e^(-2*x) + 1)^(2/3)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
```

Fricas [B] time = 1.81418, size = 335, normalized size = 20.94

$$-\frac{6\left(\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)+\cosh\left(x\right)\right)\cosh\left(x\right)^{\frac{2}{3}}\sinh\left(x\right)^{\frac{1}{3}}}{5\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{2}-2\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="fricas")
```

```
[Out] -6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(
x) + cosh(x))*cosh(x)^(2/3)*sinh(x)^(1/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**(2/3)/sinh(x)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(x\right)^{\frac{2}{3}}}{\sinh\left(x\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)^(2/3)/sinh(x)^(8/3), x)
```

3.68 $\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Rubi [A] time = 0.0296269, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.0159079, size = 16, normalized size = 1.

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (\sinh(x))^{\frac{2}{3}} (\cosh(x))^{-\frac{8}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

```
[Out] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

Maxima [B] time = 1.63048, size = 82, normalized size = 5.12

$$-\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}+\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="maxima")
```

```
[Out] -3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)*e^(-4*x)/(e^(-2*x) + 1)^(8/3) +
 3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)/(e^(-2*x) + 1)^(8/3)
```

Fricas [B] time = 1.84881, size = 333, normalized size = 20.81

$$\frac{6\left(\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)\right)\cosh\left(x\right)^{\frac{1}{3}}\sinh\left(x\right)^{\frac{2}{3}}}{5\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="fricas")
```

```
[Out] 6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x
) - cosh(x))*cosh(x)^(1/3)*sinh(x)^(2/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + c
osh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**(2/3)/cosh(x)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)^{\frac{2}{3}}}{\cosh\left(x\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

3.69 $\int \cosh(x)\mathbf{csch}^{\frac{7}{3}}(x)\,dx$

Optimal. Leaf size=10

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

```
[Out] (-3*Csch[x]^(4/3))/4
```

Rubi [A] time = 0.0259891, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2621, 30}

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[x]^(7/3),x]

[Out] (-3*Csch[x]^(4/3))/4
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx &= -\operatorname{Subst}\left(\int \sqrt[3]{x}\,dx, x, \operatorname{csch}(x)\right)\\ &= -\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)\end{aligned}$$

Mathematica [A] time = 0.0077568, size = 10, normalized size = 1.

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[x]^(7/3),x]
```

```
[Out] (-3*Csch[x]^(4/3))/4
```

Maple [A] time = 0.006, size = 7, normalized size = 0.7

$$-\frac{3}{4}\left(\operatorname{csch}\left(x\right)\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(x)^(7/3),x)
```

```
[Out] -3/4*csch(x)^(4/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(x\right) \operatorname{csch}\left(x\right)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)*csch(x)^(7/3), x)
```

Fricas [B] time = 1.82942, size = 213, normalized size = 21.3

$$-\frac{3 \cdot 2^{\frac{1}{3}} \left(\frac{\cosh(x)+\sinh(x)}{\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\sinh(x)^{2}-1}\right)^{\frac{1}{3}} \left(\cosh\left(x\right) + \sinh\left(x\right)\right)}{2 \left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/2*2^(1/3)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1))^(1/3)*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{csch}(x)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)*csch(x)^(7/3), x)
```

3.70 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0167702, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 203}

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.0117064, size = 23, normalized size = 1.

$$\frac{\sinh(a+bx)}{b} - \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Maple [A] time = 0.016, size = 24, normalized size = 1.

$$\frac{\sinh(bx+a)}{b} - 2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a),x)
```

```
[Out] sinh(b*x+a)/b-2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.58081, size = 55, normalized size = 2.39

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Fricas [B] time = 1.93584, size = 254, normalized size = 11.04

$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)\sinh\left(bx+\right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\tanh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x), x)
```

Giac [A] time = 1.2086, size = 43, normalized size = 1.87

$$-\frac{4\,\arctan\left(e^{(bx+a)}\right)-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="giac")

[Out] -1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

3.71 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rubi [A] time = 0.0264534, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.0307261, size = 21, normalized size = 1.

$$\frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Maple [A] time = 0.013, size = 32, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\sinh(bx+a))^2}{\cosh(bx+a)} + 2\cosh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(-sinh(b*x+a)^2/cosh(b*x+a)+2*cosh(b*x+a))
```

Maxima [B] time = 1.02232, size = 73, normalized size = 3.48

$$\frac{e^{(-bx-a)}}{2b} + \frac{5e^{(-2bx-2a)}+1}{2b\left(e^{(-bx-a)}+e^{(-3bx-3a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

Fricas [A] time = 1.75055, size = 85, normalized size = 4.05

$$\frac{\cosh\left(bx+a\right)^2+\sinh\left(bx+a\right)^2+3}{2\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\tanh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**2, x)
```

Giac [B] time = 1.18766, size = 62, normalized size = 2.95

$$\frac{\frac{\left(5\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-a\right)}}{e^{\left(3\,bx+2\,a\right)}+e^{\left(bx\right)}}+e^{\left(bx+a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*e^(2*b*x + 2*a) + 1)*e^(-a)/(e^(3*b*x + 2*a) + e^(b*x)) + e^(b*x +
a))/b
```

3.72 $\int \sinh(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sinh(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sinh[a + b*x])/(2*b) - (Sinh[a + b*x]
*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0297894, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 203}

$$\frac{3\sinh(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sinh[a + b*x])/(2*b) - (Sinh[a + b*x]
*Tanh[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sinh(a+bx)\right)}{b}\\ &= -\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\ &= \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\ &= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} + \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0537199, size = 48, normalized size = 0.98

$$\frac{\sinh(a+bx)\tanh^2(a+bx)}{b} - \frac{3\left(\tan^{-1}(\sinh(a+bx)) - \tanh(a+bx)\operatorname{sech}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] (Sinh[a + b*x]*Tanh[a + b*x]^2)/b - (3*(ArcTan[Sinh[a + b*x]] - Sech[a + b*
x]*Tanh[a + b*x]))/(2*b)
```

Maple [A] time = 0.019, size = 70, normalized size = 1.4

$$\frac{(\sinh(bx+a))^3}{b(\cosh(bx+a))^2} + 3\frac{\sinh(bx+a)}{b(\cosh(bx+a))^2} - \frac{3\operatorname{sech}(bx+a)\tanh(bx+a)}{2b} - 3\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*sinh(b*x+a)^3/cosh(b*x+a)^2+3/b*sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+
a)*tanh(b*x+a)/b-3*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.59881, size = 123, normalized size = 2.51

$$\frac{3 \arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)}}{2b} + \frac{4 e^{(-2bx-2a)} - e^{(-4bx-4a)} + 1}{2b\left(e^{(-bx-a)} + 2e^{(-3bx-3a)} + e^{(-5bx-5a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - 1/2*e^(-b*x - a)/b + 1/2*(4*e^(-2*b*x - 2*a) - e
^(-4*b*x - 4*a) + 1)/(b*(e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + e^(-5*b*x - 5*
a)))
```

Fricas [B] time = 1.96743, size = 1289, normalized size = 26.31

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2 + 1\right)\sinh(bx+a)^4 + 3\cosh(bx+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*si
nh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3
 + 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a
)^2 + (5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x
+ a))*arctan(cosh(b*x + a) + sinh(b*x + a)) - 3*cosh(b*x + a)^2 + 6*(cosh(b
*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b
```

```
*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 2*b*cos
h(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + 2*(5*b*cosh(b*
x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a) + (5*b*cosh
(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh{\left(a + bx \right)} \tanh^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**3, x)
```

Giac [A] time = 1.20499, size = 88, normalized size = 1.8

$$\frac{\frac{2\left(e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^{2}} - 6\,\arctan\left(e^{(bx+a)}\right) + e^{(bx+a)} - e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - 6*arctan(e
^(b*x + a)) + e^(b*x + a) - e^(-b*x - a))/b
```

3.73 $\int \sinh(a+bx)\tanh^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0329711, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0348394, size = 37, normalized size = 1.

$$\frac{\cosh(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}+\frac{2\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Maple [A] time = 0.017, size = 67, normalized size = 1.8

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^4}{(\cosh(bx+a))^3}+\frac{4\,(\sinh(bx+a))^2}{3\,(\cosh(bx+a))^3}-\frac{8\,(\sinh(bx+a))^2}{3\cosh(bx+a)}+\frac{8\cosh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^4,x)
```

```
[Out] 1/b*(sinh(b*x+a)^4/cosh(b*x+a)^3+4/3*sinh(b*x+a)^2/cosh(b*x+a)^3-8/3*sinh(b
*x+a)^2/cosh(b*x+a)+8/3*cosh(b*x+a))
```

Maxima [B] time = 1.02946, size = 132, normalized size = 3.57

$$\frac{e^{(-bx-a)}}{2\,b}+\frac{33\,e^{(-2\,bx-2\,a)}+41\,e^{(-4\,bx-4\,a)}+27\,e^{(-6\,bx-6\,a)}+3}{6\,b\left(e^{(-bx-a)}+3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/6*(33*e^(-2*b*x - 2*a) + 41*e^(-4*b*x - 4*a) + 27*e^
(-6*b*x - 6*a) + 3)/(b*(e^(-b*x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5
*a) + e^(-7*b*x - 7*a)))
```

Fricas [B] time = 1.75935, size = 257, normalized size = 6.95

$$\frac{3\cosh(bx+a)^4 + 3\sinh(bx+a)^4 + 18\left(\cosh(bx+a)^2 + 2\right)\sinh(bx+a)^2 + 36\cosh(bx+a)^2 + 25}{6\left(b\cosh(bx+a)^3 + 3\,b\cosh(bx+a)\sinh(bx+a)^2 + 3\,b\cosh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 + 2)*sinh(
b*x + a)^2 + 36*cosh(b*x + a)^2 + 25)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + 3*b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**4, x)
```

Giac [B] time = 1.27727, size = 96, normalized size = 2.59

$$\frac{\frac{8\left(3\,e^{(5\,bx+5\,a)}+4\,e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^3} + 3\,e^{(bx+a)} + 3\,e^{(-bx-a)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*e^(5*b*x + 5*a) + 4*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(e^(2*b*x +
2*a) + 1)^3 + 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

3.74 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rubi [A] time = 0.0250738, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]

[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.0185642, size = 25, normalized size = 0.89

$$-\frac{\log(\cosh(a+bx)) - \frac{1}{2}\cosh^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -((-Cosh[a + b*x]^2/2 + Log[Cosh[a + b*x]])/b)
```

Maple [A] time = 0.017, size = 27, normalized size = 1.

$$\frac{(\sinh(bx+a))^2}{2b} - \frac{\ln(\cosh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a),x)
```

```
[Out] 1/2*sinh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Maxima [B] time = 1.54706, size = 76, normalized size = 2.71

$$-\frac{bx+a}{b} + \frac{e^{(2bx+2a)}}{8b} + \frac{e^{(-2bx-2a)}}{8b} - \frac{\log\left(e^{(-2bx-2a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

Fricas [B] time = 1.97428, size = 551, normalized size = 19.68

$$\frac{8\,bx\cosh(bx+a)^2+\cosh(bx+a)^4+4\,\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(4\,bx+3\,\cosh(bx+a)^2\right)\sinh}{8\left(b\cosh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x), x)
```

Giac [B] time = 1.23303, size = 77, normalized size = 2.75

$$\frac{8\,bx-\left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+e^{(2\,bx+2\,a)}-8\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + e^(2*b*x + 2*a) - 8
*log(e^(2*b*x + 2*a) + 1))/b
```

3.75 $\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] (-3*x)/2 + (3*Tanh[a + b*x])/(2*b) + (Sinh[a + b*x]^2*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0397953, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 206}

$$\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[a + b*x])/(2*b) + (Sinh[a + b*x]^2*Tanh[a + b*x])/(2*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(a+b$$
$$= -\frac{3x}{2} + \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}$$

Mathematica [A] time = 0.110423, size = 31, normalized size = 0.78

$$\frac{-6(a+bx)+\sinh(2(a+bx))+4\tanh(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] (-6*(a + b*x) + Sinh[2*(a + b*x)] + 4*Tanh[a + b*x])/(4*b)
```

Maple [A] time = 0.015, size = 39, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^3}{2\cosh(bx+a)} - \frac{3\,bx}{2} - \frac{3\,a}{2} + \frac{3\tanh(bx+a)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*sinh(b*x+a)^3/cosh(b*x+a)-3/2*b*x-3/2*a+3/2*tanh(b*x+a))
```

Maxima [A] time = 1.01326, size = 86, normalized size = 2.15

$$-\frac{3\,(bx+a)}{2\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{17\,e^{(-2\,bx-2\,a)}+1}{8\,b\left(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b + 1/8*(17*e^(-2*b*x - 2*a) + 1)/(
b*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a)))
```

Fricas [A] time = 1.82512, size = 150, normalized size = 3.75

$$\frac{\sinh\left(bx+a\right)^{3}-4\,(3\,bx+2)\cosh\left(bx+a\right)+3\left(\cosh\left(bx+a\right)^{2}+3\right)\sinh\left(bx+a\right)}{8\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sinh(b*x + a)^3 - 4*(3*b*x + 2)*cosh(b*x + a) + 3*(cosh(b*x + a)^2 + 3
)*sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{2}\left(a+bx\right)\tanh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**2,x)
```

[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**2, x)

Giac [A] time = 1.30143, size = 92, normalized size = 2.3

$$-\frac{12\,bx-\frac{\left(3\,e^{(4\,bx+4\,a)}-14\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,a)}}{e^{(2\,bx)}+e^{(4\,bx+2\,a)}}-e^{(2\,bx+2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")

[Out] -1/8*(12*b*x - (3*e^(4*b*x + 4*a) - 14*e^(2*b*x + 2*a) - 1)*e^(-2*a)/(e^(2*b*x) + e^(4*b*x + 2*a)) - e^(2*b*x + 2*a))/b

3.76 $\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\text{sech}^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}$$

```
[Out] Cosh[a + b*x]^2/(2*b) - (2*Log[Cosh[a + b*x]])/b - Sech[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.043562, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\text{sech}^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - (2*Log[Cosh[a + b*x]])/b - Sech[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{2\log(\cosh(a+bx))}{b} - \frac{\text{sech}^2(a+bx)}{2b}$$

Mathematica [A] time = 0.050776, size = 35, normalized size = 0.81

$$-\frac{-\sinh^2(a+bx)+\text{sech}^2(a+bx)+4\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]

[Out] -(4*Log[Cosh[a + b*x]] + Sech[a + b*x]^2 - Sinh[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.017, size = 47, normalized size = 1.1

$$\frac{(\sinh(bx+a))^4}{2b(\cosh(bx+a))^2} - 2\frac{\ln(\cosh(bx+a))}{b} + \frac{(\tanh(bx+a))^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a)^3,x)

[Out] 1/2/b*sinh(b*x+a)^4/cosh(b*x+a)^2-2*ln(cosh(b*x+a))/b+tanh(b*x+a)^2/b
```

Maxima [B] time = 1.58946, size = 139, normalized size = 3.23

$$-\frac{2(bx+a)}{b} + \frac{e^{(-2bx-2a)}}{8b} - \frac{2\log\left(e^{(-2bx-2a)}+1\right)}{b} + \frac{2e^{(-2bx-2a)}-15e^{(-4bx-4a)}+1}{8b\left(e^{(-2bx-2a)}+2e^{(-4bx-4a)}+e^{(-6bx-6a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b - 2*log(e^(-2*b*x - 2*a) + 1)/b + 1
/8*(2*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 1)/(b*(e^(-2*b*x - 2*a) + 2*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Fricas [B] time = 1.89526, size = 2049, normalized size = 47.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
2*(8*b*x + 1)*cosh(b*x + a)^6 + 2*(8*b*x + 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 + 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 + 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
+ 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 + 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 + 8*b*x + 1)*sinh(b*x + a)^2
 - 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*cosh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 + 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 + (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 + 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**3, x)
```

Giac [B] time = 1.29219, size = 130, normalized size = 3.02

$$\frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + \frac{8\left(3\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2} + e^{(2\,bx+2\,a)} - 16\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(16*b*x - (8*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*(3*e^(4*b*x + 4*
a) + 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) + 1)^2 + e^(2*b*x + 2*a) - 16*
log(e^(2*b*x + 2*a) + 1))/b
```

3.77 $\int \sinh^3(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0264162, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2592, 302, 203}

$$\frac{\sinh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0144086, size = 38, normalized size = 1.

$$\frac{\sinh^3(a+bx)}{3b} - \frac{\sinh(a+bx)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.019, size = 38, normalized size = 1.

$$\frac{(\sinh(bx+a))^3}{3b} - \frac{\sinh(bx+a)}{b} + 2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a),x)
```

```
[Out] 1/3*sinh(b*x+a)^3/b-sinh(b*x+a)/b+2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.56535, size = 96, normalized size = 2.53

$$-\frac{\left(15\,e^{(-2\,bx-2\,a)}-1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{24\,b} - \frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(15*e^(-2*b*x - 2*a) - 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) -
 e^(-3*b*x - 3*a))/b - 2*arctan(e^(-b*x - a))/b
```

Fricas [B] time = 1.87216, size = 836, normalized size = 22.

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2 - 1\right)\sinh(bx+a)^4 - 15\cosh(bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 48*(cosh(b*x + a)^3 + 3*cosh(b*x + a)^2*
sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3)*arctan(c
osh(b*x + a) + sinh(b*x + a)) + 15*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 1
0*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^3
+ 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b
*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x), x)
```

Giac [A] time = 1.18248, size = 85, normalized size = 2.24

$$\frac{\left(15\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+18\,a)}-15\,e^{(bx+16\,a)}\right)e^{(-15\,a)}+48\,\arctan\left(e^{(bx+a)}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((15*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 18*a) - 15*e^
(b*x + 16*a))*e^(-15*a) + 48*arctan(e^(b*x + a)))/b
```

3.78 $\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh^3(a+bx)}{3b}-\frac{2\cosh(a+bx)}{b}-\frac{\text{sech}(a+bx)}{b}$$

```
[Out] (-2*Cosh[a + b*x])/b + Cosh[a + b*x]^3/(3*b) - Sech[a + b*x]/b
```

Rubi [A] time = 0.040545, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\cosh^3(a+bx)}{3b}-\frac{2\cosh(a+bx)}{b}-\frac{\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x])/b + Cosh[a + b*x]^3/(3*b) - Sech[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\mathrm{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{2\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b} - \frac{\mathrm{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.0334827, size = 40, normalized size = 1.05

$$-\frac{7\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b} - \frac{\mathrm{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] (-7*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Sech[a + b*x]/b
```

Maple [A] time = 0.019, size = 50, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^4}{3\cosh(bx+a)} + \frac{4(\sinh(bx+a))^2}{3\cosh(bx+a)} - \frac{8\cosh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)^4/cosh(b*x+a)+4/3*sinh(b*x+a)^2/cosh(b*x+a)-8/3*cosh(b
*x+a))
```

Maxima [B] time = 1.06418, size = 107, normalized size = 2.82

$$-\frac{21e^{(-bx-a)} - e^{(-3bx-3a)}}{24b} - \frac{20e^{(-2bx-2a)} + 69e^{(-4bx-4a)} - 1}{24b\left(e^{(-3bx-3a)} + e^{(-5bx-5a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(21*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 1/24*(20*e^(-2*b*x - 2*a) +
69*e^(-4*b*x - 4*a) - 1)/(b*(e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Fricas [A] time = 1.82863, size = 177, normalized size = 4.66

$$\frac{\cosh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}-10\right)\sinh\left(bx+a\right)^{2}-20\cosh\left(bx+a\right)^{2}-45}{24\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b
*x + a)^2 - 20*cosh(b*x + a)^2 - 45)/(b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{3}\left(a+bx\right)\tanh^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**2, x)
```

Giac [B] time = 1.2644, size = 103, normalized size = 2.71

$$-\frac{\left(21\,e^{\left(2\,bx+2\,a\right)}-1\right)e^{\left(-3\,bx-3\,a\right)}-\left(e^{\left(3\,bx+24\,a\right)}-21\,e^{\left(bx+22\,a\right)}\right)e^{\left(-21\,a\right)}+\frac{48\,e^{\left(bx+a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*((21*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - (e^(3*b*x + 24*a) - 21*e
^(b*x + 22*a))*e^(-21*a) + 48*e^(b*x + a)/(e^(2*b*x + 2*a) + 1))/b
```

3.79 $\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sinh^3(a+bx)}{6b}-\frac{5\sinh(a+bx)}{2b}-\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}$$

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sinh[a + b*x])/(2*b) + (5*Sinh[a + b*x
]^3)/(6*b) - (Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0424965, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 203}

$$\frac{5\sinh^3(a+bx)}{6b}-\frac{5\sinh(a+bx)}{2b}-\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sinh[a + b*x])/(2*b) + (5*Sinh[a + b*x
]^3)/(6*b) - (Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}$$
$$= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \sinh(a+bx)\right)}{2b}$$
$$= -\frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \ldots\right.}{}$$
$$= \frac{5\tan^{-1}(\sinh(a+bx))}{2b} - \frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2}{2b}$$

Mathematica [A] time = 0.103281, size = 65, normalized size = 0.98

$$\frac{2\sinh^3(a+bx)\tanh^2(a+bx)+15\tan^{-1}(\sinh(a+bx))-10\sinh(a+bx)\tanh^2(a+bx)-15\tanh(a+bx)\text{sech}(a+bx}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (15*ArcTan[Sinh[a + b*x]] - 15*Sech[a + b*x]*Tanh[a + b*x] - 10*Sinh[a + b*
x]*Tanh[a + b*x]^2 + 2*Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(6*b)
```

Maple [A] time = 0.02, size = 92, normalized size = 1.4

$$\frac{(\sinh(bx+a))^5}{3b(\cosh(bx+a))^2} - \frac{5(\sinh(bx+a))^3}{3b(\cosh(bx+a))^2} - 5\frac{\sinh(bx+a)}{b(\cosh(bx+a))^2} + \frac{5\,\text{sech}(bx+a)\tanh(bx+a)}{2b} + 5\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^3,x)
```

```
[Out] 1/3/b*sinh(b*x+a)^5/cosh(b*x+a)^2-5/3/b*sinh(b*x+a)^3/cosh(b*x+a)^2-5/b*sin
h(b*x+a)/cosh(b*x+a)^2+5/2*sech(b*x+a)*tanh(b*x+a)/b+5*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.63116, size = 157, normalized size = 2.38

$$\frac{27\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b}-\frac{5\,\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{25\,e^{(-2\,bx-2\,a)}+77\,e^{(-4\,bx-4\,a)}+3\,e^{(-6\,bx-6\,a)}-1}{24\,b\left(e^{(-3\,bx-3\,a)}+2\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(27*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 5*arctan(e^(-b*x - a))/b - 1/
24*(25*e^(-2*b*x - 2*a) + 77*e^(-4*b*x - 4*a) + 3*e^(-6*b*x - 6*a) - 1)/(b*
(e^(-3*b*x - 3*a) + 2*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a)))
```

Fricas [B] time = 1.94062, size = 2390, normalized size = 36.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^8 - 25*cosh(b*x + a)^8 + 40*(3*
cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 - 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 - 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 - 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 + 5)*sinh(b
*x + a)^4 + 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 - 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 - 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 120*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 +
sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^5 + 2*cosh(b*x + a
```

```
)^5 + 5*(7*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*
x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*
cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (
7*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*
arctan(cosh(b*x + a) + sinh(b*x + a)) + 25*cosh(b*x + a)^2 + 10*(cosh(b*x +
 a)^9 - 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 5*co
sh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)*sinh
(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(b*x + a)
^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sin
h(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4 + 20*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x +
 a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**3, x)
```

Giac [A] time = 1.37744, size = 130, normalized size = 1.97

$$\frac{\left(27\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+30\,a)}-27\,e^{(bx+28\,a)}\right)e^{(-27\,a)}-\frac{24\left(e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2}+120\,\arctan\left(e^{(bx+a)}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*((27*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 30*a) - 27*e^
(b*x + 28*a))*e^(-27*a) - 24*(e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*
a) + 1)^2 + 120*arctan(e^(b*x + a)))/b
```

3.80 $\int \sinh^4(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\cosh^4(a+bx)}{4b}-\frac{\cosh^2(a+bx)}{b}+\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] -(Cosh[a + b*x]^2/b) + Cosh[a + b*x]^4/(4*b) + Log[Cosh[a + b*x]]/b
```

Rubi [A] time = 0.0321603, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2590, 266, 43}

$$\frac{\cosh^4(a+bx)}{4b}-\frac{\cosh^2(a+bx)}{b}+\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

```
[Out] -(Cosh[a + b*x]^2/b) + Cosh[a + b*x]^4/(4*b) + Log[Cosh[a + b*x]]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= -\frac{\cosh^2(a+bx)}{b}+\frac{\cosh^4(a+bx)}{4b}+\frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.0302297, size = 34, normalized size = 0.85

$$\frac{\frac{1}{4}\cosh^4(a+bx)-\cosh^2(a+bx)+\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

```
[Out] (-Cosh[a + b*x]^2 + Cosh[a + b*x]^4/4 + Log[Cosh[a + b*x]])/b
```

Maple [A] time = 0.017, size = 39, normalized size = 1.

$$\frac{(\sinh(bx+a))^4}{4b}-\frac{(\sinh(bx+a))^2}{2b}+\frac{\ln(\cosh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^4*tanh(b*x+a),x)
```

```
[Out] 1/4*sinh(b*x+a)^4/b-1/2*sinh(b*x+a)^2/b+ln(cosh(b*x+a))/b
```

Maxima [B] time = 1.5845, size = 109, normalized size = 2.72

$$-\frac{\left(12\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{64\,b}+\frac{bx+a}{b}-\frac{12\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}}{64\,b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b - 1/64*(12*
e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a))/b + log(e^(-2*b*x - 2*a) + 1)/b
```

Fricas [B] time = 1.81901, size = 1270, normalized size = 31.75

$$\frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2 - 3\right)\sinh(bx+a)^6 - 64\,bx\cosh(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x - 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) - 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 - 45*cosh(b*x + a)^4 - 3)
*sinh(b*x + a)^2 - 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 - 9*cosh(b*x + a)^
5 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**4*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**4*tanh(a + b*x), x)
```

Giac [B] time = 1.19127, size = 116, normalized size = 2.9

$$-\frac{64\,bx - \left(48\,e^{(4\,bx+4\,a)} - 12\,e^{(2\,bx+2\,a)} + 1\right)e^{(-4\,bx-4\,a)} - \left(e^{(4\,bx+16\,a)} - 12\,e^{(2\,bx+14\,a)}\right)e^{(-12\,a)} - 64\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4
*a) - (e^(4*b*x + 16*a) - 12*e^(2*b*x + 14*a))*e^(-12*a) - 64*log(e^(2*b*x
+ 2*a) + 1))/b
```

3.81 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -(Sech[a + b*x]/b)
```

Rubi [A] time = 0.0130824, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0069194, size = 11, normalized size = 1.

$$-\frac{\mathrm{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Maple [A] time = 0.007, size = 12, normalized size = 1.1

$$-\frac{\mathrm{sech}\,(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a),x)
```

```
[Out] -sech(b*x+a)/b
```

Maxima [B] time = 1.134, size = 31, normalized size = 2.82

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Fricas [B] time = 1.69006, size = 154, normalized size = 14.

$$-\frac{2\,(\cosh\,(bx+a)+\sinh\,(bx+a))}{b\cosh\,(bx+a)^2+2\,b\cosh\,(bx+a)\sinh\,(bx+a)+b\sinh\,(bx+a)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [A] time = 0.320807, size = 17, normalized size = 1.55

$$\begin{cases} -\frac{\operatorname{sech}(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x)
```

```
[Out] Piecewise((-sech(a + b*x)/b, Ne(b, 0)), (x*tanh(a)*sech(a), True))
```

Giac [B] time = 1.16443, size = 32, normalized size = 2.91

$$-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

3.82 $\int \mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\mathrm{sech}^2(a+bx)}{2b}$$

[Out] -Sech[a + b*x]^2/(2*b)

Rubi [A] time = 0.0203543, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\mathrm{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]

[Out] -Sech[a + b*x]^2/(2*b)

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\mathrm{Subst}\left(\int x\,dx, x, \mathrm{sech}(a+bx)\right)}{b}\\ &= -\frac{\mathrm{sech}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0108386, size = 15, normalized size = 1.

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$-\frac{(\operatorname{sech}(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a),x)
```

```
[Out] -1/2*sech(b*x+a)^2/b
```

Maxima [A] time = 1.00907, size = 18, normalized size = 1.2

$$\frac{\tanh(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*tanh(b*x + a)^2/b
```

Fricas [B] time = 1.75892, size = 235, normalized size = 15.67

$$-\frac{2(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3+3b\cosh(bx+a)+\left(3b\cosh(bx+a)^2+b\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

Sympy [A] time = 0.678421, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\operatorname{sech}^{2}(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a),x)
```

```
[Out] Piecewise((-sech(a + b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)*sech(a)**2, True)
)
```

Giac [A] time = 1.19404, size = 36, normalized size = 2.4

$$-\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.83 $\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=16

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

Rubi [A] time = 0.0298977, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 30}

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^{-1+n}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{sech}^n(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0375077, size = 16, normalized size = 1.

$$-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

Maple [A] time = 0.007, size = 17, normalized size = 1.1

$$-\frac{(\operatorname{sech}(bx+a))^n}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^n*tanh(b*x+a),x)
```

```
[Out] -sech(b*x+a)^n/b/n
```

Maxima [B] time = 1.77211, size = 49, normalized size = 3.06

$$-\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/(b*n)
```

Fricas [B] time = 1.85129, size = 338, normalized size = 21.12

$$-\frac{\cosh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+\sinh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 + 1))))/(b*n)
```

Sympy [A] time = 0.677922, size = 39, normalized size = 2.44

$$\begin{cases} x \tanh(a) & \text{for } b = 0 \wedge n = 0 \\ x \tanh(a) \operatorname{sech}^{n}(a) & \text{for } b = 0 \\ x - \frac{\log(\tanh(a+bx)+1)}{b} & \text{for } n = 0 \\ -\frac{\operatorname{sech}^{n}(a+bx)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a),x)
```

```
[Out] Piecewise((x*tanh(a), Eq(b, 0) & Eq(n, 0)), (x*tanh(a)*sech(a)**n, Eq(b, 0)
), (x - log(tanh(a + b*x) + 1)/b, Eq(n, 0)), (-sech(a + b*x)**n/(b*n), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(bx+a)^{n} \tanh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a), x)
```

3.84 $\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] Tanh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0289947, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx &= \frac{i\,\operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0062168, size = 15, normalized size = 1.

$$\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b)
```

Maple [B] time = 0.018, size = 42, normalized size = 2.8

$$\frac{1}{b}\left(-\frac{\sinh(bx+a)}{2\left(\cosh(bx+a)\right)^3}+\frac{\tanh(bx+a)}{2}\left(\frac{2}{3}+\frac{\left(\operatorname{sech}(bx+a)\right)^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*sinh(b*x+a)/cosh(b*x+a)^3+1/2*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)
)
```

Maxima [A] time = 1.0373, size = 18, normalized size = 1.2

$$\frac{\tanh(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tanh(b*x + a)^3/b
```

Fricas [B] time = 1.86072, size = 378, normalized size = 25.2

$$-\frac{8\left(\cosh(bx+a)^2+\cosh(bx+a)\sinh(bx+a)+\sinh\right.}{3\left(b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4+4\,b\cosh(bx+a)^2+2\left(3\,b\cosh(bx+a)^2+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [B] time = 1.21848, size = 42, normalized size = 2.8

$$-\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

3.85 $\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^4(a+bx)}{4b}$$

```
[Out] Tanh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0300007, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tanh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Tanh[a + b*x]^4/(4*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^3\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0048121, size = 15, normalized size = 1.

$$\frac{\tanh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Tanh[a + b*x]^4/(4*b)
```

Maple [B] time = 0.018, size = 42, normalized size = 2.8

$$\frac{1}{b}\left(-\frac{(\sinh(bx+a))^2}{4(\cosh(bx+a))^4}+\frac{(\sinh(bx+a))^2}{4(\cosh(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/4*sinh(b*x+a)^2/cosh(b*x+a)^4+1/4*sinh(b*x+a)^2/cosh(b*x+a)^2)
```

Maxima [A] time = 1.07871, size = 18, normalized size = 1.2

$$\frac{\tanh(bx+a)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*tanh(b*x + a)^4/b
```

Fricas [B] time = 1.80813, size = 572, normalized size = 38.13

$$-\frac{2\left(\cosh(bx+a)^3+3\cosh(bx+a)\sinh(b\right.}{b\cosh(bx+a)^5+5\,b\cosh(bx+a)\sinh(bx+a)^4+b\sinh(bx+a)^5+5\,b\cosh(bx+a)^3+\left(10\,b\cosh(bx+a)^2+3\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 5*b*cosh(b*x + a)^3
 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + 5*(2*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x + a) + (5*b*cosh(b*x + a
)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [A] time = 2.38937, size = 44, normalized size = 2.93

$$\begin{cases} -\frac{\tanh^{2}(a+bx)\operatorname{sech}^{2}(a+bx)}{4b} - \frac{\operatorname{sech}^{2}(a+bx)}{4b} & \text{for } b \neq 0 \\ x\tanh^{3}(a)\operatorname{sech}^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**2/(4*b) - sech(a + b*x)**2/(4*b
), Ne(b, 0)), (x*tanh(a)**3*sech(a)**2, True))
```

Giac [B] time = 1.25667, size = 50, normalized size = 3.33

$$-\frac{2\left(e^{(6\,bx+6\,a)} + e^{(2\,bx+2\,a)}\right)}{b\left(e^{(2\,bx+2\,a)} + 1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

3.86 $\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0351163, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 32}

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int (-ix)^n\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0191072, size = 19, normalized size = 1.

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.014, size = 20, normalized size = 1.1

$$\frac{(\tanh(bx+a))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^n,x)
```

```
[Out] tanh(b*x+a)^(n+1)/b/(n+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0714, size = 190, normalized size = 10.

$$\frac{\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)\sinh(bx+a)+\sinh(bx+a)\sinh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)}{(bn+b)\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (cosh(n*log(sinh(b*x + a)/cosh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sin
h(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n + b)*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^n (a + bx) \operatorname{sech}^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh (bx + a)^n \operatorname{sech} (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n*sech(b*x + a)^2, x)
```

3.87 $\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0244432, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int\left(-1+x^2\right)\,dx,x,\operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0311728, size = 27, normalized size = 1.

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Maple [A] time = 0.011, size = 50, normalized size = 1.9

$$\frac{1}{b}\left(-\frac{(\sinh(bx+a))^2}{3(\cosh(bx+a))^3}+\frac{2(\sinh(bx+a))^2}{3\cosh(bx+a)}-\frac{2\cosh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/3*sinh(b*x+a)^2/cosh(b*x+a)^3+2/3*sinh(b*x+a)^2/cosh(b*x+a)-2/3*cos
h(b*x+a))
```

Maxima [B] time = 1.02041, size = 200, normalized size = 7.41

$$-\frac{2e^{(-bx-a)}}{b\left(3e^{(-2bx-2a)}+3e^{(-4bx-4a)}+e^{(-6bx-6a)}+1\right)}-\frac{4e^{(-3bx-3a)}}{3b\left(3e^{(-2bx-2a)}+3e^{(-4bx-4a)}+e^{(-6bx-6a)}+1\right)}-\frac{2e^{(-}}{b\left(3e^{(-2bx-2a)}+3e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6
*a) + 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a
) + e^(-6*b*x - 6*a) + 1)) - 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) + 3*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Fricas [B] time = 1.74834, size = 467, normalized size = 17.3

$$-\frac{2\left(3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2+3\sinh(bx+a)^3+\left(9\cosh(b\right.\right.}{3\left(b\cosh(bx+a)^4+4b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4+4b\cosh(bx+a)^2+2\left(3b\cosh(bx+a)^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*b*cosh(b*
x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [A] time = 1.28128, size = 41, normalized size = 1.52

$$\begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}(a+bx)}{3b} - \frac{2\operatorname{sech}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)/(3*b) - 2*sech(a + b*x)/(3*b), N
e(b, 0)), (x*tanh(a)**3*sech(a), True))
```

Giac [A] time = 1.24873, size = 66, normalized size = 2.44

$$-\frac{2\left(3e^{(5bx+5a)} + 2e^{(3bx+3a)} + 3e^{(bx+a)}\right)}{3b\left(e^{(2bx+2a)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(5*b*x + 5*a) + 2*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(b*(e^(2*b*x +
 2*a) + 1)^3)
```

3.88 $\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\operatorname{sech}^5(a+bx)}{5b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[Out] -Sech[a + b*x]^3/(3*b) + Sech[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0354046, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\operatorname{sech}^5(a+bx)}{5b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] -Sech[a + b*x]^3/(3*b) + Sech[a + b*x]^5/(5*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \mathrm{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \mathrm{sech}(a+bx)\right)}{b}$$
$$= \frac{\mathrm{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \mathrm{sech}(a+bx)\right)}{b}$$
$$= -\frac{\mathrm{sech}^3(a+bx)}{3b} + \frac{\mathrm{sech}^5(a+bx)}{5b}$$

Mathematica [A] time = 0.0559811, size = 31, normalized size = 1.

$$\frac{\mathrm{sech}^5(a+bx)}{5b} - \frac{\mathrm{sech}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] -Sech[a + b*x]^3/(3*b) + Sech[a + b*x]^5/(5*b)
```

Maple [B] time = 0.023, size = 68, normalized size = 2.2

$$\frac{1}{b}\left(-\frac{(\sinh(bx+a))^2}{5(\cosh(bx+a))^5} + \frac{2(\sinh(bx+a))^2}{15(\cosh(bx+a))^3} + \frac{2(\sinh(bx+a))^2}{15\cosh(bx+a)} - \frac{2\cosh(bx+a)}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/5*sinh(b*x+a)^2/cosh(b*x+a)^5+2/15*sinh(b*x+a)^2/cosh(b*x+a)^3+2/15
*sinh(b*x+a)^2/cosh(b*x+a)-2/15*cosh(b*x+a))
```

Maxima [B] time = 1.16571, size = 289, normalized size = 9.32

$$-\frac{8e^{(-3bx-3a)}}{3b\left(5e^{(-2bx-2a)}+10e^{(-4bx-4a)}+10e^{(-6bx-6a)}+5e^{(-8bx-8a)}+e^{(-10bx-10a)}+1\right)} + \frac{16e^{(}}{15b\left(5e^{(-2bx-2a)}+10e^{(-4bx-4a)}+10e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15*e^(-5
*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 8/3*e^(-7*b*x - 7*a)/
(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Fricas [B] time = 1.82287, size = 946, normalized size = 30.52

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}+5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+a\right)^{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 +
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x
+ a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(
b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)
^2 + 5*b)*sinh(b*x + a))
```

Sympy [A] time = 4.51699, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\tanh^{2}{\left(a+bx \right)}\operatorname{sech}^{3}{\left(a+bx \right)}}{5b}-\frac{2\operatorname{sech}^{3}{\left(a+bx \right)}}{15b} & \text{for}\: b\neq 0 \\ x\tanh^{3}{\left(a \right)}\operatorname{sech}^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**3/(5*b) - 2*sech(a + b*x)**3/(1
5*b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**3, True))
```

Giac [A] time = 1.19908, size = 70, normalized size = 2.26

$$-\frac{8\left(5\,e^{(7\,bx+7\,a)}-2\,e^{(5\,bx+5\,a)}+5\,e^{(3\,bx+3\,a)}\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*e^(7*b*x + 7*a) - 2*e^(5*b*x + 5*a) + 5*e^(3*b*x + 3*a))/(b*(e^(2*
b*x + 2*a) + 1)^5)
```

3.89 $\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\operatorname{sech}^{n+2}(a+bx)}{b(n+2)}-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

```
[Out] -(Sech[a + b*x]^n/(b*n)) + Sech[a + b*x]^(2 + n)/(b*(2 + n))
```

Rubi [A] time = 0.0485654, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 14}

$$\frac{\operatorname{sech}^{n+2}(a+bx)}{b(n+2)}-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n)) + Sech[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int x^{-1+n}\left(-1+x^2\right)\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-x^{-1+n}+x^{1+n}\right)\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{sech}^n(a+bx)}{bn} + \frac{\operatorname{sech}^{2+n}(a+bx)}{b(2+n)}$$

Mathematica [A] time = 0.122265, size = 32, normalized size = 0.89

$$\frac{\operatorname{sech}^n(a+bx)\left(\frac{\operatorname{sech}^2(a+bx)}{n+2}-\frac{1}{n}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

```
[Out] (Sech[a + b*x]^n*(-n^(-1) + Sech[a + b*x]^2/(2 + n)))/b
```

Maple [C] time = 0.134, size = 275, normalized size = 7.6

$$-\frac{n e^{4bx+4a}+2e^{4bx+4a}-2ne^{2bx+2a}+4e^{2bx+2a}+n+2}{(n+2)\,bn\left(1+e^{2bx+2a}\right)^2} e^{-\frac{n}{2}\left(i\pi\left(\operatorname{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)\right)^3 - i\pi\left(\operatorname{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)\right)^2 \operatorname{csgn}\left(ie^{bx+a}\right) - i\pi\left(\operatorname{csgn}\left(\frac{ie^{bx}}{1+e^{2b}}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^n*tanh(b*x+a)^3,x)
```

```
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)-2*n*exp(2*b*x+2*a)+4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(1+exp(2*b*x+2*a))^2*exp(-1/2*n*(I*Pi*csgn(I*exp(b*x+a)/(1+exp(
2*b*x+2*a)))^3-I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))^2*csgn(I*exp(b*x+
a))-I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))^2*csgn(I/(1+exp(2*b*x+2*a)))
+I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))*csgn(I*exp(b*x+a))*csgn(I/(1+ex
p(2*b*x+2*a)))-2*ln(exp(b*x+a))+2*ln(1+exp(2*b*x+2*a))-2*ln(2)))
```

Maxima [B] time = 2.01708, size = 466, normalized size = 12.94

$$-\frac{2^n n e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+2\left(n^2+2\,n\right)e^{(-2\,bx-2\,a)}+\left(n^2+2\,n\right)e^{(-4\,bx-4\,a)}+2\,n\right)b}+\frac{\left(2^{n+1}n-2^{n+2}\right)e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+2\left(n^2+2\,n\right)e^{(-2\,bx-2\,a)}+\left(n^2+2\,n\right)e^{(-4\,bx-4\,a)}+2\,n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*(n^2 + 2*n)
*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) + (2^(n + 1)*n -
 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((
n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*
b) - (2^n*n + 2^(n + 1))*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-2*b*x - 2*a) +
 1) - 4*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x -
 4*a) + 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/
((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n
)*b)
```

Fricas [B] time = 1.85963, size = 605, normalized size = 16.81

$$-\frac{\left((n+2)\cosh\left(bx+a\right)^2+(n+2)\sinh\left(bx+a\right)^2-n+2\right)\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+}{bn^2+\left(bn^2+2\,bn\right)\cosh\left(bx+a\right)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 - n + 2)*cosh(n*log(2*
(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 + 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*
x + a)^2 - n + 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x +
a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(b*n^2 + (b*
n^2 + 2*b*n)*cosh(b*x + a)^2 + (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(bx+a)^n \tanh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a)^3, x)
```

3.90 $\int \mathbf{sech}^4(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

```
[Out] Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.032713, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b}$$

Mathematica [A] time = 0.0407975, size = 56, normalized size = 1.81

$$\frac{2\tanh(a+bx)}{15b} - \frac{\tanh(a+bx)\operatorname{sech}^4(a+bx)}{5b} + \frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

```
[Out] (2*Tanh[a + b*x])/(15*b) + (Sech[a + b*x]^2*Tanh[a + b*x])/(15*b) - (Sech[a
 + b*x]^4*Tanh[a + b*x])/(5*b)
```

Maple [A] time = 0.019, size = 52, normalized size = 1.7

$$\frac{1}{b}\left(-\frac{\sinh(bx+a)}{4\left(\cosh(bx+a)\right)^5} + \frac{\tanh(bx+a)}{4}\left(\frac{8}{15} + \frac{\left(\operatorname{sech}(bx+a)\right)^4}{5} + \frac{4\left(\operatorname{sech}(bx+a)\right)^2}{15}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4*sinh(b*x+a)/cosh(b*x+a)^5+1/4*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b
*x+a)^2)*tanh(b*x+a))
```

Maxima [B] time = 1.12916, size = 373, normalized size = 12.03

$$\frac{4\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)} - \frac{4\,e^{(-4\,}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 4/3*e^(-4*b*
x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a)
 + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4*e^(-6*b*x - 6*a)/(b*(5
*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x
 - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4/15/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4
*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a)
 + 1))
```

Fricas [B] time = 1.85278, size = 836, normalized size = 26.97

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}+5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+a\right)^{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/15*(8*cosh(b*x + a)^3 + 24*cosh(b*x + a)*sinh(b*x + a)^2 + 7*sinh(b*x +
a)^3 + (21*cosh(b*x + a)^2 - 5)*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cos
h(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5 + (21*
b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 5*b*cos
h(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4
+ 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 50*
b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(b*x + a
) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)^2 + 5*
b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{2}\left(a+bx\right)\operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**4, x)
```

Giac [A] time = 1.25431, size = 72, normalized size = 2.32

$$-\frac{4\left(15\,e^{(6\,bx+6\,a)}-5\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/15*(15*e^(6*b*x + 6*a) - 5*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(
e^(2*b*x + 2*a) + 1)^5)
```

3.91 $\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] (2*Tanh[a + b*x]^(3/2))/(3*b) - (2*Tanh[a + b*x]^(7/2))/(7*b)
```

Rubi [A] time = 0.0366913, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 14}

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

```
[Out] (2*Tanh[a + b*x]^(3/2))/(3*b) - (2*Tanh[a + b*x]^(7/2))/(7*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = -\frac{i\,\text{Subst}\left(\int \sqrt{-ix}\left(1+x^2\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(\sqrt{-ix}-(-ix)^{5/2}\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

Mathematica [A] time = 0.105289, size = 29, normalized size = 0.83

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)\left(3\operatorname{sech}^2(a+bx)+4\right)}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]

[Out] (2*(4 + 3*Sech[a + b*x]^2)*Tanh[a + b*x]^(3/2))/(21*b)
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(bx+a))^4 \sqrt{\tanh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x)

[Out] int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x)
```

Maxima [B] time = 1.80421, size = 475, normalized size = 13.57

$$\frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-2bx-2a)}}{21\,b\left(3\,e^{(-2bx-2a)}+3\,e^{(-4bx-4a)}+e^{(-6bx-6a)}+1\right)\sqrt{e^{(-2bx-2a)}+1}} - \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-4bx-4a)}}{21\,b\left(3\,e^{(-2bx-2a)}+3\,e^{(-4bx-4a)}+e^{(-6bx-6a)}+1\right)\sqrt{e^{(-2bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-2*b*x - 2*a)/(b*(3
*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b
*x - 2*a) + 1)) - 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-
4*b*x - 4*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)
 + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) - 8/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b
*x - a) + 1)*e^(-6*b*x - 6*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) + 8/21*sqrt(e^(-b*x - a)
 + 1)*sqrt(-e^(-b*x - a) + 1)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] time = 1.8987, size = 1521, normalized size = 43.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 8/21*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5
 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + (cosh(b*x + a)^6 + 6*
cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 4)*
sinh(b*x + a)^4 + 4*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 4*cosh(b*x + a
))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 24*cosh(b*x + a)^2 - 4)*sinh(b*x
 + a)^2 - 4*cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 - 4*
cosh(b*x + a))*sinh(b*x + a) - 1)*sqrt(sinh(b*x + a)/cosh(b*x + a)) + 1)/(b
*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 +
3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2
+ 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*
cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh{\left(a + bx \right)}}\operatorname{sech}^{4}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(a + b*x))*sech(a + b*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(bx+a)^4 \sqrt{\tanh(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^4*sqrt(tanh(b*x + a)), x)
```

3.92 $\int \mathbf{sech}^4(a+bx)\tanh^n(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}-\frac{\tanh^{n+3}(a+bx)}{b(n+3)}$$

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n)) - Tanh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rubi [A] time = 0.0442714, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}-\frac{\tanh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n)) - Tanh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int(-ix)^n\left(1+x^2\right)\,dx,x,i\tanh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int\left((-ix)^n-(-ix)^{2+n}\right)\,dx,x,i\tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh^{1+n}(a+bx)}{b(1+n)} - \frac{\tanh^{3+n}(a+bx)}{b(3+n)}$$

Mathematica [A] time = 0.904107, size = 73, normalized size = 1.82

$$\frac{\tanh^{n-1}(a+bx)\left(\tanh^2(a+bx)\operatorname{sech}^2(a+bx)(\cosh(2(a+bx))+n+2)-2\tanh^2(a+bx)^{\frac{1-n}{2}}\right)}{b(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] (Tanh[a + b*x]^(-1 + n)*((2 + n + Cosh[2*(a + b*x)])*Sech[a + b*x]^2*Tanh[a
 + b*x]^2 - 2*(Tanh[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Maple [C] time = 0.204, size = 535, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^n,x)
```

```
[Out] 2*(exp(6*b*x+6*a)+2*n*exp(4*b*x+4*a)+3*exp(4*b*x+4*a)-2*n*exp(2*b*x+2*a)-3*
exp(2*b*x+2*a)-1)/b/(n+1)/(n+3)/(1+exp(2*b*x+2*a))^3*exp(-1/2*n*(I*Pi*csgn(
I*(1+exp(b*x+a))/(1+exp(2*b*x+2*a)))^3-I*Pi*csgn(I*(1+exp(b*x+a))/(1+exp(2*
b*x+2*a)))^2*csgn(I*(1+exp(b*x+a)))-I*Pi*csgn(I*(1+exp(b*x+a))/(1+exp(2*b*x
+2*a)))^2*csgn(I/(1+exp(2*b*x+2*a)))+I*Pi*csgn(I*(1+exp(b*x+a))/(1+exp(2*b*
x+2*a)))*csgn(I*(1+exp(b*x+a)))*csgn(I/(1+exp(2*b*x+2*a)))-I*Pi*csgn(I*(1+e
xp(b*x+a))/(1+exp(2*b*x+2*a)))*csgn(I*(exp(b*x+a)-1)/(1+exp(2*b*x+2*a))*(1+
exp(b*x+a)))^2+I*Pi*csgn(I*(1+exp(b*x+a))/(1+exp(2*b*x+2*a)))*csgn(I*(exp(b
*x+a)-1)/(1+exp(2*b*x+2*a))*(1+exp(b*x+a)))*csgn(I*(exp(b*x+a)-1))+I*Pi*csg
n(I*(exp(b*x+a)-1)/(1+exp(2*b*x+2*a))*(1+exp(b*x+a)))^3-I*Pi*csgn(I*(exp(b*
```

```
x+a)-1)/(1+exp(2*b*x+2*a))*(1+exp(b*x+a)))^2*csgn(I*(exp(b*x+a)-1))-2*ln(1+
exp(b*x+a))-2*ln(exp(b*x+a)-1)+2*ln(1+exp(2*b*x+2*a))))
```

Maxima [B] time = 1.64752, size = 680, normalized size = 17.

$$\frac{2\,(2\,n+3)e^{\left(-2\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+3\left(n^2+4\,n+3\right)e^{(-2\,bx-2\,a)}+3\left(n^2+4\,n+3\right)e^{(-4\,bx-4\,a)}+\left(n^2+4\,n+3\right)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}-\frac{2}{\left(n^2+3\left(n^2+4\,n\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 2*(2*n + 3)*e^(-2*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
- n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((n^2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x -
2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n + 3)*e^(-6*b*x - 6*a
) + 4*n + 3)*b) - 2*(2*n + 3)*e^(-4*b*x + n*log(e^(-b*x - a) + 1) + n*log(-
e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 3*(n^2 + 4*n
 + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) - 2*e^(-6*b*x + n*log(e^(-b*x - a) + 1) +
 n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 6*a)/((n^2 + 3*(n
^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2
+ 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) + 2*e^(n*log(e^(-b*x - a) + 1) +
n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 3*(n^2 + 4*
n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b)
```

Fricas [B] time = 1.9202, size = 477, normalized size = 11.92

$$\frac{2\left(\left(\sinh\left(bx+a\right)^3+\left(3\,\cosh\left(bx+a\right)^2+2\,n+3\right)\sinh\left(bx+a\right)\right)\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)+\left(\sinh\left(bx+a\right)^3+\left(3\,\cosh\left(b\right.\right.\right.\right.}{\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)^3+3\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 2*((sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 2*n + 3)*sinh(b*x + a))*cosh(n*l
og(sinh(b*x + a)/cosh(b*x + a))) + (sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 +
2*n + 3)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n^2 +
```

```
 4*b*n + 3*b)*cosh(b*x + a)^3 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)*sinh(
b*x + a)^2 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{n}(a+bx)\operatorname{sech}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh(bx+a)^{n}\operatorname{sech}(bx+a)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n*sech(b*x + a)^4, x)
```

3.93 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0237503, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0154904, size = 34, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] time = 0.013, size = 49, normalized size = 1.4

$$-\frac{\sinh(bx+a)}{b(\cosh(bx+a))^2}+\frac{\text{sech}(bx+a)\tanh(bx+a)}{2b}+\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^2,x)
```

```
[Out] -1/b*sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x
+a))/b
```

Maxima [B] time = 1.6376, size = 89, normalized size = 2.62

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{e^{(-bx-a)}-e^{(-3bx-3a)}}{b\left(2e^{(-2bx-2a)}+e^{(-4bx-4a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 1.82645, size = 759, normalized size = 22.32

$$-\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \right.}{b\cosh(bx+a)^4 + 4\,b\cosh(bx+a)\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x), x)
```

Giac [A] time = 1.1867, size = 63, normalized size = 1.85

$$-\frac{\frac{e^{(3\,bx+3\,a)}-e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^2} - \arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - arctan(e^(b*x +
 a)))/b
```

3.94 $\int \text{sech}(a+bx)\tanh^4(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) - (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (
Sech[a + b*x]*Tanh[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.044051, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) - (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (
Sech[a + b*x]*Tanh[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = -\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b} + \frac{3}{4}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$
$$= -\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b} + \frac{3}{8}\int \operatorname{sech}(a+bx)\,dx$$
$$= \frac{3\tan^{-1}(\sinh(a+bx))}{8b} - \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

Mathematica [A] time = 0.16624, size = 59, normalized size = 1.07

$$\frac{3\tan^{-1}(\sinh(a+bx)) - 6\tanh(a+bx)\operatorname{sech}^3(a+bx) + \left(3\tanh(a+bx) - 8\tanh^3(a+bx)\right)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]] - 6*Sech[a + b*x]^3*Tanh[a + b*x] + Sech[a + b*x]*
(3*Tanh[a + b*x] - 8*Tanh[a + b*x]^3))/(8*b)
```

Maple [A] time = 0.013, size = 90, normalized size = 1.6

$$-\frac{(\sinh(bx+a))^3}{b(\cosh(bx+a))^4} - \frac{\sinh(bx+a)}{b(\cosh(bx+a))^4} + \frac{(\operatorname{sech}(bx+a))^3\tanh(bx+a)}{4b} + \frac{3\operatorname{sech}(bx+a)\tanh(bx+a)}{8b} + \frac{3\arctan\left(e\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^4,x)
```

```
[Out] -1/b*sinh(b*x+a)^3/cosh(b*x+a)^4-1/b*sinh(b*x+a)/cosh(b*x+a)^4+1/4*sech(b*x
+a)^3*tanh(b*x+a)/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+3/4*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.74562, size = 151, normalized size = 2.75

$$-\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b} - \frac{5e^{(-bx-a)} - 3e^{(-3bx-3a)} + 3e^{(-5bx-5a)} - 5e^{(-7bx-7a)}}{4b\left(4e^{(-2bx-2a)} + 6e^{(-4bx-4a)} + 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b - 1/4*(5*e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*
e^(-5*b*x - 5*a) - 5*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x
 - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] time = 1.86569, size = 2257, normalized size = 41.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(5*cosh(b*x + a)^7 + 35*cosh(b*x + a)*sinh(b*x + a)^6 + 5*sinh(b*x + a
)^7 + 3*(35*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 3*cosh(b*x + a)^5 + 5*(3
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^4 + (175*cosh(b*x + a)^4
 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + 3*(35*cosh
(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(co
sh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)
^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x
 + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 1
0*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6
 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (35*c
osh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)
- 5*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
 b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh
(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b
)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cos
h(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6
+ 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*co
sh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] Integral(tanh(a + b*x)**4*sech(a + b*x), x)
```

Giac [A] time = 1.23525, size = 96, normalized size = 1.75

$$-\frac{\frac{5\,e^{(7\,bx+7\,a)}-3\,e^{(5\,bx+5\,a)}+3\,e^{(3\,bx+3\,a)}-5\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^4}-3\,\arctan\left(e^{(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/4*((5*e^(7*b*x + 7*a) - 3*e^(5*b*x + 5*a) + 3*e^(3*b*x + 3*a) - 5*e^(b*x
 + a))/(e^(2*b*x + 2*a) + 1)^4 - 3*arctan(e^(b*x + a)))/b
```

3.95 $\int \mathbf{sech}^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\mathbf{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\mathbf{sech}(a+bx)}{8b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rubi [A] time = 0.0451404, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\mathbf{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\mathbf{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^3(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\mathrm{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{1}{4}\int \mathrm{sech}^3(a+bx)\,dx\\ &= \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\mathrm{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{1}{8}\int \mathrm{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\mathrm{sech}^3(a+bx)\tanh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0201913, size = 55, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\mathrm{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Maple [A] time = 0.017, size = 69, normalized size = 1.3

$$-\frac{\sinh(bx+a)}{3\,b\,(\cosh(bx+a))^4}+\frac{(\mathrm{sech}\,(bx+a))^3\tanh(bx+a)}{12\,b}+\frac{\mathrm{sech}\,(bx+a)\tanh(bx+a)}{8\,b}+\frac{\arctan\left(e^{bx+a}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^2,x)
```

```
[Out] -1/3/b*sinh(b*x+a)/cosh(b*x+a)^4+1/12*sech(b*x+a)^3*tanh(b*x+a)/b+1/8*sech(
b*x+a)*tanh(b*x+a)/b+1/4*arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.8487, size = 149, normalized size = 2.71

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{e^{(-bx-a)}-7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}-e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*arctan(e^(-b*x - a))/b + 1/4*(e^(-b*x - a) - 7*e^(-3*b*x - 3*a) + 7*e^
(-5*b*x - 5*a) - e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4
*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] time = 1.87445, size = 2236, normalized size = 40.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 +
7*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 7*cosh(b*x + a)^5 + 35*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^4 + 7*(5*cosh(b*x + a)^4 - 10*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^3 + 7*cosh(b*x + a)^3 + 7*(3*cosh(b*x + a)^5
- 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (cosh(b*x + a)^8
+ 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(b*
x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)*s
inh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b*x
 + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*(c
osh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (7*cosh(b*x + a)^6
- 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a
))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)
^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*
(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^
4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh
(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cosh(b*x
```

```
+ a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 +
3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
+ b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [A] time = 1.21273, size = 90, normalized size = 1.64

$$\frac{\frac{e^{(7\,bx+7\,a)}-7\,e^{(5\,bx+5\,a)}+7\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^4}+\arctan\left(e^{(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((e^(7*b*x + 7*a) - 7*e^(5*b*x + 5*a) + 7*e^(3*b*x + 3*a) - e^(b*x + a)
)/(e^(2*b*x + 2*a) + 1)^4 + arctan(e^(b*x + a)))/b
```

3.96 $\int \operatorname{sech}(x)\tanh^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\operatorname{sech}^5(x)+\frac{2\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Rubi [A] time = 0.0186715, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 194}

$$-\frac{1}{5}\operatorname{sech}^5(x)+\frac{2\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*Tanh[x]^5,x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5}$$

Mathematica [A] time = 0.012073, size = 21, normalized size = 1.

$$-\frac{1}{5}\operatorname{sech}^5(x) + \frac{2\operatorname{sech}^3(x)}{3} - \operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*Tanh[x]^5,x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Maple [B] time = 0.007, size = 46, normalized size = 2.2

$$-\frac{(\sinh(x))^4}{(\cosh(x))^5} - \frac{4\,(\sinh(x))^2}{5\,(\cosh(x))^5} + \frac{8\,(\sinh(x))^2}{15\,(\cosh(x))^3} + \frac{8\,(\sinh(x))^2}{15\cosh(x)} - \frac{8\cosh(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*tanh(x)^5,x)
```

```
[Out] -sinh(x)^4/cosh(x)^5-4/5*sinh(x)^2/cosh(x)^5+8/15*sinh(x)^2/cosh(x)^3+8/15*
sinh(x)^2/cosh(x)-8/15*cosh(x)
```

Maxima [B] time = 1.28342, size = 258, normalized size = 12.29

$$-\frac{2\,e^{(-x)}}{5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1} - \frac{8\,e^{(-3\,x)}}{3\left(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1\right)} - \frac{}{15\,(5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x)
+ 1) - 8/3*e^(-3*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) +
e^(-10*x) + 1) - 116/15*e^(-5*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) +
5*e^(-8*x) + e^(-10*x) + 1) - 8/3*e^(-7*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e
^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 2*e^(-9*x)/(5*e^(-2*x) + 10*e^(-4*x
) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Fricas [B] time = 1.83942, size = 620, normalized size = 29.52

$$-\frac{2\left(15\cosh\left(x\right)^{5}+75\cosh\left(x\right)\sinh\left(x\right)^{4}+15\sinh\left(x\right)^{5}+5\left(30\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{3}+35\,\mathrm{c}\right.}{15\left(\cosh\left(x\right)^{6}+6\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\left(5\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)^{4}+6\cosh\left(x\right)^{4}+4\left(5\cosh\left(x\right)^{3}+4\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^5 + 75*cosh(x)*sinh(x)^4 + 15*sinh(x)^5 + 5*(30*cosh(x)^2
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 15*(10*cosh(x)^3 + 7*cosh(x))*sinh(x)^2 +
(75*cosh(x)^4 + 15*cosh(x)^2 + 38)*sinh(x) + 78*cosh(x))/(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 2)*sinh(x)^4 + 6*cosh(x)^4 +
4*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 12*cosh(x)^2 + 5)*
sinh(x)^2 + 15*cosh(x)^2 + 2*(3*cosh(x)^5 + 8*cosh(x)^3 + 5*cosh(x))*sinh(x
) + 10)
```

Sympy [A] time = 2.50233, size = 29, normalized size = 1.38

$$-\frac{\tanh^{4}\left(x\right)\operatorname{sech}\left(x\right)}{5}-\frac{4\tanh^{2}\left(x\right)\operatorname{sech}\left(x\right)}{15}-\frac{8\operatorname{sech}\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)**5,x)
```

```
[Out] -tanh(x)**4*sech(x)/5 - 4*tanh(x)**2*sech(x)/15 - 8*sech(x)/15
```

Giac [B] time = 1.21732, size = 47, normalized size = 2.24

$$-\frac{2\left(15\left(e^{(-x)}+e^{x}\right)^{4}-40\left(e^{(-x)}+e^{x}\right)^{2}+48\right)}{15\left(e^{(-x)}+e^{x}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -2/15*(15*(e^(-x) + e^x)^4 - 40*(e^(-x) + e^x)^2 + 48)/(e^(-x) + e^x)^5
```

3.97 $\int \mathbf{sech}^7(x) \tanh^5(x) \, dx$

Optimal. Leaf size=25

$$-\frac{1}{11}\operatorname{sech}^{11}(x)+\frac{2\operatorname{sech}^9(x)}{9}-\frac{\operatorname{sech}^7(x)}{7}$$

```
[Out] -Sech[x]^7/7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Rubi [A] time = 0.0286869, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$-\frac{1}{11}\operatorname{sech}^{11}(x)+\frac{2\operatorname{sech}^9(x)}{9}-\frac{\operatorname{sech}^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^7*Tanh[x]^5,x]
```

```
[Out] -Sech[x]^7/7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\operatorname{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\frac{1}{7}\operatorname{sech}^7(x)+\frac{2\operatorname{sech}^9(x)}{9}-\frac{\operatorname{sech}^{11}(x)}{11}$$

Mathematica [A] time = 0.0145422, size = 25, normalized size = 1.

$$-\frac{1}{11}\operatorname{sech}^{11}(x)+\frac{2\operatorname{sech}^9(x)}{9}-\frac{\operatorname{sech}^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^7*Tanh[x]^5,x]
```

```
[Out] -Sech[x]^7/7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Maple [B] time = 0.014, size = 76, normalized size = 3.

$$-\frac{(\sinh(x))^4}{7(\cosh(x))^{11}}-\frac{4(\sinh(x))^2}{77(\cosh(x))^{11}}+\frac{8(\sinh(x))^2}{693(\cosh(x))^9}+\frac{8(\sinh(x))^2}{693(\cosh(x))^7}+\frac{8(\sinh(x))^2}{693(\cosh(x))^5}+\frac{8(\sinh(x))^2}{693(\cosh(x))^3}+\frac{8(\sinh}{693\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^7*tanh(x)^5,x)
```

```
[Out] -1/7*sinh(x)^4/cosh(x)^11-4/77*sinh(x)^2/cosh(x)^11+8/693*sinh(x)^2/cosh(x)
^9+8/693*sinh(x)^2/cosh(x)^7+8/693*sinh(x)^2/cosh(x)^5+8/693*sinh(x)^2/cosh
(x)^3+8/693*sinh(x)^2/cosh(x)-8/693*cosh(x)
```

Maxima [B] time = 1.25601, size = 501, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -128/7*e^(-7*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x
) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-9*x)/(11*e^(-2*x) + 55*e^(-
4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^
(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) - 47
360/693*e^(-11*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x)
+ 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18
*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-13*x)/(11*e^(-2*x) + 55*e
^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330
*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) -
 128/7*e^(-15*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*
x) + 11*e^(-20*x) + e^(-22*x) + 1)
```

Fricas [B] time = 1.81562, size = 2333, normalized size = 93.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -128/693*(99*cosh(x)^8 + 792*cosh(x)*sinh(x)^7 + 99*sinh(x)^8 + 44*(63*cosh
(x)^2 - 5)*sinh(x)^6 - 220*cosh(x)^6 + 264*(21*cosh(x)^3 - 5*cosh(x))*sinh(
x)^5 + 10*(693*cosh(x)^4 - 330*cosh(x)^2 + 37)*sinh(x)^4 + 370*cosh(x)^4 +
8*(693*cosh(x)^5 - 550*cosh(x)^3 + 185*cosh(x))*sinh(x)^3 + 4*(693*cosh(x)^
6 - 825*cosh(x)^4 + 555*cosh(x)^2 - 55)*sinh(x)^2 - 220*cosh(x)^2 + 8*(99*c
osh(x)^7 - 165*cosh(x)^5 + 185*cosh(x)^3 - 55*cosh(x))*sinh(x) + 99)/(cosh(
x)^15 + 15*cosh(x)*sinh(x)^14 + sinh(x)^15 + (105*cosh(x)^2 + 11)*sinh(x)^1
3 + 11*cosh(x)^13 + 13*(35*cosh(x)^3 + 11*cosh(x))*sinh(x)^12 + (1365*cosh(
x)^4 + 858*cosh(x)^2 + 55)*sinh(x)^11 + 55*cosh(x)^11 + 11*(273*cosh(x)^5 +
 286*cosh(x)^3 + 55*cosh(x))*sinh(x)^10 + 55*(91*cosh(x)^6 + 143*cosh(x)^4
+ 55*cosh(x)^2 + 3)*sinh(x)^9 + 165*cosh(x)^9 + 33*(195*cosh(x)^7 + 429*cos
h(x)^5 + 275*cosh(x)^3 + 45*cosh(x))*sinh(x)^8 + (6435*cosh(x)^8 + 18876*co
sh(x)^6 + 18150*cosh(x)^4 + 5940*cosh(x)^2 + 329)*sinh(x)^7 + 331*cosh(x)^7
 + (5005*cosh(x)^9 + 18876*cosh(x)^7 + 25410*cosh(x)^5 + 13860*cosh(x)^3 +
2317*cosh(x))*sinh(x)^6 + (3003*cosh(x)^10 + 14157*cosh(x)^8 + 25410*cosh(x
)^6 + 20790*cosh(x)^4 + 6909*cosh(x)^2 + 451)*sinh(x)^5 + 473*cosh(x)^5 + 5
*(273*cosh(x)^11 + 1573*cosh(x)^9 + 3630*cosh(x)^7 + 4158*cosh(x)^5 + 2317*
cosh(x)^3 + 473*cosh(x))*sinh(x)^4 + (455*cosh(x)^12 + 3146*cosh(x)^10 + 90
```

```
75*cosh(x)^8 + 13860*cosh(x)^6 + 11515*cosh(x)^4 + 4510*cosh(x)^2 + 407)*si
nh(x)^3 + 517*cosh(x)^3 + (105*cosh(x)^13 + 858*cosh(x)^11 + 3025*cosh(x)^9
 + 5940*cosh(x)^7 + 6951*cosh(x)^5 + 4730*cosh(x)^3 + 1551*cosh(x))*sinh(x)
^2 + (15*cosh(x)^14 + 143*cosh(x)^12 + 605*cosh(x)^10 + 1485*cosh(x)^8 + 23
03*cosh(x)^6 + 2255*cosh(x)^4 + 1221*cosh(x)^2 + 165)*sinh(x) + 495*cosh(x)
)
```

Sympy [A] time = 41.6912, size = 34, normalized size = 1.36

$$-\frac{\tanh^4(x)\operatorname{sech}^7(x)}{11}-\frac{4\tanh^2(x)\operatorname{sech}^7(x)}{99}-\frac{8\operatorname{sech}^7(x)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**7*tanh(x)**5,x)
```

```
[Out] -tanh(x)**4*sech(x)**7/11 - 4*tanh(x)**2*sech(x)**7/99 - 8*sech(x)**7/693
```

Giac [A] time = 1.21379, size = 47, normalized size = 1.88

$$-\frac{128\left(99\left(e^{(-x)}+e^x\right)^4-616\left(e^{(-x)}+e^x\right)^2+1008\right)}{693\left(e^{(-x)}+e^x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -128/693*(99*(e^(-x) + e^x)^4 - 616*(e^(-x) + e^x)^2 + 1008)/(e^(-x) + e^x)
^11
```

3.98 $\int \mathbf{sech}^3(x) \tanh^4(x)\, dx$

Optimal. Leaf size=38

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh^3(x)\mathbf{sech}^3(x) - \frac{1}{8}\tanh(x)\mathbf{sech}^3(x) + \frac{1}{16}\tanh(x)\mathbf{sech}(x)$$

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 - (Sech[x]^3*Tanh[x])/8 - (Sech[x
]^3*Tanh[x]^3)/6
```

Rubi [A] time = 0.053362, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh^3(x)\mathbf{sech}^3(x) - \frac{1}{8}\tanh(x)\mathbf{sech}^3(x) + \frac{1}{16}\tanh(x)\mathbf{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 - (Sech[x]^3*Tanh[x])/8 - (Sech[x
]^3*Tanh[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(x)\tanh^4(x)\,dx &= -\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{2}\int \operatorname{sech}^3(x)\tanh^2(x)\,dx\\ &= -\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{8}\int \operatorname{sech}^3(x)\,dx\\ &= \frac{1}{16}\operatorname{sech}(x)\tanh(x)-\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{16}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{16}\tan^{-1}(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)-\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)\end{aligned}$$

Mathematica [A] time = 0.009206, size = 48, normalized size = 1.26

$$\frac{1}{16}\tan^{-1}(\sinh(x))-\frac{1}{3}\tanh^3(x)\operatorname{sech}^3(x)-\frac{1}{6}\tanh(x)\operatorname{sech}^5(x)+\frac{1}{24}\tanh(x)\operatorname{sech}^3(x)+\frac{1}{16}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6 - (Sech[x]^3*Tanh[x]^3)/3
```

Maple [A] time = 0.013, size = 46, normalized size = 1.2

$$-\frac{(\sinh(x))^3}{3\,(\cosh(x))^6}-\frac{\sinh(x)}{5\,(\cosh(x))^6}+\frac{\tanh(x)}{5}\left(\frac{(\operatorname{sech}(x))^5}{6}+\frac{5\,(\operatorname{sech}(x))^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)+\frac{\arctan(e^x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3*tanh(x)^4,x)
```

```
[Out] -1/3*sinh(x)^3/cosh(x)^6-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech
(x)^3+5/16*sech(x))*tanh(x)+1/8*arctan(exp(x))
```

Maxima [B] time = 1.72808, size = 115, normalized size = 3.03

$$\frac{3\,e^{(-x)} - 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} - 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1\right)} - \frac{1}{8}\,\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) - 47*e^(-3*x) + 78*e^(-5*x) - 78*e^(-7*x) + 47*e^(-9*x) - 3*
e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-10
*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Fricas [B] time = 1.80916, size = 3186, normalized size = 83.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
- 47)*sinh(x)^9 - 47*cosh(x)^9 + 9*(55*cosh(x)^3 - 47*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 - 282*cosh(x)^2 + 13)*sinh(x)^7 + 78*cosh(x)^7 + 42*(33*co
sh(x)^5 - 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 - 987*cos
h(x)^4 + 273*cosh(x)^2 - 13)*sinh(x)^5 - 78*cosh(x)^5 + 6*(165*cosh(x)^7 -
987*cosh(x)^5 + 455*cosh(x)^3 - 65*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 - 39
48*cosh(x)^6 + 2730*cosh(x)^4 - 780*cosh(x)^2 + 47)*sinh(x)^3 + 47*cosh(x)^
3 + 3*(55*cosh(x)^9 - 564*cosh(x)^7 + 546*cosh(x)^5 - 260*cosh(x)^3 + 47*co
sh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(
11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))
*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8
+ 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6
 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cos
h(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x
)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x
)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(
x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(
x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)*ar
ctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 - 141*cosh(x)^8 + 182*cosh(x)^6
```

```
- 130*cosh(x)^4 + 47*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12*c
osh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x
)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh
(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*c
osh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*s
inh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 +
5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*
cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 +
 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45
*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*
cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^4(x)\operatorname{sech}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3*tanh(x)**4,x)
```

```
[Out] Integral(tanh(x)**4*sech(x)**3, x)
```

Giac [B] time = 1.1813, size = 99, normalized size = 2.61

$$\frac{1}{32}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^{5} - 32\left(e^{(-x)} - e^{x}\right)^{3} - 48\,e^{(-x)} + 48\,e^{x}}{24\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{3}} + \frac{1}{16}\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 - 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.99 $\int \mathbf{sech}^5(x)\tanh^2(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{16}\tan^{-1}(\sinh(x))-\frac{1}{6}\tanh(x)\mathbf{sech}^5(x)+\frac{1}{24}\tanh(x)\mathbf{sech}^3(x)+\frac{1}{16}\tanh(x)\mathbf{sech}(x)$$

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Rubi [A] time = 0.0370977, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16}\tan^{-1}(\sinh(x))-\frac{1}{6}\tanh(x)\mathbf{sech}^5(x)+\frac{1}{24}\tanh(x)\mathbf{sech}^3(x)+\frac{1}{16}\tanh(x)\mathbf{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^5(x)\tanh^2(x)\,dx &= -\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{6}\int \operatorname{sech}^5(x)\,dx\\
&= \frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{8}\int \operatorname{sech}^3(x)\,dx\\
&= \frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{16}\int \operatorname{sech}(x)\,dx\\
&= \frac{1}{16}\tan^{-1}(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.0086348, size = 36, normalized size = 1.

$$\frac{1}{16}\tan^{-1}(\sinh(x))-\frac{1}{6}\tanh(x)\operatorname{sech}^5(x)+\frac{1}{24}\tanh(x)\operatorname{sech}^3(x)+\frac{1}{16}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Maple [A] time = 0.012, size = 36, normalized size = 1.

$$-\frac{\sinh(x)}{5\,(\cosh(x))^6}+\frac{\tanh(x)}{5}\left(\frac{(\operatorname{sech}(x))^5}{6}+\frac{5\,(\operatorname{sech}(x))^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)+\frac{\arctan(e^x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5*tanh(x)^2,x)
```

```
[Out] -1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech(x)^3+5/16*sech(x))*tanh
(x)+1/8*arctan(exp(x))
```

Maxima [B] time = 1.56763, size = 115, normalized size = 3.19

$$\frac{3\,e^{(-x)} + 17\,e^{(-3\,x)} - 114\,e^{(-5\,x)} + 114\,e^{(-7\,x)} - 17\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,\left(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1\right)} - \frac{1}{8}\,\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 17*e^(-3*x) - 114*e^(-5*x) + 114*e^(-7*x) - 17*e^(-9*x) -
3*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-
10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Fricas [B] time = 1.83163, size = 3189, normalized size = 88.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
+ 17)*sinh(x)^9 + 17*cosh(x)^9 + 9*(55*cosh(x)^3 + 17*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 + 102*cosh(x)^2 - 19)*sinh(x)^7 - 114*cosh(x)^7 + 42*(33*c
osh(x)^5 + 34*cosh(x)^3 - 19*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 + 357*co
sh(x)^4 - 399*cosh(x)^2 + 19)*sinh(x)^5 + 114*cosh(x)^5 + 6*(165*cosh(x)^7
+ 357*cosh(x)^5 - 665*cosh(x)^3 + 95*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 +
1428*cosh(x)^6 - 3990*cosh(x)^4 + 1140*cosh(x)^2 - 17)*sinh(x)^3 - 17*cosh(
x)^3 + 3*(55*cosh(x)^9 + 204*cosh(x)^7 - 798*cosh(x)^5 + 380*cosh(x)^3 - 17
*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 +
6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(
x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)
^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x
)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*
cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cos
h(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cos
h(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*
cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*co
sh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*co
sh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 + 51*cosh(x)^8 - 266*cosh(x)^
```

```
6 + 190*cosh(x)^4 - 17*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12
*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh
(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*co
sh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5
*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)
*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3
+ 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 2
0*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7
 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 +
45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 +
6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 +
5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^2(x)\operatorname{sech}^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5*tanh(x)**2,x)
```

```
[Out] Integral(tanh(x)**2*sech(x)**5, x)
```

Giac [B] time = 1.20283, size = 99, normalized size = 2.75

$$\frac{1}{32}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^{5} + 32\left(e^{(-x)} - e^{x}\right)^{3} - 48\,e^{(-x)} + 48\,e^{x}}{24\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{3}} + \frac{1}{16}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 + 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.100 $\int \mathbf{sech}^8(x)\tanh^6(x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rubi [A] time = 0.0303592, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 270}

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^8*Tanh[x]^6,x]
```

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = i\,\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, i\tanh(x)\right)$$
$$= i\,\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx, x, i\tanh(x)\right)$$
$$= \frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

Mathematica [B] time = 0.0254378, size = 67, normalized size = 2.03

$$\frac{16\tanh(x)}{3003}-\frac{1}{13}\tanh(x)\operatorname{sech}^{12}(x)+\frac{27}{143}\tanh(x)\operatorname{sech}^{10}(x)-\frac{53}{429}\tanh(x)\operatorname{sech}^8(x)+\frac{5\tanh(x)\operatorname{sech}^6(x)}{3003}+\frac{2\tanh(x)\mathrm{s}}{100}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^8*Tanh[x]^6,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

Maple [B] time = 0.084, size = 72, normalized size = 2.2

$$-\frac{(\sinh(x))^5}{8\,(\cosh(x))^{13}}-\frac{(\sinh(x))^3}{16\,(\cosh(x))^{13}}-\frac{\sinh(x)}{64\,(\cosh(x))^{13}}+\frac{\tanh(x)}{64}\left(\frac{1024}{3003}+\frac{(\operatorname{sech}(x))^{12}}{13}+\frac{12\,(\operatorname{sech}(x))^{10}}{143}+\frac{40\,(\operatorname{sech}(x))}{429}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^8*tanh(x)^6,x)
```

```
[Out] -1/8*sinh(x)^5/cosh(x)^13-1/16*sinh(x)^3/cosh(x)^13-1/64*sinh(x)/cosh(x)^13
+1/64*(1024/3003+1/13*sech(x)^12+12/143*sech(x)^10+40/429*sech(x)^8+320/300
3*sech(x)^6+128/1001*sech(x)^4+512/3003*sech(x)^2)*tanh(x)
```

Maxima [B] time = 1.06547, size = 1157, normalized size = 35.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="maxima")
```

```
[Out] 32/231*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/
77*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287
*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*
x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/21*e
^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(
-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) +
 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 512/21*e^(-
8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10
*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 28
6*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 768/7*e^(-10*x
)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x)
 + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e
^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 1216/7*e^(-12*x)/
(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) +
 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(
-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 192*e^(-14*x)/(13*e
^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716
*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x
) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 96*e^(-16*x)/(13*e^(-2*x
) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-1
2*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78
*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32*e^(-18*x)/(13*e^(-2*x) + 78
*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-2
2*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32/3003/(13*e^(-2*x) + 78*e^(-4*x) +
 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-1
4*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e
^(-24*x) + e^(-26*x) + 1)
```

Fricas [B] time = 1.86253, size = 3032, normalized size = 91.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^6(x)\operatorname{sech}^8(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**8*tanh(x)**6,x)
```

```
[Out] Integral(tanh(x)**6*sech(x)**8, x)
```

Giac [B] time = 1.19937, size = 89, normalized size = 2.7

$$-\frac{32\left(3003\,e^{(18\,x)}-9009\,e^{(16\,x)}+18018\,e^{(14\,x)}-16302\,e^{(12\,x)}+10296\,e^{(10\,x)}-2288\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+\right.}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="giac")
```

```
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

3.101 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rubi [A] time = 0.0180681, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cosh(a+bx)\coth(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= \frac{\cosh(a+bx)}{b} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0257619, size = 26, normalized size = 1.13

$$
\frac{\cosh(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b + Log[Tanh[(a + b*x)/2]]/b
```

Maple [A] time = 0.016, size = 21, normalized size = 0.9

$$
\frac{\cosh(bx+a) - 2\operatorname{Artanh}\left(e^{bx+a}\right)}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a),x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.20723, size = 80, normalized size = 3.48

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

Fricas [B] time = 1.84889, size = 355, normalized size = 15.43

$$\frac{\cosh\left(bx+a\right)^{2} - 2\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right) + 1\right) + 2\left(\cosh\left(bx+a\right) + \sinh\left(bx+\right.\right.}{2\left(b\cosh\left(bx+a\right) + b\sinh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(a+bx\right)\coth\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x), x)
```

Giac [A] time = 1.18726, size = 59, normalized size = 2.57

$$\frac{e^{(bx+a)} + e^{(-bx-a)} - 2 \log\left(e^{(bx+a)} + 1\right) + 2 \log\left(\left|e^{(bx+a)} - 1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*x
 + a) - 1)))/b
```

3.102 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\sinh(a+bx)}{b}-\frac{\text{csch}(a+bx)}{b}$$

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0216343, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh(a+bx)}{b}-\frac{\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```


```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```


Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\text{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.0142865, size = 22, normalized size = 1.

$$\frac{\sinh(a+bx)}{b} - \frac{\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Maple [A] time = 0.015, size = 32, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{\sinh(bx+a)} + 2\sinh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)*cosh(b*x+a)^2+2*sinh(b*x+a))
```

Maxima [B] time = 1.19467, size = 76, normalized size = 3.45

$$-\frac{e^{(-bx-a)}}{2b} - \frac{5e^{(-2bx-2a)} - 1}{2b\left(e^{(-bx-a)} - e^{(-3bx-3a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

Fricas [A] time = 1.79687, size = 85, normalized size = 3.86

$$\frac{\cosh(bx+a)^2+\sinh(bx+a)^2-3}{2\,b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**2, x)
```

Giac [B] time = 1.19667, size = 68, normalized size = 3.09

$$-\frac{\frac{\left(5\,e^{(2\,bx+2\,a)}-1\right)e^{(-a)}}{e^{(3\,bx+2\,a)}-e^{(bx)}}-e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((5*e^(2*b*x + 2*a) - 1)*e^(-a)/(e^(3*b*x + 2*a) - e^(b*x)) - e^(b*x +
 a))/b
```

3.103 $\int \cosh(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\cosh(a+bx)}{2b}-\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(2*b) + (3*Cosh[a + b*x])/(2*b) - (Cosh[a + b*x
]*Coth[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0359131, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{3\cosh(a+bx)}{2b}-\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(2*b) + (3*Cosh[a + b*x])/(2*b) - (Cosh[a + b*x
]*Coth[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\ &= \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\ &= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b} + \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0319093, size = 67, normalized size = 1.37

$$\frac{\cosh(a+bx)}{b} - \frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]/b - Csch[(a + b*x)/2]^2/(8*b) + (3*Log[Tanh[(a + b*x)/2]])/(2
*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] time = 0.02, size = 62, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3}{(\sinh(bx+a))^2} - 3\frac{\cosh(bx+a)}{(\sinh(bx+a))^2} + \frac{3\operatorname{csch}(bx+a)\coth(bx+a)}{2} - 3\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^3,x)
```

```
[Out] 1/b*(1/sinh(b*x+a)^2*cosh(b*x+a)^3-3/sinh(b*x+a)^2*cosh(b*x+a)+3/2*csch(b*x
+a)*coth(b*x+a)-3*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.21084, size = 146, normalized size = 2.98

$$\frac{e^{(-bx-a)}}{2\,b} - \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{4\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - 1}{2\,b\left(e^{(-bx-a)} - 2\,e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b - 3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1
)/b - 1/2*(4*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) - 1)/(b*(e^(-b*x - a) - 2*
e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Fricas [B] time = 2.02582, size = 1710, normalized size = 34.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^5
+ 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2
```

```
 + 1)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
1)/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)
^5 - 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + 2*
(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a)
 + (5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**3, x)
```

Giac [A] time = 1.28521, size = 107, normalized size = 2.18

$$-\frac{\frac{2\left(e^{(3\,bx+3\,a)}+e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-e^{(bx+a)}-e^{(-bx-a)}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - e^(b*x +
a) - e^(-b*x - a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

3.104 $\int \cosh(a+bx)\coth^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{2\operatorname{csch}(a+bx)}{b}$$

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0250251, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{2\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{2\text{csch}(a+bx)}{b} - \frac{\text{csch}^3(a+bx)}{3b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.0213773, size = 37, normalized size = 1.

$$\frac{\sinh(a+bx)}{b} - \frac{\text{csch}^3(a+bx)}{3b} - \frac{2\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Maple [A] time = 0.019, size = 67, normalized size = 1.8

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4}{(\sinh(bx+a))^3} - \frac{4\,(\cosh(bx+a))^2}{3\,(\sinh(bx+a))^3} - \frac{8\,(\cosh(bx+a))^2}{3\,\sinh(bx+a)} + \frac{8\,\sinh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^4,x)
```

```
[Out] 1/b*(1/sinh(b*x+a)^3*cosh(b*x+a)^4-4/3/sinh(b*x+a)^3*cosh(b*x+a)^2-8/3/sinh
(b*x+a)*cosh(b*x+a)^2+8/3*sinh(b*x+a))
```

Maxima [B] time = 1.15323, size = 135, normalized size = 3.65

$$-\frac{e^{(-bx-a)}}{2\,b} - \frac{33\,e^{(-2\,bx-2\,a)} - 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} - 3}{6\,b\left(e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/6*(33*e^(-2*b*x - 2*a) - 41*e^(-4*b*x - 4*a) + 27*e
^(-6*b*x - 6*a) - 3)/(b*(e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x -
5*a) - e^(-7*b*x - 7*a)))
```

Fricas [B] time = 1.82168, size = 238, normalized size = 6.43

$$\frac{3 \cosh (bx+a)^4 + 3 \sinh (bx+a)^4 + 18 \left(\cosh (bx+a)^2 - 2\right) \sinh (bx+a)^2 - 36 \cosh (bx+a)^2 + 25}{6 \left(b \sinh (bx+a)^3 + 3 \left(b \cosh (bx+a)^2 - b\right) \sinh (bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 - 2)*sinh(
b*x + a)^2 - 36*cosh(b*x + a)^2 + 25)/(b*sinh(b*x + a)^3 + 3*(b*cosh(b*x +
a)^2 - b)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25634, size = 96, normalized size = 2.59

$$-\frac{\frac{8 \left(3 e^{(5 bx+5 a)} - 4 e^{(3 bx+3 a)} + 3 e^{(bx+a)}\right)}{\left(e^{(2 bx+2 a)} - 1\right)^3} - 3 e^{(bx+a)} + 3 e^{(-bx-a)}}{6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(8*(3*e^(5*b*x + 5*a) - 4*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(e^(2*b*x +
 2*a) - 1)^3 - 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

3.105 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0236637, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0212238, size = 25, normalized size = 0.93

$$\frac{\sinh^2(a+bx)+2\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.017, size = 26, normalized size = 1.

$$\frac{(\cosh(bx+a))^2}{2\,b} + \frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Maxima [B] time = 1.08088, size = 95, normalized size = 3.52

$$\frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
- a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 1.79296, size = 552, normalized size = 20.44

$$-\frac{8\,bx\cosh\left(bx+a\right)^{2}-\cosh\left(bx+a\right)^{4}-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-\sinh\left(bx+a\right)^{4}+2\left(4\,bx-3\,\cosh\left(bx+a\right)^{2}\right)\sin}{8\left(b\cosh\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.16251, size = 81, normalized size = 3.

$$-\frac{8\,bx-\left(4\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-2\,bx-2\,a\right)}-e^{\left(2\,bx+2\,a\right)}-8\,\log\left(\left|e^{\left(2\,bx+2\,a\right)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) -
8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.106 $\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3x}{2}$$

```
[Out] (3*x)/2 - (3*Coth[a + b*x])/(2*b) + (Cosh[a + b*x]^2*Coth[a + b*x])/(2*b)
```

Rubi [A] time = 0.0411313, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 206}

$$-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

```
[Out] (3*x)/2 - (3*Coth[a + b*x])/(2*b) + (Cosh[a + b*x]^2*Coth[a + b*x])/(2*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\coth^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \coth(a+bx)\right)}{b}\\ &= \frac{\cosh^2(a+bx)\coth(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \coth(a+bx)\right)}{2b}\\ &= -\frac{3\coth(a+bx)}{2b} + \frac{\cosh^2(a+bx)\coth(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \coth(a+b\right.}{2b}\\ &= \frac{3x}{2} - \frac{3\coth(a+bx)}{2b} + \frac{\cosh^2(a+bx)\coth(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.10684, size = 31, normalized size = 0.78

$$\frac{6(a+bx)+\sinh(2(a+bx))-4\coth(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

```
[Out] (6*(a + b*x) - 4*Coth[a + b*x] + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.018, size = 39, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3}{2\sinh(bx+a)} + \frac{3\,bx}{2} + \frac{3\,a}{2} - \frac{3\coth(bx+a)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2/sinh(b*x+a)*cosh(b*x+a)^3+3/2*b*x+3/2*a-3/2*coth(b*x+a))
```

Maxima [A] time = 1.12189, size = 89, normalized size = 2.22

$$\frac{3\,(bx+a)}{2\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{17\,e^{(-2\,bx-2\,a)}-1}{8\,b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b - 1/8*(17*e^(-2*b*x - 2*a) - 1)/(b
*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

Fricas [A] time = 1.72865, size = 166, normalized size = 4.15

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+4\,(3\,bx+2)\sinh\left(bx+a\right)-9\,\cosh\left(bx+a\right)}{8\,b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 4*(3*b*x + 2)*sinh
(b*x + a) - 9*cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh^{2}\left(a+bx\right)\coth^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*coth(a + b*x)**2, x)
```

Giac [A] time = 1.25637, size = 90, normalized size = 2.25

$$\frac{12\,bx+\frac{\left(3\,e^{(4\,bx+4\,a)}+14\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,a)}}{e^{(2\,bx)}-e^{(4\,bx+2\,a)}}+e^{(2\,bx+2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*b*x + (3*e^(4*b*x + 4*a) + 14*e^(2*b*x + 2*a) - 1)*e^(-2*a)/(e^(2*b
*x) - e^(4*b*x + 2*a)) + e^(2*b*x + 2*a))/b
```

3.107 $\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh^2(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}$$

```
[Out] -Csch[a + b*x]^2/(2*b) + (2*Log[Sinh[a + b*x]])/b + Sinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0420806, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sinh^2(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```


```
[Out] -Csch[a + b*x]^2/(2*b) + (2*Log[Sinh[a + b*x]])/b + Sinh[a + b*x]^2/(2*b)
```


Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{csch}^2(a+bx)}{2b} + \frac{2\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0479868, size = 35, normalized size = 0.81

$$-\frac{-\sinh^2(a+bx)+\text{csch}^2(a+bx)-4\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

```
[Out] -(Csch[a + b*x]^2 - 4*Log[Sinh[a + b*x]] - Sinh[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.02, size = 48, normalized size = 1.1

$$\frac{(\cosh(bx+a))^4}{2b(\sinh(bx+a))^2} + 2\frac{\ln(\sinh(bx+a))}{b} - \frac{(\coth(bx+a))^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^3,x)
```

```
[Out] 1/2/b/sinh(b*x+a)^2*cosh(b*x+a)^4+2*ln(sinh(b*x+a))/b-coth(b*x+a)^2/b
```

Maxima [B] time = 1.07772, size = 162, normalized size = 3.77

$$\frac{2(bx+a)}{b} + \frac{e^{(-2bx-2a)}}{8b} + \frac{2\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2e^{(-2bx-2a)}+15e^{(-4bx-4a)}-1}{8b\left(e^{(-2bx-2a)}-2e^{(-4bx-4a)}+e^{(-6bx-6a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(
e^(-b*x - a) - 1)/b - 1/8*(2*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 1)/(b
*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Fricas [B] time = 1.86623, size = 2049, normalized size = 47.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 -
2*(8*b*x + 1)*cosh(b*x + a)^6 - 2*(8*b*x - 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 - 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 - 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
- 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 - 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 - 8*b*x - 1)*sinh(b*x + a)^2
 + 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*sinh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 - 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 - (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 - 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26816, size = 134, normalized size = 3.12

$$-\frac{16\,bx-\left(8\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+\frac{8\left(3\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-e^{(2\,bx+2\,a)}-16\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(16*b*x - (8*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*(3*e^(4*b*x + 4
*a) - 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) - 1)^2 - e^(2*b*x + 2*a) - 16
*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.108 $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b + Cosh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0300179, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2592, 302, 206}

$$\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b + Cosh[a + b*x]^3/(3*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0293825, size = 44, normalized size = 1.16

$$\frac{5\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

```
[Out] (5*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) + Log[Tanh[(a + b*x)/2]]
/b
```

Maple [A] time = 0.017, size = 31, normalized size = 0.8

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3}{3} + \cosh(bx+a) - 2\,\text{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a),x)
```

```
[Out] 1/b*(1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.04711, size = 117, normalized size = 3.08

$$\frac{\left(15\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{24\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/24*(15*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) +
e^(-3*b*x - 3*a))/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 1.89726, size = 1037, normalized size = 27.29

$$\frac{\cosh\left(bx+a\right)^6 + 6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5 + \sinh\left(bx+a\right)^6 + 15\left(\cosh\left(bx+a\right)^2+1\right)\sinh\left(bx+a\right)^4 + 15\,\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 15*cosh(b*x + a)^2 - 24*(cosh(b*x + a)^3
 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh
(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 24*(cosh(b*x + a)^3 +
 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(cosh(b*x + a)^5 + 10
*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^3 +
 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*
sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh^3\left(a+bx\right)\coth\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x), x)
```

Giac [B] time = 1.19812, size = 104, normalized size = 2.74

$$\frac{\left(15\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+18\,a)}+15\,e^{(bx+16\,a)}\right)e^{(-15\,a)}-24\,\log\left(e^{(bx+a)}+1\right)+24\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((15*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 18*a) + 15*e^
(b*x + 16*a))*e^(-15*a) - 24*log(e^(b*x + a) + 1) + 24*log(abs(e^(b*x + a)
- 1)))/b
```

3.109 $\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\text{csch}(a+bx)}{b}$$

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0353186, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\text{csch}(a+bx)}{b} + \frac{2\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.0182505, size = 38, normalized size = 1.

$$\frac{\sinh^3(a+bx)}{3b} + \frac{2\sinh(a+bx)}{b} - \frac{\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.017, size = 50, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4}{3\sinh(bx+a)} - \frac{4(\cosh(bx+a))^2}{3\sinh(bx+a)} + \frac{8\sinh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3/sinh(b*x+a)*cosh(b*x+a)^4-4/3/sinh(b*x+a)*cosh(b*x+a)^2+8/3*sinh(b
*x+a))
```

Maxima [B] time = 1.0822, size = 107, normalized size = 2.82

$$-\frac{21\,e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{24\,b} + \frac{20\,e^{(-2\,bx-2\,a)} - 69\,e^{(-4\,bx-4\,a)} + 1}{24\,b\left(e^{(-3\,bx-3\,a)} - e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(21*e^(-b*x - a) + e^(-3*b*x - 3*a))/b + 1/24*(20*e^(-2*b*x - 2*a) -
69*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

Fricas [A] time = 1.85744, size = 177, normalized size = 4.66

$$\frac{\cosh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}+10\right)\sinh\left(bx+a\right)^{2}+20\cosh\left(bx+a\right)^{2}-45}{24\,b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b
*x + a)^2 + 20*cosh(b*x + a)^2 - 45)/(b*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24665, size = 103, normalized size = 2.71

$$-\frac{\left(21\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-3\,bx-3\,a\right)}-\left(e^{\left(3\,bx+24\,a\right)}+21\,e^{\left(bx+22\,a\right)}\right)e^{\left(-21\,a\right)}+\frac{48\,e^{\left(bx+a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*((21*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - (e^(3*b*x + 24*a) + 21*e
^(b*x + 22*a))*e^(-21*a) + 48*e^(b*x + a)/(e^(2*b*x + 2*a) - 1))/b
```

3.110 $\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\cosh^3(a+bx)}{6b}+\frac{5\cosh(a+bx)}{2b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}$$

```
[Out] (-5*ArcTanh[Cosh[a + b*x]])/(2*b) + (5*Cosh[a + b*x])/(2*b) + (5*Cosh[a + b
*x]^3)/(6*b) - (Cosh[a + b*x]^3*Coth[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0482733, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{5\cosh^3(a+bx)}{6b}+\frac{5\cosh(a+bx)}{2b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```


```
[Out] (-5*ArcTanh[Cosh[a + b*x]])/(2*b) + (5*Cosh[a + b*x])/(2*b) + (5*Cosh[a + b
*x]^3)/(6*b) - (Cosh[a + b*x]^3*Coth[a + b*x]^2)/(2*b)
```


Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}$$
$$= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(a+bx)\right)}{2b}$$
$$= \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}$$
$$= -\frac{5\tanh^{-1}(\cosh(a+bx))}{2b} + \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+bx)\,\text{cot}}{2b}$$

Mathematica [A] time = 0.040455, size = 85, normalized size = 1.29

$$\frac{9\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b} - \frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{5\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Csch[(a + b*x)/2]^2/(8
*b) + (5*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] time = 0.021, size = 81, normalized size = 1.2

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^5}{3(\sinh(bx+a))^2} + \frac{5(\cosh(bx+a))^3}{3(\sinh(bx+a))^2} - 5\frac{\cosh(bx+a)}{(\sinh(bx+a))^2} + \frac{5\,\text{csch}(bx+a)\coth(bx+a)}{2} - 5\,\text{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^3,x)
```

```
[Out] 1/b*(1/3/sinh(b*x+a)^2*cosh(b*x+a)^5+5/3/sinh(b*x+a)^2*cosh(b*x+a)^3-5/sinh
(b*x+a)^2*cosh(b*x+a)+5/2*csch(b*x+a)*coth(b*x+a)-5*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.04491, size = 180, normalized size = 2.73

$$\frac{27\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b}-\frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{25\,e^{(-2\,bx-2\,a)}-77\,e^{(-4\,bx-4\,a)}+3\,e^{(-6\,bx-6\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-2\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(27*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - 5/2*log(e^(-b*x - a) + 1)/b +
 5/2*log(e^(-b*x - a) - 1)/b + 1/24*(25*e^(-2*b*x - 2*a) - 77*e^(-4*b*x - 4
*a) + 3*e^(-6*b*x - 6*a) + 1)/(b*(e^(-3*b*x - 3*a) - 2*e^(-5*b*x - 5*a) + e
^(-7*b*x - 7*a)))
```

Fricas [B] time = 1.97856, size = 3027, normalized size = 45.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^8 + 25*cosh(b*x + a)^8 + 40*(3*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 + 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 + 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 + 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 - 5)*sinh(b
*x + a)^4 - 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 + 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 + 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 25*cosh(b*x + a)^2 - 60*(cosh(b*x + a)^7 + 7*cosh(b*x + a
```

```
)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 - 2)*sinh(b*x + a
)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
+ a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + co
sh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))
*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 3*cosh(b*x + a
)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 60*(cosh(b*x +
 a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x +
a)^2 - 2)*sinh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*co
sh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x +
 a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a)
 - 1) + 10*(cosh(b*x + a)^9 + 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 - 20*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
2*b*cosh(b*x + a))*sinh(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a
)^4 - 20*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 - 2
0*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x +
a)^6 - 10*b*cosh(b*x + a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32919, size = 146, normalized size = 2.21

$$\frac{\left(27\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+30\,a)}+27\,e^{(bx+28\,a)}\right)e^{(-27\,a)}-\frac{24\left(e^{(3\,bx+3\,a)}+e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-60\,\log\left(e^{(bx+a)}+1\right)+60\,\log\left(\left|e^{(bx+a)}\right.\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*((27*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 30*a) + 27*e^
(b*x + 28*a))*e^(-27*a) - 24*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*
a) - 1)^2 - 60*log(e^(b*x + a) + 1) + 60*log(abs(e^(b*x + a) - 1)))/b
```

3.111 $\int \cosh^4(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^2(a+bx)}{b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/b + Sinh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0308386, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2590, 266, 43}

$$\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^2(a+bx)}{b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/b + Sinh[a + b*x]^4/(4*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{b} + \frac{\sinh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.0323827, size = 35, normalized size = 0.9

$$\frac{\sinh^4(a+bx)+4\sinh^2(a+bx)+4\log(\sinh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

```
[Out] (4*Log[Sinh[a + b*x]] + 4*Sinh[a + b*x]^2 + Sinh[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.017, size = 39, normalized size = 1.

$$\frac{(\cosh(bx+a))^4}{4\,b} + \frac{(\cosh(bx+a))^2}{2\,b} + \frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*coth(b*x+a),x)
```

```
[Out] 1/4*cosh(b*x+a)^4/b+1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Maxima [B] time = 1.09916, size = 128, normalized size = 3.28

$$\frac{\left(12\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} + \frac{12\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}}{64\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/64*(12*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b + 1/64*(12*e
^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/b + log(e^(-b*x - a) + 1)/b + log(e^(-b
*x - a) - 1)/b
```

Fricas [B] time = 1.90217, size = 1270, normalized size = 32.56

$$\frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2 + 3\right)\sinh(bx+a)^6 - 64\,bx\cosh(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x + 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) + 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 + 45*cosh(b*x + a)^4 + 3)
*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 + 9*cosh(b*x + a)^
5 + 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*coth(b*x+a),x)
```

[Out] Timed out

Giac [B] time = 1.23364, size = 117, normalized size = 3.

$$-\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}-\left(e^{(4\,bx+16\,a)}+12\,e^{(2\,bx+14\,a)}\right)e^{(-12\,a)}-64\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4
*a) - (e^(4*b*x + 16*a) + 12*e^(2*b*x + 14*a))*e^(-12*a) - 64*log(abs(e^(2*
b*x + 2*a) - 1)))/b
```

3.112 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -(Csch[a + b*x]/b)
```

Rubi [A] time = 0.0107484, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{i\,\mathrm{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0091367, size = 11, normalized size = 1.

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Maple [A] time = 0.007, size = 12, normalized size = 1.1

$$-\frac{\operatorname{csch}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*coth(b*x+a),x)
```

```
[Out] -csch(b*x+a)/b
```

Maxima [B] time = 1.04634, size = 34, normalized size = 3.09

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Fricas [B] time = 1.73605, size = 154, normalized size = 14.

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)*csch(a + b*x), x)
```

Giac [B] time = 1.20304, size = 32, normalized size = 2.91

$$-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) - 1))
```

3.113 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0200289, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0131857, size = 15, normalized size = 1.

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Maple [A] time = 0.009, size = 14, normalized size = 0.9

$$-\frac{(\operatorname{csch}(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -1/2*csch(b*x+a)^2/b
```

Maxima [A] time = 1.03158, size = 18, normalized size = 1.2

$$-\frac{\coth(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*coth(b*x + a)^2/b
```

Fricas [B] time = 1.81544, size = 232, normalized size = 15.47

$$-\frac{2(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3-b\cosh(bx+a)+3\left(b\cosh(bx+a)^2-b\right)\sinh(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

Sympy [A] time = 4.92465, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\operatorname{csch}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\coth(a)\operatorname{csch}^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Piecewise((-csch(a + b*x)**2/(2*b), Ne(b, 0)), (x*coth(a)*csch(a)**2, True)
)
```

Giac [A] time = 1.17034, size = 36, normalized size = 2.4

$$-\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.114 $\int \cosh(a+bx)\mathbf{csch}^{1+n}(a+bx)\,dx$

Optimal. Leaf size=16

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

Rubi [A] time = 0.0279535, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 30}

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x^{-1+n}\,dx, x, \operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}^n(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0198964, size = 16, normalized size = 1.

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

Maple [A] time = 0.006, size = 17, normalized size = 1.1

$$-\frac{(\operatorname{csch}(bx+a))^n}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*csch(b*x+a)^n,x)
```

```
[Out] -csch(b*x+a)^n/b/n
```

Maxima [B] time = 1.90992, size = 72, normalized size = 4.5

$$-\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))/
(b*n)
```

Fricas [B] time = 1.89532, size = 338, normalized size = 21.12

$$-\frac{\cosh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)+\sinh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right.\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1))))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a+bx)\operatorname{csch}^n(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)*csch(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^n \coth(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^n*coth(b*x + a), x)
```

3.115 $\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] -Coth[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0274867, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^3/(3*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx &= -\frac{i\,\mathrm{Subst}\left(\int x^2\,dx, x, i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0041052, size = 15, normalized size = 1.

$$-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^3/(3*b)
```

Maple [B] time = 0.017, size = 42, normalized size = 2.8

$$\frac{1}{b}\left(-\frac{\cosh(bx+a)}{2(\sinh(bx+a))^3}-\frac{\coth(bx+a)}{2}\left(\frac{2}{3}-\frac{(\operatorname{csch}(bx+a))^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^3*cosh(b*x+a)-1/2*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
)
```

Maxima [A] time = 1.10479, size = 18, normalized size = 1.2

$$-\frac{\coth(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*coth(b*x + a)^3/b
```

Fricas [B] time = 1.73432, size = 378, normalized size = 25.2

$$-\frac{8\left(\cosh(bx+a)^2+\cosh(bx+a)\sinh(bx+a)+\sinh}{3\left(b\cosh(bx+a)^4+4b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4-4b\cosh(bx+a)^2+2\left(3b\cosh(bx+a)^2-2b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**2, x)
```

Giac [B] time = 1.23816, size = 42, normalized size = 2.8

$$-\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

3.116 $\int \coth^3(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\coth^4(a+bx)}{4b}$$

```
[Out] -Coth[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0290491, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\coth^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^4/(4*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.004639, size = 15, normalized size = 1.

$$-\frac{\coth^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^4/(4*b)
```

Maple [B] time = 0.022, size = 42, normalized size = 2.8

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{4(\sinh(bx+a))^4}-\frac{(\cosh(bx+a))^2}{4(\sinh(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4*cosh(b*x+a)^2-1/4*cosh(b*x+a)^2/sinh(b*x+a)^2)
```

Maxima [A] time = 1.04203, size = 18, normalized size = 1.2

$$-\frac{\coth(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*coth(b*x + a)^4/b
```

Fricas [B] time = 1.85037, size = 568, normalized size = 37.87

$$-\frac{2\left(\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx\right.}{b\cosh(bx+a)^5+5b\cosh(bx+a)\sinh(bx+a)^4+b\sinh(bx+a)^5-3b\cosh(bx+a)^3+5\left(2b\cosh(bx+a)^2-b\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3
 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*
b*cosh(b*x + a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + 5*(b*cosh(b*x + a)^4
 - 3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**2, x)
```

Giac [B] time = 1.25696, size = 50, normalized size = 3.33

$$-\frac{2\left(e^{(6\,bx+6\,a)}+e^{(2\,bx+2\,a)}\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

3.117 $\int \coth^n(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=20

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Rubi [A] time = 0.033743, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 32}

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int(-ix)^n\,dx,x,i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.022899, size = 20, normalized size = 1.

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Maple [A] time = 0.013, size = 21, normalized size = 1.1

$$-\frac{(\coth(bx+a))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^n*csch(b*x+a)^2,x)
```

```
[Out] -coth(b*x+a)^(n+1)/b/(n+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01245, size = 192, normalized size = 9.6

$$-\frac{\cosh(bx+a)\cosh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)+\cosh(bx+a)\sinh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)}{(bn+b)\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)*cosh(n*log(cosh(b*x + a)/sinh(b*x + a))) + cosh(b*x + a)*si
nh(n*log(cosh(b*x + a)/sinh(b*x + a))))/((b*n + b)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**n*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(bx+a)^n \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)^n*csch(b*x + a)^2, x)
```

3.118 $\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0206844, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0137484, size = 27, normalized size = 1.

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Maple [A] time = 0.014, size = 50, normalized size = 1.9

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{3(\sinh(bx+a))^3}-\frac{2(\cosh(bx+a))^2}{3\sinh(bx+a)}+\frac{2\sinh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^3*cosh(b*x+a)^2-2/3/sinh(b*x+a)*cosh(b*x+a)^2+2/3*sin
h(b*x+a))
```

Maxima [B] time = 1.05293, size = 200, normalized size = 7.41

$$\frac{2e^{(-bx-a)}}{b\left(3e^{(-2bx-2a)}-3e^{(-4bx-4a)}+e^{(-6bx-6a)}-1\right)}-\frac{4e^{(-3bx-3a)}}{3b\left(3e^{(-2bx-2a)}-3e^{(-4bx-4a)}+e^{(-6bx-6a)}-1\right)}+\frac{2e^{(-5b}}{b\left(3e^{(-2bx-2a)}-3e^{(-4b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*
a) - 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a)
 + e^(-6*b*x - 6*a) - 1)) + 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e
^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

Fricas [B] time = 1.7614, size = 464, normalized size = 17.19

$$-\frac{2\left(3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2+3\sinh(bx+a)^3+\left(9\cosh(\right.\right.}{3\left(b\cosh(bx+a)^4+4b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4-4b\cosh(bx+a)^2+2\left(3b\cosh(bx+a)^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a
)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*b*cosh(b*x
+ a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)
^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x), x)
```

Giac [A] time = 1.21634, size = 66, normalized size = 2.44

$$-\frac{2\left(3\,e^{(5\,bx+5\,a)}-2\,e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(5*b*x + 5*a) - 2*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(b*(e^(2*b*x +
 2*a) - 1)^3)
```

3.119 $\int \coth^3(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=31

$$-\frac{\operatorname{csch}^5(a+bx)}{5b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

```
[Out] -Csch[a + b*x]^3/(3*b) - Csch[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0318708, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$-\frac{\operatorname{csch}^5(a+bx)}{5b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

```
[Out] -Csch[a + b*x]^3/(3*b) - Csch[a + b*x]^5/(5*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{i\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{i\,\mathrm{Subst}\left(\int \left(-x^2+x^4\right)dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}^3(a+bx)}{3b} - \frac{\operatorname{csch}^5(a+bx)}{5b}$$

Mathematica [A] time = 0.0256133, size = 31, normalized size = 1.

$$-\frac{\operatorname{csch}^5(a+bx)}{5b} - \frac{\operatorname{csch}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

```
[Out] -Csch[a + b*x]^3/(3*b) - Csch[a + b*x]^5/(5*b)
```

Maple [B] time = 0.02, size = 68, normalized size = 2.2

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{5(\sinh(bx+a))^5} - \frac{2(\cosh(bx+a))^2}{15(\sinh(bx+a))^3} + \frac{2(\cosh(bx+a))^2}{15\sinh(bx+a)} - \frac{2\sinh(bx+a)}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/5/sinh(b*x+a)^5*cosh(b*x+a)^2-2/15/sinh(b*x+a)^3*cosh(b*x+a)^2+2/15
/sinh(b*x+a)*cosh(b*x+a)^2-2/15*sinh(b*x+a))
```

Maxima [B] time = 1.0815, size = 289, normalized size = 9.32

$$\frac{8e^{(-3bx-3a)}}{3b\left(5e^{(-2bx-2a)} - 10e^{(-4bx-4a)} + 10e^{(-6bx-6a)} - 5e^{(-8bx-8a)} + e^{(-10bx-10a)} - 1\right)} + \frac{16e^{(-5}}{15b\left(5e^{(-2bx-2a)} - 10e^{(-4bx-4a)} + 10e^{(-6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15*e^(-5*
b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 8/3*e^(-7*b*x - 7*a)/(
b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8
*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

Fricas [B] time = 1.84297, size = 946, normalized size = 30.52

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}-5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+a\right)^{2}-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 9*b*cosh(b*x + a)^3 + (35*b*cosh(b*x +
 a)^4 - 50*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 - 50*b*cosh(b*x + a)^3 + 27*b*cosh(b*x + a))*sinh(b*x + a)^2 - 5*b*cosh(b
*x + a) + (7*b*cosh(b*x + a)^6 - 25*b*cosh(b*x + a)^4 + 33*b*cosh(b*x + a)^
2 - 15*b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{3}\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**3, x)
```

Giac [A] time = 1.2193, size = 70, normalized size = 2.26

$$-\frac{8\left(5\,e^{(7\,bx+7\,a)}+2\,e^{(5\,bx+5\,a)}+5\,e^{(3\,bx+3\,a)}\right)}{15\,b\left(e^{(2\,bx+2\,a)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*e^(7*b*x + 7*a) + 2*e^(5*b*x + 5*a) + 5*e^(3*b*x + 3*a))/(b*(e^(2*
b*x + 2*a) - 1)^5)
```

3.120 $\int \cosh^3(a+bx)\,\mathbf{csch}^{3+n}(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}-\frac{\operatorname{csch}^{n+2}(a+bx)}{b(n+2)}$$

```
[Out] -(Csch[a + b*x]^n/(b*n)) - Csch[a + b*x]^(2 + n)/(b*(2 + n))
```

Rubi [A] time = 0.0421384, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 14}

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}-\frac{\operatorname{csch}^{n+2}(a+bx)}{b(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n)) - Csch[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \frac{\text{Subst}\left(\int x^{-1+n}\left(-1-x^2\right)\,dx, x, \operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-x^{-1+n}-x^{1+n}\right)\,dx, x, \operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}^n(a+bx)}{bn} - \frac{\operatorname{csch}^{2+n}(a+bx)}{b(2+n)}$$

Mathematica [A] time = 0.0657785, size = 34, normalized size = 0.92

$$-\frac{\operatorname{csch}^n(a+bx)\left(n\operatorname{csch}^2(a+bx)+n+2\right)}{bn(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

```
[Out] -((Csch[a + b*x]^n*(2 + n + n*Csch[a + b*x]^2))/(b*n*(2 + n)))
```

Maple [C] time = 0.152, size = 499, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^n,x)
```

```
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)+2*n*exp(2*b*x+2*a)-4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(exp(2*b*x+2*a)-1)^2*exp(-1/2*n*(I*csgn(I/(exp(b*x+a)-1)/(1+exp
(b*x+a)))^3*Pi-I*csgn(I/(exp(b*x+a)-1)/(1+exp(b*x+a)))^2*csgn(I/(exp(b*x+a)
-1))*Pi-I*csgn(I/(exp(b*x+a)-1)/(1+exp(b*x+a)))^2*csgn(I/(1+exp(b*x+a)))*Pi
+I*csgn(I/(exp(b*x+a)-1)/(1+exp(b*x+a)))*csgn(I/(exp(b*x+a)-1))*csgn(I/(1+e
xp(b*x+a)))*Pi-I*csgn(I/(exp(b*x+a)-1)/(1+exp(b*x+a)))*csgn(I*exp(b*x+a)/(1
+exp(b*x+a))/(exp(b*x+a)-1))^2*Pi+I*csgn(I/(exp(b*x+a)-1)/(1+exp(b*x+a)))*c
sgn(I*exp(b*x+a)/(1+exp(b*x+a))/(exp(b*x+a)-1))*csgn(I*exp(b*x+a))*Pi+I*csg
n(I*exp(b*x+a)/(1+exp(b*x+a))/(exp(b*x+a)-1))^3*Pi-I*csgn(I*exp(b*x+a)/(1+e
xp(b*x+a))/(exp(b*x+a)-1))^2*csgn(I*exp(b*x+a))*Pi-2*ln(2)-2*ln(exp(b*x+a))
+2*ln(exp(b*x+a)-1)+2*ln(1+exp(b*x+a))))
```

Maxima [B] time = 1.91423, size = 559, normalized size = 15.11

$$-\frac{2^{n}ne^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{\left(n^{2}-2\left(n^{2}+2n\right)e^{(-2bx-2a)}+\left(n^{2}+2n\right)e^{(-4bx-4a)}+2n\right)b}-\frac{\left(2^{n+1}n-2^{n+2}\right)e^{\left(-(bx+a)n-2bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right.}}{\left(n^{2}-2\left(n^{2}+2n\right)e^{(-2bx-2a)}+\left(n^{2}+2n\right)e^{(-4bx-4a)}+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1)
)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2
*n)*b) - (2^(n + 1)*n - 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-b*x
- a) + 1) - n*log(-e^(-b*x - a) + 1) - 2*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x
 - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - (2^n*n + 2^(n + 1))*e^(-
(b*x + a)*n - 4*b*x - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1) -
4*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a)
+ 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^
(-b*x - a) + 1))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4
*b*x - 4*a) + 2*n)*b)
```

Fricas [B] time = 1.98283, size = 603, normalized size = 16.3

$$\frac{\left((n+2)\cosh\left(bx+a\right)^{2}+(n+2)\sinh\left(bx+a\right)^{2}+n-2\right)\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^{2}+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^{2}-1}\right)\right)+\left(\right.}{bn^{2}-\left(bn^{2}+2bn\right)\cosh\left(bx+a\right)^{2}-\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 + n - 2)*cosh(n*log(2*(
cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 - 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x
 + a)^2 + n - 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/(b*n^2 - (b*n
^2 + 2*b*n)*cosh(b*x + a)^2 - (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^n \coth(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^n*coth(b*x + a)^3, x)
```

3.121 $\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rubi [A] time = 0.0301996, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx &= -\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}+\frac{1}{2}\int \text{csch}(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0315977, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) + Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

Maple [A] time = 0.014, size = 45, normalized size = 1.3

$$\frac{1}{b}\left(-\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^2*cosh(b*x+a)+1/2*csch(b*x+a)*coth(b*x+a)-arctanh(exp(b
*x+a)))
```

Maxima [B] time = 1.07316, size = 113, normalized size = 3.32

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.84481, size = 1088, normalized size = 32.

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x), x)
```

Giac [A] time = 1.23439, size = 78, normalized size = 2.29

$$-\frac{\frac{2\left(e^{(3bx+3a)}+e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^2}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + log(e^(b*
x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

3.122 $\int \coth^2(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\mathrm{csch}^3(a+bx)}{4b}-\frac{\coth(a+bx)\mathrm{csch}(a+bx)}{8b}$$

```
[Out] ArcTanh[Cosh[a + b*x]]/(8*b) - (Coth[a + b*x]*Csch[a + b*x])/(8*b) - (Coth[
a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0596215, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\mathrm{csch}^3(a+bx)}{4b}-\frac{\coth(a+bx)\mathrm{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(8*b) - (Coth[a + b*x]*Csch[a + b*x])/(8*b) - (Coth[
a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx &= -\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{1}{4}\int \operatorname{csch}^3(a+bx)\,dx\\ &= -\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}-\frac{1}{8}\int \operatorname{csch}(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.044009, size = 95, normalized size = 1.73

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - Log[Tanh[(a + b*
x)/2]]/(8*b) - Sech[(a + b*x)/2]^2/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

Maple [A] time = 0.022, size = 58, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\cosh(bx+a)}{3\,(\sinh(bx+a))^4}-\frac{\coth(bx+a)}{3}\left(-\frac{(\operatorname{csch}(bx+a))^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)+\frac{\operatorname{Artanh}\left(e^{bx+a}\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^4*cosh(b*x+a)-1/3*(-1/4*csch(b*x+a)^3+3/8*csch(b*x+a)
)*coth(b*x+a)+1/4*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.09227, size = 174, normalized size = 3.16

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{8\,b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{8\,b}+\frac{e^{(-bx-a)}+7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*log(e^(-b*x - a) + 1)/b - 1/8*log(e^(-b*x - a) - 1)/b + 1/4*(e^(-b*x -
a) + 7*e^(-3*b*x - 3*a) + 7*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a))/(b*(4*e^(-
2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) -
 1))
```

Fricas [B] time = 2.02739, size = 3066, normalized size = 55.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*x + a)^7 + 14*cosh(b*x + a)*sinh(b*x + a)^6 + 2*sinh(b*x + a
)^7 + 14*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 14*cosh(b*x + a)^5 + 70*
(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^4 + 14*(5*cosh(b*x + a)^4 +
 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 14*(3*cosh(
b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - (cosh(
b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x +
a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*c
osh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 -
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b
*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 -
 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a
)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 1
5*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a
```

```
)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(7*cos
h(b*x + a)^6 + 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 1)*sinh(b*x + a) +
 2*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)
*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh
(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 -
 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cos
h(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))
*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**3, x)
```

Giac [A] time = 1.223, size = 108, normalized size = 1.96

$$-\frac{\frac{2\left(e^{(7\,bx+7\,a)}+7\,e^{(5\,bx+5\,a)}+7\,e^{(3\,bx+3\,a)}+e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^4}-\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(e^(7*b*x + 7*a) + 7*e^(5*b*x + 5*a) + 7*e^(3*b*x + 3*a) + e^(b*x +
 a))/(e^(2*b*x + 2*a) - 1)^4 - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) -
 1)))/b
```

3.123 $\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}-\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) - (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]^3*Csch[a + b*x])/(4*b)
```

Rubi [A] time = 0.0570593, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}-\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) - (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]^3*Csch[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = -\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}+\frac{3}{4}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}+\frac{3}{8}\int \text{csch}(a+bx)\,dx$$
$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx}{4b}$$

Mathematica [A] time = 0.0389819, size = 95, normalized size = 1.73

$$-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{5\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{5\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-5*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (3*Log[Tanh[
(a + b*x)/2]])/(8*b) - (5*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4
/(64*b)
```

Maple [A] time = 0.015, size = 74, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^3}{(\sinh(bx+a))^4}+\frac{\cosh(bx+a)}{(\sinh(bx+a))^4}+\left(-\frac{(\text{csch}(bx+a))^3}{4}+\frac{3\,\text{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\,\text{Artanh}\left(e^{bx+a}\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^4*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^4*cosh(b*x+a)^3+1/sinh(b*x+a)^4*cosh(b*x+a)+(-1/4*csch(
b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.06964, size = 180, normalized size = 3.27

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}+\frac{5\,e^{(-bx-a)}+3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}+5\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b + 1/4*(5*e^(-b*x
 - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/(b*(4
*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8
*a) - 1))
```

Fricas [B] time = 1.94066, size = 3081, normalized size = 56.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(b*x + a)^7 + 70*cosh(b*x + a)*sinh(b*x + a)^6 + 10*sinh(b*x +
 a)^7 + 6*(35*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 6*cosh(b*x + a)^5 + 10
*(35*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(175*cosh(b*x +
 a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 6*cosh(b*x + a)^3 + 6*(35
*cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 +
3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*
(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x
 + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cos
h(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x +
 a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cos
h(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4
*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*co
sh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x
+ a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*co
sh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 2*(35*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 5)*sinh(
b*x + a) + 10*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*
x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*
```

```
x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x +
a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3
 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b
*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cos
h(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**4*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**4*csch(a + b*x), x)
```

Giac [A] time = 1.31105, size = 116, normalized size = 2.11

$$-\frac{\frac{2\left(5\,e^{(7\,bx+7\,a)}+3\,e^{(5\,bx+5\,a)}+3\,e^{(3\,bx+3\,a)}+5\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^4}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(5*e^(7*b*x + 7*a) + 3*e^(5*b*x + 5*a) + 3*e^(3*b*x + 3*a) + 5*e^(b
*x + a))/(e^(2*b*x + 2*a) - 1)^4 + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*
x + a) - 1)))/b
```

3.124 $\int \coth^2(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}$$

```
[Out] Coth[x]^3/3 - Coth[x]^5/5
```

Rubi [A] time = 0.0263057, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Csch[x]^4,x]
```

```
[Out] Coth[x]^3/3 - Coth[x]^5/5
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\mathrm{csch}^4(x)\,dx &= i\,\mathrm{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\coth(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\coth(x)\right)\\ &= \frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0225496, size = 27, normalized size = 1.59

$$\frac{2\coth(x)}{15}-\frac{1}{5}\coth(x)\mathrm{csch}^4(x)-\frac{1}{15}\coth(x)\mathrm{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Csch[x]^4,x]
```

```
[Out] (2*Coth[x])/15 - (Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5
```

Maple [B] time = 0.012, size = 28, normalized size = 1.7

$$-\frac{\cosh(x)}{4\,(\sinh(x))^5}-\frac{\coth(x)}{4}\left(-\frac{8}{15}-\frac{(\mathrm{csch}(x))^4}{5}+\frac{4\,(\mathrm{csch}(x))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*csch(x)^4,x)
```

```
[Out] -1/4/sinh(x)^5*cosh(x)-1/4*(-8/15-1/5*csch(x)^4+4/15*csch(x)^2)*coth(x)
```

Maxima [B] time = 1.01819, size = 201, normalized size = 11.82

$$\frac{4\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}+\frac{4\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}+\frac{}{5\,e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*
x) - 1) + 4/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 4*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
5*e^(-8*x) + e^(-10*x) - 1)
```

Fricas [B] time = 1.83057, size = 555, normalized size = 32.65

$$-\frac{8}{15\left(\cosh\left(x\right)^{7}+7\cosh\left(x\right)\sinh\left(x\right)^{6}+\sinh\left(x\right)^{7}+\left(21\cosh\left(x\right)^{2}-5\right)\sinh\left(x\right)^{5}-5\cosh\left(x\right)^{5}+5\left(7\cosh\left(x\right)^{3}-5\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="fricas")
```

```
[Out] -8/15*(7*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 21*cosh(x)*sinh(x)^2 + 8*sinh(x
)^3 + 5*cosh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)
^2 - 5)*sinh(x)^5 - 5*cosh(x)^5 + 5*(7*cosh(x)^3 - 5*cosh(x))*sinh(x)^4 + (
35*cosh(x)^4 - 50*cosh(x)^2 + 11)*sinh(x)^3 + 9*cosh(x)^3 + (21*cosh(x)^5 -
 50*cosh(x)^3 + 27*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 25*cosh(x)^4 + 33*co
sh(x)^2 - 15)*sinh(x) - 5*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\coth^{2}\left(x\right)\operatorname{csch}^{4}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**2*csch(x)**4, x)
```

Giac [B] time = 1.18252, size = 41, normalized size = 2.41

$$-\frac{4\left(15e^{(6x)}+5e^{(4x)}+5e^{(2x)}-1\right)}{15\left(e^{(2x)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="giac")
```

```
[Out] -4/15*(15*e^(6*x) + 5*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

3.125 $\int \coth^3(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

```
[Out] -Csch[x]^4/4 - Csch[x]^6/6
```

Rubi [A] time = 0.0280708, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Csch[x]^4,x]

[Out] -Csch[x]^4/4 - Csch[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \coth^3(x)\text{csch}^4(x)\,dx = \text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx, x, -i\text{csch}(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^3+x^5\right)\,dx, x, -i\text{csch}(x)\right)$$
$$= -\frac{1}{4}\text{csch}^4(x) - \frac{\text{csch}^6(x)}{6}$$

Mathematica [A] time = 0.0090641, size = 17, normalized size = 1.

$$-\frac{1}{6}\text{csch}^6(x) - \frac{\text{csch}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Csch[x]^4,x]
```

```
[Out] -Csch[x]^4/4 - Csch[x]^6/6
```

Maple [B] time = 0.013, size = 32, normalized size = 1.9

$$-\frac{(\cosh(x))^2}{6\,(\sinh(x))^6} - \frac{(\cosh(x))^2}{12\,(\sinh(x))^4} + \frac{(\cosh(x))^2}{12\,(\sinh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*csch(x)^4,x)
```

```
[Out] -1/6/sinh(x)^6*cosh(x)^2-1/12/sinh(x)^4*cosh(x)^2+1/12*cosh(x)^2/sinh(x)^2
```

Maxima [B] time = 1.02277, size = 188, normalized size = 11.06

$$\frac{4\,e^{(-4\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1} + \frac{8\,e^{(-6\,x)}}{3\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-4*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 8/3*e^(-6*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x)
 - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1) + 4*e^(-8*x)/(6*e^(-2*x) - 15
*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1)
```

Fricas [B] time = 1.70895, size = 737, normalized size = 43.35

$$-\frac{}{3\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 2\left(14\cosh(x)^2 - 3\right)\sinh(x)^6 - 6\cosh(x)^6 + 4\left(14\cosh(x)^3 - 9\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(3*cosh(x)^3 + cosh(x))*sinh(x) + 3)/(cosh(x)
^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(14*cosh(x)^2 - 3)*sinh(x)^6 - 6*c
osh(x)^6 + 4*(14*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 45*co
sh(x)^2 + 8)*sinh(x)^4 + 16*cosh(x)^4 + 8*(7*cosh(x)^5 - 15*cosh(x)^3 + 7*c
osh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 45*cosh(x)^4 + 48*cosh(x)^2 - 13)*sin
h(x)^2 - 26*cosh(x)^2 + 4*(2*cosh(x)^7 - 9*cosh(x)^5 + 14*cosh(x)^3 - 7*cos
h(x))*sinh(x) + 15)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*csch(x)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18309, size = 39, normalized size = 2.29

$$-\frac{4\left(3e^{(8x)}+2e^{(6x)}+3e^{(4x)}\right)}{3\left(e^{(2x)}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(8*x) + 2*e^(6*x) + 3*e^(4*x))/(e^(2*x) - 1)^6
```

3.126 $\int \coth^n(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=26

$$\frac{\coth^{n+1}(x)}{n+1}-\frac{\coth^{n+3}(x)}{n+3}$$

```
[Out] Coth[x]^(1 + n)/(1 + n) - Coth[x]^(3 + n)/(3 + n)
```

Rubi [A] time = 0.0351811, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\coth^{n+1}(x)}{n+1}-\frac{\coth^{n+3}(x)}{n+3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^n*Csch[x]^4,x]
```

```
[Out] Coth[x]^(1 + n)/(1 + n) - Coth[x]^(3 + n)/(3 + n)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \coth^n(x)\text{csch}^4(x)\,dx = -\left(i\,\text{Subst}\left(\int(-ix)^n\left(1+x^2\right)\,dx, x, i\coth(x)\right)\right)$$
$$= -\left(i\,\text{Subst}\left(\int\left((-ix)^n-(-ix)^{2+n}\right)\,dx, x, i\coth(x)\right)\right)$$
$$= \frac{\coth^{1+n}(x)}{1+n} - \frac{\coth^{3+n}(x)}{3+n}$$

Mathematica [A] time = 0.0860355, size = 30, normalized size = 1.15

$$\frac{\text{csch}^2(x)(-n+\cosh(2x)-2)\coth^{n+1}(x)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^n*Csch[x]^4,x]
```

```
[Out] ((-2 - n + Cosh[2*x])*Coth[x]^(1 + n)*Csch[x]^2)/((1 + n)*(3 + n))
```

Maple [C] time = 0.208, size = 371, normalized size = 14.3

$$-2\,\frac{-e^{6x}+2\,ne^{4x}+3\,e^{4x}+2\,ne^{2x}+3\,e^{2x}-1}{(n+1)\,(n+3)\left(e^{2x}-1\right)^3}e^{-1/2\,n\left(i\pi\left(\text{csgn}\left(\frac{i\left(e^{2x}+1\right)}{e^x+1}\right)\right)^3-i\pi\left(\text{csgn}\left(\frac{i\left(e^{2x}+1\right)}{e^x+1}\right)\right)^2\text{csgn}\left(\frac{i}{e^x+1}\right)-i\pi\left(\text{csgn}\left(\frac{i\left(e^{2x}+1\right)}{e^x+1}\right)\right)^2\text{csgn}\left(i\left(e^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^n*csch(x)^4,x)
```

```
[Out] -2*(-exp(6*x)+2*n*exp(4*x)+3*exp(4*x)+2*n*exp(2*x)+3*exp(2*x)-1)/(n+1)/(n+3
)/(exp(2*x)-1)^3*exp(-1/2*n*(I*Pi*csgn(I/(exp(x)+1)*(exp(2*x)+1))^3-I*Pi*cs
gn(I/(exp(x)+1)*(exp(2*x)+1))^2*csgn(I/(exp(x)+1))-I*Pi*csgn(I/(exp(x)+1)*(
exp(2*x)+1))^2*csgn(I*(exp(2*x)+1))+I*Pi*csgn(I/(exp(x)+1)*(exp(2*x)+1))*cs
gn(I/(exp(x)+1))*csgn(I*(exp(2*x)+1))-I*Pi*csgn(I/(exp(x)+1)*(exp(2*x)+1))*
csgn(I/(exp(x)-1)*(exp(2*x)+1)/(exp(x)+1))^2+I*Pi*csgn(I/(exp(x)+1)*(exp(2*
x)+1))*csgn(I/(exp(x)-1)*(exp(2*x)+1)/(exp(x)+1))*csgn(I/(exp(x)-1))+I*Pi*c
sgn(I/(exp(x)-1)*(exp(2*x)+1)/(exp(x)+1))^3-I*Pi*csgn(I/(exp(x)-1)*(exp(2*x
)+1)/(exp(x)+1))^2*csgn(I/(exp(x)-1))+2*ln(exp(x)-1)+2*ln(exp(x)+1)-2*ln(ex
p(2*x)+1)))
```

Maxima [B] time = 1.66412, size = 497, normalized size = 19.12

$$-\frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-2\,x\right)}}{n^2-3\left(n^2+4\,n+3\right)e^{(-2\,x)}+3\left(n^2+4\,n+3\right)e^{(-4\,x)}-\left(n^2+4\,n+3\right)e^{(-6\,x)}+4\,n+3}-\frac{2\,(2\,n+3)e^{\left(-n\log\right.}}{n^2-3\left(n^2+4\,n+3\right)e^{(-2\,x)}+3\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="maxima")
```

```
[Out] -2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) +
1) - 2*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) -
(n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) - 2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n
*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 4*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(
-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) +
2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 6*x)/(
n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n
+ 3)*e^(-6*x) + 4*n + 3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n
*log(e^(-2*x) + 1))/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e
^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3)
```

Fricas [B] time = 2.00791, size = 347, normalized size = 13.35

$$\frac{2\left(\left(\cosh\left(x\right)^3+3\,\cosh\left(x\right)\sinh\left(x\right)^2-(2\,n+3)\cosh\left(x\right)\right)\cosh\left(n\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)+\left(\cosh\left(x\right)^3+3\,\cosh\left(x\right)\sinh\left(x\right)^2-(2\right.\right.}{\left(n^2+4\,n+3\right)\sinh\left(x\right)^3+3\left(\left(n^2+4\,n+3\right)\cosh\left(x\right)^2-n^2-4\,n-3\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*cosh(n*log(cosh(x)
/sinh(x))) + (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*sinh(n*l
og(cosh(x)/sinh(x))))/((n^2 + 4*n + 3)*sinh(x)^3 + 3*((n^2 + 4*n + 3)*cosh(
x)^2 - n^2 - 4*n - 3)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**n*csch(x)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(x)^n \operatorname{csch}(x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="giac")
```

```
[Out] integrate(coth(x)^n*csch(x)^4, x)
```

3.127 $\int \coth^4(x)\mathbf{csch}^3(x)\,dx$

Optimal. Leaf size=38

$$\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth^3(x)\mathbf{csch}^3(x)-\frac{1}{8}\coth(x)\mathbf{csch}^3(x)-\frac{1}{16}\coth(x)\mathbf{csch}(x)$$

```
[Out] ArcTanh[Cosh[x]]/16 - (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/8 - (Coth[
x]^3*Csch[x]^3)/6
```

Rubi [A] time = 0.0645452, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth^3(x)\mathbf{csch}^3(x)-\frac{1}{8}\coth(x)\mathbf{csch}^3(x)-\frac{1}{16}\coth(x)\mathbf{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/16 - (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/8 - (Coth[
x]^3*Csch[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^4(x)\operatorname{csch}^3(x)\,dx &= -\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)+\frac{1}{2}\int \coth^2(x)\operatorname{csch}^3(x)\,dx\\ &= -\frac{1}{8}\coth(x)\operatorname{csch}^3(x)-\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)+\frac{1}{8}\int \operatorname{csch}^3(x)\,dx\\ &= -\frac{1}{16}\coth(x)\operatorname{csch}(x)-\frac{1}{8}\coth(x)\operatorname{csch}^3(x)-\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)-\frac{1}{16}\int \operatorname{csch}(x)\,dx\\ &= \frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{16}\coth(x)\operatorname{csch}(x)-\frac{1}{8}\coth(x)\operatorname{csch}^3(x)-\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)\end{aligned}$$

Mathematica [B] time = 0.0197441, size = 84, normalized size = 2.21

$$-\frac{1}{384}\operatorname{csch}^6\left(\frac{x}{2}\right)-\frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right)-\frac{1}{64}\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{1}{384}\operatorname{sech}^6\left(\frac{x}{2}\right)+\frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right)-\frac{1}{64}\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{16}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Csch[x]^3,x]
```

```
[Out] -Csch[x/2]^2/64 - Csch[x/2]^4/64 - Csch[x/2]^6/384 - Log[Tanh[x/2]]/16 - Se
ch[x/2]^2/64 + Sech[x/2]^4/64 - Sech[x/2]^6/384
```

Maple [A] time = 0.016, size = 46, normalized size = 1.2

$$-\frac{(\cosh(x))^3}{3\,(\sinh(x))^6}+\frac{\cosh(x)}{5\,(\sinh(x))^6}+\frac{\coth(x)}{5}\left(-\frac{(\operatorname{csch}(x))^5}{6}+\frac{5\,(\operatorname{csch}(x))^3}{24}-\frac{5\operatorname{csch}(x)}{16}\right)+\frac{\operatorname{Artanh}(e^x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*csch(x)^3,x)
```

```
[Out] -1/3/sinh(x)^6*cosh(x)^3+1/5/sinh(x)^6*cosh(x)+1/5*(-1/6*csch(x)^5+5/24*csc
h(x)^3-5/16*csch(x))*coth(x)+1/8*arctanh(exp(x))
```

Maxima [B] time = 1.04158, size = 132, normalized size = 3.47

$$\frac{3\,e^{(-x)} + 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} + 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} + 3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)} + \frac{1}{16}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{16}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 47*e^(-3*x) + 78*e^(-5*x) + 78*e^(-7*x) + 47*e^(-9*x) + 3*
e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 1/16*log(e^(-x) + 1) - 1/16*log(e^(-x) - 1)
```

Fricas [B] time = 1.873, size = 4327, normalized size = 113.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cosh(x)^11 + 66*cosh(x)*sinh(x)^10 + 6*sinh(x)^11 + 2*(165*cosh(x)
^2 + 47)*sinh(x)^9 + 94*cosh(x)^9 + 18*(55*cosh(x)^3 + 47*cosh(x))*sinh(x)^
8 + 12*(165*cosh(x)^4 + 282*cosh(x)^2 + 13)*sinh(x)^7 + 156*cosh(x)^7 + 84*
(33*cosh(x)^5 + 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 12*(231*cosh(x)^6 +
987*cosh(x)^4 + 273*cosh(x)^2 + 13)*sinh(x)^5 + 156*cosh(x)^5 + 12*(165*cos
h(x)^7 + 987*cosh(x)^5 + 455*cosh(x)^3 + 65*cosh(x))*sinh(x)^4 + 2*(495*cos
h(x)^8 + 3948*cosh(x)^6 + 2730*cosh(x)^4 + 780*cosh(x)^2 + 47)*sinh(x)^3 +
94*cosh(x)^3 + 6*(55*cosh(x)^9 + 564*cosh(x)^7 + 546*cosh(x)^5 + 260*cosh(x
)^3 + 47*cosh(x))*sinh(x)^2 - 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(
x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 -
 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15
*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(23
1*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 +
 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15
*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4
+ 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6
 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^1
1 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11
```

```
 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cos
h(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x
)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7
 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cos
h(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x
)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*si
nh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 2
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*c
osh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(c
osh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(
x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(11*cosh(x)^10 + 141*cosh(x
)^8 + 182*cosh(x)^6 + 130*cosh(x)^4 + 47*cosh(x)^2 + 1)*sinh(x) + 6*cosh(x)
)/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*s
inh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33
*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5
- 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 +
105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)
^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6
+ 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x
)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6
*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7
 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*csch(x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15026, size = 96, normalized size = 2.53

$$-\frac{3\left(e^{(-x)}+e^{x}\right)^{5}+32\left(e^{(-x)}+e^{x}\right)^{3}-48\,e^{(-x)}-48\,e^{x}}{24\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{3}}+\frac{1}{32}\,\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{32}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(e^(-x) + e^x)^5 + 32*(e^(-x) + e^x)^3 - 48*e^(-x) - 48*e^x)/((e^(
-x) + e^x)^2 - 4)^3 + 1/32*log(e^(-x) + e^x + 2) - 1/32*log(e^(-x) + e^x -
2)
```

3.128 $\int \coth^4(x)\mathbf{csch}^6(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{9}\coth^9(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^5(x)}{5}$$

```
[Out] -Coth[x]^5/5 + (2*Coth[x]^7)/7 - Coth[x]^9/9
```

Rubi [A] time = 0.0293946, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 270}

$$-\frac{1}{9}\coth^9(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Csch[x]^6,x]
```

```
[Out] -Coth[x]^5/5 + (2*Coth[x]^7)/7 - Coth[x]^9/9
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \coth^4(x)\mathrm{csch}^6(x)\,dx &= i\,\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,i\coth(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(x^4+2x^6+x^8\right)\,dx,x,i\coth(x)\right)\\ &= -\frac{1}{5}\coth^5(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^9(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.027158, size = 47, normalized size = 1.88

$$-\frac{8\coth(x)}{315}-\frac{1}{9}\coth(x)\mathrm{csch}^8(x)-\frac{10}{63}\coth(x)\mathrm{csch}^6(x)-\frac{1}{105}\coth(x)\mathrm{csch}^4(x)+\frac{4}{315}\coth(x)\mathrm{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Csch[x]^6,x]
```

```
[Out] (-8*Coth[x])/315 + (4*Coth[x]*Csch[x]^2)/315 - (Coth[x]*Csch[x]^4)/105 - (1
0*Coth[x]*Csch[x]^6)/63 - (Coth[x]*Csch[x]^8)/9
```

Maple [B] time = 0.014, size = 50, normalized size = 2.

$$-\frac{(\cosh(x))^3}{6(\sinh(x))^9}+\frac{\cosh(x)}{16(\sinh(x))^9}+\frac{\coth(x)}{16}\left(-\frac{128}{315}-\frac{(\mathrm{csch}(x))^8}{9}+\frac{8(\mathrm{csch}(x))^6}{63}-\frac{16(\mathrm{csch}(x))^4}{105}+\frac{64(\mathrm{csch}(x))^2}{315}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*csch(x)^6,x)
```

```
[Out] -1/6/sinh(x)^9*cosh(x)^3+1/16/sinh(x)^9*cosh(x)+1/16*(-128/315-1/9*csch(x)^
8+8/63*csch(x)^6-16/105*csch(x)^4+64/315*csch(x)^2)*coth(x)
```

Maxima [B] time = 1.25276, size = 582, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="maxima")
```

```
[Out] -16/35*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 12
6*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) +
64/35*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126
*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 3
2/5*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 112
/5*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^
(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16*e
^(-10*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-1
0*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 32/3*e^
(-12*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10
*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/315/(
9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*
e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Fricas [B] time = 1.90838, size = 1516, normalized size = 60.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="fricas")
```

```
[Out] -16/315*(211*cosh(x)^6 + 1254*cosh(x)*sinh(x)^5 + 211*sinh(x)^6 + 3*(1055*c
osh(x)^2 + 102)*sinh(x)^4 + 306*cosh(x)^4 + 4*(1045*cosh(x)^3 + 324*cosh(x)
)*sinh(x)^3 + 3*(1055*cosh(x)^4 + 612*cosh(x)^2 + 159)*sinh(x)^2 + 477*cosh
(x)^2 + 6*(209*cosh(x)^5 + 216*cosh(x)^3 + 135*cosh(x))*sinh(x) + 126)/(cos
h(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(22*cosh(x)^2 - 3)*sinh(x)
^10 - 9*cosh(x)^10 + 10*(22*cosh(x)^3 - 9*cosh(x))*sinh(x)^9 + 9*(55*cosh(x
)^4 - 45*cosh(x)^2 + 4)*sinh(x)^8 + 36*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + 4*cosh(x))*sinh(x)^7 + (924*cosh(x)^6 - 1890*cosh(x)^4 + 1008*cos
h(x)^2 - 85)*sinh(x)^6 - 85*cosh(x)^6 + 6*(132*cosh(x)^7 - 378*cosh(x)^5 +
336*cosh(x)^3 - 83*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 126*cosh(x)^6 +
168*cosh(x)^4 - 85*cosh(x)^2 + 9)*sinh(x)^4 + 135*cosh(x)^4 + 4*(55*cosh(x)
^9 - 270*cosh(x)^7 + 504*cosh(x)^5 - 415*cosh(x)^3 + 117*cosh(x))*sinh(x)^3
 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 + 336*cosh(x)^6 - 425*cosh(x)^4 + 270*c
osh(x)^2 - 54)*sinh(x)^2 - 162*cosh(x)^2 + 6*(2*cosh(x)^11 - 15*cosh(x)^9 +
 48*cosh(x)^7 - 83*cosh(x)^5 + 78*cosh(x)^3 - 30*cosh(x))*sinh(x) + 84)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*csch(x)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17559, size = 65, normalized size = 2.6

$$-\frac{16\left(210\,e^{(12\,x)}+315\,e^{(10\,x)}+441\,e^{(8\,x)}+126\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="giac")
```

```
[Out] -16/315*(210*e^(12*x) + 315*e^(10*x) + 441*e^(8*x) + 126*e^(6*x) + 36*e^(4*
x) - 9*e^(2*x) + 1)/(e^(2*x) - 1)^9
```

3.129 $\int \coth^5(6x)\mathbf{csch}(6x)\,dx$

Optimal. Leaf size=29

$$-\frac{1}{30}\mathrm{csch}^5(6x)-\frac{1}{9}\mathrm{csch}^3(6x)-\frac{1}{6}\mathrm{csch}(6x)$$

```
[Out] -Csch[6*x]/6 - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Rubi [A] time = 0.0191935, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2606, 194}

$$-\frac{1}{30}\mathrm{csch}^5(6x)-\frac{1}{9}\mathrm{csch}^3(6x)-\frac{1}{6}\mathrm{csch}(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[6*x]^5*Csch[6*x],x]
```

```
[Out] -Csch[6*x]/6 - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\left(\frac{1}{6}i\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, -i\text{csch}(6x)\right)\right)$$
$$= -\left(\frac{1}{6}i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -i\text{csch}(6x)\right)\right)$$
$$= -\frac{1}{6}\text{csch}(6x) - \frac{1}{9}\text{csch}^3(6x) - \frac{1}{30}\text{csch}^5(6x)$$

Mathematica [A] time = 0.0171361, size = 29, normalized size = 1.

$$-\frac{1}{30}\text{csch}^5(6x) - \frac{1}{9}\text{csch}^3(6x) - \frac{1}{6}\text{csch}(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[6*x]^5*Csch[6*x],x]
```

```
[Out] -Csch[6*x]/6 - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Maple [B] time = 0.017, size = 64, normalized size = 2.2

$$-\frac{(\cosh(6x))^4}{6(\sinh(6x))^5} + \frac{2(\cosh(6x))^2}{15(\sinh(6x))^5} + \frac{4(\cosh(6x))^2}{45(\sinh(6x))^3} - \frac{4(\cosh(6x))^2}{45\sinh(6x)} + \frac{4\sinh(6x)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)^5*csch(6*x),x)
```

```
[Out] -1/6/sinh(6*x)^5*cosh(6*x)^4+2/15/sinh(6*x)^5*cosh(6*x)^2+4/45/sinh(6*x)^3*
cosh(6*x)^2-4/45/sinh(6*x)*cosh(6*x)^2+4/45*sinh(6*x)
```

Maxima [B] time = 1.09573, size = 258, normalized size = 8.9

$$\frac{e^{(-6x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-18x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="maxima")
```

```
[Out] 1/3*e^(-6*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(
-60*x) - 1) - 4/9*e^(-18*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*
e^(-48*x) + e^(-60*x) - 1) + 58/45*e^(-30*x)/(5*e^(-12*x) - 10*e^(-24*x) +
10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) - 4/9*e^(-42*x)/(5*e^(-12*x) -
10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) + 1/3*e^(-54*x)/
(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1)
```

Fricas [B] time = 1.68757, size = 705, normalized size = 24.31

$$-\frac{15\cosh(6x)^5 + 75\cosh(6x)\sinh(6x)^4 + 15\sinh(6x)^5 + 5\left(30\cosh(6x)^2 - 7\right)\sinh(6x)^3 \cdots}{45\left(\cosh(6x)^6 + 6\cosh(6x)\sinh(6x)^5 + \sinh(6x)^6 + 3\left(5\cosh(6x)^2 - 2\right)\sinh(6x)^4 - 6\cosh(6x)^4 + 4\left(5\cosh(6x)^3 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="fricas")
```

```
[Out] -1/45*(15*cosh(6*x)^5 + 75*cosh(6*x)*sinh(6*x)^4 + 15*sinh(6*x)^5 + 5*(30*c
osh(6*x)^2 - 7)*sinh(6*x)^3 - 5*cosh(6*x)^3 + 15*(10*cosh(6*x)^3 - cosh(6*x
))*sinh(6*x)^2 + 3*(25*cosh(6*x)^4 - 35*cosh(6*x)^2 + 26)*sinh(6*x) + 38*co
sh(6*x))/(cosh(6*x)^6 + 6*cosh(6*x)*sinh(6*x)^5 + sinh(6*x)^6 + 3*(5*cosh(6
*x)^2 - 2)*sinh(6*x)^4 - 6*cosh(6*x)^4 + 4*(5*cosh(6*x)^3 - 4*cosh(6*x))*si
nh(6*x)^3 + 3*(5*cosh(6*x)^4 - 12*cosh(6*x)^2 + 5)*sinh(6*x)^2 + 15*cosh(6*
x)^2 + 2*(3*cosh(6*x)^5 - 8*cosh(6*x)^3 + 5*cosh(6*x))*sinh(6*x) - 10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)**5*csch(6*x),x)
```

```
[Out] Integral(coth(6*x)**5*csch(6*x), x)
```

Giac [B] time = 1.14253, size = 63, normalized size = 2.17

$$-\frac{15\left(e^{(6x)} - e^{(-6x)}\right)^4 + 40\left(e^{(6x)} - e^{(-6x)}\right)^2 + 48}{45\left(e^{(6x)} - e^{(-6x)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="giac")
```

```
[Out] -1/45*(15*(e^(6*x) - e^(-6*x))^4 + 40*(e^(6*x) - e^(-6*x))^2 + 48)/(e^(6*x)
 - e^(-6*x))^5
```

3.130 $\int \coth^7(x)\mathbf{csch}^3(x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{9}\text{csch}^9(x)-\frac{3\text{csch}^7(x)}{7}-\frac{3\text{csch}^5(x)}{5}-\frac{\text{csch}^3(x)}{3}$$

```
[Out] -Csch[x]^3/3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Rubi [A] time = 0.0336737, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$-\frac{1}{9}\text{csch}^9(x)-\frac{3\text{csch}^7(x)}{7}-\frac{3\text{csch}^5(x)}{5}-\frac{\text{csch}^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^7*Csch[x]^3,x]
```

```
[Out] -Csch[x]^3/3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = -\left(i\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx, x, -i\operatorname{csch}(x)\right)\right)$$
$$= -\left(i\,\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+x^8\right)dx, x, -i\operatorname{csch}(x)\right)\right)$$
$$= -\frac{1}{3}\operatorname{csch}^3(x) - \frac{3\operatorname{csch}^5(x)}{5} - \frac{3\operatorname{csch}^7(x)}{7} - \frac{\operatorname{csch}^9(x)}{9}$$

Mathematica [A] time = 0.0117182, size = 33, normalized size = 1.

$$-\frac{1}{9}\operatorname{csch}^9(x) - \frac{3\operatorname{csch}^7(x)}{7} - \frac{3\operatorname{csch}^5(x)}{5} - \frac{\operatorname{csch}^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^7*Csch[x]^3,x]
```

```
[Out] -Csch[x]^3/3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Maple [B] time = 0.015, size = 76, normalized size = 2.3

$$-\frac{(\cosh(x))^6}{3(\sinh(x))^9} + \frac{2(\cosh(x))^4}{5(\sinh(x))^9} - \frac{8(\cosh(x))^2}{45(\sinh(x))^9} - \frac{16(\cosh(x))^2}{315(\sinh(x))^7} + \frac{16(\cosh(x))^2}{315(\sinh(x))^5} - \frac{16(\cosh(x))^2}{315(\sinh(x))^3} + \frac{16(\cosh(x))}{315\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7*csch(x)^3,x)
```

```
[Out] -1/3/sinh(x)^9*cosh(x)^6+2/5/sinh(x)^9*cosh(x)^4-8/45/sinh(x)^9*cosh(x)^2-1
6/315/sinh(x)^7*cosh(x)^2+16/315/sinh(x)^5*cosh(x)^2-16/315/sinh(x)^3*cosh(
x)^2+16/315/sinh(x)*cosh(x)^2-16/315*sinh(x)
```

Maxima [B] time = 1.06554, size = 587, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/
5*e^(-5*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632/3
5*e^(-7*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 2848/
315*e^(-9*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632
/35*e^(-11*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16
/5*e^(-13*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 8/3
*e^(-15*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Fricas [B] time = 1.77903, size = 1571, normalized size = 47.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(105*cosh(x)^8 + 840*cosh(x)*sinh(x)^7 + 105*sinh(x)^8 + 42*(70*cosh
(x)^2 + 3)*sinh(x)^6 + 126*cosh(x)^6 + 84*(70*cosh(x)^3 + 9*cosh(x))*sinh(x
)^5 + 6*(1225*cosh(x)^4 + 315*cosh(x)^2 + 136)*sinh(x)^4 + 816*cosh(x)^4 +
24*(245*cosh(x)^5 + 105*cosh(x)^3 + 101*cosh(x))*sinh(x)^3 + 2*(1470*cosh(x
)^6 + 945*cosh(x)^4 + 2448*cosh(x)^2 + 241)*sinh(x)^2 + 482*cosh(x)^2 + 4*(
210*cosh(x)^7 + 189*cosh(x)^5 + 606*cosh(x)^3 + 115*cosh(x))*sinh(x) + 711)
/(cosh(x)^11 + 11*cosh(x)*sinh(x)^10 + sinh(x)^11 + (55*cosh(x)^2 - 9)*sinh
(x)^9 - 9*cosh(x)^9 + 3*(55*cosh(x)^3 - 27*cosh(x))*sinh(x)^8 + (330*cosh(x
)^4 - 324*cosh(x)^2 + 37)*sinh(x)^7 + 35*cosh(x)^7 + 7*(66*cosh(x)^5 - 108*
cosh(x)^3 + 35*cosh(x))*sinh(x)^6 + 3*(154*cosh(x)^6 - 378*cosh(x)^4 + 259*
cosh(x)^2 - 31)*sinh(x)^5 - 75*cosh(x)^5 + (330*cosh(x)^7 - 1134*cosh(x)^5
+ 1225*cosh(x)^3 - 375*cosh(x))*sinh(x)^4 + (165*cosh(x)^8 - 756*cosh(x)^6
+ 1295*cosh(x)^4 - 930*cosh(x)^2 + 162)*sinh(x)^3 + 90*cosh(x)^3 + (55*cosh
(x)^9 - 324*cosh(x)^7 + 735*cosh(x)^5 - 750*cosh(x)^3 + 270*cosh(x))*sinh(x
)^2 + (11*cosh(x)^10 - 81*cosh(x)^8 + 259*cosh(x)^6 - 465*cosh(x)^4 + 486*c
osh(x)^2 - 210)*sinh(x) - 42*cosh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**7*csch(x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17492, size = 73, normalized size = 2.21

$$\frac{8\left(105\left(e^{(-x)}-e^{x}\right)^{6}+756\left(e^{(-x)}-e^{x}\right)^{4}+2160\left(e^{(-x)}-e^{x}\right)^{2}+2240\right)}{315\left(e^{(-x)}-e^{x}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="giac")
```

```
[Out] 8/315*(105*(e^(-x) - e^x)^6 + 756*(e^(-x) - e^x)^4 + 2160*(e^(-x) - e^x)^2
+ 2240)/(e^(-x) - e^x)^9
```

3.131 $\int \sinh(a+bx)\sinh(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

```
[Out] -(x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0266107, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5613, 2637}

$$\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + b*x],x]
```

```
[Out] -(x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh(a+bx)\sinh(c+bx)\,dx &= \int \left(-\frac{1}{2}\cosh(a-c)+\frac{1}{2}\cosh(a+c+2bx)\right)dx \\
&= -\frac{1}{2}x\cosh(a-c)+\frac{1}{2}\int \cosh(a+c+2bx)\,dx \\
&= -\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.033118, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx+c)-2bx\cosh(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + b*x],x]
```

```
[Out] (-2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

Maple [A] time = 0.014, size = 24, normalized size = 0.9

$$-\frac{x\cosh(a-c)}{2}+\frac{\sinh(2\,bx+a+c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(b*x+c),x)
```

```
[Out] -1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Maxima [B] time = 1.1784, size = 78, normalized size = 2.89

$$-\frac{(bx+a)\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}}{4\,b}+\frac{e^{(2\,bx+a+c)}}{8\,b}-\frac{e^{(-2\,bx-a-c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
 1/8*e^(-2*b*x - a - c)/b
```

Fricas [B] time = 1.9213, size = 232, normalized size = 8.59

$$-\frac{2\,bx\cosh\left(-a+c\right)-2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)+\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)+\sinh\left(bx+c\right)^{2}\sinh\left(-a+c\right)}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*cosh(-a + c) - 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) + cos
h(b*x + c)^2*sinh(-a + c) + sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
 - b*sinh(-a + c)^2)
```

Sympy [A] time = 1.09259, size = 58, normalized size = 2.15

$$\begin{cases}\frac{x\sinh\left(a+bx\right)\sinh\left(bx+c\right)}{2}-\frac{x\cosh\left(a+bx\right)\cosh\left(bx+c\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(bx+c\right)}{2b} & \text{for}\ b\neq 0\\ x\sinh\left(a\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x)
```

```
[Out] Piecewise((x*sinh(a + b*x)*sinh(b*x + c)/2 - x*cosh(a + b*x)*cosh(b*x + c)/
2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*sinh(a)*sinh(c), True)
)
```

Giac [B] time = 1.18586, size = 96, normalized size = 3.56

$$-\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}-1\right)e^{(-2\,bx-a-c)}-e^{(2\,bx+a+c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x +
2*c) - 1)*e^(-2*b*x - a - c) - e^(2*b*x + a + c))/b
```

3.132 $\int \sinh(c - bx) \sinh(a + bx) \, dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cosh(a+c) - \frac{\sinh(a+2bx-c)}{4b}$$

```
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0267764, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5613, 2637}

$$\frac{1}{2}x\cosh(a+c) - \frac{\sinh(a+2bx-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c - b*x]*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \int \left(\frac{1}{2}\cosh(a+c) - \frac{1}{2}\cosh(a-c+2bx)\right)dx$$
$$= \frac{1}{2}x\cosh(a+c) - \frac{1}{2}\int \cosh(a-c+2bx)\,dx$$
$$= \frac{1}{2}x\cosh(a+c) - \frac{\sinh(a-c+2bx)}{4b}$$

Mathematica [A] time = 0.0422186, size = 27, normalized size = 1.

$$\frac{1}{2}x\cosh(a+c) - \frac{\sinh(a+2bx-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c - b*x]*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Maple [A] time = 0.01, size = 24, normalized size = 0.9

$$\frac{x\cosh(a+c)}{2} - \frac{\sinh(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(b*x-c)*sinh(b*x+a),x)
```

```
[Out] 1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

Maxima [B] time = 1.04771, size = 80, normalized size = 2.96

$$\frac{(bx+a)\left(e^{(2a+2c)}+1\right)e^{(-a-c)}}{4b} - \frac{e^{(2bx+a-c)}}{8b} + \frac{e^{(-2bx-a+c)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b - 1/8*e^(2*b*x + a - c)/b +
1/8*e^(-2*b*x - a + c)/b
```

Fricas [B] time = 1.90528, size = 223, normalized size = 8.26

$$\frac{2\,bx\cosh\left(a+c\right)-2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)+\sinh\left(bx+a\right)^{2}\sinh\left(a+c\right)}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) - 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) + cosh(b
*x + a)^2*sinh(a + c) + sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

Sympy [A] time = 1.07458, size = 61, normalized size = 2.26

$$-\begin{cases}\frac{x\sinh\left(a+bx\right)\sinh\left(bx-c\right)}{2}-\frac{x\cosh\left(a+bx\right)\cosh\left(bx-c\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(bx-c\right)}{2b} & \text{for}\ b\neq 0\\ -x\sinh\left(a\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x)
```

```
[Out] -Piecewise((x*sinh(a + b*x)*sinh(b*x - c)/2 - x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (-x*sinh(a)*sinh(c), Tru
e))
```

Giac [B] time = 1.17207, size = 105, normalized size = 3.89

$$\frac{2\,bx\left(e^{\left(2\,a+2\,c\right)}+1\right)e^{\left(-a-c\right)}-\left(e^{\left(2\,bx\right)}+e^{\left(2\,bx+2\,a+2\,c\right)}-e^{\left(2\,c\right)}\right)e^{\left(-2\,bx-a-c\right)}-e^{\left(2\,bx+a-c\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) - e^(2*c))*e^(-2*b*x - a - c) - e^(2*b*x + a - c))/b
```

3.133 $\int \cosh(a+bx)\cosh(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

```
[Out] (x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0188788, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5614, 2637}

$$\frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + b*x],x]
```

```
[Out] (x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \int \left(\frac{1}{2}\cosh(a-c) + \frac{1}{2}\cosh(a+c+2bx)\right)dx$$
$$= \frac{1}{2}x\cosh(a-c) + \frac{1}{2}\int \cosh(a+c+2bx)\,dx$$
$$= \frac{1}{2}x\cosh(a-c) + \frac{\sinh(a+c+2bx)}{4b}$$

Mathematica [A] time = 0.0226661, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx+c)+2bx\cosh(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + b*x],x]
```

```
[Out] (2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$\frac{x\cosh(a-c)}{2} + \frac{\sinh(2\,bx+a+c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(b*x+c),x)
```

```
[Out] 1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Maxima [B] time = 1.13596, size = 78, normalized size = 2.89

$$\frac{(bx+a)\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}}{4\,b} + \frac{e^{(2\,bx+a+c)}}{8\,b} - \frac{e^{(-2\,bx-a-c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
1/8*e^(-2*b*x - a - c)/b
```

Fricas [B] time = 1.82386, size = 231, normalized size = 8.56

$$\frac{2\,bx\cosh\left(-a+c\right)+2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)-\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)-\sinh\left(bx+c\right)^{2}\sinh\left(-a+c\right)}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(-a + c) + 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) - cosh
(b*x + c)^2*sinh(-a + c) - sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
- b*sinh(-a + c)^2)
```

Sympy [A] time = 1.09124, size = 58, normalized size = 2.15

$$\begin{cases} -\frac{x\sinh\left(a+bx\right)\sinh\left(bx+c\right)}{2}+\frac{x\cosh\left(a+bx\right)\cosh\left(bx+c\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(bx+c\right)}{2b} & \text{for}\: b\neq 0 \\ x\cosh\left(a\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x + c)/2 + x*cosh(a + b*x)*cosh(b*x + c)
/2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))
```

Giac [B] time = 1.13134, size = 93, normalized size = 3.44

$$\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}+1\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a+c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x + 2
*c) + 1)*e^(-2*b*x - a - c) + e^(2*b*x + a + c))/b
```

3.134 $\int \cosh(c - bx)\cosh(a + bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

```
[Out] (x*Cosh[a + c])/2 + Sinh[a - c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0188699, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5614, 2637}

$$\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c - b*x]*Cosh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 + Sinh[a - c + 2*b*x]/(4*b)
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(c-bx)\cosh(a+bx)\,dx &= \int \left(\frac{1}{2}\cosh(a+c)+\frac{1}{2}\cosh(a-c+2bx)\right)dx \\
&= \frac{1}{2}x\cosh(a+c)+\frac{1}{2}\int \cosh(a-c+2bx)\,dx \\
&= \frac{1}{2}x\cosh(a+c)+\frac{\sinh(a-c+2bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.0230023, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx-c)+2bx\cosh(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c - b*x]*Cosh[a + b*x],x]
```

```
[Out] (2*b*x*Cosh[a + c] + Sinh[a - c + 2*b*x])/(4*b)
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$\frac{x\cosh(a+c)}{2}+\frac{\sinh(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x-c)*cosh(b*x+a),x)
```

```
[Out] 1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

Maxima [B] time = 1.19569, size = 80, normalized size = 2.96

$$\frac{(bx+a)\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}}{4\,b}+\frac{e^{(2\,bx+a-c)}}{8\,b}-\frac{e^{(-2\,bx-a+c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b + 1/8*e^(2*b*x + a - c)/b -
1/8*e^(-2*b*x - a + c)/b
```

Fricas [B] time = 1.79256, size = 223, normalized size = 8.26

$$\frac{2\,bx\cosh\left(a+c\right)+2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)^2\sinh\left(a+c\right)-\sinh\left(bx+a\right)^2\sinh\left(a+c\right)}{4\left(b\cosh\left(a+c\right)^2-b\sinh\left(a+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) + 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) - cosh(b
*x + a)^2*sinh(a + c) - sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

Sympy [A] time = 1.07301, size = 58, normalized size = 2.15

$$\begin{cases}-\frac{x\sinh\left(a+bx\right)\sinh\left(bx-c\right)}{2}+\frac{x\cosh\left(a+bx\right)\cosh\left(bx-c\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(bx-c\right)}{2b} & \text{for}\ b\neq 0\\ x\cosh\left(a\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x - c)/2 + x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))
```

Giac [B] time = 1.18808, size = 100, normalized size = 3.7

$$\frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}-\left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}+e^{(2\,c)}\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a-c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) + e^(2*c))*e^(-2*b*x - a - c) + e^(2*b*x + a - c))/b
```

3.135 $\int \tanh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(bx+c))}{b}+x$$

```
[Out] x - (Coth[a - c]*Log[Cosh[a + b*x]])/b + (Coth[a - c]*Log[Cosh[c + b*x]])/b
```

Rubi [A] time = 0.0695035, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5646, 5644, 3475}

$$-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(bx+c))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] x - (Coth[a - c]*Log[Cosh[a + b*x]])/b + (Coth[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 5646

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] - Dist[(b*Cosh[(b*c - a*d)/d])/d, Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x-\cosh(a-c)\int \mathbf{sech}(a+bx)\mathbf{sech}(c+bx)\,dx$$
$$= x-\coth(a-c)\int \tanh(a+bx)\,dx+\coth(a-c)\int \tanh(c+bx)\,dx$$
$$= x-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(c+bx))}{b}$$

Mathematica [A] time = 0.472242, size = 29, normalized size = 0.78

$$\frac{\coth(a-c)(\log(\cosh(bx+c))-\log(\cosh(a+bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] x + (Coth[a - c]*(-Log[Cosh[a + b*x]] + Log[Cosh[c + b*x]]))/b
```

Maple [B] time = 0.044, size = 151, normalized size = 4.1

$$x-\frac{\ln\left(1+e^{2bx+2a}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}-\frac{\ln\left(1+e^{2bx+2a}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(1+exp(2*b*x+2*a))*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(1+exp(2*b*x+2*a))*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
)+exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)+exp(2*a-
2*c))*exp(2*c)
```

Maxima [B] time = 1.71078, size = 112, normalized size = 3.03

$$x+\frac{a}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx-2a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx)}+e^{(2c)}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*
c))) + (e^(2*a) + e^(2*c))*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c))
)
```

Fricas [B] time = 1.98148, size = 701, normalized size = 18.95

$$\frac{bx\cosh(-a+c)^2 - 2\,bx\cosh(-a+c)\sinh(-a+c) + bx\sinh(-a+c)^2 - bx - \left(\cosh(-a+c)^2 - 2\,\cosh(-a+c)\sinh(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh(a+bx)\tanh(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(tanh(a + b*x)*tanh(b*x + c), x)
```

Giac [B] time = 1.47571, size = 149, normalized size = 4.03

$$\frac{bx - \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(\frac{\left|-\left|e^{(2a)}-e^{(2c)}\right|+2e^{(2bx+2a+2c)}+e^{(2a)}+e^{(2c)}\right|}{\left|e^{(2a)}-e^{(2c)}\right|+2e^{(2bx+2a+2c)}+e^{(2a)}+e^{(2c)}}\right)}{\left|e^{(2a)}-e^{(2c)}\right|}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x - (e^(2*a) + e^(2*c))*log(abs(-abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2
*a + 2*c) + e^(2*a) + e^(2*c))/(abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*a +
 2*c) + e^(2*a) + e^(2*c)))/abs(e^(2*a) - e^(2*c)))/b
```

3.136 $\int \tanh(c-bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}-x$$

```
[Out] -x - (Coth[a + c]*Log[Cosh[c - b*x]])/b + (Coth[a + c]*Log[Cosh[a + b*x]])/
b
```

Rubi [A] time = 0.0691853, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5646, 5644, 3475}

$$-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c - b*x]*Tanh[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Cosh[c - b*x]])/b + (Coth[a + c]*Log[Cosh[a + b*x]])/
b
```

Rule 5646

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] - Dist[(b*Cosh[(b*c - a*d)/d])/d, Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \tanh(c-bx)\tanh(a+bx)\,dx &= -x+\cosh(a+c)\int \mathbf{sech}(c-bx)\mathbf{sech}(a+bx)\,dx\\
&= -x+\coth(a+c)\int \tanh(c-bx)\,dx+\coth(a+c)\int \tanh(a+bx)\,dx\\
&= -x-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.460417, size = 30, normalized size = 0.83

$$\frac{\coth(a+c)(\log(\cosh(a+bx))-\log(\cosh(c-bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c - b*x]*Tanh[a + b*x],x]
```

```
[Out] -x + (Coth[a + c]*(-Log[Cosh[c - b*x]] + Log[Cosh[a + b*x]]))/b
```

Maple [B] time = 0.042, size = 149, normalized size = 4.1

$$-x-\frac{\ln\left(e^{2a+2c}+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}-\frac{\ln\left(e^{2a+2c}+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(1+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tanh(b*x-c)*tanh(b*x+a),x)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(e
xp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(1+ex
p(2*b*x+2*a))*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(1+exp(2*b*x+2*a))
```

Maxima [B] time = 1.71157, size = 117, normalized size = 3.25

$$-x-\frac{a}{b}+\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-2bx-2a)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}-\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-2bx+2c)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c)
- 1)) - (e^(2*a + 2*c) + 1)*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1
))
```

Fricas [B] time = 1.97328, size = 672, normalized size = 18.67

$$-\frac{bx\cosh(a+c)^2 - 2\,bx\cosh(a+c)\sinh(a+c) + bx\sinh(a+c)^2 - bx - \left(\cosh(a+c)^2 - 2\,\cosh(a+c)\sinh(a+c) + s\right.}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(b*x + a)*cosh(a + c) - sinh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\tanh(a+bx)\tanh(bx-c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x)
```

```
[Out] -Integral(tanh(a + b*x)*tanh(b*x - c), x)
```

Giac [B] time = 1.19637, size = 116, normalized size = 3.22

$$-\frac{bx + \frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1} + \frac{\left(e^{(2\,a)}+e^{(4\,a+2\,c)}\right)\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) +
(e^(2*a) + e^(4*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)
))/b
```

3.137 $\int \coth(a+bx)\coth(c+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(bx+c))}{b}+x$$

```
[Out] x - (Coth[a - c]*Log[Sinh[a + b*x]])/b + (Coth[a - c]*Log[Sinh[c + b*x]])/b
```

Rubi [A] time = 0.0347978, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5647, 5645, 3475}

$$-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(bx+c))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Coth[c + b*x],x]
```

```
[Out] x - (Coth[a - c]*Log[Sinh[a + b*x]])/b + (Coth[a - c]*Log[Sinh[c + b*x]])/b
```

Rule 5647

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(a+bx)\coth(c+bx)\,dx = x+\cosh(a-c)\int \mathbf{csch}(a+bx)\mathbf{csch}(c+bx)\,dx$$
$$= x-\coth(a-c)\int \coth(a+bx)\,dx+\coth(a-c)\int \coth(c+bx)\,dx$$
$$= x-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(c+bx))}{b}$$

Mathematica [A] time = 0.461867, size = 29, normalized size = 0.78

$$\frac{\coth(a-c)(\log(\sinh(bx+c))-\log(\sinh(a+bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Coth[c + b*x],x]
```

```
[Out] x + (Coth[a - c]*(-Log[Sinh[a + b*x]] + Log[Sinh[c + b*x]]))/b
```

Maple [B] time = 0.047, size = 155, normalized size = 4.2

$$x-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*coth(b*x+c),x)
```

```
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-1)*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(exp(2*b*x+2*a)-1)*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
)-exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-exp(2*a-
2*c))*exp(2*c)
```

Maxima [B] time = 1.19917, size = 212, normalized size = 5.73

$$x+\frac{a}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+e^{(2\right.}}{b\left(e^{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c)))
 - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + (e^(
2*a) + e^(2*c))*log(e^(-b*x) + e^c)/(b*(e^(2*a) - e^(2*c))) + (e^(2*a) + e^
(2*c))*log(e^(-b*x) - e^c)/(b*(e^(2*a) - e^(2*c)))
```

Fricas [B] time = 1.96398, size = 701, normalized size = 18.95

$$\frac{bx\cosh(-a+c)^2 - 2\,bx\cosh(-a+c)\sinh(-a+c) + bx\sinh(-a+c)^2 - bx - \left(\cosh(-a+c)^2 - 2\,\cosh(-a+c)\sinh(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20971, size = 159, normalized size = 4.3

$$\frac{bx+\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(\frac{\left|-\left|e^{(2a)}-e^{(2c)}\right|+2e^{(2bx+2a+2c)}-e^{(2a)}-e^{(2c)}\right|}{\left|\left|e^{(2a)}-e^{(2c)}\right|+2e^{(2bx+2a+2c)}-e^{(2a)}-e^{(2c)}\right|}\right)}{\left|e^{(2a)}-e^{(2c)}\right|}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x + (e^(2*a) + e^(2*c))*log(abs(-abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2
*a + 2*c) - e^(2*a) - e^(2*c))/abs(abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*
a + 2*c) - e^(2*a) - e^(2*c)))/abs(e^(2*a) - e^(2*c)))/b
```

3.138 $\int \coth(c-bx)\coth(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}-x$$

```
[Out] -x - (Coth[a + c]*Log[Sinh[c - b*x]])/b + (Coth[a + c]*Log[Sinh[a + b*x]])/
b
```

Rubi [A] time = 0.0362535, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5647, 5645, 3475}

$$-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c - b*x]*Coth[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Sinh[c - b*x]])/b + (Coth[a + c]*Log[Sinh[a + b*x]])/
b
```

Rule 5647

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x+\cosh(a+c)\int \mathrm{csch}(c-bx)\mathrm{csch}(a+bx)\,dx$$
$$= -x+\coth(a+c)\int \coth(c-bx)\,dx+\coth(a+c)\int \coth(a+bx)\,dx$$
$$= -x-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.444234, size = 32, normalized size = 0.89

$$\frac{\coth(a+c)(\log(-\sinh(a+bx))-\log(\sinh(c-bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c - b*x]*Coth[a + b*x],x]
```

```
[Out] -x + (Coth[a + c]*(-Log[Sinh[c - b*x]] + Log[-Sinh[a + b*x]]))/b
```

Maple [B] time = 0.047, size = 153, normalized size = 4.3

$$-x-\frac{\ln\left(-e^{2a+2c}+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}-\frac{\ln\left(-e^{2a+2c}+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(b*x-c)*coth(b*x+a),x)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(
exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(ex
p(2*b*x+2*a)-1)*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)
```

Maxima [B] time = 1.14284, size = 216, normalized size = 6.

$$-x-\frac{a}{b}+\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}+\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2a+2c)}-1\right)}-\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}-\frac{\left(e^{(2a+2c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)
) + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - (e^
(2*a + 2*c) + 1)*log(e^(-b*x + c) + 1)/(b*(e^(2*a + 2*c) - 1)) - (e^(2*a +
2*c) + 1)*log(e^(-b*x + c) - 1)/(b*(e^(2*a + 2*c) - 1))
```

Fricas [B] time = 1.91712, size = 672, normalized size = 18.67

$$-\frac{bx\cosh(a+c)^2 - 2bx\cosh(a+c)\sinh(a+c) + bx\sinh(a+c)^2 - bx - \left(\cosh(a+c)^2 - 2\cosh(a+c)\sinh(a+c) + \cdots\right.}{b\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(a + c)*sinh(b*x + a) - cosh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \coth(a+bx)\coth(bx-c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x)
```

```
[Out] -Integral(coth(a + b*x)*coth(b*x - c), x)
```

<u>Giac</u> [B] time = 1.25612, size = 122, normalized size = 3.39

$$-\frac{bx + \frac{\left(e^{(2a+2c)}+1\right)\log\left(\left|e^{(2bx)}-e^{(2c)}\right|\right)}{e^{(2a+2c)}-1} + \frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) -
1) + (e^(2*a) + e^(4*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(
4*a + 2*c)))/b
```

3.139 $\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\text{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a-c)\log(\cosh(bx+c))}{b}$$

```
[Out] (Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cosh[c + b*x]])/b
```

Rubi [A] time = 0.0246153, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5644, 3475}

$$\frac{\text{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a-c)\log(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Sech[c + b*x],x]

[Out] (Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx &= \text{csch}(a-c)\int \tanh(a+bx)\,dx-\text{csch}(a-c)\int \tanh(c+bx)\,dx\\ &= \frac{\text{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a-c)\log(\cosh(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.206235, size = 27, normalized size = 0.75

$$\frac{\operatorname{csch}(a-c)(\log(\cosh(a+bx))-\log(\cosh(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Sech[c + b*x],x]
```

```
[Out] (Csch[a - c]*(Log[Cosh[a + b*x]] - Log[Cosh[c + b*x]]))/b
```

Maple [B] time = 0.033, size = 77, normalized size = 2.1

$$-2\,\frac{\ln\left(e^{2\,bx+2\,a}+e^{2\,a-2\,c}\right)e^{a+c}}{b\left(e^{2\,a}-e^{2\,c}\right)}+2\,\frac{\ln\left(1+e^{2\,bx+2\,a}\right)e^{a+c}}{b\left(e^{2\,a}-e^{2\,c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sech(b*x+c),x)
```

```
[Out] -2/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)+exp(2*a-2*c))*exp(a+c)+2/b/(exp(
2*a)-exp(2*c))*ln(1+exp(2*b*x+2*a))*exp(a+c)
```

Maxima [A] time = 1.6957, size = 92, normalized size = 2.56

$$\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)
*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c)))
```

Fricas [B] time = 1.87307, size = 490, normalized size = 13.61

$$\frac{2\left((\cosh(-a+c)-\sinh(-a+c))\log\left(\frac{2(\cosh(bx+c)\cosh(-a+c)-\sinh(bx+c)\sinh(-a+c))}{\cosh(bx+c)\cosh(-a+c)-(\cosh(-a+c)+\sinh(-a+c))\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)-(\cosh(-a\right.}{b\cosh(-a+c)^2-2b\cosh(-a+c)\sinh(-a+c)+b\sinh(-a+c)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b
*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a
 + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh(
-a + c))*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
 c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x + c), x)
```

Giac [B] time = 1.22999, size = 135, normalized size = 3.75

$$\frac{2\,e^{(a+c)}\log\left(\frac{\left|-\left|e^{(2\,a)}-e^{(2\,c)}\right|+2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}+e^{(2\,c)}\right|}{\left|e^{(2\,a)}-e^{(2\,c)}\right|+2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}+e^{(2\,c)}}\right)}{b\left|e^{(2\,a)}-e^{(2\,c)}\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

```
[Out] 2*e^(a + c)*log(abs(-abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*a + 2*c) + e^(
2*a) + e^(2*c))/(abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*a + 2*c) + e^(2*a)
 + e^(2*c)))/(b*abs(e^(2*a) - e^(2*c)))
```

3.140 $\int \mathbf{sech}(c-bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}$$

```
[Out] -((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log[Cosh[a + b*x]])/b
```

Rubi [A] time = 0.0234886, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5644, 3475}

$$\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c - b*x]*Sech[a + b*x],x]
```

```
[Out] -((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log[Cosh[a + b*x]])/b
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx&=\operatorname{csch}(a+c)\int \tanh(c-bx)\,dx+\operatorname{csch}(a+c)\int \tanh(a+bx)\,dx\\&=-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.199106, size = 27, normalized size = 0.82

$$-\frac{\operatorname{csch}(a+c)(\log(\cosh(c-bx))-\log(\cosh(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c - b*x]*Sech[a + b*x],x]
```

```
[Out] -((Csch[a + c]*(Log[Cosh[c - b*x]] - Log[Cosh[a + b*x]]))/b)
```

Maple [B] time = 0.033, size = 75, normalized size = 2.3

$$-2\,\frac{\ln\left(e^{2\,a+2\,c}+e^{2\,bx+2\,a}\right)e^{a+c}}{b\left(e^{2\,a+2\,c}-1\right)}+2\,\frac{\ln\left(1+e^{2\,bx+2\,a}\right)e^{a+c}}{b\left(e^{2\,a+2\,c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x-c)*sech(b*x+a),x)
```

```
[Out] -2/b/(exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)+2/b/(exp(2*a
+2*c)-1)*ln(1+exp(2*b*x+2*a))*exp(a+c)
```

Maxima [A] time = 1.61928, size = 90, normalized size = 2.73

$$\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx+2\,c)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)
*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1))
```

Fricas [B] time = 2.14801, size = 471, normalized size = 14.27

$$\frac{2\left(\left(\cosh\left(a+c\right)-\sinh\left(a+c\right)\right)\log\left(\frac{2\left(\cosh(bx+a)\cosh(a+c)-\sinh(bx+a)\sinh(a+c)\right)}{\cosh(bx+a)\cosh(a+c)-\left(\cosh(a+c)+\sinh(a+c)\right)\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-\left(\cosh\left(a+c\right)-\mathrm{s}\right.\right.}{b\cosh\left(a+c\right)^{2}-2\,b\cosh\left(a+c\right)\sinh\left(a+c\right)+b\sinh\left(a+c\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(b*x + a)*cosh(a + c) - sinh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a+bx\right)\operatorname{sech}\left(bx-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x - c), x)
```

Giac [B] time = 1.17126, size = 95, normalized size = 2.88

$$-\frac{2\left(\frac{e^{(a+c)}\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) + e^(3*a + c)*lo
g(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)))/b
```

3.141 $\int \text{csch}(a+bx)\text{csch}(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\text{csch}(a-c)\log(\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}$$

```
[Out] -((Csch[a - c]*Log[Sinh[a + b*x]])/b) + (Csch[a - c]*Log[Sinh[c + b*x]])/b
```

Rubi [A] time = 0.0237326, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5645, 3475}

$$\frac{\text{csch}(a-c)\log(\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Csch[c + b*x],x]
```

```
[Out] -((Csch[a - c]*Log[Sinh[a + b*x]])/b) + (Csch[a - c]*Log[Sinh[c + b*x]])/b
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{csch}(a+bx)\text{csch}(c+bx)\,dx &= -\left(\text{csch}(a-c)\int \coth(a+bx)\,dx\right)+\text{csch}(a-c)\int \coth(c+bx)\,dx\\ &= -\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}+\frac{\text{csch}(a-c)\log(\sinh(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.220831, size = 28, normalized size = 0.78

$$-\frac{\operatorname{csch}(a-c)(\log(\sinh(a+bx))-\log(\sinh(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Csch[c + b*x],x]
```

```
[Out] -((Csch[a - c]*(Log[Sinh[a + b*x]] - Log[Sinh[c + b*x]]))/b)
```

Maple [B] time = 0.035, size = 79, normalized size = 2.2

$$2\,\frac{\ln\left(e^{2\,bx+2\,a}-e^{2\,a-2\,c}\right)e^{a+c}}{b\left(e^{2\,a}-e^{2\,c}\right)}-2\,\frac{\ln\left(e^{2\,bx+2\,a}-1\right)e^{a+c}}{b\left(e^{2\,a}-e^{2\,c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*csch(b*x+c),x)
```

```
[Out] 2/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-exp(2*a-2*c))*exp(a+c)-2/b/(exp(2
*a)-exp(2*c))*ln(exp(2*b*x+2*a)-1)*exp(a+c)
```

Maxima [B] time = 1.15714, size = 180, normalized size = 5.

$$-\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}-e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

```
[Out] -2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)*lo
g(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) + e^
c)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) - e^c)/(b*(e^(2*a) -
e^(2*c)))
```

Fricas [B] time = 2.14778, size = 491, normalized size = 13.64

$$-\frac{2\left((\cosh(-a+c)-\sinh(-a+c))\log\left(\frac{2(\cosh(-a+c)\sinh(bx+c)-\cosh(bx+c)\sinh(-a+c))}{\cosh(bx+c)\cosh(-a+c)-(\cosh(-a+c)+\sinh(-a+c))\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)-(\cosh(-a+c)-\dots\right.}{b\cosh(-a+c)^2-2b\cosh(-a+c)\sinh(-a+c)+b\sinh(-a+c)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

```
[Out] -2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(
b*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-
a + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh
(-a + c))*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a
+ c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x)
```

```
[Out] Integral(csch(a + b*x)*csch(b*x + c), x)
```

Giac [B] time = 1.20016, size = 147, normalized size = 4.08

$$\frac{2\,e^{(a+c)}\log\left(\frac{\left|-\left|e^{(2a)}-e^{(2c)}\right|+2\,e^{(2bx+2a+2c)}-e^{(2a)}-e^{(2c)}\right|}{\left|\left|e^{(2a)}-e^{(2c)}\right|+2\,e^{(2bx+2a+2c)}-e^{(2a)}-e^{(2c)}\right|}\right)}{b\left|e^{(2a)}-e^{(2c)}\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] 2*e^(a + c)*log(abs(-abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*a + 2*c) - e^(
2*a) - e^(2*c))/abs(abs(e^(2*a) - e^(2*c)) + 2*e^(2*b*x + 2*a + 2*c) - e^(2
*a) - e^(2*c)))/(b*abs(e^(2*a) - e^(2*c)))
```

3.142 $\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}$$

```
[Out] -((Csch[a + c]*Log[Sinh[c - b*x]])/b) + (Csch[a + c]*Log[Sinh[a + b*x]])/b
```

Rubi [A] time = 0.0236297, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5645, 3475}

$$\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c - b*x]*Csch[a + b*x],x]
```

```
[Out] -((Csch[a + c]*Log[Sinh[c - b*x]])/b) + (Csch[a + c]*Log[Sinh[a + b*x]])/b
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx&=\operatorname{csch}(a+c)\int \coth(c-bx)\,dx+\operatorname{csch}(a+c)\int \coth(a+bx)\,dx\\&=-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.202135, size = 29, normalized size = 0.88

$$-\frac{\operatorname{csch}(a+c)(\log(\sinh(c-bx))-\log(-\sinh(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c - b*x]*Csch[a + b*x],x]
```

```
[Out] -((Csch[a + c]*(Log[Sinh[c - b*x]] - Log[-Sinh[a + b*x]]))/b)
```

Maple [B] time = 0.033, size = 77, normalized size = 2.3

$$2\,\frac{\ln\left(e^{2\,bx+2\,a}-1\right)e^{a+c}}{b\left(e^{2\,a+2\,c}-1\right)}-2\,\frac{\ln\left(-e^{2\,a+2\,c}+e^{2\,bx+2\,a}\right)e^{a+c}}{b\left(e^{2\,a+2\,c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csch(b*x-c)*csch(b*x+a),x)
```

```
[Out] 2/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)*exp(a+c)-2/b/(exp(2*a+2*c)-1)*ln(
-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)
```

Maxima [B] time = 1.11593, size = 174, normalized size = 5.27

$$\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}-1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)) + 2*e^(a + c)*log
(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) +
 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) - 1)/(b*(e^(2*a
+ 2*c) - 1))
```

Fricas [B] time = 2.1594, size = 471, normalized size = 14.27

$$\frac{2\left(\left(\cosh\left(a+c\right)-\sinh\left(a+c\right)\right)\log\left(\frac{2\left(\cosh(a+c)\sinh(bx+a)-\cosh(bx+a)\sinh(a+c)\right)}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-\left(\cosh\left(a+c\right)-s\right.}{b\cosh\left(a+c\right)^{2}-2\,b\cosh\left(a+c\right)\sinh\left(a+c\right)+b\sinh\left(a+c\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(a + c)*sinh(b*x + a) - cosh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\operatorname{csch}\left(a+bx\right)\operatorname{csch}\left(bx-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x)
```

```
[Out] -Integral(csch(a + b*x)*csch(b*x - c), x)
```

Giac [B] time = 1.20042, size = 100, normalized size = 3.03

$$-\frac{2\left(\frac{e^{(a+c)}\log\left(\left|e^{(2\,bx)}-e^{(2\,c)}\right|\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) - 1) + e^(3*a +
c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(4*a + 2*c)))/b
```

3.143 $\int \sinh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0228972, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5620, 2637, 3770}

$$\frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = -(\cosh(a-c)\int \mathbf{sech}(c+bx)\,dx) + \int \cosh(a+bx)\,dx$$
$$= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [B] time = 0.0579334, size = 86, normalized size = 2.97

$$-\frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\cosh(bx)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [C] time = 0.078, size = 167, normalized size = 5.8

$$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{\frac{i}{2}\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}e^{2a}}{b} + \frac{\frac{i}{2}\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}e^{2c}}{b} - \frac{\frac{i}{2}\ln\left(e^{bx+a} + ie^{a-c}\right)e^{-a-c}e^{2a}}{b} - \frac{\frac{i}{2}\ln\left(e^{bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a
-c)*exp(2*a)+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*c)-1/2*I/b*l
n(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(2*a)-1/2*I/b*ln(exp(b*x+a)+I*exp(a-c
))*exp(-a-c)*exp(2*c)
```

Maxima [A] time = 1.65073, size = 77, normalized size = 2.66

$$\frac{\left(e^{(2a)} + e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} - \frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b -
 1/2*e^(-b*x - a)/b
```

Fricas [B] time = 2.19109, size = 914, normalized size = 31.52

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)^2 + \left(\cosh(-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\tanh(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c), x)
```

Giac [A] time = 1.20172, size = 66, normalized size = 2.28

$$-\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) +
e^(-b*x - a))/b
```

3.144 $\int \sinh(a+bx)\tanh^2(c+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*Si
nh[a - c])/b
```

Rubi [A] time = 0.0551535, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5620, 5623, 2638, 3770, 2606, 8}

$$-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*Si
nh[a - c])/b
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx\right) + \int \cosh(a+bx)\tanh(c+bx)\,dx$$
$$= \frac{\cosh(a-c)\,\text{Subst}\left(\int 1\,dx, x, \operatorname{sech}(c+bx)\right)}{b} - \sinh(a-c)\int \operatorname{sech}(c+bx)\,dx + \int \text{si}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} - \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}$$

Mathematica [B] time = 0.10009, size = 102, normalized size = 2.27

$$\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b} - \frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b} + \text{cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b + (Cosh[a - c]*Sech[c + b*x])/b - (2*ArcTan[((Cosh[c]
 - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[
(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [C] time = 0.089, size = 205, normalized size = 4.6

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}\left(e^{a}\right)^{2}}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}\left(e^{c}\right)^{2}}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}+i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(a)
^2-1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(c)^2-1/2*I/b*ln(exp(b*x+
a)+I*exp(a-c))*exp(-a-c)*exp(a)^2+1/2*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-
c)*exp(c)^2
```

Maxima [B] time = 1.73557, size = 142, normalized size = 3.16

$$\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}+\frac{e^{(-bx-a)}}{2b}+\frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a)}+e^{(2c)}}{2b\left(e^{(-bx-a+2c)}+e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

Fricas [B] time = 2.24425, size = 2491, normalized size = 55.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 + 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\tanh^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**2, x)
```

Giac [A] time = 1.24064, size = 119, normalized size = 2.64

$$-\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(2\,e^{(2bx+2a)}+3\,e^{(2bx+2c)}+1\right)e^{(-a)}}{e^{(3bx+2c)}+e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
2*a) + 3*e^(2*b*x + 2*c) + 1)*e^(-a)/(e^(3*b*x + 2*c) + e^(b*x)) - e^(b*x +
 a))/b
```

3.145 $\int \sinh(a+bx)\tanh^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}-\frac{3\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\tanh(bx+c)\mathbf{sech}(bx+c)}{2b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] (-3*ArcTan[Sinh[c + b*x]]*Cosh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c])/
b + Sinh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x]*Tanh[c + b*x])/(2*b)
```

Rubi [A] time = 0.082597, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5620, 5623, 2637, 3770, 2606, 8, 2611}

$$\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}-\frac{3\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\tanh(bx+c)\mathbf{sech}(bx+c)}{2b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[c + b*x]]*Cosh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c])/
b + Sinh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x]*Tanh[c + b*x])/(2*b)
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh^3(c+bx)\,dx &= -\left(\cosh(a-c)\int \mathbf{sech}(c+bx)\tanh^2(c+bx)\,dx\right)+\int \cosh(a+bx)\tanh^2(c+bx)\,dx\\ &= \frac{\cosh(a-c)\mathbf{sech}(c+bx)\tanh(c+bx)}{2b}-\frac{1}{2}\cosh(a-c)\int \mathbf{sech}(c+bx)\,dx-\sinh(a-\\ &= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{2b}+\frac{\cosh(a-c)\mathbf{sech}(c+bx)\tanh(c+bx)}{2b}-\cosh(a\\ &= -\frac{3\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{2b}+\frac{\mathbf{sech}(c+bx)\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}+\frac{\text{co}}{}\end{aligned}$$

Mathematica [A] time = 0.334092, size = 70, normalized size = 0.97

$$\frac{\mathbf{sech}^2(bx+c)(2\sinh(a-bx-2c)+\sinh(a+3bx+2c)+5\sinh(a+bx))-12\cosh(a-c)\tan^{-1}\left(\cosh(c)\tanh\left(\frac{bx}{2}\right)+\text{si}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + Sech[c + b*x]^2*
(2*Sinh[a - 2*c - b*x] + 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] time = 0.093, size = 240, normalized size = 3.3

$$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(3\,e^{2\,bx+4\,a+2\,c} - e^{2\,bx+2\,a+4\,c} + e^{4\,a} - 3\,e^{2\,a+2\,c}\right)}{2\,b\left(e^{2\,bx+2\,a+2\,c} + e^{2\,a}\right)^2} + \frac{\frac{3\,i}{4}\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}e^{2\,a}}{b} + \frac{\frac{3\,i}{4}\ln\left(e^{bx+a} - ie^{\cdots}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)-exp
(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*a)+3/4*I/b*ln(exp(b*x+a)-
I*exp(a-c))*exp(-a-c)*exp(2*c)-3/4*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*
exp(2*a)-3/4*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.78294, size = 201, normalized size = 2.79

$$\frac{3\left(e^{(2\,a)} + e^{(2\,c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b} + \frac{\left(5\,e^{(2\,a+2\,c)} - e^{(4\,c)}\right)e^{(-2\,bx-2\,a)} + \left(2\,e^{(4\,a)} - 3\,e^{(2\,a+2\,c)}\right)e^{(-4\,bx-4\,a)} + e^{(4\,c)}}{2\,b\left(e^{(-bx-a+4\,c)} + 2\,e^{(-3\,bx-a+2\,c)} + e^{(-5\,bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) - 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

Fricas [B] time = 2.31111, size = 4772, normalized size = 66.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
- 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 + 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 + 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
```

```
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a + bx)\tanh^3(bx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**3, x)
```

Giac [A] time = 1.22407, size = 151, normalized size = 2.1

$$-\frac{3\left(e^{(2a)} + e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(3e^{(3bx+2a+2c)} - e^{(3bx+4c)} + e^{(bx+2a)} - 3e^{(bx+2c)}\right)e^{(-a)}}{\left(e^{(2bx+2c)}+1\right)^2} - e^{(bx+a)} + e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
2*a + 2*c) - e^(3*b*x + 4*c) + e^(b*x + 2*a) - 3*e^(b*x + 2*c))*e^(-a)/(e^(
2*b*x + 2*c) + 1)^2 - e^(b*x + a) + e^(-b*x - a))/b
```

3.146 $\int \coth(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(a+bx)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0204265, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5622, 2637, 3770}

$$\frac{\sinh(a+bx)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \sinh(a-c)\int \mathbf{csch}(c+bx)\,dx + \int \cosh(a+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [C] time = 0.0530698, size = 93, normalized size = 3.21

$$-\frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\cosh(bx)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b
*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]
*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [B] time = 0.041, size = 155, normalized size = 5.3

$$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}-e^{a-c}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*
exp(2*a)+1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*c)+1/2/b*ln(exp(b*x+
a)-exp(a-c))*exp(-a-c)*exp(2*a)-1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp
(2*c)
```

Maxima [B] time = 1.25286, size = 127, normalized size = 4.38

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(bx+a)}}{2b} - \frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x
 - a)/b
```

Fricas [B] time = 2.33157, size = 1233, normalized size = 42.52

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)^2 + \left(\cosh(-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
- 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)
^2 - 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sin
h(-a + c)^2 - 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*
(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) +
 cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a
+ c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sin
h(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\coth(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(b*x + c), x)
```

Giac [B] time = 1.16616, size = 117, normalized size = 4.03

$$-\frac{\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+c)}-1\right|\right)-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) - e^(b*x + a) + e^(-b
*x - a))/b
```

3.147 $\int \coth^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - (Csch[c + b*x
]*Sinh[a - c])/b
```

Rubi [A] time = 0.0460025, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5622, 5621, 2638, 3770, 2606, 8}

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - (Csch[c + b*x
]*Sinh[a - c])/b
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \sinh(a-c)\int \coth(c+bx)\mathbf{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth(c+bx)\,dx$$
$$= \cosh(a-c)\int \mathbf{csch}(c+bx)\,dx - \frac{(i\sinh(a-c))\,\mathrm{Subst}(\int 1\,dx, x, -i\mathbf{csch}(c+bx))}{b} +$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b} - \frac{\mathbf{csch}(c+bx)\sinh(a-c)}{b}$$

Mathematica [C] time = 0.0942187, size = 110, normalized size = 2.39

$$-\frac{\sinh(a-c)\mathbf{csch}(bx+c)}{b} - \frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b} + \frac{\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b - (Csch[c + b*x]*Sinh[a - c])/b + (Sinh[a]*Sinh[
b*x])/b
```

Maple [B] time = 0.046, size = 197, normalized size = 4.3

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*a)+
1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)-1/2/b*ln(exp(b*x+a)+exp(a-
c))*exp(-a-c)*exp(2*a)-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.17949, size = 189, normalized size = 4.11

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{e^{(-bx-a)}}{2b}-\frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a)}-e^{(2}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

Fricas [B] time = 2.40071, size = 3363, normalized size = 73.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\coth^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(b*x + c)**2, x)
```

Giac [B] time = 1.18693, size = 163, normalized size = 3.54

$$-\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{\left(2e^{(2bx+2a)}-3e^{(2bx+2c)}+1\right)e^{(-a)}}{e^{(3bx+2c)}-e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + (2*e^(2*b*x + 2*a)
- 3*e^(2*b*x + 2*c) + 1)*e^(-a)/(e^(3*b*x + 2*c) - e^(b*x)) - e^(b*x + a))/
b
```

3.148 $\int \coth^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{3\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\coth(bx+c)\mathbf{csch}(bx+c)}{2b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (3*ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/(
2*b) - (Coth[c + b*x]*Csch[c + b*x]*Sinh[a - c])/(2*b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0874259, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5622, 5621, 2637, 3770, 2606, 8, 2611}

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{3\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\coth(bx+c)\mathbf{csch}(bx+c)}{2b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (3*ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/(
2*b) - (Coth[c + b*x]*Csch[c + b*x]*Sinh[a - c])/(2*b) + Sinh[a + b*x]/b
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \sinh(a-c)\int \coth^2(c+bx)\mathbf{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth^2(c+bx)\,dx$$
$$= -\frac{\coth(c+bx)\mathbf{csch}(c+bx)\sinh(a-c)}{2b} + \cosh(a-c)\int \coth(c+bx)\mathbf{csch}(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\mathbf{csch}(c+bx)\sinh(a-c)}{2b} - \frac{(i\cosh$$
$$= -\frac{\cosh(a-c)\mathbf{csch}(c+bx)}{b} - \frac{3\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\mathbf{csch}$$

Mathematica [A] time = 0.327516, size = 70, normalized size = 0.96

$$\frac{\mathbf{csch}^2(bx+c)(2\sinh(a-bx-2c)+\sinh(a+3bx+2c)-5\sinh(a+bx))-12\sinh(a-c)\tanh^{-1}\left(\sinh(c)\tanh\left(\frac{bx}{2}\right)+c\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Sinh[a - c] + Csch[c + b*x]^2
*(2*Sinh[a - 2*c - b*x] - 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

Maple [B] time = 0.053, size = 230, normalized size = 3.2

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}-e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}-\frac{3\,\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{4b}+\frac{3\,\ln\left(e^{bx+a}+e^{a}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)-ex
p(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2-3/4/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*a)+3/4/b*ln(exp(b*x+a)+exp(
a-c))*exp(-a-c)*exp(2*c)+3/4/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*a)-3
/4/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.22753, size = 251, normalized size = 3.44

$$-\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b}+\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b}-\frac{e^{(-bx-a)}}{2b}-\frac{\left(5\,e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a}}{2b\left(e^{(-bx-a+4c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) + 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

Fricas [B] time = 2.35647, size = 6496, normalized size = 88.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) - 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 + 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 - 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) + 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 - 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
 + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
```

```
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) - 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21814, size = 207, normalized size = 2.84

$$-\frac{3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(3e^{(3bx+2a+2c)}+e^{(3bx+4c)}-e^{(bx+2a)}-3e\right.}{\left(e^{(2bx+2c)}-1\right)^{2}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - 3*(e^(2
*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(3*e^(3*b*x +
 2*a + 2*c) + e^(3*b*x + 4*c) - e^(b*x + 2*a) - 3*e^(b*x + 2*c))*e^(-a)/(e^
(2*b*x + 2*c) - 1)^2 - 2*e^(b*x + a) + 2*e^(-b*x - a))/b
```

3.149 $\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Rubi [A] time = 0.0158705, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5624, 3475, 8}

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \tanh(c+bx)\,dx + \sinh(a-c)\int 1\,dx$$
$$= \frac{\cosh(a-c)\log(\cosh(c+bx))}{b} + x\sinh(a-c)$$

Mathematica [A] time = 0.11894, size = 26, normalized size = 1.

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b} + x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Maple [B] time = 0.036, size = 148, normalized size = 5.7

$$xe^{a-c} - e^{-a-c}e^{2a}x - e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} - \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{-a-c}e^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)*sinh(b*x+a),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2/b*ln(exp(2*b*x+2*a)+exp(2*a-2*c))*exp(-a-c)
*exp(2*a)+1/2/b*ln(exp(2*b*x+2*a)+exp(2*a-2*c))*exp(-a-c)*exp(2*c)
```

Maxima [A] time = 1.26344, size = 66, normalized size = 2.54

$$\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-2bx)}+e^{(2c)}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

Fricas [B] time = 2.15378, size = 231, normalized size = 8.88

$$-\frac{2\,bx - \left(\cosh\left(-a+c\right)^2 - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^2 + 1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c), x)
```

Giac [A] time = 1.14728, size = 66, normalized size = 2.54

$$-\frac{2\,bxe^{(-a+c)} - \left(e^{(2\,a+c)} + e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)} + 1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x
+ 2*c) + 1))/b
```

3.150 $\int \mathbf{sech}^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}$$

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rubi [A] time = 0.0327126, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5624, 2606, 8, 3770}

$$\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^2(c+bx)\sinh(a+bx)\,dx &= \cosh(a-c)\int \mathrm{sech}(c+bx)\tanh(c+bx)\,dx+\sinh(a-c)\int \mathrm{sech}(c+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}-\frac{\cosh(a-c)\,\mathrm{Subst}\left(\int 1\,dx,x,\mathrm{sech}(c+bx)\right)}{b}\\ &= -\frac{\cosh(a-c)\mathrm{sech}(c+bx)}{b}+\frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] time = 0.0872576, size = 83, normalized size = 2.37

$$\frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\cosh(a-c)\mathrm{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*
x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)
/2]*Sinh[c])]*Sinh[a - c])/b
```

Maple [C] time = 0.08, size = 181, normalized size = 5.2

$$-\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\frac{i}{2}\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}\left(e^{a}\right)^2}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}\left(e^{c}\right)^2}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}\left(e^{a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I/b*l
n(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(a)^2-1/2*I/b*ln(exp(b*x+a)+I*exp(a-c
))*exp(-a-c)*exp(c)^2-1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(a)^2+
```

```
1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(c)^2
```

Maxima [B] time = 1.88148, size = 95, normalized size = 2.71

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) + e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

Fricas [B] time = 2.07769, size = 1129, normalized size = 32.26

$$\frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-\cosh(bx+c)\sinh(-a+c)^2+\left(\left(\cosh(-a+c)^2-1\right)\cosh(bx+c)^2+\left(\cosh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) -
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 + 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh (a+bx) \operatorname{sech}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**2, x)
```

Giac [A] time = 1.18654, size = 92, normalized size = 2.63

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(e^{(bx+2a)} + e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] ((e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) + e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

3.151 $\int \mathbf{sech}^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\mathbf{sech}^2(bx+c)}{2b}$$

```
[Out] -(Cosh[a - c]*Sech[c + b*x]^2)/(2*b) + (Sinh[a - c]*Tanh[c + b*x])/b
```

Rubi [A] time = 0.0440048, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5624, 2606, 30, 3767, 8}

$$\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\mathbf{sech}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -(Cosh[a - c]*Sech[c + b*x]^2)/(2*b) + (Sinh[a - c]*Tanh[c + b*x])/b
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \operatorname{sech}^2(c+bx)\tanh(c+bx)\,dx + \sinh(a-c)\int \operatorname{sech}^2(c+bx)\,dx$$
$$= -\frac{\cosh(a-c)\operatorname{Subst}(\int x\,dx, x, \operatorname{sech}(c+bx))}{b} + \frac{(i\sinh(a-c))\operatorname{Subst}(\int 1\,dx, x, -i\operatorname{ta}}{b}$$
$$= -\frac{\cosh(a-c)\operatorname{sech}^2(c+bx)}{2b} + \frac{\sinh(a-c)\tanh(c+bx)}{b}$$

Mathematica [A] time = 0.162124, size = 35, normalized size = 0.92

$$-\frac{\operatorname{sech}(c)\operatorname{sech}^2(bx+c)(\cosh(a)-\sinh(a-c)\sinh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -(Sech[c]*Sech[c + b*x]^2*(Cosh[a] - Sinh[a - c]*Sinh[c + 2*b*x]))/(2*b)
```

Maple [A] time = 0.03, size = 58, normalized size = 1.5

$$-\frac{\left(2\,e^{2\,bx+2\,a+2\,c}+e^{2\,a}-e^{2\,c}\right)e^{3\,a-c}}{b\left(e^{2\,bx+2\,a+2\,c}+e^{2\,a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] -(2*exp(2*b*x+2*a+2*c)+exp(2*a)-exp(2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2
*exp(3*a-c)
```

Maxima [B] time = 1.25841, size = 162, normalized size = 4.26

$$-\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}-\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4
*c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

Fricas [B] time = 1.99303, size = 644, normalized size = 16.95

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}+3\,b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)\sinh\left(bx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + cosh(-a + c)*sinh(b*x + c) - 2*cosh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\operatorname{sech}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**3, x)
```

Giac [A] time = 1.14558, size = 69, normalized size = 1.82

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

3.152 $\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rubi [A] time = 0.0149365, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5626, 3475, 8}

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int 1\,dx + \sinh(a-c)\int \coth(c+bx)\,dx$$
$$= x\cosh(a-c) + \frac{\log(\sinh(c+bx))\sinh(a-c)}{b}$$

Mathematica [A] time = 0.111104, size = 26, normalized size = 1.

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b} + x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Maple [B] time = 0.036, size = 150, normalized size = 5.8

$$xe^{a-c} - e^{-a-c}e^{2a}x + e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} + \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln\left(e^{2bx+2a} - e^{2a-2c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{2bx+2a} - e^{2a-2c}\right)e^{-a-c}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)*sinh(b*x+a),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a+1/b*exp(-a-c)*exp(2*c)*a+1/2/b*ln(exp(2*b*x+2*a)-exp(2*a-2*c))*exp(-a-c)
*exp(2*a)-1/2/b*ln(exp(2*b*x+2*a)-exp(2*a-2*c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.26277, size = 113, normalized size = 4.35

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2b} + \frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) - e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

Fricas [B] time = 2.12788, size = 230, normalized size = 8.85

$$\frac{2\,bx + \left(\cosh\left(-a+c\right)^{2} - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^{2} - 1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c), x)
```

Giac [A] time = 1.20695, size = 69, normalized size = 2.65

$$\frac{2\,bxe^{(-a+c)} + \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)} - 1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*b
*x + 2*c) - 1)))/b
```

3.153 $\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rubi [A] time = 0.0317207, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5626, 2606, 8, 3770}

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \mathrm{csch}^2(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \mathrm{csch}(c+bx)\,dx + \sinh(a-c)\int \coth(c+bx)\mathrm{csch}(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} - \frac{(i\sinh(a-c))\,\mathrm{Subst}(\int 1\,dx, x, -i\mathrm{csch}(c+bx))}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} - \frac{\mathrm{csch}(c+bx)\sinh(a-c)}{b}$$

Mathematica [C] time = 0.0799594, size = 90, normalized size = 2.5

$$-\frac{\sinh(a-c)\mathrm{csch}(bx+c)}{b} - \frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b - (Csch[c + b*x]*Sinh[a - c])/b
```

Maple [B] time = 0.039, size = 172, normalized size = 4.8

$$\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{bx+a}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2/b*ln(
exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*a)-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-
a-c)*exp(2*c)+1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*a)+1/2/b*ln(exp
```

```
(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.13385, size = 139, normalized size = 3.86

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) - e^(2*c))*e^(-b*x - a
)/(b*(e^(-2*b*x) - e^(2*c)))
```

Fricas [B] time = 2.19585, size = 1709, normalized size = 47.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c) - ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
```

```
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\operatorname{csch}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**2, x)
```

Giac [B] time = 1.18783, size = 140, normalized size = 3.89

$$-\frac{\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2\,a)}-e^{(bx+2\,c)}\right)e^{(-a)}}{e^{(2\,bx+2\,c)}-1}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) -
e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

3.154 $\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\operatorname{csch}^2(bx+c)}{2b}$$

```
[Out] -((Cosh[a - c]*Coth[c + b*x])/b) - (Csch[c + b*x]^2*Sinh[a - c])/(2*b)
```

Rubi [A] time = 0.0443071, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5626, 2606, 30, 3767, 8}

$$-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\operatorname{csch}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Coth[c + b*x])/b) - (Csch[c + b*x]^2*Sinh[a - c])/(2*b)
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \operatorname{csch}^2(c+bx)\,dx + \sinh(a-c)\int \coth(c+bx)\operatorname{csch}^2(c+bx)\,dx$$
$$= -\frac{(i\cosh(a-c))\operatorname{Subst}\left(\int 1\,dx, x, -i\coth(c+bx)\right)}{b} + \frac{\sinh(a-c)\operatorname{Subst}\left(\int x\,dx, x, -i\operatorname{cs}\right.}{b}$$
$$= -\frac{\cosh(a-c)\coth(c+bx)}{b} - \frac{\operatorname{csch}^2(c+bx)\sinh(a-c)}{2b}$$

Mathematica [A] time = 0.172487, size = 35, normalized size = 0.9

$$-\frac{\operatorname{csch}(c)\operatorname{csch}^2(bx+c)(\cosh(a)-\cosh(a-c)\cosh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a] - Cosh[a - c]*Cosh[c + 2*b*x])*Csch[c]*Csch[c + b*x]^2)/(2*b)
```

Maple [A] time = 0.028, size = 57, normalized size = 1.5

$$\frac{\left(-2\,e^{2\,bx+2\,a+2\,c}+e^{2\,a}+e^{2\,c}\right)e^{3\,a-c}}{b\left(-e^{2\,bx+2\,a+2\,c}+e^{2\,a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] (-2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2*exp(3*a-c)
```

Maxima [B] time = 1.08096, size = 177, normalized size = 4.54

$$-\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4
*c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

Fricas [B] time = 2.11493, size = 620, normalized size = 15.9

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}-b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)\sinh\left(bx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(-a + c) - sinh(-a + c))*sinh(b*x + c) - cosh(b*x + c)*sinh(-a +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx\right)\operatorname{csch}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**3, x)
```

Giac [A] time = 1.17676, size = 72, normalized size = 1.85

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}-e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

3.155 $\int \cosh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cosh(a+bx)}{b}-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

```
[Out] Cosh[a + b*x]/b - (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rubi [A] time = 0.0193314, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5623, 2638, 3770}

$$\frac{\cosh(a+bx)}{b}-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] Cosh[a + b*x]/b - (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = -(\sinh(a-c)\int \mathbf{sech}(c+bx)\,dx) + \int \sinh(a+bx)\,dx$$
$$= \frac{\cosh(a+bx)}{b} - \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}$$

Mathematica [B] time = 0.057125, size = 86, normalized size = 2.97

$$-\frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b} + \frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b - (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[
c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c]
)]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [C] time = 0.08, size = 167, normalized size = 5.8

$$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{\frac{i}{2}\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}\,(e^{a})^2}{b} - \frac{\frac{i}{2}\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}\,(e^{c})^2}{b} - \frac{\frac{i}{2}\ln\left(e^{bx+a} + ie^{a-c}\right)e^{-a-c}\,(e^{a})^2}{b} + \frac{\frac{i}{2}\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a
-c)*exp(a)^2-1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(c)^2-1/2*I/b*l
n(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(a)^2+1/2*I/b*ln(exp(b*x+a)+I*exp(a-c
))*exp(-a-c)*exp(c)^2
```

Maxima [B] time = 1.64584, size = 80, normalized size = 2.76

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b +
 1/2*e^(-b*x - a)/b
```

Fricas [B] time = 2.15508, size = 914, normalized size = 31.52

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)^2 + \left(\cosh(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\tanh(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c), x)
```

<u>Giac</u> [A] time = 1.18435, size = 72, normalized size = 2.48

$$-\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) -
e^(-b*x - a))/b
```

3.156 $\int \cosh(a+bx)\tanh^2(c+bx)\,dx$

Optimal. Leaf size=45

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
Sinh[a + b*x]/b
```

Rubi [A] time = 0.0424318, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5623, 5620, 2637, 3770, 2606, 8}

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
Sinh[a + b*x]/b
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\tanh^2(c+bx)\,dx &= -\left(\sinh(a-c)\int \mathbf{sech}(c+bx)\tanh(c+bx)\,dx\right)+\int \sinh(a+bx)\tanh(c+bx)\,dx\\ &= -\left(\cosh(a-c)\int \mathbf{sech}(c+bx)\,dx\right)+\frac{\sinh(a-c)\,\mathrm{Subst}(\int 1\,dx,x,\mathbf{sech}(c+bx))}{b}+\int\\ &= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\mathbf{sech}(c+bx)\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.0976402, size = 102, normalized size = 2.27

$$\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}-\frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sinh(a)\cosh(bx)}{b}+\frac{\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Sech[c + b*x]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [C] time = 0.089, size = 207, normalized size = 4.6

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{b}+\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}+ie^{a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*
a)+1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*c)-1/2*I/b*ln(exp(b*x+
a)+I*exp(a-c))*exp(-a-c)*exp(2*a)-1/2*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-
c)*exp(2*c)
```

Maxima [B] time = 1.75529, size = 139, normalized size = 3.09

$$\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}-\frac{e^{(-bx-a)}}{2b}+\frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a)}+e^{(2c)}}{2b\left(e^{(-bx-a+2c)}+e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

Fricas [B] time = 2.2155, size = 2491, normalized size = 55.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 - 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\tanh^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**2, x)
```

Giac [A] time = 1.19028, size = 116, normalized size = 2.58

$$-\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(2e^{(2bx+2a)}-3e^{(2bx+2c)}-1\right)e^{(-a)}}{e^{(3bx+2c)}+e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
2*a) - 3*e^(2*b*x + 2*c) - 1)*e^(-a)/(e^(3*b*x + 2*c) + e^(b*x)) - e^(b*x +
 a))/b
```

3.157 $\int \cosh(a+bx)\tanh^3(c+bx)\,dx$

Optimal. Leaf size=72

$$-\frac{3\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}+\frac{\sinh(a-c)\tanh(bx+c)\mathbf{sech}(bx+c)}{2b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (3*ArcTan[Sinh[c + b*x]]*
Sinh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c]*Tanh[c + b*x])/(2*b)
```

Rubi [A] time = 0.0787169, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5623, 5620, 2638, 3770, 2606, 8, 2611}

$$-\frac{3\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\mathbf{sech}(bx+c)}{b}+\frac{\sinh(a-c)\tanh(bx+c)\mathbf{sech}(bx+c)}{2b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (3*ArcTan[Sinh[c + b*x]]*
Sinh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c]*Tanh[c + b*x])/(2*b)
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\tanh^3(c+bx)\,dx &= -\left(\sinh(a-c)\int \mathbf{sech}(c+bx)\tanh^2(c+bx)\,dx\right)+\int \sinh(a+bx)\tanh^2(c+bx)\,d\\ &= \frac{\mathbf{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}-\cosh(a-c)\int \mathbf{sech}(c+bx)\tanh(c+bx)\,d\\ &= -\frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{2b}+\frac{\mathbf{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}+\frac{\cosh}{}\\ &= \frac{\cosh(a+bx)}{b}+\frac{\cosh(a-c)\mathbf{sech}(c+bx)}{b}-\frac{3\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{2b}+\frac{\mathrm{se}}{}\end{aligned}$$

Mathematica [A] time = 0.308795, size = 115, normalized size = 1.6

$$\frac{\mathbf{sech}(c)\mathbf{sech}^2(bx+c)(-\cosh(a-bx-c))+\mathbf{sech}(c)\mathbf{sech}^2(bx+c)\cosh(a+bx-c)+\mathbf{sech}(c)\cosh(a-2c)\mathbf{sech}(bx+c)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (Cosh[a - 2*c]*Sech[c]*Sech[c + b*x] - Cosh[a - c - b*x]*Sech[c]*Sech[c + b
*x]^2 + Cosh[a - c + b*x]*Sech[c]*Sech[c + b*x]^2 + Cosh[a]*(4*Cosh[b*x] +
3*Sech[c]*Sech[c + b*x]) - 12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Sinh[
a - c] + 4*Sinh[a]*Sinh[b*x])/(4*b)
```

Maple [C] time = 0.091, size = 238, normalized size = 3.3

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c}+e^{2a}\right)^{2}}+\frac{\frac{3i}{4}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}\left(e^{a}\right)^{2}}{b}-\frac{\frac{3i}{4}\ln\left(e^{bx+a}-ie\cdots\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)+exp
(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(a)^2-3/4*I/b*ln(exp(b*x+a)-
I*exp(a-c))*exp(-a-c)*exp(c)^2-3/4*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*
exp(a)^2+3/4*I/b*ln(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(c)^2
```

Maxima [B] time = 1.7538, size = 201, normalized size = 2.79

$$\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b}+\frac{e^{(-bx-a)}}{2b}+\frac{\left(5\,e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)}+\left(2\,e^{(4a)}+3\,e^{(2a+2c)}\right)e^{(-4bx-4a)}+e^{(4c)}}{2b\left(e^{(-bx-a+4c)}+2\,e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) + 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

Fricas [B] time = 2.00538, size = 4772, normalized size = 66.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
+ 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 - 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 - 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
```

```
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\tanh^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**3, x)
```

Giac [A] time = 1.25087, size = 154, normalized size = 2.14

$$-\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(3e^{(3bx+2a+2c)}+e^{(3bx+4c)}+e^{(bx+2a)}+3e^{(bx+2c)}\right)e^{(-a)}}{\left(e^{(2bx+2c)}+1\right)^{2}}-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
2*a + 2*c) + e^(3*b*x + 4*c) + e^(b*x + 2*a) + 3*e^(b*x + 2*c))*e^(-a)/(e^(
2*b*x + 2*c) + 1)^2 - e^(b*x + a) - e^(-b*x - a))/b
```

3.158 $\int \cosh(a+bx)\coth(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b
```

Rubi [A] time = 0.0199443, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5621, 2638, 3770}

$$\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \cosh(a-c)\int \operatorname{\mathbf{csch}}(c+bx)\,dx + \int \sinh(a+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b}$$

Mathematica [C] time = 0.0538084, size = 93, normalized size = 3.21

$$-\frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b} + \frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [B] time = 0.042, size = 155, normalized size = 5.3

$$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*
exp(2*a)-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*c)+1/2/b*ln(exp(b*x+
a)-exp(a-c))*exp(-a-c)*exp(2*a)+1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp
(2*c)
```

Maxima [B] time = 1.07457, size = 122, normalized size = 4.21

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b + 1/2*e^(-b*x
 - a)/b
```

Fricas [B] time = 1.9522, size = 1233, normalized size = 42.52

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)^2 + \left(\cosh(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh
(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
+ 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)
^2 + 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sin
h(-a + c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*
(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) +
 cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a
+ c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sin
h(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\coth(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(b*x + c), x)
```

Giac [B] time = 1.18309, size = 115, normalized size = 3.97

$$-\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) - e^(b*x + a) - e^(-b
*x - a))/b
```

3.159 $\int \cosh(a+bx)\coth^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b +
 Sinh[a + b*x]/b
```

Rubi [A] time = 0.0426559, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5621, 5622, 2637, 3770, 2606, 8}

$$-\frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b +
 Sinh[a + b*x]/b
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\mathbf{csch}(c+bx)\,dx + \int \coth(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{(i\cosh(a-c))\,\mathrm{Subst}(\int 1\,dx, x, -i\mathbf{csch}(c+bx))}{b} + \sinh(a-c)\int \mathbf{csch}(c+bx)\,dx +$$
$$= -\frac{\cosh(a-c)\mathbf{csch}(c+bx)}{b} - \frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [C] time = 0.0955164, size = 110, normalized size = 2.39

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b} - \frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\cosh(bx)}{b} + \frac{\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) + (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((
Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh
[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sin
h[b*x])/b
```

Maple [B] time = 0.047, size = 195, normalized size = 4.2

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))-1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*a)+
1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*c)+1/2/b*ln(exp(b*x+a)-exp(a-
c))*exp(-a-c)*exp(2*a)-1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.15535, size = 194, normalized size = 4.22

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}-\frac{e^{(-bx-a)}}{2b}-\frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a)}-e^{(2c)}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

Fricas [B] time = 2.06523, size = 3363, normalized size = 73.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\coth^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(b*x + c)**2, x)
```

Giac [B] time = 1.18202, size = 169, normalized size = 3.67

$$-\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{\left(2e^{(2bx+2a)}+3e^{(2bx+2c)}-1\right)e^{(-a)}}{e^{(3bx+2c)}-e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + (2*e^(2*b*x + 2*a)
+ 3*e^(2*b*x + 2*c) - 1)*e^(-a)/(e^(3*b*x + 2*c) - e^(b*x)) - e^(b*x + a))/
b
```

3.160 $\int \cosh(a+bx)\coth^3(c+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{3\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{\cosh(a-c)\coth(bx+c)\mathbf{csch}(bx+c)}{2b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] (-3*ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/(2*b) + Cosh[a + b*x]/b - (Cosh[a -
 c]*Coth[c + b*x]*Csch[c + b*x])/(2*b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rubi [A] time = 0.0861191, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5621, 5622, 2638, 3770, 2606, 8, 2611}

$$-\frac{3\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{\cosh(a-c)\coth(bx+c)\mathbf{csch}(bx+c)}{2b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/(2*b) + Cosh[a + b*x]/b - (Cosh[a -
 c]*Coth[c + b*x]*Csch[c + b*x])/(2*b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \cosh(a-c)\int \coth^2(c+bx)\mathbf{csch}(c+bx)\,dx + \int \coth^2(c+bx)\sinh(a+bx)\,dx$$

$$= -\frac{\cosh(a-c)\coth(c+bx)\mathbf{csch}(c+bx)}{2b} + \frac{1}{2}\cosh(a-c)\int \mathbf{csch}(c+bx)\,dx + \sinh($$

$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{2b} - \frac{\cosh(a-c)\coth(c+bx)\mathbf{csch}(c+bx)}{2b} + \cosh$$

$$= -\frac{3\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{2b} + \frac{\cosh(a+bx)}{b} - \frac{\cosh(a-c)\coth(c+bx)\mathrm{cs}}{2b}$$

Mathematica [A] time = 0.330845, size = 70, normalized size = 0.96

$$\frac{\mathbf{csch}^2(bx+c)(2\cosh(a-bx-2c)+\cosh(a+3bx+2c)-5\cosh(a+bx))-12\cosh(a-c)\tanh^{-1}\left(\sinh(c)\tanh\left(\frac{bx}{2}\right)\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + (2*Cosh[a - 2*c
 - b*x] - 5*Cosh[a + b*x] + Cosh[a + 2*c + 3*b*x])*Csch[c + b*x]^2)/(4*b)
```

Maple [B] time = 0.056, size = 228, normalized size = 3.1

$$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c} + e^{2bx+2a+4c} + e^{4a} - 3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c} + e^{2a}\right)^2} - \frac{3\,\ln\left(e^{bx+a} + e^{a-c}\right)e^{-a-c}e^{2a}}{4b} - \frac{3\,\ln\left(e^{bx+a} + e^{a-c}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)+ex
p(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2-3/4/b*ln(exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*a)-3/4/b*ln(exp(b*x+a)+exp(
a-c))*exp(-a-c)*exp(2*c)+3/4/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*a)+3
/4/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.10902, size = 248, normalized size = 3.4

$$-\frac{3\left(e^{(2a)} + e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{4b} + \frac{3\left(e^{(2a)} + e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{4b} + \frac{e^{(-bx-a)}}{2b} - \frac{\left(5\,e^{(2a+2c)} - e^{(4c)}\right)e^{(-2bx-2a)}}{2b\left(e^{(-bx-a+4c)} -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) - 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

Fricas [B] time = 2.15675, size = 6496, normalized size = 88.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) + 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 - 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 + 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) - 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 + 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
 + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
```

```
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) + 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24403, size = 204, normalized size = 2.79

$$-\frac{3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(3e^{(3bx+2a+2c)}-e^{(3bx+4c)}-e^{(bx+2a)}+3e^{(b}\right.}{\left(e^{(2bx+2c)}-1\right)^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - 3*(e^(2
*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(3*e^(3*b*x +
 2*a + 2*c) - e^(3*b*x + 4*c) - e^(b*x + 2*a) + 3*e^(b*x + 2*c))*e^(-a)/(e^
(2*b*x + 2*c) - 1)^2 - 2*e^(b*x + a) - 2*e^(-b*x - a))/b
```

3.161 $\int \cosh(a+bx)\,\mathbf{sech}(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rubi [A] time = 0.014617, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5627, 3475, 8}

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = \cosh(a-c)\int 1\,dx + \sinh(a-c)\int \tanh(c+bx)\,dx$$
$$= x\cosh(a-c) + \frac{\log(\cosh(c+bx))\sinh(a-c)}{b}$$

Mathematica [A] time = 0.108671, size = 26, normalized size = 1.

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b} + x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Maple [B] time = 0.038, size = 146, normalized size = 5.6

$$xe^{a-c} - e^{-a-c}e^{2a}x + e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} + \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln\left(e^{2bx+2a} + e^{2a-2c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{2bx+2a} + e^{2a-2c}\right)e^{-a-c}e}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a+1/b*exp(-a-c)*exp(2*c)*a+1/2/b*ln(exp(2*b*x+2*a)+exp(2*a-2*c))*exp(-a-c)
*exp(2*a)-1/2/b*ln(exp(2*b*x+2*a)+exp(2*a-2*c))*exp(-a-c)*exp(2*c)
```

Maxima [A] time = 1.11685, size = 69, normalized size = 2.65

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-2bx)} + e^{(2c)}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

Fricas [B] time = 1.86597, size = 230, normalized size = 8.85

$$\frac{2\,bx+\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}-1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c), x)
```

Giac [A] time = 1.16317, size = 68, normalized size = 2.62

$$\frac{2\,bxe^{(-a+c)}+\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x +
 2*c) + 1))/b
```

3.162 $\int \cosh(a+bx)\mathbf{sech}^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}$$

```
[Out] (ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b - (Sech[c + b*x]*Sinh[a - c])/b
```

Rubi [A] time = 0.0308974, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5627, 2606, 8, 3770}

$$\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b - (Sech[c + b*x]*Sinh[a - c])/b
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\mathbf{sech}^2(c+bx)\,dx &= \cosh(a-c)\int \mathbf{sech}(c+bx)\,dx+\sinh(a-c)\int \mathbf{sech}(c+bx)\tanh(c+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b}-\frac{\sinh(a-c)\,\mathrm{Subst}(\int 1\,dx,x,\mathbf{sech}(c+bx))}{b}\\ &= \frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b}-\frac{\mathbf{sech}(c+bx)\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] time = 0.0822096, size = 83, normalized size = 2.37

$$\frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sinh(a-c)\mathbf{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/
2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b - (Sec
h[c + b*x]*Sinh[a - c])/b
```

Maple [C] time = 0.08, size = 183, normalized size = 5.2

$$-\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\frac{i}{2}\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{b}+\frac{\frac{i}{2}\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{b}-\frac{\frac{i}{2}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c)^2,x)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I/b*l
n(exp(b*x+a)+I*exp(a-c))*exp(-a-c)*exp(2*a)+1/2*I/b*ln(exp(b*x+a)+I*exp(a-c
))*exp(-a-c)*exp(2*c)-1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*a)-
```

```
1/2*I/b*ln(exp(b*x+a)-I*exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.63424, size = 95, normalized size = 2.71

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) - e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

Fricas [B] time = 1.83236, size = 1129, normalized size = 32.26

$$\frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-\cosh(bx+c)\sinh(-a+c)^2+\left(\left(\cosh(-a+c)^2+1\right)\cosh(bx+c)^2+\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) +
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 - 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\operatorname{sech}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**2, x)
```

Giac [A] time = 1.14912, size = 92, normalized size = 2.63

$$\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(e^{(bx+2a)}-e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="giac")
```

```
[Out] ((e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) - e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

3.163 $\int \cosh(a+bx)\mathbf{sech}^3(c+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh(a-c)\tanh(bx+c)}{b}-\frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}$$

```
[Out] -(Sech[c + b*x]^2*Sinh[a - c])/(2*b) + (Cosh[a - c]*Tanh[c + b*x])/b
```

Rubi [A] time = 0.0418814, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5627, 2606, 30, 3767, 8}

$$\frac{\cosh(a-c)\tanh(bx+c)}{b}-\frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^3,x]
```

```
[Out] -(Sech[c + b*x]^2*Sinh[a - c])/(2*b) + (Cosh[a - c]*Tanh[c + b*x])/b
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = \cosh(a-c)\int \text{sech}^2(c+bx)\,dx + \sinh(a-c)\int \text{sech}^2(c+bx)\tanh(c+bx)\,dx$$
$$= \frac{(i\cosh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\tanh(c+bx))}{b} - \frac{\sinh(a-c)\,\text{Subst}(\int x\,dx, x, \text{sech}(}{b}$$
$$= -\frac{\text{sech}^2(c+bx)\sinh(a-c)}{2b} + \frac{\cosh(a-c)\tanh(c+bx)}{b}$$

Mathematica [A] time = 0.167805, size = 35, normalized size = 0.92

$$-\frac{\text{sech}(c)\text{sech}^2(bx+c)(\sinh(a)-\cosh(a-c)\sinh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^3,x]
```

```
[Out] -(Sech[c]*Sech[c + b*x]^2*(Sinh[a] - Cosh[a - c]*Sinh[c + 2*b*x]))/(2*b)
```

Maple [A] time = 0.03, size = 56, normalized size = 1.5

$$-\frac{\left(2\,e^{2\,bx+2\,a+2\,c}+e^{2\,a}+e^{2\,c}\right)e^{3\,a-c}}{b\left(e^{2\,bx+2\,a+2\,c}+e^{2\,a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c)^3,x)
```

```
[Out] -(2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2
*exp(3*a-c)
```

Maxima [B] time = 1.101, size = 161, normalized size = 4.24

$$\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

Fricas [B] time = 1.82322, size = 644, normalized size = 16.95

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}+3\,b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)\sinh\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -2*(2*cosh(b*x + c)*cosh(-a + c) - cosh(b*x + c)*sinh(-a + c) - sinh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(a+bx\right)\operatorname{sech}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**3, x)
```

Giac [A] time = 1.18868, size = 66, normalized size = 1.74

$$-\frac{\left(2\,e^{\left(2\,bx+2\,a+2\,c\right)}+e^{\left(2\,a\right)}+e^{\left(2\,c\right)}\right)e^{\left(-a-c\right)}}{b\left(e^{\left(2\,bx+2\,c\right)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

3.164 $\int \cosh(a+bx)\mathbf{csch}(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b}+x\sinh(a-c)$$

[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]

Rubi [A] time = 0.0147932, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5625, 3475, 8}

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]*Csch[c + b*x],x]

[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh(a+bx)\mathbf{csch}(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\,dx + \sinh(a-c)\int 1\,dx$$
$$= \frac{\cosh(a-c)\log(\sinh(c+bx))}{b} + x\sinh(a-c)$$

Mathematica [A] time = 0.107218, size = 26, normalized size = 1.

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b} + x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

Maple [B] time = 0.036, size = 152, normalized size = 5.9

$$xe^{a-c} - e^{-a-c}e^{2a}x - e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} - \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln\left(e^{2bx+2a} - e^{2a-2c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{2bx+2a} - e^{2a-2c}\right)e^{-a-c}e^{2c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2/b*ln(exp(2*b*x+2*a)-exp(2*a-2*c))*exp(-a-c)
*exp(2*a)+1/2/b*ln(exp(2*b*x+2*a)-exp(2*a-2*c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.11762, size = 108, normalized size = 4.15

$$\frac{\left(e^{(2a)} + e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2b} + \frac{\left(e^{(2a)} + e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) + e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

Fricas [B] time = 1.85695, size = 231, normalized size = 8.88

$$-\frac{2\,bx-\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}+1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c), x)
```

Giac [A] time = 1.19731, size = 68, normalized size = 2.62

$$-\frac{2\,bxe^{(-a+c)}-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*
b*x + 2*c) - 1)))/b
```

3.165 $\int \cosh(a+bx)\mathbf{csch}^2(c+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rubi [A] time = 0.0306922, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5625, 2606, 8, 3770}

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\mathbf{csch}^2(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\mathbf{csch}(c+bx)\,dx + \sinh(a-c)\int \mathbf{csch}(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b} - \frac{(i\cosh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\mathbf{csch}(c+bx$$
$$= -\frac{\cosh(a-c)\mathbf{csch}(c+bx)}{b} - \frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b}$$

Mathematica [C] time = 0.0682005, size = 90, normalized size = 2.5

$$-\frac{\cosh(a-c)\mathbf{csch}(bx+c)}{b} - \frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh
[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Co
sh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b
```

Maple [B] time = 0.039, size = 170, normalized size = 4.7

$$\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c)^2,x)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2/b*ln(
exp(b*x+a)+exp(a-c))*exp(-a-c)*exp(2*a)+1/2/b*ln(exp(b*x+a)+exp(a-c))*exp(-
a-c)*exp(2*c)+1/2/b*ln(exp(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*a)-1/2/b*ln(exp
```

```
(b*x+a)-exp(a-c))*exp(-a-c)*exp(2*c)
```

Maxima [B] time = 1.0882, size = 142, normalized size = 3.94

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) + e^(2*c))*e^(-b*x - a
)/(b*(e^(-2*b*x) - e^(2*c)))
```

Fricas [B] time = 1.8243, size = 1709, normalized size = 47.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c) - ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
```

```
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\operatorname{csch}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c)**2, x)
```

Giac [B] time = 1.19119, size = 143, normalized size = 3.97

$$-\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}-1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) +
e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

3.166 $\int \cosh(a+bx)\mathbf{csch}^3(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cosh(a-c)\mathbf{csch}^2(bx+c)}{2b}-\frac{\sinh(a-c)\coth(bx+c)}{b}$$

```
[Out] -(Cosh[a - c]*Csch[c + b*x]^2)/(2*b) - (Coth[c + b*x]*Sinh[a - c])/b
```

Rubi [A] time = 0.0430111, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5625, 2606, 30, 3767, 8}

$$-\frac{\cosh(a-c)\mathbf{csch}^2(bx+c)}{2b}-\frac{\sinh(a-c)\coth(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

```
[Out] -(Cosh[a - c]*Csch[c + b*x]^2)/(2*b) - (Coth[c + b*x]*Sinh[a - c])/b
```

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\operatorname{csch}^2(c+bx)\,dx + \sinh(a-c)\int \operatorname{csch}^2(c+bx)\,dx$$
$$= \frac{\cosh(a-c)\,\text{Subst}(\int x\,dx, x, -i\operatorname{csch}(c+bx))}{b} - \frac{(i\sinh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\operatorname{co}}{b}$$
$$= -\frac{\cosh(a-c)\operatorname{csch}^2(c+bx)}{2b} - \frac{\coth(c+bx)\sinh(a-c)}{b}$$

Mathematica [A] time = 0.169359, size = 35, normalized size = 0.9

$$-\frac{\operatorname{csch}(c)\operatorname{csch}^2(bx+c)(\sinh(a)-\sinh(a-c)\cosh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

```
[Out] -(Csch[c]*Csch[c + b*x]^2*(Sinh[a] - Cosh[c + 2*b*x]*Sinh[a - c]))/(2*b)
```

Maple [A] time = 0.032, size = 59, normalized size = 1.5

$$\frac{\left(-2\,e^{2\,bx+2\,a+2\,c}+e^{2\,a}-e^{2\,c}\right)e^{3\,a-c}}{b\left(-e^{2\,bx+2\,a+2\,c}+e^{2\,a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c)^3,x)
```

```
[Out] (-2*exp(2*b*x+2*a+2*c)+exp(2*a)-exp(2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2*exp(3*a-c)
```

Maxima [B] time = 1.06859, size = 178, normalized size = 4.56

$$\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}-\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

Fricas [B] time = 1.83185, size = 620, normalized size = 15.9

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}-b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)\sinh\left(bx+c\right)^{3}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + (cosh(-a + c) - 2*sinh(-a + c))*sinh(b*x +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(a+bx\right)\operatorname{csch}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c)**3, x)
```

Giac [A] time = 1.1989, size = 69, normalized size = 1.77

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}-e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

3.167 $\int \sinh(a+bx)\sinh(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

```
[Out] -Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0421003, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5613, 2637}

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x],x]
```

```
[Out] -Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \int \left(-\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)dx$$
$$= -\left(\frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx\right)+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx$$
$$= -\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

Mathematica [A] time = 0.21916, size = 43, normalized size = 1.

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x],x]
```

```
[Out] -Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.018, size = 40, normalized size = 0.9

$$-\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c),x)
```

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86633, size = 170, normalized size = 3.95

$$-\frac{d\cosh(dx+c)\sinh(bx+a)-b\cosh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(d*x + c)*sinh(b*x + a) - b*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^
2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 2.02849, size = 153, normalized size = 3.56

$$\begin{cases} x\sinh(a)\sinh(c) & \text{for } b=0\wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2}+\frac{x\cosh(a-dx)\cosh(c+dx)}{2}+\frac{\sinh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sinh(a+dx)\sinh(c+dx)}{2}-\frac{x\cosh(a+dx)\cosh(c+dx)}{2}+\frac{\sinh(a+dx)\cosh(c+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(c+dx)\cosh(a+bx)}{b^2-d^2}-\frac{d\sinh(a+bx)\cosh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 + sinh(a - d*x)*cosh(c + d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*sinh(c + d*x)/2 - x*cosh(a + d*x)*cosh(c
 + d*x)/2 + sinh(a + d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*sinh(c + d*x)*
cosh(a + b*x)/(b**2 - d**2) - d*sinh(a + b*x)*cosh(c + d*x)/(b**2 - d**2),
True))
```

Giac [B] time = 1.16566, size = 115, normalized size = 2.67

$$\frac{e^{(bx+dx+a+c)}}{4(b+d)}-\frac{e^{(bx-dx+a-c)}}{4(b-d)}+\frac{e^{(-bx+dx-a+c)}}{4(b-d)}-\frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) - 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.168 $\int \sinh(a+bx)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

```
[Out] -Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0626973, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5613, 2638}

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

```
[Out] -Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \int \left(-\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)x)\right) dx$$
$$= \frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx + \frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx - \frac{1}{2}\int \sinh(a+$$
$$= -\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] time = 0.76852, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cosh(a+bx-2c-2dx)}{b-2d}+\frac{\cosh(a+bx+2c+2dx)}{b+2d}-\frac{2\sinh(a)\sinh(bx)}{b}-\frac{2\cosh(a)\cosh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

```
[Out] ((-2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a
+ 2*c + b*x + 2*d*x]/(b + 2*d) - (2*Sinh[a]*Sinh[b*x])/b)/4
```

Maple [A] time = 0.013, size = 57, normalized size = 0.9

$$-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^2,x)
```

```
[Out] -1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95177, size = 304, normalized size = 4.9

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2 - 4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c) + b^2\cosh(bx+a)\sinh(dx+c)^2 - \left(b^2 - 4\right.}{2\left(\left(b^3 - 4\,bd^2\right)\cosh(bx+a)^2 - \left(b^3 - 4\,bd^2\right)\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 9.54679, size = 401, normalized size = 6.47

$$\begin{cases} x\sinh(a)\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} - \frac{3\sinh^2(c+dx)\cosh(a-2dx)}{8d} + \frac{\cosh(a-2dx)\cos}{8d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} - \frac{\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{4d} + \sinh^2(\\ \frac{b^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} - \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} + \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2
/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq(
b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 - 3*sinh(c + d*
x)**2*cosh(a - 2*d*x)/(8*d) + cosh(a - 2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b,
 -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d*x)*cosh(c +
 d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 - sinh(a + 2*d
```

```
*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a + 2*d*x)/(2
*d), Eq(b, 2*d)), (b**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) -
2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*
sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cosh(a + b*x)*cos
h(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

Giac [B] time = 1.17574, size = 162, normalized size = 2.61

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)}-\frac{e^{(bx+a)}}{4\,b}+\frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)}+\frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

3.169 $\int \sinh(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=91

$$-\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}-\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8
*(b - d)) - (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3
*d)*x]/(8*(b + 3*d))
```

Rubi [A] time = 0.0746128, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5613, 2637}

$$-\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}-\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```


```
[Out] -Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8
*(b - d)) - (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3
*d)*x]/(8*(b + 3*d))
```


Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \int \left(-\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)-\frac{3}{8}\cosh(a+c+(b+d$$

$$= -\left(\frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \cos$$

$$= -\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+$$

Mathematica [A] time = 0.465432, size = 86, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\sinh(a+bx-3c-3dx)}{b-3d}+\frac{3\sinh(a+bx-c-dx)}{b-d}+\frac{\sinh(a+bx+3c+3dx)}{b+3d}-\frac{3\sinh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

```
[Out] (-(Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sinh[a - c + b*x - d*x])/(b
- d) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sinh[a + c + (b + d)*x])/
(b + d))/8
```

Maple [A] time = 0.015, size = 84, normalized size = 0.9

$$-\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8b-8d}-\frac{3\sinh(a+c+(b+d)x)}{8b+8d}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^3,x)
```

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90875, size = 516, normalized size = 5.67

$$-\frac{9\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(b^3-bd^2\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+3\left(\left(b^2d-d^3\right)\cosh\left(dx\right.\right.}{4\left(\left(b^4-10\,b^2d^2+9\,d^4\right)\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*(b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3 -
b*d^2)*cosh(b*x + a)*sinh(d*x + c)^3 + 3*((b^2*d - d^3)*cosh(d*x + c)^3 - (
b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b^3 - b*d^2)*cosh(b*x + a
)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a))*sinh(d*x + c))/((b^4 - 1
0*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x +
a)^2)
```

Sympy [A] time = 50.452, size = 932, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 + sinh(a - 3*d*x)*cosh(c + d*x)**3/(8*d) - 7*sinh(c + d*x)
**3*cosh(a - 3*d*x)/(24*d) + sinh(c + d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2
/(4*d), Eq(b, -3*d)), (3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a -
d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)*
cosh(c + d*x)/8 - 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a - d*x)*co
sh(c + d*x)**3/(8*d) - 5*sinh(c + d*x)**3*cosh(a - d*x)/(8*d) + 3*sinh(c +
d*x)*cosh(a - d*x)*cosh(c + d*x)**2/(4*d), Eq(b, -d)), (3*x*sinh(a + d*x)*s
```

```
inh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*
x*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(c
 + d*x)**3/8 + 3*sinh(a + d*x)*cosh(c + d*x)**3/(8*d) + 5*sinh(c + d*x)**3*
cosh(a + d*x)/(8*d) - 3*sinh(c + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(4*d),
 Eq(b, d)), (x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a + 3*d*x)*sin
h(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c
 + d*x)/8 - x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 + 7*sinh(a + 3*d*x)*sinh(c
 + d*x)**2*cosh(c + d*x)/(8*d) + 5*sinh(a + 3*d*x)*cosh(c + d*x)**3/(12*d)
- 9*sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, 3*d)), (b**
3*sinh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*
sinh(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4)
 - 7*b*d**2*sinh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) +
 6*b*d**2*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2
 + 9*d**4) + 9*d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10
*b**2*d**2 + 9*d**4) - 6*d**3*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**
2*d**2 + 9*d**4), True))
```

Giac [B] time = 1.27518, size = 242, normalized size = 2.66

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} + \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} + \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) - 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

3.170 $\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] x/4 - Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
- Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.0685166, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] x/4 - Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
- Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \int \left(\frac{1}{4} - \frac{1}{4}\cosh(2a+2bx) + \frac{1}{8}\cosh(2(a-c)+2(b-d)x) - \frac{1}{4}\cosh(2c+2dx) + \frac{1}{8}\right.$$
$$= \frac{x}{4} + \frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx + \frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx - \frac{1}{4}\int$$
$$= \frac{x}{4} - \frac{\sinh(2a+2bx)}{8b} + \frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} - \frac{\sinh(2c+2dx)}{8d} + \frac{\sinh(2(a+c}{16(}$$

Mathematica [A] time = 0.723742, size = 106, normalized size = 1.2

$$\frac{\left(2d^3-2b^2d\right)\sinh(2(a+bx)) + bd(b+d)\sinh(2(a+x(b-d)-c)) + b(b-d)(d(\sinh(2(a+x(b+d)+c))+4x(b+d)) -}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)
*x)] + b*(b - d)*(-2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a
 + c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.023, size = 83, normalized size = 0.9

$$\frac{x}{4} - \frac{\sinh(2bx+2a)}{8b} - \frac{\sinh(2dx+2c)}{8d} + \frac{\sinh((2b-2d)x+2a-2c)}{16b-16d} + \frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^2,x)
```

```
[Out] 1/4*x-1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d)
*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88068, size = 455, normalized size = 5.17

$$\frac{b^2 d \cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2 + \left(b^3 d - b d^3\right)x + \left(b^2 d \cosh(bx+a)\cosh(dx+c)^2 - \left(b^2 d - d^3\right)\cosh(bx+\right.}{4\left(\left(b^3 d - b d^3\right)\cosh(bx+a)^2 - \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 - (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 +
b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

Sympy [A] time = 37.4244, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)
**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)**
2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**2
*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c +
d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh(
c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) - sinh(a
 - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(8*d) + 3*sinh(a - d*x)*cosh(a - d*x
)*cosh(c + d*x)**2/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)**
2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x)
*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3*
x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)**2*sinh(c + d*x)*cosh
```

```
(c + d*x)/(2*d) + sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)/(8*d) - 3*si
nh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d), Eq(b, d)), ((x*sinh(a + b
*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*sinh(c
)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*
b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3)
- b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d
*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(a +
b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c + d
*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(a
+ b*x)*sinh(c + d*x)**2*cosh(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh
(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b*x
)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*cos
h(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c + d*
x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*cosh(
c + d*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a
 + b*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d
*x)**2/(4*b**3*d - 4*b*d**3), True))
```

Giac [B] time = 1.2311, size = 211, normalized size = 2.4

$$\frac{1}{4}x+\frac{e^{(2bx+2dx+2a+2c)}}{32(b+d)}+\frac{e^{(2bx-2dx+2a-2c)}}{32(b-d)}-\frac{e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx+2dx-2a+2c)}}{32(b-d)}-\frac{e^{(-2bx-2dx-2a-2c)}}{32(b+d)}+\frac{e^{(-2bx-2a)}}{16b}-\frac{e^{(2dx+2c)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) + 1/16*e^(-2
*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

3.171 $\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=144

$$-\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\cosh(2a+x(2b+3d)+3c)}{16(2b+3d)}+$$

```
[Out] -Cosh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Cosh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) + (3*Cosh[c + d*x])/(8*d) - Cosh[3*c + 3*d*x]/(24*d
) - (3*Cosh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Cosh[2*a + 3*c + (2*b
+ 3*d)*x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.121526, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2638}

$$-\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\cosh(2a+x(2b+3d)+3c)}{16(2b+3d)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```


```
[Out] -Cosh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Cosh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) + (3*Cosh[c + d*x])/(8*d) - Cosh[3*c + 3*d*x]/(24*d
) - (3*Cosh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Cosh[2*a + 3*c + (2*b
+ 3*d)*x]/(16*(2*b + 3*d))
```


Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \int \left(-\frac{1}{16}\sinh(2a-3c+(2b-3d)x) + \frac{3}{16}\sinh(2a-c+(2b-d)x) + \frac{3}{8}\sinh(c+dx\right.$$
$$= -\left(\frac{1}{16}\int \sinh(2a-3c+(2b-3d)x)\,dx\right) + \frac{1}{16}\int \sinh(2a+3c+(2b+3d)x)\,dx - \frac{1}{8}$$
$$= -\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\cosh(c+dx)}{8d} - \frac{\cos}{}$$

Mathematica [A] time = 1.62358, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(-\frac{3\cosh(2a+2bx-3c-3dx)}{2b-3d} + \frac{9\cosh(2a+2bx-c-dx)}{2b-d} - \frac{9\cosh(2a+2bx+c+dx)}{2b+d} + \frac{3\cosh(2a+2bx+3c+}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[c]*Cosh[d*x])/d - (2*Cosh[3*c]*Cosh[3*d*x])/d - (3*Cosh[2*a - 3*c
 + 2*b*x - 3*d*x])/(2*b - 3*d) + (9*Cosh[2*a - c + 2*b*x - d*x])/(2*b - d)
- (9*Cosh[2*a + c + 2*b*x + d*x])/(2*b + d) + (3*Cosh[2*a + 3*c + 2*b*x + 3
*d*x])/(2*b + 3*d) + (18*Sinh[c]*Sinh[d*x])/d - (2*Sinh[3*c]*Sinh[3*d*x])/d
)/48
```

Maple [A] time = 0.017, size = 133, normalized size = 0.9

$$-\frac{\cosh(2a-3c+(2b-3d)x)}{32b-48d} + \frac{3\cosh(2a-c+(2b-d)x)}{32b-16d} + \frac{3\cosh(dx+c)}{8d} - \frac{\cosh(3dx+3c)}{24d} - \frac{3\cosh(2a+c}{32b+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^3,x)
```

```
[Out] -1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*b+
d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89329, size = 961, normalized size = 6.67

$$\frac{12\left(4\,b^3d-bd^3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^3-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-d^4\right)\cosh\left(bx+a\right)^2\right)\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(4*b^3*d - b*d^3)*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^3 - (1
6*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x
+ c)^3 - 9*((4*b^2*d^2 - d^4)*cosh(d*x + c)^3 - (4*b^2*d^2 - 9*d^4)*cosh(d*
x + c))*sinh(b*x + a)^2 + 36*((4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)
^2 - (4*b^3*d - 9*b*d^3)*cosh(b*x + a))*sinh(b*x + a)*sinh(d*x + c) - 3*(9*
(4*b^2*d^2 - d^4)*cosh(d*x + c)*sinh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*
d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
 9*(16*b^4 - 40*b^2*d^2 + 9*d^4 + (4*b^2*d^2 - 9*d^4)*cosh(b*x + a)^2)*cosh
(d*x + c))/((16*b^4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 4
0*b^2*d^3 + 9*d^5)*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20852, size = 351, normalized size = 2.44

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}-\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}-\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}+\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) + 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) + 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

3.172 $\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}-\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh}{}$$

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) - Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) - (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) - (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.144429, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2637}

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}-\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) - Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) - (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) - (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x) - \frac{9}{32}\cosh(a-c+(b-d)x) - \frac{1}{32}\cosh(3(a-c)+\right.$$
$$= -\left(\frac{1}{32}\int \cosh(3(a-c)+3(b-d)x)\,dx\right) + \frac{1}{32}\int \cosh(3(a+c)+3(b+d)x)\,dx + \frac{3}{3}$$
$$= \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} - \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)}$$

Mathematica [A] time = 1.57155, size = 177, normalized size = 0.91

$$\frac{1}{96}\left(\frac{9\sinh(a+bx-3c-3dx)}{b-3d} - \frac{27\sinh(a+bx-c-dx)}{b-d} - \frac{\sinh(3(a+bx-c-dx))}{b-d} + \frac{9\sinh(3a+3bx-c-dx)}{3b-d} - \frac{9}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

```
[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Sinh[a - c + b*x - d*x])/(
b - d) - Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*
x])/(3*b - d) - (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Sinh[a + 3*c
 + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*
(a + c + (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.023, size = 184, normalized size = 0.9

$$\frac{3\sinh(a-3c+(b-3d)x)}{32b-96d} - \frac{9\sinh(a-c+(b-d)x)}{32b-32d} + \frac{9\sinh(a+c+(b+d)x)}{32b+32d} - \frac{3\sinh(a+3c+(b+3d)x)}{32b+96d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*sinh(d*x+c)^3,x)
```

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a
+c+(b+d)*x)/(b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/96/(b-d)*sinh((3*b-3
*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)-3/32*sinh(3*a+c+(3*b+d)*x
)/(3*b+d)+1/96/(b+d)*sinh((3*b+3*d)*x+3*a+3*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08868, size = 1710, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)^3 - 9*(b^4*d - 10*b^2*
d^3 + 9*d^5)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^
4)*cosh(b*x + a)^3 + 3*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*sinh(b*
x + a)^2 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(d*x + c)^3 +
3*((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(81*b^4
*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*c
osh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 - 3*((81*b^4*d - 90*b^2*d^3 + 9
*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^3 - 9*
(9*b^4*d - 82*b^2*d^3 + 9*d^5 - (b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^
2)*cosh(d*x + c))*sinh(b*x + a) + 3*(9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*
x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*
b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^2 - 3*((9*b^5 - 82*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^2 - 9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cos
h(b*x + a))*sinh(b*x + a)^2 - 9*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a
))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^
4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x +
a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24565, size = 504, normalized size = 2.58

$$\frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} - \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} - \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} - \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} + \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} - \frac{9\,e^{(bx-dx+a}}{64\,(b-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) - 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) - 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) + 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) + 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

3.173 $\int \cosh(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0398964, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x],x]
```

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \int \left(\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)dx$$
$$= \frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx$$
$$= \frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

Mathematica [A] time = 0.173012, size = 43, normalized size = 1.

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x],x]
```

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.01, size = 40, normalized size = 0.9

$$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92083, size = 169, normalized size = 3.93

$$\frac{b\cosh(dx+c)\sinh(bx+a)-d\cosh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)*sinh(b*x + a) - d*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 2.04283, size = 153, normalized size = 3.56

$$\begin{cases} x\cosh(a)\cosh(c) & \text{for } b=0\wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2}+\frac{x\cosh(a-dx)\cosh(c+dx)}{2}-\frac{\sinh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ -\frac{x\sinh(a+dx)\sinh(c+dx)}{2}+\frac{x\cosh(a+dx)\cosh(c+dx)}{2}+\frac{\sinh(a+dx)\cosh(c+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(a+bx)\cosh(c+dx)}{b^2-d^2}-\frac{d\sinh(c+dx)\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 - sinh(a - d*x)*cosh(c + d*x)/(
2*d), Eq(b, -d)), (-x*sinh(a + d*x)*sinh(c + d*x)/2 + x*cosh(a + d*x)*cosh(
c + d*x)/2 + sinh(a + d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*sinh(a + b*x)
*cosh(c + d*x)/(b**2 - d**2) - d*sinh(c + d*x)*cosh(a + b*x)/(b**2 - d**2),
 True))
```

Giac [B] time = 1.17167, size = 115, normalized size = 2.67

$$\frac{e^{(bx+dx+a+c)}}{4(b+d)}+\frac{e^{(bx-dx+a-c)}}{4(b-d)}-\frac{e^{(-bx+dx-a+c)}}{4(b-d)}-\frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) - 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.174 $\int \cosh(a+bx)\cosh^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sinh(a+bx)}{2b}$$

```
[Out] Sinh[a + b*x]/(2*b) + Sinh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sinh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0550744, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sinh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

```
[Out] Sinh[a + b*x]/(2*b) + Sinh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sinh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \int \left(\frac{1}{2}\cosh(a+bx) + \frac{1}{4}\cosh(a-2c+(b-2d)x) + \frac{1}{4}\cosh(a+2c+(b+2d)x)\right) dx$$
$$= \frac{1}{4}\int \cosh(a-2c+(b-2d)x)\,dx + \frac{1}{4}\int \cosh(a+2c+(b+2d)x)\,dx + \frac{1}{2}\int \cosh(a$$
$$= \frac{\sinh(a+bx)}{2b} + \frac{\sinh(a-2c+(b-2d)x)}{4(b-2d)} + \frac{\sinh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] time = 0.688158, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\sinh(a+bx-2c-2dx)}{b-2d} + \frac{\sinh(a+bx+2c+2dx)}{b+2d} + \frac{2\sinh(a)\cosh(bx)}{b} + \frac{2\cosh(a)\sinh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

```
[Out] ((2*Cosh[b*x]*Sinh[a])/b + (2*Cosh[a]*Sinh[b*x])/b + Sinh[a - 2*c + b*x - 2
*d*x]/(b - 2*d) + Sinh[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

Maple [A] time = 0.019, size = 57, normalized size = 0.9

$$\frac{\sinh(bx+a)}{2b} + \frac{\sinh(a-2c+(b-2d)x)}{4b-8d} + \frac{\sinh(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c)^2,x)
```

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90188, size = 286, normalized size = 4.61

$$-\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)-b^{2}\sinh\left(bx+a\right)\sinh\left(dx+c\right)^{2}-\left(b^{2}\cosh\left(dx+c\right)^{2}+b^{2}-4\,d^{2}\right)\sinh\left(bx\right.}{2\left(\left(b^{3}-4\,bd^{2}\right)\cosh\left(bx+a\right)^{2}-\left(b^{3}-4\,bd^{2}\right)\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(d*x + c) - b^2*sinh(b*x + a)*s
inh(d*x + c)^2 - (b^2*cosh(d*x + c)^2 + b^2 - 4*d^2)*sinh(b*x + a))/((b^3 -
 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 8.7833, size = 398, normalized size = 6.42

$$\begin{cases} x\cosh\left(a\right)\cosh^{2}\left(c\right) \\ \left(-\frac{x\sinh^{2}\left(c+dx\right)}{2}+\frac{x\cosh^{2}\left(c+dx\right)}{2}+\frac{\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}\right)\cosh\left(a\right) \\ \frac{x\sinh\left(a-2dx\right)\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2}+\frac{x\sinh^{2}\left(c+dx\right)\cosh\left(a-2dx\right)}{4}+\frac{x\cosh\left(a-2dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{\sinh\left(a-2dx\right)\sinh^{2}\left(c+dx\right)}{8d}-\frac{3\sinh\left(a-2dx\right)\cos\ldots}{8d} \\ -\frac{x\sinh\left(a+2dx\right)\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2}+\frac{x\sinh^{2}\left(c+dx\right)\cosh\left(a+2dx\right)}{4}+\frac{x\cosh\left(a+2dx\right)\cosh^{2}\left(c+dx\right)}{4}-\frac{\sinh\left(a+2dx\right)\sinh^{2}\left(c+dx\right)}{8d}+\frac{3\sinh\left(a+2dx\right)\ldots}{8d} \\ \frac{b^{2}\sinh\left(a+bx\right)\cosh^{2}\left(c+dx\right)}{b^{3}-4bd^{2}}-\frac{2bd\sinh\left(c+dx\right)\cosh\left(a+bx\right)\cosh\left(c+dx\right)}{b^{3}-4bd^{2}}+\frac{2d^{2}\sinh\left(a+bx\right)\sinh^{2}\left(c+dx\right)}{b^{3}-4bd^{2}}-\frac{2d^{2}\sinh\left(a+bx\right)\cosh^{2}\left(c+dx\right)}{b^{3}-4bd^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**
2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a), Eq
(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d*x)
**2*cosh(a - 2*d*x)/4 + x*cosh(a - 2*d*x)*cosh(c + d*x)**2/4 + sinh(a - 2*d
*x)*sinh(c + d*x)**2/(8*d) - 3*sinh(a - 2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b
, -2*d)), (-x*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d*
x)**2*cosh(a + 2*d*x)/4 + x*cosh(a + 2*d*x)*cosh(c + d*x)**2/4 - sinh(a + 2
```

```
*d*x)*sinh(c + d*x)**2/(8*d) + 3*sinh(a + 2*d*x)*cosh(c + d*x)**2/(8*d), Eq
(b, 2*d)), (b**2*sinh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*s
inh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sinh(a
+ b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*sinh(a + b*x)*cosh(c + d
*x)**2/(b**3 - 4*b*d**2), True))
```

Giac [B] time = 1.21939, size = 162, normalized size = 2.61

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} + \frac{e^{(bx+a)}}{4\,b} - \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} - \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} - \frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b - 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

3.175 $\int \cosh(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}+\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rubi [A] time = 0.0693878, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}+\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)+\frac{3}{8}\cosh(a+c+(b+d)\right.$$
$$= \frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \cosh(a$$
$$= \frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\mathrm{si}}{}$$

Mathematica [A] time = 0.427321, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\sinh(a+bx-3c-3dx)}{b-3d}+\frac{3\sinh(a+bx-c-dx)}{b-d}+\frac{\sinh(a+bx+3c+3dx)}{b+3d}+\frac{3\sinh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] (Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sinh[a - c + b*x - d*x])/(b - d
) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sinh[a + c + (b + d)*x])/(b
+ d))/8
```

Maple [A] time = 0.011, size = 84, normalized size = 0.9

$$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8b-8d}+\frac{3\sinh(a+c+(b+d)x)}{8b+8d}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c)^3,x)
```

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90776, size = 520, normalized size = 5.71

$$\frac{3\left(b^{3}-bd^{2}\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^{2}-3\left(b^{2}d-d^{3}\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^{3}+\left(\left(b^{3}-bd^{2}\right)\cosh\left(dx\right.\right.}{4\left(\left(b^{4}-10\,b^{2}d^{2}+9\,d^{4}\right)\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - 3*(b^2*d
 - d^3)*cosh(b*x + a)*sinh(d*x + c)^3 + ((b^3 - b*d^2)*cosh(d*x + c)^3 + 3*
(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^2*d - d^3)*cosh(b*x
+ a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a))*sinh(d*x + c))/((b^4
- 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x
 + a)^2)
```

Sympy [A] time = 46.3635, size = 926, normalized size = 10.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 - 3*sinh(a - 3*d*x)*cosh(c + d*x)**3/(8*d) + sinh(c + d*x)
**3*cosh(a - 3*d*x)/(24*d) - sinh(c + d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2
/(4*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a -
 d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a - d*x)
*cosh(c + d*x)/8 + 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + sinh(a - d*x)*cos
h(c + d*x)**3/(8*d) - 3*sinh(c + d*x)**3*cosh(a - d*x)/(8*d) + 3*sinh(c + d
*x)*cosh(a - d*x)*cosh(c + d*x)**2/(4*d), Eq(b, -d)), (3*x*sinh(a + d*x)*si
```

```
nh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x
*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(c
+ d*x)**3/8 - sinh(a + d*x)*cosh(c + d*x)**3/(8*d) - 3*sinh(c + d*x)**3*cos
h(a + d*x)/(8*d) + 3*sinh(c + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(4*d), Eq
(b, d)), (-x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a + 3*d*x)*sinh(
c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c +
 d*x)/8 + x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 + sinh(a + 3*d*x)*sinh(c + d
*x)**2*cosh(c + d*x)/(8*d) + 5*sinh(a + 3*d*x)*cosh(c + d*x)**3/(12*d) - 3*
sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, 3*d)), (b**3*si
nh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh
(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6
*b*d**2*sinh(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 +
 9*d**4) - 7*b*d**2*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9
*d**4) - 6*d**3*sinh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**
4) + 9*d**3*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d*
*2 + 9*d**4), True))
```

Giac [B] time = 1.2004, size = 242, normalized size = 2.66

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)}-\frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)}-\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}-\frac{e^{(-}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) - 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

3.176 $\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
+ Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.0668682, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
+ Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)+\frac{1}{4}\cosh(2c+2dx)+\frac{1}{8}\right.$$
$$=\frac{x}{4}+\frac{1}{8}\int\cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int\cosh(2(a+c)+2(b+d)x)\,dx+\frac{1}{4}$$
$$=\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+}{16}$$

Mathematica [A] time = 0.68262, size = 105, normalized size = 1.19

$$\frac{2d\left(b^2-d^2\right)\sinh(2(a+bx))+bd(b+d)\sinh(2(a+x(b-d)-c))+b(b-d)(d(\sinh(2(a+x(b+d)+c))+4x(b+d))+}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] + b*(b - d)*(2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.012, size = 83, normalized size = 0.9

$$\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}+\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*cosh(d*x+c)^2,x)
```

```
[Out] 1/4*x+1/8*sinh(2*b*x+2*a)/b+1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d)
*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84235, size = 455, normalized size = 5.17

$$\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+\left(b^3d-bd^3\right)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+\left(b^2d-d^3\right)\cosh(bx+\right.}{4\left(\left(b^3d-bd^3\right)\cosh(bx+a)^2-\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 -
b^3 + b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

Sympy [A] time = 34.1966, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cosh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a)*
*2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**
2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c +
 d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh
(c + d*x)**2/8 - sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) - sinh(
a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(8*d) - 5*sinh(a - d*x)*cosh(a - d*
x)*cosh(c + d*x)**2/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x
)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3
*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 - sinh(a + d*x)**2*sinh(c + d*x)*cos
```

```
h(c + d*x)/(2*d) + sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)/(8*d) + 5*s
inh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d), Eq(b, d)), ((-x*sinh(a +
 b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cosh
(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d -
4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3
) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3
*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b**3*sinh(a
+ b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*sinh(c +
 d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(
a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*si
nh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b
*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*c
osh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c +
d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cos
h(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh
(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c +
 d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

Giac [B] time = 1.17973, size = 211, normalized size = 2.4

$$\frac{1}{4}x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} + \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} - \frac{e^{(-2\,bx-2\,a)}}{16\,b} + \frac{e^{(2\,dx+2\,c)}}{16\,d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-2
*b*x - 2*a)/b + 1/16*e^(2*d*x + 2*c)/d - 1/16*e^(-2*d*x - 2*c)/d
```

3.177 $\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3}{}$$

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sinh[c + d*x])/(8*d) + Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.0998268, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```


```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sinh[c + d*x])/(8*d) + Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```


Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x) + \frac{3}{16}\cosh(2a-c+(2b-d)x) + \frac{3}{8}\cosh(c+dx\right.$$
$$= \frac{1}{16}\int \cosh(2a-3c+(2b-3d)x)\,dx + \frac{1}{16}\int \cosh(2a+3c+(2b+3d)x)\,dx + \frac{1}{8}\int$$
$$= \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sinh(c+dx)}{8d} + \sinh$$

Mathematica [A] time = 1.61184, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(\frac{3\sinh(2a+2bx-3c-3dx)}{2b-3d} + \frac{9\sinh(2a+2bx-c-dx)}{2b-d} + \frac{9\sinh(2a+2bx+c+dx)}{2b+d} + \frac{3\sinh(2a+2bx+3c+3}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[d*x]*Sinh[c])/d + (2*Cosh[3*d*x]*Sinh[3*c])/d + (18*Cosh[c]*Sinh[
d*x])/d + (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x])
/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
+ 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d)
)/48
```

Maple [A] time = 0.03, size = 133, normalized size = 0.9

$$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d} + \frac{3\sinh(2a-c+(2b-d)x)}{32b-16d} + \frac{3\sinh(dx+c)}{8d} + \frac{\sinh(3dx+3c)}{24d} + \frac{3\sinh(2a+c+}{32b+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*cosh(d*x+c)^3,x)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92245, size = 926, normalized size = 6.43

$$\frac{36\left(4\,b^3d-bd^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2+\left(16\,b^4-40\,b^2d^2+9\,d^4-9\left(4\,b^2d^2-d^4\right)\cosh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 - 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) + 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 - 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 - 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.21821, size = 351, normalized size = 2.44

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}+\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}+\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) + 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d - 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

3.178 $\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}+\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh}{}$$

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) + Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) + (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.132814, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}+\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```


```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) + Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) + (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```


Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x) + \frac{9}{32}\cosh(a-c+(b-d)x) + \frac{1}{32}\cosh(3(a-c)+\right.$$
$$= \frac{1}{32}\int \cosh(3(a-c)+3(b-d)x)\,dx + \frac{1}{32}\int \cosh(3(a+c)+3(b+d)x)\,dx + \frac{3}{32}\int$$
$$= \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} + \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} + \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)}$$

Mathematica [A] time = 1.50947, size = 176, normalized size = 0.9

$$\frac{1}{96}\left(\frac{9\sinh(a+bx-3c-3dx)}{b-3d} + \frac{27\sinh(a+bx-c-dx)}{b-d} + \frac{\sinh(3(a+bx-c-dx))}{b-d} + \frac{9\sinh(3a+3bx-c-dx)}{3b-d} + \frac{9}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

```
[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sinh[a - c + b*x - d*x])/(
b - d) + Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*
x])/(3*b - d) + (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sinh[a + 3*c
 + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*
(a + c + (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.013, size = 184, normalized size = 0.9

$$\frac{3\sinh(a-3c+(b-3d)x)}{32b-96d} + \frac{9\sinh(a-c+(b-d)x)}{32b-32d} + \frac{9\sinh(a+c+(b+d)x)}{32b+32d} + \frac{3\sinh(a+3c+(b+3d)x)}{32b+96d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*cosh(d*x+c)^3,x)
```

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a
+c+(b+d)*x)/(b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sinh((3*b-3
*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sinh(3*a+c+(3*b+d)*x
)/(3*b+d)+1/96/(b+d)*sinh((3*b+3*d)*x+3*a+3*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98312, size = 1716, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^
2 + 9*b*d^4)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^
5)*cosh(b*x + a)^3 + 3*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)*sinh(b*
x + a)^2 + 27*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(d*x + c)^3 +
 3*((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)*sinh(b*x + a)^3 + 3*(27*b^
5 - 30*b^3*d^2 + 3*b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*
cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((27*b^5 - 30*b^3*d^2 + 3*
b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^3 + 9
*(9*b^5 - 82*b^3*d^2 + 9*b*d^4 + 3*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*x +
a)^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^4*d - 10*b^2*d^3 + 9*d^5)*cosh
(b*x + a)^3 + ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^3 + 27*(9*b^4*d
 - 10*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*((9*b^4*d - 82*b^2*
d^3 + 9*d^5)*cosh(b*x + a)*cosh(d*x + c)^2 + 3*(b^4*d - 10*b^2*d^3 + 9*d^5)
*cosh(b*x + a))*sinh(b*x + a)^2 + 9*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x
 + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
 a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*
x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*cosh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19269, size = 504, normalized size = 2.58

$$\frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)}+\frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)}+\frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)}+\frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)}+\frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)}+\frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)}+\frac{9\,e^{(bx-dx+}}{64\,(b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) + 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) + 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) - 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

3.179 $\int \cosh(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0451606, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \int \left(\frac{1}{2}\sinh(a-c+(b-d)x)+\frac{1}{2}\sinh(a+c+(b+d)x)\right)dx$$
$$= \frac{1}{2}\int \sinh(a-c+(b-d)x)\,dx + \frac{1}{2}\int \sinh(a+c+(b+d)x)\,dx$$
$$= \frac{\cosh(a-c+(b-d)x)}{2(b-d)} + \frac{\cosh(a+c+(b+d)x)}{2(b+d)}$$

Mathematica [A] time = 0.19596, size = 43, normalized size = 1.

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)} + \frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.01, size = 40, normalized size = 0.9

$$\frac{\cosh(a-c+(b-d)x)}{2b-2d} + \frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a),x)
```

```
[Out] 1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66595, size = 169, normalized size = 3.93

$$\frac{b\cosh(bx+a)\cosh(dx+c)-d\sinh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(d*x + c) - d*sinh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 2.0004, size = 153, normalized size = 3.56

$$\begin{cases} x\sinh(a)\cosh(c) & \text{for } b=0\wedge d=0 \\ \frac{x\sinh(a-dx)\cosh(c+dx)}{2}+\frac{x\sinh(c+dx)\cosh(a-dx)}{2}-\frac{\cosh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sinh(a+dx)\cosh(c+dx)}{2}-\frac{x\sinh(c+dx)\cosh(a+dx)}{2}+\frac{\cosh(a+dx)\cosh(c+dx)}{2d} & \text{for } b=d \\ \frac{b\cosh(a+bx)\cosh(c+dx)}{b^2-d^2}-\frac{d\sinh(a+bx)\sinh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*cosh(c
 + d*x)/2 + x*sinh(c + d*x)*cosh(a - d*x)/2 - cosh(a - d*x)*cosh(c + d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)*cosh(a
 + d*x)/2 + cosh(a + d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*cosh(a + b*x)*
cosh(c + d*x)/(b**2 - d**2) - d*sinh(a + b*x)*sinh(c + d*x)/(b**2 - d**2),
True))
```

Giac [B] time = 1.20533, size = 115, normalized size = 2.67

$$\frac{e^{(bx+dx+a+c)}}{4(b+d)}+\frac{e^{(bx-dx+a-c)}}{4(b-d)}+\frac{e^{(-bx+dx-a+c)}}{4(b-d)}+\frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) + 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.180 $\int \cosh^2(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

```
[Out] Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0570709, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \int \left(\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)x)\right)dx$$
$$= \frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sinh(a$$
$$= \frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] time = 0.677511, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cosh(a+bx-2c-2dx)}{b-2d}+\frac{\cosh(a+bx+2c+2dx)}{b+2d}+\frac{2\sinh(a)\sinh(bx)}{b}+\frac{2\cosh(a)\cosh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a +
 2*c + b*x + 2*d*x]/(b + 2*d) + (2*Sinh[a]*Sinh[b*x])/b)/4
```

Maple [A] time = 0.007, size = 57, normalized size = 0.9

$$\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85953, size = 304, normalized size = 4.9

$$\frac{b^2 \cosh(bx+a)\cosh(dx+c)^2 - 4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c) + b^2\cosh(bx+a)\sinh(dx+c)^2 + \left(b^2 - 4\right.}{2\left(\left(b^3 - 4\,bd^2\right)\cosh(bx+a)^2 - \left(b^3 - 4\,bd^2\right)\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 8.54137, size = 403, normalized size = 6.5

$$\begin{cases} x\sinh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2} + \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} + \frac{\sinh^2(c+dx)\cosh(a-2dx)}{8d} - \frac{3\cosh(a-2dx)\cos}{8d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} + \frac{3\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{4d} - \frac{\sinh^2}{} \\ \frac{b^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} + \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**
2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq
(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
 d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 + sinh(c + d*x
)**2*cosh(a - 2*d*x)/(8*d) - 3*cosh(a - 2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b
, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d*x)*cosh(c
+ d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 + 3*sinh(a +
```

```
2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh(a + 2*d*x)
/(2*d), Eq(b, 2*d)), (b**2*cosh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2)
 - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d*
*2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) - 2*d**2*cosh(a + b*x)*
cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

Giac [B] time = 1.23527, size = 162, normalized size = 2.61

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)}+\frac{e^{(bx+a)}}{4\,b}+\frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)}+\frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)}+\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) + 1/4*e^(-b*x - a)/b
```

3.181 $\int \cosh^3(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] Cosh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Cosh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rubi [A] time = 0.0817464, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Cosh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \int \left(\frac{1}{8}\sinh(a-3c+(b-3d)x)+\frac{3}{8}\sinh(a-c+(b-d)x)+\frac{3}{8}\sinh(a+c+(b+d)x\right.$$
$$= \frac{1}{8}\int \sinh(a-3c+(b-3d)x)\,dx + \frac{1}{8}\int \sinh(a+3c+(b+3d)x)\,dx + \frac{3}{8}\int \sinh(a$$
$$= \frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)} + \frac{3\cosh(a-c+(b-d)x)}{8(b-d)} + \frac{3\cosh(a+c+(b+d)x)}{8(b+d)} +$$

Mathematica [A] time = 0.44746, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\cosh(a+bx-3c-3dx)}{b-3d}+\frac{3\cosh(a+bx-c-dx)}{b-d}+\frac{\cosh(a+bx+3c+3dx)}{b+3d}+\frac{3\cosh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Cosh[a - c + b*x - d*x])/(b - d
) + Cosh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Cosh[a + c + (b + d)*x])/(b
+ d))/8
```

Maple [A] time = 0.012, size = 84, normalized size = 0.9

$$\frac{\cosh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\cosh(a-c+(b-d)x)}{8b-8d}+\frac{3\cosh(a+c+(b+d)x)}{8b+8d}+\frac{\cosh(a+3c+(b+3d)x)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+
(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.85347, size = 514, normalized size = 5.65

$$\frac{\left(b^3 - bd^2\right)\cosh\left(bx + a\right)\cosh\left(dx + c\right)^3 + 3\left(b^3 - bd^2\right)\cosh\left(bx + a\right)\cosh\left(dx + c\right)\sinh\left(dx + c\right)^2 - 3\left(b^2d - d^3\right)\sinh\left(bx + \right.}{4\left(\left(b^4 - 10\,b^2d^2 + 9\,d^4\right)\cosh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(b^3 - b*d^2)*cosh(b*x
 + a)*cosh(d*x + c)*sinh(d*x + c)^2 - 3*(b^2*d - d^3)*sinh(b*x + a)*sinh(d*
x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(b*x + a)*cosh(d*x + c) - 3*(b^2*d - 9*d^3
 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a)*sinh(d*x + c))/((b^4 - 10
*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x + a
)^2)
```

Sympy [A] time = 46.3861, size = 940, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*sinh(b*x+a),x)

```
[Out] Piecewise((x*sinh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d*x)
*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sinh(a - 3*d*x)*cosh(c + d*x)**3/8 +
x*sinh(c + d*x)**3*cosh(a - 3*d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - 3*d*x)*co
sh(c + d*x)**2/8 - 3*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/(8*d) -
 sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/(8*d) - 5*cosh(a - 3*d*x)*c
osh(c + d*x)**3/(12*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**2*
cosh(c + d*x)/8 + 3*x*sinh(a - d*x)*cosh(c + d*x)**3/8 - 3*x*sinh(c + d*x)*
*3*cosh(a - d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)**2/8 + 3
*sinh(a - d*x)*sinh(c + d*x)*cosh(c + d*x)**2/(8*d) + 3*sinh(c + d*x)**2*co
sh(a - d*x)*cosh(c + d*x)/(8*d) - cosh(a - d*x)*cosh(c + d*x)**3/(4*d), Eq(
```

```
b, -d)), (-3*x*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(a
+ d*x)*cosh(c + d*x)**3/8 + 3*x*sinh(c + d*x)**3*cosh(a + d*x)/8 - 3*x*sinh
(c + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/8 + 3*sinh(a + d*x)*sinh(c + d*x)*
cosh(c + d*x)**2/(8*d) - 3*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/(8*
d) + cosh(a + d*x)*cosh(c + d*x)**3/(4*d), Eq(b, d)), (3*x*sinh(a + 3*d*x)*
sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sinh(a + 3*d*x)*cosh(c + d*x)**3/8 - x
*sinh(c + d*x)**3*cosh(a + 3*d*x)/8 - 3*x*sinh(c + d*x)*cosh(a + 3*d*x)*cos
h(c + d*x)**2/8 + sinh(a + 3*d*x)*sinh(c + d*x)**3/(8*d) - sinh(c + d*x)**2
*cosh(a + 3*d*x)*cosh(c + d*x)/(4*d) + 7*cosh(a + 3*d*x)*cosh(c + d*x)**3/(
24*d), Eq(b, 3*d)), (b**3*cosh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d*
*2 + 9*d**4) - 3*b**2*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4
- 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh(c + d*x)**2*cosh(a + b*x)*cosh(c +
 d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*cosh(a + b*x)*cosh(c + d*x)
**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d**3*sinh(a + b*x)*sinh(c + d*x)**3/
(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*sinh(a + b*x)*sinh(c + d*x)*cosh(c
+ d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

Giac [B] time = 1.19169, size = 242, normalized size = 2.66

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)}+\frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)}+\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}+\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) + 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) + 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

3.182 $\int \cosh(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=68

$$\frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\sinh(c+dx)}{2d}$$

```
[Out] Sinh[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Sinh[c + d*x]/(2*d) + Sinh[2*a
+ c + (2*b + d)*x]/(4*(2*b + d))
```

Rubi [A] time = 0.0536795, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2637}

$$\frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\sinh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Sinh[c + d*x]/(2*d) + Sinh[2*a
+ c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \int \left(\frac{1}{4}\cosh(2a-c+(2b-d)x) - \frac{1}{2}\cosh(c+dx) + \frac{1}{4}\cosh(2a+c+(2b+d)x)\right) dx$$
$$= \frac{1}{4}\int \cosh(2a-c+(2b-d)x)\,dx + \frac{1}{4}\int \cosh(2a+c+(2b+d)x)\,dx - \frac{1}{2}\int \cosh(c$$
$$= \frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)} - \frac{\sinh(c+dx)}{2d} + \frac{\sinh(2a+c+(2b+d)x)}{4(2b+d)}$$

Mathematica [A] time = 0.720775, size = 74, normalized size = 1.09

$$\frac{1}{4}\left(\frac{\sinh(2a+2bx-c-dx)}{2b-d} + \frac{\sinh(2a+2bx+c+dx)}{2b+d} - \frac{2\sinh(c)\cosh(dx)}{d} - \frac{2\cosh(c)\sinh(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] ((-2*Cosh[d*x]*Sinh[c])/d - (2*Cosh[c]*Sinh[d*x])/d + Sinh[2*a - c + 2*b*x
- d*x]/(2*b - d) + Sinh[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Maple [A] time = 0.01, size = 63, normalized size = 0.9

$$\frac{\sinh(2a-c+(2b-d)x)}{8b-4d} - \frac{\sinh(dx+c)}{2d} + \frac{\sinh(2a+c+(2b+d)x)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^2,x)
```

```
[Out] 1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83368, size = 266, normalized size = 3.91

$$\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)-\left(d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\,b^2-d^2\right)\sinh\left(dx+c\right)}{2\left(\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^2-\left(4\,b^2d-d^3\right)\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - (d^2*cosh(b*x + a)^2
 + d^2*sinh(b*x + a)^2 + 4*b^2 - d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(
b*x + a)^2 - (4*b^2*d - d^3)*sinh(b*x + a)^2)
```

Sympy [A] time = 9.19713, size = 405, normalized size = 5.96

$$\begin{cases} x\sinh^2\left(a\right)\cosh\left(c\right) \\ \frac{x\sinh^2\left(a-\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{x\sinh\left(a-\frac{dx}{2}\right)\sinh\left(c+dx\right)\cosh\left(a-\frac{dx}{2}\right)}{2}+\frac{x\cosh^2\left(a-\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{\sinh^2\left(a-\frac{dx}{2}\right)\sinh\left(c+dx\right)}{d}+\frac{\sinh\left(a-\frac{dx}{2}\right)\cosh\left(\right.}{2 } \\ \frac{x\sinh^2\left(a+\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}-\frac{x\sinh\left(a+\frac{dx}{2}\right)\sinh\left(c+dx\right)\cosh\left(a+\frac{dx}{2}\right)}{2}+\frac{x\cosh^2\left(a+\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{3\sinh^2\left(a+\frac{dx}{2}\right)\sinh\left(c+dx\right)}{4d}-\frac{\sinh\left(c+dx\right)\cosh}{4d} \\ \left(\frac{x\sinh^2\left(a+bx\right)}{2}-\frac{x\cosh^2\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}\right)\cosh\left(c\right) \\ \frac{2b^2\sinh^2\left(a+bx\right)\sinh\left(c+dx\right)}{4b^2d-d^3}-\frac{2b^2\sinh\left(c+dx\right)\cosh^2\left(a+bx\right)}{4b^2d-d^3}+\frac{2bd\sinh\left(a+bx\right)\cosh\left(a+bx\right)\cosh\left(c+dx\right)}{4b^2d-d^3}-\frac{d^2\sinh^2\left(a+bx\right)\sinh\left(c+dx\right)}{4b^2d-d^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x/2)**
2*cosh(c + d*x)/4 + x*sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)/2 + x*c
osh(a - d*x/2)**2*cosh(c + d*x)/4 + sinh(a - d*x/2)**2*sinh(c + d*x)/d + si
nh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)/(2*d), Eq(b, -d/2)), (x*sinh(a
+ d*x/2)**2*cosh(c + d*x)/4 - x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/
2)/2 + x*cosh(a + d*x/2)**2*cosh(c + d*x)/4 + 3*sinh(a + d*x/2)**2*sinh(c +
```

```
d*x)/(4*d) - sinh(c + d*x)*cosh(a + d*x/2)**2/(4*d), Eq(b, d/2)), ((x*sinh
(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*
cosh(c), Eq(d, 0)), (2*b**2*sinh(a + b*x)**2*sinh(c + d*x)/(4*b**2*d - d**3
) - 2*b**2*sinh(c + d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3) + 2*b*d*sinh(a
+ b*x)*cosh(a + b*x)*cosh(c + d*x)/(4*b**2*d - d**3) - d**2*sinh(a + b*x)**
2*sinh(c + d*x)/(4*b**2*d - d**3), True))
```

Giac [A] time = 1.16759, size = 167, normalized size = 2.46

$$\frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)}+\frac{e^{(2\,bx-dx+2\,a-c)}}{8\,(2\,b-d)}-\frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)}-\frac{e^{(-2\,bx-dx-2\,a-c)}}{8\,(2\,b+d)}-\frac{e^{(dx+c)}}{4\,d}+\frac{e^{(-dx-c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/8*e^(2*b*x - d*x + 2*a - c)/(2*
b - d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 1/8*e^(-2*b*x - d*x - 2
*a - c)/(2*b + d) - 1/4*e^(d*x + c)/d + 1/4*e^(-d*x - c)/d
```

3.183 $\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

```
[Out] -x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
 - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.0655807, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
 - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)-\frac{1}{4}\cosh(2c+2dx)+\right.$$
$$= -\frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx+\frac{1}{4}$$
$$= -\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a-}{1}$$

Mathematica [A] time = 0.70163, size = 107, normalized size = 1.22

$$\frac{2d\left(b^2-d^2\right)\sinh(2(a+bx))+bd(b+d)\sinh(2(a+x(b-d)-c))-b(b-d)(-d\sinh(2(a+x(b+d)+c))+2(b+d)\sin}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] - b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sinh[2*(c + d*x)] - d*Sinh[2*(a +
c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.01, size = 83, normalized size = 0.9

$$-\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/4*x+1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d
)*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86773, size = 455, normalized size = 5.17

$$\frac{b^2 d \cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2 - \left(b^3 d - bd^3\right)x + \left(b^2 d \cosh(bx+a)\cosh(dx+c)^2 + \left(b^2 d - d^3\right)\cosh(bx+}{4\left(\left(b^3 d - bd^3\right)\cosh(bx+a)^2 - \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 +
b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

Sympy [A] time = 34.4958, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)*
*2, Eq(b, 0)), (-x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 + 3*x*sinh(a - d*x)*
*2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c
+ d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 - x*cosh(a - d*x)**2*cos
h(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 3*si
nh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(8*d) - sinh(a - d*x)*cosh(a - d
*x)*cosh(c + d*x)**2/(8*d), Eq(b, -d)), (-x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 + 3*x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d
*x)*cosh(a + d*x)*cosh(c + d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a + d*x)**2/8
 - x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)**2*sinh(c + d*x)*c
```

```
osh(c + d*x)/(2*d) - 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)/(8*d) +
 sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d), Eq(b, d)), ((x*sinh(a
+ b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cos
h(c)**2, Eq(d, 0)), (-b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d
- 4*b*d**3) + b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d*
*3) + b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*
*3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(
a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c
 + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sin
h(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*
sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(a +
 b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(c + d*x)**2
*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*cosh(a + b*x)**2*cosh(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*c
osh(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*co
sh(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c
 + d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

Giac [B] time = 1.18476, size = 211, normalized size = 2.4

$$-\frac{1}{4}x+\frac{e^{(2bx+2dx+2a+2c)}}{32(b+d)}+\frac{e^{(2bx-2dx+2a-2c)}}{32(b-d)}+\frac{e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx+2dx-2a+2c)}}{32(b-d)}-\frac{e^{(-2bx-2dx-2a-2c)}}{32(b+d)}-\frac{e^{(-2bx-2a)}}{16b}-\frac{e^{(2dx+2c)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
 + 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2
*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-
2*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

3.184 $\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}-3$$

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) - (3*Sinh[c + d*x])/(8*d) - Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.0925019, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2637}

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}-3$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) - (3*Sinh[c + d*x])/(8*d) - Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \int \left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x) + \frac{3}{16}\cosh(2a-c+(2b-d)x) - \frac{3}{8}\cosh(c+dx)\right.$$
$$= \frac{1}{16}\int \cosh(2a-3c+(2b-3d)x)\,dx + \frac{1}{16}\int \cosh(2a+3c+(2b+3d)x)\,dx - \frac{1}{8}\int$$
$$= \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\sinh(c+dx)}{8d} - \frac{\sinh(}{}$$

Mathematica [A] time = 1.56935, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(\frac{3\sinh(2a+2bx-3c-3dx)}{2b-3d} + \frac{9\sinh(2a+2bx-c-dx)}{2b-d} + \frac{9\sinh(2a+2bx+c+dx)}{2b+d} + \frac{3\sinh(2a+2bx+3c+3}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] ((-18*Cosh[d*x]*Sinh[c])/d - (2*Cosh[3*d*x]*Sinh[3*c])/d - (18*Cosh[c]*Sinh
[d*x])/d - (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x]
)/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
 + 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d
))/48
```

Maple [A] time = 0.01, size = 133, normalized size = 0.9

$$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d} + \frac{3\sinh(2a-c+(2b-d)x)}{32b-16d} - \frac{3\sinh(dx+c)}{8d} - \frac{\sinh(3dx+3c)}{24d} + \frac{3\sinh(2a+c+}{32b+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92453, size = 926, normalized size = 6.43

$$\frac{36\left(4\,b^3d - bd^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2 - \left(16\,b^4 - 40\,b^2d^2 + 9\,d^4 + 9\left(4\,b^2d^2 - d^4\right)\cosh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 - (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 + 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) - 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 + 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 + 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21426, size = 351, normalized size = 2.44

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}+\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}+\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d - 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d + 1/48*e^(-3
*d*x - 3*c)/d
```

3.185 $\int \cosh(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] (-3*Cosh[a - c + (b - d)*x])/(8*(b - d)) + Cosh[3*a - c + (3*b - d)*x]/(8*(
3*b - d)) - (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[3*a + c + (3*b +
 d)*x]/(8*(3*b + d))
```

Rubi [A] time = 0.0805228, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a - c + (b - d)*x])/(8*(b - d)) + Cosh[3*a - c + (3*b - d)*x]/(8*(
3*b - d)) - (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[3*a + c + (3*b +
 d)*x]/(8*(3*b + d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{8}\sinh(a-c+(b-d)x)+\frac{1}{8}\sinh(3a-c+(3b-d)x)-\frac{3}{8}\sinh(a+c+(b+d)\right.$$
$$= \frac{1}{8}\int \sinh(3a-c+(3b-d)x)\,dx+\frac{1}{8}\int \sinh(3a+c+(3b+d)x)\,dx-\frac{3}{8}\int \sinh(a-$$
$$= -\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+$$

Mathematica [A] time = 0.495546, size = 90, normalized size = 0.93

$$\frac{1}{8}\left(-\frac{3\cosh(a+bx-c-dx)}{b-d}+\frac{\cosh(3a+3bx-c-dx)}{3b-d}+\frac{\cosh(3a+3bx+c+dx)}{3b+d}-\frac{3\cosh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] ((-3*Cosh[a - c + b*x - d*x])/(b - d) + Cosh[3*a - c + 3*b*x - d*x]/(3*b -
d) + Cosh[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cosh[a + c + (b + d)*x])/(b
 + d))/8
```

Maple [A] time = 0.013, size = 90, normalized size = 0.9

$$-\frac{3\cosh(a-c+(b-d)x)}{8b-8d}+\frac{\cosh(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cosh(a+c+(b+d)x)}{8b+8d}+\frac{\cosh(3a+c+(3b+d)x)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^3,x)
```

```
[Out] -3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14674, size = 562, normalized size = 5.79

$$\frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2+3\left(\left(b^3-bd^2\right)\cosh\left(bx+a\right)^3-\left(9\,b^3-bd^2\right)\cosh\left(bx+a\right)\right)\cosh\left(\right.}{4\left(\left(9\,b^4-10\,b^2d^2+d^4\right)\cosh\left(bx+a\right)^4-2\left(9\,b^4-10\,b^2d^2+d^4\right)\cosh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((b^3
- b*d^2)*cosh(b*x + a)^3 - (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) - (
(b^2*d - d^3)*sinh(b*x + a)^3 - 3*(9*b^2*d - d^3 - (b^2*d - d^3)*cosh(b*x +
 a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x +
 a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b
^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [A] time = 46.8247, size = 942, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c), Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - d*x)**
3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)/8 - 3*
x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a
 - d*x)**3/8 - 5*sinh(a - d*x)**2*cosh(a - d*x)*cosh(c + d*x)/(8*d) - sinh(
a - d*x)*sinh(c + d*x)*cosh(a - d*x)**2/(8*d) + cosh(a - d*x)**3*cosh(c + d
*x)/(4*d), Eq(b, -d)), (x*sinh(a - d*x/3)**3*cosh(c + d*x)/8 + 3*x*sinh(a -
 d*x/3)**2*sinh(c + d*x)*cosh(a - d*x/3)/8 + 3*x*sinh(a - d*x/3)*cosh(a - d
*x/3)**2*cosh(c + d*x)/8 + x*sinh(c + d*x)*cosh(a - d*x/3)**3/8 + 9*sinh(a
- d*x/3)**3*sinh(c + d*x)/(8*d) + 3*sinh(a - d*x/3)**2*cosh(a - d*x/3)*cosh
(c + d*x)/(4*d) - cosh(a - d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, -d/3)), (x*
```

```
sinh(a + d*x/3)**3*cosh(c + d*x)/8 - 3*x*sinh(a + d*x/3)**2*sinh(c + d*x)*c
osh(a + d*x/3)/8 + 3*x*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)/8 -
 x*sinh(c + d*x)*cosh(a + d*x/3)**3/8 + 9*sinh(a + d*x/3)**3*sinh(c + d*x)/
(8*d) - 3*sinh(a + d*x/3)**2*cosh(a + d*x/3)*cosh(c + d*x)/(4*d) + cosh(a +
 d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sinh(a + d*x)**3*cosh(c +
 d*x)/8 - 3*x*sinh(a + d*x)**2*sinh(c + d*x)*cosh(a + d*x)/8 - 3*x*sinh(a +
 d*x)*cosh(a + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(c + d*x)*cosh(a + d*x)**3
/8 + 5*sinh(a + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/(8*d) - sinh(a + d*x)*s
inh(c + d*x)*cosh(a + d*x)**2/(8*d) - cosh(a + d*x)**3*cosh(c + d*x)/(4*d),
 Eq(b, d)), (9*b**3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4 -
10*b**2*d**2 + d**4) - 6*b**3*cosh(a + b*x)**3*cosh(c + d*x)/(9*b**4 - 10*b
**2*d**2 + d**4) - 7*b**2*d*sinh(a + b*x)**3*sinh(c + d*x)/(9*b**4 - 10*b**
2*d**2 + d**4) + 6*b**2*d*sinh(a + b*x)*sinh(c + d*x)*cosh(a + b*x)**2/(9*b
**4 - 10*b**2*d**2 + d**4) - 3*b*d**2*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c
 + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*sinh(a + b*x)**3*sinh(c + d*x
)/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Giac [B] time = 1.17978, size = 247, normalized size = 2.55

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{16\,(3\,b+d)}+\frac{e^{(3\,bx-dx+3\,a-c)}}{16\,(3\,b-d)}-\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}-\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}-\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}+\frac{e^{(-3\,bx+dx-3\,a+c)}}{16\,(3\,b-d)}+\frac{e^{(-}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/16*e^(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/16*e^(b*x + d*x + a + c)/(b + d) - 3/16*e^(b*x - d*x + a - c)/
(b - d) - 3/16*e^(-b*x + d*x - a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)
/(b + d) + 1/16*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/16*e^(-3*b*x - d*x
 - 3*a - c)/(3*b + d)
```

3.186 $\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cosh(3a+x(3b+2d)+2c)}{16(3b+2d)}-$$

```
[Out] (-3*Cosh[a + b*x])/(8*b) + Cosh[3*a + 3*b*x]/(24*b) - (3*Cosh[a - 2*c + (b
- 2*d)*x])/(16*(b - 2*d)) + Cosh[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)
) - (3*Cosh[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cosh[3*a + 2*c + (3*b
+ 2*d)*x]/(16*(3*b + 2*d))
```

Rubi [A] time = 0.113771, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cosh(3a+x(3b+2d)+2c)}{16(3b+2d)}-$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```


```
[Out] (-3*Cosh[a + b*x])/(8*b) + Cosh[3*a + 3*b*x]/(24*b) - (3*Cosh[a - 2*c + (b
- 2*d)*x])/(16*(b - 2*d)) + Cosh[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)
) - (3*Cosh[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cosh[3*a + 2*c + (3*b
+ 2*d)*x]/(16*(3*b + 2*d))
```


Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{8}\sinh(a+bx)+\frac{1}{8}\sinh(3a+3bx)-\frac{3}{16}\sinh(a-2c+(b-2d)x)+\frac{1}{16}\sinh(\right.$$
$$=\frac{1}{16}\int \sinh(3a-2c+(3b-2d)x)\,dx+\frac{1}{16}\int \sinh(3a+2c+(3b+2d)x)\,dx+\frac{1}{8}\int$$
$$=-\frac{3\cosh(a+bx)}{8b}+\frac{\cosh(3a+3bx)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cosh(3a-2c}{16(3b}$$

Mathematica [A] time = 1.64109, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{9\cosh(a+bx-2c-2dx)}{b-2d}+\frac{3\cosh(3a+3bx-2c-2dx)}{3b-2d}-\frac{9\cosh(a+bx+2c+2dx)}{b+2d}+\frac{3\cosh(3a+3bx+2c}{3b+2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] ((-18*Cosh[a]*Cosh[b*x])/b + (2*Cosh[3*a]*Cosh[3*b*x])/b - (9*Cosh[a - 2*c
+ b*x - 2*d*x])/(b - 2*d) + (3*Cosh[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d)
 - (9*Cosh[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cosh[3*a + 2*c + 3*b*x +
2*d*x])/(3*b + 2*d) - (18*Sinh[a]*Sinh[b*x])/b + (2*Sinh[3*a]*Sinh[3*b*x])/
b)/48
```

Maple [A] time = 0.017, size = 127, normalized size = 0.9

$$-\frac{3\cosh(bx+a)}{8b}+\frac{\cosh(3bx+3a)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16b-32d}+\frac{\cosh(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cosh(a+2}{16b+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a)^3,x)
```

```
[Out] -3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*d
)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+2*
d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25953, size = 1054, normalized size = 7.64

$$\frac{\left(9\,b^4-40\,b^2d^2+16\,d^4\right)\cosh\left(bx+a\right)^3+9\left(\left(b^4-4\,b^2d^2\right)\cosh\left(bx+a\right)^3-\left(9\,b^4-4\,b^2d^2\right)\cosh\left(bx+a\right)\right)\cosh\left(dx+c\right)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*((9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x + a)^3 + 9*((b^4 - 4*b^2*d^2)*
cosh(b*x + a)^3 - (9*b^4 - 4*b^2*d^2)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*(9
*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 + (9*b^4 - 40*b^2*d^2 + 16
*d^4)*cosh(b*x + a))*sinh(b*x + a)^2 + 9*((b^4 - 4*b^2*d^2)*cosh(b*x + a)^3
 + 3*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b^4 - 4*b^2*d^2)*
cosh(b*x + a))*sinh(d*x + c)^2 - 9*(9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x +
 a) - 12*((b^3*d - 4*b*d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3*d - 4*
b*d^3 - (b^3*d - 4*b*d^3)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a))*sin
h(d*x + c))/((9*b^5 - 40*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^4 - 2*(9*b^5 - 4
0*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^5 - 40*b^3*d^2
 + 16*b*d^4)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18607, size = 346, normalized size = 2.51

$$\frac{e^{(3\,bx+2\,dx+3\,a+2\,c)}}{32\,(3\,b+2\,d)}+\frac{e^{(3\,bx-2\,dx+3\,a-2\,c)}}{32\,(3\,b-2\,d)}+\frac{e^{(3\,bx+3\,a)}}{48\,b}-\frac{3\,e^{(bx+2\,dx+a+2\,c)}}{32\,(b+2\,d)}-\frac{3\,e^{(bx-2\,dx+a-2\,c)}}{32\,(b-2\,d)}-\frac{3\,e^{(bx+a)}}{16\,b}-\frac{3\,e^{(-bx+2\,dx-a+2\,c)}}{32\,(b-2\,d)}-3\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/32*e^(3*b*x - 2*d*x + 3*
a - 2*c)/(3*b - 2*d) + 1/48*e^(3*b*x + 3*a)/b - 3/32*e^(b*x + 2*d*x + a + 2
*c)/(b + 2*d) - 3/32*e^(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/16*e^(b*x + a)
/b - 3/32*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 3/32*e^(-b*x - 2*d*x - a -
 2*c)/(b + 2*d) - 3/16*e^(-b*x - a)/b + 1/32*e^(-3*b*x + 2*d*x - 3*a + 2*c)
/(3*b - 2*d) + 1/32*e^(-3*b*x - 2*d*x - 3*a - 2*c)/(3*b + 2*d) + 1/48*e^(-3
*b*x - 3*a)/b
```

3.187 $\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cosh}{}$$

```
[Out] (-3*Cosh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cosh[a - c + (b - d)*x
])/(32*(b - d)) + Cosh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cosh[3*a
- c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cosh[a + c + (b + d)*x])/(32*(b + d
)) + Cosh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cosh[3*a + c + (3*b +
d)*x])/(32*(3*b + d)) - (3*Cosh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.153086, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cosh[a - c + (b - d)*x
])/(32*(b - d)) + Cosh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cosh[3*a
- c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cosh[a + c + (b + d)*x])/(32*(b + d
)) + Cosh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cosh[3*a + c + (3*b +
d)*x])/(32*(3*b + d)) - (3*Cosh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{32}\sinh(a-3c+(b-3d)x) - \frac{9}{32}\sinh(a-c+(b-d)x) + \frac{1}{32}\sinh(3(a-c)+\right.$$
$$= \frac{1}{32}\int \sinh(3(a-c)+3(b-d)x)\,dx + \frac{1}{32}\int \sinh(3(a+c)+3(b+d)x)\,dx - \frac{3}{32}\int$$
$$= -\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\cosh(a-c+(b-d)x)}{32(b-d)} + \frac{\cosh(3(a-c)+3(b-d)}{96(b-d)}$$

Mathematica [A] time = 1.67264, size = 176, normalized size = 0.9

$$\frac{1}{96}\left(-\frac{9\cosh(a+bx-3c-3dx)}{b-3d} - \frac{27\cosh(a+bx-c-dx)}{b-d} + \frac{\cosh(3(a+bx-c-dx))}{b-d} + \frac{9\cosh(3a+3bx-c-dx)}{3b-d} +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] ((-9*Cosh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cosh[a - c + b*x - d*x])/
(b - d) + Cosh[3*(a - c + b*x - d*x)]/(b - d) + (9*Cosh[3*a - c + 3*b*x - d
*x])/(3*b - d) + (9*Cosh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cosh[a + 3*
c + b*x + 3*d*x])/(b + 3*d) - (27*Cosh[a + c + (b + d)*x])/(b + d) + Cosh[3
*(a + c + (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.013, size = 184, normalized size = 0.9

$$-\frac{3\cosh(a-3c+(b-3d)x)}{32b-96d} - \frac{9\cosh(a-c+(b-d)x)}{32b-32d} - \frac{9\cosh(a+c+(b+d)x)}{32b+32d} - \frac{3\cosh(a+3c+(b+3d)x)}{32b+96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^3,x)
```

```
[Out] -3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)-9/32*cosh(
a+c+(b+d)*x)/(b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)+1/96*cosh((3*b-3*d)*x
+3*a-3*c)/(b-d)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cosh(3*a+c+(3*b+d)*
x)/(3*b+d)+1/96*cosh((3*b+3*d)*x+3*a+3*c)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36054, size = 1715, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5
)*sinh(b*x + a)^3 - 3*(81*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^
3 + 9*d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*((9*b^5 - 82
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c))*sinh(b*x + a)^2 + 3*(3*(9*b^5 - 82*b^
3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + ((9*b^5 - 82
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b
*x + a))*cosh(d*x + c))*sinh(d*x + c)^2 + 27*((b^5 - 10*b^3*d^2 + 9*b*d^4)*
cosh(b*x + a)^3 - (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a))*cosh(d*x +
c) - 3*((3*b^4*d - 30*b^2*d^3 + 27*d^5 + (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos
h(d*x + c)^2)*sinh(b*x + a)^3 - 3*(27*b^4*d - 246*b^2*d^3 + 27*d^5 - 3*(b^4
*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 + (81*b^4*d - 90*b^2*d^3 + 9*d^5 -
 (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^2)*sinh(b*x
+ a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19088, size = 504, normalized size = 2.58

$$\frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)}+\frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)}+\frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)}+\frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)}-\frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)}-\frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)}-\frac{9\,e^{(bx-dx+a}}{64\,(b-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) - 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) + 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) + 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

3.188 $\int \sinh(a+bx)\tanh(c+dx)\,dx$

Optimal. Leaf size=121

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rubi [A] time = 0.106408, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5601, 2194, 2251}

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 5601

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh(c+dx)\,dx &= \int \left(-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}+\frac{e^{-a-bx}}{1+e^{2(c+dx)}}-\frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right)+\frac{1}{2}\int e^{a+bx}\,dx+\int \frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx\\ &= \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};-e^{2(c+\right.}}{b}\end{aligned}$$

Mathematica [B] time = 14.1913, size = 278, normalized size = 2.3

$$\frac{e^{-a-bx-c}\left((b-2d)\left(2b\,\mathrm{sech}(c)e^{2(a+x(b+d)+c)}\,{}_2F_1\left(1,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)-(b+2d)\left(\left(e^{2a}+2e^{2c}+1\right)\mathrm{sech}(c)\,{}_2F_1\left(1,-\frac{b}{2d}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - c - b*x)*(-(b*(b + 2*d)*E^(2*(c + d*x))*(-1 + E^(2*a))*Hypergeomet
ric2F1[1, 1 - b/(2*d), 2 - b/(2*d), -E^(2*(c + d*x))]*Sech[c]) + (b - 2*d)*
(2*b*E^(2*(a + c + (b + d)*x))*Hypergeometric2F1[1, 1 + b/(2*d), 2 + b/(2*d
), -E^(2*(c + d*x))]*Sech[c] - (b + 2*d)*(-Sech[c] - E^(2*a)*Sech[c] + (1 +
 E^(2*a) + 2*E^(2*c))*Hypergeometric2F1[1, -b/(2*d), 1 - b/(2*d), -E^(2*(c
+ d*x))]*Sech[c] + 2*E^(2*(a + c + b*x))*Hypergeometric2F1[1, b/(2*d), 1 +
b/(2*d), -E^(2*(c + d*x))]*Sech[c] - 4*E^(a + c + b*x)*Cosh[a + b*x]*Tanh[c
]))))/(4*(b^3 - 4*b*d^2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] int(sinh(b*x+a)*tanh(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int\frac{2\left(e^{(2\,bx+2\,a)}-1\right)}{e^{(bx+2\,dx+a+2\,c)}+e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 - 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(bx+a\right)\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)*tanh(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(a+bx\right)\tanh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh{(bx+a)}\tanh{(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)*tanh(d*x + c), x)
```

3.189 $\int \coth(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2F
1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rubi [A] time = 0.108786, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5603, 2194, 2251}

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2F
1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 5603

```
Int[Coth[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \left(-\frac{1}{2}e^{-a-bx} + \frac{1}{2}e^{a+bx} + \frac{e^{-a-bx}}{1-e^{2(c+dx)}} - \frac{e^{a+bx}}{1-e^{2(c+dx)}}\right) dx$$

$$= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right) + \frac{1}{2}\int e^{a+bx}\,dx + \int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx - \int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx$$

$$= \frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2(c+dx)}\right)}{b}$$

Mathematica [B] time = 6.7317, size = 240, normalized size = 2.05

$$\frac{e^{-a-bx+2c}\left(be^{2dx}\,{}_2F_1\left(1,1-\frac{b}{2d};2-\frac{b}{2d};e^{2(c+dx)}\right)-(b-2d)\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)\right)}{b\left(e^{2c}-1\right)(b-2d)} - \frac{e^{a+2c}\left(\frac{e^{bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b} - \cdots\right)}{e^{2c}-1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x]*Coth[c])/b + (E^(-a + 2*c - b*x)*(b*E^(2*d*x)*Hypergeome
tric2F1[1, 1 - b/(2*d), 2 - b/(2*d), E^(2*(c + d*x))] - (b - 2*d)*Hypergeom
etric2F1[1, -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))]))/(b*(b - 2*d)*(-1 + E^
(2*c))) - (E^(a + 2*c)*(-((E^((b + 2*d)*x)*Hypergeometric2F1[1, 1 + b/(2*d)
, 2 + b/(2*d), E^(2*(c + d*x))])/(b + 2*d)) + (E^(b*x)*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b))/(-1 + E^(2*c)) + (Coth[c]*Sinh
[a]*Sinh[b*x])/b
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \coth(dx+c)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(coth(d*x+c)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}-1}{e^{(bx+dx+a+c)}+e^{(bx+a)}}\,dx+\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}-1}{e^{(bx+dx+a+c)}-e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) -
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) - 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(dx+c\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)*sinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(a+bx\right)\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(dx+c)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)*sinh(b*x + a), x)
```

3.190 $\int \cosh(a + bx) \coth(c + dx) \, dx$

Optimal. Leaf size=116

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rubi [A] time = 0.10577, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5602, 2194, 2251}

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + d*x],x]
```

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 5602

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \cosh(a+bx)\coth(c+dx)\,dx &= \int \left(\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}-\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}\right)dx\\
&= \frac{1}{2}\int e^{-a-bx}\,dx+\frac{1}{2}\int e^{a+bx}\,dx-\int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx\\
&= -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2(c+d}\right.}{b}
\end{aligned}$$

Mathematica [A] time = 9.16386, size = 99, normalized size = 0.85

$$\frac{e^{-a-bx}\left(-2e^{2(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)+e^{2(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -b/(2*d), 1 -
b/(2*d), E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^(2*(c + d*x))]))/(2*b)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int \cosh(bx+a)\coth(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(d*x+c),x)
```

```
[Out] int(cosh(b*x+a)*coth(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}+1}{e^{(bx+dx+a+c)}+e^{(bx+a)}}\,dx+\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}+1}{e^{(bx+dx+a+c)}-e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) +
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) + 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(bx+a\right)\coth\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*coth(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(a+bx\right)\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(c + d*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh (bx + a) \coth (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*coth(d*x + c), x)
```

3.191 $\int \cosh(a+bx)\tanh(c+dx)\,dx$

Optimal. Leaf size=120

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric
2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rubi [A] time = 0.100245, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5604, 2194, 2251}

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric
2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 5604

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\tanh(c+dx)\,dx &= \int \left(\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}-\frac{e^{-a-bx}}{1+e^{2(c+dx)}}-\frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)dx\\ &= \frac{1}{2}\int e^{-a-bx}\,dx+\frac{1}{2}\int e^{a+bx}\,dx-\int \frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx\\ &= -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};-e^{2}\right.}{b}\end{aligned}$$

Mathematica [A] time = 11.3145, size = 103, normalized size = 0.86

$$\frac{e^{-a-bx}\left(-2e^{2(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)+e^{2(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -b/(2*d), 1 -
b/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d)
, 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \cosh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] int(cosh(b*x+a)*tanh(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int \frac{2\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(bx+2\,dx+a+2\,c)}+e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 + 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(bx+a\right)\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*tanh(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(a+bx\right)\tanh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)\tanh(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*tanh(d*x + c), x)
```

3.192 $\int \sinh(x)\sinh(2x)\,dx$

Optimal. Leaf size=8

$$\frac{2\sinh^3(x)}{3}$$

```
[Out] (2*Sinh[x]^3)/3
```

Rubi [A] time = 0.0090377, antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{6}\sinh(3x)-\frac{\sinh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[2*x],x]
```

```
[Out] -Sinh[x]/2 + Sinh[3*x]/6
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(2x)\,dx=-\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Mathematica [A] time = 0.0052166, size = 15, normalized size = 1.88

$$\frac{1}{6}\sinh(3x)-\frac{\sinh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[2*x],x]
```

```
[Out] -Sinh[x]/2 + Sinh[3*x]/6
```

Maple [A] time = 0.007, size = 7, normalized size = 0.9

$$\frac{2\left(\sinh\left(x\right)\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(2*x),x)
```

```
[Out] 2/3*sinh(x)^3
```

Maxima [B] time = 1.04915, size = 36, normalized size = 4.5

$$-\frac{1}{12}\left(3\,e^{(-2\,x)}-1\right)e^{(3\,x)}+\frac{1}{4}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) + 1/4*e^(-x) - 1/12*e^(-3*x)
```

Fricas [B] time = 2.03822, size = 61, normalized size = 7.62

$$\frac{1}{6}\,\sinh\left(x\right)^{3}+\frac{1}{2}\left(\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="fricas")
```

```
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 - 1)*sinh(x)
```

Sympy [B] time = 0.643073, size = 20, normalized size = 2.5

$$\frac{2\sinh(x)\cosh(2x)}{3}-\frac{\sinh(2x)\cosh(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x)
```

```
[Out] 2*sinh(x)*cosh(2*x)/3 - sinh(2*x)*cosh(x)/3
```

Giac [B] time = 1.25114, size = 34, normalized size = 4.25

$$\frac{1}{12}\left(3e^{(2x)}-1\right)e^{(-3x)}+\frac{1}{12}e^{(3x)}-\frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="giac")
```

```
[Out] 1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

3.193 $\int \sinh(x)\sinh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\sinh(4x)-\frac{1}{4}\sinh(2x)$$

```
[Out] -Sinh[2*x]/4 + Sinh[4*x]/8
```

Rubi [A] time = 0.0094791, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{8}\sinh(4x)-\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[3*x],x]
```

```
[Out] -Sinh[2*x]/4 + Sinh[4*x]/8
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(3x)\,dx=-\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Mathematica [A] time = 0.0064357, size = 17, normalized size = 1.

$$\frac{1}{8}\sinh(4x)-\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[3*x],x]
```

```
[Out] -Sinh[2*x]/4 + Sinh[4*x]/8
```

Maple [A] time = 0.019, size = 14, normalized size = 0.8

$$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(3*x),x)
```

```
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)
```

Maxima [A] time = 1.05222, size = 36, normalized size = 2.12

$$-\frac{1}{16}\left(2e^{(-2x)}-1\right)e^{(4x)}+\frac{1}{8}e^{(-2x)}-\frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="maxima")
```

```
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) + 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

Fricas [A] time = 2.11509, size = 80, normalized size = 4.71

$$\frac{1}{2}\cosh(x)\sinh(x)^{3}+\frac{1}{2}\left(\cosh(x)^{3}-\cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 - cosh(x))*sinh(x)
```

Sympy [A] time = 0.632957, size = 20, normalized size = 1.18

$$\frac{3\sinh(x)\cosh(3x)}{8} - \frac{\sinh(3x)\cosh(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x)
```

```
[Out] 3*sinh(x)*cosh(3*x)/8 - sinh(3*x)*cosh(x)/8
```

Giac [A] time = 1.21103, size = 36, normalized size = 2.12

$$\frac{1}{16}\left(2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{16}\,e^{(4\,x)} - \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/16*(2*e^(2*x) - 1)*e^(-4*x) + 1/16*e^(4*x) - 1/8*e^(2*x)
```

3.194 $\int \sinh(x) \sinh(4x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{10}\sinh(5x) - \frac{1}{6}\sinh(3x)$$

```
[Out] -Sinh[3*x]/6 + Sinh[5*x]/10
```

Rubi [A] time = 0.0101462, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{10}\sinh(5x) - \frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[4*x],x]
```

```
[Out] -Sinh[3*x]/6 + Sinh[5*x]/10
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

Mathematica [A] time = 0.0066184, size = 17, normalized size = 1.

$$\frac{1}{10}\sinh(5x) - \frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[4*x],x]
```

```
[Out] -Sinh[3*x]/6 + Sinh[5*x]/10
```

Maple [A] time = 0.02, size = 14, normalized size = 0.8

$$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(4*x),x)
```

```
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)
```

Maxima [A] time = 1.0524, size = 36, normalized size = 2.12

$$-\frac{1}{60}\left(5e^{(-2x)}-3\right)e^{(5x)}+\frac{1}{12}e^{(-3x)}-\frac{1}{20}e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) + 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

Fricas [B] time = 2.03812, size = 119, normalized size = 7.

$$\frac{1}{10}\sinh(x)^5+\frac{1}{6}\left(6\cosh(x)^2-1\right)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^4-\cosh(x)^2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 - 1)*sinh(x)^3 + 1/2*(cosh(x)^4 - cosh(x)
^2)*sinh(x)
```

Sympy [A] time = 0.608297, size = 20, normalized size = 1.18

$$\frac{4\sinh(x)\cosh(4x)}{15} - \frac{\sinh(4x)\cosh(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x)
```

```
[Out] 4*sinh(x)*cosh(4*x)/15 - sinh(4*x)*cosh(x)/15
```

Giac [A] time = 1.1564, size = 36, normalized size = 2.12

$$\frac{1}{60}\left(5e^{(2x)} - 3\right)e^{(-5x)} + \frac{1}{20}e^{(5x)} - \frac{1}{12}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

3.195 $\int \sinh(x)\sinh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh((m+1)x)}{2(m+1)} - \frac{\sinh((1-m)x)}{2(1-m)}$$

```
[Out] -Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0319325, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5613, 2637}

$$\frac{\sinh((m+1)x)}{2(m+1)} - \frac{\sinh((1-m)x)}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[m*x],x]
```

```
[Out] -Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\sinh(mx)\,dx &= \int \left(-\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right)dx\\ &= -\left(\frac{1}{2}\int \cosh((1-m)x)\,dx\right)+\frac{1}{2}\int \cosh((1+m)x)\,dx\\ &= -\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0406219, size = 25, normalized size = 0.71

$$\frac{m\sinh(x)\cosh(mx)-\cosh(x)\sinh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[m*x],x]
```

```
[Out] (m*Cosh[m*x]*Sinh[x] - Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] time = 0.022, size = 28, normalized size = 0.8

$$-\frac{\sinh\left((-1+m)\,x\right)}{-2+2\,m}+\frac{\sinh\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(m*x),x)
```

```
[Out] -1/2/(-1+m)*sinh((-1+m)*x)+1/2*sinh((1+m)*x)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19077, size = 117, normalized size = 3.34

$$\frac{m\cosh(mx)\sinh(x)-\cosh(x)\sinh(mx)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(m*x)*sinh(x) - cosh(x)*sinh(m*x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] time = 1.09108, size = 78, normalized size = 2.23

$$\begin{cases} -\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}-\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=-1 \\ \frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=1 \\ \frac{m\sinh(x)\cosh(mx)}{m^2-1}-\frac{\sinh(mx)\cosh(x)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 - sinh(x)*cosh(x)/2, Eq(m, -1))
, (x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, 1)), (m*sinh(
x)*cosh(m*x)/(m**2 - 1) - sinh(m*x)*cosh(x)/(m**2 - 1), True))
```

Giac [B] time = 1.15118, size = 80, normalized size = 2.29

$$\frac{e^{(mx+x)}}{4(m+1)}-\frac{e^{(mx-x)}}{4(m-1)}+\frac{e^{(-mx+x)}}{4(m-1)}-\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

3.196 $\int \cosh(2x)\sinh(x)\,dx$

Optimal. Leaf size=15

$$\frac{1}{6}\cosh(3x)-\frac{\cosh(x)}{2}$$

```
[Out] -Cosh[x]/2 + Cosh[3*x]/6
```

Rubi [A] time = 0.009962, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cosh(3x)-\frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[2*x]*Sinh[x],x]
```

```
[Out] -Cosh[x]/2 + Cosh[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(2x)\sinh(x)\,dx=-\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Mathematica [A] time = 0.0050589, size = 15, normalized size = 1.

$$\frac{1}{6}\cosh(3x)-\frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[2*x]*Sinh[x],x]
```

```
[Out] -Cosh[x]/2 + Cosh[3*x]/6
```

Maple [A] time = 0.01, size = 12, normalized size = 0.8

$$-\frac{\cosh(x)}{2} + \frac{\cosh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*sinh(x),x)
```

```
[Out] -1/2*cosh(x)+1/6*cosh(3*x)
```

Maxima [B] time = 1.09112, size = 36, normalized size = 2.4

$$-\frac{1}{12}\left(3e^{(-2x)} - 1\right)e^{(3x)} - \frac{1}{4}e^{(-x)} + \frac{1}{12}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) - 1/4*e^(-x) + 1/12*e^(-3*x)
```

Fricas [A] time = 2.07606, size = 72, normalized size = 4.8

$$\frac{1}{6}\cosh(x)^3 + \frac{1}{2}\cosh(x)\sinh(x)^2 - \frac{1}{2}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 - 1/2*cosh(x)
```

Sympy [A] time = 0.608357, size = 20, normalized size = 1.33

$$\frac{2\sinh(x)\sinh(2x)}{3}-\frac{\cosh(x)\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x)
```

```
[Out] 2*sinh(x)*sinh(2*x)/3 - cosh(x)*cosh(2*x)/3
```

Giac [B] time = 1.1749, size = 34, normalized size = 2.27

$$-\frac{1}{12}\left(3e^{(2x)}-1\right)e^{(-3x)}+\frac{1}{12}e^{(3x)}-\frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

3.197 $\int \cosh(3x)\sinh(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\cosh(4x)-\frac{1}{4}\cosh(2x)$$

```
[Out] -Cosh[2*x]/4 + Cosh[4*x]/8
```

Rubi [A] time = 0.0105323, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{8}\cosh(4x)-\frac{1}{4}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[3*x]*Sinh[x],x]
```

```
[Out] -Cosh[2*x]/4 + Cosh[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(3x)\sinh(x)\,dx=-\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Mathematica [A] time = 0.0065284, size = 17, normalized size = 1.

$$\frac{1}{8}\cosh(4x)-\frac{\cosh^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[3*x]*Sinh[x],x]
```

```
[Out] -Cosh[x]^2/2 + Cosh[4*x]/8
```

Maple [A] time = 0.019, size = 14, normalized size = 0.8

$$-\frac{\cosh(2x)}{4} + \frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*sinh(x),x)
```

```
[Out] -1/4*cosh(2*x)+1/8*cosh(4*x)
```

Maxima [A] time = 1.02542, size = 36, normalized size = 2.12

$$-\frac{1}{16}\left(2e^{(-2x)} - 1\right)e^{(4x)} - \frac{1}{8}e^{(-2x)} + \frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) - 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

Fricas [B] time = 1.98447, size = 109, normalized size = 6.41

$$\frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - \frac{1}{4}\cosh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 - 1)*sinh(x)^2 - 1/4*cosh(
x)^2
```

Sympy [A] time = 0.586712, size = 20, normalized size = 1.18

$$\frac{3\sinh(x)\sinh(3x)}{8} - \frac{\cosh(x)\cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x)
```

```
[Out] 3*sinh(x)*sinh(3*x)/8 - cosh(x)*cosh(3*x)/8
```

Giac [A] time = 1.17471, size = 35, normalized size = 2.06

$$\frac{1}{16}\left(e^{(2x)} + e^{(-2x)}\right)^2 - \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 - 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.198 $\int \cosh(4x)\sinh(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

```
[Out] -Cosh[3*x]/6 + Cosh[5*x]/10
```

Rubi [A] time = 0.0108626, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[4*x]*Sinh[x],x]
```

```
[Out] -Cosh[3*x]/6 + Cosh[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(4x)\sinh(x)\,dx=-\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Mathematica [A] time = 0.0061861, size = 17, normalized size = 1.

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[4*x]*Sinh[x],x]
```

```
[Out] -Cosh[3*x]/6 + Cosh[5*x]/10
```

Maple [A] time = 0.027, size = 14, normalized size = 0.8

$$-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*sinh(x),x)
```

```
[Out] -1/6*cosh(3*x)+1/10*cosh(5*x)
```

Maxima [A] time = 1.04297, size = 36, normalized size = 2.12

$$-\frac{1}{60}\left(5\,e^{(-2\,x)}-3\right)e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) - 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

Fricas [B] time = 2.05455, size = 130, normalized size = 7.65

$$\frac{1}{10}\cosh(x)^5+\frac{1}{2}\cosh(x)\sinh(x)^4-\frac{1}{6}\cosh(x)^3+\frac{1}{2}\left(2\cosh(x)^3-\cosh(x)\right)\sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 - 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 -
 cosh(x))*sinh(x)^2
```

Sympy [A] time = 0.596578, size = 20, normalized size = 1.18

$$\frac{4\sinh\left(x\right)\sinh\left(4x\right)}{15}-\frac{\cosh\left(x\right)\cosh\left(4x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x)
```

```
[Out] 4*sinh(x)*sinh(4*x)/15 - cosh(x)*cosh(4*x)/15
```

Giac [A] time = 1.16258, size = 36, normalized size = 2.12

$$-\frac{1}{60}\left(5\,e^{(2\,x)}-3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

3.199 $\int \cosh(mx)\sinh(x)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((m+1)x)}{2(m+1)}$$

```
[Out] Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.034527, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5618, 2638}

$$\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[m*x]*Sinh[x],x]
```

```
[Out] Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh(mx)\sinh(x)\,dx = \int \left(\frac{1}{2}\sinh((1-m)x) + \frac{1}{2}\sinh((1+m)x)\right)dx$$
$$= \frac{1}{2}\int \sinh((1-m)x)\,dx + \frac{1}{2}\int \sinh((1+m)x)\,dx$$
$$= \frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

Mathematica [A] time = 0.0391172, size = 25, normalized size = 0.71

$$\frac{m\sinh(x)\sinh(mx) - \cosh(x)\cosh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[m*x]*Sinh[x],x]
```

```
[Out] (-(Cosh[x]*Cosh[m*x]) + m*Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] time = 0.009, size = 28, normalized size = 0.8

$$-\frac{\cosh\left((-1+m)\,x\right)}{-2+2\,m} + \frac{\cosh\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(m*x)*sinh(x),x)
```

```
[Out] -1/2*cosh((-1+m)*x)/(-1+m)+1/2*cosh((1+m)*x)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13578, size = 117, normalized size = 3.34

$$\frac{m\sinh\left(mx\right)\sinh\left(x\right)-\cosh\left(mx\right)\cosh\left(x\right)}{\left(m^{2}-1\right)\cosh\left(x\right)^{2}-\left(m^{2}-1\right)\sinh\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] (m*sinh(m*x)*sinh(x) - cosh(m*x)*cosh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] time = 1.35055, size = 37, normalized size = 1.06

$$\begin{cases}\frac{\cosh^{2}\left(x\right)}{2} & \text{for}\: m = -1 \vee m = 1\\\frac{m\sinh\left(x\right)\sinh\left(mx\right)}{m^{2}-1}-\frac{\cosh\left(x\right)\cosh\left(mx\right)}{m^{2}-1} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x)
```

```
[Out] Piecewise((cosh(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sinh(x)*sinh(m*x)/(m**2
- 1) - cosh(x)*cosh(m*x)/(m**2 - 1), True))
```

Giac [B] time = 1.13905, size = 80, normalized size = 2.29

$$\frac{e^{\left(mx+x\right)}}{4\left(m+1\right)}-\frac{e^{\left(mx-x\right)}}{4\left(m-1\right)}-\frac{e^{\left(-mx+x\right)}}{4\left(m-1\right)}+\frac{e^{\left(-mx-x\right)}}{4\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

3.200 $\int \sinh(x)\tanh(2x)\,dx$

Optimal. Leaf size=19

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

Rubi [A] time = 0.025886, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 203}

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[2*x],x]
```

```
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sinh(x)\tanh(2x)\,dx = -\,\text{Subst}\left(\int -\frac{2x^2}{1+2x^2}\,dx, x, \sinh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{1+2x^2}\,dx, x, \sinh(x)\right)$$
$$= \sinh(x) - \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)$$

Mathematica [A] time = 0.0151152, size = 19, normalized size = 1.

$$\sinh(x) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[2*x],x]
```

```
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

Maple [C] time = 0.048, size = 54, normalized size = 2.8

$$\frac{e^x}{2} - \frac{e^{-x}}{2} + \frac{i}{4}\sqrt{2}\ln\left(e^{2x} - i\sqrt{2}e^x - 1\right) - \frac{i}{4}\sqrt{2}\ln\left(e^{2x} + i\sqrt{2}e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(2*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/4*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/4*I*
2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)
```

Maxima [B] time = 1.54249, size = 72, normalized size = 3.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) - 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 2.13494, size = 420, normalized size = 22.11

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(-\frac{\sqrt{2}\cosh(x}{}\right.}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) - cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) +
 sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(x\right)\tanh\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x)
```

```
[Out] Integral(sinh(x)*tanh(2*x), x)
```

Giac [B] time = 1.19434, size = 49, normalized size = 2.58

$$-\frac{1}{4}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

3.201 $\int \sinh(x)\tanh(3x)\,dx$

Optimal. Leaf size=19

$$\sinh(x)-\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))$$

```
[Out] -ArcTan[Sinh[x]]/3 - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Rubi [A] time = 0.0433314, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1279, 1163, 203}

$$\sinh(x)-\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[3*x],x]
```

```
[Out] -ArcTan[Sinh[x]]/3 - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\tanh(3x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^2\left(-3-4x^2\right)}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\ &= \sinh(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-4-8x^2}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\ &= \sinh(x)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))+\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.028378, size = 19, normalized size = 1.

$$\sinh(x)-\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[3*x],x]
```

```
[Out] -ArcTan[Sinh[x]]/3 - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Maple [C] time = 0.066, size = 60, normalized size = 3.2

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i}{3}\ln\left(e^{x}-i\right)-\frac{i}{3}\ln\left(e^{x}+i\right)+\frac{i}{6}\ln\left(e^{2x}-ie^{x}-1\right)-\frac{i}{6}\ln\left(e^{2x}+ie^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(3*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/3*I*ln(exp(x)-I)-1/3*I*ln(exp(x)+I)+1/6*I*ln(exp(2
*x)-I*exp(x)-1)-1/6*I*ln(exp(2*x)+I*exp(x)-1)
```

Maxima [B] time = 1.61363, size = 62, normalized size = 3.26

$$\frac{1}{3}\arctan\left(\sqrt{3}+2e^{(-x)}\right)+\frac{1}{3}\arctan\left(-\sqrt{3}+2e^{(-x)}\right)+\frac{2}{3}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(sqrt(3) + 2*e^(-x)) + 1/3*arctan(-sqrt(3) + 2*e^(-x)) + 2/3*arct
an(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 2.01357, size = 302, normalized size = 15.89

$$\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(-\frac{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}}{\cosh(x)-\sinh(x)}\right)-6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)/(cosh(x) - sinh(x))) - 6*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x))
+ 3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\tanh\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x)
```

```
[Out] Integral(sinh(x)*tanh(3*x), x)
```

Giac [B] time = 1.19821, size = 58, normalized size = 3.05

$$-\frac{1}{3}\,\pi - \frac{1}{3}\,\arctan\left(\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - \frac{1}{3}\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] -1/3*pi - 1/3*arctan((e^(2*x) - 1)*e^(-x)) - 1/3*arctan(1/2*(e^(2*x) - 1)*e
^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.202 $\int \sinh(x)\tanh(4x)\,dx$

Optimal. Leaf size=69

$$\sinh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + Sq
rt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Rubi [A] time = 0.0962809, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 203}

$$\sinh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[4*x],x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + Sq
rt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh(x)\tanh(4x)\,dx &= -\text{Subst}\left(\int \frac{4x^2\left(-1-2x^2\right)}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{x^2\left(-1-2x^2\right)}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\right)\\
&= \sinh(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{-2-8x^2}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x)+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{4+2\sqrt{2}
\end{aligned}$$

$$= -\frac{1}{4}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)+\sinh(x)$$

Mathematica [A] time = 0.134683, size = 69, normalized size = 1.

$$\sinh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[4*x],x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + Sq
rt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Maple [C] time = 0.083, size = 42, normalized size = 0.6

$$\frac{\mathrm{e}^{x}}{2}-\frac{\mathrm{e}^{-x}}{2}+\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}+128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,\mathrm{e}^{x}+\mathrm{e}^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(4*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)-1),_R=RootOf(2048*_Z
^4+128*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{2}\int\frac{2\left(e^{(7x)}+e^{x}\right)}{e^{(8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate(2*(e^(7*x) + e^x)/(e^(8*x) + 1), x
)
```

Fricas [B] time = 2.35069, size = 504, normalized size = 7.3

$$-\frac{1}{2}\left(\sqrt{\sqrt{2}+2}\arctan\left(\frac{1}{2}\left(\sqrt{\sqrt{2}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{2}+2}\left(\sqrt{2}-2\right)-\left(\left(\sqrt{2}-2\right)e^{(2x)}-\sqrt{2}+2\right)\sqrt{\sqrt{2}+2}\right)e^{(-x)}\right)e^{x}-\sqrt{-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(sqrt(2) + 2)*arctan(1/2*(sqrt(sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqr
t(sqrt(2) + 2)*(sqrt(2) - 2) - ((sqrt(2) - 2)*e^(2*x) - sqrt(2) + 2)*sqrt(s
qrt(2) + 2))*e^(-x))*e^x - sqrt(-sqrt(2) + 2)*arctan(1/2*(sqrt(-sqrt(2)*e^(
2*x) + e^(4*x) + 1)*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - ((sqrt(2) + 2)*e^(2*
```

```
x) - sqrt(2) - 2)*sqrt(-sqrt(2) + 2))*e^(-x))*e^x - e^(2*x) + 1)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x) \tanh(4x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x)
```

```
[Out] Integral(sinh(x)*tanh(4*x), x)
```

Giac [A] time = 1.25248, size = 96, normalized size = 1.39

$$-\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(sqrt(2) + 2)) - 1/4*sqrt
(-sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(-sqrt(2) + 2)) - 1/2*e^(-x) + 1/
2*e^x
```

3.203 $\int \sinh(x)\tanh(5x)\,dx$

Optimal. Leaf size=87

$$\sinh(x) - \frac{1}{5}\tan^{-1}(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x\right.$$

```
[Out] -ArcTan[Sinh[x]]/5 - (Sqrt[(3 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]])/5 - (Sqrt[(3 - Sqrt[5])/2]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]])
/5 + Sinh[x]
```

Rubi [A] time = 0.290608, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6742, 2073, 203, 1166}

$$\sinh(x) - \frac{1}{5}\tan^{-1}(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[5*x],x]
```


```
[Out] -ArcTan[Sinh[x]]/5 - (Sqrt[(3 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]])/5 - (Sqrt[(3 - Sqrt[5])/2]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]])
/5 + Sinh[x]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \sinh(x)\tanh(5x)\,dx &= -\mathrm{Subst}\left(\int \frac{x^2\left(-5-20x^2-16x^4\right)}{1+13x^2+28x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= -\mathrm{Subst}\left(\int \left(-1+\frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\right)dx, x, \sinh(x)\right)\\
&= \sinh(x)-\mathrm{Subst}\left(\int \frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \sinh(x)-\mathrm{Subst}\left(\int \left(\frac{1}{5\left(1+x^2\right)}+\frac{4\left(1+6x^2\right)}{5\left(1+12x^2+16x^4\right)}\right)dx, x, \sinh(x)\right)\\
&= \sinh(x)-\frac{1}{5}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)-\frac{4}{5}\mathrm{Subst}\left(\int \frac{1+6x^2}{1+12x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{5}\tan^{-1}(\sinh(x))+\sinh(x)-\frac{1}{5}\left(4\left(3-\sqrt{5}\right)\right)\mathrm{Subst}\left(\int \frac{1}{6-2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)-\frac{1}{5}\Big(\\
&= -\frac{1}{5}\tan^{-1}(\sinh(x))-\frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\Big(\sqrt{2}
\end{aligned}$$

Mathematica [A] time = 0.187241, size = 81, normalized size = 0.93

$$\frac{1}{10}\left(10\sinh(x)-2\tan^{-1}(\sinh(x))-\sqrt{2\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\sqrt{6-2\sqrt{5}}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[5*x],x]
```

```
[Out] (-2*ArcTan[Sinh[x]] - Sqrt[2*(3 + Sqrt[5])]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]] - Sqrt[6 - 2*Sqrt[5]]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]] + 10*S
```

```
inh[x])/10
```

Maple [C] time = 0.089, size = 60, normalized size = 0.7

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i}{5}\ln\left(e^{x}-i\right)-\frac{i}{5}\ln\left(e^{x}+i\right)+\sum_{\_R=\mathrm{RootOf}\left(10000\_Z^{4}+300\_Z^{2}+1\right)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(5*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/5*I*ln(exp(x)-I)-1/5*I*ln(exp(x)+I)+sum(_R*ln(-10*
_R*exp(x)+exp(2*x)-1),_R=RootOf(10000*_Z^4+300*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{2}{5}\arctan\left(e^{x}\right)-\frac{1}{2}\int\frac{2\left(3e^{(7x)}-e^{(5x)}-e^{(3x)}+3e^{x}\right)}{5\left(e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 2/5*arctan(e^x) - 1/2*integrate(2/5*(3*e^(7*x) -
 e^(5*x) - e^(3*x) + 3*e^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

Fricas [B] time = 2.4281, size = 724, normalized size = 8.32

$$-\frac{1}{10}\left(2\sqrt{2}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{8}\left(\sqrt{2\left(\sqrt{5}-1\right)e^{(2x)}+4e^{(4x)}+4}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{5}+3}-2\left(\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)e^{(2x)}-\sqrt{5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(2)*sqrt(sqrt(5) + 3)*arctan(1/8*(sqrt(2*(sqrt(5) - 1)*e^(2*x)
 + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(5) + 3) - 2*((sqr
```

```
t(5)*sqrt(2) - 3*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(
5) + 3))*e^(-x))*e^x - 2*sqrt(2)*sqrt(-sqrt(5) + 3)*arctan(1/8*(sqrt(-2*(sq
rt(5) + 1)*e^(2*x) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-sqr
t(5) + 3) - 2*((sqrt(5)*sqrt(2) + 3*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) - 3*
sqrt(2))*sqrt(-sqrt(5) + 3))*e^(-x))*e^x + 4*arctan(e^x)*e^x - 5*e^(2*x) +
5)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\tanh(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x)
```

```
[Out] Integral(sinh(x)*tanh(5*x), x)
```

Giac [A] time = 1.24147, size = 109, normalized size = 1.25

$$-\frac{1}{10}\pi - \frac{1}{10}\left(\sqrt{5}+1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}+1}\right) - \frac{1}{10}\left(\sqrt{5}-1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}-1}\right) - \frac{1}{5}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="giac")
```

```
[Out] -1/10*pi - 1/10*(sqrt(5) + 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) + 1)) - 1/1
0*(sqrt(5) - 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) - 1)) - 1/5*arctan(1/2*(e
^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.204 $\int \sinh(x)\tanh(6x)\,dx$

Optimal. Leaf size=87

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Rubi [A] time = 0.258683, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 203, 1166}

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[6*x],x]
```

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
```

```
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \sinh(x)\tanh(6x)\,dx &= -\operatorname{Subst}\left(\int \frac{2x^2\left(-3-16x^2-16x^4\right)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \frac{x^2\left(-3-16x^2-16x^4\right)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \left(-\frac{1}{2}+\frac{1+12x^2+16x^4}{2\left(1+18x^2+48x^4+32x^6\right)}\right)dx, x, \sinh(x)\right)\right)\\
&= \sinh(x)-\operatorname{Subst}\left(\int \frac{1+12x^2+16x^4}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= \sinh(x)-\operatorname{Subst}\left(\int \left(\frac{1}{3\left(1+2x^2\right)}+\frac{2\left(1+8x^2\right)}{3\left(1+16x^2+16x^4\right)}\right)dx, x, \sinh(x)\right)\\
&= \sinh(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1+8x^2}{1+16x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\sinh(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\operatorname{Subst}\left(\int \frac{1}{8-4\sqrt{3}+16x^2}\,dx, x, \sinh(x)\right)-\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)+\sinh(
\end{aligned}
$$

Mathematica [A] time = 0.123729, size = 87, normalized size = 1.

$$\sinh(x) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[6*x],x]
```

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Maple [C] time = 0.105, size = 84, normalized size = 1.

$$\frac{e^{x}}{2} - \frac{e^{-x}}{2} + \frac{i}{12}\sqrt{2}\ln\left(e^{2x} - i\sqrt{2}e^{x} - 1\right) - \frac{i}{12}\sqrt{2}\ln\left(e^{2x} + i\sqrt{2}e^{x} - 1\right) + \sum_{\_R=\mathrm{RootOf}\left(20736\_Z^{4}+576\_Z^{2}+1\right)} \_R\ \ln\left(-12\_R\,e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(6*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*
I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)+sum(_R*ln(-12*_R*exp(x)+exp(2*x)-
1),_R=RootOf(20736*_Z^4+576*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)} - \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{x}\right)\right) - \frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{x}\right)\right) - \frac{1}{2}\int \frac{2\left(2e^{(7x)} - e^{(5x)} - e^{(3x)}\right.}{3\left(e^{(8x)} - e^{(4x)} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)
) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/2*integrate(2/3*
```

```
(2*e^(7*x) - e^(5*x) - e^(3*x) + 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Fricas [B] time = 2.37504, size = 633, normalized size = 7.28

$$-\frac{1}{6}\left(2\sqrt{\sqrt{3}+2}\arctan\left(\left(\sqrt{\sqrt{3}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-\left(\left(\sqrt{3}-2\right)e^{(2x)}-\sqrt{3}+2\right)\sqrt{\sqrt{3}+2}\right)e^{(-x)}\right)e^{x}-2\sqrt{-\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(sqrt(3) + 2)*arctan((sqrt(sqrt(3)*e^(2*x) + e^(4*x) + 1)*sqrt(
sqrt(3) + 2)*(sqrt(3) - 2) - ((sqrt(3) - 2)*e^(2*x) - sqrt(3) + 2)*sqrt(sqr
t(3) + 2))*e^(-x))*e^x - 2*sqrt(-sqrt(3) + 2)*arctan((sqrt(-sqrt(3)*e^(2*x)
 + e^(4*x) + 1)*(sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - ((sqrt(3) + 2)*e^(2*x) -
 sqrt(3) - 2)*sqrt(-sqrt(3) + 2))*e^(-x))*e^x + sqrt(2)*arctan(1/2*sqrt(2)*
e^(3*x) + 1/2*sqrt(2)*e^x)*e^x + sqrt(2)*arctan(1/2*sqrt(2)*e^x)*e^x - 3*e^
(2*x) + 3)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\tanh(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x)
```

```
[Out] Integral(sinh(x)*tanh(6*x), x)
```

Giac [A] time = 1.19229, size = 135, normalized size = 1.55

$$-\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6) + sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) + sqrt(2))) - 1
/12*(sqrt(6) - sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) - sqrt(2))) - 1/1
2*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x) +
1/2*e^x
```

3.205 $\int \sinh(x)\tanh(nx)\,dx$

Optimal. Leaf size=81

$$-e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)+\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

```
[Out] 1/(2*E^x) + E^x/2 - Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), -E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -E^(2*n*x)]
```

Rubi [A] time = 0.0696508, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5601, 2194, 2251}

$$-e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)+\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[n*x],x]
```


```
[Out] 1/(2*E^x) + E^x/2 - Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), -E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -E^(2*n*x)]
```


Rule 5601

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
```

```
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\tanh(nx)\,dx &= \int\left(-\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{e^{-x}}{1+e^{2nx}}-\frac{e^{x}}{1+e^{2nx}}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-x}\,dx\right)+\frac{\int e^{x}\,dx}{2}+\int\frac{e^{-x}}{1+e^{2nx}}\,dx-\int\frac{e^{x}}{1+e^{2nx}}\,dx\\ &= \frac{e^{-x}}{2}+\frac{e^{x}}{2}-e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)\end{aligned}$$

Mathematica [B] time = 0.170531, size = 164, normalized size = 2.02

$$\frac{1}{2}e^{-2x}\left(-\frac{e^{2nx+x}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};-e^{2nx}\right)}{2n-1}+\frac{e^{(2n+3)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};-e^{2nx}\right)}{2n+1}-e^{x}\left({}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[n*x],x]
```

```
[Out] (-((E^(x + 2*n*x)*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), -E^(2*n*x)
])/(-1 + 2*n)) + (E^((3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n), 2 + 1/(
2*n), -E^(2*n*x)])/(1 + 2*n) - E^x*(Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2
*n), -E^(2*n*x)] + E^(2*x)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^(2
*n*x)]))/(2*E^(2*x))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \sinh(x)\tanh(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(n*x),x)
```

```
[Out] int(sinh(x)*tanh(n*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}-\frac{1}{2}\int\frac{2\left(e^{(2x)}-1\right)}{e^{(2nx+x)}+e^{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/2*integrate(2*(e^(2*x) - 1)/(e^(2*n*x + x) + e
^x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(x\right)\tanh\left(nx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="fricas")
```

```
[Out] integral(sinh(x)*tanh(n*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\tanh\left(nx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x)
```

```
[Out] Integral(sinh(x)*tanh(n*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\tanh\left(nx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)*tanh(n*x), x)
```

3.206 $\int \coth(2x)\sinh(x)\,dx$

Optimal. Leaf size=10

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

```
[Out] -ArcTan[Sinh[x]]/2 + Sinh[x]
```

Rubi [A] time = 0.0230417, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 203}

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[2*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/2 + Sinh[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth(2x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1+2x^2}{2+2x^2}\,dx, x, \sinh(x)\right)$$
$$= \sinh(x) - \text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{2}\tan^{-1}(\sinh(x)) + \sinh(x)$$

Mathematica [A] time = 0.0084669, size = 10, normalized size = 1.

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[2*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/2 + Sinh[x]
```

Maple [A] time = 0.019, size = 9, normalized size = 0.9

$$\sinh(x) - \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(2*x)*sinh(x),x)
```

```
[Out] sinh(x)-arctan(exp(x))
```

Maxima [A] time = 1.51668, size = 22, normalized size = 2.2

$$\arctan\left(e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 2.05331, size = 167, normalized size = 16.7

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-\cosh\left(x\right)^{2}-2\,\cosh\left(x\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(
x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(x\right)\coth\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(2*x), x)
```

Giac [A] time = 1.15817, size = 22, normalized size = 2.2

$$-\arctan\left(e^{x}\right)-\frac{1}{2}\,e^{\left(-x\right)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -arctan(e^x) - 1/2*e^(-x) + 1/2*e^x
```

3.207 $\int \coth(3x)\sinh(x)\,dx$

Optimal. Leaf size=20

$$\sinh(x)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

Rubi [A] time = 0.0269765, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 203}

$$\sinh(x)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[3*x]*Sinh[x],x]
```

```
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(3x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1+4x^2}{3+4x^2}\,dx, x, \sinh(x)\right)\\ &= \sinh(x) - 2\,\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0157836, size = 20, normalized size = 1.

$$\sinh(x) - \frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[3*x]*Sinh[x],x]
```

```
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

Maple [B] time = 0.049, size = 51, normalized size = 2.6

$$-\frac{\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\tanh\left(\frac{x}{2}\right)\right) - \left(\tanh\left(\frac{x}{2}\right) - 1\right)^{-1} - \frac{\sqrt{3}}{3}\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right) - \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(3*x)*sinh(x),x)
```

```
[Out] -1/3*3^(1/2)*arctan(1/3*tanh(1/2*x)*3^(1/2))-1/(tanh(1/2*x)-1)-1/3*3^(1/2)*
arctan(tanh(1/2*x)*3^(1/2))-1/(tanh(1/2*x)+1)
```

Maxima [B] time = 1.57482, size = 66, normalized size = 3.3

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)} + 1\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)} - 1\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) + 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1)) - 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 2.21664, size = 433, normalized size = 21.65

$$-\frac{2\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh\left(x\right)+\frac{1}{3}\sqrt{3}\sinh\left(x\right)\right)-2\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\arctan\left(-\frac{\sqrt{3}\mathrm{co}}{}\right.}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/
3*sqrt(3)*sinh(x)) - 2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqr
t(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))
/(cosh(x) - sinh(x))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/
(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(x\right)\coth\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(3*x), x)
```

Giac [B] time = 1.17834, size = 49, normalized size = 2.45

$$-\frac{1}{6}\sqrt{3}\left(\pi+2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

3.208 $\int \coth(4x)\sinh(x)\,dx$

Optimal. Leaf size=28

$$\sinh(x) - \frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[Sinh[x]]/4 - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Rubi [A] time = 0.0531672, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 203}

$$\sinh(x) - \frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(4x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1+8x^2+8x^4}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(1-\frac{3+4x^2}{4+12x^2+8x^4}\right)dx, x, \sinh(x)\right)\\ &= \sinh(x)-\text{Subst}\left(\int \frac{3+4x^2}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)\\ &= \sinh(x)-2\,\text{Subst}\left(\int \frac{1}{4+8x^2}\,dx, x, \sinh(x)\right)-2\,\text{Subst}\left(\int \frac{1}{8+8x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{4}\tan^{-1}(\sinh(x))-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}+\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.029861, size = 28, normalized size = 1.

$$\sinh(x)-\frac{1}{4}\tan^{-1}(\sinh(x))-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Maple [B] time = 0.092, size = 143, normalized size = 5.1

$$-\frac{\sqrt{2}}{4+4\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)-\frac{1}{2+2\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)+\frac{\sqrt{2}}{-4+4\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{-2+2\sqrt{2}}\right)-\frac{1}{-2+2\sqrt{2}}\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(4*x)*sinh(x),x)
```

```
[Out] -1/2*2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))-1/(2+2*2^(1/
2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))+1/2*2^(1/2)/(-2+2*2^(1/2))*arctan(2
*tanh(1/2*x)/(-2+2*2^(1/2)))-1/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^
(1/2)))-1/2*arctan(tanh(1/2*x))-1/(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)
```

Maxima [B] time = 1.56177, size = 81, normalized size = 2.89

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)+\frac{1}{2}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/4*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e
^x
```

Fricas [B] time = 2.31449, size = 493, normalized size = 17.61

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(-\frac{\sqrt{2}\cosh(x}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) + 2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 2*cosh(
x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\coth\left(4x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(4*x), x)
```

Giac [B] time = 1.21276, size = 73, normalized size = 2.61

$$-\frac{1}{8}\pi - \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.209 $\int \coth(5x)\sinh(x)\,dx$

Optimal. Leaf size=82

$$\sinh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTan[Sqrt[(2*(5 + Sqrt[5]))/5]*Sinh[x]])/5 + Sinh[x]
```

Rubi [A] time = 0.187413, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 203}

$$\sinh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[5*x]*Sinh[x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTan[Sqrt[(2*(5 + Sqrt[5]))/5]*Sinh[x]])/5 + Sinh[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(5x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1+12x^2+16x^4}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(1-\frac{4\left(1+2x^2\right)}{5+20x^2+16x^4}\right)dx, x, \sinh(x)\right)\\ &= \sinh(x)-4\,\text{Subst}\left(\int \frac{1+2x^2}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)\\ &= \sinh(x)-\frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10-2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)-\frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Subst}\Bigg(\\ &= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)\end{aligned}$$

Mathematica [A] time = 0.223715, size = 76, normalized size = 0.93

$$\frac{1}{10}\left(10\sinh(x)-\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sinh(x)\right)-\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[5*x]*Sinh[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTan[Sqrt[2 + 2/Sqrt[5]]*Sinh[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]] + 10*Sinh[x])/10
```

Maple [B] time = 0.155, size = 246, normalized size = 3.

$$\frac{\sqrt{5}}{10\sqrt{5-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{5-2\sqrt{5}}}\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2\sqrt{5-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{5-2\sqrt{5}}}\tanh\left(\frac{x}{2}\right)\right)-\frac{\sqrt{5}}{10\sqrt{5+2\sqrt{5}}}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(5*x)*sinh(x),x)
```

```
[Out] 1/10*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5-2*5^(1/2))^(1/2))-1/
2/(5-2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5-2*5^(1/2))^(1/2))-1/10*5^(1/2)/
(5+2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5+2*5^(1/2))^(1/2))-1/2/(5+2*5^(1/2
))^(1/2)*arctan(tanh(1/2*x)/(5+2*5^(1/2))^(1/2))+1/2*5^(1/2)/(25-10*5^(1/2)
)^(1/2)*arctan(5*tanh(1/2*x)/(25-10*5^(1/2))^(1/2))-3/2/(25-10*5^(1/2))^(1/
2)*arctan(5*tanh(1/2*x)/(25-10*5^(1/2))^(1/2))-1/2*5^(1/2)/(25+10*5^(1/2))^
(1/2)*arctan(5*tanh(1/2*x)/(25+10*5^(1/2))^(1/2))-3/2/(25+10*5^(1/2))^(1/2)
*arctan(5*tanh(1/2*x)/(25+10*5^(1/2))^(1/2))-1/(tanh(1/2*x)-1)-1/(tanh(1/2*
x)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{2}\int\frac{e^{(3x)}+e^{(2x)}+e^{x}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx-\frac{1}{2}\int\frac{e^{(3x)}-e^{(2x)}+e^{x}}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(3*x) + e^(2*x) + e^x)/(e^(4*x)
 + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/2*integrate((e^(3*x) - e^(2*x) + e^
x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x)
```

Fricas [B] time = 2.38223, size = 699, normalized size = 8.52

$$-\frac{1}{10}\left(2\sqrt{2}\sqrt{\sqrt{5}+5}\arctan\left(\frac{1}{40}\left(\sqrt{2\left(\sqrt{5}+1\right)e^{(2x)}+4e^{(4x)}+4}\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+5}-2\left(\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)e^{(2x)}-\sqrt{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/40*(sqrt(2*(sqrt(5) + 1)*e^(2*x
) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5) - 2*((sq
rt(5)*sqrt(2) - 5*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt
(5) + 5))*e^(-x))*e^x - 2*sqrt(2)*sqrt(-sqrt(5) + 5)*arctan(1/40*(sqrt(-2*(
sqrt(5) - 1)*e^(2*x) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(-s
```

```
qrt(5) + 5) - 2*((sqrt(5)*sqrt(2) + 5*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) -
5*sqrt(2))*sqrt(-sqrt(5) + 5))*e^(-x))*e^x - 5*e^(2*x) + 5)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\coth(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(5*x), x)
```

Giac [A] time = 1.25231, size = 101, normalized size = 1.23

$$-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(1/2*sqrt(5) + 5/2))
- 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(-1/2*sqrt(5) + 5/2
)) - 1/2*e^(-x) + 1/2*e^x
```

3.210 $\int \coth(6x)\sinh(x)\,dx$

Optimal. Leaf size=38

$$\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[Sinh[x]]/6 - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*S
qrt[3]) + Sinh[x]
```

Rubi [A] time = 0.0773604, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 203}

$$\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[6*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/6 - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*S
qrt[3]) + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \coth(6x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2-\frac{1}{3\left(1+x^2\right)}-\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)dx, x, \sinh(x)\right)\\
&= \sinh(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\text{Subst}\Bigg(\int
\\
&= -\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0578494, size = 38, normalized size = 1.

$$
\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[6*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/6 - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*S
qrt[3]) + Sinh[x]
```

Maple [B] time = 0.112, size = 172, normalized size = 4.5

$$
\frac{\sqrt{3}}{12-6\sqrt{3}}\arctan\left(2\frac{\tanh(x/2)}{4-2\sqrt{3}}\right)-\frac{2}{12-6\sqrt{3}}\arctan\left(2\frac{\tanh(x/2)}{4-2\sqrt{3}}\right)-\frac{\sqrt{3}}{12+6\sqrt{3}}\arctan\left(2\frac{\tanh(x/2)}{4+2\sqrt{3}}\right)-\frac{2}{12+6\sqrt{3}}\text{ar}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)*sinh(x),x)
```

```
[Out] 1/3*3^(1/2)/(4-2*3^(1/2))*arctan(2*tanh(1/2*x)/(4-2*3^(1/2)))-2/3/(4-2*3^(1
/2))*arctan(2*tanh(1/2*x)/(4-2*3^(1/2)))-1/3*3^(1/2)/(4+2*3^(1/2))*arctan(2
```

```
*tanh(1/2*x)/(4+2*3^(1/2)))-2/3/(4+2*3^(1/2))*arctan(2*tanh(1/2*x)/(4+2*3^(
1/2)))-1/6*3^(1/2)*arctan(1/3*tanh(1/2*x)*3^(1/2))-1/3*arctan(tanh(1/2*x))-
1/(tanh(1/2*x)-1)-1/6*3^(1/2)*arctan(tanh(1/2*x)*3^(1/2))-1/(tanh(1/2*x)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)-\frac{1}{3}\arctan\left(e^{x}\right)-\frac{1}{2}\int\frac{e^{(3x)}+}{3\left(e^{(4x)}-e^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/
6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x - 1)) - 1/3*arctan(e^x) - 1/2*integrate
(1/3*(e^(3*x) + e^x)/(e^(4*x) - e^(2*x) + 1), x)
```

Fricas [B] time = 2.23425, size = 624, normalized size = 16.42

$$-\frac{\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right)-\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\arctan\left(-\frac{\sqrt{3}\cosh(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/3*
sqrt(3)*sinh(x)) - (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqrt(3)
*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))/(co
sh(x) - sinh(x))) - (cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 3*(cosh(x) + sinh(x))*arctan(cosh(x
) + sinh(x)) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/(cosh(x)
+ sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh(x)\coth(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(6*x), x)
```

Giac [B] time = 1.2043, size = 92, normalized size = 2.42

$$-\frac{1}{6}\pi - \frac{1}{12}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{6}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) -
1/6*arctan((e^(2*x) - 1)*e^(-x)) - 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1
/2*e^(-x) + 1/2*e^x
```

3.211 $\int \operatorname{sech}(2x)\sinh(x)\,dx$

Optimal. Leaf size=16

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2])
```

Rubi [A] time = 0.0194854, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*x]*Sinh[x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.28133, size = 155, normalized size = 9.69

$$\frac{-4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\log\left(\sqrt{2}-2\cosh(x)\right)-\log\left(2\cosh(x)+\sqrt{2}\right)-2i\tan^{-1}\left(\frac{\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cosh\left(\frac{x}{2}\right)-\left(\sqrt{2}-1\right)\sinh\left(\frac{x}{2}\right)}\right)+2}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*x]*Sinh[x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Lo
g[Sqrt[2] - 2*Cosh[x]] - Log[Sqrt[2] + 2*Cosh[x]])/(4*Sqrt[2])
```

Maple [B] time = 0.033, size = 39, normalized size = 2.4

$$\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*x)*sinh(x),x)
```

```
[Out] 1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2))
*2^(1/2)
```

Maxima [B] time = 1.52406, size = 57, normalized size = 3.56

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1)
```

Fricas [B] time = 1.98553, size = 122, normalized size = 7.62

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}-2\sqrt{2}\cosh\left(x\right)+2}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\operatorname{sech}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(2*x), x)
```

Giac [B] time = 1.19838, size = 51, normalized size = 3.19

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

3.212 $\int \operatorname{sech}(3x)\sinh(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(3-4\cosh^2(x)\right)-\frac{1}{3}\log(\cosh(x))$$

```
[Out] -Log[Cosh[x]]/3 + Log[3 - 4*Cosh[x]^2]/6
```

Rubi [A] time = 0.0301864, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4357, 266, 36, 29, 31}

$$\frac{1}{6}\log\left(3-4\cosh^2(x)\right)-\frac{1}{3}\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[3*x]*Sinh[x],x]
```

```
[Out] -Log[Cosh[x]]/3 + Log[3 - 4*Cosh[x]^2]/6
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}(3x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cosh^2(x)\right)\\
&= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cosh^2(x)\right)\\
&= -\frac{1}{3}\log(\cosh(x))+\frac{1}{6}\log\left(3-4\cosh^2(x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0086202, size = 17, normalized size = 0.81

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(8\sinh^2(x)+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[3*x]*Sinh[x],x]
```

```
[Out] -ArcTanh[(5 + 8*Sinh[x]^2)/3]/3
```

Maple [A] time = 0.034, size = 26, normalized size = 1.2

$$-\frac{\ln\left(e^{2x}+1\right)}{3}+\frac{\ln\left(e^{4x}-e^{2x}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(3*x)*sinh(x),x)
```

```
[Out] -1/3*ln(exp(2*x)+1)+1/6*ln(exp(4*x)-exp(2*x)+1)
```

Maxima [B] time = 1.56583, size = 61, normalized size = 2.9

$$\frac{1}{6}\log\left(\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{3}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/6*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/6*log(-sqrt(3)*e^(-x) + e^(-2*x)
 + 1) - 1/3*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.05395, size = 171, normalized size = 8.14

$$\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)-\frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh(x)\operatorname{sech}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(3*x), x)
```

Giac [B] time = 1.15759, size = 55, normalized size = 2.62

$$\frac{1}{6}\,\log\left(\sqrt{3}e^{x}+e^{(2\,x)}+1\right)+\frac{1}{6}\,\log\left(-\sqrt{3}e^{x}+e^{(2\,x)}+1\right)-\frac{1}{3}\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1) -
1/3*log(e^(2*x) + 1)
```

3.213 $\int \textbf{sech}(4x)\sinh(x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Cosh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rubi [A] time = 0.0735708, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4357, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[4*x]*Sinh[x],x]
```

```
[Out] ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Cosh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(4x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \sqrt{2}\operatorname{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)-\sqrt{2}\operatorname{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [C] time = 0.0237442, size = 110, normalized size = 1.55

$$\frac{1}{16}\operatorname{RootSum}\left[\#1^8+1\&, \frac{\#1^2 x+2\#1^2\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)-2\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#\right.}{\#1^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[4*x]*Sinh[x],x]
```

```
[Out] RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Si
nh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x
/2]*#1]*#1^2)/#1^5 & ]/16
```

Maple [C] time = 0.046, size = 40, normalized size = 0.6

$$2\sum_{\_R=\operatorname{RootOf}\left(32768\_Z^4-512\_Z^2+1\right)}\_R\ln\left(e^{2x}+\left(4096\_R^3-48\_R\right)e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(4*x)*sinh(x),x)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(4096*_R^3-48*_R)*exp(x)+1),_R=RootOf(32768*_Z^4-512*_
Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(4x)\sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] integrate(sech(4*x)*sinh(x), x)
```

Fricas [B] time = 2.16482, size = 776, normalized size = 10.93

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqr
t(2) - 1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqr
t(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) -
1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqrt(-sqrt
(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) + 1)*cos
h(x) + (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(-sqrt(2) +
 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) + 1)*cosh(x)
+ (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(4*x), x)
```

Giac [B] time = 1.22065, size = 155, normalized size = 2.18

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt
(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sqrt(sqrt(2) + 2)*log
(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

3.214 $\int \mathbf{sech}(5x)\sinh(x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\cosh(x))$$

```
[Out] Log[Cosh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cosh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] - 8*Cosh[x]^2])/20
```

Rubi [A] time = 0.0833591, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4357, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[5*x]*Sinh[x],x]
```

```
[Out] Log[Cosh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cosh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] - 8*Cosh[x]^2])/20
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}(5x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{5}\log(\cosh(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cosh^2(x)\right)-\frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\\
&= \frac{1}{5}\log(\cosh(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cosh^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cosh^2(\right.
\end{aligned}
$$

Mathematica [A] time = 0.083173, size = 57, normalized size = 0.92

$$
\frac{1}{20}\left(\left(\sqrt{5}-1\right)\log\left(8\sinh^2(x)-\sqrt{5}+3\right)-\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+3\right)+4\log(\cosh(x))\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[5*x]*Sinh[x],x]
```

```
[Out] (4*Log[Cosh[x]] + (-1 + Sqrt[5])*Log[3 - Sqrt[5] + 8*Sinh[x]^2] - (1 + Sqrt
[5])*Log[3 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Maple [B] time = 0.058, size = 101, normalized size = 1.6

$$\frac{\ln\left(\mathrm{e}^{2x}+1\right)}{5}-\frac{\ln\left(\mathrm{e}^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\mathrm{e}^{2x}+1\right)}{20}+\frac{\ln\left(\mathrm{e}^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\mathrm{e}^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(\mathrm{e}^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\mathrm{e}^{2x}+1\right)}{20}-\frac{\ln\left(\mathrm{e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(5*x)*sinh(x),x)
```

```
[Out] 1/5*ln(exp(2*x)+1)-1/20*ln(exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(
exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2*5^(1/
2)-1/2)*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2*5^(1/2)-1/2)*exp(2*x)+1)*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{5}\int\frac{\left(e^{(6x)}-e^{(4x)}+e^{(2x)}-1\right)e^{(2x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx+\frac{2}{5}\int\frac{e^{(6x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx+\frac{1}{5}\int\frac{e^{(4x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -2/5*integrate((e^(6*x) - e^(4*x) + e^(2*x) - 1)*e^(2*x)/(e^(8*x) - e^(6*x)
 + e^(4*x) - e^(2*x) + 1), x) + 2/5*integrate(e^(6*x)/(e^(8*x) - e^(6*x) +
e^(4*x) - e^(2*x) + 1), x) + 1/5*integrate(e^(4*x)/(e^(8*x) - e^(6*x) + e^(
4*x) - e^(2*x) + 1), x) - 4/5*integrate(e^(2*x)/(e^(8*x) - e^(6*x) + e^(4*x
) - e^(2*x) + 1), x) + 1/5*log(e^(2*x) + 1)
```

Fricas [B] time = 2.19283, size = 583, normalized size = 9.4

$$\frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh\left(x\right)^{4}+4\sinh\left(x\right)^{4}-4\left(\sqrt{5}+1\right)\cosh\left(x\right)^{2}+4\left(6\cosh\left(x\right)^{2}-\sqrt{5}-1\right)\sinh\left(x\right)^{2}+\sqrt{5}+7}{2\cosh\left(x\right)^{4}+2\sinh\left(x\right)^{4}+2\left(6\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{2}-2\cosh\left(x\right)^{2}+1}\right)-\frac{1}{20}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 - 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 - sqrt(5) - 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{sech}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(5*x), x)
```

Giac [B] time = 1.20215, size = 159, normalized size = 2.56

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right)+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/20*(
sqrt(5) - 1)*log(-1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(
5) + 1)*log(1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5) +
1)*log(-1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/5*log(e^(2*x) + 1)
```

3.215 $\int \mathbf{sech}(6x)\sinh(x)\,dx$

Optimal. Leaf size=85

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) - ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Rubi [A] time = 0.0791248, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4357, 2057, 207, 1166}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[6*x]*Sinh[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) - ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(6x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx, x, \cosh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}+\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cosh(x)\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\right)+\frac{4}{3}\operatorname{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \cosh(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \cosh(x)\right)+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-8+4\sqrt{\ldots}}\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 0.26666, size = 385, normalized size = 4.53

$$\sqrt{2}\operatorname{RootSum}\left[\#1^8-\#1^4+1\&, \frac{\#1^6 x-\#1^4 x+\#1^2 x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)-2\#1^4\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sin\ldots\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[6*x]*Sinh[x],x]
```

```
[Out] ((4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqrt
[2])*Sinh[x/2])] - (4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Cos
h[x/2] - (1 + Sqrt[2])*Sinh[x/2])] + 8*ArcTanh[Sqrt[2] - I*Tanh[x/2]] - 2*L
og[Sqrt[2] - 2*Cosh[x]] + 2*Log[Sqrt[2] + 2*Cosh[x]] + Sqrt[2]*RootSum[1 -
#1^4 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/
2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#
1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*
#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]
*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/(24*Sqrt[2])
```

Maple [C] time = 0.068, size = 78, normalized size = 0.9

$$\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{12}+2\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^4-2304\_Z^2+1\right)}\_R\ln\left(e^{2x}+\left(13824\_R^3-96\_R\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(6*x)*sinh(x),x)
```

```
[Out] 1/12*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+exp(2*x)-exp(x)*2^(1/2
))*2^(1/2)+2*sum(_R*ln(exp(2*x)+(13824*_R^3-96*_R)*exp(x)+1),_R=RootOf(3317
76*_Z^4-2304*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)+\int\frac{e^{(7x)}-e^{(5x)}+e^{(3x)}-e^{x}}{3\left(e^{(8x)}-e^{(4x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
 + e^(2*x) + 1) + integrate(1/3*(e^(7*x) - e^(5*x) + e^(3*x) - e^x)/(e^(8*x
) - e^(4*x) + 1), x)
```

Fricas [B] time = 2.33375, size = 906, normalized size = 10.66

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2}+1\right)-\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sq
rt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*s
qrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3)
- 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*sqrt(-s
qrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) + 2)*
cosh(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(-sqrt(
3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) + 2)*cosh
(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(2)*log((co
sh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{sech}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(6*x), x)
```

Giac [B] time = 1.24073, size = 208, normalized size = 2.45

$$-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^x+e^{(2x)}+1\right)+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^x+e^{(2x)}+1\right)-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - sqrt(2))*log(1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) +
1/24*(sqrt(6) - sqrt(2))*log(-1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) -
1/24*(sqrt(6) + sqrt(2))*log(1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/24*(sqrt(6) + sqrt(2))*log(-1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

3.216 $\int \operatorname{csch}(2x)\sinh(x)\,dx$

Optimal. Leaf size=7

$$\frac{1}{2}\tan^{-1}(\sinh(x))$$

```
[Out] ArcTan[Sinh[x]]/2
```

Rubi [A] time = 0.013489, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4288, 3770}

$$\frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/2
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(2x)\sinh(x)\,dx &= \frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0028293, size = 7, normalized size = 1.

$$\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*x]*Sinh[x],x]
```

```
[Out] ArcTan[Tanh[x/2]]
```

Maple [A] time = 0.013, size = 4, normalized size = 0.6

$$\arctan\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*x)*sinh(x),x)
```

```
[Out] arctan(exp(x))
```

Maxima [A] time = 1.57405, size = 9, normalized size = 1.29

$$-\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -arctan(e^(-x))
```

Fricas [A] time = 2.07055, size = 36, normalized size = 5.14

$$\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(2*x), x)
```

Giac [A] time = 1.20529, size = 4, normalized size = 0.57

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] arctan(e^x)
```

3.217 $\int \mathrm{csch}(3x)\sinh(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0362462, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[3*x]*Sinh[x],x]
```

```
[Out] ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}(3x)\sinh(x)\,dx &= \mathrm{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0241455, size = 44, normalized size = 2.93

$$-\frac{1}{4}e^{2x}\left(e^{2x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{6x}\right)-2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[3*x]*Sinh[x],x]
```

```
[Out] -(E^(2*x)*(-2*Hypergeometric2F1[1/3, 1, 4/3, E^(6*x)] + E^(2*x)*Hypergeomet
ric2F1[2/3, 1, 5/3, E^(6*x)]))/4
```

Maple [C] time = 0.033, size = 40, normalized size = 2.7

$$\frac{i}{6}\sqrt{3}\ln\left(e^{2x}+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\frac{i}{6}\sqrt{3}\ln\left(e^{2x}+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(3*x)*sinh(x),x)
```

```
[Out] 1/6*I*3^(1/2)*ln(exp(2*x)+1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)+1/2-
1/2*I*3^(1/2))
```

Maxima [B] time = 1.53863, size = 53, normalized size = 3.53

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{(-x)}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{(-x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) - 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1))
```

Fricas [B] time = 2.12863, size = 115, normalized size = 7.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{csch}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(3*x), x)
```

Giac [A] time = 1.20302, size = 26, normalized size = 1.73

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1))
```

3.218 $\int \mathrm{csch}(4x)\sinh(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -ArcTan[Sinh[x]]/4 + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0268189, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{4+8x^2}\,dx, x, \sinh(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{8+8x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) + \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0214198, size = 26, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} - \frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Maple [C] time = 0.04, size = 62, normalized size = 2.4

$$\frac{i}{4}\ln\left(e^x - i\right) - \frac{i}{4}\ln\left(e^x + i\right) + \frac{i}{8}\sqrt{2}\ln\left(e^{2x} + i\sqrt{2}e^x - 1\right) - \frac{i}{8}\sqrt{2}\ln\left(e^{2x} - i\sqrt{2}e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(4*x)*sinh(x),x)
```

```
[Out] 1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*e
xp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)
```

Maxima [B] time = 1.52082, size = 68, normalized size = 2.62

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right) + \frac{1}{2}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x))
```

Fricas [B] time = 2.18787, size = 297, normalized size = 11.42

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/4*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x))) - 1/2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(x\right)\operatorname{csch}\left(4x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(4*x), x)
```

Giac [B] time = 1.2097, size = 59, normalized size = 2.27

$$-\frac{1}{8}\pi+\frac{1}{8}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)-\frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*pi + 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.219 $\int \operatorname{csch}(5x)\sinh(x)\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 - 2*Sqrt[5]]])/5 - (Sqrt[(5 +
Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 + 2*Sqrt[5]]])/5
```

Rubi [A] time = 0.108097, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 203}

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[5*x]*Sinh[x],x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 - 2*Sqrt[5]]])/5 - (Sqrt[(5 +
Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 + 2*Sqrt[5]]])/5
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \mathbf{csch}(5x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1-x^2}{5+10x^2+x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{10}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5-2\sqrt{5}+x^2}\,dx, x, \tanh(x)\right) - \frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5+2\sqrt{5}+\cdots}\right.$$
$$= \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

Mathematica [A] time = 0.103277, size = 84, normalized size = 1.12

$$\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-3\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{\left(3+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[5*x]*Sinh[x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((-3 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTan[((3 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [C] time = 0.038, size = 41, normalized size = 0.6

$$2\sum_{\_R=\text{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)}\_R\ln\left(4000\_R^3-200\_R^2+e^{2x}+30\_R-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(5*x)*sinh(x),x)
```

```
[Out] 2*sum(_R*ln(4000*_R^3-200*_R^2+exp(2*x)+30*_R-1),_R=RootOf(32000*_Z^4+400*_
Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/10*(-1)^(3/5)*log((-1)^(1/5) + e^(-2*x)) + 1/10*sqrt(5)*(-1)^(3/5)*log((s
qrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*e^(-2*
x))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*
e^(-2*x)))/sqrt(2*sqrt(5) - 10) - 1/10*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)
^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))/(sqrt(
5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))
)/sqrt(-2*sqrt(5) - 10) - 1/10*log(-(sqrt(5)*(-1)^(1/5) + (-1)^(1/5))*e^(-2
*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) + 1/10*l
og((sqrt(5)*(-1)^(1/5) - (-1)^(1/5))*e^(-2*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/
(sqrt(5)*(-1)^(2/5) - (-1)^(2/5)) - 1/10*integrate((e^(3*x) + 2*e^(2*x) + 3
*e^x + 4)*e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(
(e^(3*x) - 2*e^(2*x) + 3*e^x - 4)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x +
1), x) + 1/10*log(e^x + 1) + 1/10*log(e^x - 1)
```

Fricas [B] time = 2.35452, size = 558, normalized size = 7.44

$$-\frac{1}{5}\sqrt{2}\sqrt{-\sqrt{5}+5}\arctan\left(\frac{1}{40}\sqrt{5}\sqrt{2}\sqrt{-32\left(\sqrt{5}-1\right)e^{(2x)}+64e^{(4x)}+64}\sqrt{-\sqrt{5}+5}-\frac{1}{20}\left(4\sqrt{5}\sqrt{2}e^{(2x)}+\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(2)*sqrt(-sqrt(5) + 5)*arctan(1/40*sqrt(5)*sqrt(2)*sqrt(-32*(sqrt(
5) - 1)*e^(2*x) + 64*e^(4*x) + 64)*sqrt(-sqrt(5) + 5) - 1/20*(4*sqrt(5)*sqr
t(2)*e^(2*x) + sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(5) + 5)) + 1/5*sqrt(
2)*sqrt(sqrt(5) + 5)*arctan(-1/20*(4*sqrt(5)*sqrt(2)*e^(2*x) + sqrt(5)*sqrt
(2) + 5*sqrt(2))*sqrt(sqrt(5) + 5) + 1/5*sqrt(5)*sqrt((sqrt(5) + 1)*e^(2*x)
 + 2*e^(4*x) + 2)*sqrt(sqrt(5) + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\operatorname{csch}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(5*x), x)
```

Giac [A] time = 1.18576, size = 92, normalized size = 1.23

$$\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4e^{(2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4e^{(2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) - 1/10*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) + 1)/sqrt(-
2*sqrt(5) + 10))
```

3.220 $\int \mathrm{csch}(6x)\sinh(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTan[Sinh[x]]/6 + ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rubi [A] time = 0.0454255, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 203}

$$\frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[6*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/6 + ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(6x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{3\left(1+x^2\right)}+\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)dx, x, \sinh(x)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\text{Subst}\left(\int \frac{1}{3+4x\right.\\ &= \frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0325924, size = 30, normalized size = 0.83

$$\frac{1}{6}\left(\tan^{-1}(\sinh(x))+\tan^{-1}(2\sinh(x))-\sqrt{3}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[6*x]*Sinh[x],x]
```

```
[Out] (ArcTan[Sinh[x]] + ArcTan[2*Sinh[x]] - Sqrt[3]*ArcTan[(2*Sinh[x])/Sqrt[3]])
/6
```

Maple [C] time = 0.054, size = 92, normalized size = 2.6

$$\frac{i}{6}\ln\left(e^x+i\right)-\frac{i}{6}\ln\left(e^x-i\right)+\frac{i}{12}\ln\left(e^{2x}+ie^x-1\right)-\frac{i}{12}\ln\left(e^{2x}-ie^x-1\right)+\frac{i}{12}\sqrt{3}\ln\left(e^{2x}-i\sqrt{3}e^x-1\right)-\frac{i}{12}\sqrt{3}\ln\left(e^{2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(6*x)*sinh(x),x)
```

```
[Out] 1/6*I*ln(exp(x)+I)-1/6*I*ln(exp(x)-I)+1/12*I*ln(exp(2*x)+I*exp(x)-1)-1/12*I
*ln(exp(2*x)-I*exp(x)-1)+1/12*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2)*exp(x)-1)-1/1
```

```
2*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp(x)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right)+\frac{1}{3}\arctan\left(e^x\right)+\int\frac{e^{(3x)}+e^x}{6\left(e^{(4x)}-e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + 1/3*arctan(e^x) + integrate(1/6*(e^(3*x) + e^x)/(e^(4*x)
- e^(2*x) + 1), x)
```

Fricas [B] time = 2.15969, size = 408, normalized size = 11.33

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2+}{3(\cosh(x)-\sinh(x))}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*cosh(x) + 1/3*sqrt(3)*sinh(x)) + 1/6*sqrt(3
)*arctan(-1/3*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh
(x)^2 + 2*sqrt(3))/(cosh(x) - sinh(x))) - 1/6*arctan(-(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 1/2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh(x)\operatorname{csch}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(6*x), x)
```

Giac [B] time = 1.19096, size = 78, normalized size = 2.17

$$\frac{1}{6}\pi - \frac{1}{12}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) + \frac{1}{6}\arctan\left(\left(e^{(2x)} - 1\right)e^{(-x)}\right) + \frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) + 1
/6*arctan((e^(2*x) - 1)*e^(-x)) + 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.221 $\int \cosh(x)\sinh(2x)\,dx$

Optimal. Leaf size=8

$$\frac{2\cosh^3(x)}{3}$$

```
[Out] (2*Cosh[x]^3)/3
```

Rubi [A] time = 0.0105073, antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[2*x],x]
```

```
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(2x)\,dx=\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Mathematica [A] time = 0.0052221, size = 15, normalized size = 1.88

$$\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[2*x],x]
```

```
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

Maple [A] time = 0.009, size = 7, normalized size = 0.9

$$\frac{2 (\cosh(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(2*x),x)
```

```
[Out] 2/3*cosh(x)^3
```

Maxima [B] time = 1.00849, size = 36, normalized size = 4.5

$$\frac{1}{12} \left(3 e^{(-2x)} + 1\right) e^{(3x)} + \frac{1}{4} e^{(-x)} + \frac{1}{12} e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="maxima")
```

```
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) + 1/4*e^(-x) + 1/12*e^(-3*x)
```

Fricas [B] time = 1.98672, size = 72, normalized size = 9.

$$\frac{1}{6} \cosh(x)^3 + \frac{1}{2} \cosh(x) \sinh(x)^2 + \frac{1}{2} \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="fricas")
```

```
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 + 1/2*cosh(x)
```

Sympy [B] time = 0.647082, size = 20, normalized size = 2.5

$$-\frac{\sinh(x)\sinh(2x)}{3}+\frac{2\cosh(x)\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x)
```

```
[Out] -sinh(x)*sinh(2*x)/3 + 2*cosh(x)*cosh(2*x)/3
```

Giac [B] time = 1.22339, size = 34, normalized size = 4.25

$$\frac{1}{12}\left(3\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{12}\,e^{(3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="giac")
```

```
[Out] 1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

3.222 $\int \cosh(x)\sinh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

```
[Out] Cosh[2*x]/4 + Cosh[4*x]/8
```

Rubi [A] time = 0.0106027, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[3*x],x]
```

```
[Out] Cosh[2*x]/4 + Cosh[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(3x)\,dx=\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Mathematica [A] time = 0.0069289, size = 17, normalized size = 1.

$$\frac{\cosh^2(x)}{2}+\frac{1}{8}\cosh(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[3*x],x]
```

```
[Out] Cosh[x]^2/2 + Cosh[4*x]/8
```

Maple [A] time = 0.014, size = 12, normalized size = 0.7

$$(\cosh(x))^4 - \frac{(\cosh(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(3*x),x)
```

```
[Out] cosh(x)^4-1/2*cosh(x)^2
```

Maxima [A] time = 1.04745, size = 36, normalized size = 2.12

$$\frac{1}{16}\left(2e^{(-2x)} + 1\right)e^{(4x)} + \frac{1}{8}e^{(-2x)} + \frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="maxima")
```

```
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) + 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

Fricas [B] time = 2.1121, size = 109, normalized size = 6.41

$$\frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + \frac{1}{4}\cosh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 + 1)*sinh(x)^2 + 1/4*cosh(
x)^2
```

Sympy [A] time = 0.558174, size = 20, normalized size = 1.18

$$-\frac{\sinh\left(x\right)\sinh\left(3x\right)}{8}+\frac{3\cosh\left(x\right)\cosh\left(3x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x)
```

```
[Out] -sinh(x)*sinh(3*x)/8 + 3*cosh(x)*cosh(3*x)/8
```

Giac [A] time = 1.19394, size = 35, normalized size = 2.06

$$\frac{1}{16}\left(e^{(2x)}+e^{(-2x)}\right)^2+\frac{1}{8}e^{(2x)}+\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 + 1/8*e^(2*x) + 1/8*e^(-2*x)
```

3.223 $\int \cosh(x)\sinh(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

```
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Rubi [A] time = 0.0108065, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[4*x],x]
```

```
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(4x)\,dx=\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Mathematica [A] time = 0.0059692, size = 17, normalized size = 1.

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[4*x],x]
```

```
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Maple [A] time = 0.011, size = 14, normalized size = 0.8

$$\frac{8\left(\cosh\left(x\right)\right)^{5}}{5}-\frac{4\left(\cosh\left(x\right)\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(4*x),x)
```

```
[Out] 8/5*cosh(x)^5-4/3*cosh(x)^3
```

Maxima [A] time = 1.03567, size = 36, normalized size = 2.12

$$\frac{1}{60}\left(5\,e^{(-2\,x)}+3\right)e^{(5\,x)}+\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) + 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

Fricas [B] time = 2.0702, size = 130, normalized size = 7.65

$$\frac{1}{10}\cosh\left(x\right)^{5}+\frac{1}{2}\cosh\left(x\right)\sinh\left(x\right)^{4}+\frac{1}{6}\cosh\left(x\right)^{3}+\frac{1}{2}\left(2\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 + 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 +
 cosh(x))*sinh(x)^2
```

Sympy [A] time = 0.586667, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\sinh(4x)}{15}+\frac{4\cosh(x)\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x)
```

```
[Out] -sinh(x)*sinh(4*x)/15 + 4*cosh(x)*cosh(4*x)/15
```

Giac [A] time = 1.15576, size = 36, normalized size = 2.12

$$\frac{1}{60}\left(5e^{(2x)}+3\right)e^{(-5x)}+\frac{1}{20}e^{(5x)}+\frac{1}{12}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

3.224 $\int \cosh(x)\sinh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh((m+1)x)}{2(m+1)} - \frac{\cosh((1-m)x)}{2(1-m)}$$

```
[Out] -Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0330954, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5618, 2638}

$$\frac{\cosh((m+1)x)}{2(m+1)} - \frac{\cosh((1-m)x)}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[m*x],x]
```

```
[Out] -Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cosh(x)\sinh(mx)\,dx = \int \left(-\frac{1}{2}\sinh((1-m)x) + \frac{1}{2}\sinh((1+m)x)\right) dx$$
$$= -\left(\frac{1}{2}\int \sinh((1-m)x)\,dx\right) + \frac{1}{2}\int \sinh((1+m)x)\,dx$$
$$= -\frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

Mathematica [A] time = 0.0376786, size = 25, normalized size = 0.71

$$\frac{m\cosh(x)\cosh(mx) - \sinh(x)\sinh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[m*x],x]
```

```
[Out] (m*Cosh[x]*Cosh[m*x] - Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] time = 0.006, size = 28, normalized size = 0.8

$$\frac{\cosh\left((-1+m)\,x\right)}{-2+2\,m} + \frac{\cosh\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(m*x),x)
```

```
[Out] 1/2*cosh((-1+m)*x)/(-1+m)+1/2*cosh((1+m)*x)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10308, size = 117, normalized size = 3.34

$$\frac{m \cosh(mx) \cosh(x) - \sinh(mx) \sinh(x)}{(m^2 - 1) \cosh(x)^2 - (m^2 - 1) \sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(m*x)*cosh(x) - sinh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] time = 1.04207, size = 42, normalized size = 1.2

$$\begin{cases} -\frac{\cosh^2(x)}{2} & \text{for } m = -1 \\ \frac{\cosh^2(x)}{2} & \text{for } m = 1 \\ \frac{m \cosh(x) \cosh(mx)}{m^2 - 1} - \frac{\sinh(x) \sinh(mx)}{m^2 - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-cosh(x)**2/2, Eq(m, -1)), (cosh(x)**2/2, Eq(m, 1)), (m*cosh(x)*
cosh(m*x)/(m**2 - 1) - sinh(x)*sinh(m*x)/(m**2 - 1), True))
```

Giac [B] time = 1.14923, size = 80, normalized size = 2.29

$$\frac{e^{(mx+x)}}{4(m+1)} + \frac{e^{(mx-x)}}{4(m-1)} + \frac{e^{(-mx+x)}}{4(m-1)} + \frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

3.225 $\int \cosh(x)\cosh(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Rubi [A] time = 0.0088413, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[2*x],x]
```

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(2x)\,dx=\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Mathematica [A] time = 0.005055, size = 15, normalized size = 1.

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[2*x],x]
```

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Maple [A] time = 0.013, size = 12, normalized size = 0.8

$$\frac{\sinh(x)}{2} + \frac{\sinh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(2*x),x)
```

```
[Out] 1/2*sinh(x)+1/6*sinh(3*x)
```

Maxima [B] time = 1.03474, size = 36, normalized size = 2.4

$$\frac{1}{12}\left(3\,e^{(-2\,x)}+1\right)e^{(3\,x)} - \frac{1}{4}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) - 1/4*e^(-x) - 1/12*e^(-3*x)
```

Fricas [A] time = 1.96405, size = 61, normalized size = 4.07

$$\frac{1}{6}\,\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^2+1\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="fricas")
```

```
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 + 1)*sinh(x)
```

Sympy [A] time = 0.618163, size = 20, normalized size = 1.33

$$-\frac{\sinh(x)\cosh(2x)}{3} + \frac{2\sinh(2x)\cosh(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x)
```

```
[Out] -sinh(x)*cosh(2*x)/3 + 2*sinh(2*x)*cosh(x)/3
```

Giac [B] time = 1.16954, size = 34, normalized size = 2.27

$$-\frac{1}{12}\left(3\,e^{(2\,x)} + 1\right)e^{(-3\,x)} + \frac{1}{12}\,e^{(3\,x)} + \frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

3.226 $\int \cosh(x)\cosh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

```
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Rubi [A] time = 0.0087817, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[3*x],x]
```

```
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(3x)\,dx=\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Mathematica [A] time = 0.0056586, size = 17, normalized size = 1.

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[3*x],x]
```

```
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Maple [A] time = 0.018, size = 14, normalized size = 0.8

$$\frac{\sinh(2x)}{4} + \frac{\sinh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(3*x),x)
```

```
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)
```

Maxima [A] time = 1.0541, size = 36, normalized size = 2.12

$$\frac{1}{16}\left(2\,e^{(-2\,x)} + 1\right)e^{(4\,x)} - \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{16}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="maxima")
```

```
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) - 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

Fricas [A] time = 2.13975, size = 80, normalized size = 4.71

$$\frac{1}{2}\cosh(x)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^3 + \cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 + cosh(x))*sinh(x)
```

Sympy [A] time = 0.571234, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\cosh(3x)}{8}+\frac{3\sinh(3x)\cosh(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x)
```

```
[Out] -sinh(x)*cosh(3*x)/8 + 3*sinh(3*x)*cosh(x)/8
```

Giac [A] time = 1.17635, size = 36, normalized size = 2.12

$$-\frac{1}{16}\left(2\,e^{(2\,x)}+1\right)e^{(-4\,x)}+\frac{1}{16}\,e^{(4\,x)}+\frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] -1/16*(2*e^(2*x) + 1)*e^(-4*x) + 1/16*e^(4*x) + 1/8*e^(2*x)
```

3.227 $\int \cosh(x)\cosh(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Rubi [A] time = 0.0087368, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[4*x],x]
```

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(4x)\,dx=\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Mathematica [A] time = 0.0043738, size = 17, normalized size = 1.

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[4*x],x]
```

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Maple [A] time = 0.037, size = 14, normalized size = 0.8

$$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(4*x),x)
```

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)
```

Maxima [A] time = 1.0425, size = 36, normalized size = 2.12

$$\frac{1}{60}\left(5\,e^{(-2\,x)}+3\right)e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="maxima")
```

```
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) - 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

Fricas [B] time = 2.0215, size = 119, normalized size = 7.

$$\frac{1}{10}\sinh(x)^5+\frac{1}{6}\left(6\cosh(x)^2+1\right)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^4+\cosh(x)^2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 + 1)*sinh(x)^3 + 1/2*(cosh(x)^4 + cosh(x)
^2)*sinh(x)
```

Sympy [A] time = 0.558227, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\cosh(4x)}{15}+\frac{4\sinh(4x)\cosh(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x)
```

```
[Out] -sinh(x)*cosh(4*x)/15 + 4*sinh(4*x)*cosh(x)/15
```

Giac [A] time = 1.18915, size = 36, normalized size = 2.12

$$-\frac{1}{60}\left(5\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}+\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="giac")
```

```
[Out] -1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

3.228 $\int \cosh(x)\cosh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((m+1)x)}{2(m+1)}$$

```
[Out] Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0286342, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5614, 2637}

$$\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[m*x],x]
```

```
[Out] Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\cosh(mx)\,dx &= \int \left(\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right)dx\\ &= \frac{1}{2}\int \cosh((1-m)x)\,dx+\frac{1}{2}\int \cosh((1+m)x)\,dx\\ &= \frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0341788, size = 25, normalized size = 0.71

$$\frac{m\cosh(x)\sinh(mx)-\sinh(x)\cosh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[m*x],x]
```

```
[Out] (-(Cosh[m*x]*Sinh[x]) + m*Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] time = 0.009, size = 28, normalized size = 0.8

$$\frac{\sinh((-1+m)x)}{-2+2m}+\frac{\sinh((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(m*x),x)
```

```
[Out] 1/2/(-1+m)*sinh((-1+m)*x)+1/2*sinh((1+m)*x)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06575, size = 117, normalized size = 3.34

$$\frac{m\cosh(x)\sinh(mx)-\cosh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(x)*sinh(m*x) - cosh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] time = 1.26294, size = 56, normalized size = 1.6

$$\begin{cases} -\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sinh(mx)\cosh(x)}{m^2-1}-\frac{\sinh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, -1)
| Eq(m, 1)), (m*sinh(m*x)*cosh(x)/(m**2 - 1) - sinh(x)*cosh(m*x)/(m**2 - 1)
, True))
```

Giac [B] time = 1.16816, size = 80, normalized size = 2.29

$$\frac{e^{(mx+x)}}{4(m+1)}+\frac{e^{(mx-x)}}{4(m-1)}-\frac{e^{(-mx+x)}}{4(m-1)}-\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

3.229 $\int \cosh(x)\tanh(2x)\,dx$

Optimal. Leaf size=19

$$\cosh(x) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2]) + Cosh[x]
```

Rubi [A] time = 0.031566, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 207}

$$\cosh(x) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[2*x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cosh(x)\tanh(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cosh(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cosh(x)\right)\\
&= \cosh(x) + \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}} + \cosh(x)
\end{aligned}
$$

Mathematica [C] time = 0.17027, size = 164, normalized size = 8.63

$$\frac{4\sqrt{2}\cosh(x) - 4\tanh^{-1}\left(\sqrt{2} - i\tanh\left(\frac{x}{2}\right)\right) + \log\left(\sqrt{2} - 2\cosh(x)\right) - \log\left(2\cosh(x) + \sqrt{2}\right) - 2i\tan^{-1}\left(\frac{\sinh\left(\frac{x}{2}\right)+\cos\cdots}{\left(1+\sqrt{2}\right)\cosh\left(\frac{x}{2}\right)-(\sqrt{\cdots}}\cdots\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[2*x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 4*
Sqrt[2]*Cosh[x] + Log[Sqrt[2] - 2*Cosh[x]] - Log[Sqrt[2] + 2*Cosh[x]])/(4*S
qrt[2])
```

Maple [A] time = 0.013, size = 16, normalized size = 0.8

$$\cosh(x) - \frac{\text{Artanh}\left(\cosh(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(2*x),x)
```

```
[Out] cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.7082, size = 70, normalized size = 3.68

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 2.11186, size = 259, normalized size = 13.63

$$\frac{2\cosh(x)^2+\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)+4\cosh(x)\sinh(x)+2\sinh(x)^2+2}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) + 4*cosh(x)*sinh(x
) + 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\tanh(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x)
```

```
[Out] Integral(cosh(x)*tanh(2*x), x)
```

<u>Giac</u> [B] time = 1.14523, size = 61, normalized size = 3.21

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

3.230 $\int \cosh(x)\tanh(3x)\,dx$

Optimal. Leaf size=20

$$\cosh(x) - \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[(2*Cosh[x])/Sqrt[3]]/Sqrt[3]) + Cosh[x]
```

Rubi [A] time = 0.0299257, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\cosh(x) - \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[3*x],x]
```

```
[Out] -(ArcTanh[(2*Cosh[x])/Sqrt[3]]/Sqrt[3]) + Cosh[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\tanh(3x)\,dx = \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \cosh(x)\right)$$
$$= \cosh(x) - 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

Mathematica [C] time = 0.0578828, size = 55, normalized size = 2.75

$$\cosh(x) - \frac{\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[3*x],x]
```

```
[Out] -(ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]]/Sqrt[3]) - ArcTanh[(2 + I*Tanh[x/2])/S
qrt[3]]/Sqrt[3] + Cosh[x]
```

Maple [A] time = 0.017, size = 17, normalized size = 0.9

$$\cosh(x) - \frac{\sqrt{3}}{3}\text{Artanh}\left(\frac{2\cosh(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(3*x),x)
```

```
[Out] cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

Maxima [B] time = 1.69763, size = 207, normalized size = 10.35

$$-\frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) - \frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{12}\sqrt{3}\log\left(-\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/12*sqrt(3)*log(-sqrt(3
)*e^(-x) + e^(-2*x) + 1) - 1/12*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/
12*sqrt(3)*log(-sqrt(3)*e^x + e^(2*x) + 1) + 1/6*arctan(sqrt(3) + 2*e^(-x))
 + 1/6*arctan(sqrt(3) + 2*e^x) + 1/6*arctan(-sqrt(3) + 2*e^(-x)) + 1/6*arct
an(-sqrt(3) + 2*e^x) + 1/3*arctan(e^(-x)) + 1/3*arctan(e^x) + 1/2*e^(-x) +
1/2*e^x
```

Fricas [B] time = 2.05551, size = 275, normalized size = 13.75

$$\frac{3 \cosh\left(x\right)^{2} + \left(\sqrt{3} \cosh\left(x\right) + \sqrt{3} \sinh\left(x\right)\right) \log\left(\frac{2 \cosh\left(x\right)^{2} + 2 \sinh\left(x\right)^{2} - 4 \sqrt{3} \cosh\left(x\right) + 5}{2 \cosh\left(x\right)^{2} + 2 \sinh\left(x\right)^{2} - 1}\right) + 6 \cosh\left(x\right) \sinh\left(x\right) + 3 \sinh\left(x\right)^{2} + 3}{6 \left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 + 2
*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) + 6*co
sh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(x\right) \tanh\left(3 x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x)
```

```
[Out] Integral(cosh(x)*tanh(3*x), x)
```

<u>Giac</u> [B] time = 1.15087, size = 61, normalized size = 3.05

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

3.231 $\int \cosh(x)\tanh(4x)\,dx$

Optimal. Leaf size=69

$$\cosh(x) - \frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + S
qrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]])/4 + Cosh[x]
```

Rubi [A] time = 0.0867444, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 207}

$$\cosh(x) - \frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[4*x],x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + S
qrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]])/4 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\tanh(4x)\,dx &= \text{Subst}\left(\int \frac{4x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= 4\,\text{Subst}\left(\int \frac{x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x)-\frac{1}{2}\,\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x)-\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)+\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{}{-4-}\right.\\ &= -\frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)+\cosh(x)\end{aligned}$$

Mathematica [C] time = 0.0229881, size = 113, normalized size = 1.64

$$\frac{1}{16}\text{RootSum}\left[\#1^8+1\&, \frac{\#1^6 x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)-2\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\right.}{\#1^7}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[4*x],x]
```

```
[Out] Cosh[x] + RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/
2]*#1 - Sinh[x/2]*#1] + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#
```

```
1 - Sinh[x/2]*#1]*#1^6)/#1^7 & ]/16
```

Maple [A] time = 0.054, size = 66, normalized size = 1.

$$\cosh(x) - \frac{(\sqrt{2}-1)\sqrt{2}}{4\sqrt{2-\sqrt{2}}}\operatorname{Artanh}\left(2\frac{\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{(1+\sqrt{2})\sqrt{2}}{4\sqrt{2+\sqrt{2}}}\operatorname{Artanh}\left(2\frac{\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(4*x),x)
```

```
[Out] cosh(x)-1/4*(2^(1/2)-1)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cosh(x)/(2-2^(1
/2))^(1/2))-1/4*(1+2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*cosh(x)/(2+
2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)} + \frac{1}{2}\int \frac{2\left(e^{(7x)}-e^{x}\right)}{e^{(8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^x)/(e^(8*x) + 1), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\tanh(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x)
```

```
[Out] Integral(cosh(x)*tanh(4*x), x)
```

Giac [B] time = 1.23464, size = 161, normalized size = 2.33

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(sqr
t(2) + 2)*log(-sqrt(sqrt(2) + 2) + e^(-x) + e^x) - 1/8*sqrt(-sqrt(2) + 2)*l
og(sqrt(-sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(-s
qrt(2) + 2) + e^(-x) + e^x) + 1/2*e^(-x) + 1/2*e^x
```

3.232 $\int \cosh(x)\tanh(5x)\,dx$

Optimal. Leaf size=82

$$\cosh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right)$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cosh[x]])/5 - (Sqrt
[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cosh[x]])/5 + Cosh[x]
```

Rubi [A] time = 0.116861, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cosh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[5*x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cosh[x]])/5 - (Sqrt
[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cosh[x]])/5 + Cosh[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\tanh(5x)\,dx = \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx, x, \cosh(x)\right)$$
$$= \cosh(x)-4\,\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)$$
$$= \cosh(x)+\frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \cosh(x)\right)+\frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Sub}$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh\right.$$

Mathematica [C] time = 0.0275745, size = 249, normalized size = 3.04

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^6+\#1^4-\#1^2+1\&, \frac{\#1^6x-\#1^4x+\#1^2x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\text{co}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[5*x],x]
```

```
[Out] Cosh[x] + RootSum[1 - #1^2 + #1^4 - #1^6 + #1^8 & , (-x - 2*Log[-Cosh[x/2]
- Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Si
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - S
inh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] -
Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1 + 2*#1^3 - 3*#1^5 + 4*#
1^7) & ]/4
```

Maple [A] time = 0.061, size = 70, normalized size = 0.9

$$\cosh(x)-\frac{\left(\sqrt{5}-1\right)\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\text{Artanh}\left(4\frac{\cosh(x)}{\sqrt{10-2\sqrt{5}}}\right)-\frac{\sqrt{5}\left(\sqrt{5}+1\right)}{5\sqrt{10+2\sqrt{5}}}\text{Artanh}\left(4\frac{\cosh(x)}{\sqrt{10+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(5*x),x)
```

```
[Out] cosh(x)-1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*cosh(x)/(10-
2*5^(1/2))^(1/2))-1/5*5^(1/2)*(5^(1/2)+1)/(10+2*5^(1/2))^(1/2)*arctanh(4*co
sh(x)/(10+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}+\frac{1}{2}\int\frac{2\left(e^{(7x)}-e^{(5x)}+e^{(3x)}-e^{x}\right)}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^(5*x) + e^(3*x) - e
^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

Fricas [B] time = 2.15884, size = 1019, normalized size = 12.43

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}+5}\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) + 5)*log(2*cosh(x)^
2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*s
qrt(sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) +
5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + s
qrt(2)*sinh(x))*sqrt(sqrt(5) + 5) + 2) + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x)
)*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (s
qrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x
) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
 + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2
```

```
) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 - 10)/(cosh(x) + sinh(
x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\tanh(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x)
```

```
[Out] Integral(cosh(x)*tanh(5*x), x)
```

Giac [B] time = 1.23672, size = 171, normalized size = 2.09

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(-\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/
20*sqrt(2*sqrt(5) + 10)*log(-sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) - 1/20
*sqrt(-2*sqrt(5) + 10)*log(sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/20*
sqrt(-2*sqrt(5) + 10)*log(-sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/2*e
^(-x) + 1/2*e^x
```

3.233 $\int \cosh(x)\tanh(6x)\,dx$

Optimal. Leaf size=87

$$\cosh(x)-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] -ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cosh[
x])/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cosh[x])/Sqrt[2 +
 Sqrt[3]]])/6 + Cosh[x]
```

Rubi [A] time = 0.251012, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$\cosh(x)-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[6*x],x]
```


```
[Out] -ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cosh[
x])/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cosh[x])/Sqrt[2 +
 Sqrt[3]]])/6 + Cosh[x]
```


Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
```

```
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \cosh(x)\tanh(6x)\,dx = \text{Subst}\left(\int \frac{2x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \cosh(x)\right)$$

$$= \cosh(x)-\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)$$

$$= \cosh(x)-\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cosh(x)\right)$$

$$= \cosh(x)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x, \cosh(\right.$$

$$= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\cosh(x)+\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \cosh(\right.$$

$$= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)+$$

Mathematica [C] time = 0.28289, size = 395, normalized size = 4.54

$$\sqrt{2}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{2\#1^6x+\#1^4x-\#1^2x+4\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)+2\#1^4\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sin\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[6*x],x]
```

```
[Out] ((-4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 8*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 24
*Sqrt[2]*Cosh[x] + 2*Log[Sqrt[2] - 2*Cosh[x]] - 2*Log[Sqrt[2] + 2*Cosh[x]]
+ Sqrt[2]*RootSum[1 - #1^4 + #1^8 & , (-2*x - 4*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1] - x*#1^2 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + x*#1^4 + 2*Log[-Cosh[x/2] - Sinh[x/2] + C
osh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 2*x*#1^6 + 4*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/(24*Sqrt[2])
```

Maple [A] time = 0.069, size = 102, normalized size = 1.2

$$\cosh(x)-\frac{2\sqrt{3}\left(3+2\sqrt{3}\right)}{18\sqrt{6}+18\sqrt{2}}\text{Artanh}\left(8\frac{\cosh(x)}{2\sqrt{6}+2\sqrt{2}}\right)-\frac{\left(-6+4\sqrt{3}\right)\sqrt{3}}{18\sqrt{6}-18\sqrt{2}}\text{Artanh}\left(8\frac{\cosh(x)}{2\sqrt{6}-2\sqrt{2}}\right)-\frac{\text{Artanh}\left(\cosh(x)\sqrt{2}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(6*x),x)
```

```
[Out] cosh(x)-2/9*3^(1/2)*(3+2*3^(1/2))/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cosh(x)/(
2*6^(1/2)+2*2^(1/2)))-2/9*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arct
anh(8*cosh(x)/(2*6^(1/2)-2*2^(1/2)))-1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{2}\int\frac{2\left(2e^{(7x)}+e^{(5x)}-e^{(3x)}-2e^{x}\right)}{3\left(e^{(8x)}-e^{(4x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/
12*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + 1/2*integrate(2/3*(2*e^(7*x) +
 e^(5*x) - e^(3*x) - 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Fricas [B] time = 2.19526, size = 976, normalized size = 11.22

$$-\frac{\sqrt{\sqrt{3}+2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{3}+2}(\cosh(x)+\sinh(x))+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(3)
 + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - s
qrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(3) + 2)*(cosh(x) + s
inh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3) + 2)*
(cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x
)) + 1) - 6*cosh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2
+ sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 12*cosh(x)*
sinh(x) - 6*sinh(x)^2 - 6)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\tanh(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x)
```

```
[Out] Integral(cosh(x)*tanh(6*x), x)
```

<u>Giac</u> [B] time = 1.22039, size = 212, normalized size = 2.44

$$-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + sqrt(2))*log(1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) - 1
/24*(sqrt(6) - sqrt(2))*log(1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/2
4*(sqrt(6) - sqrt(2))*log(-1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) + 1/24
*(sqrt(6) + sqrt(2))*log(-1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/12*
sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e^(-x
) + 1/2*e^x
```

3.234 $\int \cosh(x)\coth(2x)\,dx$

Optimal. Leaf size=10

$$\cosh(x) - \frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]/2 + Cosh[x]
```

Rubi [A] time = 0.0264988, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 388, 206}

$$\cosh(x) - \frac{1}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[2*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cosh(x)\coth(2x)\,dx &= -\operatorname{Subst}\left(\int \frac{-1+2x^2}{2(1-x^2)}\,dx, x, \cosh(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cosh(x)\right)\right)\\
&= \cosh(x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{2}\tanh^{-1}(\cosh(x)) + \cosh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0144016, size = 14, normalized size = 1.4

$$\cosh(x) + \frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[2*x],x]
```

```
[Out] Cosh[x] + Log[Tanh[x/2]]/2
```

Maple [A] time = 0.023, size = 9, normalized size = 0.9

$$\cosh(x) - \operatorname{Artanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(2*x),x)
```

```
[Out] cosh(x)-arctanh(exp(x))
```

Maxima [B] time = 1.11396, size = 39, normalized size = 3.9

$$\frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] time = 2.20459, size = 231, normalized size = 23.1

$$\frac{\cosh(x)^2 - (\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) + 1) + (\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) - 1) + 2\,\mathrm{c}}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^2 - (cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + (cosh(x)
+ sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/
(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\coth(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x)
```

```
[Out] Integral(cosh(x)*coth(2*x), x)
```

Giac [B] time = 1.10755, size = 35, normalized size = 3.5

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.235 $\int \cosh(x)\coth(3x)\,dx$

Optimal. Leaf size=45

$$\cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(\cosh(x)+1)-\frac{1}{6}\log(2\cosh(x)+1)$$

```
[Out] Cosh[x] + Log[1 - 2*Cosh[x]]/6 + Log[1 - Cosh[x]]/6 - Log[1 + Cosh[x]]/6 -
Log[1 + 2*Cosh[x]]/6
```

Rubi [A] time = 0.063326, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$\cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(\cosh(x)+1)-\frac{1}{6}\log(2\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] + Log[1 - 2*Cosh[x]]/6 + Log[1 - Cosh[x]]/6 - Log[1 + Cosh[x]]/6 -
Log[1 + 2*Cosh[x]]/6
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh(x)\coth(3x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx,x,\cosh(x)\right)\\
&= \cosh(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx,x,\cosh(x)\right)\\
&= \cosh(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx,x,\cosh(x)\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx,x,\cosh(x\right.\\
&= \cosh(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cosh(x)\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx,x,\cosh(x)\right)-\frac{1}{6}\operatorname{Sub}\\
&= \cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(1+\cosh(x))-\frac{1}{6}\log(1+2\cosh
\end{aligned}$$

Mathematica [A] time = 0.017799, size = 47, normalized size = 1.04

$$\cosh(x)+\frac{1}{3}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cosh(x))-\frac{1}{6}\log(2\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] - Log[Cosh[x/2]]/3 + Log[1 - 2*Cosh[x]]/6 - Log[1 + 2*Cosh[x]]/6 +
Log[Sinh[x/2]]/3
```

Maple [A] time = 0.053, size = 50, normalized size = 1.1

$$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln\left(e^{x}-1\right)}{3}-\frac{\ln\left(e^{x}+1\right)}{3}-\frac{\ln\left(e^{2x}+e^{x}+1\right)}{6}+\frac{\ln\left(e^{2x}-e^{x}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(3*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/3*ln(exp(x)-1)-1/3*ln(exp(x)+1)-1/6*ln(exp(2*x)+ex
p(x)+1)+1/6*ln(exp(2*x)-exp(x)+1)
```

Maxima [A] time = 1.69428, size = 77, normalized size = 1.71

$$\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-\frac{1}{6}\,\log\left(e^{(-x)}+e^{(-2\,x)}+1\right)-\frac{1}{3}\,\log\left(e^{(-x)}+1\right)+\frac{1}{3}\,\log\left(e^{(-x)}-1\right)+\frac{1}{6}\,\log\left(-e^{(-x)}+e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^(-2*x) + 1) - 1/3*log(e^(-x) + 1)
 + 1/3*log(e^(-x) - 1) + 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

Fricas [B] time = 2.15193, size = 412, normalized size = 9.16

$$\frac{3\,\cosh\left(x\right)^{2}-\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\frac{2\,\cosh\left(x\right)+1}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\frac{2\,\cosh\left(x\right)-1}{\cosh\left(x\right)-\sinh\left(x\right)}\right)-2\left(\cosh\left(x\right)+\sinh\left(x\right.}{6\left(\cosh\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 - (cosh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(
x))) + (cosh(x) + sinh(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(co
sh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cos
h(x) + sinh(x) - 1) + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(
x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh (x) \coth (3x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x)
```

```
[Out] Integral(cosh(x)*coth(3*x), x)
```

Giac [A] time = 1.17656, size = 74, normalized size = 1.64

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^x + 2) - 1/6*log(e^(-x) + e^x + 1
) + 1/6*log(e^(-x) + e^x - 1) + 1/6*log(e^(-x) + e^x - 2)
```

3.236 $\int \cosh(x)\coth(4x)\,dx$

Optimal. Leaf size=28

$$\cosh(x) - \frac{1}{4}\tanh^{-1}(\cosh(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

```
[Out] -ArcTanh[Cosh[x]]/4 - ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2]) + Cosh[x]
```

Rubi [A] time = 0.0567339, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cosh(x) - \frac{1}{4}\tanh^{-1}(\cosh(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[4*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/4 - ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2]) + Cosh[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\coth(4x)\,dx &= -\operatorname{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)dx, x, \cosh(x)\right)\\ &= \cosh(x)-\operatorname{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x)+2\operatorname{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \cosh(x)\right)+2\operatorname{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\cosh(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}+\cosh(x)\end{aligned}$$

Mathematica [C] time = 0.224495, size = 192, normalized size = 6.86

$$\frac{8\sqrt{2}\cosh(x)-4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\sqrt{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sqrt{2}-2\cosh(x)\right)-\log\left(2\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[4*x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 8*
Sqrt[2]*Cosh[x] - 2*Sqrt[2]*Log[Cosh[x/2]] + Log[Sqrt[2] - 2*Cosh[x]] - Log
[Sqrt[2] + 2*Cosh[x]] + 2*Sqrt[2]*Log[Sinh[x/2]])/(8*Sqrt[2])
```

Maple [B] time = 0.073, size = 63, normalized size = 2.3

$$\frac{e^x}{2}+\frac{e^{-x}}{2}-\frac{\ln\left(e^x+1\right)}{4}+\frac{\ln\left(e^x-1\right)}{4}+\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(4*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/8*ln(1+exp(2*x)-
exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.66876, size = 95, normalized size = 3.39

$$-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{4}\log\left(e^{(-x)}+1\right)+\frac{1}{4}\log\left(e^{(-x)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x - 1/4*log(e^(-x) + 1) + 1/4*l
og(e^(-x) - 1)
```

Fricas [B] time = 2.20536, size = 397, normalized size = 14.18

$$\frac{4\cosh(x)^2+\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)-2\left(\cosh(x)+\sinh(x)\right)\log\left(\cosh(x)+\sin\right.}{8\left(\cosh(x)+\sinh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 2*(cosh(x) + sin
h(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh
(x) - 1) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 + 4)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\coth(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x)
```

```
[Out] Integral(cosh(x)*coth(4*x), x)
```

Giac [B] time = 1.16048, size = 90, normalized size = 3.21

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{8}\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{8}\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x - 1/8*log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

3.237 $\int \cosh(x)\coth(5x)\,dx$

Optimal. Leaf size=110

$$\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cosh(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cosh(x\right.$$

```
[Out] -ArcTanh[Cosh[x]]/5 + Cosh[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cosh[x]]
)/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]])/20 - ((1 - Sqrt[5])*Log
[1 - Sqrt[5] + 4*Cosh[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cosh[x]]
)/20
```

Rubi [A] time = 0.176159, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cosh(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cosh(x\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[5*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/5 + Cosh[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cosh[x]]
)/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]])/20 - ((1 - Sqrt[5])*Log
[1 - Sqrt[5] + 4*Cosh[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cosh[x]]
)/20
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh(x)\coth(5x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \cosh(x)\right)\\
&= -\operatorname{Subst}\left(\int\left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{2}{5}\operatorname{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{5}\tanh^{-1}(\cosh(x))+\cosh(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cosh(x)\right)+\frac{1}{5}\left(1-\right.\\
&= -\frac{1}{5}\tanh^{-1}(\cosh(x))+\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1\right.
\end{aligned}$$

Mathematica [A] time = 0.114537, size = 133, normalized size = 1.21

$$\frac{1}{100}\left(100\cosh(x)+20\log\left(\sinh\left(\frac{x}{2}\right)\right)-20\log\left(\cosh\left(\frac{x}{2}\right)\right)+\sqrt{5}\left(\sqrt{5}-5\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\sqrt{5}\left(5+\sqrt{5}\right)\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[5*x],x]
```

```
[Out] (100*Cosh[x] - 20*Log[Cosh[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] -
 4*Cosh[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]] - Sqrt[5]*
(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 4*Cosh[x]] + 20*Log[Sinh[x/2]])/100
```

Maple [B] time = 0.083, size = 190, normalized size = 1.7

$$\frac{e^x}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^x+1)}{5}+\frac{\ln(e^x-1)}{5}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^x+1\right)}{20}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^x+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{2x}+\left(\frac{\sqrt{5}}{2}-\right.\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(5*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)-1/5*ln(exp(x)+1)+1/5*ln(exp(x)-1)+1/20*ln(exp(2*x)+(
-1/2-1/2*5^(1/2))*exp(x)+1)+1/20*ln(exp(2*x)+(-1/2-1/2*5^(1/2))*exp(x)+1)*5
^(1/2)+1/20*ln(exp(2*x)+(1/2*5^(1/2)-1/2)*exp(x)+1)-1/20*ln(exp(2*x)+(1/2*5
^(1/2)-1/2)*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)+
1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+(1/2+
1/2*5^(1/2))*exp(x)+1)-1/20*ln(exp(2*x)+(1/2+1/2*5^(1/2))*exp(x)+1)*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}-\frac{1}{5}\int\frac{\left(e^{(3x)}+e^{(2x)}+e^x+1\right)e^x}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^x+1}\,dx+\frac{1}{5}\int\frac{\left(e^{(3x)}-e^{(2x)}+e^x-1\right)e^x}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^x+1}\,dx+\frac{3}{10}\int\frac{e^{(3x)}}{e^{(4x)}+e^{(3x)}+e^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/
(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 1/5*integrate((e^(3*x) - e^(2
*x) + e^x - 1)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integ
rate(e^(3*x)/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 3/10*integrate(e
^(3*x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)
/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^(2*x)/(e^(4
*x) - e^(3*x) + e^(2*x) - e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3
*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2
*x) - e^x + 1), x) - 1/5*log(e^x + 1) + 1/5*log(e^x - 1)
```

Fricas [B] time = 2.23793, size = 980, normalized size = 8.91

$$10\cosh(x)^2+\left(\sqrt{5}\cosh(x)+\sqrt{5}\sinh(x)\right)\log\left(-\frac{4\left(\sqrt{5}-1\right)\cosh(x)-4\cosh(x)^2-4\sinh(x)^2+\sqrt{5}-7}{2\cosh(x)^2+2\sinh(x)^2+2\cosh(x)+1}\right)+\left(\sqrt{5}\cosh(x)+\sqrt{5}\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*(10*cosh(x)^2 + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqrt(5) -
 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 + sqrt(5) - 7)/(2*cosh(x)^2 + 2*sin
h(x)^2 + 2*cosh(x) + 1)) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqr
t(5) + 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 - sqrt(5) - 7)/(2*cosh(x)^2 +
 2*sinh(x)^2 - 2*cosh(x) + 1)) - (cosh(x) + sinh(x))*log((2*cosh(x)^2 + 2*s
inh(x)^2 + 2*cosh(x) + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + (c
osh(x) + sinh(x))*log((2*cosh(x)^2 + 2*sinh(x)^2 - 2*cosh(x) + 1)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x) + sinh(x))*log(cosh(x) + s
inh(x) + 1) + 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 20*cosh(x)
*sinh(x) + 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\coth(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x)
```

```
[Out] Integral(cosh(x)*coth(5*x), x)
```

Giac [A] time = 1.18942, size = 212, normalized size = 1.93

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2e^{(-x)}-2e^{x}+1}{\sqrt{5}+2e^{(-x)}+2e^{x}-1}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2e^{(-x)}-2e^{x}-1}{\sqrt{5}+2e^{(-x)}+2e^{x}+1}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{20}\log\left(\left(e^{(-x)}+e^{x}\right)^{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x + 1)/(sqrt(5) + 2*e^(-x) + 2*
e^x - 1)) + 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x - 1)/(sqrt(5) + 2
*e^(-x) + 2*e^x + 1)) + 1/2*e^(-x) + 1/2*e^x - 1/20*log((e^(-x) + e^x)^2 +
e^(-x) + e^x - 1) + 1/20*log((e^(-x) + e^x)^2 - e^(-x) - e^x - 1) - 1/10*lo
g(e^(-x) + e^x + 2) + 1/10*log(e^(-x) + e^x - 2)
```

3.238 $\int \cosh(x)\coth(6x)\,dx$

Optimal. Leaf size=38

$$\cosh(x) - \frac{1}{6}\tanh^{-1}(\cosh(x)) - \frac{1}{6}\tanh^{-1}(2\cosh(x)) - \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 - ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3]) + Cosh[x]
```

Rubi [A] time = 0.0859348, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\cosh(x) - \frac{1}{6}\tanh^{-1}(\cosh(x)) - \frac{1}{6}\tanh^{-1}(2\cosh(x)) - \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[6*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 - ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\coth(6x)\,dx = -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cosh(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-2-\frac{1}{3\left(-1+x^2\right)}-\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cosh(x)\right)\right)$$
$$= \cosh(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x)\right)+\text{Su}$$
$$= -\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cosh(x)$$

Mathematica [C] time = 0.0711355, size = 95, normalized size = 2.5

$$\frac{1}{12}\left(12\cosh(x)-2\sqrt{3}\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\sqrt{3}\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[6*x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + I*T
anh[x/2])/Sqrt[3]] + 12*Cosh[x] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - L
og[1 + 2*Cosh[x]] + 2*Log[Sinh[x/2]])/12
```

Maple [B] time = 0.097, size = 87, normalized size = 2.3

$$\frac{e^x}{2}+\frac{e^{-x}}{2}+\frac{\ln\left(e^x-1\right)}{6}-\frac{\ln\left(e^x+1\right)}{6}+\frac{\ln\left(e^{2x}-e^x+1\right)}{12}-\frac{\ln\left(e^{2x}+e^x+1\right)}{12}+\frac{\sqrt{3}\ln\left(e^{2x}-\sqrt{3}e^x+1\right)}{12}-\frac{\sqrt{3}\ln\left(e^{2x}+\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(6*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/6*ln(exp(x)-1)-1/6*ln(exp(x)+1)+1/12*ln(exp(2*x)-e
xp(x)+1)-1/12*ln(exp(2*x)+exp(x)+1)+1/12*3^(1/2)*ln(exp(2*x)-3^(1/2)*exp(x)
+1)-1/12*3^(1/2)*ln(exp(2*x)+3^(1/2)*exp(x)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}+\frac{1}{2}\int\frac{e^{(3x)}-e^{x}}{e^{(4x)}-e^{(2x)}+1}\,dx-\frac{1}{12}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{6}\log\left(e^{x}+1\right)+\frac{1}{6}\log\left(e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate((e^(3*x) - e^x)/(e^(4*x) - e^(2*x)
 + 1), x) - 1/12*log(e^(2*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6
*log(e^x + 1) + 1/6*log(e^x - 1)
```

Fricas [B] time = 2.12742, size = 586, normalized size = 15.42

$$6\cosh(x)^{2}+\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^{2}+2\sinh(x)^{2}-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^{2}+2\sinh(x)^{2}-1}\right)-\left(\cosh(x)+\sinh(x)\right)\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*(6*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 +
2*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) - (co
sh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + (cosh(x) + sinh
(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x) + sinh(x))*log(c
osh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) +
12*cosh(x)*sinh(x) + 6*sinh(x)^2 + 6)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\coth(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x)
```

```
[Out] Integral(cosh(x)*coth(6*x), x)
```

Giac [B] time = 1.13602, size = 120, normalized size = 3.16

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x - 1/12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1)
+ 1/12*log(e^(-x) + e^x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

3.239 $\int \cosh(x)\coth(nx)\,dx$

Optimal. Leaf size=76

$$e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^x}{2}$$

```
[Out] -1/(2*E^x) + E^x/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, E^(2*n*x)]
```

Rubi [A] time = 0.0722335, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5602, 2194, 2251}

$$e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[n*x],x]
```

```
[Out] -1/(2*E^x) + E^x/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, E^(2*n*x)]
```

Rule 5602

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
```

```
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\coth(nx)\,dx &= \int \left(\frac{e^{-x}}{2}+\frac{e^x}{2}-\frac{e^{-x}}{1-e^{2nx}}-\frac{e^x}{1-e^{2nx}}\right)dx\\ &= \frac{1}{2}\int e^{-x}\,dx+\frac{\int e^x\,dx}{2}-\int \frac{e^{-x}}{1-e^{2nx}}\,dx-\int \frac{e^x}{1-e^{2nx}}\,dx\\ &= -\frac{e^{-x}}{2}+\frac{e^x}{2}+e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)\end{aligned}$$

Mathematica [B] time = 0.169179, size = 156, normalized size = 2.05

$$\frac{1}{2}e^{-2x}\left(-\frac{e^{2nx+x}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};e^{2nx}\right)}{2n-1}-\frac{e^{(2n+3)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};e^{2nx}\right)}{2n+1}+e^x\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^{3x}\,{}_2F\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[n*x],x]
```

```
[Out] (-((E^(x + 2*n*x)*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^(2*n*x)]
)/(-1 + 2*n)) - (E^((3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n), 2 + 1/(2
*n), E^(2*n*x)])/(1 + 2*n) + E^x*Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n)
, E^(2*n*x)] - E^(3*x)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), E^(2*n*x)
])/(2*E^(2*x))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \cosh(x)\coth(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(n*x),x)
```

```
[Out] int(cosh(x)*coth(n*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{2}\int\frac{e^{(2x)}+1}{e^{(nx+x)}+e^{x}}\,dx+\frac{1}{2}\int\frac{e^{(2x)}+1}{e^{(nx+x)}-e^{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) + e^x),
 x) + 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) - e^x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(x\right)\coth\left(nx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(x)*coth(n*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(x\right)\coth\left(nx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x)
```

```
[Out] Integral(cosh(x)*coth(n*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh\left(x\right)\coth\left(nx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)*coth(n*x), x)
```

3.240 $\int \cosh(x)\mathbf{sech}(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rubi [A] time = 0.0164232, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4356, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[2*x],x]
```

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\mathrm{sech}(2x)\,dx = \mathrm{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0079444, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[2*x],x]
```

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Maple [C] time = 0.033, size = 44, normalized size = 2.9

$$\frac{i}{4}\sqrt{2}\ln\left(e^{2x} + i\sqrt{2}e^{x} - 1\right) - \frac{i}{4}\sqrt{2}\ln\left(e^{2x} - i\sqrt{2}e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(2*x),x)
```

```
[Out] 1/4*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)-1/4*I*2^(1/2)*ln(exp(2*x)-I*2
^(1/2)*exp(x)-1)
```

Maxima [B] time = 1.76938, size = 58, normalized size = 3.87

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

Fricas [B] time = 2.02641, size = 254, normalized size = 16.93

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+}{2(\cosh(x)-\sinh(x))}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/2*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x)
```

```
[Out] Integral(cosh(x)*sech(2*x), x)
```

Giac [B] time = 1.12215, size = 53, normalized size = 3.53

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x))
```

3.241 $\int \cosh(x)\mathbf{sech}(3x)\, dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

```
[Out] ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

Rubi [A] time = 0.0317943, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203}

$$\frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[3*x],x]
```

```
[Out] ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh(x)\mathbf{sech}(3x)\, dx &= \text{Subst}\left(\int \frac{1}{1+3x^2}\, dx, x, \tanh(x)\right) \\ &= \frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.0218634, size = 48, normalized size = 3.2

$$\frac{1}{4}e^{2x}\left(2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-e^{6x}\right)+e^{2x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[3*x],x]
```

```
[Out] (E^(2*x)*(2*Hypergeometric2F1[1/3, 1, 4/3, -E^(6*x)] + E^(2*x)*Hypergeometr
ic2F1[2/3, 1, 5/3, -E^(6*x)]))/4
```

Maple [C] time = 0.035, size = 40, normalized size = 2.7

$$\frac{i}{6}\sqrt{3}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\frac{i}{6}\sqrt{3}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(3*x),x)
```

```
[Out] 1/6*I*3^(1/2)*ln(exp(2*x)-1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)-1/2-
1/2*I*3^(1/2))
```

Maxima [B] time = 1.76901, size = 154, normalized size = 10.27

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-2x)}-1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{6}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{12}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-2*x) - 1)) - 1/6*sqrt(3)*arctan(sqrt
(3) + 2*e^x) + 1/6*sqrt(3)*arctan(-sqrt(3) + 2*e^x) + 1/12*log(sqrt(3)*e^x
+ e^(2*x) + 1) + 1/12*log(-sqrt(3)*e^x + e^(2*x) + 1) - 1/6*log(e^(2*x) + 1
) + 1/6*log(e^(-2*x) + 1) - 1/12*log(-e^(-2*x) + e^(-4*x) + 1)
```

Fricas [B] time = 2.06588, size = 115, normalized size = 7.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh\left(x\right)+3\sqrt{3}\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh (x) \operatorname{sech} (3x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x)
```

```
[Out] Integral(cosh(x)*sech(3*x), x)
```

Giac [A] time = 1.13974, size = 26, normalized size = 1.73

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,e^{(2\,x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1))
```

3.242 $\int \cosh(x)\mathbf{sech}(4x)\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(2
*Sinh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rubi [A] time = 0.0389463, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4356, 1093, 203}

$$\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[4*x],x]
```

```
[Out] ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(2
*Sinh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\mathbf{sech}(4x)\,dx &= \text{Subst}\left(\int \frac{1}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\ &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{4+2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [A] time = 0.0967995, size = 67, normalized size = 0.94

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[4*x],x]
```

```
[Out] (Sqrt[2 + Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTan[(2*Sin
h[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Maple [C] time = 0.056, size = 40, normalized size = 0.6

$$2\sum_{\_R=\text{RootOf}\left(32768\_Z^4+512\_Z^2+1\right)}\_R\ln\left(e^{2x}+\left(-4096\_R^3-48\_R\right)e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(4*x),x)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(-4096*_R^3-48*_R)*exp(x)-1),_R=RootOf(32768*_Z^4+512*
_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)*sech(4*x), x)
```

Fricas [B] time = 2.23634, size = 427, normalized size = 6.01

$$-\frac{1}{2}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{1}{2}\left(\left(\sqrt{2}e^{(2x)}-\sqrt{2}\right)\sqrt{\sqrt{2}+2}-\sqrt{2}\sqrt{-\sqrt{2}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{2}+2}\right)e^{(-x)}\right)+\frac{1}{2}\sqrt{-\sqrt{2}+2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(2) + 2)*arctan(-1/2*((sqrt(2)*e^(2*x) - sqrt(2))*sqrt(sqrt(2
) + 2) - sqrt(2)*sqrt(-sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqrt(sqrt(2) + 2))*e^
(-x)) + 1/2*sqrt(-sqrt(2) + 2)*arctan(-1/2*((sqrt(2)*e^(2*x) - sqrt(2))*sqr
t(-sqrt(2) + 2) - sqrt(2)*sqrt(sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqrt(-sqrt(2)
 + 2))*e^(-x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x)
```

```
[Out] Integral(cosh(x)*sech(4*x), x)
```

Giac [B] time = 1.21649, size = 182, normalized size = 2.56

$$\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2}{\sqrt{\sqrt{2}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2))
```

3.243 $\int \cosh(x)\mathbf{sech}(5x)\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)$$

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[5 - 2*Sqrt[5]]*Tanh[x]])/5 + (Sqrt[(5 +
 Sqrt[5])/2]*ArcTan[Sqrt[5 + 2*Sqrt[5]]*Tanh[x]])/5
```

Rubi [A] time = 0.125132, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 203}

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[5*x],x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[5 - 2*Sqrt[5]]*Tanh[x]])/5 + (Sqrt[(5 +
 Sqrt[5])/2]*ArcTan[Sqrt[5 + 2*Sqrt[5]]*Tanh[x]])/5
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\mathbf{sech}(5x)\,dx = \text{Subst}\left(\int \frac{1-x^2}{1+10x^2+5x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\left(-1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5+2\sqrt{5}+5x^2}\,dx, x, \tanh(x)\right) + \frac{1}{2}\left(-1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5-2\sqrt{5}+5
$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)$$

Mathematica [A] time = 0.0997007, size = 84, normalized size = 1.12

$$\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\frac{\left(5+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right) + \sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-5\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[5*x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((5 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTan[((-5 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [C] time = 0.037, size = 41, normalized size = 0.6

$$2\sum_{\_R=\text{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)} \_R \ln\left(-4000\_R^3+200\_R^2+e^{2x}-30\_R+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(5*x),x)
```

```
[Out] 2*sum(_R*ln(-4000*_R^3+200*_R^2+exp(2*x)-30*_R+1),_R=RootOf(32000*_Z^4+400*
_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}+4e^{(-2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{\sqrt{5}\arctan\left(-\frac{\sqrt{5}-4e^{(-2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\log\left(-\left(\sqrt{5}+1\right)e^{(-2x)}+2e^{(-4x)}+2\right)}{10\left(\sqrt{5}+1\right)}+\frac{\log\left(\left(\sqrt{5}-1\right)e^{(-2x)}+2\right.}{10\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*arctan((sqrt(5) + 4*e^(-2*x) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*
sqrt(5) + 10) - 1/5*sqrt(5)*arctan(-(sqrt(5) - 4*e^(-2*x) + 1)/sqrt(-2*sqrt
(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*log(-(sqrt(5) + 1)*e^(-2*x) + 2*e^(
-4*x) + 2)/(sqrt(5) + 1) + 1/10*log((sqrt(5) - 1)*e^(-2*x) + 2*e^(-4*x) + 2
)/(sqrt(5) - 1) - 1/5*integrate((e^(7*x) - 2*e^(5*x) - 2*e^(3*x) + e^x)*e^x
/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x) + 1/10*log(e^(2*x) + 1) -
1/10*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.30668, size = 558, normalized size = 7.44

$$-\frac{1}{5}\sqrt{2}\sqrt{\sqrt{5}+5}\arctan\left(\frac{1}{40}\sqrt{5}\sqrt{2}\sqrt{-32\left(\sqrt{5}+1\right)e^{(2x)}+64e^{(4x)}+64}\sqrt{\sqrt{5}+5}-\frac{1}{20}\left(4\sqrt{5}\sqrt{2}e^{(2x)}-\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/40*sqrt(5)*sqrt(2)*sqrt(-32*(sqrt(5
) + 1)*e^(2*x) + 64*e^(4*x) + 64)*sqrt(sqrt(5) + 5) - 1/20*(4*sqrt(5)*sqrt(
2)*e^(2*x) - sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5)) + 1/5*sqrt(2)*
sqrt(-sqrt(5) + 5)*arctan(-1/20*(4*sqrt(5)*sqrt(2)*e^(2*x) - sqrt(5)*sqrt(2
) + 5*sqrt(2))*sqrt(-sqrt(5) + 5) + 1/5*sqrt(5)*sqrt((sqrt(5) - 1)*e^(2*x)
+ 2*e^(4*x) + 2)*sqrt(-sqrt(5) + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{sech}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x)
```

```
[Out] Integral(cosh(x)*sech(5*x), x)
```

Giac [A] time = 1.17706, size = 92, normalized size = 1.23

$$-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4e^{(2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4e^{(2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) + 1)/sqrt(
-2*sqrt(5) + 10))
```

3.244 $\int \cosh(x)\mathbf{sech}(6x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) + ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rubi [A] time = 0.0576444, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4356, 2057, 203, 1166}

$$-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[6*x],x]
```

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) + ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(x)\mathbf{sech}(6x)\,dx &= \text{Subst}\left(\int \frac{1}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{3\left(1+2x^2\right)}+\frac{4\left(1+2x^2\right)}{3\left(1+16x^2+16x^4\right)}\right)dx, x, \sinh(x)\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1+2x^2}{1+16x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{8-4\sqrt{3}+16x^2}\,dx, x, \sinh(x)\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{8+4\sqrt{3}\right.\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [A] time = 0.0843409, size = 81, normalized size = 0.95

$$
\frac{1}{6}\left(-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sinh(x)\right)+\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[6*x],x]
```

```
[Out] (-(Sqrt[2]*ArcTan[Sqrt[2]*Sinh[x]]) + Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/
Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]
])/6
```

Maple [C] time = 0.083, size = 83, normalized size = 1.

$$\frac{i}{12}\sqrt{2}\ln\left(\mathrm{e}^{2x}-i\sqrt{2}\mathrm{e}^{x}-1\right)-\frac{i}{12}\sqrt{2}\ln\left(\mathrm{e}^{2x}+i\sqrt{2}\mathrm{e}^{x}-1\right)+2\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^4+2304\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(13824\_R^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(6*x),x)
```

```
[Out] 1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*I*2^(1/2)*ln(exp(2*x)+I
*2^(1/2)*exp(x)-1)+2*sum(_R*ln(exp(2*x)+(13824*_R^3+96*_R)*exp(x)-1),_R=Roo
tOf(331776*_Z^4+2304*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\int\frac{e^{(7x)}+e^{(5x)}+e^{(3x)}+e^{x}}{3\left(e^{(8x)}-e^{(4x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/6*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + integrate(1/3*(e^(7*x) + e^(5*x) + e^(3*x) +
 e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Fricas [B] time = 2.38837, size = 489, normalized size = 5.75

$$-\frac{1}{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(\sqrt{\sqrt{3}+2}\left(e^{(2x)}-1\right)-\sqrt{-\sqrt{3}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{3}+2}\right)e^{(-x)}\right)-\frac{1}{3}\sqrt{-\sqrt{3}+2}\arctan\left(-\left(\sqrt{-\sqrt{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(sqrt(3) + 2)*arctan(-(sqrt(sqrt(3) + 2)*(e^(2*x) - 1) - sqrt(-sqr
t(3)*e^(2*x) + e^(4*x) + 1)*sqrt(sqrt(3) + 2))*e^(-x)) - 1/3*sqrt(-sqrt(3)
+ 2)*arctan(-(sqrt(-sqrt(3) + 2)*(e^(2*x) - 1) - sqrt(sqrt(3)*e^(2*x) + e^(
4*x) + 1)*sqrt(-sqrt(3) + 2))*e^(-x)) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*e^(3
*x) + 1/2*sqrt(2)*e^x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{sech}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x)
```

```
[Out] Integral(cosh(x)*sech(6*x), x)
```

Giac [B] time = 1.20785, size = 239, normalized size = 2.81

$$\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}-\sqrt{2}+4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}-\sqrt{2}-4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}+\sqrt{}}{\sqrt{6}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - sqrt(2))*arctan((sqrt(6) - sqrt(2) + 4*e^x)/(sqrt(6) + sqrt
(2))) + 1/12*(sqrt(6) - sqrt(2))*arctan(-(sqrt(6) - sqrt(2) - 4*e^x)/(sqrt(
6) + sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan((sqrt(6) + sqrt(2) + 4*e^x
)/(sqrt(6) - sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan(-(sqrt(6) + sqrt(2
) - 4*e^x)/(sqrt(6) - sqrt(2))) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*e^x)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x))
```

3.245 $\int \cosh(x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=7

$$-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]/2
```

Rubi [A] time = 0.0150622, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[2*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/2
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{csch}(2x)\,dx &= \frac{1}{2}\int \text{csch}(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0022791, size = 11, normalized size = 1.57

$$\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[2*x],x]
```

```
[Out] Log[Tanh[x/2]]/2
```

Maple [A] time = 0.013, size = 6, normalized size = 0.9

$$-\text{Artanh}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(2*x),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [B] time = 1.07637, size = 26, normalized size = 3.71

$$-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] time = 1.98568, size = 89, normalized size = 12.71

$$-\frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x)
```

```
[Out] Integral(cosh(x)*csch(2*x), x)
```

Giac [B] time = 1.1409, size = 22, normalized size = 3.14

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.246 $\int \cosh(x)\mathbf{csch}(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sinh(x))-\frac{1}{6}\log\left(4\sinh^2(x)+3\right)$$

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Rubi [A] time = 0.0305192, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 29, 31}

$$\frac{1}{3}\log(\sinh(x))-\frac{1}{6}\log\left(4\sinh^2(x)+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[3*x],x]
```

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\mathbf{csch}(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3+4x^2\right)}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(3+4x)}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{3+4x}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0092672, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(4\sinh^2(x)+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[3*x],x]
```

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Maple [A] time = 0.038, size = 24, normalized size = 1.1

$$\frac{\ln\left(e^{2x}-1\right)}{3} - \frac{\ln\left(e^{4x}+e^{2x}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(3*x),x)
```

```
[Out] 1/3*ln(exp(2*x)-1)-1/6*ln(exp(4*x)+exp(2*x)+1)
```

Maxima [B] time = 1.63466, size = 63, normalized size = 3.

$$-\frac{1}{6}\log\left(e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}-1\right)-\frac{1}{6}\log\left(-e^{(-x)}+e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="maxima")
```

```
[Out] -1/6*log(e^(-x) + e^(-2*x) + 1) + 1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 1)
 - 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

Fricas [B] time = 2.0435, size = 173, normalized size = 8.24

$$-\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\operatorname{csch}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x)
```

```
[Out] Integral(cosh(x)*csch(3*x), x)
```

Giac [B] time = 1.15222, size = 54, normalized size = 2.57

$$-\frac{1}{6}\log\left(e^{(2x)}+e^{x}+1\right)-\frac{1}{6}\log\left(e^{(2x)}-e^{x}+1\right)+\frac{1}{3}\log\left(e^{x}+1\right)+\frac{1}{3}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="giac")
```

```
[Out] -1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) + 1/3*log(e^x + 1)
 + 1/3*log(abs(e^x - 1))
```

3.247 $\int \cosh(x)\mathbf{csch}(4x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]/4 + ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.03261, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[4*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/4 + ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\mathbf{csch}(4x)\,dx = -\operatorname{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx, x, \cosh(x)\right)$$
$$= 2\operatorname{Subst}\left(\int \frac{1}{4-8x^2}\,dx, x, \cosh(x)\right) - 2\operatorname{Subst}\left(\int \frac{1}{8-8x^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\cosh(x)) + \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.294351, size = 183, normalized size = 7.04

$$\frac{4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\sqrt{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sqrt{2}-2\cosh(x)\right)+\log\left(2\cosh(x)+\sqrt{2}\right)+}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[4*x],x]
```

```
[Out] ((2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqrt
[2])*Sinh[x/2])] - (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Cos
h[x/2] - (1 + Sqrt[2])*Sinh[x/2])] + 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] - 2*S
qrt[2]*Log[Cosh[x/2]] - Log[Sqrt[2] - 2*Cosh[x]] + Log[Sqrt[2] + 2*Cosh[x]]
 + 2*Sqrt[2]*Log[Sinh[x/2]])/(8*Sqrt[2])
```

Maple [B] time = 0.039, size = 53, normalized size = 2.

$$-\frac{\ln(e^x+1)}{4}+\frac{\ln(e^x-1)}{4}+\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(4*x),x)
```

```
[Out] -1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2
)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.68406, size = 81, normalized size = 3.12

$$\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{4}\log\left(e^{(-x)}+1\right)+\frac{1}{4}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) - 1/8*sqrt(2)*log(-sqrt(2)*e
^(-x) + e^(-2*x) + 1) - 1/4*log(e^(-x) + 1) + 1/4*log(e^(-x) - 1)
```

Fricas [B] time = 2.10151, size = 211, normalized size = 8.12

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2+2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)-\frac{1}{4}\log(\cosh(x)+\sinh(x)+1)+\frac{1}{4}\log(\cosh(x)+\sinh(x)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2)) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sinh(x)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\operatorname{csch}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x)
```

```
[Out] Integral(cosh(x)*csch(4*x), x)
```

Giac [B] time = 1.14539, size = 77, normalized size = 2.96

$$-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)-\frac{1}{8}\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{8}\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) - 1/8*
log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

3.248 $\int \cosh(x)\mathbf{csch}(5x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sinh(x))$$

```
[Out] Log[Sinh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] + 8*Sinh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Rubi [A] time = 0.0779144, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4356, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[5*x],x]
```

```
[Out] Log[Sinh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] + 8*Sinh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\mathbf{csch}(5x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(5+20x^2+16x^4\right)}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5+20x+16x^2\right)}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{-20-16x}{5+20x+16x^2}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{5}\log(\sinh(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10+2\sqrt{5}+16x}\,dx, x, \sinh^2(x)\right)-\frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\text{S}\\ &= \frac{1}{5}\log(\sinh(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}+8\sinh^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}+8\sinh^2(x\right.\end{aligned}$$

Mathematica [A] time = 0.0590466, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(4\log(\sinh(x))-\left(1+\sqrt{5}\right)\log\left(4\cosh(2x)-\sqrt{5}+1\right)+\left(\sqrt{5}-1\right)\log\left(4\cosh(2x)+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[2*x]]) + (-1 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cosh[2*x]] + 4*Log[Sinh[x]])/20
```

Maple [B] time = 0.063, size = 101, normalized size = 1.6

$$\frac{\ln\left(e^{2x}-1\right)}{5}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}+\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}-\frac{\ln\left(e^{4x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(5*x),x)
```

```
[Out] 1/5*ln(exp(2*x)-1)-1/20*ln(exp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(e
xp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2-1/2*5^(
1/2))*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{5}\int\frac{\left(e^{(3x)}+e^{(2x)}+e^{x}+1\right)e^{x}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx-\frac{1}{5}\int\frac{\left(e^{(3x)}-e^{(2x)}+e^{x}-1\right)e^{x}}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^{x}+1}\,dx+\frac{3}{10}\int\frac{e^{(3x)}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="maxima")
```

```
[Out] -1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/(e^(4*x) + e^(3*x) + e^(2*
x) + e^x + 1), x) - 1/5*integrate((e^(3*x) - e^(2*x) + e^x - 1)*e^x/(e^(4*x
) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integrate(e^(3*x)/(e^(4*x) + e^
(3*x) + e^(2*x) + e^x + 1), x) - 3/10*integrate(e^(3*x)/(e^(4*x) - e^(3*x)
+ e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) + e^(3*x) + e^(2
*x) + e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) - e^(3*x) + e^(2*x) -
e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1),
x) + 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/5*l
og(e^x + 1) + 1/5*log(e^x - 1)
```

Fricas [B] time = 2.16057, size = 583, normalized size = 9.4

$$\frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4+4\sinh(x)^4+4\left(\sqrt{5}+1\right)\cosh(x)^2+4\left(6\cosh(x)^2+\sqrt{5}+1\right)\sinh(x)^2+\sqrt{5}+7}{2\cosh(x)^4+2\sinh(x)^4+2\left(6\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+1}\right)-\frac{1}{20}\log\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 + 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 + sqrt(5) + 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cosh(x)\operatorname{csch}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x)
```

```
[Out] Integral(cosh(x)*csch(5*x), x)
```

Giac [B] time = 1.1892, size = 146, normalized size = 2.35

$$-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right)+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\left(\sqrt{5}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="giac")
```

```
[Out] -1/20*(sqrt(5) + 1)*log(1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5
) + 1)*log(-1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(1
/2*(sqrt(5) - 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(-1/2*(sqrt(5)
- 1)*e^x + e^(2*x) + 1) + 1/5*log(e^x + 1) + 1/5*log(abs(e^x - 1))
```

3.249 $\int \cosh(x)\mathbf{csch}(6x)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 + ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3])
```

Rubi [A] time = 0.05212, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[6*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 + ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(x)\mathrm{csch}(6x)\,dx = -\mathrm{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cosh(x)\right)$$
$$= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)$$
$$= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cosh(x)\right)\right)$$
$$= \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x)\right)-\mathrm{Subst}\left(\int \frac{1}{-3+\cdots}\right.$$
$$= -\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [C] time = 0.0615614, size = 91, normalized size = 2.53

$$\frac{1}{12}\left(2\sqrt{3}\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\sqrt{3}\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log(1-2\cosh\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[6*x],x]
```

```
[Out] (2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + I*Ta
nh[x/2])/Sqrt[3]] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - Log[1 + 2*Cosh[
x]] + 2*Log[Sinh[x/2]])/12
```

Maple [B] time = 0.046, size = 77, normalized size = 2.1

$$\frac{\ln\left(e^{x}-1\right)}{6}-\frac{\ln\left(e^{x}+1\right)}{6}-\frac{\ln\left(e^{2x}+e^{x}+1\right)}{12}+\frac{\sqrt{3}\ln\left(e^{2x}+\sqrt{3}e^{x}+1\right)}{12}-\frac{\sqrt{3}\ln\left(e^{2x}-\sqrt{3}e^{x}+1\right)}{12}+\frac{\ln\left(e^{2x}-e^{x}+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(6*x),x)
```

```
[Out] 1/6*ln(exp(x)-1)-1/6*ln(exp(x)+1)-1/12*ln(exp(2*x)+exp(x)+1)+1/12*3^(1/2)*l
n(exp(2*x)+3^(1/2)*exp(x)+1)-1/12*3^(1/2)*ln(exp(2*x)-3^(1/2)*exp(x)+1)+1/1
```

```
2*ln(exp(2*x)-exp(x)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^{(3x)} - e^{x}}{2\left(e^{(4x)} - e^{(2x)} + 1\right)}\,dx - \frac{1}{12}\log\left(e^{(2x)} + e^{x} + 1\right) + \frac{1}{12}\log\left(e^{(2x)} - e^{x} + 1\right) - \frac{1}{6}\log\left(e^{x} + 1\right) + \frac{1}{6}\log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*(e^(3*x) - e^x)/(e^(4*x) - e^(2*x) + 1), x) - 1/12*log(e^(2*
x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6*log(e^x + 1) + 1/6*log(e^
x - 1)
```

Fricas [B] time = 2.17203, size = 358, normalized size = 9.94

$$\frac{1}{12}\sqrt{3}\log\left(\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 4\sqrt{3}\cosh(x) + 5}{2\cosh(x)^2 + 2\sinh(x)^2 - 1}\right) - \frac{1}{12}\log\left(\frac{2\cosh(x) + 1}{\cosh(x) - \sinh(x)}\right) + \frac{1}{12}\log\left(\frac{2\cosh(x) - 1}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((2*cosh(x)^2 + 2*sinh(x)^2 + 4*sqrt(3)*cosh(x) + 5)/(2*cos
h(x)^2 + 2*sinh(x)^2 - 1)) - 1/12*log((2*cosh(x) + 1)/(cosh(x) - sinh(x)))
+ 1/12*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/6*log(cosh(x) + sinh(x)
 + 1) + 1/6*log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\operatorname{csch}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x)
```

```
[Out] Integral(cosh(x)*csch(6*x), x)
```

Giac [B] time = 1.12417, size = 107, normalized size = 2.97

$$-\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{12}\log\left(e^{(-x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) - 1/1
2*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1) + 1/12*log(e^(-x) + e^
x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

3.250 $\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}$$

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.117687, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 12, 3308, 2181}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^m \cosh(a+bx)\sinh(a+bx)\,dx &= \int \frac{1}{2}x^m \sinh(2a+2bx)\,dx\\
&= \frac{1}{2}\int x^m \sinh(2a+2bx)\,dx\\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx\\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0459349, size = 66, normalized size = 0.94

$$\frac{e^{-2a}2^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left(e^{4a}(bx)^m\text{Gamma}(m+1,-2bx)+(-bx)^m\text{Gamma}(m+1,2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*x^m*(E^(4*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*Gamma[1
+ m, 2*b*x]))/(b*E^(2*a)*(-(b^2*x^2))^m)
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)

Maxima [A] time = 1.26501, size = 80, normalized size = 1.14

$$\frac{1}{4}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma\,(m+1,2\,bx)-\frac{1}{4}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma\,(m+1,-2\,bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")

[Out] 1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x)

Fricas [A] time = 2.20564, size = 258, normalized size = 3.69

$$\frac{\cosh\,(m\log\,(2\,b)+2\,a)\,\Gamma\,(m+1,2\,bx)+\cosh\,(m\log\,(-2\,b)-2\,a)\,\Gamma\,(m+1,-2\,bx)-\Gamma\,(m+1,2\,bx)\sinh\,(m\log\,(2\,b)+2}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")

[Out] 1/8*(cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) + cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) - gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - gamma(m + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a))/b

---

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a),x)

[Out] Exception raised: TypeError

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a), x)
```

3.251 $\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}$$

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b^4) - (3*x^
2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh[a + b*x]^2)/(4*b^3) + (x
^3*Sinh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0691723, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5372, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b^4) - (3*x^
2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh[a + b*x]^2)/(4*b^3) + (x
^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh(a+bx)\sinh(a+bx)\,dx &= \frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}\\ &= -\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int \sinh^2\\ &= \frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+\\ &= \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sin\end{aligned}$$

Mathematica [A] time = 0.103852, size = 50, normalized size = 0.53

$$\frac{\left(4b^3x^3+6bx\right)\cosh(2(a+bx))-3\left(2b^2x^2+1\right)\sinh(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((6*b*x + 4*b^3*x^3)*Cosh[2*(a + b*x)] - 3*(1 + 2*b^2*x^2)*Sinh[2*(a + b*x)
])/(16*b^4)
```

Maple [B] time = 0.004, size = 203, normalized size = 2.2

$$\frac{1}{b^4}\left(\frac{(bx+a)^3(\cosh(bx+a))^2}{2}-\frac{3(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{4}-\frac{(bx+a)^3}{4}+\frac{(3bx+3a)(\cosh(bx+a))^2}{4}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/2*(b*x+a)^3*cosh(b*x+a)^2-3/4*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/
4*(b*x+a)^3+3/4*(b*x+a)*cosh(b*x+a)^2-3/8*cosh(b*x+a)*sinh(b*x+a)-3/8*b*x-3
/8*a-3*a*(1/2*(b*x+a)^2*cosh(b*x+a)^2-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1
/4*(b*x+a)^2+1/4*cosh(b*x+a)^2)+3*a^2*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b
*x+a)*sinh(b*x+a)-1/4*b*x-1/4*a)-1/2*a^3*cosh(b*x+a)^2)
```

Maxima [A] time = 1.10756, size = 116, normalized size = 1.23

$$\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 3*e^(2*a))*e^
(2*b*x)/b^4 + 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

Fricas [A] time = 2.05681, size = 180, normalized size = 1.91

$$\frac{\left(2\,b^3x^3+3\,bx\right)\cosh(bx+a)^2-3\left(2\,b^2x^2+1\right)\cosh(bx+a)\sinh(bx+a)+\left(2\,b^3x^3+3\,bx\right)\sinh(bx+a)^2}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 3*(2*b^2*x^2 + 1)*cosh(b*x + a)*
sinh(b*x + a) + (2*b^3*x^3 + 3*b*x)*sinh(b*x + a)^2)/b^4
```

Sympy [A] time = 2.38946, size = 119, normalized size = 1.27

$$\begin{cases} \frac{x^3\sinh^2(a+bx)}{4b} + \frac{x^3\cosh^2(a+bx)}{4b} - \frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{8b^3} + \frac{3x\cosh^2(a+bx)}{8b^3} - \frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4} & \text{for } b \neq 0 \\ \frac{x^4\sinh(a)\cosh(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**2/(4*b) + x**3*cosh(a + b*x)**2/(4*b) - 3*x*
*2*sinh(a + b*x)*cosh(a + b*x)/(4*b**2) + 3*x*sinh(a + b*x)**2/(8*b**3) + 3
*x*cosh(a + b*x)**2/(8*b**3) - 3*sinh(a + b*x)*cosh(a + b*x)/(8*b**4), Ne(b
, 0)), (x**4*sinh(a)*cosh(a)/4, True))
```

Giac [A] time = 1.16116, size = 99, normalized size = 1.05

$$\frac{\left(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3\right)e^{(2\,bx+2\,a)}}{32\,b^4} + \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/32*(4*b^3*
x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

3.252 $\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{x^2}{4b}$$

```
[Out] x^2/(4*b) - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^
3) + (x^2*Sinh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0427648, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5372, 3310, 30}

$$\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x^2/(4*b) - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^
3) + (x^2*Sinh[a + b*x]^2)/(2*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh(a+bx)\sinh(a+bx)\,dx &= \frac{x^2\sinh^2(a+bx)}{2b}-\frac{\int x\sinh^2(a+bx)\,dx}{b}\\ &= -\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\int x\,dx}{2b}\\ &= \frac{x^2}{4b}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0673341, size = 39, normalized size = 0.61

$$\frac{(2b^2x^2+1)\cosh(2(a+bx))-2bx\sinh(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 2*b*x*Sinh[2*(a + b*x)])/(8*b^3)
```

Maple [B] time = 0.004, size = 114, normalized size = 1.8

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\cosh(bx+a))^2}{2}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^2}{4}+\frac{(\cosh(bx+a))^2}{4}-2a\left(1/2\ (bx+a)\,(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/2*(b*x+a)^2*cosh(b*x+a)^2-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*
(b*x+a)^2+1/4*cosh(b*x+a)^2-2*a*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b*x+a)*
sinh(b*x+a)-1/4*b*x-1/4*a)+1/2*a^2*cosh(b*x+a)^2)
```

Maxima [A] time = 1.08568, size = 86, normalized size = 1.34

$$\frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3}+\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/16*(2*
b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3
```

Fricas [A] time = 2.123, size = 154, normalized size = 2.41

$$-\frac{4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)^2-\left(2\,b^2x^2+1\right)\sinh\left(bx+a\right)^2}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*x^2 + 1)*cosh(b*x + a)^2 -
 (2*b^2*x^2 + 1)*sinh(b*x + a)^2)/b^3
```

Sympy [A] time = 1.21291, size = 75, normalized size = 1.17

$$\begin{cases}\frac{x^2\sinh^2\left(a+bx\right)}{4b}+\frac{x^2\cosh^2\left(a+bx\right)}{4b}-\frac{x\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b^2}+\frac{\sinh^2\left(a+bx\right)}{4b^3} & \text{for}\ b\neq 0\\ \frac{x^3\sinh\left(a\right)\cosh\left(a\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2/(4*b) + x**2*cosh(a + b*x)**2/(4*b) - x*si
nh(a + b*x)*cosh(a + b*x)/(2*b**2) + sinh(a + b*x)**2/(4*b**3), Ne(b, 0)),
(x**3*sinh(a)*cosh(a)/3, True))
```

Giac [A] time = 1.14793, size = 77, normalized size = 1.2

$$\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{16\,b^3}+\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(2*b^2*x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 + 1/16*(2*b^2*x^2 + 2*b*x
+ 1)*e^(-2*b*x - 2*a)/b^3
```

3.253 $\int x \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=44

$$-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}$$

```
[Out] x/(4*b) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0234004, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5372, 2635, 8}

$$-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x/(4*b) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(2*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{x\sinh^2(a+bx)}{2b} - \frac{\int \sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} + \frac{\int 1\,dx}{4b}$$
$$= \frac{x}{4b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0549291, size = 28, normalized size = 0.64

$$-\frac{\sinh(2(a+bx)) - 2bx\cosh(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] -(-2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*b^2)
```

Maple [A] time = 0.003, size = 53, normalized size = 1.2

$$\frac{1}{b^2}\left(\frac{(bx+a)(\cosh(bx+a))^2}{2} - \frac{\cosh(bx+a)\sinh(bx+a)}{4} - \frac{bx}{4} - \frac{a}{4} - \frac{a(\cosh(bx+a))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b*x+a)*sinh(b*x+a)-1/4*b*x-1/4*a-
1/2*a*cosh(b*x+a)^2)
```

Maxima [A] time = 1.19771, size = 62, normalized size = 1.41

$$\frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x -
 2*a)/b^2
```

Fricas [A] time = 1.99045, size = 112, normalized size = 2.55

$$\frac{bx\cosh(bx+a)^2 + bx\sinh(bx+a)^2 - \cosh(bx+a)\sinh(bx+a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a
))/b^2
```

Sympy [A] time = 0.610406, size = 56, normalized size = 1.27

$$\begin{cases} \frac{x\sinh^2(a+bx)}{4b} + \frac{x\cosh^2(a+bx)}{4b} - \frac{\sinh(a+bx)\cosh(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2\sinh(a)\cosh(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2/(4*b) + x*cosh(a + b*x)**2/(4*b) - sinh(a + b
*x)*cosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)/2, True))
```

Giac [A] time = 1.16262, size = 55, normalized size = 1.25

$$\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(2*b*x - 1)*e^(2*b*x + 2*a)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^
2
```

3.254 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0137316, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sinh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0096956, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.9

$$\frac{(\cosh(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b
```

Maxima [A] time = 1.0907, size = 18, normalized size = 1.2

$$\frac{\cosh(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(b*x + a)^2/b
```

Fricas [A] time = 1.99963, size = 58, normalized size = 3.87

$$\frac{\cosh(bx+a)^2+\sinh(bx+a)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

Sympy [A] time = 0.22203, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh^{2}(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

Giac [A] time = 1.12654, size = 32, normalized size = 2.13

$$\frac{e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/b
```

3.255 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sinh(2a)\text{Chi}(2bx) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Rubi [A] time = 0.0745568, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5448, 12, 3303, 3298, 3301}

$$\frac{1}{2}\sinh(2a)\text{Chi}(2bx) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx &= \int \frac{\sinh(2a+2bx)}{2x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx \\
&= \frac{1}{2}\mathbf{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\mathbf{Shi}(2bx)
\end{aligned}$$

Mathematica [A] time = 0.0254978, size = 25, normalized size = 0.93

$$\frac{1}{2}(\sinh(2a)\mathbf{Chi}(2bx) + \cosh(2a)\mathbf{Shi}(2bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a] + Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Maple [A] time = 0.021, size = 26, normalized size = 1.

$$\frac{e^{-2a}\mathrm{Ei}(1,2bx)}{4} - \frac{e^{2a}\mathrm{Ei}(1,-2bx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] 1/4*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.32902, size = 31, normalized size = 1.15

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-\frac{1}{4}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

Fricas [A] time = 2.02355, size = 109, normalized size = 4.04

$$\frac{1}{4}\left(\mathrm{Ei}\left(2\,bx\right)-\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\frac{1}{4}\left(\mathrm{Ei}\left(2\,bx\right)+\mathrm{Ei}\left(-2\,bx\right)\right)\sinh\left(2\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a) + 1/4*(Ei(2*b*x) + Ei(-2*b*x))*sinh(
2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x, x)
```

Giac [A] time = 1.14183, size = 31, normalized size = 1.15

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,bx\right)e^{\left(2\,a\right)} - \frac{1}{4}\,\mathrm{Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

3.256 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=39

$$b\cosh(2a)\text{Chi}(2bx) + b\sinh(2a)\text{Shi}(2bx) - \frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x]
```

Rubi [A] time = 0.0948402, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$b\cosh(2a)\text{Chi}(2bx) + b\sinh(2a)\text{Shi}(2bx) - \frac{\sinh(2a+2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x} + b\int \frac{\cosh(2a+2bx)}{x}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x} + (b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx + (b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx\\
&= b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx)
\end{aligned}$$

Mathematica [A] time = 0.0709192, size = 42, normalized size = 1.08

$$\frac{1}{2}\left(2b\cosh(2a)\text{Chi}(2bx) + 2b\sinh(2a)\text{Shi}(2bx) - \frac{\sinh(2(a+bx))}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] (2*b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*(a + b*x)]/x + 2*b*Sinh[2*a]*Si
nhIntegral[2*b*x])/2
```

Maple [A] time = 0.026, size = 56, normalized size = 1.4

$$\frac{e^{-2bx-2a}}{4x} - \frac{be^{-2a}\mathrm{Ei}(1, 2bx)}{2} - \frac{e^{2bx+2a}}{4x} - \frac{be^{2a}\mathrm{Ei}(1, -2bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/4*exp(-2*b*x-2*a)/x-1/2*b*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*b*x+2*a)/x-1/2*
b*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.34828, size = 36, normalized size = 0.92

$$\frac{1}{2}be^{(-2a)}\Gamma(-1, 2bx) + \frac{1}{2}be^{(2a)}\Gamma(-1, -2bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x)
```

Fricas [A] time = 1.89209, size = 174, normalized size = 4.46

$$\frac{(bx\mathrm{Ei}(2bx) + bx\mathrm{Ei}(-2bx))\cosh(2a) - 2\cosh(bx+a)\sinh(bx+a) + (bx\mathrm{Ei}(2bx) - bx\mathrm{Ei}(-2bx))\sinh(2a)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) - 2*cosh(b*x + a)*sinh(b*x
+ a) + (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.13854, size = 70, normalized size = 1.79

$$\frac{2\,bx\mathrm{Ei}\left(2\,bx\right)e^{\left(2\,a\right)} + 2\,bx\mathrm{Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)} - e^{\left(2\,bx+2\,a\right)} + e^{\left(-2\,bx-2\,a\right)}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x*Ei(2*b*x)*e^(2*a) + 2*b*x*Ei(-2*b*x)*e^(-2*a) - e^(2*b*x + 2*a)
+ e^(-2*b*x - 2*a))/x
```

3.257 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=60

$$b^2\sinh(2a)\text{Chi}(2bx)+b^2\cosh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{4x^2}-\frac{b\cosh(2a+2bx)}{2x}$$

```
[Out] -(b*Cosh[2*a + 2*b*x])/(2*x) + b^2*CoshIntegral[2*b*x]*Sinh[2*a] - Sinh[2*a
 + 2*b*x]/(4*x^2) + b^2*Cosh[2*a]*SinhIntegral[2*b*x]
```

Rubi [A] time = 0.123143, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$b^2\sinh(2a)\text{Chi}(2bx)+b^2\cosh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{4x^2}-\frac{b\cosh(2a+2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```


```
[Out] -(b*Cosh[2*a + 2*b*x])/(2*x) + b^2*CoshIntegral[2*b*x]*Sinh[2*a] - Sinh[2*a
 + 2*b*x]/(4*x^2) + b^2*Cosh[2*a]*SinhIntegral[2*b*x]
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^3}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^3}\,dx \\
&= -\frac{\sinh(2a+2bx)}{4x^2} + \frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{2x} - \frac{\sinh(2a+2bx)}{4x^2} + b^2\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{2x} - \frac{\sinh(2a+2bx)}{4x^2} + \left(b^2\cosh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx + \left(b^2\sinh(2\right. \\
&= -\frac{b\cosh(2a+2bx)}{2x} + b^2\operatorname{Chi}(2bx)\sinh(2a) - \frac{\sinh(2a+2bx)}{4x^2} + b^2\cosh(2a)\operatorname{Shi}(2bx
\end{aligned}$$

Mathematica [A] time = 0.174611, size = 61, normalized size = 1.02

$$\frac{1}{2}\left(2b^2\sinh(2a)\operatorname{Chi}(2bx) + 2b^2\cosh(2a)\operatorname{Shi}(2bx) - \frac{\sinh(2(a+bx)) + 2bx\cosh(2(a+bx))}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

```
[Out] (2*b^2*CoshIntegral[2*b*x]*Sinh[2*a] - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a
 + b*x)])/(2*x^2) + 2*b^2*Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Maple [A] time = 0.032, size = 90, normalized size = 1.5

$$-\frac{b\mathrm{e}^{-2\,bx-2\,a}}{4\,x}+\frac{\mathrm{e}^{-2\,bx-2\,a}}{8\,x^{2}}+\frac{b^{2}\mathrm{e}^{-2\,a}\mathrm{Ei}\left(1,2\,bx\right)}{2}-\frac{\mathrm{e}^{2\,bx+2\,a}}{8\,x^{2}}-\frac{b\mathrm{e}^{2\,bx+2\,a}}{4\,x}-\frac{b^{2}\mathrm{e}^{2\,a}\mathrm{Ei}\left(1,-2\,bx\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^3,x)
```

```
[Out] -1/4*b*exp(-2*b*x-2*a)/x+1/8*exp(-2*b*x-2*a)/x^2+1/2*b^2*exp(-2*a)*Ei(1,2*b
*x)-1/8*exp(2*b*x+2*a)/x^2-1/4*b*exp(2*b*x+2*a)/x-1/2*b^2*exp(2*a)*Ei(1,-2*
b*x)
```

Maxima [A] time = 1.23321, size = 41, normalized size = 0.68

$$b^{2}e^{(-2\,a)}\Gamma\left(-2,2\,bx\right)-b^{2}e^{(2\,a)}\Gamma\left(-2,-2\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-2*a)*gamma(-2, 2*b*x) - b^2*e^(2*a)*gamma(-2, -2*b*x)
```

Fricas [A] time = 1.94455, size = 257, normalized size = 4.28

$$-\frac{bx\cosh\left(bx+a\right)^{2}+bx\sinh\left(bx+a\right)^{2}-\left(b^{2}x^{2}\mathrm{Ei}\left(2\,bx\right)-b^{2}x^{2}\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(b^{2}x^{2}\right.}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - (b^2*x^2*Ei(2*b*x) - b^2*
x^2*Ei(-2*b*x))*cosh(2*a) + cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2*Ei(2*b*x
) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.13612, size = 116, normalized size = 1.93

$$\frac{4\,b^2x^2\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-4\,b^2x^2\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-2\,bxe^{(2\,bx+2\,a)}-2\,bxe^{(-2\,bx-2\,a)}-e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*x^2*Ei(2*b*x)*e^(2*a) - 4*b^2*x^2*Ei(-2*b*x)*e^(-2*a) - 2*b*x*e^
(2*b*x + 2*a) - 2*b*x*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
)/x^2
```

3.258 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=85

$$\frac{2}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{2}{3}b^3\sinh(2a)\mathrm{Shi}(2bx)-\frac{b^2\sinh(2a+2bx)}{3x}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b\cosh(2a+2bx)}{6x^2}$$

```
[Out] -(b*Cosh[2*a + 2*b*x])/(6*x^2) + (2*b^3*Cosh[2*a]*CoshIntegral[2*b*x])/3 -
Sinh[2*a + 2*b*x]/(6*x^3) - (b^2*Sinh[2*a + 2*b*x])/(3*x) + (2*b^3*Sinh[2*a
]*SinhIntegral[2*b*x])/3
```

Rubi [A] time = 0.150576, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$\frac{2}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{2}{3}b^3\sinh(2a)\mathrm{Shi}(2bx)-\frac{b^2\sinh(2a+2bx)}{3x}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b\cosh(2a+2bx)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(6*x^2) + (2*b^3*Cosh[2*a]*CoshIntegral[2*b*x])/3 -
Sinh[2*a + 2*b*x]/(6*x^3) - (b^2*Sinh[2*a + 2*b*x])/(3*x) + (2*b^3*Sinh[2*a
]*SinhIntegral[2*b*x])/3
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^4}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= -\frac{\sinh(2a+2bx)}{6x^3}+\frac{1}{3}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{6x^2}-\frac{\sinh(2a+2bx)}{6x^3}+\frac{1}{3}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{6x^2}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b^2\sinh(2a+2bx)}{3x}+\frac{1}{3}\left(2b^3\right)\int \frac{\cosh(2a+2}{x} \\
&= -\frac{b\cosh(2a+2bx)}{6x^2}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b^2\sinh(2a+2bx)}{3x}+\frac{1}{3}\left(2b^3\cosh(2a)\right)\int \frac{\text{co}}{} \\
&= -\frac{b\cosh(2a+2bx)}{6x^2}+\frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx)-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b^2\sinh(2a+2bx)}{3x}
\end{aligned}$$

Mathematica [A] time = 0.155667, size = 77, normalized size = 0.91

$$-\frac{-4b^3x^3\cosh(2a)\text{Chi}(2bx)-4b^3x^3\sinh(2a)\text{Shi}(2bx)+2b^2x^2\sinh(2(a+bx))+\sinh(2(a+bx))+bx\cosh(2(a+bx))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*x*Cosh[2*(a + b*x)] - 4*b^3*x^3*Cosh[2*a]*CoshIntegral[2*b*x] + Sinh[2*
(a + b*x)] + 2*b^2*x^2*Sinh[2*(a + b*x)] - 4*b^3*x^3*Sinh[2*a]*SinhIntegral
[2*b*x])/(6*x^3)
```

Maple [A] time = 0.034, size = 124, normalized size = 1.5

$$\frac{b^2 e^{-2bx-2a}}{6x} - \frac{b e^{-2bx-2a}}{12x^2} + \frac{e^{-2bx-2a}}{12x^3} - \frac{b^3 e^{-2a} \mathrm{Ei}(1, 2bx)}{3} - \frac{e^{2bx+2a}}{12x^3} - \frac{b e^{2bx+2a}}{12x^2} - \frac{b^2 e^{2bx+2a}}{6x} - \frac{b^3 e^{2a} \mathrm{Ei}(1, -2bx)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^4,x)
```

```
[Out] 1/6*b^2*exp(-2*b*x-2*a)/x-1/12*b*exp(-2*b*x-2*a)/x^2+1/12*exp(-2*b*x-2*a)/x
^3-1/3*b^3*exp(-2*a)*Ei(1,2*b*x)-1/12*exp(2*b*x+2*a)/x^3-1/12*b*exp(2*b*x+2
*a)/x^2-1/6*b^2*exp(2*b*x+2*a)/x-1/3*b^3*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.39122, size = 42, normalized size = 0.49

$$2 b^3 e^{(-2a)} \Gamma(-3, 2bx) + 2 b^3 e^{(2a)} \Gamma(-3, -2bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 2*b^3*e^(-2*a)*gamma(-3, 2*b*x) + 2*b^3*e^(2*a)*gamma(-3, -2*b*x)
```

Fricas [A] time = 1.96662, size = 286, normalized size = 3.36

$$-\frac{bx \cosh(bx+a)^2 + bx \sinh(bx+a)^2 + 2\left(2 b^2 x^2 + 1\right) \cosh(bx+a) \sinh(bx+a) - 2\left(b^3 x^3 \mathrm{Ei}(2bx) + b^3 x^3 \mathrm{Ei}(-2bx)\right) c}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 + 2*(2*b^2*x^2 + 1)*cosh(b*
x + a)*sinh(b*x + a) - 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a)
 - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**4,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.11924, size = 162, normalized size = 1.91

$$\frac{4\,b^3x^3\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}+4\,b^3x^3\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-2\,b^2x^2e^{(2\,bx+2\,a)}+2\,b^2x^2e^{(-2\,bx-2\,a)}-bxe^{(2\,bx+2\,a)}-bxe^{(-2\,bx-2\,a)}-e^{(2\,bx+2\,a)}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*x^3*Ei(2*b*x)*e^(2*a) + 4*b^3*x^3*Ei(-2*b*x)*e^(-2*a) - 2*b^2*x
^2*e^(2*b*x + 2*a) + 2*b^2*x^2*e^(-2*b*x - 2*a) - b*x*e^(2*b*x + 2*a) - b*x
*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/x^3
```

3.259 $\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}+\frac{e^{\cdot}}{}$$

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) + (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) + (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rubi [A] time = 0.196015, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}+\frac{e^{\cdot}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) + (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) + (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx &= \int \left(\frac{1}{4}x^m \sinh(a+bx)+\frac{1}{4}x^m \sinh(3a+3bx)\right)dx \\
&= \frac{1}{4}\int x^m \sinh(a+bx)\,dx+\frac{1}{4}\int x^m \sinh(3a+3bx)\,dx \\
&= \frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx-\frac{1}{8}\int e^{i(3ia+3ib\ldots} \\
&= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\ldots}{8\ldots}
\end{aligned}$$

Mathematica [A] time = 0.173345, size = 114, normalized size = 0.85

$$\frac{e^{-3a}x^m\left(3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\text{Gamma}(m+1,-3bx)+(-bx)^m\text{Gamma}(m+1,3bx)\right)+3e^{2a}\left(e^{2a}(-bx)^{-m}\text{Gamma}(m\ldots\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (x^m*(3*E^(2*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b
*x]/(b*x)^m) + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x))^m*Gamma[1 +
 m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int x^m\left(\cosh(bx+a)\right)^2\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

Maxima [A] time = 1.3533, size = 153, normalized size = 1.14

$$\frac{1}{8}\left(3\,bx\right)^{-m-1}x^{m+1}e^{\left(-3\,a\right)}\Gamma\left(m+1,3\,bx\right)+\frac{1}{8}\left(bx\right)^{-m-1}x^{m+1}e^{\left(-a\right)}\Gamma\left(m+1,bx\right)-\frac{1}{8}\left(-bx\right)^{-m-1}x^{m+1}e^{a}\Gamma\left(m+1,-bx\right)-\frac{1}{8}\left(-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-m
 - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e^
a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1,
 -3*b*x)
```

Fricas [A] time = 2.2133, size = 486, normalized size = 3.63

$$\frac{\cosh\left(m\log\left(3\,b\right)+3\,a\right)\Gamma\left(m+1,3\,bx\right)+3\,\cosh\left(m\log\left(b\right)+a\right)\Gamma\left(m+1,bx\right)+3\,\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+1,-bx\right)+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) + 3*cosh(m*log(b) + a)*gam
ma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) + cosh(m*log(-3*b
) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
- 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, -3*b*x)*sinh(m*lo
g(-3*b) - 3*a) - 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a), x)
```

3.260 $\int x^3 \cosh^2(a+bx) \sinh(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{2\sinh^3(a+bx)}{27b^4}-\frac{14\sinh(a+bx)}{9b^4}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{4x\cosh(\ldots}{3b^{\ldots}}$$

```
[Out] (4*x*Cosh[a + b*x])/(3*b^3) + (2*x*Cosh[a + b*x]^3)/(9*b^3) + (x^3*Cosh[a +
 b*x]^3)/(3*b) - (14*Sinh[a + b*x])/(9*b^4) - (2*x^2*Sinh[a + b*x])/(3*b^2)
 - (x^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b^2) - (2*Sinh[a + b*x]^3)/(27*b^
4)
```

Rubi [A] time = 0.120894, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5373, 3311, 3296, 2637, 2633}

$$-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{2\sinh^3(a+bx)}{27b^4}-\frac{14\sinh(a+bx)}{9b^4}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{4x\cosh(\ldots}{3b^{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*x*Cosh[a + b*x])/(3*b^3) + (2*x*Cosh[a + b*x]^3)/(9*b^3) + (x^3*Cosh[a +
 b*x]^3)/(3*b) - (14*Sinh[a + b*x])/(9*b^4) - (2*x^2*Sinh[a + b*x])/(3*b^2)
 - (x^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b^2) - (2*Sinh[a + b*x]^3)/(27*b^
4)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{x^3\cosh^3(a+bx)}{3b}-\frac{\int x^2\cosh^3(a+bx)\,dx}{b}\\ &= \frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2}-\frac{2\int \cos\\ &= \frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\cosh^2(a+bx)\sin\\ &= \frac{4x\cosh(a+bx)}{3b^3}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{2\sinh(a+bx)}{9b^4}-\frac{2x^2\\ &= \frac{4x\cosh(a+bx)}{3b^3}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{14\sinh(a+bx)}{9b^4}-\frac{2x\end{aligned}$$

Mathematica [A] time = 0.380855, size = 86, normalized size = 0.74

$$\frac{27bx\left(b^2x^2+6\right)\cosh(a+bx)+\left(9b^3x^3+6bx\right)\cosh(3(a+bx))-2\sinh(a+bx)\left(\left(9b^2x^2+2\right)\cosh(2(a+bx))+45b^2x^2}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*b*x*(6 + b^2*x^2)*Cosh[a + b*x] + (6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)]
 - 2*(82 + 45*b^2*x^2 + (2 + 9*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(
108*b^4)
```

Maple [B] time = 0.007, size = 334, normalized size = 2.9

$$\frac{1}{b^4}\left(\frac{(bx+a)^3\cosh(bx+a)(\sinh(bx+a))^2}{3}+\frac{(bx+a)^3\cosh(bx+a)}{3}-\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^2}{3}-\frac{2\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+1/3*(b*x+a)^3*cosh(b*x+a)-1/
3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-2/3*(b*x+a)^2*sinh(b*x+a)+2/9*(b*x+a)
*sinh(b*x+a)^2*cosh(b*x+a)+14/9*(b*x+a)*cosh(b*x+a)-2/27*sinh(b*x+a)*cosh(b
*x+a)^2-40/27*sinh(b*x+a)-3*a*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+1/3*
(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-4/9*(b*x+a)*sin
h(b*x+a)+2/27*cosh(b*x+a)*sinh(b*x+a)^2+14/27*cosh(b*x+a))+3*a^2*(1/3*(b*x+
a)*sinh(b*x+a)^2*cosh(b*x+a)+1/3*(b*x+a)*cosh(b*x+a)-1/9*sinh(b*x+a)*cosh(b
*x+a)^2-2/9*sinh(b*x+a))-a^3*(1/3*cosh(b*x+a)*sinh(b*x+a)^2+1/3*cosh(b*x+a)
))
```

Maxima [A] time = 1.16696, size = 216, normalized size = 1.85

$$\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4}+\frac{\left(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a\right)e^{(bx)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 + 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

Fricas [A] time = 1.99839, size = 332, normalized size = 2.84

$$\frac{3\left(3\,b^3x^3+2\,bx\right)\cosh\left(bx+a\right)^3+9\left(3\,b^3x^3+2\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3+27\left(b^3x^3\right.}{108\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/108*(3*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 9*(3*b^3*x^3 + 2*b*x)*cosh(b
*x + a)*sinh(b*x + a)^2 - (9*b^2*x^2 + 2)*sinh(b*x + a)^3 + 27*(b^3*x^3 + 6
*b*x)*cosh(b*x + a) - 3*(27*b^2*x^2 + (9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 54)
*sinh(b*x + a))/b^4
```

Sympy [A] time = 4.19104, size = 146, normalized size = 1.25

$$\begin{cases}\frac{x^3\cosh^3\left(a+bx\right)}{3b}+\frac{2x^2\sinh^3\left(a+bx\right)}{3b^2}-\frac{x^2\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{b^2}-\frac{4x\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{3b^3}+\frac{14x\cosh^3\left(a+bx\right)}{9b^3}+\frac{40\sinh^3\left(a+bx\right)}{27b^4}-\frac{14\sinh}{} \\ \frac{x^4\sinh\left(a\right)\cosh^2\left(a\right)}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**3*cosh(a + b*x)**3/(3*b) + 2*x**2*sinh(a + b*x)**3/(3*b**2) -
 x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 4*x*sinh(a + b*x)**2*cosh(a + b
*x)/(3*b**3) + 14*x*cosh(a + b*x)**3/(9*b**3) + 40*sinh(a + b*x)**3/(27*b**
4) - 14*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), (x**4*sinh(a)*c
osh(a)**2/4, True))
```

Giac [A] time = 1.16019, size = 189, normalized size = 1.62

$$\frac{\left(9\,b^3x^3-9\,b^2x^2+6\,bx-2\right)e^{\left(3\,bx+3\,a\right)}}{216\,b^4}+\frac{\left(b^3x^3-3\,b^2x^2+6\,bx-6\right)e^{\left(bx+a\right)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{\left(-bx-a\right)}}{8\,b^4}+\frac{\left(9\,b^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 + 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

3.261 $\int x^2 \cosh^2(a+bx) \sinh(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{4x\sinh(a+bx)}{9b^2}+\frac{2\cosh^3(a+bx)}{27b^3}+\frac{4\cosh(a+bx)}{9b^3}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{9b^2}+\frac{x^2\cosh^3(a+bx)}{3b}$$

```
[Out] (4*Cosh[a + b*x])/(9*b^3) + (2*Cosh[a + b*x]^3)/(27*b^3) + (x^2*Cosh[a + b*
x]^3)/(3*b) - (4*x*Sinh[a + b*x])/(9*b^2) - (2*x*Cosh[a + b*x]^2*Sinh[a + b
*x])/(9*b^2)
```

Rubi [A] time = 0.0705722, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5373, 3310, 3296, 2638}

$$-\frac{4x\sinh(a+bx)}{9b^2}+\frac{2\cosh^3(a+bx)}{27b^3}+\frac{4\cosh(a+bx)}{9b^3}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{9b^2}+\frac{x^2\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*Cosh[a + b*x])/(9*b^3) + (2*Cosh[a + b*x]^3)/(27*b^3) + (x^2*Cosh[a + b*
x]^3)/(3*b) - (4*x*Sinh[a + b*x])/(9*b^2) - (2*x*Cosh[a + b*x]^2*Sinh[a + b
*x])/(9*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\int x\cosh^3(a+bx)\,dx}{3b}\\ &= \frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2}-\frac{4\int x\cosh\\ &= \frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{4x\sinh(a+bx)}{9b^2}-\frac{2x\cosh^2(a+bx)\sinh(a\\ &= \frac{4\cosh(a+bx)}{9b^3}+\frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{4x\sinh(a+bx)}{9b^2}-\frac{2x\cos\end{aligned}$$

Mathematica [A] time = 0.199815, size = 65, normalized size = 0.78

$$\frac{27\left(b^2x^2+2\right)\cosh(a+bx)+\left(9b^2x^2+2\right)\cosh(3(a+bx))-6bx(9\sinh(a+bx)+\sinh(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*(2 + b^2*x^2)*Cosh[a + b*x] + (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] - 6*b*x
*(9*Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(108*b^3)
```

Maple [B] time = 0.006, size = 193, normalized size = 2.3

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\sinh(bx+a))^2\cosh(bx+a)}{3}+\frac{(bx+a)^2\cosh(bx+a)}{3}-\frac{(2bx+2a)\sinh(bx+a)(\cosh(bx+a))^2}{9}-(4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+1/3*(b*x+a)^2*cosh(b*x+a)-2/
9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-4/9*(b*x+a)*sinh(b*x+a)+2/27*cosh(b*x+a
)*sinh(b*x+a)^2+14/27*cosh(b*x+a)-2*a*(1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a
)+1/3*(b*x+a)*cosh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2-2/9*sinh(b*x+a))+a^
2*(1/3*cosh(b*x+a)*sinh(b*x+a)^2+1/3*cosh(b*x+a)))
```

Maxima [A] time = 1.11215, size = 165, normalized size = 1.99

$$\frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^3} + \frac{\left(b^2x^2e^a - 2\,bxe^a + 2\,e^a\right)e^{(bx)}}{8\,b^3} + \frac{\left(b^2x^2 + 2\,bx + 2\right)e^{(-bx-a)}}{8\,b^3} + \frac{\left(9\,b^2x^2 + 6\,bx + 2\right)e}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 + 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 + 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Fricas [A] time = 1.97761, size = 273, normalized size = 3.29

$$-\frac{6\,bx\sinh\left(bx + a\right)^3 - \left(9\,b^2x^2 + 2\right)\cosh\left(bx + a\right)^3 - 3\left(9\,b^2x^2 + 2\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right)^2 - 27\left(b^2x^2 + 2\right)\cosh\left(bx\right.}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*sinh(b*x + a)^3 - (9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 3*(9*b^2*
x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 27*(b^2*x^2 + 2)*cosh(b*x + a) + 1
8*(b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a))/b^3
```

Sympy [A] time = 2.25007, size = 105, normalized size = 1.27

$$\begin{cases} \frac{x^2\cosh^3\left(a+bx\right)}{3b} + \frac{4x\sinh^3\left(a+bx\right)}{9b^2} - \frac{2x\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{3b^2} - \frac{4\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{9b^3} + \frac{14\cosh^3\left(a+bx\right)}{27b^3} & \text{for}\: b \neq 0 \\ \frac{x^3\sinh\left(a\right)\cosh^2\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x)**3/(3*b) + 4*x*sinh(a + b*x)**3/(9*b**2) - 2*
x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 4*sinh(a + b*x)**2*cosh(a + b*x
)/(9*b**3) + 14*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), (x**3*sinh(a)*cosh(a
)**2/3, True))
```

Giac [A] time = 1.14968, size = 146, normalized size = 1.76

$$\frac{\left(9\,b^2x^2 - 6\,bx + 2\right)e^{(3\,bx+3\,a)}}{216\,b^3} + \frac{\left(b^2x^2 - 2\,bx + 2\right)e^{(bx+a)}}{8\,b^3} + \frac{\left(b^2x^2 + 2\,bx + 2\right)e^{(-bx-a)}}{8\,b^3} + \frac{\left(9\,b^2x^2 + 6\,bx + 2\right)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 + 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.262 $\int x \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)}{3b^2}+\frac{x\cosh^3(a+bx)}{3b}$$

```
[Out] (x*Cosh[a + b*x]^3)/(3*b) - Sinh[a + b*x]/(3*b^2) - Sinh[a + b*x]^3/(9*b^2)
```

Rubi [A] time = 0.0313528, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 2633}

$$-\frac{\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)}{3b^2}+\frac{x\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + b*x]^3)/(3*b) - Sinh[a + b*x]/(3*b^2) - Sinh[a + b*x]^3/(9*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{x\cosh^3(a+bx)}{3b}-\frac{\int\cosh^3(a+bx)\,dx}{3b}$$
$$=\frac{x\cosh^3(a+bx)}{3b}-\frac{i\,\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-i\sinh(a+bx)\right)}{3b^2}$$
$$=\frac{x\cosh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{3b^2}-\frac{\sinh^3(a+bx)}{9b^2}$$

Mathematica [A] time = 0.136738, size = 46, normalized size = 1.02

$$-\frac{9\sinh(a+bx)+\sinh(3(a+bx))-9bx\cosh(a+bx)-3bx\cosh(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -(-9*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + 9*Sinh[a + b*x] + Sinh[3
*(a + b*x)])/(36*b^2)
```

Maple [B] time = 0.007, size = 92, normalized size = 2.

$$\frac{1}{b^2}\left(\frac{(bx+a)(\sinh(bx+a))^2\cosh(bx+a)}{3}+\frac{(bx+a)\cosh(bx+a)}{3}-\frac{\sinh(bx+a)(\cosh(bx+a))^2}{9}-\frac{2\sinh(bx+a)}{9}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+1/3*(b*x+a)*cosh(b*x+a)-1/9*si
nh(b*x+a)*cosh(b*x+a)^2-2/9*sinh(b*x+a)-a*(1/3*cosh(b*x+a)*sinh(b*x+a)^2+1/
3*cosh(b*x+a)))
```

Maxima [B] time = 1.19734, size = 113, normalized size = 2.51

$$\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}+\frac{(bxe^a-e^a)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

Fricas [A] time = 2.30944, size = 205, normalized size = 4.56

$$\frac{3\,bx\cosh\left(bx+a\right)^{3}+9\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+9\,bx\cosh\left(bx+a\right)-\sinh\left(bx+a\right)^{3}-3\left(\cosh\left(bx+a\right)^{2}+3\right)\sin}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b*x*cosh(b*x + a)^3 + 9*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 9*b*x*c
osh(b*x + a) - sinh(b*x + a)^3 - 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b^2
```

Sympy [A] time = 1.13427, size = 61, normalized size = 1.36

$$\begin{cases} \frac{x\cosh^{3}\left(a+bx\right)}{3b}+\frac{2\sinh^{3}\left(a+bx\right)}{9b^{2}}-\frac{\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{3b^{2}} & \text{for}\: b\neq 0\\ \frac{x^{2}\sinh\left(a\right)\cosh^{2}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a + b*x)**3/(3*b) + 2*sinh(a + b*x)**3/(9*b**2) - sinh(a
+ b*x)*cosh(a + b*x)**2/(3*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)**2/2, Tr
ue))
```

Giac [A] time = 1.16303, size = 103, normalized size = 2.29

$$\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^{2}}+\frac{(bx-1)e^{(bx+a)}}{8\,b^{2}}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^{2}}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 + 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

3.263 $\int \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] Cosh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0205782, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^3/(3*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0033687, size = 15, normalized size = 1.

$$\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\cosh(bx+a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*cosh(b*x+a)^2,x)
```

```
[Out] 1/3*cosh(b*x+a)^3/b
```

Maxima [A] time = 1.17164, size = 18, normalized size = 1.2

$$\frac{\cosh(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*cosh(b*x + a)^3/b
```

Fricas [B] time = 2.2081, size = 105, normalized size = 7.

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + 3\cosh(bx+a)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 3*cosh(b*x + a))/
b
```

Sympy [A] time = 0.516506, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cosh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**2, True))
```

Giac [B] time = 1.13505, size = 62, normalized size = 4.13

$$\frac{\left(3\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + e^(3*b*x + 3*a) + 3*e^(b*x
 + a))/b
```

3.264 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x} dx$

Optimal. Leaf size=47

$$\frac{1}{4}\sinh(a)\mathbf{Chi}(bx) + \frac{1}{4}\sinh(3a)\mathbf{Chi}(3bx) + \frac{1}{4}\cosh(a)\mathbf{Shi}(bx) + \frac{1}{4}\cosh(3a)\mathbf{Shi}(3bx)$$

```
[Out] (CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (Cosh[a
]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.141382, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{4}\sinh(a)\mathbf{Chi}(bx) + \frac{1}{4}\sinh(3a)\mathbf{Chi}(3bx) + \frac{1}{4}\cosh(a)\mathbf{Shi}(bx) + \frac{1}{4}\cosh(3a)\mathbf{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (Cosh[a
]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x}+\frac{\sinh(3a+3bx)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x}\,dx\\ &= \frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{4}\sinh(a)\int \frac{\cosh(bx)}{x}\,d\\ &= \frac{1}{4}\mathbf{Chi}(bx)\sinh(a)+\frac{1}{4}\mathbf{Chi}(3bx)\sinh(3a)+\frac{1}{4}\cosh(a)\mathbf{Shi}(bx)+\frac{1}{4}\cosh(3a)\mathbf{Shi}(3b\end{aligned}$$

Mathematica [A] time = 0.0702908, size = 39, normalized size = 0.83

$$\frac{1}{4}(\sinh(a)\mathbf{Chi}(bx)+\sinh(3a)\mathbf{Chi}(3bx)+\cosh(a)\mathbf{Shi}(bx)+\cosh(3a)\mathbf{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + Cosh[a]*SinhIn
tegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] time = 0.057, size = 47, normalized size = 1.

$$\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{8}+\frac{e^{-a}\mathrm{Ei}(1,bx)}{8}-\frac{e^{a}\mathrm{Ei}(1,-bx)}{8}-\frac{e^{3a}\mathrm{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x,x)
```

```
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)-1/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.36931, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}-\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{1}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

Fricas [A] time = 2.38121, size = 204, normalized size = 4.34

$$\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)-\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(bx\right)-\mathrm{Ei}\left(-bx\right)\right)\cosh\left(a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(bx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 1/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 1/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x, x)
```

Giac [A] time = 1.18299, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{1}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}-\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

3.265 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2} dx$

Optimal. Leaf size=80

$$\frac{1}{4}b\cosh(a)\mathrm{Chi}(bx)+\frac{3}{4}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{1}{4}b\sinh(a)\mathrm{Shi}(bx)+\frac{3}{4}b\sinh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3b}{4x}$$

```
[Out] (b*Cosh[a]*CoshIntegral[b*x])/4 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/4 - S
inh[a + b*x]/(4*x) - Sinh[3*a + 3*b*x]/(4*x) + (b*Sinh[a]*SinhIntegral[b*x]
)/4 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.185551, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{4}b\cosh(a)\mathrm{Chi}(bx)+\frac{3}{4}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{1}{4}b\sinh(a)\mathrm{Shi}(bx)+\frac{3}{4}b\sinh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3b}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x])/4 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/4 - S
inh[a + b*x]/(4*x) - Sinh[3*a + 3*b*x]/(4*x) + (b*Sinh[a]*SinhIntegral[b*x]
)/4 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^2}+\frac{\sinh(3a+3bx)}{4x^2}\right)dx\\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^2}\,dx\\
&= -\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}b\int \frac{\cosh(a+bx)}{x}\,dx+\frac{1}{4}(3b)\int \frac{\cosh(3a+3
\end{aligned}$$

$$= -\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{4}(3b\cosh(3a))$$

$$= \frac{1}{4}b\cosh(a)\mathbf{Chi}(bx)+\frac{3}{4}b\cosh(3a)\mathbf{Chi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}$$

Mathematica [A] time = 0.148197, size = 70, normalized size = 0.88

$$\frac{bx\cosh(a)\mathbf{Chi}(bx)+3bx\cosh(3a)\mathbf{Chi}(3bx)+bx\sinh(a)\mathbf{Shi}(bx)+3bx\sinh(3a)\mathbf{Shi}(3bx)-\sinh(a+bx)-\sinh(3(a+}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*x*Cosh[a]*CoshIntegral[b*x] + 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] - Sinh
[a + b*x] - Sinh[3*(a + b*x)] + b*x*Sinh[a]*SinhIntegral[b*x] + 3*b*x*Sinh[
```

```
3*a]*SinhIntegral[3*b*x])/(4*x)
```

Maple [A] time = 0.066, size = 104, normalized size = 1.3

$$\frac{e^{-3bx-3a}}{8x} - \frac{3be^{-3a}\mathrm{Ei}(1,3bx)}{8} + \frac{e^{-bx-a}}{8x} - \frac{be^{-a}\mathrm{Ei}(1,bx)}{8} - \frac{e^{bx+a}}{8x} - \frac{be^{a}\mathrm{Ei}(1,-bx)}{8} - \frac{e^{3bx+3a}}{8x} - \frac{3be^{3a}\mathrm{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/8*exp(-3*b*x-3*a)/x-3/8*b*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-b*x-a)/x-1/8*b*e
xp(-a)*Ei(1,b*x)-1/8/x*exp(b*x+a)-1/8*b*exp(a)*Ei(1,-b*x)-1/8/x*exp(3*b*x+3
*a)-3/8*b*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.38231, size = 68, normalized size = 0.85

$$\frac{3}{8}\,be^{(-3a)}\Gamma(-1,3bx) + \frac{1}{8}\,be^{(-a)}\Gamma(-1,bx) + \frac{1}{8}\,be^{a}\Gamma(-1,-bx) + \frac{3}{8}\,be^{(3a)}\Gamma(-1,-3bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) + 1/8*b*e^a*g
amma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

Fricas [A] time = 1.89148, size = 327, normalized size = 4.09

$$-\frac{2\sinh(bx+a)^3 - 3(bx\mathrm{Ei}(3bx) + bx\mathrm{Ei}(-3bx))\cosh(3a) - (bx\mathrm{Ei}(bx) + bx\mathrm{Ei}(-bx))\cosh(a) + 2\left(3\cosh(bx+a)^2 + \right.}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sinh(b*x + a)^3 - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*cosh(3*a) - (b
*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
```

```
) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) - (b*x*Ei(b*x) - b*x*Ei(-b
*x))*sinh(a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**2, x)
```

Giac [A] time = 1.13776, size = 122, normalized size = 1.52

$$\frac{3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+bx\mathrm{Ei}\left(-bx\right)e^{(-a)}+3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+bx\mathrm{Ei}\left(bx\right)e^{a}-e^{(3\,bx+3\,a)}-e^{(bx+a)}+e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) + 3*b*x*Ei(-3*b*x)*e^(-3
*a) + b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) - e^(b*x + a) + e^(-b*x - a) + e^(-
3*b*x - 3*a))/x
```

3.266 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=119

$$\frac{1}{8}b^2\sinh(a)\mathrm{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\mathrm{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\mathrm{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+}{8x^2}$$

```
[Out] -(b*Cosh[a + b*x])/(8*x) - (3*b*Cosh[3*a + 3*b*x])/(8*x) + (b^2*CoshIntegra
l[b*x]*Sinh[a])/8 + (9*b^2*CoshIntegral[3*b*x]*Sinh[3*a])/8 - Sinh[a + b*x]
/(8*x^2) - Sinh[3*a + 3*b*x]/(8*x^2) + (b^2*Cosh[a]*SinhIntegral[b*x])/8 +
(9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rubi [A] time = 0.248809, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{8}b^2\sinh(a)\mathrm{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\mathrm{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\mathrm{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*Cosh[a + b*x])/(8*x) - (3*b*Cosh[3*a + 3*b*x])/(8*x) + (b^2*CoshIntegra
l[b*x]*Sinh[a])/8 + (9*b^2*CoshIntegral[3*b*x]*Sinh[3*a])/8 - Sinh[a + b*x]
/(8*x^2) - Sinh[3*a + 3*b*x]/(8*x^2) + (b^2*Cosh[a]*SinhIntegral[b*x])/8 +
(9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^3}+\frac{\sinh(3a+3bx)}{4x^3}\right)dx\\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^3}\,dx\\
&= -\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}b\int \frac{\cosh(a+bx)}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\cosh(3a+3}{x^2}\\
&= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}b^2\int\\
&= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}\left(b^2\,\text{co}\right.\\
&= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}+\frac{1}{8}b^2\textbf{Chi}(bx)\sinh(a)+\frac{9}{8}b^2\textbf{Chi}(3bx)\sinh(\end{aligned}$$

Mathematica [A] time = 0.283141, size = 105, normalized size = 0.88

$$-\frac{-b^2x^2\sinh(a)\textbf{Chi}(bx)-9b^2x^2\sinh(3a)\textbf{Chi}(3bx)-b^2x^2\cosh(a)\textbf{Shi}(bx)-9b^2x^2\cosh(3a)\textbf{Shi}(3bx)+\sinh(a+bx)+\text{s}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^2*x^2*CoshIntegral[b*x]*S
inh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*a] + Sinh[a + b*x] + Sinh[3*(
a + b*x)] - b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*SinhInt
egral[3*b*x])/(8*x^2)
```

Maple [A] time = 0.069, size = 169, normalized size = 1.4

$$-\frac{3\,b\mathrm{e}^{-3\,bx-3\,a}}{16\,x}+\frac{\mathrm{e}^{-3\,bx-3\,a}}{16\,x^{2}}+\frac{9\,b^{2}\mathrm{e}^{-3\,a}\mathrm{Ei}\left(1,3\,bx\right)}{16}-\frac{b\mathrm{e}^{-bx-a}}{16\,x}+\frac{\mathrm{e}^{-bx-a}}{16\,x^{2}}+\frac{b^{2}\mathrm{e}^{-a}\mathrm{Ei}\left(1,bx\right)}{16}-\frac{\mathrm{e}^{bx+a}}{16\,x^{2}}-\frac{b\mathrm{e}^{bx+a}}{16\,x}-\frac{b^{2}\mathrm{e}^{a}\mathrm{Ei}\left(1,-bx\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x)
```

```
[Out] -3/16*b*exp(-3*b*x-3*a)/x+1/16*exp(-3*b*x-3*a)/x^2+9/16*b^2*exp(-3*a)*Ei(1,
3*b*x)-1/16*b*exp(-b*x-a)/x+1/16*exp(-b*x-a)/x^2+1/16*b^2*exp(-a)*Ei(1,b*x)
-1/16/x^2*exp(b*x+a)-1/16*b/x*exp(b*x+a)-1/16*b^2*exp(a)*Ei(1,-b*x)-1/16/x^
2*exp(3*b*x+3*a)-3/16*b/x*exp(3*b*x+3*a)-9/16*b^2*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.25599, size = 78, normalized size = 0.66

$$\frac{9}{8}\,b^{2}e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)+\frac{1}{8}\,b^{2}e^{(-a)}\Gamma\left(-2,bx\right)-\frac{1}{8}\,b^{2}e^{a}\Gamma\left(-2,-bx\right)-\frac{9}{8}\,b^{2}e^{(3\,a)}\Gamma\left(-2,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) - 1/8*b^2
*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

Fricas [A] time = 1.83628, size = 489, normalized size = 4.11

$$-\frac{6\,bx\cosh\left(bx+a\right)^{3}+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\,bx\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)^{3}-9\left(b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)-b^{2}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*b*x
*cosh(b*x + a) + 2*sinh(b*x + a)^3 - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b
*x))*cosh(3*a) - (b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(3*cosh(b
*x + a)^2 + 1)*sinh(b*x + a) - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*s
inh(3*a) - (b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**3, x)
```

Giac [A] time = 1.15733, size = 211, normalized size = 1.77

$$\frac{9\,b^2x^2\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} - b^2x^2\mathrm{Ei}\left(-bx\right)e^{(-a)} - 9\,b^2x^2\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} + b^2x^2\mathrm{Ei}\left(bx\right)e^{a} - 3\,bxe^{(3\,bx+3\,a)} - bxe^{(bx+a)} - bxe^{(-bx-a)}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) - 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) + b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) - b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - e^(b*
x + a) + e^(-b*x - a) + e^(-3*b*x - 3*a))/x^2
```

3.267 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$\frac{1}{24}b^3\cosh(a)\mathbf{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\mathbf{Chi}(3bx)+\frac{1}{24}b^3\sinh(a)\mathbf{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\mathbf{Shi}(3bx)-\frac{b^2\sinh(a+bx)}{24x}-\frac{3b^2}{}$$

```
[Out] -(b*Cosh[a + b*x])/(24*x^2) - (b*Cosh[3*a + 3*b*x])/(8*x^2) + (b^3*Cosh[a]*
CoshIntegral[b*x])/24 + (9*b^3*Cosh[3*a]*CoshIntegral[3*b*x])/8 - Sinh[a +
b*x]/(12*x^3) - (b^2*Sinh[a + b*x])/(24*x) - Sinh[3*a + 3*b*x]/(12*x^3) - (
3*b^2*Sinh[3*a + 3*b*x])/(8*x) + (b^3*Sinh[a]*SinhIntegral[b*x])/24 + (9*b^
3*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rubi [A] time = 0.289493, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{24}b^3\cosh(a)\mathbf{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\mathbf{Chi}(3bx)+\frac{1}{24}b^3\sinh(a)\mathbf{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\mathbf{Shi}(3bx)-\frac{b^2\sinh(a+bx)}{24x}-\frac{3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```


```
[Out] -(b*Cosh[a + b*x])/(24*x^2) - (b*Cosh[3*a + 3*b*x])/(8*x^2) + (b^3*Cosh[a]*
CoshIntegral[b*x])/24 + (9*b^3*Cosh[3*a]*CoshIntegral[3*b*x])/8 - Sinh[a +
b*x]/(12*x^3) - (b^2*Sinh[a + b*x])/(24*x) - Sinh[3*a + 3*b*x]/(12*x^3) - (
3*b^2*Sinh[3*a + 3*b*x])/(8*x) + (b^3*Sinh[a]*SinhIntegral[b*x])/24 + (9*b^
3*Sinh[3*a]*SinhIntegral[3*b*x])/8
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^4}+\frac{\sinh(3a+3bx)}{4x^4}\right)dx \\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^4}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^4}\,dx \\
&= -\frac{\sinh(a+bx)}{12x^3}-\frac{\sinh(3a+3bx)}{12x^3}+\frac{1}{12}b\int \frac{\cosh(a+bx)}{x^3}\,dx+\frac{1}{4}b\int \frac{\cosh(3a+3b
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{\sinh(3a+3bx)}{12x^3}+\frac{1}{24}b^2\int \\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\frac{\sinh(3a}{12x} \\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\frac{\sinh(3a}{12x} \\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}+\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3
\end{aligned}$$

Mathematica [A] time = 0.349048, size = 138, normalized size = 0.9

$$-\frac{-b^3x^3\cosh(a)\text{Chi}(bx)-27b^3x^3\cosh(3a)\text{Chi}(3bx)-b^3x^3\sinh(a)\text{Shi}(bx)-27b^3x^3\sinh(3a)\text{Shi}(3bx)+b^2x^2\sinh(a+}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^3*x^3*Cosh[a]*CoshIntegra
l[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b*x] + 2*Sinh[a + b*x] + b^2*x
^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] - b^3*
x^3*Sinh[a]*SinhIntegral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*x])/(
24*x^3)
```

Maple [A] time = 0.076, size = 234, normalized size = 1.5

$$\frac{3\,b^2\mathrm{e}^{-3\,bx-3\,a}}{16\,x}-\frac{b\mathrm{e}^{-3\,bx-3\,a}}{16\,x^2}+\frac{\mathrm{e}^{-3\,bx-3\,a}}{24\,x^3}-\frac{9\,b^3\mathrm{e}^{-3\,a}\mathrm{Ei}\,(1,3\,bx)}{16}+\frac{b^2\mathrm{e}^{-bx-a}}{48\,x}-\frac{b\mathrm{e}^{-bx-a}}{48\,x^2}+\frac{\mathrm{e}^{-bx-a}}{24\,x^3}-\frac{b^3\mathrm{e}^{-a}\mathrm{Ei}\,(1,bx)}{48}-\frac{\mathrm{e}^{bx+a}}{24\,x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x)
```

```
[Out] 3/16*b^2*exp(-3*b*x-3*a)/x-1/16*b*exp(-3*b*x-3*a)/x^2+1/24*exp(-3*b*x-3*a)/
x^3-9/16*b^3*exp(-3*a)*Ei(1,3*b*x)+1/48*b^2*exp(-b*x-a)/x-1/48*b*exp(-b*x-a
)/x^2+1/24*exp(-b*x-a)/x^3-1/48*b^3*exp(-a)*Ei(1,b*x)-1/24/x^3*exp(b*x+a)-1
/48*b/x^2*exp(b*x+a)-1/48*b^2/x*exp(b*x+a)-1/48*b^3*exp(a)*Ei(1,-b*x)-1/24/
x^3*exp(3*b*x+3*a)-1/16*b/x^2*exp(3*b*x+3*a)-3/16*b^2/x*exp(3*b*x+3*a)-9/16
*b^3*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.26281, size = 78, normalized size = 0.51

$$\frac{27}{8}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{8}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)+\frac{1}{8}\,b^3e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{8}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) + 1/8*b^
3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

Fricas [A] time = 1.75587, size = 548, normalized size = 3.56

$$-\frac{6\,bx\cosh\left(bx+a\right)^{3}+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\left(9\,b^{2}x^{2}+2\right)\sinh\left(bx+a\right)^{3}+2\,bx\cosh\left(bx+a\right)-27\left(b^{3}x^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(9*
b^2*x^2 + 2)*sinh(b*x + a)^3 + 2*b*x*cosh(b*x + a) - 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) - (b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*cosh
(a) + 2*(b^2*x^2 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a) - (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**4, x)
```

Giac [A] time = 1.16996, size = 301, normalized size = 1.95

$$\frac{27\,b^{3}x^{3}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+b^{3}x^{3}\mathrm{Ei}\left(-bx\right)e^{(-a)}+27\,b^{3}x^{3}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+b^{3}x^{3}\mathrm{Ei}\left(bx\right)e^{a}-9\,b^{2}x^{2}e^{(3\,bx+3\,a)}-b^{2}x^{2}e^{(bx+a)}+b^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) + 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) + b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) - b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) + 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
```

```
*e^(3*b*x + 3*a) - b*x*e^(b*x + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) - 2*e^(b*x + a) + 2*e^(-b*x - a) + 2*e^(-3*b*x - 3*
a))/x^3
```

3.268 $\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}+\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamm}}{b}$$

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) + (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rubi [A] time = 0.237333, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}+\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamm}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```



```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) + (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx &= \int \left(\frac{1}{4}x^m \sinh(2a+2bx)+\frac{1}{8}x^m \sinh(4a+4bx)\right)dx\\ &= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx+\frac{1}{4}\int x^m \sinh(2a+2bx)\,dx\\ &= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx-\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx-\frac{1}{8}\int e^{i(2i\cdots\\ &= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}+\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-4-m}\cdots}{}\end{aligned}$$

Mathematica [A] time = 0.125787, size = 110, normalized size = 0.79

$$\frac{e^{-4a}4^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(e^{2a}2^{m+2}\text{Gamma}(m+1,2bx)+\text{Gamma}(m+1,4bx)\right)+e^{8a}(bx)^m\text{Gamma}(m+1,-4bx\cdots}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] + 2^(2 + m)*E^(6*a)*(
b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(2^(2 + m)*E^(2*a)*Gamma[1 + m, 2*
b*x] + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x^m\left(\cosh\left(bx+a\right)\right)^3\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

Maxima [A] time = 1.26874, size = 158, normalized size = 1.14

$$\frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma\left(m+1,4\,bx\right)+\frac{1}{8}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma\left(m+1,2\,bx\right)-\frac{1}{8}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma\left(m+1,-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) + 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

Fricas [A] time = 1.9086, size = 518, normalized size = 3.73

$$\frac{\cosh\left(m\log\left(4\,b\right)+4\,a\right)\Gamma\left(m+1,4\,bx\right)+4\,\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)+4\,\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) + 4*cosh(m*log(2*b) + 2*a)
*gamma(m + 1, 2*b*x) + 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cos
h(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(
4*b) + 4*a) - 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - 4*gamma(m + 1,
 -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) -
4*a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx + a)^3 \sinh(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a), x)
```

3.269 $\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{3x^2\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{9x\cosh^2(a+bx)}{32b^3}-\frac{3\sinh(a+}{}$$

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (9*x*Cosh[a + b*x]^2)/(32*b^3) + (3*x*
Cosh[a + b*x]^4)/(32*b^3) + (x^3*Cosh[a + b*x]^4)/(4*b) - (45*Cosh[a + b*x]
*Sinh[a + b*x])/(256*b^4) - (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) -
(3*Cosh[a + b*x]^3*Sinh[a + b*x])/(128*b^4) - (3*x^2*Cosh[a + b*x]^3*Sinh[a
 + b*x])/(16*b^2)
```

Rubi [A] time = 0.141297, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5373, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{9x\cosh^2(a+bx)}{32b^3}-\frac{3\sinh(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```


```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (9*x*Cosh[a + b*x]^2)/(32*b^3) + (3*x*
Cosh[a + b*x]^4)/(32*b^3) + (x^3*Cosh[a + b*x]^4)/(4*b) - (45*Cosh[a + b*x]
*Sinh[a + b*x])/(256*b^4) - (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) -
(3*Cosh[a + b*x]^3*Sinh[a + b*x])/(128*b^4) - (3*x^2*Cosh[a + b*x]^3*Sinh[a
 + b*x])/(16*b^2)
```


Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{x^3\cosh^4(a+bx)}{4b} - \frac{3\int x^2\cosh^4(a+bx)\,dx}{4b}$$
$$= \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\int \cos$$
$$= \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{9x^2\cosh(a+bx)\sinh}{32b^2}$$
$$= -\frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45\cosh(a+}{25}$$
$$= -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45\,c}{}$$

Mathematica [A] time = 0.599431, size = 91, normalized size = 0.59

$$\frac{32bx\left(2b^2x^2+3\right)\cosh(2(a+bx))+2bx\left(8b^2x^2+3\right)\cosh(4(a+bx))-3\sinh(2(a+bx))\left(\left(8b^2x^2+1\right)\cosh(2(a+bx))+3\right.}{512b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (32*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + 2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a
 + b*x)] - 3*(16 + 32*b^2*x^2 + (1 + 8*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[2*(
a + b*x)])/(512*b^4)
```

Maple [B] time = 0.009, size = 414, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b*x+a)^3*cosh(b*x+a)^
2-3/16*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3-9/32*(b*x+a)^2*cosh(b*x+a)*sinh(
b*x+a)-3/32*(b*x+a)^3+3/32*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-3/128*cosh(b
*x+a)^3*sinh(b*x+a)-45/256*cosh(b*x+a)*sinh(b*x+a)-45/256*b*x-45/256*a+3/8*
(b*x+a)*cosh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b
*x+a)^2*cosh(b*x+a)^2-1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-3/16*(b*x+a)*co
sh(b*x+a)*sinh(b*x+a)-3/32*(b*x+a)^2+1/32*cosh(b*x+a)^2*sinh(b*x+a)^2+1/8*c
osh(b*x+a)^2)+3*a^2*(1/4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b*x+a)*co
sh(b*x+a)^2-1/16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(b*x+a)*sinh(b*x+a)-3/3
2*b*x-3/32*a)-a^3*(1/4*cosh(b*x+a)^2*sinh(b*x+a)^2+1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.082, size = 231, normalized size = 1.49

$$\frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} + \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4} + \frac{\left(4\,b^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 + 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

Fricas [A] time = 1.77661, size = 468, normalized size = 3.02

$$\frac{\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4+16\left(2\,b^3x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 + 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 + 2*(16*b^3*x^3 + 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] time = 8.53415, size = 226, normalized size = 1.46

$$\begin{cases} -\frac{3x^3\sinh^4(a+bx)}{32b}+\frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b}+\frac{5x^3\cosh^4(a+bx)}{32b}+\frac{9x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2}-\frac{15x^2\sinh(a+bx)\cosh^3(a+bx)}{32b^2}-\frac{45x\sinh^4}{256b} \\ \frac{x^4\sinh(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**3*cosh(a + b*x)**4/(32*b) + 9*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) - 15*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2
) - 45*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**
2/(128*b**3) + 51*x*cosh(a + b*x)**4/(256*b**3) + 45*sinh(a + b*x)**3*cosh(
a + b*x)/(256*b**4) - 51*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0
)), (x**4*sinh(a)*cosh(a)**3/4, True))
```

Giac [A] time = 1.15055, size = 196, normalized size = 1.26

$$\frac{\left(32\,b^3x^3-24\,b^2x^2+12\,bx-3\right)e^{(4\,bx+4\,a)}}{2048\,b^4}+\frac{\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{64\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{64\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 + 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.270 $\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{\cosh^4(a+bx)}{32b^3}+\frac{3\cosh^2(a+bx)}{32b^3}-\frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b^2}-\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\cosh^4(a+bx)}{4b}-$$

```
[Out] (-3*x^2)/(32*b) + (3*Cosh[a + b*x]^2)/(32*b^3) + Cosh[a + b*x]^4/(32*b^3) +
 (x^2*Cosh[a + b*x]^4)/(4*b) - (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) -
 (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(8*b^2)
```

Rubi [A] time = 0.0760245, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 3310, 30}

$$\frac{\cosh^4(a+bx)}{32b^3}+\frac{3\cosh^2(a+bx)}{32b^3}-\frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b^2}-\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\cosh^4(a+bx)}{4b}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x^2)/(32*b) + (3*Cosh[a + b*x]^2)/(32*b^3) + Cosh[a + b*x]^4/(32*b^3) +
 (x^2*Cosh[a + b*x]^4)/(4*b) - (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) -
 (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(8*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{x^2\cosh^4(a+bx)}{4b} - \frac{\int x\cosh^4(a+bx)\,dx}{2b}$$
$$= \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2} - \frac{3\int x\cosh^2}{8}$$
$$= \frac{3\cosh^2(a+bx)}{32b^3} + \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{3x\cosh(a+bx)\sinh(a-}{16b^2}$$
$$= -\frac{3x^2}{32b} + \frac{3\cosh^2(a+bx)}{32b^3} + \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{3x\cosh(a+bx)}{16b}$$

Mathematica [A] time = 0.231325, size = 70, normalized size = 0.69

$$\frac{16\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(8b^2x^2+1\right)\cosh(4(a+bx))-4bx(8\sinh(2(a+bx))+\sinh(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] -
 4*b*x*(8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)]))/(256*b^3)
```

Maple [B] time = 0.006, size = 237, normalized size = 2.4

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\sinh(bx+a))^2(\cosh(bx+a))^2}{4} + \frac{(bx+a)^2(\cosh(bx+a))^2}{4} - \frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^3}{8} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b*x+a)^2*cosh(b*x+a)^
2-1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-3/16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a
)-3/32*(b*x+a)^2+1/32*cosh(b*x+a)^2*sinh(b*x+a)^2+1/8*cosh(b*x+a)^2-2*a*(1/
```

```
4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b*x+a)*cosh(b*x+a)^2-1/16*cosh(b
*x+a)^3*sinh(b*x+a)-3/32*cosh(b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a)+a^2*(1/4*
cosh(b*x+a)^2*sinh(b*x+a)^2+1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.20604, size = 171, normalized size = 1.69

$$\frac{\left(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3} + \frac{\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3} + \frac{\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{\left(8\,b^2x^2 + 4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 + 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

Fricas [A] time = 1.75687, size = 394, normalized size = 3.9

$$-\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 - \left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4 - \left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4 - 16\left(2\,b^2x^2+1\right)\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
- 2*(16*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] time = 4.30484, size = 150, normalized size = 1.49

$$\begin{cases} -\frac{3x^2\sinh^4(a+bx)}{32b} + \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b} + \frac{5x^2\cosh^4(a+bx)}{32b} + \frac{3x\sinh^3(a+bx)\cosh(a+bx)}{16b^2} - \frac{5x\sinh(a+bx)\cosh^3(a+bx)}{16b^2} - \frac{3\sinh^4(a+bx}{64b^3} \\ \frac{x^3\sinh(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**2*cosh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**3*c
osh(a + b*x)/(16*b**2) - 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) - 3*s
inh(a + b*x)**4/(64*b**3) + 5*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*
sinh(a)*cosh(a)**3/3, True))
```

Giac [A] time = 1.17006, size = 153, normalized size = 1.51

$$\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}+\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{32\,b^3}+\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 + 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.271 $\int x \cosh^3(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=65

$$-\frac{\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3x}{32b}$$

```
[Out] (-3*x)/(32*b) + (x*Cosh[a + b*x]^4)/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])
/(32*b^2) - (Cosh[a + b*x]^3*Sinh[a + b*x])/(16*b^2)
```

Rubi [A] time = 0.042589, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5373, 2635, 8}

$$-\frac{\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3x}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x)/(32*b) + (x*Cosh[a + b*x]^4)/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])
/(32*b^2) - (Cosh[a + b*x]^3*Sinh[a + b*x])/(16*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{x\cosh^4(a+bx)}{4b} - \frac{\int \cosh^4(a+bx)\,dx}{4b}$$
$$= \frac{x\cosh^4(a+bx)}{4b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\int \cosh^2(a+bx)\,dx}{16b}$$
$$= \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - 3$$
$$= -\frac{3x}{32b} + \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sinh(a+}{16b^2}$$

Mathematica [A] time = 0.154184, size = 50, normalized size = 0.77

$$-\frac{8\sinh(2(a+bx)) + \sinh(4(a+bx)) - 16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -(-16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 8*Sinh[2*(a + b*x)]
 + Sinh[4*(a + b*x)])/(128*b^2)
```

Maple [A] time = 0.007, size = 113, normalized size = 1.7

$$\frac{1}{b^2}\left(\frac{(bx+a)(\sinh(bx+a))^2(\cosh(bx+a))^2}{4} + \frac{(bx+a)(\cosh(bx+a))^2}{4} - \frac{(\cosh(bx+a))^3\sinh(bx+a)}{16} - \frac{3\cosh(bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*(b*x+a)*cosh(b*x+a)^2-1/
16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-a
*(1/4*cosh(b*x+a)^2*sinh(b*x+a)^2+1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.1851, size = 123, normalized size = 1.89

$$\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Fricas [A] time = 1.80991, size = 289, normalized size = 4.45

$$\frac{bx\cosh\left(bx+a\right)^4+bx\sinh\left(bx+a\right)^4+4\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\left(3\,bx\cosh\left(bx+a\right)^2+2\,bx\right.}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 + 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] time = 2.23581, size = 110, normalized size = 1.69

$$\begin{cases} -\frac{3x\sinh^4(a+bx)}{32b}+\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b}+\frac{5x\cosh^4(a+bx)}{32b}+\frac{3\sinh^3(a+bx)\cosh(a+bx)}{32b^2}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{for } b\neq 0 \\ \frac{x^2\sinh(a)\cosh^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x
)**2/(16*b) + 5*x*cosh(a + b*x)**4/(32*b) + 3*sinh(a + b*x)**3*cosh(a + b*x
)/(32*b**2) - 5*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*
```

```
sinh(a)*cosh(a)**3/2, True))
```

Giac [A] time = 1.15961, size = 109, normalized size = 1.68

$$\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}+\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

3.272 $\int \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\cosh^4(a+bx)}{4b}$$

[Out] Cosh[a + b*x]^4/(4*b)

Rubi [A] time = 0.0202069, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x],x]

[Out] Cosh[a + b*x]^4/(4*b)

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0024895, size = 15, normalized size = 1.

$$\frac{\cosh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^4/(4*b)
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\frac{(\cosh(bx+a))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/4*cosh(b*x+a)^4/b
```

Maxima [A] time = 1.06118, size = 18, normalized size = 1.2

$$\frac{\cosh(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*cosh(b*x + a)^4/b
```

Fricas [B] time = 1.66286, size = 146, normalized size = 9.73

$$\frac{\cosh(bx+a)^4 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 + 2\right)\sinh(bx+a)^2 + 4\cosh(bx+a)^2}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 2)*sinh(b*
x + a)^2 + 4*cosh(b*x + a)^2)/b
```

Sympy [A] time = 1.05049, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cosh^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x \sinh(a) \cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**3, True))
```

Giac [B] time = 1.14367, size = 66, normalized size = 4.4

$$\frac{\left(e^{(2bx+2a)} + e^{(-2bx-2a)}\right)^2 + 4e^{(2bx+2a)} + 4e^{(-2bx-2a)}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/64*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 4*e^(2*b*x + 2*a) + 4*e^(-2*
b*x - 2*a))/b
```

3.273 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 + (Co
sh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rubi [A] time = 0.14056, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 + (Co
sh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx &= \int \left(\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= \frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx+\frac{1}{4}\sinh(2a)\int \frac{\cosh(2b}{x}\\ &= \frac{1}{4}\mathbf{Chi}(2bx)\sinh(2a)+\frac{1}{8}\mathbf{Chi}(4bx)\sinh(4a)+\frac{1}{4}\cosh(2a)\mathbf{Shi}(2bx)+\frac{1}{8}\cosh(4a)\mathbf{Shi}\end{aligned}$$

Mathematica [A] time = 0.0815933, size = 47, normalized size = 0.89

$$\frac{1}{8}(2\sinh(2a)\mathbf{Chi}(2bx)+\sinh(4a)\mathbf{Chi}(4bx)+2\cosh(2a)\mathbf{Shi}(2bx)+\cosh(4a)\mathbf{Shi}(4bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

```
[Out] (2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] + 2*Cosh[2
*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Maple [A] time = 0.06, size = 50, normalized size = 0.9

$$\frac{e^{-4a}\mathrm{Ei}(1,4bx)}{16}+\frac{e^{-2a}\mathrm{Ei}(1,2bx)}{8}-\frac{e^{4a}\mathrm{Ei}(1,-4bx)}{16}-\frac{e^{2a}\mathrm{Ei}(1,-2bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x,x)
```

```
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*a)*Ei(1,2*b*x)-1/16*exp(4*a)*Ei(1,-4*
b*x)-1/8*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.2708, size = 61, normalized size = 1.15

$$\frac{1}{16}\,\text{Ei}\left(4\,bx\right)e^{\left(4\,a\right)} + \frac{1}{8}\,\text{Ei}\left(2\,bx\right)e^{\left(2\,a\right)} - \frac{1}{8}\,\text{Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)} - \frac{1}{16}\,\text{Ei}\left(-4\,bx\right)e^{\left(-4\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Fricas [A] time = 1.71797, size = 223, normalized size = 4.21

$$\frac{1}{16}\left(\text{Ei}\left(4\,bx\right) - \text{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right) + \frac{1}{8}\left(\text{Ei}\left(2\,bx\right) - \text{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right) + \frac{1}{16}\left(\text{Ei}\left(4\,bx\right) + \text{Ei}\left(-4\,bx\right)\right)\sinh\left(4\,a\right) + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) + 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) + 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + bx\right)\cosh^{3}\left(a + bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x, x)
```

Giac [A] time = 1.14895, size = 61, normalized size = 1.15

$$\frac{1}{16}\,\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}+\frac{1}{8}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

3.274 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\text{sinh}$$

```
[Out] (b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2 -
 Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) + (b*Sinh[2*a]*SinhInteg
ral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rubi [A] time = 0.189175, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\text{sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```


```
[Out] (b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2 -
 Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) + (b*Sinh[2*a]*SinhInteg
ral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx &= \int \left(\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx \\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= -\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}+\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cosh(4a+4
\end{aligned}$$

$$= -\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}+\frac{1}{2}(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{2}(b\cosh(4a$$

$$= \frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}$$

Mathematica [A] time = 0.204442, size = 80, normalized size = 0.9

$$\frac{4bx\cosh(2a)\text{Chi}(2bx)+4bx\cosh(4a)\text{Chi}(4bx)+4bx\sinh(2a)\text{Shi}(2bx)+4bx\sinh(4a)\text{Shi}(4bx)-2\sinh(2(a+bx))-\text{s}}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

```
[Out] (4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] + 4*b*x*Cosh[4*a]*CoshIntegral[4*b*x]
- 2*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegral[2*
```

```
b*x] + 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/(8*x)
```

Maple [A] time = 0.069, size = 110, normalized size = 1.2

$$\frac{e^{-4bx-4a}}{16x} - \frac{be^{-4a}\text{Ei}(1,4bx)}{4} + \frac{e^{-2bx-2a}}{8x} - \frac{be^{-2a}\text{Ei}(1,2bx)}{4} - \frac{e^{4bx+4a}}{16x} - \frac{be^{4a}\text{Ei}(1,-4bx)}{4} - \frac{e^{2bx+2a}}{8x} - \frac{be^{2a}\text{Ei}(1,-2b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/16*exp(-4*b*x-4*a)/x-1/4*b*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*b*x-2*a)/x-1/
4*b*exp(-2*a)*Ei(1,2*b*x)-1/16/x*exp(4*b*x+4*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
-1/8*exp(2*b*x+2*a)/x-1/4*b*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.33478, size = 72, normalized size = 0.81

$$\frac{1}{4}\,be^{(-4\,a)}\Gamma\left(-1,4\,bx\right) + \frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right) + \frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right) + \frac{1}{4}\,be^{(4\,a)}\Gamma\left(-1,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

Fricas [A] time = 1.78677, size = 370, normalized size = 4.16

$$-\frac{2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3} - \left(bx\text{Ei}\left(4\,bx\right) + bx\text{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right) - \left(bx\text{Ei}\left(2\,bx\right) + bx\text{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right) + 2\,\big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) - (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 +
```

```
 cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
- (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**2, x)
```

Giac [A] time = 1.20084, size = 135, normalized size = 1.52

$$\frac{4\,bx\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} + 4\,bx\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} + 4\,bx\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} + 4\,bx\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)} - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 2\,e^{(-2\,bx-2\,a}}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) + 4*b*x*Ei(2*b*x)*e^(2*a) + 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a)
+ 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

3.275 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{1}{2}b^2\sinh(2a)\mathbf{Chi}(2bx)+b^2\sinh(4a)\mathbf{Chi}(4bx)+\frac{1}{2}b^2\cosh(2a)\mathbf{Shi}(2bx)+b^2\cosh(4a)\mathbf{Shi}(4bx)-\frac{\sinh(2a+2bx)}{8x^2}-\mathrm{sin}$$

```
[Out] -(b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) + (b^2*CoshInteg
ral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] - Sinh[2*a + 2*
b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rubi [A] time = 0.247623, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b^2\sinh(2a)\mathbf{Chi}(2bx)+b^2\sinh(4a)\mathbf{Chi}(4bx)+\frac{1}{2}b^2\cosh(2a)\mathbf{Shi}(2bx)+b^2\cosh(4a)\mathbf{Shi}(4bx)-\frac{\sinh(2a+2bx)}{8x^2}-\mathrm{sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) + (b^2*CoshInteg
ral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] - Sinh[2*a + 2*
b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \left(\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx$$
$$= -\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}+\frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx+\frac{1}{4}b\int \frac{\cosh(4a+4}{x^2}$$
$$= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}+\frac{1}{2}b^2\int$$
$$= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}+\frac{1}{2}\left(b^2\right.$$
$$= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}+\frac{1}{2}b^2\mathrm{Chi}(2bx)\sinh(2a)+b^2\mathrm{Chi}(4bx)\sinh($$

Mathematica [A] time = 0.589403, size = 112, normalized size = 0.9

$$b^2\sinh(4a)\mathrm{Chi}(4bx)+b^2\sinh(a)\cosh(a)\mathrm{Chi}(2bx)+\frac{1}{2}b^2\cosh(2a)\mathrm{Shi}(2bx)+b^2\cosh(4a)\mathrm{Shi}(4bx)-\frac{\sinh(2(a+bx))+}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + b^2*CoshIntegral[4*b*x]*Sinh[4*a]
 - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4*(a
 + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b*x]
)/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Maple [A] time = 0.069, size = 178, normalized size = 1.4

$$-\frac{b\mathrm{e}^{-4\,bx-4\,a}}{8\,x}+\frac{\mathrm{e}^{-4\,bx-4\,a}}{32\,x^{2}}+\frac{b^{2}\mathrm{e}^{-4\,a}\mathrm{Ei}\left(1,4\,bx\right)}{2}-\frac{b\mathrm{e}^{-2\,bx-2\,a}}{8\,x}+\frac{\mathrm{e}^{-2\,bx-2\,a}}{16\,x^{2}}+\frac{b^{2}\mathrm{e}^{-2\,a}\mathrm{Ei}\left(1,2\,bx\right)}{4}-\frac{\mathrm{e}^{4\,bx+4\,a}}{32\,x^{2}}-\frac{b\mathrm{e}^{4\,bx+4\,a}}{8\,x}-\frac{b^{2}\mathrm{e}^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x)
```

```
[Out] -1/8*b*exp(-4*b*x-4*a)/x+1/32*exp(-4*b*x-4*a)/x^2+1/2*b^2*exp(-4*a)*Ei(1,4*
b*x)-1/8*b*exp(-2*b*x-2*a)/x+1/16*exp(-2*b*x-2*a)/x^2+1/4*b^2*exp(-2*a)*Ei(
1,2*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a)*Ei
(1,-4*b*x)-1/16*exp(2*b*x+2*a)/x^2-1/8*b*exp(2*b*x+2*a)/x-1/4*b^2*exp(2*a)*
Ei(1,-2*b*x)
```

Maxima [A] time = 1.38813, size = 81, normalized size = 0.65

$$b^{2}e^{(-4\,a)}\Gamma\left(-2,4\,bx\right)+\frac{1}{2}\,b^{2}e^{(-2\,a)}\Gamma\left(-2,2\,bx\right)-\frac{1}{2}\,b^{2}e^{(2\,a)}\Gamma\left(-2,-2\,bx\right)-b^{2}e^{(4\,a)}\Gamma\left(-2,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) + 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) - 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

Fricas [A] time = 1.87652, size = 570, normalized size = 4.56

$$-\frac{bx\cosh\left(bx+a\right)^{4}+bx\sinh\left(bx+a\right)^{4}+bx\cosh\left(bx+a\right)^{2}+\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\left(6\,bx\cosh\left(bx+a\right)^{2}+bx\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + b*x*cosh(b*x + a)^2 + cos
h(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2
- 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) - (b^2*x^2*Ei(2*b*x)
 - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) - (b^2*x^2*E
i(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**3, x)
```

Giac [A] time = 1.18339, size = 227, normalized size = 1.82

$$\frac{16\,b^2x^2\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}+8\,b^2x^2\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-8\,b^2x^2\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-16\,b^2x^2\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-4\,bxe^{(4\,bx+4\,a)}-4\,bxe^{(2\,bx+}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 8*b^2*x^2*Ei(2*b*x)*e^(2*a) - 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) - 4*b*x*e^(2*b*x + 2*a) - 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

3.276 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$\frac{1}{3}b^3\cosh(2a)\mathbf{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\mathbf{Chi}(4bx)+\frac{1}{3}b^3\sinh(2a)\mathbf{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\mathbf{Shi}(4bx)-\frac{b^2\sinh(2a+2bx)}{6x}$$

```
[Out] -(b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) + (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 - S
inh[2*a + 2*b*x]/(12*x^3) - (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*
x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) + (b^3*Sinh[2*a]*SinhIntegral[2
*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rubi [A] time = 0.29719, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{3}b^3\cosh(2a)\mathbf{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\mathbf{Chi}(4bx)+\frac{1}{3}b^3\sinh(2a)\mathbf{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\mathbf{Shi}(4bx)-\frac{b^2\sinh(2a+2bx)}{6x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) + (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 - S
inh[2*a + 2*b*x]/(12*x^3) - (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*
x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) + (b^3*Sinh[2*a]*SinhIntegral[2
*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \left(\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx + \frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx$$
$$= -\frac{\sinh(2a+2bx)}{12x^3} - \frac{\sinh(4a+4bx)}{24x^3} + \frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx + \frac{1}{6}b\int \frac{\cosh(4a+4}{x^3}$$
$$= -\frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} - \frac{\sinh(2a+2bx)}{12x^3} - \frac{\sinh(4a+4bx)}{24x^3} + \frac{1}{6}b^2\int$$
$$= -\frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} - \frac{\sinh(2a+2bx)}{12x^3} - \frac{b^2\sinh(2a+2bx)}{6x} - \text{sin}$$
$$= -\frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} - \frac{\sinh(2a+2bx)}{12x^3} - \frac{b^2\sinh(2a+2bx)}{6x} - \text{sin}$$
$$= -\frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} + \frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx) + \frac{4}{3}b^3\cosh(4a)\text{Ch}$$

Mathematica [A] time = 0.534986, size = 150, normalized size = 0.89

$$-\frac{-8b^3x^3\cosh(2a)\text{Chi}(2bx) - 32b^3x^3\cosh(4a)\text{Chi}(4bx) - 8b^3x^3\sinh(2a)\text{Shi}(2bx) - 32b^3x^3\sinh(4a)\text{Shi}(4bx) + 4b^2x^2\text{si}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] - 8*b^3*x^3*Cosh[2*a]*C
oshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] + 2*Sinh[2*(a
 + b*x)] + 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2*Sinh
[4*(a + b*x)] - 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*Sinh[4
*a]*SinhIntegral[4*b*x])/(24*x^3)
```

Maple [A] time = 0.072, size = 246, normalized size = 1.5

$$\frac{b^2 e^{-4bx-4a}}{6x} - \frac{b e^{-4bx-4a}}{24x^2} + \frac{e^{-4bx-4a}}{48x^3} - \frac{2b^3 e^{-4a} \mathrm{Ei}(1,4bx)}{3} + \frac{b^2 e^{-2bx-2a}}{12x} - \frac{b e^{-2bx-2a}}{24x^2} + \frac{e^{-2bx-2a}}{24x^3} - \frac{b^3 e^{-2a} \mathrm{Ei}(1,2bx)}{6} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x)
```

```
[Out] 1/6*b^2*exp(-4*b*x-4*a)/x-1/24*b*exp(-4*b*x-4*a)/x^2+1/48*exp(-4*b*x-4*a)/x
^3-2/3*b^3*exp(-4*a)*Ei(1,4*b*x)+1/12*b^2*exp(-2*b*x-2*a)/x-1/24*b*exp(-2*b
*x-2*a)/x^2+1/24*exp(-2*b*x-2*a)/x^3-1/6*b^3*exp(-2*a)*Ei(1,2*b*x)-1/48/x^3
*exp(4*b*x+4*a)-1/24*b/x^2*exp(4*b*x+4*a)-1/6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*
exp(4*a)*Ei(1,-4*b*x)-1/24*exp(2*b*x+2*a)/x^3-1/24*b*exp(2*b*x+2*a)/x^2-1/1
2*b^2*exp(2*b*x+2*a)/x-1/6*b^3*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.38147, size = 80, normalized size = 0.47

$$4\,b^3 e^{(-4\,a)}\Gamma\left(-3, 4\,bx\right) + b^3 e^{(-2\,a)}\Gamma\left(-3, 2\,bx\right) + b^3 e^{(2\,a)}\Gamma\left(-3, -2\,bx\right) + 4\,b^3 e^{(4\,a)}\Gamma\left(-3, -4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + b^3*e^(-2*a)*gamma(-3, 2*b*x) + b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

Fricas [A] time = 1.85127, size = 647, normalized size = 3.83

$$-\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 + 2\left(8b^2x^2+1\right)\cosh(bx+a)\sinh(bx+a)^3 + bx\cosh(bx+a)^2 + \left(6bx\cosh(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 + b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 + b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) -
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 + (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**4, x)
```

Giac [A] time = 1.15062, size = 319, normalized size = 1.89

$$\frac{32b^3x^3\mathrm{Ei}(4bx)e^{(4a)} + 8b^3x^3\mathrm{Ei}(2bx)e^{(2a)} + 8b^3x^3\mathrm{Ei}(-2bx)e^{(-2a)} + 32b^3x^3\mathrm{Ei}(-4bx)e^{(-4a)} - 8b^2x^2e^{(4bx+4a)} - 4b^2x^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) + 8*b^3*x^3*Ei(2*b*x)*e^(2*a) + 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
```

```
 + 4*a) - 4*b^2*x^2*e^(2*b*x + 2*a) + 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a) - 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x +
2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

3.277 $\int \frac{\cosh(x)\sinh(x)}{x}\,dx$

Optimal. Leaf size=8

$$\frac{\mathrm{Shi}(2x)}{2}$$

[Out] SinhIntegral[2*x]/2

Rubi [A] time = 0.0301795, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3298}

$$\frac{\mathrm{Shi}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[x]*Sinh[x])/x,x]

[Out] SinhIntegral[2*x]/2

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\sinh(2x)}{2x}\,dx$$
$$= \frac{1}{2}\int \frac{\sinh(2x)}{x}\,dx$$
$$= \frac{\text{Shi}(2x)}{2}$$

Mathematica [A] time = 0.0057429, size = 8, normalized size = 1.

$$\frac{\text{Shi}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x,x]
```

```
[Out] SinhIntegral[2*x]/2
```

Maple [A] time = 0.017, size = 7, normalized size = 0.9

$$\frac{\text{Shi}\,(2\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x,x)
```

```
[Out] 1/2*Shi(2*x)
```

Maxima [B] time = 1.28468, size = 18, normalized size = 2.25

$$\frac{1}{4}\,\text{Ei}\,(2\,x) - \frac{1}{4}\,\text{Ei}\,(-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

Fricas [B] time = 1.97821, size = 38, normalized size = 4.75

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,x\right)-\frac{1}{4}\,\mathrm{Ei}\left(-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="fricas")
```

```
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(x\right)\cosh\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x, x)
```

Giac [B] time = 1.14652, size = 18, normalized size = 2.25

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,x\right)-\frac{1}{4}\,\mathrm{Ei}\left(-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="giac")
```

```
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

3.278 $\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx$

Optimal. Leaf size=16

$$\mathrm{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

```
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

Rubi [A] time = 0.0474439, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5448, 12, 3297, 3301}

$$\mathrm{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/x^2,x]
```

```
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx &= \int \frac{\sinh(2x)}{2x^2}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2x)}{x^2}\,dx\\ &= -\frac{\sinh(2x)}{2x}+\int \frac{\cosh(2x)}{x}\,dx\\ &= \text{Chi}(2x)-\frac{\sinh(2x)}{2x}\end{aligned}$$

Mathematica [A] time = 0.0061389, size = 16, normalized size = 1.

$$\text{Chi}(2x)-\frac{\sinh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^2,x]
```

```
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

Maple [A] time = 0.007, size = 15, normalized size = 0.9

$$\text{Chi}\,(2\,x)-\frac{\sinh\,(2\,x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x^2,x)
```

```
[Out] Chi(2*x)-1/2*sinh(2*x)/x
```

Maxima [A] time = 1.23678, size = 20, normalized size = 1.25

$$\frac{1}{2}\,\Gamma\left(-1,2\,x\right)+\frac{1}{2}\,\Gamma\left(-1,-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, 2*x) + 1/2*gamma(-1, -2*x)
```

Fricas [A] time = 2.02682, size = 70, normalized size = 4.38

$$\frac{x\mathrm{Ei}\left(2\,x\right)+x\mathrm{Ei}\left(-2\,x\right)-2\,\cosh\left(x\right)\sinh\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*Ei(2*x) + x*Ei(-2*x) - 2*cosh(x)*sinh(x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(x\right)\cosh\left(x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x**2,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x**2, x)
```

Giac [B] time = 1.12932, size = 41, normalized size = 2.56

$$\frac{2\,x\mathrm{Ei}\left(2\,x\right)+2\,x\mathrm{Ei}\left(-2\,x\right)-e^{(2\,x)}+e^{(-2\,x)}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x*Ei(2*x) + 2*x*Ei(-2*x) - e^(2*x) + e^(-2*x))/x
```

3.279 $\int \frac{\cosh(x)\sinh(x)}{x^3} dx$

Optimal. Leaf size=27

$$\text{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

```
[Out] -Cosh[2*x]/(2*x) - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Rubi [A] time = 0.0629465, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5448, 12, 3297, 3298}

$$\text{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/x^3,x]
```

```
[Out] -Cosh[2*x]/(2*x) - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx &= \int \frac{\sinh(2x)}{2x^3}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2x)}{x^3}\,dx\\ &= -\frac{\sinh(2x)}{4x^2}+\frac{1}{2}\int \frac{\cosh(2x)}{x^2}\,dx\\ &= -\frac{\cosh(2x)}{2x}-\frac{\sinh(2x)}{4x^2}+\int \frac{\sinh(2x)}{x}\,dx\\ &= -\frac{\cosh(2x)}{2x}-\frac{\sinh(2x)}{4x^2}+\mathrm{Shi}(2x)\end{aligned}$$

Mathematica [A] time = 0.007349, size = 27, normalized size = 1.

$$\mathrm{Shi}(2x)-\frac{\sinh(2x)}{4x^2}-\frac{\cosh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^3,x]
```

```
[Out] -Cosh[2*x]/(2*x) - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Maple [A] time = 0.007, size = 24, normalized size = 0.9

$$-\frac{\cosh(2x)}{2x}+\mathrm{Shi}(2x)-\frac{\sinh(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x^3,x)
```

```
[Out] -1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

Maxima [A] time = 1.20613, size = 18, normalized size = 0.67

$$\Gamma\left(-2, 2\,x\right) - \Gamma\left(-2, -2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="maxima")
```

```
[Out] gamma(-2, 2*x) - gamma(-2, -2*x)
```

Fricas [A] time = 2.00945, size = 113, normalized size = 4.19

$$\frac{x^2 \mathrm{Ei}\left(2\,x\right) - x^2 \mathrm{Ei}\left(-2\,x\right) - x\cosh\left(x\right)^2 - x\sinh\left(x\right)^2 - \cosh\left(x\right)\sinh\left(x\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*Ei(2*x) - x^2*Ei(-2*x) - x*cosh(x)^2 - x*sinh(x)^2 - cosh(x)*sinh(
x))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)\cosh\left(x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x**3,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x**3, x)
```

Giac [B] time = 1.18049, size = 65, normalized size = 2.41

$$\frac{4\,x^2 \mathrm{Ei}\left(2\,x\right) - 4\,x^2 \mathrm{Ei}\left(-2\,x\right) - 2\,xe^{(2\,x)} - 2\,xe^{(-2\,x)} - e^{(2\,x)} + e^{(-2\,x)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*x^2*Ei(2*x) - 4*x^2*Ei(-2*x) - 2*x*e^(2*x) - 2*x*e^(-2*x) - e^(2*x)
+ e^(-2*x))/x^2
```

3.280 $\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}-$$

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) - (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rubi [A] time = 0.183457, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3307, 2181}

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}-$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) - (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{4}x^m\cosh(a+bx)+\frac{1}{4}x^m\cosh(3a+3bx)\right)dx$$

$$= -\left(\frac{1}{4}\int x^m\cosh(a+bx)\,dx\right)+\frac{1}{4}\int x^m\cosh(3a+3bx)\,dx$$

$$= -\left(\frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx\right)-\frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx+\frac{1}{8}\int e^{i(3ia+}$$

$$= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b}-\frac{e^{a}x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma}{8b}$$

Mathematica [A] time = 0.226951, size = 114, normalized size = 0.85

$$\frac{e^{-3a}x^m\left(3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\text{Gamma}(m+1,-3bx)-(-bx)^m\text{Gamma}(m+1,3bx)\right)-3e^{4a}(-bx)^{-m}\text{Gamma}(m+1,-}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-3*E^(4*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (3*E^(2*a)*Gamma[1 + m
, b*x])/(b*x)^m + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b*x))^m*Gamma[
1 + m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^m\cosh(bx+a)(\sinh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

Maxima [A] time = 1.36379, size = 153, normalized size = 1.14

$$-\frac{1}{8}\left(3\,bx\right)^{-m-1}x^{m+1}e^{(-3\,a)}\Gamma\left(m+1,3\,bx\right)+\frac{1}{8}\left(bx\right)^{-m-1}x^{m+1}e^{(-a)}\Gamma\left(m+1,bx\right)+\frac{1}{8}\left(-bx\right)^{-m-1}x^{m+1}e^{a}\Gamma\left(m+1,-bx\right)-\frac{1}{8}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-
m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e
^a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1
, -3*b*x)
```

Fricas [A] time = 1.95132, size = 487, normalized size = 3.63

$$-\frac{\cosh\left(m\log\left(3\,b\right)+3\,a\right)\Gamma\left(m+1,3\,bx\right)-3\,\cosh\left(m\log\left(b\right)+a\right)\Gamma\left(m+1,bx\right)+3\,\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+1,-bx\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) - 3*cosh(m*log(b) + a)*ga
mma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) - cosh(m*log(-3*
b) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
 - 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + gamma(m + 1, -3*b*x)*sinh(m*l
og(-3*b) - 3*a) + 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^2, x)
```

3.281 $\int x^3 \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=117

$$\frac{2x^2\cosh(a+bx)}{3b^2} - \frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} - \frac{4x\sinh(a+bx)}{3b^3} - \frac{2\cosh^3(a+bx)}{27b^4} + \frac{14\cosh(}{9b}$$

```
[Out] (14*Cosh[a + b*x])/(9*b^4) + (2*x^2*Cosh[a + b*x])/(3*b^2) - (2*Cosh[a + b*
x]^3)/(27*b^4) - (4*x*Sinh[a + b*x])/(3*b^3) - (x^2*Cosh[a + b*x]*Sinh[a +
b*x]^2)/(3*b^2) + (2*x*Sinh[a + b*x]^3)/(9*b^3) + (x^3*Sinh[a + b*x]^3)/(3*
b)
```

Rubi [A] time = 0.14488, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5372, 3311, 3296, 2638, 2633}

$$\frac{2x^2\cosh(a+bx)}{3b^2} - \frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} - \frac{4x\sinh(a+bx)}{3b^3} - \frac{2\cosh^3(a+bx)}{27b^4} + \frac{14\cosh(}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (14*Cosh[a + b*x])/(9*b^4) + (2*x^2*Cosh[a + b*x])/(3*b^2) - (2*Cosh[a + b*
x]^3)/(27*b^4) - (4*x*Sinh[a + b*x])/(3*b^3) - (x^2*Cosh[a + b*x]*Sinh[a +
b*x]^2)/(3*b^2) + (2*x*Sinh[a + b*x]^3)/(9*b^3) + (x^3*Sinh[a + b*x]^3)/(3*
b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx &= \frac{x^3\sinh^3(a+bx)}{3b}-\frac{\int x^2\sinh^3(a+bx)\,dx}{b}\\ &= -\frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2}+\frac{2x\sinh^3(a+bx)}{9b^3}+\frac{x^3\sinh^3(a+bx)}{3b}-\frac{2\int \sinh\\ &= \frac{2x^2\cosh(a+bx)}{3b^2}-\frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2}+\frac{2x\sinh^3(a+bx)}{9b^3}+\frac{x^3\sinh^3(}{3b}\\ &= \frac{2\cosh(a+bx)}{9b^4}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{2\cosh^3(a+bx)}{27b^4}-\frac{4x\sinh(a+bx)}{3b^3}-\frac{x^2\cos}{}\\ &= \frac{14\cosh(a+bx)}{9b^4}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{2\cosh^3(a+bx)}{27b^4}-\frac{4x\sinh(a+bx)}{3b^3}-\frac{x^2\cos}{}\end{aligned}$$

Mathematica [A] time = 0.398752, size = 84, normalized size = 0.72

$$\frac{81\left(b^2x^2+2\right)\cosh(a+bx)-\left(9b^2x^2+2\right)\cosh(3(a+bx))+6bx\sinh(a+bx)\left(\left(3b^2x^2+2\right)\cosh(2(a+bx))-3b^2x^2-26\right)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (81*(2 + b^2*x^2)*Cosh[a + b*x] - (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] + 6*b*x
*(-26 - 3*b^2*x^2 + (2 + 3*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(108*
b^4)
```

Maple [B] time = 0.005, size = 334, normalized size = 2.9

$$\frac{1}{b^4}\left(\frac{(bx+a)^3\sinh(bx+a)(\cosh(bx+a))^2}{3}-\frac{(bx+a)^3\sinh(bx+a)}{3}-\frac{(bx+a)^2(\sinh(bx+a))^2\cosh(bx+a)}{3}+\frac{2\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/3*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2-1/3*(b*x+a)^3*sinh(b*x+a)-1/
3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+2/3*(b*x+a)^2*cosh(b*x+a)+2/9*(b*x+a)
*sinh(b*x+a)*cosh(b*x+a)^2-14/9*(b*x+a)*sinh(b*x+a)-2/27*cosh(b*x+a)*sinh(b
*x+a)^2+40/27*cosh(b*x+a)-3*a*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-1/3*
(b*x+a)^2*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+4/9*(b*x+a)*cos
h(b*x+a)+2/27*sinh(b*x+a)*cosh(b*x+a)^2-14/27*sinh(b*x+a))+3*a^2*(1/3*(b*x+
a)*sinh(b*x+a)*cosh(b*x+a)^2-1/3*(b*x+a)*sinh(b*x+a)-1/9*cosh(b*x+a)*sinh(b
*x+a)^2+2/9*cosh(b*x+a))-a^3*(1/3*sinh(b*x+a)*cosh(b*x+a)^2-1/3*sinh(b*x+a)
))
```

Maxima [A] time = 1.13066, size = 216, normalized size = 1.85

$$\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4}-\frac{\left(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a\right)e^{(bx)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 - 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

Fricas [A] time = 1.76391, size = 332, normalized size = 2.84

$$-\frac{\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3+3\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\left(3\,b^3x^3+2\,bx\right)\sinh\left(bx+a\right)^3-81\left(b^2x^2+\right.}{108\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*((9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)*s
inh(b*x + a)^2 - 3*(3*b^3*x^3 + 2*b*x)*sinh(b*x + a)^3 - 81*(b^2*x^2 + 2)*c
osh(b*x + a) + 9*(3*b^3*x^3 - (3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 18*b*x)
*sinh(b*x + a))/b^4
```

Sympy [A] time = 4.19061, size = 146, normalized size = 1.25

$$\begin{cases} \frac{x^3\sinh^3(a+bx)}{3b}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{b^2}+\frac{2x^2\cosh^3(a+bx)}{3b^2}+\frac{14x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)\cosh^2(a+bx)}{3b^3}-\frac{14\sinh^2(a+bx)\cosh(a+bx)}{9b^4}+ \\ \frac{x^4\sinh^2(a)\cosh(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise(((x**3*sinh(a + b*x)**3/(3*b) - x**2*sinh(a + b*x)**2*cosh(a + b*x
)/b**2 + 2*x**2*cosh(a + b*x)**3/(3*b**2) + 14*x*sinh(a + b*x)**3/(9*b**3)
- 4*x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 14*sinh(a + b*x)**2*cosh(a
+ b*x)/(9*b**4) + 40*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), (x**4*sinh(a)**
2*cosh(a)/4, True))
```

Giac [A] time = 1.162, size = 189, normalized size = 1.62

$$\frac{\left(9\,b^3x^3-9\,b^2x^2+6\,bx-2\right)e^{(3\,bx+3\,a)}}{216\,b^4}-\frac{\left(b^3x^3-3\,b^2x^2+6\,bx-6\right)e^{(bx+a)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx-a)}}{8\,b^4}-\frac{\left(9\,b^3x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

3.282 $\int x^2 \cosh(a+bx) \sinh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{2\sinh^3(a+bx)}{27b^3}-\frac{4\sinh(a+bx)}{9b^3}+\frac{4x\cosh(a+bx)}{9b^2}-\frac{2x\sinh^2(a+bx)\cosh(a+bx)}{9b^2}+\frac{x^2\sinh^3(a+bx)}{3b}$$

```
[Out] (4*x*Cosh[a + b*x])/(9*b^2) - (4*Sinh[a + b*x])/(9*b^3) - (2*x*Cosh[a + b*x
]*Sinh[a + b*x]^2)/(9*b^2) + (2*Sinh[a + b*x]^3)/(27*b^3) + (x^2*Sinh[a + b
*x]^3)/(3*b)
```

Rubi [A] time = 0.07921, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 3310, 3296, 2637}

$$\frac{2\sinh^3(a+bx)}{27b^3}-\frac{4\sinh(a+bx)}{9b^3}+\frac{4x\cosh(a+bx)}{9b^2}-\frac{2x\sinh^2(a+bx)\cosh(a+bx)}{9b^2}+\frac{x^2\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (4*x*Cosh[a + b*x])/(9*b^2) - (4*Sinh[a + b*x])/(9*b^3) - (2*x*Cosh[a + b*x
]*Sinh[a + b*x]^2)/(9*b^2) + (2*Sinh[a + b*x]^3)/(27*b^3) + (x^2*Sinh[a + b
*x]^3)/(3*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{x^2\sinh^3(a+bx)}{3b} - \frac{2\int x\sinh^3(a+bx)\,dx}{3b}$$
$$= -\frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2} + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(a+bx)}{3b} + \frac{4\int x\,\text{si}$$
$$= \frac{4x\cosh(a+bx)}{9b^2} - \frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2} + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(}{3b}$$
$$= \frac{4x\cosh(a+bx)}{9b^2} - \frac{4\sinh(a+bx)}{9b^3} - \frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2} + \frac{2\sinh^3(a\,-}{27b^3}$$

Mathematica [A] time = 0.425285, size = 66, normalized size = 0.8

$$\frac{\sinh(a+bx)\left(\left(9b^2x^2+2\right)\cosh(2(a+bx))-9b^2x^2-26\right)+27bx\cosh(a+bx)-3bx\cosh(3(a+bx))}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (27*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + (-26 - 9*b^2*x^2 + (2 + 9
*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(54*b^3)
```

Maple [B] time = 0.005, size = 193, normalized size = 2.3

$$\frac{1}{b^3}\left(\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^2}{3} - \frac{(bx+a)^2\sinh(bx+a)}{3} - \frac{(2\,bx+2\,a)(\sinh(bx+a))^2\cosh(bx+a)}{9} + \frac{(4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-1/3*(b*x+a)^2*sinh(b*x+a)-2/
9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+4/9*(b*x+a)*cosh(b*x+a)+2/27*sinh(b*x+a
)*cosh(b*x+a)^2-14/27*sinh(b*x+a)-2*a*(1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^
2-1/3*(b*x+a)*sinh(b*x+a)-1/9*cosh(b*x+a)*sinh(b*x+a)^2+2/9*cosh(b*x+a))+a^
2*(1/3*sinh(b*x+a)*cosh(b*x+a)^2-1/3*sinh(b*x+a)))
```

Maxima [A] time = 1.15764, size = 165, normalized size = 1.99

$$\frac{\left(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^3}-\frac{\left(b^2x^2e^{a}-2\,bxe^{a}+2\,e^{a}\right)e^{(bx)}}{8\,b^3}+\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{8\,b^3}-\frac{\left(9\,b^2x^2+6\,bx+2\right)e^{(-\cdots}}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Fricas [A] time = 1.82602, size = 271, normalized size = 3.27

$$-\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3-54\,bx\cosh\left(bx+a\right)+3\left(9\,b^2x^2-\cdots\right.}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b
^2*x^2 + 2)*sinh(b*x + a)^3 - 54*b*x*cosh(b*x + a) + 3*(9*b^2*x^2 - (9*b^2*
x^2 + 2)*cosh(b*x + a)^2 + 18)*sinh(b*x + a))/b^3
```

Sympy [A] time = 2.21602, size = 105, normalized size = 1.27

$$\begin{cases} \frac{x^2\sinh^3\left(a+bx\right)}{3b}-\frac{2x\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{3b^2}+\frac{4x\cosh^3\left(a+bx\right)}{9b^2}+\frac{14\sinh^3\left(a+bx\right)}{27b^3}-\frac{4\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{9b^3} & \text{for } b\neq 0 \\ \frac{x^3\sinh^2\left(a\right)\cosh\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**3/(3*b) - 2*x*sinh(a + b*x)**2*cosh(a + b*x)
/(3*b**2) + 4*x*cosh(a + b*x)**3/(9*b**2) + 14*sinh(a + b*x)**3/(27*b**3) -
 4*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cos
h(a)/3, True))
```

Giac [A] time = 1.20118, size = 146, normalized size = 1.76

$$\frac{\left(9\,b^2x^2-6\,bx+2\right)e^{(3\,bx+3\,a)}}{216\,b^3}-\frac{\left(b^2x^2-2\,bx+2\right)e^{(bx+a)}}{8\,b^3}+\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{8\,b^3}-\frac{\left(9\,b^2x^2+6\,bx+2\right)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.283 $\int x \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=45

$$-\frac{\cosh^3(a+bx)}{9b^2}+\frac{\cosh(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

```
[Out] Cosh[a + b*x]/(3*b^2) - Cosh[a + b*x]^3/(9*b^2) + (x*Sinh[a + b*x]^3)/(3*b)
```

Rubi [A] time = 0.0347695, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5372, 2633}

$$-\frac{\cosh^3(a+bx)}{9b^2}+\frac{\cosh(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(3*b^2) - Cosh[a + b*x]^3/(9*b^2) + (x*Sinh[a + b*x]^3)/(3*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{x\sinh^3(a+bx)}{3b} - \frac{\int \sinh^3(a+bx)\,dx}{3b}$$
$$= \frac{x\sinh^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(a+bx)\right)}{3b^2}$$
$$= \frac{\cosh(a+bx)}{3b^2} - \frac{\cosh^3(a+bx)}{9b^2} + \frac{x\sinh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.136516, size = 38, normalized size = 0.84

$$\frac{12bx\sinh^3(a+bx) + 9\cosh(a+bx) - \cosh(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (9*Cosh[a + b*x] - Cosh[3*(a + b*x)] + 12*b*x*Sinh[a + b*x]^3)/(36*b^2)
```

Maple [B] time = 0.006, size = 92, normalized size = 2.

$$\frac{1}{b^2}\left(\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^2}{3} - \frac{(bx+a)\sinh(bx+a)}{3} - \frac{\cosh(bx+a)(\sinh(bx+a))^2}{9} + \frac{2\cosh(bx+a)}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-1/3*(b*x+a)*sinh(b*x+a)-1/9*co
sh(b*x+a)*sinh(b*x+a)^2+2/9*cosh(b*x+a)-a*(1/3*sinh(b*x+a)*cosh(b*x+a)^2-1/
3*sinh(b*x+a)))
```

Maxima [B] time = 1.23595, size = 113, normalized size = 2.51

$$\frac{\left(3\,bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2} - \frac{(bxe^a - e^a)e^{(bx)}}{8\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{8\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

Fricas [A] time = 1.80617, size = 203, normalized size = 4.51

$$\frac{3\,bx\sinh\left(bx+a\right)^{3}-\cosh\left(bx+a\right)^{3}-3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+9\left(bx\cosh\left(bx+a\right)^{2}-bx\right)\sinh\left(bx+a\right)+9\,\cosh}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b*x*sinh(b*x + a)^3 - cosh(b*x + a)^3 - 3*cosh(b*x + a)*sinh(b*x +
a)^2 + 9*(b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) + 9*cosh(b*x + a))/b^2
```

Sympy [A] time = 1.11243, size = 61, normalized size = 1.36

$$\begin{cases} \frac{x\sinh^{3}\left(a+bx\right)}{3b}-\frac{\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{3b^{2}}+\frac{2\cosh^{3}\left(a+bx\right)}{9b^{2}} & \text{for}\ b\neq 0 \\ \frac{x^{2}\sinh^{2}\left(a\right)\cosh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**3/(3*b) - sinh(a + b*x)**2*cosh(a + b*x)/(3*b**
2) + 2*cosh(a + b*x)**3/(9*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)/2, Tr
ue))
```

Giac [A] time = 1.17366, size = 103, normalized size = 2.29

$$\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^{2}}-\frac{(bx-1)e^{(bx+a)}}{8\,b^{2}}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^{2}}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 - 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

3.284 $\int \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] Sinh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0183197, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int x^2\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0033657, size = 15, normalized size = 1.

$$\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{(\sinh(bx+a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/3*sinh(b*x+a)^3/b
```

Maxima [A] time = 1.12684, size = 18, normalized size = 1.2

$$\frac{\sinh(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sinh(b*x + a)^3/b
```

Fricas [B] time = 1.72191, size = 89, normalized size = 5.93

$$\frac{\sinh(bx+a)^3+3\left(\cosh(bx+a)^2-1\right)\sinh(bx+a)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a))/b
```

Sympy [A] time = 0.508895, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sinh^{3}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^{2}(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((sinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a), True))
```

Giac [B] time = 1.15681, size = 62, normalized size = 4.13

$$\frac{\left(3\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + e^(3*b*x + 3*a) - 3*e^(b*x
 + a))/b
```

3.285 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{1}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (Sinh[
a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.122942, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{1}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (Sinh[
a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x}+\frac{\cosh(3a+3bx)}{4x}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x}\,dx\\
&= -\left(\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx-\frac{1}{4}\sinh(a)\int \frac{\sinh(bx}{x}\\
&= -\frac{1}{4}\cosh(a)\mathbf{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathbf{Chi}(3bx)-\frac{1}{4}\sinh(a)\mathbf{Shi}(bx)+\frac{1}{4}\sinh(3a)\mathbf{Shi}(3bx
\end{aligned}
$$

Mathematica [A] time = 0.0696584, size = 41, normalized size = 0.87

$$\frac{1}{4}(-\cosh(a)\mathbf{Chi}(bx)+\cosh(3a)\mathbf{Chi}(3bx)-\sinh(a)\mathbf{Shi}(bx)+\sinh(3a)\mathbf{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (-(Cosh[a]*CoshIntegral[b*x]) + Cosh[3*a]*CoshIntegral[3*b*x] - Sinh[a]*Sin
hIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] time = 0.068, size = 47, normalized size = 1.

$$-\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{8}+\frac{e^{-a}\mathrm{Ei}(1,bx)}{8}+\frac{e^{a}\mathrm{Ei}(1,-bx)}{8}-\frac{e^{3a}\mathrm{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)+1/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.29796, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{1}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-\frac{1}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

Fricas [A] time = 1.74424, size = 204, normalized size = 4.34

$$\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{1}{8}\left(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right)\right)\cosh\left(a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)-\mathrm{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)-\frac{1}{8}\left(\mathrm{Ei}\left(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 1/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x, x)
```

Giac [A] time = 1.12384, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

3.286 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{4}b\sinh(a)\text{Chi}(bx)+\frac{3}{4}b\sinh(3a)\text{Chi}(3bx)-\frac{1}{4}b\cosh(a)\text{Shi}(bx)+\frac{3}{4}b\cosh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+}{4x}$$

```
[Out] Cosh[a + b*x]/(4*x) - Cosh[3*a + 3*b*x]/(4*x) - (b*CoshIntegral[b*x]*Sinh[a
])/4 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/4 - (b*Cosh[a]*SinhIntegral[b*x]
)/4 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.16738, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{4}b\sinh(a)\text{Chi}(bx)+\frac{3}{4}b\sinh(3a)\text{Chi}(3bx)-\frac{1}{4}b\cosh(a)\text{Shi}(bx)+\frac{3}{4}b\cosh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(4*x) - Cosh[3*a + 3*b*x]/(4*x) - (b*CoshIntegral[b*x]*Sinh[a
])/4 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/4 - (b*Cosh[a]*SinhIntegral[b*x]
)/4 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x^2}+\frac{\cosh(3a+3bx)}{4x^2}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^2}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^2}\,dx\\
&= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}b\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}(3b)\int \frac{\sinh(3a+3bx)}{x}\\
&= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}(3b\cosh(3a))\int\\
&= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}b\mathbf{Chi}(bx)\sinh(a)+\frac{3}{4}b\mathbf{Chi}(3bx)\sinh(3a)-\frac{1}{4}b
\end{aligned}$$

Mathematica [A] time = 0.206659, size = 68, normalized size = 0.85

$$-\frac{bx\sinh(a)\mathbf{Chi}(bx)-3bx\sinh(3a)\mathbf{Chi}(3bx)+bx\cosh(a)\mathbf{Shi}(bx)-3bx\cosh(3a)\mathbf{Shi}(3bx)-\cosh(a+bx)+\cosh(3(a+}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -(-Cosh[a + b*x] + Cosh[3*(a + b*x)] + b*x*CoshIntegral[b*x]*Sinh[a] - 3*b*
x*CoshIntegral[3*b*x]*Sinh[3*a] + b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x*Cos
```

```
h[3*a]*SinhIntegral[3*b*x])/(4*x)
```

Maple [A] time = 0.072, size = 104, normalized size = 1.3

$$-\frac{e^{-3bx-3a}}{8x}+\frac{3be^{-3a}\text{Ei}(1,3bx)}{8}+\frac{e^{-bx-a}}{8x}-\frac{be^{-a}\text{Ei}(1,bx)}{8}+\frac{e^{bx+a}}{8x}+\frac{be^{a}\text{Ei}(1,-bx)}{8}-\frac{e^{3bx+3a}}{8x}-\frac{3be^{3a}\text{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

```
[Out] -1/8*exp(-3*b*x-3*a)/x+3/8*b*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-b*x-a)/x-1/8*b*
exp(-a)*Ei(1,b*x)+1/8/x*exp(b*x+a)+1/8*b*exp(a)*Ei(1,-b*x)-1/8/x*exp(3*b*x+
3*a)-3/8*b*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.278, size = 68, normalized size = 0.85

$$-\frac{3}{8}\,be^{(-3\,a)}\Gamma(-1,3\,bx)+\frac{1}{8}\,be^{(-a)}\Gamma(-1,bx)-\frac{1}{8}\,be^{a}\Gamma(-1,-bx)+\frac{3}{8}\,be^{(3\,a)}\Gamma(-1,-3\,bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) - 1/8*b*e^a*
gamma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

Fricas [A] time = 1.80012, size = 340, normalized size = 4.25

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2-3\,(bx\text{Ei}(3\,bx)-bx\text{Ei}(-3\,bx))\cosh(3\,a)+(bx\text{Ei}(bx)-bx\text{Ei}(-bx}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*x*Ei(3*b*x
) - b*x*Ei(-3*b*x))*cosh(3*a) + (b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 3*(b
```

```
*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + (b*x*Ei(b*x) + b*x*Ei(-b*x))*sin
h(a) - 2*cosh(b*x + a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**2, x)
```

Giac [A] time = 1.13309, size = 123, normalized size = 1.54

$$\frac{3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + bx\mathrm{Ei}\left(-bx\right)e^{(-a)} - 3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - bx\mathrm{Ei}\left(bx\right)e^{a} - e^{(3\,bx+3\,a)} + e^{(bx+a)} + e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) - 3*b*x*Ei(-3*b*x)*e^(-3
*a) - b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) + e^(b*x + a) + e^(-b*x - a) - e^(-
3*b*x - 3*a))/x
```

3.287 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(}{}$$

```
[Out] Cosh[a + b*x]/(8*x^2) - Cosh[3*a + 3*b*x]/(8*x^2) - (b^2*Cosh[a]*CoshIntegr
al[b*x])/8 + (9*b^2*Cosh[3*a]*CoshIntegral[3*b*x])/8 + (b*Sinh[a + b*x])/(8
*x) - (3*b*Sinh[3*a + 3*b*x])/(8*x) - (b^2*Sinh[a]*SinhIntegral[b*x])/8 + (
9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rubi [A] time = 0.217915, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```


```
[Out] Cosh[a + b*x]/(8*x^2) - Cosh[3*a + 3*b*x]/(8*x^2) - (b^2*Cosh[a]*CoshIntegr
al[b*x])/8 + (9*b^2*Cosh[3*a]*CoshIntegral[3*b*x])/8 + (b*Sinh[a + b*x])/(8
*x) - (3*b*Sinh[3*a + 3*b*x])/(8*x) - (b^2*Sinh[a]*SinhIntegral[b*x])/8 + (
9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/8
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x^3}+\frac{\cosh(3a+3bx)}{4x^3}\right)dx \\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^3}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^3}\,dx \\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}-\frac{1}{8}b\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\sinh(3a+3bx)}{x^2} \\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}-\frac{1}{8}b^2\int \frac{\cos}{} \\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}-\frac{1}{8}\left(b^2\cosh\right. \\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}-\frac{1}{8}b^2\cosh(a)\mathbf{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\mathbf{Chi}(3bx)+
\end{aligned}$$

Mathematica [A] time = 0.255259, size = 107, normalized size = 0.9

$$\frac{-b^2x^2\cosh(a)\mathbf{Chi}(bx)+9b^2x^2\cosh(3a)\mathbf{Chi}(3bx)-b^2x^2\sinh(a)\mathbf{Shi}(bx)+9b^2x^2\sinh(3a)\mathbf{Shi}(3bx)+bx\sinh(a+bx)-3}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (Cosh[a + b*x] - Cosh[3*(a + b*x)] - b^2*x^2*Cosh[a]*CoshIntegral[b*x] + 9*
b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a
 + b*x)] - b^2*x^2*Sinh[a]*SinhIntegral[b*x] + 9*b^2*x^2*Sinh[3*a]*SinhInte
gral[3*b*x])/(8*x^2)
```

Maple [A] time = 0.075, size = 169, normalized size = 1.4

$$\frac{3\,b\mathrm{e}^{-3\,bx-3\,a}}{16\,x}-\frac{\mathrm{e}^{-3\,bx-3\,a}}{16\,x^{2}}-\frac{9\,b^{2}\mathrm{e}^{-3\,a}\mathrm{Ei}\left(1,3\,bx\right)}{16}-\frac{b\mathrm{e}^{-bx-a}}{16\,x}+\frac{\mathrm{e}^{-bx-a}}{16\,x^{2}}+\frac{b^{2}\mathrm{e}^{-a}\mathrm{Ei}\left(1,bx\right)}{16}+\frac{\mathrm{e}^{bx+a}}{16\,x^{2}}+\frac{b\mathrm{e}^{bx+a}}{16\,x}+\frac{b^{2}\mathrm{e}^{a}\mathrm{Ei}\left(1,-bx\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 3/16*b*exp(-3*b*x-3*a)/x-1/16*exp(-3*b*x-3*a)/x^2-9/16*b^2*exp(-3*a)*Ei(1,3
*b*x)-1/16*b*exp(-b*x-a)/x+1/16*exp(-b*x-a)/x^2+1/16*b^2*exp(-a)*Ei(1,b*x)+
1/16/x^2*exp(b*x+a)+1/16*b/x*exp(b*x+a)+1/16*b^2*exp(a)*Ei(1,-b*x)-1/16/x^2
*exp(3*b*x+3*a)-3/16*b/x*exp(3*b*x+3*a)-9/16*b^2*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.33273, size = 78, normalized size = 0.66

$$-\frac{9}{8}\,b^{2}e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)+\frac{1}{8}\,b^{2}e^{(-a)}\Gamma\left(-2,bx\right)+\frac{1}{8}\,b^{2}e^{a}\Gamma\left(-2,-bx\right)-\frac{9}{8}\,b^{2}e^{(3\,a)}\Gamma\left(-2,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) + 1/8*b^
2*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

Fricas [A] time = 1.76983, size = 485, normalized size = 4.08

$$-\frac{6\,bx\sinh\left(bx+a\right)^{3}+2\,\cosh\left(bx+a\right)^{3}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\left(b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)+b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x
 + a)^2 - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*cosh(3*a) + (b^2*x^2*E
i(b*x) + b^2*x^2*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b
*x + a) - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + (b^2*x^2*E
i(b*x) - b^2*x^2*Ei(-b*x))*sinh(a) - 2*cosh(b*x + a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**3, x)
```

Giac [A] time = 1.18011, size = 211, normalized size = 1.77

$$\frac{9\,b^2x^2\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} - b^2x^2\mathrm{Ei}\left(-bx\right)e^{(-a)} + 9\,b^2x^2\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - b^2x^2\mathrm{Ei}\left(bx\right)e^{a} - 3\,bxe^{(3\,bx+3\,a)} + bxe^{(bx+a)} - bxe^{(-bx-a)}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) - b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) + b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + e^(b*
x + a) + e^(-b*x - a) - e^(-3*b*x - 3*a))/x^2
```

3.288 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{24}b^3\sinh(a)\mathbf{Chi}(bx)+\frac{9}{8}b^3\sinh(3a)\mathbf{Chi}(3bx)-\frac{1}{24}b^3\cosh(a)\mathbf{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\mathbf{Shi}(3bx)+\frac{b^2\cosh(a+bx)}{24x}-$$

```
[Out] Cosh[a + b*x]/(12*x^3) + (b^2*Cosh[a + b*x])/(24*x) - Cosh[3*a + 3*b*x]/(12
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(8*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/2
4 + (9*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/8 + (b*Sinh[a + b*x])/(24*x^2) -
(b*Sinh[3*a + 3*b*x])/(8*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/24 + (9*b^3
*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rubi [A] time = 0.281335, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{24}b^3\sinh(a)\mathbf{Chi}(bx)+\frac{9}{8}b^3\sinh(3a)\mathbf{Chi}(3bx)-\frac{1}{24}b^3\cosh(a)\mathbf{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\mathbf{Shi}(3bx)+\frac{b^2\cosh(a+bx)}{24x}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```


```
[Out] Cosh[a + b*x]/(12*x^3) + (b^2*Cosh[a + b*x])/(24*x) - Cosh[3*a + 3*b*x]/(12
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(8*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/2
4 + (9*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/8 + (b*Sinh[a + b*x])/(24*x^2) -
(b*Sinh[3*a + 3*b*x])/(8*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/24 + (9*b^3
*Cosh[3*a]*SinhIntegral[3*b*x])/8
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x^4}+\frac{\cosh(3a+3bx)}{4x^4}\right)dx \\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^4}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^4}\,dx \\
&= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{1}{12}b\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}b\int \frac{\sinh(3a+3bx)}{x^3}\,d \\
&= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}+\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}-\frac{1}{24}b^2\int \frac{\cos}{} \\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}+\frac{b\sinh(}{24x} \\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}+\frac{b\sinh(}{24x} \\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}-\frac{1}{24}b^3\mathrm{Ch}
\end{aligned}
$$

Mathematica [A] time = 0.305473, size = 138, normalized size = 0.9

$$
\frac{-b^3x^3\sinh(a)\mathrm{Chi}(bx)+27b^3x^3\sinh(3a)\mathrm{Chi}(3bx)-b^3x^3\cosh(a)\mathrm{Shi}(bx)+27b^3x^3\cosh(3a)\mathrm{Shi}(3bx)+b^2x^2\cosh(a+bx}{24x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^2*
Cosh[3*(a + b*x)] - b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshInte
gral[3*b*x]*Sinh[3*a] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a + b*x)] - b^3*x
^3*Cosh[a]*SinhIntegral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x])/(2
4*x^3)
```

Maple [A] time = 0.076, size = 234, normalized size = 1.5

$$-\frac{3\,b^2\mathrm{e}^{-3\,bx-3\,a}}{16\,x}+\frac{b\mathrm{e}^{-3\,bx-3\,a}}{16\,x^2}-\frac{\mathrm{e}^{-3\,bx-3\,a}}{24\,x^3}+\frac{9\,b^3\mathrm{e}^{-3\,a}\mathrm{Ei}\,(1,3\,bx)}{16}+\frac{b^2\mathrm{e}^{-bx-a}}{48\,x}-\frac{b\mathrm{e}^{-bx-a}}{48\,x^2}+\frac{\mathrm{e}^{-bx-a}}{24\,x^3}-\frac{b^3\mathrm{e}^{-a}\mathrm{Ei}\,(1,bx)}{48}+\frac{\mathrm{e}^{bx+a}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x)
```

```
[Out] -3/16*b^2*exp(-3*b*x-3*a)/x+1/16*b*exp(-3*b*x-3*a)/x^2-1/24*exp(-3*b*x-3*a)
/x^3+9/16*b^3*exp(-3*a)*Ei(1,3*b*x)+1/48*b^2*exp(-b*x-a)/x-1/48*b*exp(-b*x-
a)/x^2+1/24*exp(-b*x-a)/x^3-1/48*b^3*exp(-a)*Ei(1,b*x)+1/24/x^3*exp(b*x+a)+
1/48*b/x^2*exp(b*x+a)+1/48*b^2/x*exp(b*x+a)+1/48*b^3*exp(a)*Ei(1,-b*x)-1/24
/x^3*exp(3*b*x+3*a)-1/16*b/x^2*exp(3*b*x+3*a)-3/16*b^2/x*exp(3*b*x+3*a)-9/1
6*b^3*exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.34099, size = 78, normalized size = 0.51

$$-\frac{27}{8}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{8}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{8}\,b^3e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{8}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) - 1/8*b
^3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

Fricas [A] time = 1.63779, size = 549, normalized size = 3.56

$$-\frac{6\,bx\sinh\left(bx+a\right)^3+2\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3+6\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-2\left(b^2x^2+2\right)\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(6*b*x*sinh(b*x + a)^3 + 2*(9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 6*(9*b^2
*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) -
27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*sinh(3*a) + (b^3*x^3*Ei(b*x) +
b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**4, x)
```

Giac [A] time = 1.1941, size = 300, normalized size = 1.95

$$\frac{27\,b^3x^3\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+b^3x^3\mathrm{Ei}\left(-bx\right)e^{(-a)}-27\,b^3x^3\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-b^3x^3\mathrm{Ei}\left(bx\right)e^{a}-9\,b^2x^2e^{(3\,bx+3\,a)}+b^2x^2e^{(bx+a)}+b^2x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) - b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) + b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) - 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
```

```
*e^(3*b*x + 3*a) + b*x*e^(b*x + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a) - 2*e^(-3*b*x - 3*
a))/x^3
```

3.289 $\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\text{Gamma}(m+1,4bx)}{b}-\frac{x^{m+1}}{8(m+1)}$$

```
[Out] -x^(1 + m)/(8*(1 + m)) + (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*
b*(-(b*x))^m) - (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + m))*b*E^(4*a)*(b*x)^m)
```

Rubi [A] time = 0.12789, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {5448, 3307, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\text{Gamma}(m+1,4bx)}{b}-\frac{x^{m+1}}{8(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(1 + m)/(8*(1 + m)) + (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*
b*(-(b*x))^m) - (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + m))*b*E^(4*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{x^m}{8}+\frac{1}{8}x^m\cosh(4a+4bx)\right)dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{8}\int x^m\cosh(4a+4bx)\,dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx+\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}-\frac{4^{-3-m}e^{-4a}x^m(bx)^{-m}\Gamma(1+m}{b}$$

Mathematica [A] time = 0.205554, size = 76, normalized size = 0.89

$$\frac{1}{64}x^m\left(\frac{e^{4a}4^{-m}(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{-4a}4^{-m}(bx)^{-m}\text{Gamma}(m+1,4bx)}{b}-\frac{8x}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-8*x)/(1 + m) + (E^(4*a)*Gamma[1 + m, -4*b*x])/(4^m*b*(-(b*x))^m) -
Gamma[1 + m, 4*b*x]/(4^m*b*E^(4*a)*(b*x)^m)))/64
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^m(\cosh(bx+a))^2(\sinh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88802, size = 377, normalized size = 4.44

$$-\frac{8\,bx\cosh\left(m\log\left(x\right)\right)+\left(m+1\right)\cosh\left(m\log\left(4\,b\right)+4\,a\right)\Gamma\left(m+1,4\,bx\right)-\left(m+1\right)\cosh\left(m\log\left(-4\,b\right)-4\,a\right)\Gamma\left(m+1,-4\,b\right.}{64\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(8*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4
*b*x) - (m + 1)*cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - (m + 1)*gamm
a(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) + (m + 1)*gamma(m + 1, -4*b*x)*sinh(
m*log(-4*b) - 4*a) + 8*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh^2\left(a+bx\right)\cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.290 $\int x^3 \cosh^2(a+bx) \sinh^2(a+bx)\,dx$

Optimal. Leaf size=79

$$-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}-\frac{3\cosh(4a+4bx)}{1024b^4}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{x^4}{32}$$

```
[Out] -x^4/32 - (3*Cosh[4*a + 4*b*x])/(1024*b^4) - (3*x^2*Cosh[4*a + 4*b*x])/(128
*b^2) + (3*x*Sinh[4*a + 4*b*x])/(256*b^3) + (x^3*Sinh[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.114185, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2638}

$$-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}-\frac{3\cosh(4a+4bx)}{1024b^4}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^4/32 - (3*Cosh[4*a + 4*b*x])/(1024*b^4) - (3*x^2*Cosh[4*a + 4*b*x])/(128
*b^2) + (3*x*Sinh[4*a + 4*b*x])/(256*b^3) + (x^3*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3 \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{x^3}{8}+\frac{1}{8}x^3\cosh(4a+4bx)\right)dx$$
$$= -\frac{x^4}{32}+\frac{1}{8}\int x^3\cosh(4a+4bx)\,dx$$
$$= -\frac{x^4}{32}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{3\int x^2\sinh(4a+4bx)\,dx}{32b}$$
$$= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{x^3\sinh(4a+4bx)}{32b}+\frac{3\int x\cosh(4a+4bx)\,dx}{64b^2}$$
$$= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{3\int \sin$$
$$= -\frac{x^4}{32}-\frac{3\cosh(4a+4bx)}{1024b^4}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+\frac{x^3\sinh($$

Mathematica [A] time = 0.198099, size = 58, normalized size = 0.73

$$\frac{4bx\left(8b^2x^2+3\right)\sinh(4(a+bx))-3\left(8b^2x^2+1\right)\cosh(4(a+bx))-32b^4x^4}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^4*x^4 - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 4*b*x*(3 + 8*b^2*x^2)*
Sinh[4*(a + b*x)])/(1024*b^4)
```

Maple [B] time = 0.007, size = 384, normalized size = 4.9

$$\frac{1}{b^4}\left(\frac{(bx+a)^3\sinh(bx+a)(\cosh(bx+a))^3}{4}-\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^4}{32}-\frac{3\ (bx+a)^2(\sinh(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)^3*cosh(b*x+a)*si
nh(b*x+a)-1/32*(b*x+a)^4-3/16*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2+3/32*(b
```

```
*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-3/64*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-3/128*
(b*x+a)^2-3/128*cosh(b*x+a)^2*sinh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*sinh(b*x+a)*
cosh(b*x+a)^3-1/8*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/24*(b*x+a)^3-1/8*(b*x
+a)*sinh(b*x+a)^2*cosh(b*x+a)^2+1/32*cosh(b*x+a)^3*sinh(b*x+a)-1/64*cosh(b*
x+a)*sinh(b*x+a)-1/64*b*x-1/64*a)+3*a^2*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a
)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/16*(b*x+a)^2-1/16*cosh(b*x+a)^2*s
inh(b*x+a)^2)-a^3*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a
)-1/8*b*x-1/8*a))
```

Maxima [A] time = 1.17843, size = 123, normalized size = 1.56

$$-\frac{1}{32}\,x^4+\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}-\frac{\left(32\,b^3x^3+24\,b^2x^2+12\,bx+3\right)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a
) - 3*e^(4*a))*e^(4*b*x)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3
)*e^(-4*b*x - 4*a)/b^4
```

Fricas [B] time = 1.85244, size = 352, normalized size = 4.46

$$-\frac{32\,b^4x^4+3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-16\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+18\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^2}{1024\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(32*b^4*x^4 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 - 16*(8*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^3*sinh(b*x + a) + 18*(8*b^2*x^2 + 1)*cosh(b*x + a)^2*si
nh(b*x + a)^2 - 16*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(8
*b^2*x^2 + 1)*sinh(b*x + a)^4)/b^4
```

Sympy [A] time = 8.077, size = 250, normalized size = 3.16

$$\begin{cases} -\frac{x^4\sinh^4(a+bx)}{32} + \frac{x^4\sinh^2(a+bx)\cosh^2(a+bx)}{16} - \frac{x^4\cosh^4(a+bx)}{32} + \frac{x^3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{8b} - \frac{3x^2\sinh^4(a+b}{128b^2} \\ \frac{x^4\sinh^2(a)\cosh^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**4*sinh(a + b*x)**4/32 + x**4*sinh(a + b*x)**2*cosh(a + b*x)*
*2/16 - x**4*cosh(a + b*x)**4/32 + x**3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - 3*x**2*sinh(a + b*x)**4/(12
8*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 3*x**2*cosh(
a + b*x)**4/(128*b**2) + 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 3*x
*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) - 3*sinh(a + b*x)**4/(256*b**4) -
 3*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*sinh(a)**2*cosh(a)**2/4, T
rue))
```

Giac [A] time = 1.17165, size = 105, normalized size = 1.33

$$-\frac{1}{32}x^4 + \frac{\left(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3\right)e^{(4\,bx+4\,a)}}{2048\,b^4} - \frac{\left(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3\right)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b
^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.291 $\int x^2 \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=60

$$\frac{\sinh(4a+4bx)}{256b^3} - \frac{x\cosh(4a+4bx)}{64b^2} + \frac{x^2\sinh(4a+4bx)}{32b} - \frac{x^3}{24}$$

```
[Out] -x^3/24 - (x*Cosh[4*a + 4*b*x])/(64*b^2) + Sinh[4*a + 4*b*x]/(256*b^3) + (x
^2*Sinh[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.10117, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {5448, 3296, 2637}

$$\frac{\sinh(4a+4bx)}{256b^3} - \frac{x\cosh(4a+4bx)}{64b^2} + \frac{x^2\sinh(4a+4bx)}{32b} - \frac{x^3}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^3/24 - (x*Cosh[4*a + 4*b*x])/(64*b^2) + Sinh[4*a + 4*b*x]/(256*b^3) + (x
^2*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \int\left(-\frac{x^2}{8}+\frac{1}{8}x^2\cosh(4a+4bx)\right)dx\\ &= -\frac{x^3}{24}+\frac{1}{8}\int x^2\cosh(4a+4bx)\,dx\\ &= -\frac{x^3}{24}+\frac{x^2\sinh(4a+4bx)}{32b}-\frac{\int x\sinh(4a+4bx)\,dx}{16b}\\ &= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{x^2\sinh(4a+4bx)}{32b}+\frac{\int\cosh(4a+4bx)\,dx}{64b^2}\\ &= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{\sinh(4a+4bx)}{256b^3}+\frac{x^2\sinh(4a+4bx)}{32b}\end{aligned}$$

Mathematica [A] time = 0.164927, size = 48, normalized size = 0.8

$$\frac{3\left(8b^2x^2+1\right)\sinh(4(a+bx))-12bx\cosh(4(a+bx))-32b^3x^3}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^3*x^3 - 12*b*x*Cosh[4*(a + b*x)] + 3*(1 + 8*b^2*x^2)*Sinh[4*(a + b*x
)])/(768*b^3)
```

Maple [B] time = 0.007, size = 232, normalized size = 3.9

$$\frac{1}{b^3}\left(\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^3}{4}-\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^3}{24}-\frac{(bx+a)(\sinh(bx+a}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)^2*cosh(b*x+a)*si
nh(b*x+a)-1/24*(b*x+a)^3-1/8*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2+1/32*cosh(
b*x+a)^3*sinh(b*x+a)-1/64*cosh(b*x+a)*sinh(b*x+a)-1/64*b*x-1/64*a-2*a*(1/4*
(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/16*
```

```
(b*x+a)^2-1/16*cosh(b*x+a)^2*sinh(b*x+a)^2)+a^2*(1/4*cosh(b*x+a)^3*sinh(b*x
+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a))
```

Maxima [A] time = 1.16365, size = 93, normalized size = 1.55

$$-\frac{1}{24}\,x^3+\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b
^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Fricas [B] time = 1.85527, size = 288, normalized size = 4.8

$$-\frac{8\,b^3x^3+3\,bx\cosh\left(bx+a\right)^4+18\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+3\,bx\sinh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^3\mathrm{s}}{192\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(8*b^3*x^3 + 3*b*x*cosh(b*x + a)^4 + 18*b*x*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + 3*b*x*sinh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*
x + a) - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3)/b^3
```

Sympy [A] time = 4.66419, size = 204, normalized size = 3.4

$$\begin{cases}-\frac{x^3\sinh^4(a+bx)}{24}+\frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{12}-\frac{x^3\cosh^4(a+bx)}{24}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{8b}-\frac{x\sinh^4(a+bx)}{64b^2}-\\ \frac{x^3\sinh^2(a)\cosh^2(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**4/24 + x**3*sinh(a + b*x)**2*cosh(a + b*x)*
*2/12 - x**3*cosh(a + b*x)**4/24 + x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - x*sinh(a + b*x)**4/(64*b**2
) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - x*cosh(a + b*x)**4/(6
4*b**2) + sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + sinh(a + b*x)*cosh(a +
 b*x)**3/(64*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cosh(a)**2/3, True))
```

Giac [A] time = 1.29334, size = 84, normalized size = 1.4

$$-\frac{1}{24}x^3 + \frac{\left(8b^2x^2 - 4bx + 1\right)e^{(4bx+4a)}}{512b^3} - \frac{\left(8b^2x^2 + 4bx + 1\right)e^{(-4bx-4a)}}{512b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/512*(8*b^
2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.292 $\int x \cosh^2(a + bx) \sinh^2(a + bx) \, dx$

Optimal. Leaf size=41

$$-\frac{\cosh(4a + 4bx)}{128b^2} + \frac{x \sinh(4a + 4bx)}{32b} - \frac{x^2}{16}$$

```
[Out] -x^2/16 - Cosh[4*a + 4*b*x]/(128*b^2) + (x*Sinh[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.0504448, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2638}

$$-\frac{\cosh(4a + 4bx)}{128b^2} + \frac{x \sinh(4a + 4bx)}{32b} - \frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^2/16 - Cosh[4*a + 4*b*x]/(128*b^2) + (x*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \int\left(-\frac{x}{8}+\frac{1}{8}x\cosh(4a+4bx)\right)dx\\ &= -\frac{x^2}{16}+\frac{1}{8}\int x\cosh(4a+4bx)\,dx\\ &= -\frac{x^2}{16}+\frac{x\sinh(4a+4bx)}{32b}-\frac{\int\sinh(4a+4bx)\,dx}{32b}\\ &= -\frac{x^2}{16}-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}\end{aligned}$$

Mathematica [A] time = 0.135183, size = 41, normalized size = 1.

$$-\frac{-8a^2-4bx\sinh(4(a+bx))+\cosh(4(a+bx))+8b^2x^2}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -(-8*a^2 + 8*b^2*x^2 + Cosh[4*(a + b*x)] - 4*b*x*Sinh[4*(a + b*x)])/(128*b^
2)
```

Maple [B] time = 0.005, size = 114, normalized size = 2.8

$$\frac{1}{b^2}\left(\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^2}{16}-\frac{(\cosh(bx+a))^2(\sinh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(b
*x+a)-1/16*(b*x+a)^2-1/16*cosh(b*x+a)^2*sinh(b*x+a)^2-a*(1/4*cosh(b*x+a)^3*
sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a))
```

Maxima [A] time = 1.10955, size = 69, normalized size = 1.68

$$-\frac{1}{16}x^2+\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Fricas [B] time = 1.83311, size = 234, normalized size = 5.71

$$\frac{16\,bx\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-8\,b^{2}x^{2}-\cosh\left(bx+a\right)^{4}-6\,\cosh\left(bx+a\right)^{2}\sinh\left(\right.}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 - 8*b^2*x^2 - cosh(b*x + a)^4 - 6*cosh(b*x + a)^2*sinh(b*x + a)^2 -
 sinh(b*x + a)^4)/b^2
```

Sympy [A] time = 2.43515, size = 131, normalized size = 3.2

$$\begin{cases} -\frac{x^{2}\sinh^{4}\left(a+bx\right)}{16}+\frac{x^{2}\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{8}-\frac{x^{2}\cosh^{4}\left(a+bx\right)}{16}+\frac{x\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{8b}+\frac{x\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{8b}-\frac{\sinh^{4}\left(a+bx\right)}{32b^{2}}-\cdots \\ \frac{x^{2}\sinh^{2}\left(a\right)\cosh^{2}\left(a\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**4/16 + x**2*sinh(a + b*x)**2*cosh(a + b*x)*
*2/8 - x**2*cosh(a + b*x)**4/16 + x*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) +
x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - sinh(a + b*x)**4/(32*b**2) - cosh(
a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)**2/2, True))
```

Giac [A] time = 1.22371, size = 62, normalized size = 1.51

$$-\frac{1}{16}\,x^{2}+\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^{2}}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/256*(4*b*x + 1)*e^(-4
*b*x - 4*a)/b^2
```

3.293 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rubi [A] time = 0.0391155, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}-\frac{1}{4}\int \cosh^2(a+bx)\,dx\\ &= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}-\frac{\int 1\,dx}{8}\\ &= -\frac{x}{8}-\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0222987, size = 23, normalized size = 0.5

$$\frac{\sinh(4(a+bx))-4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0., size = 43, normalized size = 0.9

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a)
```

Maxima [A] time = 1.06715, size = 53, normalized size = 1.15

$$-\frac{bx+a}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A] time = 1.72788, size = 104, normalized size = 2.26

$$\frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

Sympy [A] time = 1.19662, size = 92, normalized size = 2.

$$\begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b\neq 0 \\ x\sinh^2(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

Giac [A] time = 1.12838, size = 65, normalized size = 1.41

$$-\frac{8\,bx-\left(2\,e^{(4\,bx+4\,a)}-1\right)e^{(-4\,bx-4\,a)}+8\,a-e^{(4\,bx+4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*b*x - (2*e^(4*b*x + 4*a) - 1)*e^(-4*b*x - 4*a) + 8*a - e^(4*b*x +
4*a))/b
```

3.294 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)-\frac{\log(x)}{8}$$

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x])/8 - Log[x]/8 + (Sinh[4*a]*SinhIntegral[4*b*
x])/8
```

Rubi [A] time = 0.0848538, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x])/8 - Log[x]/8 + (Sinh[4*a]*SinhIntegral[4*b*
x])/8
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx &= \int \left(-\frac{1}{8x}+\frac{\cosh(4a+4bx)}{8x}\right)dx\\ &= -\frac{\log(x)}{8}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x}\,dx\\ &= -\frac{\log(x)}{8}+\frac{1}{8}\cosh(4a)\int \frac{\cosh(4bx)}{x}\,dx+\frac{1}{8}\sinh(4a)\int \frac{\sinh(4bx)}{x}\,dx\\ &= \frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)-\frac{\log(x)}{8}+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)\end{aligned}$$

Mathematica [A] time = 0.0974682, size = 32, normalized size = 0.97

$$\frac{1}{8}(\cosh(4a)\mathrm{Chi}(4bx)+\sinh(4a)\mathrm{Shi}(4bx)-\log(2bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x] - Log[2*b*x] + Sinh[4*a]*SinhIntegral[4*b*x]
)/8
```

Maple [A] time = 0.049, size = 30, normalized size = 0.9

$$-\frac{\ln(x)}{8}-\frac{e^{-4a}\mathrm{Ei}(1,4bx)}{16}-\frac{e^{4a}\mathrm{Ei}(1,-4bx)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/8*ln(x)-1/16*exp(-4*a)*Ei(1,4*b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

Maxima [A] time = 1.26553, size = 36, normalized size = 1.09

$$\frac{1}{16}\,\mathrm{Ei}\,(4\,bx)\,e^{(4\,a)}+\frac{1}{16}\,\mathrm{Ei}\,(-4\,bx)\,e^{(-4\,a)}-\frac{1}{8}\,\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

Fricas [A] time = 1.83623, size = 130, normalized size = 3.94

$$\frac{1}{16}\,(\mathrm{Ei}\,(4\,bx)+\mathrm{Ei}\,(-4\,bx))\cosh\,(4\,a)+\frac{1}{16}\,(\mathrm{Ei}\,(4\,bx)-\mathrm{Ei}\,(-4\,bx))\sinh\,(4\,a)-\frac{1}{8}\,\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) + Ei(-4*b*x))*cosh(4*a) + 1/16*(Ei(4*b*x) - Ei(-4*b*x))*sin
h(4*a) - 1/8*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x, x)
```

Giac [A] time = 1.15044, size = 36, normalized size = 1.09

$$\frac{1}{16}\,\mathrm{Ei}\left(4\,bx\right)e^{\left(4\,a\right)}+\frac{1}{16}\,\mathrm{Ei}\left(-4\,bx\right)e^{\left(-4\,a\right)}-\frac{1}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

3.295 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}b\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b\cosh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{8x}$$

```
[Out] 1/(8*x) - Cosh[4*a + 4*b*x]/(8*x) + (b*CoshIntegral[4*b*x]*Sinh[4*a])/2 + (
b*Cosh[4*a]*SinhIntegral[4*b*x])/2
```

Rubi [A] time = 0.111748, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b\cosh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] 1/(8*x) - Cosh[4*a + 4*b*x]/(8*x) + (b*CoshIntegral[4*b*x]*Sinh[4*a])/2 + (
b*Cosh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{1}{8x^2}+\frac{\cosh(4a+4bx)}{8x^2}\right)dx \\
&= \frac{1}{8x}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^2}\,dx \\
&= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}b\int \frac{\sinh(4a+4bx)}{x}\,dx \\
&= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}(b\cosh(4a))\int \frac{\sinh(4bx)}{x}\,dx+\frac{1}{2}(b\sinh(4a))\int \frac{\cosh(4}{x} \\
&= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}b\mathbf{Chi}(4bx)\sinh(4a)+\frac{1}{2}b\cosh(4a)\mathbf{Shi}(4bx)
\end{aligned}
$$

Mathematica [A] time = 0.0932268, size = 45, normalized size = 0.87

$$\frac{4bx\sinh(4a)\mathbf{Chi}(4bx)+4bx\cosh(4a)\mathbf{Shi}(4bx)-\cosh(4(a+bx))+1}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] (1 - Cosh[4*(a + b*x)] + 4*b*x*CoshIntegral[4*b*x]*Sinh[4*a] + 4*b*x*Cosh[4
*a]*SinhIntegral[4*b*x])/(8*x)
```

Maple [A] time = 0.049, size = 61, normalized size = 1.2

$$\frac{1}{8x} - \frac{e^{-4bx-4a}}{16x} + \frac{be^{-4a}\mathrm{Ei}(1,4bx)}{4} - \frac{e^{4bx+4a}}{16x} - \frac{be^{4a}\mathrm{Ei}(1,-4bx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

```
[Out] 1/8/x-1/16*exp(-4*b*x-4*a)/x+1/4*b*exp(-4*a)*Ei(1,4*b*x)-1/16/x*exp(4*b*x+4
*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
```

Maxima [A] time = 1.3941, size = 43, normalized size = 0.83

$$-\frac{1}{4}be^{(-4a)}\Gamma(-1,4bx) + \frac{1}{4}be^{(4a)}\Gamma(-1,-4bx) + \frac{1}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x) + 1/8/x
```

Fricas [B] time = 1.87895, size = 240, normalized size = 4.62

$$-\frac{\cosh(bx+a)^4 + 6\cosh(bx+a)^2\sinh(bx+a)^2 + \sinh(bx+a)^4 - 2(bx\mathrm{Ei}(4bx) - bx\mathrm{Ei}(-4bx))\cosh(4a) - 2(bx\mathrm{Ei}(\ldots}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x + a)^4
 - 2*(b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*cosh(4*a) - 2*(b*x*Ei(4*b*x) + b*x*Ei
(-4*b*x))*sinh(4*a) - 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**2, x)
```

Giac [A] time = 1.1669, size = 74, normalized size = 1.42

$$\frac{4\,bx\text{Ei}\,(4\,bx)\,e^{(4\,a)} - 4\,bx\text{Ei}\,(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} - e^{(-4\,bx-4\,a)} + 2}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a)
 - e^(-4*b*x - 4*a) + 2)/x
```

3.296 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=67

$$b^2\cosh(4a)\text{Chi}(4bx)+b^2\sinh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\frac{1}{16x^2}$$

```
[Out] 1/(16*x^2) - Cosh[4*a + 4*b*x]/(16*x^2) + b^2*Cosh[4*a]*CoshIntegral[4*b*x]
 - (b*Sinh[4*a + 4*b*x])/(4*x) + b^2*Sinh[4*a]*SinhIntegral[4*b*x]
```

Rubi [A] time = 0.135611, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$b^2\cosh(4a)\text{Chi}(4bx)+b^2\sinh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\frac{1}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```



```
[Out] 1/(16*x^2) - Cosh[4*a + 4*b*x]/(16*x^2) + b^2*Cosh[4*a]*CoshIntegral[4*b*x]
 - (b*Sinh[4*a + 4*b*x])/(4*x) + b^2*Sinh[4*a]*SinhIntegral[4*b*x]
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{1}{8x^3}+\frac{\cosh(4a+4bx)}{8x^3}\right)dx\\ &= \frac{1}{16x^2}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^3}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+\frac{1}{4}b\int \frac{\sinh(4a+4bx)}{x^2}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+b^2\int \frac{\cosh(4a+4bx)}{x}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\left(b^2\cosh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx+\left(b\right.\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+b^2\cosh(4a)\mathbf{Chi}(4bx)-\frac{b\sinh(4a+4bx)}{4x}+b^2\sinh(4a)\end{aligned}$$

Mathematica [A] time = 0.10449, size = 65, normalized size = 0.97

$$\frac{16b^2x^2\cosh(4a)\mathbf{Chi}(4bx)+16b^2x^2\sinh(4a)\mathbf{Shi}(4bx)-4bx\sinh(4(a+bx))-\cosh(4(a+bx))+1}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (1 - Cosh[4*(a + b*x)] + 16*b^2*x^2*Cosh[4*a]*CoshIntegral[4*b*x] - 4*b*x*S
inh[4*(a + b*x)] + 16*b^2*x^2*Sinh[4*a]*SinhIntegral[4*b*x])/(16*x^2)
```

Maple [A] time = 0.055, size = 95, normalized size = 1.4

$$\frac{1}{16\,x^2}+\frac{b\mathrm{e}^{-4\,bx-4\,a}}{8\,x}-\frac{\mathrm{e}^{-4\,bx-4\,a}}{32\,x^2}-\frac{b^2\mathrm{e}^{-4\,a}\mathrm{Ei}\,(1,4\,bx)}{2}-\frac{\mathrm{e}^{4\,bx+4\,a}}{32\,x^2}-\frac{b\mathrm{e}^{4\,bx+4\,a}}{8\,x}-\frac{b^2\mathrm{e}^{4\,a}\mathrm{Ei}\,(1,-4\,bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 1/16/x^2+1/8*b*exp(-4*b*x-4*a)/x-1/32*exp(-4*b*x-4*a)/x^2-1/2*b^2*exp(-4*a)
*Ei(1,4*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a
)*Ei(1,-4*b*x)
```

Maxima [A] time = 1.20587, size = 49, normalized size = 0.73

$$-b^2e^{(-4\,a)}\Gamma\,(-2,4\,bx)-b^2e^{(4\,a)}\Gamma\,(-2,-4\,bx)+\frac{1}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -b^2*e^(-4*a)*gamma(-2, 4*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x) + 1/16/x^2
```

Fricas [B] time = 1.76163, size = 371, normalized size = 5.54

$$-\frac{16\,bx\cosh\,(bx+a)^3\sinh\,(bx+a)+16\,bx\cosh\,(bx+a)\sinh\,(bx+a)^3+\cosh\,(bx+a)^4+6\,\cosh\,(bx+a)^2\sinh\,(bx+a}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 + cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x +
a)^4 - 8*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*cosh(4*a) - 8*(b^2*x^2*Ei
(4*b*x) - b^2*x^2*Ei(-4*b*x))*sinh(4*a) - 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**3, x)
```

Giac [A] time = 1.18005, size = 120, normalized size = 1.79

$$\frac{16\,b^2x^2\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}+16\,b^2x^2\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-4\,bxe^{(4\,bx+4\,a)}+4\,bxe^{(-4\,bx-4\,a)}-e^{(4\,bx+4\,a)}-e^{(-4\,bx-4\,a)}+2}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x
*e^(4*b*x + 4*a) + 4*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4
*a) + 2)/x^2
```

3.297 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=92

$$\frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{24x^3}$$

```
[Out] 1/(24*x^3) - Cosh[4*a + 4*b*x]/(24*x^3) - (b^2*Cosh[4*a + 4*b*x])/(3*x) + (
4*b^3*CoshIntegral[4*b*x]*Sinh[4*a])/3 - (b*Sinh[4*a + 4*b*x])/(12*x^2) + (
4*b^3*Cosh[4*a]*SinhIntegral[4*b*x])/3
```

Rubi [A] time = 0.169529, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] 1/(24*x^3) - Cosh[4*a + 4*b*x]/(24*x^3) - (b^2*Cosh[4*a + 4*b*x])/(3*x) + (
4*b^3*CoshIntegral[4*b*x]*Sinh[4*a])/3 - (b*Sinh[4*a + 4*b*x])/(12*x^2) + (
4*b^3*Cosh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx &= \int \left(-\frac{1}{8x^4}+\frac{\cosh(4a+4bx)}{8x^4}\right)dx \\
&= \frac{1}{24x^3}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^4}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{6}b\int \frac{\sinh(4a+4bx)}{x^3}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}b^2\int \frac{\cosh(4a+4bx)}{x^2}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}\left(4b^3\right)\int \mathrm{sin} \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}\left(4b^3\cosh(4\right. \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}+\frac{4}{3}b^3\mathbf{Chi}(4bx)\sinh(4a)-\frac{b\sinh(4}{12}
\end{aligned}
$$

Mathematica [A] time = 0.191029, size = 79, normalized size = 0.86

$$
-\frac{-32b^3x^3\sinh(4a)\mathbf{Chi}(4bx)-32b^3x^3\cosh(4a)\mathbf{Shi}(4bx)+8b^2x^2\cosh(4(a+bx))+2bx\sinh(4(a+bx))+\cosh(4(a+b}{24x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] -(-1 + Cosh[4*(a + b*x)] + 8*b^2*x^2*Cosh[4*(a + b*x)] - 32*b^3*x^3*CoshInt
egral[4*b*x]*Sinh[4*a] + 2*b*x*Sinh[4*(a + b*x)] - 32*b^3*x^3*Cosh[4*a]*Sin
hIntegral[4*b*x])/(24*x^3)
```

Maple [A] time = 0.054, size = 129, normalized size = 1.4

$$\frac{1}{24\,x^3}-\frac{b^2\mathrm{e}^{-4\,bx-4\,a}}{6\,x}+\frac{b\mathrm{e}^{-4\,bx-4\,a}}{24\,x^2}-\frac{\mathrm{e}^{-4\,bx-4\,a}}{48\,x^3}+\frac{2\,b^3\mathrm{e}^{-4\,a}\mathrm{Ei}\,(1,4\,bx)}{3}-\frac{\mathrm{e}^{4\,bx+4\,a}}{48\,x^3}-\frac{b\mathrm{e}^{4\,bx+4\,a}}{24\,x^2}-\frac{b^2\mathrm{e}^{4\,bx+4\,a}}{6\,x}-\frac{2\,b^3\mathrm{e}^{4\,a}\mathrm{Ei}\,(1,-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x)
```

```
[Out] 1/24/x^3-1/6*b^2*exp(-4*b*x-4*a)/x+1/24*b*exp(-4*b*x-4*a)/x^2-1/48*exp(-4*b
*x-4*a)/x^3+2/3*b^3*exp(-4*a)*Ei(1,4*b*x)-1/48/x^3*exp(4*b*x+4*a)-1/24*b/x^
2*exp(4*b*x+4*a)-1/6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*exp(4*a)*Ei(1,-4*b*x)
```

Maxima [A] time = 1.33638, size = 49, normalized size = 0.53

$$-4\,b^3e^{(-4\,a)}\Gamma\,(-3,4\,bx)+4\,b^3e^{(4\,a)}\Gamma\,(-3,-4\,bx)+\frac{1}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x) + 1/24/x
^3
```

Fricas [B] time = 1.8123, size = 436, normalized size = 4.74

$$-\frac{8\,bx\cosh\,(bx+a)^3\sinh\,(bx+a)+8\,bx\cosh\,(bx+a)\sinh\,(bx+a)^3+\left(8\,b^2x^2+1\right)\cosh\,(bx+a)^4+6\left(8\,b^2x^2+1\right)\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/24*(8*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 8*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + (8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 6*(8*b^2*x^2 + 1)*cosh(b*x + a)^2
*sinh(b*x + a)^2 + (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(b^3*x^3*Ei(4*b*x)
- b^3*x^3*Ei(-4*b*x))*cosh(4*a) - 16*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x
))*sinh(4*a) - 1)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**4, x)
```

Giac [A] time = 1.1609, size = 166, normalized size = 1.8

$$\frac{32\,b^3x^3\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} - 32\,b^3x^3\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)} - 8\,b^2x^2e^{(4\,bx+4\,a)} - 8\,b^2x^2e^{(-4\,bx-4\,a)} - 2\,bxe^{(4\,bx+4\,a)} + 2\,bxe^{(-4\,bx-4\,a)} - e^{(4}}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2
*x^2*e^(4*b*x + 4*a) - 8*b^2*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) +
 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4*a) + 2)/x^3
```

3.298 $\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=209

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}+\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\text{Gamma}(m+1,$$

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) + (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) - (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rubi [A] time = 0.28686, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3307, 2181}

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}+\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\text{Gamma}(m+1,$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) + (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) - (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{8}x^m\cosh(a+bx)+\frac{1}{16}x^m\cosh(3a+3bx)+\frac{1}{16}x^m\cosh(5a+5bx)\right)dx$$

$$= \frac{1}{16}\int x^m\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^m\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^m\cosh(a+$$

$$= \frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{-i(5ia+5ibx)}x^m\,dx+\frac{1}{32}$$

$$= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b}+\frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b}-\frac{e^a x}{}$$

Mathematica [A] time = 0.294794, size = 175, normalized size = 0.84

$$\frac{e^{-5a}x^m\left(5e^{2a}3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\text{Gamma}(m+1,-3bx)-(-bx)^m\text{Gamma}(m+1,3bx)\right)+3\,5^{-m}\left(-b^2x^2\right)^{-m}\left(e^{10a}(b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-30*E^(6*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (30*E^(4*a)*Gamma[1 +
 m, b*x])/(b*x)^m + (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b
*x))^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Ga
mma[1 + m, -5*b*x] - (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m))
)/(480*b*E^(5*a))
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^m\left(\cosh(bx+a)\right)^3\left(\sinh(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

Maxima [A] time = 1.36135, size = 231, normalized size = 1.11

$$-\frac{1}{32}(5bx)^{-m-1}x^{m+1}e^{(-5a)}\Gamma(m+1,5bx)-\frac{1}{32}(3bx)^{-m-1}x^{m+1}e^{(-3a)}\Gamma(m+1,3bx)+\frac{1}{16}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x
)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/16*(b*x)^(-m - 1)*x^(
m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(
m + 1, -b*x) - 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x
) - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

Fricas [A] time = 2.00391, size = 764, normalized size = 3.66

$$-\frac{3\cosh(m\log(5b)+5a)\Gamma(m+1,5bx)+5\cosh(m\log(3b)+3a)\Gamma(m+1,3bx)-30\cosh(m\log(b)+a)\Gamma(m+1,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) + 5*cosh(m*log(3*b) +
3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) + 30*cos
h(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1
, -3*b*x) - 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1,
 5*b*x)*sinh(m*log(5*b) + 5*a) - 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*
a) - 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sin
h(m*log(-3*b) - 3*a) + 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*
gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.299 $\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=202

$$\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3}+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{3x\sinh(5a+5bx)}{1000b^3}$$

```
[Out] (3*Cosh[a + b*x])/(4*b^4) + (3*x^2*Cosh[a + b*x])/(8*b^2) - Cosh[3*a + 3*b*
x]/(216*b^4) - (x^2*Cosh[3*a + 3*b*x])/(48*b^2) - (3*Cosh[5*a + 5*b*x])/(50
00*b^4) - (3*x^2*Cosh[5*a + 5*b*x])/(400*b^2) - (3*x*Sinh[a + b*x])/(4*b^3)
 - (x^3*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(72*b^3) + (x^3*Sinh[3
*a + 3*b*x])/(48*b) + (3*x*Sinh[5*a + 5*b*x])/(1000*b^3) + (x^3*Sinh[5*a +
5*b*x])/(80*b)
```

Rubi [A] time = 0.267255, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2638}

$$\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3}+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{3x\sinh(5a+5bx)}{1000b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```


```
[Out] (3*Cosh[a + b*x])/(4*b^4) + (3*x^2*Cosh[a + b*x])/(8*b^2) - Cosh[3*a + 3*b*
x]/(216*b^4) - (x^2*Cosh[3*a + 3*b*x])/(48*b^2) - (3*Cosh[5*a + 5*b*x])/(50
00*b^4) - (3*x^2*Cosh[5*a + 5*b*x])/(400*b^2) - (3*x*Sinh[a + b*x])/(4*b^3)
 - (x^3*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(72*b^3) + (x^3*Sinh[3
*a + 3*b*x])/(48*b) + (3*x*Sinh[5*a + 5*b*x])/(1000*b^3) + (x^3*Sinh[5*a +
5*b*x])/(80*b)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}x^3\cosh(a+bx)+\frac{1}{16}x^3\cosh(3a+3bx)+\frac{1}{16}x^3\cosh(5a+5bx)\right)dx$$
$$=\frac{1}{16}\int x^3\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^3\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^3\cosh(a+bx$$
$$=-\frac{x^3\sinh(a+bx)}{8b}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{x^3\sinh(5a+5bx)}{80b}-\frac{3\int x^2\sinh(5a+}{80b}$$
$$=\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{x^3\sinh(a+bx)}{8b}$$
$$=\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3}$$
$$=\frac{3\cosh(a+bx)}{4b^4}+\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{216b^4}-\frac{x^2\cosh(3a+3bx)}{48b^2}-3$$

Mathematica [A] time = 1.08862, size = 125, normalized size = 0.62

$$\frac{101250\left(b^2x^2+2\right)\cosh(a+bx)-625\left(9b^2x^2+2\right)\cosh(3(a+bx))-81\left(25b^2x^2+2\right)\cosh(5(a+bx))+30bx\sinh(a+b}{270000b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (101250*(2 + b^2*x^2)*Cosh[a + b*x] - 625*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
 - 81*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(-6598 - 825*b^2*x^2 + 8*
(38 + 75*b^2*x^2)*Cosh[2*(a + b*x)] + 9*(6 + 25*b^2*x^2)*Cosh[4*(a + b*x)])
*Sinh[a + b*x])/(270000*b^4)
```

Maple [B] time = 0.012, size = 534, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/5*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^3*sinh(b*x+a)-1
/15*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2-3/25*(b*x+a)^2*sinh(b*x+a)^2*cosh(b
*x+a)^3-4/75*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+26/75*(b*x+a)^2*cosh(b*x+a
)+6/125*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4-856/1125*(b*x+a)*sinh(b*x+a)+22/1
125*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-6/625*cosh(b*x+a)^3*sinh(b*x+a)^2-272
/16875*cosh(b*x+a)*sinh(b*x+a)^2+12568/16875*cosh(b*x+a)-3*a*(1/5*(b*x+a)^2
*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^2*sinh(b*x+a)-1/15*(b*x+a)^2*sinh(b
*x+a)*cosh(b*x+a)^2-2/25*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-8/225*(b*x+a)*
sinh(b*x+a)^2*cosh(b*x+a)+52/225*(b*x+a)*cosh(b*x+a)+2/125*cosh(b*x+a)^4*si
nh(b*x+a)-856/3375*sinh(b*x+a)+22/3375*sinh(b*x+a)*cosh(b*x+a)^2)+3*a^2*(1/
5*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+a)-1/15*(b*x+a)*s
inh(b*x+a)*cosh(b*x+a)^2-1/25*cosh(b*x+a)^3*sinh(b*x+a)^2-4/225*cosh(b*x+a)
*sinh(b*x+a)^2+26/225*cosh(b*x+a))-a^3*(1/5*cosh(b*x+a)^4*sinh(b*x+a)-1/5*(
2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)))
```

Maxima [A] time = 1.08415, size = 331, normalized size = 1.64

$$\frac{\left(125\,b^3x^3e^{(5\,a)}-75\,b^2x^2e^{(5\,a)}+30\,bxe^{(5\,a)}-6\,e^{(5\,a)}\right)e^{(5\,bx)}}{20000\,b^4}+\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^4}-\frac{\left(b^3x^3e^{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 + 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

Fricas [A] time = 1.75494, size = 716, normalized size = 3.54

$$-\frac{81\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+405\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-135\left(25\,b^3x^3+6\,bx\right)\sinh\left(bx+a\right)^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/270000*(81*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 405*(25*b^2*x^2 + 2)*cosh(
b*x + a)*sinh(b*x + a)^4 - 135*(25*b^3*x^3 + 6*b*x)*sinh(b*x + a)^5 + 625*(
9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 75*(75*b^3*x^3 + 18*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^2 + 50*b*x)*sinh(b*x + a)^3 + 15*(54*(25*b^2*x^2 + 2)*cosh(b*x
 + a)^3 + 125*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2 - 101250*(b^2*
x^2 + 2)*cosh(b*x + a) + 225*(150*b^3*x^3 - 3*(25*b^3*x^3 + 6*b*x)*cosh(b*x
 + a)^4 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 900*b*x)*sinh(b*x + a))/
b^4
```

Sympy [A] time = 13.5314, size = 253, normalized size = 1.25

$$\begin{cases} -\frac{2x^3\sinh^5(a+bx)}{15b} + \frac{x^3\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{2x^2\sinh^4(a+bx)\cosh(a+bx)}{5b^2} - \frac{13x^2\sinh^2(a+bx)\cosh^3(a+bx)}{15b^2} + \frac{26x^2\cosh^5(a+bx)}{75b^2} - \frac{856x\sinh \ldots}{1\ldots} \\ \frac{x^4\sinh^2(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**3*sinh(a + b*x)**5/(15*b) + x**3*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 2*x**2*sinh(a + b*x)**4*cosh(a + b*x)/(5*b**2) - 13*x**2*s
inh(a + b*x)**2*cosh(a + b*x)**3/(15*b**2) + 26*x**2*cosh(a + b*x)**5/(75*b
**2) - 856*x*sinh(a + b*x)**5/(1125*b**3) + 338*x*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(225*b**3) - 52*x*sinh(a + b*x)*cosh(a + b*x)**4/(75*b**3) + 856*s
inh(a + b*x)**4*cosh(a + b*x)/(1125*b**4) - 5114*sinh(a + b*x)**2*cosh(a +
b*x)**3/(3375*b**4) + 12568*cosh(a + b*x)**5/(16875*b**4), Ne(b, 0)), (x**4
*sinh(a)**2*cosh(a)**3/4, True))
```

Giac [A] time = 1.1853, size = 286, normalized size = 1.42

$$\frac{\left(125\,b^3x^3 - 75\,b^2x^2 + 30\,bx - 6\right)e^{(5\,bx+5\,a)}}{20000\,b^4} + \frac{\left(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2\right)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{\left(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6\right)e^{(bx+a)}}{16\,b^4} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 + 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

3.300 $\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{\sinh(a+bx)}{4b^3}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{\sinh(5a+5bx)}{1000b^3}+\frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2\sinh}{}$$

```
[Out] (x*Cosh[a + b*x])/(4*b^2) - (x*Cosh[3*a + 3*b*x])/(72*b^2) - (x*Cosh[5*a +
5*b*x])/(200*b^2) - Sinh[a + b*x]/(4*b^3) - (x^2*Sinh[a + b*x])/(8*b) + Sin
h[3*a + 3*b*x]/(216*b^3) + (x^2*Sinh[3*a + 3*b*x])/(48*b) + Sinh[5*a + 5*b*
x]/(1000*b^3) + (x^2*Sinh[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.180373, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2637}

$$-\frac{\sinh(a+bx)}{4b^3}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{\sinh(5a+5bx)}{1000b^3}+\frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```


```
[Out] (x*Cosh[a + b*x])/(4*b^2) - (x*Cosh[3*a + 3*b*x])/(72*b^2) - (x*Cosh[5*a +
5*b*x])/(200*b^2) - Sinh[a + b*x]/(4*b^3) - (x^2*Sinh[a + b*x])/(8*b) + Sin
h[3*a + 3*b*x]/(216*b^3) + (x^2*Sinh[3*a + 3*b*x])/(48*b) + Sinh[5*a + 5*b*
x]/(1000*b^3) + (x^2*Sinh[5*a + 5*b*x])/(80*b)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}x^2\cosh(a+bx)+\frac{1}{16}x^2\cosh(3a+3bx)+\frac{1}{16}x^2\cosh(5a+5bx)\right)dx$$
$$= \frac{1}{16}\int x^2\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^2\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^2\cosh(a+bx)$$
$$= -\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{x^2\sinh(5a+5bx)}{80b}-\frac{\int x\sinh(5a+5bx}{40b}$$
$$= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2 s}{}$$
$$= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{\sinh(a+bx)}{4b^3}-\frac{x^2\sinh}{}$$

Mathematica [A] time = 0.308318, size = 105, normalized size = 0.71

$$\frac{-6750\left(\left(b^2x^2+2\right)\sinh(a+bx)-2bx\cosh(a+bx)\right)+125\left(\left(9b^2x^2+2\right)\sinh(3(a+bx))-6bx\cosh(3(a+bx))\right)+27\left(\left(25\right.\right.}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-6750*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 125*(-6*b*x*C
osh[3*(a + b*x)] + (2 + 9*b^2*x^2)*Sinh[3*(a + b*x)]) + 27*(-10*b*x*Cosh[5*
(a + b*x)] + (2 + 25*b^2*x^2)*Sinh[5*(a + b*x)]))/(54000*b^3)
```

Maple [B] time = 0.008, size = 306, normalized size = 2.1

$$\frac{1}{b^3}\left(\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^4}{5}-\frac{2\ (bx+a)^2\sinh(bx+a)}{15}-\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^2}{15}-\frac{(2\,bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/5*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^2*sinh(b*x+a)-1
/15*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-2/25*(b*x+a)*sinh(b*x+a)^2*cosh(b*x
+a)^3-8/225*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+52/225*(b*x+a)*cosh(b*x+a)+2/
125*cosh(b*x+a)^4*sinh(b*x+a)-856/3375*sinh(b*x+a)+22/3375*sinh(b*x+a)*cosh
(b*x+a)^2-2*a*(1/5*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+
a)-1/15*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-1/25*cosh(b*x+a)^3*sinh(b*x+a)^2-
4/225*cosh(b*x+a)*sinh(b*x+a)^2+26/225*cosh(b*x+a))+a^2*(1/5*cosh(b*x+a)^4*
sinh(b*x+a)-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)))
```

Maxima [A] time = 1.06848, size = 252, normalized size = 1.7

$$\frac{\left(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3} + \frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3} - \frac{\left(b^2x^2e^{a} - 2\,bxe^{a} + 2\,e^{a}\right)e^{(bx)}}{16\,b^3} + \frac{\left(b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 + 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 - 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

Fricas [A] time = 1.75437, size = 579, normalized size = 3.91

$$-\frac{270\,bx\cosh\left(bx + a\right)^5 + 1350\,bx\cosh\left(bx + a\right)\sinh\left(bx + a\right)^4 - 27\left(25\,b^2x^2 + 2\right)\sinh\left(bx + a\right)^5 + 750\,bx\cosh\left(bx + a\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/54000*(270*b*x*cosh(b*x + a)^5 + 1350*b*x*cosh(b*x + a)*sinh(b*x + a)^4
- 27*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 + 750*b*x*cosh(b*x + a)^3 - 5*(225*b^
2*x^2 + 54*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 50)*sinh(b*x + a)^3 - 13500*b
*x*cosh(b*x + a) + 450*(6*b*x*cosh(b*x + a)^3 + 5*b*x*cosh(b*x + a))*sinh(b
*x + a)^2 - 15*(9*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 450*b^2*x^2 + 25*(9*b^
2*x^2 + 2)*cosh(b*x + a)^2 - 900)*sinh(b*x + a))/b^3
```

Sympy [A] time = 7.97986, size = 182, normalized size = 1.23

$$\begin{cases} -\frac{2x^2\sinh^5(a+bx)}{15b} + \frac{x^2\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{4x\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{26x\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{52x\cosh^5(a+bx)}{225b^2} - \frac{856\sinh^5(a}{3375b^3} \\ \frac{x^3\sinh^2(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**2*sinh(a + b*x)**5/(15*b) + x**2*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 4*x*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 26*x*sinh(a
 + b*x)**2*cosh(a + b*x)**3/(45*b**2) + 52*x*cosh(a + b*x)**5/(225*b**2) -
856*sinh(a + b*x)**5/(3375*b**3) + 338*sinh(a + b*x)**3*cosh(a + b*x)**2/(6
75*b**3) - 52*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**3), Ne(b, 0)), (x**3*s
inh(a)**2*cosh(a)**3/3, True))
```

Giac [A] time = 1.18424, size = 221, normalized size = 1.49

$$\frac{\left(25\,b^2x^2-10\,bx+2\right)e^{(5\,bx+5\,a)}}{4000\,b^3}+\frac{\left(9\,b^2x^2-6\,bx+2\right)e^{(3\,bx+3\,a)}}{864\,b^3}-\frac{\left(b^2x^2-2\,bx+2\right)e^{(bx+a)}}{16\,b^3}+\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{16\,b^3}-\frac{(9\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 + 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
+ 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 - 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

3.301 $\int x \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

```
[Out] Cosh[a + b*x]/(8*b^2) - Cosh[3*a + 3*b*x]/(144*b^2) - Cosh[5*a + 5*b*x]/(40
0*b^2) - (x*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(48*b) + (x*Sinh[5
*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.0958022, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2638}

$$\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(8*b^2) - Cosh[3*a + 3*b*x]/(144*b^2) - Cosh[5*a + 5*b*x]/(40
0*b^2) - (x*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(48*b) + (x*Sinh[5
*a + 5*b*x])/(80*b)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx &= \int\left(-\frac{1}{8}x\cosh(a+bx)+\frac{1}{16}x\cosh(3a+3bx)+\frac{1}{16}x\cosh(5a+5bx)\right)dx\\ &= \frac{1}{16}\int x\cosh(3a+3bx)\,dx+\frac{1}{16}\int x\cosh(5a+5bx)\,dx-\frac{1}{8}\int x\cosh(a+bx)\,dx\\ &= -\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}-\frac{\int\sinh(5a+5bx)\,dx}{80b}-\\ &= \frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a-}{48b}\end{aligned}$$

Mathematica [A] time = 0.192816, size = 70, normalized size = 0.74

$$\frac{-450bx\sinh(a+bx)+75bx\sinh(3(a+bx))+45bx\sinh(5(a+bx))+450\cosh(a+bx)-25\cosh(3(a+bx))-9\cosh(5}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (450*Cosh[a + b*x] - 25*Cosh[3*(a + b*x)] - 9*Cosh[5*(a + b*x)] - 450*b*x*S
inh[a + b*x] + 75*b*x*Sinh[3*(a + b*x)] + 45*b*x*Sinh[5*(a + b*x)])/(3600*b
^2)
```

Maple [A] time = 0.007, size = 143, normalized size = 1.5

$$\frac{1}{b^2}\left(\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^4}{5}-\frac{(2\,bx+2\,a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^2}{15}-(\cosh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/5*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+a)-1/15*
(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-1/25*cosh(b*x+a)^3*sinh(b*x+a)^2-4/225*co
```

```
sh(b*x+a)*sinh(b*x+a)^2+26/225*cosh(b*x+a)-a*(1/5*cosh(b*x+a)^4*sinh(b*x+a)
-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)))
```

Maxima [A] time = 1.06624, size = 174, normalized size = 1.85

$$\frac{\left(5\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2} + \frac{\left(3\,bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2} - \frac{(bxe^{a} - e^{a})e^{(bx)}}{16\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} - \frac{(5\,bx+1}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 + 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 + 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

Fricas [A] time = 1.834, size = 425, normalized size = 4.52

$$\frac{45\,bx\sinh\left(bx+a\right)^5 - 9\,\cosh\left(bx+a\right)^5 - 45\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 + 75\left(6\,bx\cosh\left(bx+a\right)^2 + bx\right)\sinh\left(bx+a\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*sinh(b*x + a)^5 - 9*cosh(b*x + a)^5 - 45*cosh(b*x + a)*sinh(
b*x + a)^4 + 75*(6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 - 25*cosh(b*x
 + a)^3 - 15*(6*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^2 + 225*(b
*x*cosh(b*x + a)^4 + b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) + 450*cosh(
b*x + a))/b^2
```

Sympy [A] time = 4.41347, size = 112, normalized size = 1.19

$$\begin{cases} -\frac{2x\sinh^5(a+bx)}{15b} + \frac{x\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{2\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{13\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{26\cosh^5(a+bx)}{225b^2} & \text{for } b \neq 0 \\ \frac{x^2\sinh^2(a)\cosh^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x*sinh(a + b*x)**5/(15*b) + x*sinh(a + b*x)**3*cosh(a + b*x)*
*2/(3*b) + 2*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 13*sinh(a + b*x)**2
*cosh(a + b*x)**3/(45*b**2) + 26*cosh(a + b*x)**5/(225*b**2), Ne(b, 0)), (x
**2*sinh(a)**2*cosh(a)**3/2, True))
```

Giac [A] time = 1.16221, size = 157, normalized size = 1.67

$$\frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2}+\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2}-\frac{(bx-1)e^{(bx+a)}}{16\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}-\frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 + 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 + 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

3.302 $\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^5(a+bx)}{5b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] Sinh[a + b*x]^3/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0329313, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^5(a+bx)}{5b}+\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{i\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{i\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^3(a+bx)}{3b} + \frac{\sinh^5(a+bx)}{5b}$$

Mathematica [A] time = 0.0593704, size = 27, normalized size = 0.87

$$\frac{\sinh^3(a+bx)(3\cosh(2(a+bx))+7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] ((7 + 3*Cosh[2*(a + b*x)])*Sinh[a + b*x]^3)/(30*b)
```

Maple [A] time = 0.009, size = 42, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4\sinh(bx+a)}{5} - \frac{\sinh(bx+a)}{5}\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^4*sinh(b*x+a)-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

Maxima [B] time = 1.01285, size = 105, normalized size = 3.39

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-30\,e^{(-4\,bx-4\,a)}+3\right)e^{(5\,bx+5\,a)}}{480\,b}+\frac{30\,e^{(-bx-a)}-5\,e^{(-3\,bx-3\,a)}-3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*e^(-2*b*x - 2*a) - 30*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
480*(30*e^(-b*x - a) - 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

Fricas [B] time = 1.79896, size = 178, normalized size = 5.74

$$\frac{3 \sinh (bx+a)^5 + 5 \left(6 \cosh (bx+a)^2 + 1\right) \sinh (bx+a)^3 + 15 \left(\cosh (bx+a)^4 + \cosh (bx+a)^2 - 2\right) \sinh (bx+a)}{240 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + cosh(b*x + a)^2 - 2)*sinh(b*x + a))/b
```

Sympy [A] time = 2.11098, size = 44, normalized size = 1.42

$$\begin{cases} -\frac{2 \sinh^5 (a+bx)}{15b} + \frac{\sinh^3 (a+bx) \cosh^2 (a+bx)}{3b} & \text{for } b \neq 0 \\ x \sinh^2 (a) \cosh^3 (a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*sinh(a + b*x)**5/(15*b) + sinh(a + b*x)**3*cosh(a + b*x)**2/(
3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**3, True))
```

Giac [B] time = 1.15691, size = 95, normalized size = 3.06

$$\frac{\left(30 e^{(4 bx+4 a)} - 5 e^{(2 bx+2 a)} - 3\right) e^{(-5 bx-5 a)} + 3 e^{(5 bx+5 a)} + 5 e^{(3 bx+3 a)} - 30 e^{(bx+a)}}{480 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/480*((30*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) - 3)*e^(-5*b*x - 5*a) + 3*e^
(5*b*x + 5*a) + 5*e^(3*b*x + 3*a) - 30*e^(b*x + a))/b
```

3.303 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{8}\cosh(a)\mathbf{Chi}(bx)+\frac{1}{16}\cosh(3a)\mathbf{Chi}(3bx)+\frac{1}{16}\cosh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}\sinh(a)\mathbf{Shi}(bx)+\frac{1}{16}\sinh(3a)\mathbf{Shi}(3bx)+\frac{1}{16}\,\mathrm{s}$$

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/8 + (Cosh[3*a]*CoshIntegral[3*b*x])/16 + (Cosh
[5*a]*CoshIntegral[5*b*x])/16 - (Sinh[a]*SinhIntegral[b*x])/8 + (Sinh[3*a]*
SinhIntegral[3*b*x])/16 + (Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Rubi [A] time = 0.190071, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{8}\cosh(a)\mathbf{Chi}(bx)+\frac{1}{16}\cosh(3a)\mathbf{Chi}(3bx)+\frac{1}{16}\cosh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}\sinh(a)\mathbf{Shi}(bx)+\frac{1}{16}\sinh(3a)\mathbf{Shi}(3bx)+\frac{1}{16}\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/8 + (Cosh[3*a]*CoshIntegral[3*b*x])/16 + (Cosh
[5*a]*CoshIntegral[5*b*x])/16 - (Sinh[a]*SinhIntegral[b*x])/8 + (Sinh[3*a]*
SinhIntegral[3*b*x])/16 + (Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \left(-\frac{\cosh(a+bx)}{8x}+\frac{\cosh(3a+3bx)}{16x}+\frac{\cosh(5a+5bx)}{16x}\right)dx$$
$$= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x}\,dx$$
$$= -\left(\frac{1}{8}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right)+\frac{1}{16}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx+\frac{1}{16}\cosh(5a)\int \frac{\text{co}}{}$$
$$= -\frac{1}{8}\cosh(a)\mathbf{Chi}(bx)+\frac{1}{16}\cosh(3a)\mathbf{Chi}(3bx)+\frac{1}{16}\cosh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}\sinh(a)\mathbf{Sh}$$

Mathematica [A] time = 0.104992, size = 61, normalized size = 0.84

$$\frac{1}{16}(-2\cosh(a)\mathbf{Chi}(bx)+\cosh(3a)\mathbf{Chi}(3bx)+\cosh(5a)\mathbf{Chi}(5bx)-2\sinh(a)\mathbf{Shi}(bx)+\sinh(3a)\mathbf{Shi}(3bx)+\sinh(5a)\mathbf{Shi}($$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (-2*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] + Cosh[5*a]*C
oshIntegral[5*b*x] - 2*Sinh[a]*SinhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3
*b*x] + Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Maple [A] time = 0.105, size = 71, normalized size = 1.

$$-\frac{e^{-5a}\mathrm{Ei}(1,5bx)}{32}-\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{32}+\frac{e^{-a}\mathrm{Ei}(1,bx)}{16}+\frac{e^{a}\mathrm{Ei}(1,-bx)}{16}-\frac{e^{3a}\mathrm{Ei}(1,-3bx)}{32}-\frac{e^{5a}\mathrm{Ei}(1,-5bx)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-a)*Ei(1,b*
x)+1/16*exp(a)*Ei(1,-b*x)-1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*
b*x)
```

Maxima [A] time = 1.30402, size = 86, normalized size = 1.18

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Fricas [A] time = 1.74682, size = 323, normalized size = 4.42

$$\frac{1}{32}\,\left(\mathrm{Ei}\left(5\,bx\right)+\mathrm{Ei}\left(-5\,bx\right)\right)\cosh\left(5\,a\right)+\frac{1}{32}\,\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{1}{16}\,\left(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right)\right)\cosh\left(a\right)+\frac{1}{32}\,(\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) + Ei(-5*b*x))*cosh(5*a) + 1/32*(Ei(3*b*x) + Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) + Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) - Ei(-5*b*x))*
sinh(5*a) + 1/32*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) - Ei(-b
*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x, x)
```

Giac [A] time = 1.15999, size = 86, normalized size = 1.18

$$\frac{1}{32}\operatorname{Ei}(5\,bx)\,e^{(5\,a)} + \frac{1}{32}\operatorname{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{16}\operatorname{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\operatorname{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{32}\operatorname{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\operatorname{Ei}(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.304 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{1}{8}b\sinh(a)\mathbf{Chi}(bx)+\frac{3}{16}b\sinh(3a)\mathbf{Chi}(3bx)+\frac{5}{16}b\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}b\cosh(a)\mathbf{Shi}(bx)+\frac{3}{16}b\cosh(3a)\mathbf{Shi}(3bx)$$

```
[Out] Cosh[a + b*x]/(8*x) - Cosh[3*a + 3*b*x]/(16*x) - Cosh[5*a + 5*b*x]/(16*x) -
 (b*CoshIntegral[b*x]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/16 +
 (5*b*CoshIntegral[5*b*x]*Sinh[5*a])/16 - (b*Cosh[a]*SinhIntegral[b*x])/8 +
 (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Cosh[5*a]*SinhIntegral[5*b*x
])/16
```

Rubi [A] time = 0.263126, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b\sinh(a)\mathbf{Chi}(bx)+\frac{3}{16}b\sinh(3a)\mathbf{Chi}(3bx)+\frac{5}{16}b\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}b\cosh(a)\mathbf{Shi}(bx)+\frac{3}{16}b\cosh(3a)\mathbf{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(8*x) - Cosh[3*a + 3*b*x]/(16*x) - Cosh[5*a + 5*b*x]/(16*x) -
 (b*CoshIntegral[b*x]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/16 +
 (5*b*CoshIntegral[5*b*x]*Sinh[5*a])/16 - (b*Cosh[a]*SinhIntegral[b*x])/8 +
 (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Cosh[5*a]*SinhIntegral[5*b*x
])/16
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{\cosh(a+bx)}{8x^2}+\frac{\cosh(3a+3bx)}{16x^2}+\frac{\cosh(5a+5bx)}{16x^2}\right)dx \\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^2}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^2}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^2}\,dx \\
&= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}b\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{16}(3 \\
&= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+ \\
&= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}b\,\mathbf{Chi}(bx)\sinh(a)+\frac{3}{16}b\,\mathbf{Chi}
\end{aligned}$$

Mathematica [A] time = 0.354474, size = 104, normalized size = 0.84

$$-\frac{2bx\sinh(a)\mathbf{Chi}(bx)-3bx\sinh(3a)\mathbf{Chi}(3bx)-5bx\sinh(5a)\mathbf{Chi}(5bx)+2bx\cosh(a)\mathbf{Shi}(bx)-3bx\cosh(3a)\mathbf{Shi}(3bx)-}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b*x*CoshInte
gral[b*x]*Sinh[a] - 3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] - 5*b*x*CoshIntegra
l[5*b*x]*Sinh[5*a] + 2*b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x*Cosh[3*a]*Sinh
Integral[3*b*x] - 5*b*x*Cosh[5*a]*SinhIntegral[5*b*x])/(16*x)
```

Maple [A] time = 0.109, size = 158, normalized size = 1.3

$$-\frac{e^{-5bx-5a}}{32x}+\frac{5be^{-5a}\mathrm{Ei}(1,5bx)}{32}-\frac{e^{-3bx-3a}}{32x}+\frac{3be^{-3a}\mathrm{Ei}(1,3bx)}{32}+\frac{e^{-bx-a}}{16x}-\frac{be^{-a}\mathrm{Ei}(1,bx)}{16}+\frac{e^{bx+a}}{16x}+\frac{be^{a}\mathrm{Ei}(1,-bx)}{16}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

```
[Out] -1/32*exp(-5*b*x-5*a)/x+5/32*b*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*b*x-3*a)/x
+3/32*b*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-b*x-a)/x-1/16*b*exp(-a)*Ei(1,b*x)+1
/16/x*exp(b*x+a)+1/16*b*exp(a)*Ei(1,-b*x)-1/32/x*exp(3*b*x+3*a)-3/32*b*exp(
3*a)*Ei(1,-3*b*x)-1/32/x*exp(5*b*x+5*a)-5/32*b*exp(5*a)*Ei(1,-5*b*x)
```

Maxima [A] time = 1.26234, size = 103, normalized size = 0.83

$$-\frac{5}{32}be^{(-5a)}\Gamma(-1,5bx)-\frac{3}{32}be^{(-3a)}\Gamma(-1,3bx)+\frac{1}{16}be^{(-a)}\Gamma(-1,bx)-\frac{1}{16}be^{a}\Gamma(-1,-bx)+\frac{3}{32}be^{(3a)}\Gamma(-1,-3bx)+\frac{5}{32}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/16
*b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) + 3/32*b*e^(3*a)*gamm
a(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

Fricas [B] time = 1.76466, size = 582, normalized size = 4.69

$$-\frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\cosh(bx+a)^3+2\left(10\cosh(bx+a)^3+3\cosh(bx+a)\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*cosh(b*x +
a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(b*x*Ei
(5*b*x) - b*x*Ei(-5*b*x))*cosh(5*a) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*co
sh(3*a) + 2*(b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 5*(b*x*Ei(5*b*x) + b*x*E
i(-5*b*x))*sinh(5*a) - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*
x*Ei(b*x) + b*x*Ei(-b*x))*sinh(a) - 4*cosh(b*x + a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**2, x)
```

Giac [A] time = 1.21371, size = 194, normalized size = 1.56

$$\frac{5\,bx\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} + 3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + 2\,bx\mathrm{Ei}\left(-bx\right)e^{(-a)} - 3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - 5\,bx\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)} - 2\,bx\mathrm{Ei}\left(bx\right)e^{a} -}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) + 3*b*x*Ei(3*b*x)*e^(3*a) + 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) - 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x
```

3.305 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{Sh}$$

```
[Out] Cosh[a + b*x]/(16*x^2) - Cosh[3*a + 3*b*x]/(32*x^2) - Cosh[5*a + 5*b*x]/(32
*x^2) - (b^2*Cosh[a]*CoshIntegral[b*x])/16 + (9*b^2*Cosh[3*a]*CoshIntegral[
3*b*x])/32 + (25*b^2*Cosh[5*a]*CoshIntegral[5*b*x])/32 + (b*Sinh[a + b*x])/
(16*x) - (3*b*Sinh[3*a + 3*b*x])/(32*x) - (5*b*Sinh[5*a + 5*b*x])/(32*x) -
(b^2*Sinh[a]*SinhIntegral[b*x])/16 + (9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Sinh[5*a]*SinhIntegral[5*b*x])/32
```

Rubi [A] time = 0.343936, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{Sh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```


```
[Out] Cosh[a + b*x]/(16*x^2) - Cosh[3*a + 3*b*x]/(32*x^2) - Cosh[5*a + 5*b*x]/(32
*x^2) - (b^2*Cosh[a]*CoshIntegral[b*x])/16 + (9*b^2*Cosh[3*a]*CoshIntegral[
3*b*x])/32 + (25*b^2*Cosh[5*a]*CoshIntegral[5*b*x])/32 + (b*Sinh[a + b*x])/
(16*x) - (3*b*Sinh[3*a + 3*b*x])/(32*x) - (5*b*Sinh[5*a + 5*b*x])/(32*x) -
(b^2*Sinh[a]*SinhIntegral[b*x])/16 + (9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Sinh[5*a]*SinhIntegral[5*b*x])/32
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{\cosh(a+bx)}{8x^3}+\frac{\cosh(3a+3bx)}{16x^3}+\frac{\cosh(5a+5bx)}{16x^3}\right)dx \\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^3}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^3}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^3}\,dx \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{32}(\\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+}{32x} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+}{32x} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b^2\cosh(a)\mathbf{Chi}(bx)+\frac{9}{32}b^2
\end{aligned}$$

Mathematica [A] time = 0.558273, size = 162, normalized size = 0.88

$$-\frac{2b^2x^2\cosh(a)\mathbf{Chi}(bx)-9b^2x^2\cosh(3a)\mathbf{Chi}(3bx)-25b^2x^2\cosh(5a)\mathbf{Chi}(5bx)+2b^2x^2\sinh(a)\mathbf{Shi}(bx)-9b^2x^2\sinh(3a)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] -(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b^2*x^2*Cosh
[a]*CoshIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] - 25*b^2*x^
2*Cosh[5*a]*CoshIntegral[5*b*x] - 2*b*x*Sinh[a + b*x] + 3*b*x*Sinh[3*(a + b
*x)] + 5*b*x*Sinh[5*(a + b*x)] + 2*b^2*x^2*Sinh[a]*SinhIntegral[b*x] - 9*b^
2*x^2*Sinh[3*a]*SinhIntegral[3*b*x] - 25*b^2*x^2*Sinh[5*a]*SinhIntegral[5*b
*x])/(32*x^2)
```

Maple [A] time = 0.115, size = 257, normalized size = 1.4

$$\frac{5\,b\mathrm{e}^{-5\,bx-5\,a}}{64\,x}-\frac{\mathrm{e}^{-5\,bx-5\,a}}{64\,x^2}-\frac{25\,b^2\mathrm{e}^{-5\,a}\mathrm{Ei}\,(1,5\,bx)}{64}+\frac{3\,b\mathrm{e}^{-3\,bx-3\,a}}{64\,x}-\frac{\mathrm{e}^{-3\,bx-3\,a}}{64\,x^2}-\frac{9\,b^2\mathrm{e}^{-3\,a}\mathrm{Ei}\,(1,3\,bx)}{64}-\frac{b\mathrm{e}^{-bx-a}}{32\,x}+\frac{\mathrm{e}^{-bx-a}}{32\,x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 5/64*b*exp(-5*b*x-5*a)/x-1/64*exp(-5*b*x-5*a)/x^2-25/64*b^2*exp(-5*a)*Ei(1,
5*b*x)+3/64*b*exp(-3*b*x-3*a)/x-1/64*exp(-3*b*x-3*a)/x^2-9/64*b^2*exp(-3*a)
*Ei(1,3*b*x)-1/32*b*exp(-b*x-a)/x+1/32*exp(-b*x-a)/x^2+1/32*b^2*exp(-a)*Ei(
1,b*x)+1/32/x^2*exp(b*x+a)+1/32*b/x*exp(b*x+a)+1/32*b^2*exp(a)*Ei(1,-b*x)-1
/64/x^2*exp(3*b*x+3*a)-3/64*b/x*exp(3*b*x+3*a)-9/64*b^2*exp(3*a)*Ei(1,-3*b*
x)-1/64/x^2*exp(5*b*x+5*a)-5/64*b/x*exp(5*b*x+5*a)-25/64*b^2*exp(5*a)*Ei(1,
-5*b*x)
```

Maxima [A] time = 1.31934, size = 119, normalized size = 0.65

$$-\frac{25}{32}\,b^2e^{(-5\,a)}\Gamma\,(-2,5\,bx)-\frac{9}{32}\,b^2e^{(-3\,a)}\Gamma\,(-2,3\,bx)+\frac{1}{16}\,b^2e^{(-a)}\Gamma\,(-2,bx)+\frac{1}{16}\,b^2e^{a}\Gamma\,(-2,-bx)-\frac{9}{32}\,b^2e^{(3\,a)}\Gamma\,(-2,-3\,bx)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) +
 1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) - 9/32*b^2*e
```

```
^(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

Fricas [B] time = 1.80532, size = 859, normalized size = 4.67

$$-\frac{10\,bx\sinh\left(bx+a\right)^{5}+2\,\cosh\left(bx+a\right)^{5}+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+2\left(50\,bx\cosh\left(bx+a\right)^{2}+3\,bx\right)\sinh\left(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*sinh(b*x + a)^5 + 2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b
*x + a)^4 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*cosh(b*x
 + a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 25*(b^
2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*cosh(5*a) - 9*(b^2*x^2*Ei(3*b*x) + b^
2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*cosh(a
) + 2*(25*b*x*cosh(b*x + a)^4 + 9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a
) - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*sinh(5*a) - 9*(b^2*x^2*Ei(3
*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*
x))*sinh(a) - 4*cosh(b*x + a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**3, x)
```

Giac [A] time = 1.24275, size = 328, normalized size = 1.78

$$\frac{25\,b^{2}x^{2}\text{Ei}\left(5\,bx\right)e^{(5\,a)}+9\,b^{2}x^{2}\text{Ei}\left(3\,bx\right)e^{(3\,a)}-2\,b^{2}x^{2}\text{Ei}\left(-bx\right)e^{(-a)}+9\,b^{2}x^{2}\text{Ei}\left(-3\,bx\right)e^{(-3\,a)}+25\,b^{2}x^{2}\text{Ei}\left(-5\,bx\right)e^{(-5\,a)}-2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) + 9*b^2*x^2*Ei(3*b*x)*e^(3*a) - 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) + 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) - 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) + 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) +
2*e^(-b*x - a) - e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x^2
```

3.306 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{1}{48}b^3\sinh(a)\mathbf{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\mathbf{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\mathbf{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)\mathbf{Sh}$$

```
[Out] Cosh[a + b*x]/(24*x^3) + (b^2*Cosh[a + b*x])/(48*x) - Cosh[3*a + 3*b*x]/(48
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(32*x) - Cosh[5*a + 5*b*x]/(48*x^3) - (25
*b^2*Cosh[5*a + 5*b*x])/(96*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/48 + (9*b^
3*CoshIntegral[3*b*x]*Sinh[3*a])/32 + (125*b^3*CoshIntegral[5*b*x]*Sinh[5*a
])/96 + (b*Sinh[a + b*x])/(48*x^2) - (b*Sinh[3*a + 3*b*x])/(32*x^2) - (5*b*
Sinh[5*a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/48 + (9*b^3*C
osh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*SinhIntegral[5*b*x])/
96
```

Rubi [A] time = 0.444285, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{48}b^3\sinh(a)\mathbf{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\mathbf{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\mathbf{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)\mathbf{Sh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```


```
[Out] Cosh[a + b*x]/(24*x^3) + (b^2*Cosh[a + b*x])/(48*x) - Cosh[3*a + 3*b*x]/(48
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(32*x) - Cosh[5*a + 5*b*x]/(48*x^3) - (25
*b^2*Cosh[5*a + 5*b*x])/(96*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/48 + (9*b^
3*CoshIntegral[3*b*x]*Sinh[3*a])/32 + (125*b^3*CoshIntegral[5*b*x]*Sinh[5*a
])/96 + (b*Sinh[a + b*x])/(48*x^2) - (b*Sinh[3*a + 3*b*x])/(32*x^2) - (5*b*
Sinh[5*a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/48 + (9*b^3*C
osh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*SinhIntegral[5*b*x])/
96
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \left(-\frac{\cosh(a+bx)}{8x^4} + \frac{\cosh(3a+3bx)}{16x^4} + \frac{\cosh(5a+5bx)}{16x^4}\right) dx$$
$$= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^4}\,dx + \frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^4}\,dx - \frac{1}{8}\int \frac{\cosh(a+bx)}{x^4}\,dx$$
$$= \frac{\cosh(a+bx)}{24x^3} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{\cosh(5a+5bx)}{48x^3} - \frac{1}{24}b\int \frac{\sinh(a+bx)}{x^3}\,dx + \frac{1}{16}b$$
$$= \frac{\cosh(a+bx)}{24x^3} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{\cosh(5a+5bx)}{48x^3} + \frac{b\sinh(a+bx)}{48x^2} - \frac{b\sinh(3a+}{32x^2}$$
$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x} - \frac{\cosh(5}{48}$$
$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x} - \frac{\cosh(5}{48}$$
$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x} - \frac{\cosh(5}{48}$$

Mathematica [A] time = 0.57048, size = 212, normalized size = 0.89

$$\frac{-2b^3x^3\sinh(a)\mathrm{Chi}(bx) + 27b^3x^3\sinh(3a)\mathrm{Chi}(3bx) + 125b^3x^3\sinh(5a)\mathrm{Chi}(5bx) - 2b^3x^3\cosh(a)\mathrm{Shi}(bx) + 27b^3x^3\cosh(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (4*Cosh[a + b*x] + 2*b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^
2*Cosh[3*(a + b*x)] - 2*Cosh[5*(a + b*x)] - 25*b^2*x^2*Cosh[5*(a + b*x)] -
2*b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshIntegral[3*b*x]*Sinh[3
*a] + 125*b^3*x^3*CoshIntegral[5*b*x]*Sinh[5*a] + 2*b*x*Sinh[a + b*x] - 3*b
*x*Sinh[3*(a + b*x)] - 5*b*x*Sinh[5*(a + b*x)] - 2*b^3*x^3*Cosh[a]*SinhInte
gral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Cosh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

Maple [A] time = 0.122, size = 356, normalized size = 1.5

$$-\frac{25\,b^2e^{-5\,bx-5\,a}}{192\,x} + \frac{5\,be^{-5\,bx-5\,a}}{192\,x^2} - \frac{e^{-5\,bx-5\,a}}{96\,x^3} + \frac{125\,b^3e^{-5\,a}\mathrm{Ei}\,(1,5\,bx)}{192} - \frac{3\,b^2e^{-3\,bx-3\,a}}{64\,x} + \frac{be^{-3\,bx-3\,a}}{64\,x^2} - \frac{e^{-3\,bx-3\,a}}{96\,x^3} + \frac{9\,b^3e^{-3\,a}\mathrm{Ei}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x)
```

```
[Out] -25/192*b^2*exp(-5*b*x-5*a)/x+5/192*b*exp(-5*b*x-5*a)/x^2-1/96*exp(-5*b*x-5
*a)/x^3+125/192*b^3*exp(-5*a)*Ei(1,5*b*x)-3/64*b^2*exp(-3*b*x-3*a)/x+1/64*b
*exp(-3*b*x-3*a)/x^2-1/96*exp(-3*b*x-3*a)/x^3+9/64*b^3*exp(-3*a)*Ei(1,3*b*x
)+1/96*b^2*exp(-b*x-a)/x-1/96*b*exp(-b*x-a)/x^2+1/48*exp(-b*x-a)/x^3-1/96*b
^3*exp(-a)*Ei(1,b*x)+1/48/x^3*exp(b*x+a)+1/96*b/x^2*exp(b*x+a)+1/96*b^2/x*e
xp(b*x+a)+1/96*b^3*exp(a)*Ei(1,-b*x)-1/96/x^3*exp(3*b*x+3*a)-1/64*b/x^2*exp
(3*b*x+3*a)-3/64*b^2/x*exp(3*b*x+3*a)-9/64*b^3*exp(3*a)*Ei(1,-3*b*x)-1/96/x
^3*exp(5*b*x+5*a)-5/192*b/x^2*exp(5*b*x+5*a)-25/192*b^2/x*exp(5*b*x+5*a)-12
5/192*b^3*exp(5*a)*Ei(1,-5*b*x)
```

Maxima [A] time = 1.30088, size = 119, normalized size = 0.5

$$-\frac{125}{32}\,b^3e^{(-5\,a)}\Gamma\left(-3,5\,bx\right)-\frac{27}{32}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{16}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{16}\,b^3e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{32}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
 + 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) + 27/32*b^
3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

Fricas [A] time = 1.79037, size = 996, normalized size = 4.18

$$-\frac{10\,bx\sinh\left(bx+a\right)^5+2\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+10\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(9\,b^2x^2+2\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*sinh(b*x + a)^5 + 2*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 10*(2
5*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(9*b^2*x^2 + 2)*cosh(b*x +
 a)^3 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*(10*(25*b^2*
x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2
 - 4*(b^2*x^2 + 2)*cosh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei(-5*b
*x))*cosh(5*a) - 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*
```

```
(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*cosh(a) + 2*(25*b*x*cosh(b*x + a)^4 +
9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) + b^3
*x^3*Ei(-5*b*x))*sinh(5*a) - 27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*si
nh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**4, x)
```

Giac [A] time = 1.16954, size = 462, normalized size = 1.94

$$\frac{125\,b^3x^3\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} + 27\,b^3x^3\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + 2\,b^3x^3\mathrm{Ei}\left(-bx\right)e^{(-a)} - 27\,b^3x^3\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - 125\,b^3x^3\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) + 27*b^3*x^3*Ei(3*b*x)*e^(3*a) + 2*b^3
*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) - 125*b^3*x^3*Ei(-5*b
*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) - 9*b^2*x
^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) + 2*b^2*x^2*e^(-b*x - a) - 9*b^2
*x^2*e^(-3*b*x - 3*a) - 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 - 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) + 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) - 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) + 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) - 2*e^(-5*b*x -
 5*a))/x^3
```

3.307 $\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=141

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamm}}{b}$$

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) - (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rubi [A] time = 0.210249, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamm}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) - (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx &= \int \left(-\frac{1}{4}x^m \sinh(2a+2bx)+\frac{1}{8}x^m \sinh(4a+4bx)\right)dx\\ &= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx-\frac{1}{4}\int x^m \sinh(2a+2bx)\,dx\\ &= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx-\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx-\frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx+\frac{1}{8}\int e^{i(2i\cdots}\\ &= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-4-m}\cdots}{}\end{aligned}$$

Mathematica [A] time = 0.136832, size = 112, normalized size = 0.79

$$\frac{e^{-4a}4^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(\text{Gamma}(m+1,4bx)-e^{2a}2^{m+2}\text{Gamma}(m+1,2bx)\right)+e^{8a}(bx)^m\text{Gamma}(m+1,-4bx\cdots\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] - 2^(2 + m)*E^(6*a)*(
b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(2^(2 + m)*E^(2*a)*Gamma[1 + m,
2*b*x]) + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)(\sinh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.22765, size = 158, normalized size = 1.12

$$\frac{1}{16}\left(4\,bx\right)^{-m-1}x^{m+1}e^{(-4\,a)}\Gamma\left(m+1,4\,bx\right)-\frac{1}{8}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma\left(m+1,2\,bx\right)+\frac{1}{8}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma\left(m+1,-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

Fricas [A] time = 1.86706, size = 518, normalized size = 3.67

$$\frac{\cosh\left(m\log\left(4\,b\right)+4\,a\right)\Gamma\left(m+1,4\,bx\right)-4\,\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)-4\,\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) - 4*cosh(m*log(2*b) + 2*a)
*gamma(m + 1, 2*b*x) - 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cos
h(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(
4*b) + 4*a) + 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 4*gamma(m + 1,
 -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) -
4*a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^3, x)
```

3.308 $\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\sinh^4(a+bx)}{32b^3}-\frac{9x\sinh^2(a+bx)}{32b^3}-\frac{3\sinh^3(a\cdots}{}$$

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (45*Cosh[a + b*x]*Sinh[a + b*x])/(256*
b^4) + (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (9*x*Sinh[a + b*x]^2)
/(32*b^3) - (3*Cosh[a + b*x]*Sinh[a + b*x]^3)/(128*b^4) - (3*x^2*Cosh[a + b
*x]*Sinh[a + b*x]^3)/(16*b^2) + (3*x*Sinh[a + b*x]^4)/(32*b^3) + (x^3*Sinh[
a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.13935, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5372, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\sinh^4(a+bx)}{32b^3}-\frac{9x\sinh^2(a+bx)}{32b^3}-\frac{3\sinh^3(a\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```


```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (45*Cosh[a + b*x]*Sinh[a + b*x])/(256*
b^4) + (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (9*x*Sinh[a + b*x]^2)
/(32*b^3) - (3*Cosh[a + b*x]*Sinh[a + b*x]^3)/(128*b^4) - (3*x^2*Cosh[a + b
*x]*Sinh[a + b*x]^3)/(16*b^2) + (3*x*Sinh[a + b*x]^4)/(32*b^3) + (x^3*Sinh[
a + b*x]^4)/(4*b)
```


Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x^3\sinh^4(a+bx)}{4b} - \frac{3\int x^2\sinh^4(a+bx)\,dx}{4b}$$
$$= -\frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b} - \frac{3\int \sin$$
$$= \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4}$$
$$= -\frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2}{32}$$
$$= -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} -$$

Mathematica [A] time = 0.635343, size = 95, normalized size = 0.61

$$\frac{48\left(2b^2x^2+1\right)\sinh(2(a+bx)) + 2bx\left(8b^2x^2+3\right)\cosh(4(a+bx)) - \left(3\left(8b^2x^2+1\right)\sinh(2(a+bx)) + 32bx\left(2b^2x^2+3\right)\right)\mathrm{c}}{512b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 48*(1 + 2*b^2*x^2)*Sinh[2*(a + b
*x)] - Cosh[2*(a + b*x)]*(32*b*x*(3 + 2*b^2*x^2) + 3*(1 + 8*b^2*x^2)*Sinh[2
*(a + b*x)]))/(512*b^4)
```

Maple [B] time = 0.006, size = 414, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(b*x+a)^3*cosh(b*x+a)^
2-3/16*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3+15/32*(b*x+a)^2*cosh(b*x+a)*sinh
(b*x+a)+5/32*(b*x+a)^3+3/32*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-3/128*cosh(
b*x+a)^3*sinh(b*x+a)+51/256*cosh(b*x+a)*sinh(b*x+a)+51/256*b*x+51/256*a-3/8
*(b*x+a)*cosh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(
b*x+a)^2*cosh(b*x+a)^2-1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+5/16*(b*x+a)*c
osh(b*x+a)*sinh(b*x+a)+5/32*(b*x+a)^2+1/32*cosh(b*x+a)^2*sinh(b*x+a)^2-1/8*
cosh(b*x+a)^2)+3*a^2*(1/4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(b*x+a)*c
osh(b*x+a)^2-1/16*cosh(b*x+a)^3*sinh(b*x+a)+5/32*cosh(b*x+a)*sinh(b*x+a)+5/
32*b*x+5/32*a)-a^3*(1/4*cosh(b*x+a)^2*sinh(b*x+a)^2-1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.07176, size = 231, normalized size = 1.49

$$\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}-\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4}-\frac{\left(4\,b^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 - 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

Fricas [A] time = 1.69249, size = 468, normalized size = 3.02

$$\frac{\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4-16\left(2\,b^3x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 - 2*(16*b^3*x^3 - 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 - 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] time = 7.94228, size = 226, normalized size = 1.46

$$\begin{cases} \frac{5x^3\sinh^4(a+bx)}{32b}+\frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b}-\frac{3x^3\cosh^4(a+bx)}{32b}-\frac{15x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2}+\frac{9x^2\sinh(a+bx)\cosh^3(a+bx)}{32b^2}+\frac{51x\sinh^4(a}{256b^3} \\ \frac{x^4\sinh^3(a)\cosh(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**3*cosh(a + b*x)**4/(32*b) - 15*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) + 9*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2)
 + 51*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**2
/(128*b**3) - 45*x*cosh(a + b*x)**4/(256*b**3) - 51*sinh(a + b*x)**3*cosh(a
 + b*x)/(256*b**4) + 45*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0)
), (x**4*sinh(a)**3*cosh(a)/4, True))
```

Giac [A] time = 1.2151, size = 196, normalized size = 1.26

$$\frac{\left(32\,b^3x^3-24\,b^2x^2+12\,bx-3\right)e^{(4\,bx+4\,a)}}{2048\,b^4}-\frac{\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{64\,b^4}-\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{64\,b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 - 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.309 $\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{\sinh^4(a+bx)}{32b^3}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b^2}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\sinh^4(a+bx)}{4b}-\frac{3}{3}$$

```
[Out] (-3*x^2)/(32*b) + (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) - (3*Sinh[a +
b*x]^2)/(32*b^3) - (x*Cosh[a + b*x]*Sinh[a + b*x]^3)/(8*b^2) + Sinh[a + b*x
]^4/(32*b^3) + (x^2*Sinh[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0769786, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 3310, 30}

$$\frac{\sinh^4(a+bx)}{32b^3}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b^2}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\sinh^4(a+bx)}{4b}-\frac{3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```


```
[Out] (-3*x^2)/(32*b) + (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) - (3*Sinh[a +
b*x]^2)/(32*b^3) - (x*Cosh[a + b*x]*Sinh[a + b*x]^3)/(8*b^2) + Sinh[a + b*x
]^4/(32*b^3) + (x^2*Sinh[a + b*x]^4)/(4*b)
```


Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{x^2\sinh^4(a+bx)}{4b}-\frac{\int x\sinh^4(a+bx)\,dx}{2b}\\ &= -\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2}+\frac{\sinh^4(a+bx)}{32b^3}+\frac{x^2\sinh^4(a+bx)}{4b}+\frac{3\int x\sinh\\ &= \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2}\\ &= -\frac{3x^2}{32b}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\cosh(a+bx)\sinh^3(\end{aligned}$$

Mathematica [A] time = 0.233394, size = 72, normalized size = 0.71

$$\frac{-16\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(8b^2x^2+1\right)\cosh(4(a+bx))+4bx(8\sinh(2(a+bx))-\sinh(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)]
+ 4*b*x*(8*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)]))/(256*b^3)
```

Maple [B] time = 0.006, size = 237, normalized size = 2.4

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\sinh(bx+a))^2(\cosh(bx+a))^2}{4}-\frac{(bx+a)^2(\cosh(bx+a))^2}{4}-\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^3}{8}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(b*x+a)^2*cosh(b*x+a)^
2-1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+5/16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a
)+5/32*(b*x+a)^2+1/32*cosh(b*x+a)^2*sinh(b*x+a)^2-1/8*cosh(b*x+a)^2-2*a*(1/
```

```
4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(b*x+a)*cosh(b*x+a)^2-1/16*cosh(b
*x+a)^3*sinh(b*x+a)+5/32*cosh(b*x+a)*sinh(b*x+a)+5/32*b*x+5/32*a)+a^2*(1/4*
cosh(b*x+a)^2*sinh(b*x+a)^2-1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.06434, size = 171, normalized size = 1.69

$$\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}-\frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2x^2+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 - 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

Fricas [A] time = 1.74295, size = 394, normalized size = 3.9

$$-\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-\left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4+16\left(2\,b^2x^2+1\right)\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 + 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
+ 2*(16*b^2*x^2 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 - 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] time = 4.44581, size = 150, normalized size = 1.49

$$\begin{cases}\frac{5x^2\sinh^4(a+bx)}{32b}+\frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b}-\frac{3x^2\cosh^4(a+bx)}{32b}-\frac{5x\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{3x\sinh(a+bx)\cosh^3(a+bx)}{16b^2}+\frac{5\sinh^4(a+bx)}{64b^3} \\ \frac{x^3\sinh^3(a)\cosh(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**2*cosh(a + b*x)**4/(32*b) - 5*x*sinh(a + b*x)**3*co
sh(a + b*x)/(16*b**2) + 3*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) + 5*si
nh(a + b*x)**4/(64*b**3) - 3*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*s
inh(a)**3*cosh(a)/3, True))
```

Giac [A] time = 1.19116, size = 153, normalized size = 1.51

$$\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}-\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{32\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 - 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.310 $\int x \cosh(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=65

$$-\frac{\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\sinh^4(a+bx)}{4b}-\frac{3x}{32b}$$

```
[Out] (-3*x)/(32*b) + (3*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (Cosh[a + b*x]*S
inh[a + b*x]^3)/(16*b^2) + (x*Sinh[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0430793, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5372, 2635, 8}

$$-\frac{\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\sinh^4(a+bx)}{4b}-\frac{3x}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x)/(32*b) + (3*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (Cosh[a + b*x]*S
inh[a + b*x]^3)/(16*b^2) + (x*Sinh[a + b*x]^4)/(4*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x\sinh^4(a+bx)}{4b} - \frac{\int \sinh^4(a+bx)\,dx}{4b}$$
$$= -\frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b} + \frac{3\int \sinh^2(a+bx)\,dx}{16b}$$
$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b} - \frac{3}{}$$
$$= -\frac{3x}{32b} + \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+}{4b}$$

Mathematica [A] time = 0.156814, size = 50, normalized size = 0.77

$$-\frac{-8\sinh(2(a+bx)) + \sinh(4(a+bx)) + 16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -(16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] - 8*Sinh[2*(a + b*x)]
+ Sinh[4*(a + b*x)])/(128*b^2)
```

Maple [A] time = 0.005, size = 113, normalized size = 1.7

$$\frac{1}{b^2}\left(\frac{(bx+a)(\sinh(bx+a))^2(\cosh(bx+a))^2}{4} - \frac{(bx+a)(\cosh(bx+a))^2}{4} - \frac{(\cosh(bx+a))^3\sinh(bx+a)}{16} + \frac{5\cosh(bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*(b*x+a)*cosh(b*x+a)^2-1/
16*cosh(b*x+a)^3*sinh(b*x+a)+5/32*cosh(b*x+a)*sinh(b*x+a)+5/32*b*x+5/32*a-a
*(1/4*cosh(b*x+a)^2*sinh(b*x+a)^2-1/4*cosh(b*x+a)^2))
```

Maxima [A] time = 1.12009, size = 123, normalized size = 1.89

$$\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Fricas [A] time = 1.67987, size = 289, normalized size = 4.45

$$\frac{bx\cosh\left(bx+a\right)^4+bx\sinh\left(bx+a\right)^4-4\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\left(3\,bx\cosh\left(bx+a\right)^2-2\,bx\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] time = 2.32101, size = 110, normalized size = 1.69

$$\begin{cases}\frac{5x\sinh^4\left(a+bx\right)}{32b}+\frac{3x\sinh^2\left(a+bx\right)\cosh^2\left(a+bx\right)}{16b}-\frac{3x\cosh^4\left(a+bx\right)}{32b}-\frac{5\sinh^3\left(a+bx\right)\cosh\left(a+bx\right)}{32b^2}+\frac{3\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{32b^2} & \text{for } b\neq 0\\ \frac{x^2\sinh^3\left(a\right)\cosh\left(a\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x)
**2/(16*b) - 3*x*cosh(a + b*x)**4/(32*b) - 5*sinh(a + b*x)**3*cosh(a + b*x)
/(32*b**2) + 3*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*s
```

```
inh(a)**3*cosh(a)/2, True))
```

Giac [A] time = 1.17258, size = 109, normalized size = 1.68

$$\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}-\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

3.311 $\int \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^4(a+bx)}{4b}$$

```
[Out] Sinh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0207488, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.002444, size = 15, normalized size = 1.

$$\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\sinh(bx+a))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*sinh(b*x+a)^4/b
```

Maxima [A] time = 1.01369, size = 18, normalized size = 1.2

$$\frac{\sinh(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sinh(b*x + a)^4/b
```

Fricas [B] time = 1.83843, size = 146, normalized size = 9.73

$$\frac{\cosh(bx+a)^4 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 - 2\right)\sinh(bx+a)^2 - 4\cosh(bx+a)^2}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a)^2 - 4*cosh(b*x + a)^2)/b
```

Sympy [A] time = 1.12259, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sinh^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x \sinh^3(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a), True))
```

Giac [B] time = 1.20192, size = 66, normalized size = 4.4

$$\frac{\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}\right)^2 - 4\,e^{(2\,bx+2\,a)} - 4\,e^{(-2\,bx-2\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 4*e^(2*b*x + 2*a) - 4*e^(-2*
b*x - 2*a))/b
```

3.312 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] -(CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 - (C
osh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rubi [A] time = 0.132692, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -(CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 - (C
osh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= -\left(\frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx-\frac{1}{4}\sinh(2a)\int \frac{\cosh\ldots}{\ldots}\\ &= -\frac{1}{4}\mathbf{Chi}(2bx)\sinh(2a)+\frac{1}{8}\mathbf{Chi}(4bx)\sinh(4a)-\frac{1}{4}\cosh(2a)\mathbf{Shi}(2bx)+\frac{1}{8}\cosh(4a)\mathbf{Sh}\ldots\end{aligned}$$

Mathematica [A] time = 0.0798878, size = 47, normalized size = 0.89

$$\frac{1}{8}(-2\sinh(2a)\mathbf{Chi}(2bx)+\sinh(4a)\mathbf{Chi}(4bx)-2\cosh(2a)\mathbf{Shi}(2bx)+\cosh(4a)\mathbf{Shi}(4bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] - 2*Cosh[
2*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Maple [A] time = 0.067, size = 50, normalized size = 0.9

$$\frac{e^{-4a}\mathrm{Ei}(1,4bx)}{16}-\frac{e^{-2a}\mathrm{Ei}(1,2bx)}{8}-\frac{e^{4a}\mathrm{Ei}(1,-4bx)}{16}+\frac{e^{2a}\mathrm{Ei}(1,-2bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*a)*Ei(1,2*b*x)-1/16*exp(4*a)*Ei(1,-4*
b*x)+1/8*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.21239, size = 61, normalized size = 1.15

$$\frac{1}{16}\,\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} - \frac{1}{8}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Fricas [A] time = 1.79897, size = 223, normalized size = 4.21

$$\frac{1}{16}\left(\mathrm{Ei}\left(4\,bx\right) - \mathrm{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right) - \frac{1}{8}\left(\mathrm{Ei}\left(2\,bx\right) - \mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right) + \frac{1}{16}\left(\mathrm{Ei}\left(4\,bx\right) + \mathrm{Ei}\left(-4\,bx\right)\right)\sinh\left(4\,a\right) - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) - 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) - 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3\left(a + bx\right)\cosh\left(a + bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x, x)
```

Giac [A] time = 1.21327, size = 61, normalized size = 1.15

$$\frac{1}{16}\,\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}+\frac{1}{8}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

3.313 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}b\cosh(2a)\mathrm{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\mathrm{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\mathrm{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\mathrm{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sin}{}$$

```
[Out] -(b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2
+ Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) - (b*Sinh[2*a]*SinhInte
gral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rubi [A] time = 0.175784, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b\cosh(2a)\mathrm{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\mathrm{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\mathrm{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\mathrm{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -(b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2
+ Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) - (b*Sinh[2*a]*SinhInte
gral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\ &= \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cosh(4a+4b}{x}\\ &= \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{2}(b\cosh(4a))\\ &= -\frac{1}{2}b\cosh(2a)\mathrm{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\mathrm{Chi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}\end{aligned}$$

Mathematica [A] time = 0.249716, size = 78, normalized size = 0.88

$$-\frac{4bx\cosh(2a)\mathrm{Chi}(2bx)-4bx\cosh(4a)\mathrm{Chi}(4bx)+4bx\sinh(2a)\mathrm{Shi}(2bx)-4bx\sinh(4a)\mathrm{Shi}(4bx)-2\sinh(2(a+bx))+}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -(4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 4*b*x*Cosh[4*a]*CoshIntegral[4*b*x]
 - 2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegral[2
```

```
*b*x] - 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/(8*x)
```

Maple [A] time = 0.066, size = 110, normalized size = 1.2

$$\frac{e^{-4\,bx-4\,a}}{16\,x}-\frac{b e^{-4\,a}\mathrm{Ei}\left(1,4\,bx\right)}{4}-\frac{e^{-2\,bx-2\,a}}{8\,x}+\frac{b e^{-2\,a}\mathrm{Ei}\left(1,2\,bx\right)}{4}-\frac{e^{4\,bx+4\,a}}{16\,x}-\frac{b e^{4\,a}\mathrm{Ei}\left(1,-4\,bx\right)}{4}+\frac{e^{2\,bx+2\,a}}{8\,x}+\frac{b e^{2\,a}\mathrm{Ei}\left(1,-2\,b\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/16*exp(-4*b*x-4*a)/x-1/4*b*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*b*x-2*a)/x+1/
4*b*exp(-2*a)*Ei(1,2*b*x)-1/16/x*exp(4*b*x+4*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
+1/8*exp(2*b*x+2*a)/x+1/4*b*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.24912, size = 72, normalized size = 0.81

$$\frac{1}{4}\,be^{(-4\,a)}\Gamma\left(-1,4\,bx\right)-\frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right)-\frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right)+\frac{1}{4}\,be^{(4\,a)}\Gamma\left(-1,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) - 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) - 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

Fricas [A] time = 1.80503, size = 370, normalized size = 4.16

$$-\frac{2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-\left(bx\mathrm{Ei}\left(4\,bx\right)+bx\mathrm{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right)+\left(bx\mathrm{Ei}\left(2\,bx\right)+bx\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) + (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 -
```

```
cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
+ (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**2, x)
```

Giac [A] time = 1.1774, size = 135, normalized size = 1.52

$$\frac{4\,bx\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} - 4\,bx\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} - 4\,bx\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} + 4\,bx\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)} - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)} - 2\,e^{(-2\,bx-2\,a)}}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(2*b*x)*e^(2*a) - 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a)
- 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

3.314 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sin}{}$$

```
[Out] (b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) - (b^2*CoshIntegr
al[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] + Sinh[2*a + 2*b
*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b*
x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rubi [A] time = 0.229893, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) - (b^2*CoshIntegr
al[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] + Sinh[2*a + 2*b
*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b*
x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \left(-\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx - \frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx$$
$$= \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{4}b\int \frac{\cosh(4a+4b}{x^2}$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}b^2\int$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}\left(b^2 c\right.$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{1}{2}b^2\mathbf{Chi}(2bx)\sinh(2a) + b^2\mathbf{Chi}(4bx)\sinh(4$$

Mathematica [A] time = 0.545119, size = 113, normalized size = 0.9

$$b^2\sinh(4a)\mathbf{Chi}(4bx) + b^2\sinh(a)(-\cosh(a))\mathbf{Chi}(2bx) - \frac{1}{2}b^2\cosh(2a)\mathbf{Shi}(2bx) + b^2\cosh(4a)\mathbf{Shi}(4bx) + \frac{\sinh(2(a+bx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -(b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a]) + b^2*CoshIntegral[4*b*x]*Sinh[4
*a] + (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4
*(a + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Maple [A] time = 0.069, size = 178, normalized size = 1.4

$$-\frac{b\mathrm{e}^{-4bx-4a}}{8x}+\frac{\mathrm{e}^{-4bx-4a}}{32x^2}+\frac{b^2\mathrm{e}^{-4a}\mathrm{Ei}(1,4bx)}{2}+\frac{b\mathrm{e}^{-2bx-2a}}{8x}-\frac{\mathrm{e}^{-2bx-2a}}{16x^2}-\frac{b^2\mathrm{e}^{-2a}\mathrm{Ei}(1,2bx)}{4}-\frac{\mathrm{e}^{4bx+4a}}{32x^2}-\frac{b\mathrm{e}^{4bx+4a}}{8x}-\frac{b^2\mathrm{e}^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -1/8*b*exp(-4*b*x-4*a)/x+1/32*exp(-4*b*x-4*a)/x^2+1/2*b^2*exp(-4*a)*Ei(1,4*
b*x)+1/8*b*exp(-2*b*x-2*a)/x-1/16*exp(-2*b*x-2*a)/x^2-1/4*b^2*exp(-2*a)*Ei(
1,2*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a)*Ei
(1,-4*b*x)+1/16*exp(2*b*x+2*a)/x^2+1/8*b*exp(2*b*x+2*a)/x+1/4*b^2*exp(2*a)*
Ei(1,-2*b*x)
```

Maxima [A] time = 1.22558, size = 81, normalized size = 0.65

$$b^2e^{(-4a)}\Gamma(-2,4bx)-\frac{1}{2}b^2e^{(-2a)}\Gamma(-2,2bx)+\frac{1}{2}b^2e^{(2a)}\Gamma(-2,-2bx)-b^2e^{(4a)}\Gamma(-2,-4bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) - 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

Fricas [B] time = 1.85163, size = 570, normalized size = 4.56

$$-\frac{bx\cosh(bx+a)^4+bx\sinh(bx+a)^4-bx\cosh(bx+a)^2+\cosh(bx+a)\sinh(bx+a)^3+\left(6bx\cosh(bx+a)^2-bx\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + cos
h(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2
- 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) + (b^2*x^2*Ei(2*b*x)
 - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b
*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) + (b^2*x^2*E
i(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**3, x)
```

Giac [A] time = 1.19316, size = 227, normalized size = 1.82

$$\frac{16\,b^2x^2\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} - 8\,b^2x^2\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} + 8\,b^2x^2\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)} - 4\,bxe^{(4\,bx+4\,a)} + 4\,bxe^{(2\,bx-}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) - 8*b^2*x^2*Ei(2*b*x)*e^(2*a) + 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) + 4*b*x*e^(2*b*x + 2*a) + 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

3.315 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)-\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)+\frac{b^2\sinh(2a+2bx}{6x}$$

```
[Out] (b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 + Si
nh[2*a + 2*b*x]/(12*x^3) + (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*x
]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) - (b^3*Sinh[2*a]*SinhIntegral[2*
b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rubi [A] time = 0.286577, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)-\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)+\frac{b^2\sinh(2a+2bx}{6x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```


```
[Out] (b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 + Si
nh[2*a + 2*b*x]/(12*x^3) + (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*x
]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) - (b^3*Sinh[2*a]*SinhIntegral[2*
b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \left(-\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx$$
$$= \frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{6}b\int \frac{\cosh(4a+4b}{x^3}$$
$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{1}{6}b^2\int$$
$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx)}{6x}-\sinh$$
$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx)}{6x}-\sinh$$
$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{1}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\mathrm{Chi}($$

Mathematica [A] time = 0.531421, size = 150, normalized size = 0.89

$$-\frac{8b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-32b^3x^3\cosh(4a)\mathrm{Chi}(4bx)+8b^3x^3\sinh(2a)\mathrm{Shi}(2bx)-32b^3x^3\sinh(4a)\mathrm{Shi}(4bx)-4b^2x^2\sin}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -(-2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] + 8*b^3*x^3*Cosh[2*a]*
CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] - 2*Sinh[2*(
a + b*x)] - 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2*Sin
h[4*(a + b*x)] + 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*Sinh[
4*a]*SinhIntegral[4*b*x])/(24*x^3)
```

Maple [A] time = 0.081, size = 246, normalized size = 1.5

$$\frac{b^2 \mathrm{e}^{-4\,bx-4\,a}}{6\,x}-\frac{b \mathrm{e}^{-4\,bx-4\,a}}{24\,x^2}+\frac{\mathrm{e}^{-4\,bx-4\,a}}{48\,x^3}-\frac{2\,b^3 \mathrm{e}^{-4\,a}\mathrm{Ei}\,(1,4\,bx)}{3}-\frac{b^2 \mathrm{e}^{-2\,bx-2\,a}}{12\,x}+\frac{b \mathrm{e}^{-2\,bx-2\,a}}{24\,x^2}-\frac{\mathrm{e}^{-2\,bx-2\,a}}{24\,x^3}+\frac{b^3 \mathrm{e}^{-2\,a}\mathrm{Ei}\,(1,2\,bx)}{6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 1/6*b^2*exp(-4*b*x-4*a)/x-1/24*b*exp(-4*b*x-4*a)/x^2+1/48*exp(-4*b*x-4*a)/x
^3-2/3*b^3*exp(-4*a)*Ei(1,4*b*x)-1/12*b^2*exp(-2*b*x-2*a)/x+1/24*b*exp(-2*b
*x-2*a)/x^2-1/24*exp(-2*b*x-2*a)/x^3+1/6*b^3*exp(-2*a)*Ei(1,2*b*x)+1/24*exp
(2*b*x+2*a)/x^3+1/24*b*exp(2*b*x+2*a)/x^2+1/12*b^2*exp(2*b*x+2*a)/x+1/6*b^3
*exp(2*a)*Ei(1,-2*b*x)-1/48/x^3*exp(4*b*x+4*a)-1/24*b/x^2*exp(4*b*x+4*a)-1/
6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*exp(4*a)*Ei(1,-4*b*x)
```

Maxima [A] time = 1.21077, size = 82, normalized size = 0.49

$$4\,b^3 e^{(-4\,a)}\Gamma\left(-3,4\,bx\right)-b^3 e^{(-2\,a)}\Gamma\left(-3,2\,bx\right)-b^3 e^{(2\,a)}\Gamma\left(-3,-2\,bx\right)+4\,b^3 e^{(4\,a)}\Gamma\left(-3,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) - b^3*e^(-2*a)*gamma(-3, 2*b*x) - b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

Fricas [A] time = 1.76732, size = 647, normalized size = 3.83

$$-\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 + 2\left(8b^2x^2+1\right)\cosh(bx+a)\sinh(bx+a)^3 - bx\cosh(bx+a)^2 + \left(6bx\cosh(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 - b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) +
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 - (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) + 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**4, x)
```

Giac [A] time = 1.21678, size = 319, normalized size = 1.89

$$\frac{32b^3x^3\mathrm{Ei}(4bx)e^{(4a)} - 8b^3x^3\mathrm{Ei}(2bx)e^{(2a)} - 8b^3x^3\mathrm{Ei}(-2bx)e^{(-2a)} + 32b^3x^3\mathrm{Ei}(-4bx)e^{(-4a)} - 8b^2x^2e^{(4bx+4a)} + 4b^2x^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 8*b^3*x^3*Ei(2*b*x)*e^(2*a) - 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
```

```
 + 4*a) + 4*b^2*x^2*e^(2*b*x + 2*a) - 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) + 2*b*x*e^(2*b*x + 2*a) + 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x +
2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

3.316 $\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=209

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}-\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\text{Gamma}(m+1,}{16b}$$

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) - (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) + (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rubi [A] time = 0.280038, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3308, 2181}

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}-\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\text{Gamma}(m+1,}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) - (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) + (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int \left(-\frac{1}{8}x^m\sinh(a+bx)-\frac{1}{16}x^m\sinh(3a+3bx)+\frac{1}{16}x^m\sinh(5a+5bx)\right)dx$$
$$= -\left(\frac{1}{16}\int x^m\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^m\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^m\sinh(a$$
$$= -\left(\frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx\right)+\frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{-i(5ia+5ibx)}x^m\,dx-$$
$$= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b}-\frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b}-\frac{e^a x$$

Mathematica [A] time = 0.26055, size = 174, normalized size = 0.83

$$\frac{e^{-5a}x^m\left(-5e^{2a}3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\text{Gamma}(m+1,-3bx)+(-bx)^m\text{Gamma}(m+1,3bx)\right)+3\ 5^{-m}\left(-b^2x^2\right)^{-m}\left(e^{10a}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (x^m*(-30*E^(4*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m,
 b*x]/(b*x)^m) - (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x)
)^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Gamma
[1 + m, -5*b*x] + (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m)))/(
480*b*E^(5*a))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int x^m\left(\cosh(bx+a)\right)^2\left(\sinh(bx+a)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.34418, size = 231, normalized size = 1.11

$$\frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma\,(m+1,5\,bx)-\frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma\,(m+1,3\,bx)-\frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma\,(m+1,bx)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x)
^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) - 1/16*(b*x)^(-m - 1)*x^(m
 + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(m
 + 1, -b*x) + 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x)
 - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

Fricas [A] time = 1.95095, size = 763, normalized size = 3.65

$$\frac{3\,\cosh\,(m\log\,(5\,b)+5\,a)\,\Gamma\,(m+1,5\,bx)-5\,\cosh\,(m\log\,(3\,b)+3\,a)\,\Gamma\,(m+1,3\,bx)-30\,\cosh\,(m\log\,(b)+a)\,\Gamma\,(m+1,bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) - 5*cosh(m*log(3*b) + 3
*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) - 30*cosh
(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1,
 -3*b*x) + 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1,
5*b*x)*sinh(m*log(5*b) + 5*a) + 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a
) + 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sinh
(m*log(-3*b) - 3*a) - 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*g
amma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.317 $\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=202

$$\frac{3x^2\sinh(a+bx)}{8b^2}+\frac{x^2\sinh(3a+3bx)}{48b^2}-\frac{3x^2\sinh(5a+5bx)}{400b^2}+\frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4}-\frac{3\sinh(5a+5bx)}{5000b^4}-3$$

```
[Out] (-3*x*Cosh[a + b*x])/(4*b^3) - (x^3*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*
b*x])/(72*b^3) - (x^3*Cosh[3*a + 3*b*x])/(48*b) + (3*x*Cosh[5*a + 5*b*x])/(
1000*b^3) + (x^3*Cosh[5*a + 5*b*x])/(80*b) + (3*Sinh[a + b*x])/(4*b^4) + (3
*x^2*Sinh[a + b*x])/(8*b^2) + Sinh[3*a + 3*b*x]/(216*b^4) + (x^2*Sinh[3*a +
 3*b*x])/(48*b^2) - (3*Sinh[5*a + 5*b*x])/(5000*b^4) - (3*x^2*Sinh[5*a + 5*
b*x])/(400*b^2)
```

Rubi [A] time = 0.262065, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2637}

$$\frac{3x^2\sinh(a+bx)}{8b^2}+\frac{x^2\sinh(3a+3bx)}{48b^2}-\frac{3x^2\sinh(5a+5bx)}{400b^2}+\frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4}-\frac{3\sinh(5a+5bx)}{5000b^4}-3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```


```
[Out] (-3*x*Cosh[a + b*x])/(4*b^3) - (x^3*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*
b*x])/(72*b^3) - (x^3*Cosh[3*a + 3*b*x])/(48*b) + (3*x*Cosh[5*a + 5*b*x])/(
1000*b^3) + (x^3*Cosh[5*a + 5*b*x])/(80*b) + (3*Sinh[a + b*x])/(4*b^4) + (3
*x^2*Sinh[a + b*x])/(8*b^2) + Sinh[3*a + 3*b*x]/(216*b^4) + (x^2*Sinh[3*a +
 3*b*x])/(48*b^2) - (3*Sinh[5*a + 5*b*x])/(5000*b^4) - (3*x^2*Sinh[5*a + 5*
b*x])/(400*b^2)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}x^3\sinh(a+bx)-\frac{1}{16}x^3\sinh(3a+3bx)+\frac{1}{16}x^3\sinh(5a+5bx)\right)dx$$
$$= -\left(\frac{1}{16}\int x^3\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^3\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^3\sinh(a+$$
$$= -\frac{x^3\cosh(a+bx)}{8b}-\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}-\frac{3\int x^2\cosh(5a+}{80b}$$
$$= -\frac{x^3\cosh(a+bx)}{8b}-\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}+\frac{3x^2\sinh(a+bx)}{8b^2}$$
$$= -\frac{3x\cosh(a+bx)}{4b^3}-\frac{x^3\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{72b^3}-\frac{x^3\cosh(3a+3bx)}{48b}$$
$$= -\frac{3x\cosh(a+bx)}{4b^3}-\frac{x^3\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{72b^3}-\frac{x^3\cosh(3a+3bx)}{48b}$$

Mathematica [A] time = 0.490951, size = 136, normalized size = 0.67

$$\frac{-33750\left(bx\left(b^2x^2+6\right)\cosh(a+bx)-3\left(b^2x^2+2\right)\sinh(a+bx)\right)-625\left(\left(9b^3x^3+6bx\right)\cosh(3(a+bx))-\left(9b^2x^2+2\right)\text{si}\right.}{270000b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-33750*(b*x*(6 + b^2*x^2)*Cosh[a + b*x] - 3*(2 + b^2*x^2)*Sinh[a + b*x]) -
 625*((6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)] - (2 + 9*b^2*x^2)*Sinh[3*(a + b
*x)]) + 27*(5*b*x*(6 + 25*b^2*x^2)*Cosh[5*(a + b*x)] - 3*(2 + 25*b^2*x^2)*S
inh[5*(a + b*x)]))/(270000*b^4)
```

Maple [B] time = 0.01, size = 542, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/5*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^3*cosh(b*x+a)
*sinh(b*x+a)^2-2/15*(b*x+a)^3*cosh(b*x+a)-3/25*(b*x+a)^2*sinh(b*x+a)*cosh(b
*x+a)^4+26/75*(b*x+a)^2*sinh(b*x+a)+13/75*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)
^2+6/125*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-76/1125*(b*x+a)*sinh(b*x+a)^2*
cosh(b*x+a)-856/1125*(b*x+a)*cosh(b*x+a)-6/625*cosh(b*x+a)^4*sinh(b*x+a)+12
568/16875*sinh(b*x+a)+434/16875*sinh(b*x+a)*cosh(b*x+a)^2-3*a*(1/5*(b*x+a)^
2*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-2/15
*(b*x+a)^2*cosh(b*x+a)-2/25*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4+52/225*(b*x+a
)*sinh(b*x+a)+26/225*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+2/125*cosh(b*x+a)^3*
sinh(b*x+a)^2-76/3375*cosh(b*x+a)*sinh(b*x+a)^2-856/3375*cosh(b*x+a))+3*a^2
*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*sinh(b*x+a)^2*cosh(b
*x+a)-2/15*(b*x+a)*cosh(b*x+a)-1/25*cosh(b*x+a)^4*sinh(b*x+a)+26/225*sinh(b
*x+a)+13/225*sinh(b*x+a)*cosh(b*x+a)^2)-a^3*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^
2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15*cosh(b*x+a)))
```

Maxima [A] time = 1.09067, size = 331, normalized size = 1.64

$$\frac{\left(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)}\right)e^{(5\,bx)}}{20000\,b^4} - \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^4} - \frac{\left(b^3x^3e^{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 - 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

Fricas [A] time = 1.77431, size = 722, normalized size = 3.57

$$\frac{135\left(25\,b^3x^3 + 6\,bx\right)\cosh\left(bx + a\right)^5 + 675\left(25\,b^3x^3 + 6\,bx\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right)^4 - 81\left(25\,b^2x^2 + 2\right)\sinh\left(bx + a\right)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270000*(135*(25*b^3*x^3 + 6*b*x)*cosh(b*x + a)^5 + 675*(25*b^3*x^3 + 6*b*
x)*cosh(b*x + a)*sinh(b*x + a)^4 - 81*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 18
75*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 5*(1125*b^2*x^2 - 162*(25*b^2*x^2
+ 2)*cosh(b*x + a)^2 + 250)*sinh(b*x + a)^3 + 225*(6*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^3 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^2 - 33
750*(b^3*x^3 + 6*b*x)*cosh(b*x + a) - 15*(27*(25*b^2*x^2 + 2)*cosh(b*x + a)
^4 - 6750*b^2*x^2 - 125*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 - 13500)*sinh(b*x +
 a))/b^4
```

Sympy [A] time = 13.8801, size = 253, normalized size = 1.25

$$\begin{cases} \frac{x^3\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x^3\cosh^5(a+bx)}{15b} + \frac{26x^2\sinh^5(a+bx)}{75b^2} - \frac{13x^2\sinh^3(a+bx)\cosh^2(a+bx)}{15b^2} + \frac{2x^2\sinh(a+bx)\cosh^4(a+bx)}{5b^2} - \frac{52x\sinh^4}{} \\ \frac{x^4\sinh^3(a)\cosh^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**3*cosh(a + b
*x)**5/(15*b) + 26*x**2*sinh(a + b*x)**5/(75*b**2) - 13*x**2*sinh(a + b*x)*
*3*cosh(a + b*x)**2/(15*b**2) + 2*x**2*sinh(a + b*x)*cosh(a + b*x)**4/(5*b*
*2) - 52*x*sinh(a + b*x)**4*cosh(a + b*x)/(75*b**3) + 338*x*sinh(a + b*x)**
2*cosh(a + b*x)**3/(225*b**3) - 856*x*cosh(a + b*x)**5/(1125*b**3) + 12568*
sinh(a + b*x)**5/(16875*b**4) - 5114*sinh(a + b*x)**3*cosh(a + b*x)**2/(337
5*b**4) + 856*sinh(a + b*x)*cosh(a + b*x)**4/(1125*b**4), Ne(b, 0)), (x**4*
sinh(a)**3*cosh(a)**2/4, True))
```

Giac [A] time = 1.17057, size = 286, normalized size = 1.42

$$\frac{\left(125\,b^3x^3 - 75\,b^2x^2 + 30\,bx - 6\right)e^{(5\,bx+5\,a)}}{20000\,b^4} - \frac{\left(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2\right)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{\left(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6\right)e^{(bx+a)}}{16\,b^4} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 - 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

3.318 $\int x^2 \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=148

$$\frac{x\sinh(a+bx)}{4b^2}+\frac{x\sinh(3a+3bx)}{72b^2}-\frac{x\sinh(5a+5bx)}{200b^2}-\frac{\cosh(a+bx)}{4b^3}-\frac{\cosh(3a+3bx)}{216b^3}+\frac{\cosh(5a+5bx)}{1000b^3}-\frac{x^2\cosh}{8}$$

```
[Out] -Cosh[a + b*x]/(4*b^3) - (x^2*Cosh[a + b*x])/(8*b) - Cosh[3*a + 3*b*x]/(216
*b^3) - (x^2*Cosh[3*a + 3*b*x])/(48*b) + Cosh[5*a + 5*b*x]/(1000*b^3) + (x^
2*Cosh[5*a + 5*b*x])/(80*b) + (x*Sinh[a + b*x])/(4*b^2) + (x*Sinh[3*a + 3*b
*x])/(72*b^2) - (x*Sinh[5*a + 5*b*x])/(200*b^2)
```

Rubi [A] time = 0.181898, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2638}

$$\frac{x\sinh(a+bx)}{4b^2}+\frac{x\sinh(3a+3bx)}{72b^2}-\frac{x\sinh(5a+5bx)}{200b^2}-\frac{\cosh(a+bx)}{4b^3}-\frac{\cosh(3a+3bx)}{216b^3}+\frac{\cosh(5a+5bx)}{1000b^3}-\frac{x^2\cosh}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -Cosh[a + b*x]/(4*b^3) - (x^2*Cosh[a + b*x])/(8*b) - Cosh[3*a + 3*b*x]/(216
*b^3) - (x^2*Cosh[3*a + 3*b*x])/(48*b) + Cosh[5*a + 5*b*x]/(1000*b^3) + (x^
2*Cosh[5*a + 5*b*x])/(80*b) + (x*Sinh[a + b*x])/(4*b^2) + (x*Sinh[3*a + 3*b
*x])/(72*b^2) - (x*Sinh[5*a + 5*b*x])/(200*b^2)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\int\left(-\frac{1}{8}x^2\sinh(a+bx)-\frac{1}{16}x^2\sinh(3a+3bx)+\frac{1}{16}x^2\sinh(5a+5bx)\right)dx$$
$$=-\left(\frac{1}{16}\int x^2\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^2\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^2\sinh(a+b$$
$$=-\frac{x^2\cosh(a+bx)}{8b}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{x^2\cosh(5a+5bx)}{80b}-\frac{\int x\cosh(5a+5bx}{40b}$$
$$=-\frac{x^2\cosh(a+bx)}{8b}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{x^2\cosh(5a+5bx)}{80b}+\frac{x\sinh(a+bx)}{4b^2}+$$
$$=-\frac{\cosh(a+bx)}{4b^3}-\frac{x^2\cosh(a+bx)}{8b}-\frac{\cosh(3a+3bx)}{216b^3}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{\cosh}{}$$

Mathematica [A] time = 0.445034, size = 98, normalized size = 0.66

$$\frac{-6750\left(b^2x^2+2\right)\cosh(a+bx)-125\left(9b^2x^2+2\right)\cosh(3(a+bx))+27\left(25b^2x^2+2\right)\cosh(5(a+bx))+30bx(450\sinh(a}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-6750*(2 + b^2*x^2)*Cosh[a + b*x] - 125*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
+ 27*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(450*Sinh[a + b*x] + 25*Si
nh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)]))/(54000*b^3)
```

Maple [B] time = 0.007, size = 314, normalized size = 2.1

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\sinh(bx+a))^2(\cosh(bx+a))^3}{5}-\frac{2\,(bx+a)^2(\sinh(bx+a))^2\cosh(bx+a)}{15}-\frac{2\,(bx+a)^2\cosh(bx+a)}{15}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(1/5*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^2*sinh(b*x+a)
^2*cosh(b*x+a)-2/15*(b*x+a)^2*cosh(b*x+a)-2/25*(b*x+a)*sinh(b*x+a)*cosh(b*x
+a)^4+52/225*(b*x+a)*sinh(b*x+a)+26/225*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+2
/125*cosh(b*x+a)^3*sinh(b*x+a)^2-76/3375*cosh(b*x+a)*sinh(b*x+a)^2-856/3375
*cosh(b*x+a)-2*a*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*sinh
(b*x+a)^2*cosh(b*x+a)-2/15*(b*x+a)*cosh(b*x+a)-1/25*cosh(b*x+a)^4*sinh(b*x+
a)+26/225*sinh(b*x+a)+13/225*sinh(b*x+a)*cosh(b*x+a)^2)+a^2*(1/5*cosh(b*x+a
)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15*cosh(b*x+a)))
```

Maxima [A] time = 1.06107, size = 252, normalized size = 1.7

$$\frac{\left(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3} - \frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3} - \frac{\left(b^2x^2e^{a} - 2\,bxe^{a} + 2\,e^{a}\right)e^{(bx)}}{16\,b^3} - \frac{\left(b^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 - 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 + 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

Fricas [A] time = 1.78869, size = 581, normalized size = 3.93

$$-\frac{270\,bx\sinh\left(bx+a\right)^5 - 27\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5 - 135\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 + 125\left(9\,b^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/54000*(270*b*x*sinh(b*x + a)^5 - 27*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 - 1
35*(25*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 125*(9*b^2*x^2 + 2)*cos
h(b*x + a)^3 + 150*(18*b*x*cosh(b*x + a)^2 - 5*b*x)*sinh(b*x + a)^3 - 15*(1
8*(25*b^2*x^2 + 2)*cosh(b*x + a)^3 - 25*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh
(b*x + a)^2 + 6750*(b^2*x^2 + 2)*cosh(b*x + a) + 450*(3*b*x*cosh(b*x + a)^4
 - 5*b*x*cosh(b*x + a)^2 - 30*b*x)*sinh(b*x + a))/b^3
```

Sympy [A] time = 8.0461, size = 182, normalized size = 1.23

$$\begin{cases} \frac{x^2\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x^2\cosh^5(a+bx)}{15b} + \frac{52x\sinh^5(a+bx)}{225b^2} - \frac{26x\sinh^3(a+bx)\cosh^2(a+bx)}{45b^2} + \frac{4x\sinh(a+bx)\cosh^4(a+bx)}{15b^2} - \frac{52\sinh^4(a+bx)}{225} \\ \frac{x^3\sinh^3(a)\cosh^2(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**2*cosh(a + b
*x)**5/(15*b) + 52*x*sinh(a + b*x)**5/(225*b**2) - 26*x*sinh(a + b*x)**3*co
sh(a + b*x)**2/(45*b**2) + 4*x*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2) - 5
2*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**3) + 338*sinh(a + b*x)**2*cosh(a +
 b*x)**3/(675*b**3) - 856*cosh(a + b*x)**5/(3375*b**3), Ne(b, 0)), (x**3*si
nh(a)**3*cosh(a)**2/3, True))
```

Giac [A] time = 1.22521, size = 221, normalized size = 1.49

$$\frac{\left(25\,b^2x^2 - 10\,bx + 2\right)e^{(5\,bx+5\,a)}}{4000\,b^3} - \frac{\left(9\,b^2x^2 - 6\,bx + 2\right)e^{(3\,bx+3\,a)}}{864\,b^3} - \frac{\left(b^2x^2 - 2\,bx + 2\right)e^{(bx+a)}}{16\,b^3} - \frac{\left(b^2x^2 + 2\,bx + 2\right)e^{(-bx-a)}}{16\,b^3} - \frac{(9\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 - 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
- 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 + 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

3.319 $\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(3a+3bx)}{144b^2}-\frac{\sinh(5a+5bx)}{400b^2}-\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}$$

```
[Out] -(x*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*b*x])/(48*b) + (x*Cosh[5*a + 5*b
*x])/(80*b) + Sinh[a + b*x]/(8*b^2) + Sinh[3*a + 3*b*x]/(144*b^2) - Sinh[5*
a + 5*b*x]/(400*b^2)
```

Rubi [A] time = 0.0903493, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2637}

$$\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(3a+3bx)}{144b^2}-\frac{\sinh(5a+5bx)}{400b^2}-\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -(x*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*b*x])/(48*b) + (x*Cosh[5*a + 5*b
*x])/(80*b) + Sinh[a + b*x]/(8*b^2) + Sinh[3*a + 3*b*x]/(144*b^2) - Sinh[5*
a + 5*b*x]/(400*b^2)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}x\sinh(a+bx)-\frac{1}{16}x\sinh(3a+3bx)+\frac{1}{16}x\sinh(5a+5bx)\right)dx$$
$$= -\left(\frac{1}{16}\int x\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x\sinh(5a+5bx)\,dx-\frac{1}{8}\int x\sinh(a+bx)\,d$$
$$= -\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}-\frac{\int\cosh(5a+5bx)\,dx}{80b}$$
$$= -\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}+\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(}{1}$$

Mathematica [A] time = 0.188888, size = 70, normalized size = 0.74

$$\frac{450\sinh(a+bx)+25\sinh(3(a+bx))-9\sinh(5(a+bx))-450bx\cosh(a+bx)-75bx\cosh(3(a+bx))+45bx\cosh(5(}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-450*b*x*Cosh[a + b*x] - 75*b*x*Cosh[3*(a + b*x)] + 45*b*x*Cosh[5*(a + b*x
)] + 450*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)])/(3600*
b^2)
```

Maple [A] time = 0.007, size = 149, normalized size = 1.6

$$\frac{1}{b^2}\left(\frac{(bx+a)(\sinh(bx+a))^2(\cosh(bx+a))^3}{5}-\frac{(2bx+2a)(\sinh(bx+a))^2\cosh(bx+a)}{15}-\frac{(2bx+2a)\cosh(bx+a)}{15}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*sinh(b*x+a)^2*c
osh(b*x+a)-2/15*(b*x+a)*cosh(b*x+a)-1/25*cosh(b*x+a)^4*sinh(b*x+a)+26/225*s
```

```
inh(b*x+a)+13/225*sinh(b*x+a)*cosh(b*x+a)^2-a*(1/5*cosh(b*x+a)^3*sinh(b*x+a
)^2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15*cosh(b*x+a)))
```

Maxima [A] time = 1.20229, size = 174, normalized size = 1.85

$$\frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2}-\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2}-\frac{(bxe^{a}-e^{a})e^{(bx)}}{16\,b^2}-\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}+\frac{(5\,bx+1)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 - 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 - 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

Fricas [A] time = 1.69211, size = 433, normalized size = 4.61

$$\frac{45\,bx\cosh\left(bx+a\right)^5+225\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-75\,bx\cosh\left(bx+a\right)^3-9\,\sinh\left(bx+a\right)^5-5\left(18\,\cosh\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*cosh(b*x + a)^5 + 225*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 75
*b*x*cosh(b*x + a)^3 - 9*sinh(b*x + a)^5 - 5*(18*cosh(b*x + a)^2 - 5)*sinh(
b*x + a)^3 - 450*b*x*cosh(b*x + a) + 225*(2*b*x*cosh(b*x + a)^3 - b*x*cosh(
b*x + a))*sinh(b*x + a)^2 - 15*(3*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 30)
*sinh(b*x + a))/b^2
```

Sympy [A] time = 4.18373, size = 112, normalized size = 1.19

$$\begin{cases} \frac{x\sinh^{2}{\left(a+bx\right)}\cosh^{3}{\left(a+bx\right)}}{3b}-\frac{2x\cosh^{5}{\left(a+bx\right)}}{15b}+\frac{26\sinh^{5}{\left(a+bx\right)}}{225b^{2}}-\frac{13\sinh^{3}{\left(a+bx\right)}\cosh^{2}{\left(a+bx\right)}}{45b^{2}}+\frac{2\sinh{\left(a+bx\right)}\cosh^{4}{\left(a+bx\right)}}{15b^{2}} & \text{for}\: b \neq 0 \\ \frac{x^{2}\sinh^{3}{\left(a\right)}\cosh^{2}{\left(a\right)}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x*cosh(a + b*x)**5
/(15*b) + 26*sinh(a + b*x)**5/(225*b**2) - 13*sinh(a + b*x)**3*cosh(a + b*x
)**2/(45*b**2) + 2*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**2/2, True))
```

Giac [A] time = 1.1768, size = 157, normalized size = 1.67

$$\frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2}-\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2}-\frac{(bx-1)e^{(bx+a)}}{16\,b^2}-\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}+\frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 - 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 - 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

3.320 $\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cosh^5(a+bx)}{5b}-\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] -Cosh[a + b*x]^3/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0348252, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cosh^5(a+bx)}{5b}-\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]

[Out] -Cosh[a + b*x]^3/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

Mathematica [A] time = 0.069528, size = 27, normalized size = 0.87

$$\frac{\cosh^3(a+bx)(3\cosh(2(a+bx))-7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (Cosh[a + b*x]^3*(-7 + 3*Cosh[2*(a + b*x)]))/(30*b)
```

Maple [A] time = 0.009, size = 48, normalized size = 1.6

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3(\sinh(bx+a))^2}{5}-\frac{2\cosh(bx+a)(\sinh(bx+a))^2}{15}-\frac{2\cosh(bx+a)}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15*co
sh(b*x+a))
```

Maxima [B] time = 1.02092, size = 105, normalized size = 3.39

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}+30\,e^{(-4\,bx-4\,a)}-3\right)e^{(5\,bx+5\,a)}}{480\,b}-\frac{30\,e^{(-bx-a)}+5\,e^{(-3\,bx-3\,a)}-3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/480*(5*e^(-2*b*x - 2*a) + 30*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/480*(30*e^(-b*x - a) + 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

Fricas [B] time = 1.884, size = 216, normalized size = 6.97

$$\frac{3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 - 5\cosh(bx+a)^3 + 15\left(2\cosh(bx+a)^3 - \cosh(bx+a)\right)\sinh(bx}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 5*cosh(b*x +
a)^3 + 15*(2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 30*cosh(b*x
 + a))/b
```

Sympy [A] time = 2.12358, size = 44, normalized size = 1.42

$$\begin{cases} \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2\cosh^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*cosh(a + b*x)**5/(15
*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**2, True))
```

Giac [B] time = 1.15514, size = 95, normalized size = 3.06

$$-\frac{\left(30\,e^{(4\,bx+4\,a)} + 5\,e^{(2\,bx+2\,a)} - 3\right)e^{(-5\,bx-5\,a)} - 3\,e^{(5\,bx+5\,a)} + 5\,e^{(3\,bx+3\,a)} + 30\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/480*((30*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) - 3)*e^(-5*b*x - 5*a) - 3*e
^(5*b*x + 5*a) + 5*e^(3*b*x + 3*a) + 30*e^(b*x + a))/b
```

3.321 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{8}\sinh(a)\mathbf{Chi}(bx)-\frac{1}{16}\sinh(3a)\mathbf{Chi}(3bx)+\frac{1}{16}\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}\cosh(a)\mathbf{Shi}(bx)-\frac{1}{16}\cosh(3a)\mathbf{Shi}(3bx)+\frac{1}{16}\mathrm{c}$$

```
[Out] -(CoshIntegral[b*x]*Sinh[a])/8 - (CoshIntegral[3*b*x]*Sinh[3*a])/16 + (Cosh
Integral[5*b*x]*Sinh[5*a])/16 - (Cosh[a]*SinhIntegral[b*x])/8 - (Cosh[3*a]*
SinhIntegral[3*b*x])/16 + (Cosh[5*a]*SinhIntegral[5*b*x])/16
```

Rubi [A] time = 0.182328, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{8}\sinh(a)\mathbf{Chi}(bx)-\frac{1}{16}\sinh(3a)\mathbf{Chi}(3bx)+\frac{1}{16}\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{8}\cosh(a)\mathbf{Shi}(bx)-\frac{1}{16}\cosh(3a)\mathbf{Shi}(3bx)+\frac{1}{16}\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -(CoshIntegral[b*x]*Sinh[a])/8 - (CoshIntegral[3*b*x]*Sinh[3*a])/16 + (Cosh
Integral[5*b*x]*Sinh[5*a])/16 - (Cosh[a]*SinhIntegral[b*x])/8 - (Cosh[3*a]*
SinhIntegral[3*b*x])/16 + (Cosh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x}-\frac{\sinh(3a+3bx)}{16x}+\frac{\sinh(5a+5bx)}{16x}\right)dx\\ &= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x}\,dx\right)+\frac{1}{16}\int \frac{\sinh(5a+5bx)}{x}\,dx-\frac{1}{8}\int \frac{\sinh(a+bx)}{x}\,dx\\ &= -\left(\frac{1}{8}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx\right)-\frac{1}{16}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{16}\cosh(5a)\int \frac{\sin}{}\\ &= -\frac{1}{8}\mathbf{Chi}(bx)\sinh(a)-\frac{1}{16}\mathbf{Chi}(3bx)\sinh(3a)+\frac{1}{16}\mathbf{Chi}(5bx)\sinh(5a)-\frac{1}{8}\cosh(a)\mathbf{Sh}\end{aligned}$$

Mathematica [A] time = 0.10901, size = 63, normalized size = 0.86

$$\frac{1}{16}(-2\sinh(a)\mathbf{Chi}(bx)-\sinh(3a)\mathbf{Chi}(3bx)+\sinh(5a)\mathbf{Chi}(5bx)-2\cosh(a)\mathbf{Shi}(bx)-\cosh(3a)\mathbf{Shi}(3bx)+\cosh(5a)\mathbf{Shi}($$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-2*CoshIntegral[b*x]*Sinh[a] - CoshIntegral[3*b*x]*Sinh[3*a] + CoshIntegra
l[5*b*x]*Sinh[5*a] - 2*Cosh[a]*SinhIntegral[b*x] - Cosh[3*a]*SinhIntegral[3
*b*x] + Cosh[5*a]*SinhIntegral[5*b*x])/16
```

Maple [A] time = 0.091, size = 71, normalized size = 1.

$$\frac{e^{-5a}\mathrm{Ei}(1,5bx)}{32}-\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{32}-\frac{e^{-a}\mathrm{Ei}(1,bx)}{16}+\frac{e^{a}\mathrm{Ei}(1,-bx)}{16}+\frac{e^{3a}\mathrm{Ei}(1,-3bx)}{32}-\frac{e^{5a}\mathrm{Ei}(1,-5bx)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-a)*Ei(1,b*x
)+1/16*exp(a)*Ei(1,-b*x)+1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*b
*x)
```

Maxima [A] time = 1.30887, size = 86, normalized size = 1.18

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + \frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Fricas [A] time = 1.80776, size = 323, normalized size = 4.42

$$\frac{1}{32}\left(\mathrm{Ei}\left(5\,bx\right) - \mathrm{Ei}\left(-5\,bx\right)\right)\cosh\left(5\,a\right) - \frac{1}{32}\left(\mathrm{Ei}\left(3\,bx\right) - \mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right) - \frac{1}{16}\left(\mathrm{Ei}\left(bx\right) - \mathrm{Ei}\left(-bx\right)\right)\cosh\left(a\right) + \frac{1}{32}\,(\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) - Ei(-5*b*x))*cosh(5*a) - 1/32*(Ei(3*b*x) - Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) - Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) + Ei(-5*b*x))*
sinh(5*a) - 1/32*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) + Ei(-b
*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{3}\left(a + bx\right)\cosh^{2}\left(a + bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x, x)
```

Giac [A] time = 1.17892, size = 86, normalized size = 1.18

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}-\frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+\frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-\frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.322 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2} dx$

Optimal. Leaf size=124

$$-\frac{1}{8}b\cosh(a)\text{Chi}(bx) - \frac{3}{16}b\cosh(3a)\text{Chi}(3bx) + \frac{5}{16}b\cosh(5a)\text{Chi}(5bx) - \frac{1}{8}b\sinh(a)\text{Shi}(bx) - \frac{3}{16}b\sinh(3a)\text{Shi}(3bx) -$$

```
[Out] -(b*Cosh[a]*CoshIntegral[b*x])/8 - (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/16 +
 (5*b*Cosh[5*a]*CoshIntegral[5*b*x])/16 + Sinh[a + b*x]/(8*x) + Sinh[3*a +
3*b*x]/(16*x) - Sinh[5*a + 5*b*x]/(16*x) - (b*Sinh[a]*SinhIntegral[b*x])/8
- (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Sinh[5*a]*SinhIntegral[5*b*
x])/16
```

Rubi [A] time = 0.249725, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b\cosh(a)\text{Chi}(bx) - \frac{3}{16}b\cosh(3a)\text{Chi}(3bx) + \frac{5}{16}b\cosh(5a)\text{Chi}(5bx) - \frac{1}{8}b\sinh(a)\text{Shi}(bx) - \frac{3}{16}b\sinh(3a)\text{Shi}(3bx) -$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```



```
[Out] -(b*Cosh[a]*CoshIntegral[b*x])/8 - (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/16 +
 (5*b*Cosh[5*a]*CoshIntegral[5*b*x])/16 + Sinh[a + b*x]/(8*x) + Sinh[3*a +
3*b*x]/(16*x) - Sinh[5*a + 5*b*x]/(16*x) - (b*Sinh[a]*SinhIntegral[b*x])/8
- (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Sinh[5*a]*SinhIntegral[5*b*
x])/16
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{16x^2}+\frac{\sinh(5a+5bx)}{16x^2}\right)dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^2}\,dx\right)+\frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^2}\,dx-\frac{1}{8}\int \frac{\sinh(a+bx)}{x^2}\,dx \\
&= \frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}-\frac{1}{8}b\int \frac{\cosh(a+bx)}{x}\,dx-\frac{1}{16}(3b \\
&= \frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}-\frac{1}{8}(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx- \\
&= -\frac{1}{8}b\cosh(a)\mathrm{Chi}(bx)-\frac{3}{16}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{5}{16}b\cosh(5a)\mathrm{Chi}(5bx)+\frac{\sinh(a+}{8x}
\end{aligned}$$

Mathematica [A] time = 0.264244, size = 106, normalized size = 0.85

$$\frac{-2bx\cosh(a)\mathrm{Chi}(bx)-3bx\cosh(3a)\mathrm{Chi}(3bx)+5bx\cosh(5a)\mathrm{Chi}(5bx)-2bx\sinh(a)\mathrm{Shi}(bx)-3bx\sinh(3a)\mathrm{Shi}(3bx)+}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-2*b*x*Cosh[a]*CoshIntegral[b*x] - 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] + 5
*b*x*Cosh[5*a]*CoshIntegral[5*b*x] + 2*Sinh[a + b*x] + Sinh[3*(a + b*x)] -
Sinh[5*(a + b*x)] - 2*b*x*Sinh[a]*SinhIntegral[b*x] - 3*b*x*Sinh[3*a]*SinhI
ntegral[3*b*x] + 5*b*x*Sinh[5*a]*SinhIntegral[5*b*x])/(16*x)
```

Maple [A] time = 0.095, size = 158, normalized size = 1.3

$$\frac{e^{-5bx-5a}}{32x} - \frac{5be^{-5a}\mathrm{Ei}(1,5bx)}{32} - \frac{e^{-3bx-3a}}{32x} + \frac{3be^{-3a}\mathrm{Ei}(1,3bx)}{32} - \frac{e^{-bx-a}}{16x} + \frac{be^{-a}\mathrm{Ei}(1,bx)}{16} + \frac{e^{bx+a}}{16x} + \frac{be^{a}\mathrm{Ei}(1,-bx)}{16} + \frac{e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/32*exp(-5*b*x-5*a)/x-5/32*b*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*b*x-3*a)/x+
3/32*b*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-b*x-a)/x+1/16*b*exp(-a)*Ei(1,b*x)+1/
16/x*exp(b*x+a)+1/16*b*exp(a)*Ei(1,-b*x)+1/32/x*exp(3*b*x+3*a)+3/32*b*exp(3
*a)*Ei(1,-3*b*x)-1/32/x*exp(5*b*x+5*a)-5/32*b*exp(5*a)*Ei(1,-5*b*x)
```

Maxima [A] time = 1.2915, size = 103, normalized size = 0.83

$$\frac{5}{32}be^{(-5a)}\Gamma(-1,5bx) - \frac{3}{32}be^{(-3a)}\Gamma(-1,3bx) - \frac{1}{16}be^{(-a)}\Gamma(-1,bx) - \frac{1}{16}be^{a}\Gamma(-1,-bx) - \frac{3}{32}be^{(3a)}\Gamma(-1,-3bx) + \frac{5}{32}be^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) - 1/16*
b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) - 3/32*b*e^(3*a)*gamma
(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

Fricas [A] time = 1.83772, size = 548, normalized size = 4.42

$$-\frac{2\sinh(bx+a)^5 + 2\left(10\cosh(bx+a)^2 - 1\right)\sinh(bx+a)^3 - 5\left(bx\mathrm{Ei}(5bx) + bx\mathrm{Ei}(-5bx)\right)\cosh(5a) + 3\left(bx\mathrm{Ei}(3bx)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 5*(
b*x*Ei(5*b*x) + b*x*Ei(-5*b*x))*cosh(5*a) + 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*
x))*cosh(3*a) + 2*(b*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)
^4 - 3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 5*(b*x*Ei(5*b*x) - b*x*Ei(-5*b*
x))*sinh(5*a) + 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*x*Ei(b*
x) - b*x*Ei(-b*x))*sinh(a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**2, x)
```

Giac [A] time = 1.19002, size = 189, normalized size = 1.52

$$\frac{5\,bx\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} - 3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} - 2\,bx\mathrm{Ei}\left(-bx\right)e^{(-a)} - 3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} + 5\,bx\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)} - 2\,bx\mathrm{Ei}\left(bx\right)e^{a} -}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) - 3*b*x*Ei(3*b*x)*e^(3*a) - 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) + 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) - 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x
```

3.323 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{16}b^2\sinh(a)\mathbf{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\mathbf{Chi}(3bx)+\frac{25}{32}b^2\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\mathbf{Shi}(bx)-\frac{9}{32}b^2\cosh(3a)\mathbf{Sh}$$

```
[Out] (b*Cosh[a + b*x])/(16*x) + (3*b*Cosh[3*a + 3*b*x])/(32*x) - (5*b*Cosh[5*a +
 5*b*x])/(32*x) - (b^2*CoshIntegral[b*x]*Sinh[a])/16 - (9*b^2*CoshIntegral[
3*b*x]*Sinh[3*a])/32 + (25*b^2*CoshIntegral[5*b*x]*Sinh[5*a])/32 + Sinh[a +
 b*x]/(16*x^2) + Sinh[3*a + 3*b*x]/(32*x^2) - Sinh[5*a + 5*b*x]/(32*x^2) -
(b^2*Cosh[a]*SinhIntegral[b*x])/16 - (9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Cosh[5*a]*SinhIntegral[5*b*x])/32
```

Rubi [A] time = 0.339063, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{16}b^2\sinh(a)\mathbf{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\mathbf{Chi}(3bx)+\frac{25}{32}b^2\sinh(5a)\mathbf{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\mathbf{Shi}(bx)-\frac{9}{32}b^2\cosh(3a)\mathbf{Sh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[a + b*x])/(16*x) + (3*b*Cosh[3*a + 3*b*x])/(32*x) - (5*b*Cosh[5*a +
 5*b*x])/(32*x) - (b^2*CoshIntegral[b*x]*Sinh[a])/16 - (9*b^2*CoshIntegral[
3*b*x]*Sinh[3*a])/32 + (25*b^2*CoshIntegral[5*b*x]*Sinh[5*a])/32 + Sinh[a +
 b*x]/(16*x^2) + Sinh[3*a + 3*b*x]/(32*x^2) - Sinh[5*a + 5*b*x]/(32*x^2) -
(b^2*Cosh[a]*SinhIntegral[b*x])/16 - (9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Cosh[5*a]*SinhIntegral[5*b*x])/32
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x^3}-\frac{\sinh(3a+3bx)}{16x^3}+\frac{\sinh(5a+5bx)}{16x^3}\right)dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^3}\,dx\right)+\frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^3}\,dx-\frac{1}{8}\int \frac{\sinh(a+bx)}{x^3}\,dx \\
&= \frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3a+3bx)}{32x^2}-\frac{\sinh(5a+5bx)}{32x^2}-\frac{1}{16}b\int \frac{\cosh(a+bx)}{x^2}\,dx-\frac{1}{32}(\\
&= \frac{b\cosh(a+bx)}{16x}+\frac{3b\cosh(3a+3bx)}{32x}-\frac{5b\cosh(5a+5bx)}{32x}+\frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3}{3} \\
&= \frac{b\cosh(a+bx)}{16x}+\frac{3b\cosh(3a+3bx)}{32x}-\frac{5b\cosh(5a+5bx)}{32x}+\frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3}{3} \\
&= \frac{b\cosh(a+bx)}{16x}+\frac{3b\cosh(3a+3bx)}{32x}-\frac{5b\cosh(5a+5bx)}{32x}-\frac{1}{16}b^2\mathbf{Chi}(bx)\sinh(a)-
\end{aligned}$$

Mathematica [A] time = 0.438801, size = 164, normalized size = 0.89

$$\frac{-2b^2x^2\sinh(a)\mathbf{Chi}(bx)-9b^2x^2\sinh(3a)\mathbf{Chi}(3bx)+25b^2x^2\sinh(5a)\mathbf{Chi}(5bx)-2b^2x^2\cosh(a)\mathbf{Shi}(bx)-9b^2x^2\cosh(3a)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^2*x^2*CoshIntegral[b*x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*
a] + 25*b^2*x^2*CoshIntegral[5*b*x]*Sinh[5*a] + 2*Sinh[a + b*x] + Sinh[3*(a
 + b*x)] - Sinh[5*(a + b*x)] - 2*b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*
x^2*Cosh[3*a]*SinhIntegral[3*b*x] + 25*b^2*x^2*Cosh[5*a]*SinhIntegral[5*b*x
])/(32*x^2)
```

Maple [A] time = 0.097, size = 257, normalized size = 1.4

$$-\frac{5\,b\mathrm{e}^{-5\,bx-5\,a}}{64\,x}+\frac{\mathrm{e}^{-5\,bx-5\,a}}{64\,x^2}+\frac{25\,b^2\mathrm{e}^{-5\,a}\mathrm{Ei}\,(1,5\,bx)}{64}+\frac{3\,b\mathrm{e}^{-3\,bx-3\,a}}{64\,x}-\frac{\mathrm{e}^{-3\,bx-3\,a}}{64\,x^2}-\frac{9\,b^2\mathrm{e}^{-3\,a}\mathrm{Ei}\,(1,3\,bx)}{64}+\frac{b\mathrm{e}^{-bx-a}}{32\,x}-\frac{\mathrm{e}^{-bx-a}}{32\,x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -5/64*b*exp(-5*b*x-5*a)/x+1/64*exp(-5*b*x-5*a)/x^2+25/64*b^2*exp(-5*a)*Ei(1
,5*b*x)+3/64*b*exp(-3*b*x-3*a)/x-1/64*exp(-3*b*x-3*a)/x^2-9/64*b^2*exp(-3*a
)*Ei(1,3*b*x)+1/32*b*exp(-b*x-a)/x-1/32*exp(-b*x-a)/x^2-1/32*b^2*exp(-a)*Ei
(1,b*x)+1/32/x^2*exp(b*x+a)+1/32*b/x*exp(b*x+a)+1/32*b^2*exp(a)*Ei(1,-b*x)+
1/64/x^2*exp(3*b*x+3*a)+3/64*b/x*exp(3*b*x+3*a)+9/64*b^2*exp(3*a)*Ei(1,-3*b
*x)-1/64/x^2*exp(5*b*x+5*a)-5/64*b/x*exp(5*b*x+5*a)-25/64*b^2*exp(5*a)*Ei(1
,-5*b*x)
```

Maxima [A] time = 1.32324, size = 119, normalized size = 0.65

$$\frac{25}{32}\,b^2e^{(-5\,a)}\Gamma\,(-2,5\,bx)-\frac{9}{32}\,b^2e^{(-3\,a)}\Gamma\,(-2,3\,bx)-\frac{1}{16}\,b^2e^{(-a)}\Gamma\,(-2,bx)+\frac{1}{16}\,b^2e^{a}\Gamma\,(-2,-bx)+\frac{9}{32}\,b^2e^{(3\,a)}\Gamma\,(-2,-3\,bx)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) -
1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) + 9/32*b^2*e^
```

```
(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

Fricas [B] time = 1.81288, size = 857, normalized size = 4.66

$$-\frac{10\,bx\cosh\left(bx+a\right)^{5}+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-6\,bx\cosh\left(bx+a\right)^{3}+2\,\sinh\left(bx+a\right)^{5}+2\left(10\,\cosh\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 6*b*
x*cosh(b*x + a)^3 + 2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a)^3 - 4*b*x*cosh(b*x + a) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x
+ a))*sinh(b*x + a)^2 - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*cosh(5*
a) + 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b
*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2
- 2)*sinh(b*x + a) - 25*(b^2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*sinh(5*a)
+ 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x)
 + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**3, x)
```

Giac [A] time = 1.1854, size = 323, normalized size = 1.76

$$\frac{25\,b^{2}x^{2}\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}-9\,b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+2\,b^{2}x^{2}\mathrm{Ei}\left(-bx\right)e^{(-a)}+9\,b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-25\,b^{2}x^{2}\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) - 9*b^2*x^2*Ei(3*b*x)*e^(3*a) + 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) - 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) + 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) -
2*e^(-b*x - a) - e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x^2
```

3.324 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{1}{48}b^3\cosh(a)\mathbf{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\mathbf{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\mathbf{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\mathbf{Shi}(bx)-\frac{9}{32}b^3\sinh(3a)\mathbf{Sh}$$

```
[Out] (b*Cosh[a + b*x])/(48*x^2) + (b*Cosh[3*a + 3*b*x])/(32*x^2) - (5*b*Cosh[5*a
 + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*CoshIntegral[b*x])/48 - (9*b^3*Cosh[3*a]
*CoshIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*CoshIntegral[5*b*x])/96 + Sin
h[a + b*x]/(24*x^3) + (b^2*Sinh[a + b*x])/(48*x) + Sinh[3*a + 3*b*x]/(48*x^
3) + (3*b^2*Sinh[3*a + 3*b*x])/(32*x) - Sinh[5*a + 5*b*x]/(48*x^3) - (25*b^
2*Sinh[5*a + 5*b*x])/(96*x) - (b^3*Sinh[a]*SinhIntegral[b*x])/48 - (9*b^3*S
inh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Sinh[5*a]*SinhIntegral[5*b*x])/
96
```

Rubi [A] time = 0.413544, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{48}b^3\cosh(a)\mathbf{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\mathbf{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\mathbf{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\mathbf{Shi}(bx)-\frac{9}{32}b^3\sinh(3a)\mathbf{Sh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```


```
[Out] (b*Cosh[a + b*x])/(48*x^2) + (b*Cosh[3*a + 3*b*x])/(32*x^2) - (5*b*Cosh[5*a
 + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*CoshIntegral[b*x])/48 - (9*b^3*Cosh[3*a]
*CoshIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*CoshIntegral[5*b*x])/96 + Sin
h[a + b*x]/(24*x^3) + (b^2*Sinh[a + b*x])/(48*x) + Sinh[3*a + 3*b*x]/(48*x^
3) + (3*b^2*Sinh[3*a + 3*b*x])/(32*x) - Sinh[5*a + 5*b*x]/(48*x^3) - (25*b^
2*Sinh[5*a + 5*b*x])/(96*x) - (b^3*Sinh[a]*SinhIntegral[b*x])/48 - (9*b^3*S
inh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Sinh[5*a]*SinhIntegral[5*b*x])/
96
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \left(-\frac{\sinh(a+bx)}{8x^4} - \frac{\sinh(3a+3bx)}{16x^4} + \frac{\sinh(5a+5bx)}{16x^4}\right)dx$$
$$= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^4}\,dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^4}\,dx - \frac{1}{8}\int \frac{\sinh(a+bx)}{x^4}\,dx$$
$$= \frac{\sinh(a+bx)}{24x^3} + \frac{\sinh(3a+3bx)}{48x^3} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{1}{24}b\int \frac{\cosh(a+bx)}{x^3}\,dx - \frac{1}{16}b$$
$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} + \frac{\sinh(3a}{48}$$
$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} + \frac{b^2\sinh}{48}$$
$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} + \frac{b^2\sinh}{48}$$
$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} - \frac{1}{48}b^3\cosh(a)\mathbf{Chi}(bx) -$$

Mathematica [A] time = 0.520538, size = 212, normalized size = 0.89

$$\frac{-2b^3x^3\cosh(a)\mathbf{Chi}(bx) - 27b^3x^3\cosh(3a)\mathbf{Chi}(3bx) + 125b^3x^3\cosh(5a)\mathbf{Chi}(5bx) - 2b^3x^3\sinh(a)\mathbf{Shi}(bx) - 27b^3x^3\sinh(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^3*x^3*Cosh[a]*CoshIntegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b
*x] + 125*b^3*x^3*Cosh[5*a]*CoshIntegral[5*b*x] + 4*Sinh[a + b*x] + 2*b^2*x
^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] - 2*Si
nh[5*(a + b*x)] - 25*b^2*x^2*Sinh[5*(a + b*x)] - 2*b^3*x^3*Sinh[a]*SinhInte
gral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Sinh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

Maple [A] time = 0.102, size = 356, normalized size = 1.5

$$\frac{25\,b^2{\rm e}^{-5\,bx-5\,a}}{192\,x} - \frac{5\,b{\rm e}^{-5\,bx-5\,a}}{192\,x^2} + \frac{{\rm e}^{-5\,bx-5\,a}}{96\,x^3} - \frac{125\,b^3{\rm e}^{-5\,a}{\rm Ei}\,(1,5\,bx)}{192} - \frac{3\,b^2{\rm e}^{-3\,bx-3\,a}}{64\,x} + \frac{b{\rm e}^{-3\,bx-3\,a}}{64\,x^2} - \frac{{\rm e}^{-3\,bx-3\,a}}{96\,x^3} + \frac{9\,b^3{\rm e}^{-3\,a}{\rm Ei}\,(1}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 25/192*b^2*exp(-5*b*x-5*a)/x-5/192*b*exp(-5*b*x-5*a)/x^2+1/96*exp(-5*b*x-5*
a)/x^3-125/192*b^3*exp(-5*a)*Ei(1,5*b*x)-3/64*b^2*exp(-3*b*x-3*a)/x+1/64*b*
exp(-3*b*x-3*a)/x^2-1/96*exp(-3*b*x-3*a)/x^3+9/64*b^3*exp(-3*a)*Ei(1,3*b*x)
-1/96*b^2*exp(-b*x-a)/x+1/96*b*exp(-b*x-a)/x^2-1/48*exp(-b*x-a)/x^3+1/96*b^
3*exp(-a)*Ei(1,b*x)+1/48/x^3*exp(b*x+a)+1/96*b/x^2*exp(b*x+a)+1/96*b^2/x*ex
p(b*x+a)+1/96*b^3*exp(a)*Ei(1,-b*x)+1/96/x^3*exp(3*b*x+3*a)+1/64*b/x^2*exp(
3*b*x+3*a)+3/64*b^2/x*exp(3*b*x+3*a)+9/64*b^3*exp(3*a)*Ei(1,-3*b*x)-1/96/x^
3*exp(5*b*x+5*a)-5/192*b/x^2*exp(5*b*x+5*a)-25/192*b^2/x*exp(5*b*x+5*a)-125
/192*b^3*exp(5*a)*Ei(1,-5*b*x)
```

Maxima [A] time = 1.30185, size = 119, normalized size = 0.5

$$\frac{125}{32}\,b^3e^{(-5\,a)}\Gamma\left(-3,5\,bx\right)-\frac{27}{32}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)-\frac{1}{16}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{16}\,b^3e^{a}\Gamma\left(-3,-bx\right)-\frac{27}{32}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
- 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) - 27/32*b^3
*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

Fricas [A] time = 1.87401, size = 987, normalized size = 4.15

$$-\frac{10\,bx\cosh\left(bx+a\right)^5+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5-6\,bx\cosh\left(bx+a\right)^3-2\left(9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(
25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 6*b*x*cosh(b*x + a)^3 - 2*(9*b^2*x^2 - 10
*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^3 - 4*b*x*cosh(b*x + a
) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 125*
(b^3*x^3*Ei(5*b*x) + b^3*x^3*Ei(-5*b*x))*cosh(5*a) + 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*co
```

```
sh(a) + 2*(5*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 2*b^2*x^2 - 3*(9*b^2*x^2 +
2)*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei
(-5*b*x))*sinh(5*a) + 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a)
 + 2*(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**4, x)
```

Giac [A] time = 1.17844, size = 462, normalized size = 1.94

$$\frac{125\,b^3x^3\mathrm{Ei}\,(5\,bx)\,e^{(5\,a)} - 27\,b^3x^3\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)} - 2\,b^3x^3\mathrm{Ei}\,(-bx)\,e^{(-a)} - 27\,b^3x^3\mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)} + 125\,b^3x^3\mathrm{Ei}\,(-5\,bx)\,e^{(-5\,a)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) - 27*b^3*x^3*Ei(3*b*x)*e^(3*a) - 2*b^3
*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) + 125*b^3*x^3*Ei(-5*b
*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) + 9*b^2*x
^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) - 2*b^2*x^2*e^(-b*x - a) - 9*b^2
*x^2*e^(-3*b*x - 3*a) + 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 + 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) - 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) + 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) - 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + 2*e^(-5*b*x -
 5*a))/x^3
```

3.325 $\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=155

$$\frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-6bx)}{b}-\frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{3e^{-2a}2^{-m-7}x^m(bx)^{-m}}{b}$$

```
[Out] (2^(-7 - m)*3^(-1 - m)*E^(6*a)*x^m*Gamma[1 + m, -6*b*x])/(b*(-(b*x))^m) - (
3*2^(-7 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (3*2^(-7 -
m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (2^(-7 - m)*3^(-1 - m)*x^
m*Gamma[1 + m, 6*b*x])/(b*E^(6*a)*(b*x)^m)
```

Rubi [A] time = 0.243486, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3308, 2181}

$$\frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-6bx)}{b}-\frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{3e^{-2a}2^{-m-7}x^m(bx)^{-m}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```


```
[Out] (2^(-7 - m)*3^(-1 - m)*E^(6*a)*x^m*Gamma[1 + m, -6*b*x])/(b*(-(b*x))^m) - (
3*2^(-7 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (3*2^(-7 -
m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (2^(-7 - m)*3^(-1 - m)*x^
m*Gamma[1 + m, 6*b*x])/(b*E^(6*a)*(b*x)^m)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{32}x^m \sinh(2a+2bx) + \frac{1}{32}x^m \sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int x^m \sinh(6a+6bx)\,dx - \frac{3}{32}\int x^m \sinh(2a+2bx)\,dx$$
$$= \frac{1}{64}\int e^{-i(6ia+6ibx)}x^m\,dx - \frac{1}{64}\int e^{i(6ia+6ibx)}x^m\,dx - \frac{3}{64}\int e^{-i(2ia+2ibx)}x^m\,dx + \frac{3}{64}\int$$
$$= \frac{2^{-7-m}3^{-1-m}e^{6a}x^m(-bx)^{-m}\Gamma(1+m,-6bx)}{b} - \frac{3\ 2^{-7-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}$$

Mathematica [A] time = 0.152997, size = 119, normalized size = 0.77

$$\frac{e^{-6a}2^{-m-7}3^{-m-1}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(\text{Gamma}(m+1,6bx)-e^{4a}3^{m+2}\text{Gamma}(m+1,2bx)\right)+e^{12a}(bx)^m\text{Gamma}(m+1\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*x^m*(E^(12*a)*(b*x)^m*Gamma[1 + m, -6*b*x] - 3^(2 +
m)*E^(8*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(3^(2 + m)*E^(4*a)*G
amma[1 + m, 2*b*x]) + Gamma[1 + m, 6*b*x])))/(b*E^(6*a)*(-(b^2*x^2))^m)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int x^m(\cosh(bx+a))^3(\sinh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.19729, size = 158, normalized size = 1.02

$$\frac{1}{64}(6bx)^{-m-1}x^{m+1}e^{(-6a)}\Gamma(m+1,6bx)-\frac{3}{64}(2bx)^{-m-1}x^{m+1}e^{(-2a)}\Gamma(m+1,2bx)+\frac{3}{64}(-2bx)^{-m-1}x^{m+1}e^{(2a)}\Gamma(m+1,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(6*b*x)^(-m - 1)*x^(m + 1)*e^(-6*a)*gamma(m + 1, 6*b*x) - 3/64*(2*b*x)
^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 3/64*(-2*b*x)^(-m - 1)*x
^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/64*(-6*b*x)^(-m - 1)*x^(m + 1)*e^
(6*a)*gamma(m + 1, -6*b*x)
```

Fricas [A] time = 1.8499, size = 520, normalized size = 3.35

$$\frac{\cosh(m\log(6b)+6a)\Gamma(m+1,6bx)-9\cosh(m\log(2b)+2a)\Gamma(m+1,2bx)-9\cosh(m\log(-2b)-2a)\Gamma(m+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(m*log(6*b) + 6*a)*gamma(m + 1, 6*b*x) - 9*cosh(m*log(2*b) + 2*a
)*gamma(m + 1, 2*b*x) - 9*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + co
sh(m*log(-6*b) - 6*a)*gamma(m + 1, -6*b*x) - gamma(m + 1, 6*b*x)*sinh(m*log
(6*b) + 6*a) + 9*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 9*gamma(m + 1
, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -6*b*x)*sinh(m*log(-6*b) -
 6*a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.326 $\int x^3 \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=143

$$\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6bx)}{384b^2}+\frac{9\sinh(2a+2bx)}{256b^4}-\frac{\sinh(6a+6bx)}{6912b^4}-\frac{9x\cosh(2a+2bx)}{128b^3}+\frac{x\cosh(6a+6bx)}{1152b^3}$$

```
[Out] (-9*x*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^3*Cosh[2*a + 2*b*x])/(64*b) + (x*
Cosh[6*a + 6*b*x])/(1152*b^3) + (x^3*Cosh[6*a + 6*b*x])/(192*b) + (9*Sinh[2
*a + 2*b*x])/(256*b^4) + (9*x^2*Sinh[2*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6
*b*x]/(6912*b^4) - (x^2*Sinh[6*a + 6*b*x])/(384*b^2)
```

Rubi [A] time = 0.197204, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2637}

$$\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6bx)}{384b^2}+\frac{9\sinh(2a+2bx)}{256b^4}-\frac{\sinh(6a+6bx)}{6912b^4}-\frac{9x\cosh(2a+2bx)}{128b^3}+\frac{x\cosh(6a+6bx)}{1152b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-9*x*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^3*Cosh[2*a + 2*b*x])/(64*b) + (x*
Cosh[6*a + 6*b*x])/(1152*b^3) + (x^3*Cosh[6*a + 6*b*x])/(192*b) + (9*Sinh[2
*a + 2*b*x])/(256*b^4) + (9*x^2*Sinh[2*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6
*b*x]/(6912*b^4) - (x^2*Sinh[6*a + 6*b*x])/(384*b^2)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}x^3\sinh(2a+2bx)+\frac{1}{32}x^3\sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int x^3\sinh(6a+6bx)\,dx - \frac{3}{32}\int x^3\sinh(2a+2bx)\,dx$$
$$= -\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}-\frac{\int x^2\cosh(6a+6bx)\,dx}{64b}+\frac{9\int x^2\,\text{co}}{}$$
$$= -\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}+\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+}{384b^2}$$
$$= -\frac{9x\cosh(2a+2bx)}{128b^3}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{x^3\cosh(6a+6}{192b}$$
$$= -\frac{9x\cosh(2a+2bx)}{128b^3}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{x^3\cosh(6a+6}{192b}$$

Mathematica [A] time = 0.88507, size = 90, normalized size = 0.63

$$-\frac{81bx\left(2b^2x^2+3\right)\cosh(2(a+bx))-3\left(6b^3x^3+bx\right)\cosh(6(a+bx))+\sinh(2(a+bx))\left(\left(18b^2x^2+1\right)\cosh(4(a+bx))-2\right.}{3456b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] -(81*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(b*x + 6*b^3*x^3)*Cosh[6*(a
+ b*x)] + (-121 - 234*b^2*x^2 + (1 + 18*b^2*x^2)*Cosh[4*(a + b*x)])*Sinh[2*
(a + b*x)])/(3456*b^4)
```

Maple [B] time = 0.01, size = 622, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/6*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)^3*sinh(b*x+a)
^2*cosh(b*x+a)^2-1/12*(b*x+a)^3*cosh(b*x+a)^2-1/12*(b*x+a)^2*sinh(b*x+a)*co
sh(b*x+a)^5+1/12*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3+1/36*(b*x+a)*sinh(b*x+
a)^2*cosh(b*x+a)^4-1/72*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-5/36*(b*x+a)*co
sh(b*x+a)^2-1/216*sinh(b*x+a)*cosh(b*x+a)^5+1/216*cosh(b*x+a)^3*sinh(b*x+a)
+5/72*cosh(b*x+a)*sinh(b*x+a)+5/72*b*x+5/72*a+1/8*(b*x+a)^2*cosh(b*x+a)*sin
h(b*x+a)+1/24*(b*x+a)^3-3*a*(1/6*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12
*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2-1/12*(b*x+a)^2*cosh(b*x+a)^2-1/18*(b
*x+a)*sinh(b*x+a)*cosh(b*x+a)^5+1/18*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+1/10
8*cosh(b*x+a)^4*sinh(b*x+a)^2-1/216*cosh(b*x+a)^2*sinh(b*x+a)^2-5/108*cosh(
b*x+a)^2+1/12*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/24*(b*x+a)^2)+3*a^2*(1/6*(b
*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-
1/12*(b*x+a)*cosh(b*x+a)^2-1/36*sinh(b*x+a)*cosh(b*x+a)^5+1/36*cosh(b*x+a)^
3*sinh(b*x+a)+1/24*cosh(b*x+a)*sinh(b*x+a)+1/24*b*x+1/24*a)-a^3*(1/6*cosh(b
*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2))
```

Maxima [A] time = 1.08534, size = 231, normalized size = 1.62

$$\frac{\left(36\,b^3x^3e^{(6\,a)}-18\,b^2x^2e^{(6\,a)}+6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{13824\,b^4}-\frac{3\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{512\,b^4}-\frac{3\left(4\,b^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13824*(36*b^3*x^3*e^(6*a) - 18*b^2*x^2*e^(6*a) + 6*b*x*e^(6*a) - e^(6*a))
*e^(6*b*x)/b^4 - 3/512*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*
a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 3/512*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2*x^2 + 6*b*x + 1)*e^(-6*b
*x - 6*a)/b^4
```

Fricas [A] time = 1.76797, size = 617, normalized size = 4.31

$$\frac{3\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^6-10\left(18\,b^2x^2+1\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+45\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^2\sinh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3456*(3*(6*b^3*x^3 + b*x)*cosh(b*x + a)^6 - 10*(18*b^2*x^2 + 1)*cosh(b*x
+ a)^3*sinh(b*x + a)^3 + 45*(6*b^3*x^3 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)
^4 - 3*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(6*b^3*x^3 + b*x)
*sinh(b*x + a)^6 - 81*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 9*(18*b^3*x^3 -
 5*(6*b^3*x^3 + b*x)*cosh(b*x + a)^4 + 27*b*x)*sinh(b*x + a)^2 - 3*((18*b^2
*x^2 + 1)*cosh(b*x + a)^5 - 81*(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a)
)/b^4
```

Sympy [A] time = 23.1727, size = 314, normalized size = 2.2

$$\begin{cases} -\frac{x^3\sinh^6(a+bx)}{24b} + \frac{x^3\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^3\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^3\cosh^6(a+bx)}{24b} + \frac{x^2\sinh^5(a+bx)\cosh(a+bx)}{8b^2} - \frac{x^2\sinh^3(a+bx)}{3b^2} \\ \frac{x^4\sinh^3(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**6/(24*b) + x**3*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**3*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**3*cosh(a +
 b*x)**6/(24*b) + x**2*sinh(a + b*x)**5*cosh(a + b*x)/(8*b**2) - x**2*sinh(
a + b*x)**3*cosh(a + b*x)**3/(3*b**2) + x**2*sinh(a + b*x)*cosh(a + b*x)**5
/(8*b**2) - 5*x*sinh(a + b*x)**6/(72*b**3) + x*sinh(a + b*x)**4*cosh(a + b*
x)**2/(12*b**3) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(12*b**3) - 5*x*cosh(
a + b*x)**6/(72*b**3) + 5*sinh(a + b*x)**5*cosh(a + b*x)/(72*b**4) - 31*sin
h(a + b*x)**3*cosh(a + b*x)**3/(216*b**4) + 5*sinh(a + b*x)*cosh(a + b*x)**
5/(72*b**4), Ne(b, 0)), (x**4*sinh(a)**3*cosh(a)**3/4, True))
```

Giac [A] time = 1.30538, size = 196, normalized size = 1.37

$$\frac{\left(36\,b^3x^3 - 18\,b^2x^2 + 6\,bx - 1\right)e^{(6\,bx+6\,a)}}{13824\,b^4} - \frac{3\left(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3\right)e^{(2\,bx+2\,a)}}{512\,b^4} - \frac{3\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-2\,a}}{512\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/13824*(36*b^3*x^3 - 18*b^2*x^2 + 6*b*x - 1)*e^(6*b*x + 6*a)/b^4 - 3/512*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 3/512*(4*b^3*x^3 +
```

```
 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2
*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^4
```

3.327 $\int x^2 \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=105

$$\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}-\frac{3\cosh(2a+2bx)}{128b^3}+\frac{\cosh(6a+6bx)}{3456b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}$$

```
[Out] (-3*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^2*Cosh[2*a + 2*b*x])/(64*b) + Cosh[
6*a + 6*b*x]/(3456*b^3) + (x^2*Cosh[6*a + 6*b*x])/(192*b) + (3*x*Sinh[2*a +
 2*b*x])/(64*b^2) - (x*Sinh[6*a + 6*b*x])/(576*b^2)
```

Rubi [A] time = 0.140026, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5448, 3296, 2638}

$$\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}-\frac{3\cosh(2a+2bx)}{128b^3}+\frac{\cosh(6a+6bx)}{3456b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^2*Cosh[2*a + 2*b*x])/(64*b) + Cosh[
6*a + 6*b*x]/(3456*b^3) + (x^2*Cosh[6*a + 6*b*x])/(192*b) + (3*x*Sinh[2*a +
 2*b*x])/(64*b^2) - (x*Sinh[6*a + 6*b*x])/(576*b^2)
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx &= \int\left(-\frac{3}{32}x^2\sinh(2a+2bx)+\frac{1}{32}x^2\sinh(6a+6bx)\right)dx\\
&= \frac{1}{32}\int x^2\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^2\sinh(2a+2bx)\,dx\\
&= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}-\frac{\int x\cosh(6a+6bx)\,dx}{96b}+\frac{3\int x\cos
\end{aligned}$$

$$= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}+\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+}{576b^2}$$

$$= -\frac{3\cosh(2a+2bx)}{128b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{\cosh(6a+6bx)}{3456b^3}+\frac{x^2\cosh(6a+6b}{192b}$$

Mathematica [A] time = 0.219113, size = 72, normalized size = 0.69

$$\frac{-81\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(18b^2x^2+1\right)\cosh(6(a+bx))+6bx(27\sinh(2(a+bx))-\sinh(6(a+bx)))}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-81*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 18*b^2*x^2)*Cosh[6*(a + b*x)]
 + 6*b*x*(27*Sinh[2*(a + b*x)] - Sinh[6*(a + b*x)]))/(3456*b^3)
```

Maple [B] time = 0.009, size = 358, normalized size = 3.4

$$\frac{1}{b^3}\left(\frac{(bx+a)^2(\sinh(bx+a))^2(\cosh(bx+a))^4}{6}-\frac{(bx+a)^2(\sinh(bx+a))^2(\cosh(bx+a))^2}{12}-\frac{(bx+a)^2(\cosh(bx+a))^2}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(1/6*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)^2*sinh(b*x+a)
^2*cosh(b*x+a)^2-1/12*(b*x+a)^2*cosh(b*x+a)^2-1/18*(b*x+a)*sinh(b*x+a)*cosh
```

```
(b*x+a)^5+1/18*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+1/108*cosh(b*x+a)^4*sinh(b
*x+a)^2-1/216*cosh(b*x+a)^2*sinh(b*x+a)^2-5/108*cosh(b*x+a)^2+1/12*(b*x+a)*
cosh(b*x+a)*sinh(b*x+a)+1/24*(b*x+a)^2-2*a*(1/6*(b*x+a)*sinh(b*x+a)^2*cosh(
b*x+a)^4-1/12*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-1/12*(b*x+a)*cosh(b*x+a)^
2-1/36*sinh(b*x+a)*cosh(b*x+a)^5+1/36*cosh(b*x+a)^3*sinh(b*x+a)+1/24*cosh(b
*x+a)*sinh(b*x+a)+1/24*b*x+1/24*a)+a^2*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/1
2*cosh(b*x+a)^2*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2))
```

Maxima [A] time = 1.06654, size = 171, normalized size = 1.63

$$\frac{\left(18\,b^2x^2e^{(6\,a)} - 6\,bxe^{(6\,a)} + e^{(6\,a)}\right)e^{(6\,bx)}}{6912\,b^3} - \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^3} - \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{\left(18\,b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6912*(18*b^2*x^2*e^(6*a) - 6*b*x*e^(6*a) + e^(6*a))*e^(6*b*x)/b^3 - 3/256
*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 3/256*(2*b^2
*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^
(-6*b*x - 6*a)/b^3
```

Fricas [B] time = 1.75665, size = 531, normalized size = 5.06

$$-\frac{120\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3 + 36\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5 - \left(18\,b^2x^2+1\right)\cosh\left(bx+a\right)^6 - 15\left(18\,b^2x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3456*(120*b*x*cosh(b*x + a)^3*sinh(b*x + a)^3 + 36*b*x*cosh(b*x + a)*sin
h(b*x + a)^5 - (18*b^2*x^2 + 1)*cosh(b*x + a)^6 - 15*(18*b^2*x^2 + 1)*cosh(
b*x + a)^2*sinh(b*x + a)^4 - (18*b^2*x^2 + 1)*sinh(b*x + a)^6 + 81*(2*b^2*x
^2 + 1)*cosh(b*x + a)^2 - 3*(5*(18*b^2*x^2 + 1)*cosh(b*x + a)^4 - 54*b^2*x^
2 - 27)*sinh(b*x + a)^2 + 36*(b*x*cosh(b*x + a)^5 - 9*b*x*cosh(b*x + a))*si
nh(b*x + a))/b^3
```

Sympy [A] time = 13.4604, size = 223, normalized size = 2.12

$$\begin{cases} -\frac{x^2\sinh^6(a+bx)}{24b} + \frac{x^2\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^2\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^2\cosh^6(a+bx)}{24b} + \frac{x\sinh^5(a+bx)\cosh(a+bx)}{12b^2} - \frac{2x\sinh^3(a+bx)}{9b} \\ \frac{x^3\sinh^3(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**6/(24*b) + x**2*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**2*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**2*cosh(a +
 b*x)**6/(24*b) + x*sinh(a + b*x)**5*cosh(a + b*x)/(12*b**2) - 2*x*sinh(a +
 b*x)**3*cosh(a + b*x)**3/(9*b**2) + x*sinh(a + b*x)*cosh(a + b*x)**5/(12*b
**2) - 5*sinh(a + b*x)**6/(108*b**3) + 7*sinh(a + b*x)**4*cosh(a + b*x)**2/
(72*b**3) - sinh(a + b*x)**2*cosh(a + b*x)**4/(24*b**3), Ne(b, 0)), (x**3*s
inh(a)**3*cosh(a)**3/3, True))
```

Giac [A] time = 1.22813, size = 153, normalized size = 1.46

$$\frac{\left(18\,b^2x^2 - 6\,bx + 1\right)e^{(6\,bx+6\,a)}}{6912\,b^3} - \frac{3\left(2\,b^2x^2 - 2\,bx + 1\right)e^{(2\,bx+2\,a)}}{256\,b^3} - \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{\left(18\,b^2x^2 + 6\,bx + 1\right)e^{(-}}{6912\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/6912*(18*b^2*x^2 - 6*b*x + 1)*e^(6*b*x + 6*a)/b^3 - 3/256*(2*b^2*x^2 - 2*
b*x + 1)*e^(2*b*x + 2*a)/b^3 - 3/256*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*
a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^3
```

3.328 $\int x \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}-\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}$$

```
[Out] (-3*x*Cosh[2*a + 2*b*x])/(64*b) + (x*Cosh[6*a + 6*b*x])/(192*b) + (3*Sinh[2
*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6*b*x]/(1152*b^2)
```

Rubi [A] time = 0.0726938, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2637}

$$\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}-\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```



```
[Out] (-3*x*Cosh[2*a + 2*b*x])/(64*b) + (x*Cosh[6*a + 6*b*x])/(192*b) + (3*Sinh[2
*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6*b*x]/(1152*b^2)
```


Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{32}x\sinh(2a+2bx)+\frac{1}{32}x\sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int x\sinh(6a+6bx)\,dx-\frac{3}{32}\int x\sinh(2a+2bx)\,dx$$
$$= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}-\frac{\int\cosh(6a+6bx)\,dx}{192b}+\frac{3\int\cosh(2a}{64b}$$
$$= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}+\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}$$

Mathematica [A] time = 0.143971, size = 50, normalized size = 0.75

$$-\frac{-27\sinh(2(a+bx))+\sinh(6(a+bx))+54bx\cosh(2(a+bx))-6bx\cosh(6(a+bx))}{1152b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] -(54*b*x*Cosh[2*(a + b*x)] - 6*b*x*Cosh[6*(a + b*x)] - 27*Sinh[2*(a + b*x)]
 + Sinh[6*(a + b*x)])/(1152*b^2)
```

Maple [B] time = 0.009, size = 170, normalized size = 2.5

$$\frac{1}{b^2}\left(\frac{(bx+a)(\sinh(bx+a))^2(\cosh(bx+a))^4}{6}-\frac{(bx+a)(\sinh(bx+a))^2(\cosh(bx+a))^2}{12}-\frac{(bx+a)(\cosh(bx+a))^2}{12}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/6*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)*sinh(b*x+a)^2*c
osh(b*x+a)^2-1/12*(b*x+a)*cosh(b*x+a)^2-1/36*sinh(b*x+a)*cosh(b*x+a)^5+1/36
*cosh(b*x+a)^3*sinh(b*x+a)+1/24*cosh(b*x+a)*sinh(b*x+a)+1/24*b*x+1/24*a-a*(
1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2*sinh(b*x+a)^2-1/12*cosh(
b*x+a)^2))
```

Maxima [A] time = 1.0983, size = 123, normalized size = 1.84

$$\frac{\left(6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{2304\,b^2}-\frac{3\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2304*(6*b*x*e^(6*a) - e^(6*a))*e^(6*b*x)/b^2 - 3/256*(2*b*x*e^(2*a) - e^(
2*a))*e^(2*b*x)/b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*
x + 1)*e^(-6*b*x - 6*a)/b^2
```

Fricas [B] time = 1.80394, size = 408, normalized size = 6.09

$$3\,bx\cosh\left(bx+a\right)^{6}+45\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+3\,bx\sinh\left(bx+a\right)^{6}-10\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{3}-3\,\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/576*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 - 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 27*b*x*cosh(b*x + a)^2 + 9*(5*b*x*cosh(b*x + a)^4 - 3*b*x)
*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^5 - 9*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] time = 7.62976, size = 148, normalized size = 2.21

$$\begin{cases}-\frac{x\sinh^{6}(a+bx)}{24b}+\frac{x\sinh^{4}(a+bx)\cosh^{2}(a+bx)}{8b}+\frac{x\sinh^{2}(a+bx)\cosh^{4}(a+bx)}{8b}-\frac{x\cosh^{6}(a+bx)}{24b}+\frac{\sinh^{5}(a+bx)\cosh(a+bx)}{24b^2}-\frac{\sinh^{3}(a+bx)\cosh^{3}(a+}{9b^2}\\ \frac{x^2\sinh^{3}(a)\cosh^{3}(a)}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/(24*b) + x*sinh(a + b*x)**4*cosh(a + b*x)**2
/(8*b) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x*cosh(a + b*x)**6/(24
```

```
*b) + sinh(a + b*x)**5*cosh(a + b*x)/(24*b**2) - sinh(a + b*x)**3*cosh(a +
b*x)**3/(9*b**2) + sinh(a + b*x)*cosh(a + b*x)**5/(24*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**3/2, True))
```

Giac [A] time = 1.15879, size = 109, normalized size = 1.63

$$\frac{(6\,bx-1)e^{(6\,bx+6\,a)}}{2304\,b^2}-\frac{3\,(2\,bx-1)e^{(2\,bx+2\,a)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2304*(6*b*x - 1)*e^(6*b*x + 6*a)/b^2 - 3/256*(2*b*x - 1)*e^(2*b*x + 2*a)/
b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*x + 1)*e^(-6*b*x
 - 6*a)/b^2
```

3.329 $\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^6(a+bx)}{6b}+\frac{\sinh^4(a+bx)}{4b}$$

```
[Out] Sinh[a + b*x]^4/(4*b) + Sinh[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.036688, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^6(a+bx)}{6b}+\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b) + Sinh[a + b*x]^6/(6*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^3-x^5\right)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^4(a+bx)}{4b} + \frac{\sinh^6(a+bx)}{6b}$$

Mathematica [A] time = 0.0121394, size = 35, normalized size = 1.13

$$\frac{1}{8}\left(\frac{\cosh(6(a+bx))}{24b} - \frac{3\cosh(2(a+bx))}{8b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] ((-3*Cosh[2*(a + b*x)])/(8*b) + Cosh[6*(a + b*x)]/(24*b))/8
```

Maple [A] time = 0.007, size = 52, normalized size = 1.7

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4(\sinh(bx+a))^2}{6} - \frac{(\cosh(bx+a))^2(\sinh(bx+a))^2}{12} - \frac{(\cosh(bx+a))^2}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2*sinh(b*x+a)^2-1/12*
cosh(b*x+a)^2)
```

Maxima [B] time = 1.01174, size = 76, normalized size = 2.45

$$-\frac{\left(9\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{9\,e^{(-2\,bx-2\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/384*(9*e^(-2*b*x - 2*
a) - e^(-6*b*x - 6*a))/b
```

Fricas [B] time = 1.75977, size = 197, normalized size = 6.35

$$\frac{\cosh(bx+a)^6 + 15\cosh(bx+a)^2\sinh(bx+a)^4 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^4 - 3\right)\sinh(bx+a)^2 - 9\cosh(bx}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(cosh(b*x + a)^6 + 15*cosh(b*x + a)^2*sinh(b*x + a)^4 + sinh(b*x + a)
^6 + 3*(5*cosh(b*x + a)^4 - 3)*sinh(b*x + a)^2 - 9*cosh(b*x + a)^2)/b
```

Sympy [A] time = 3.74648, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{\sinh^6(a+bx)}{12b} + \frac{\sinh^4(a+bx)\cosh^2(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-sinh(a + b*x)**6/(12*b) + sinh(a + b*x)**4*cosh(a + b*x)**2/(4*
b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**3, True))
```

Giac [A] time = 1.16595, size = 66, normalized size = 2.13

$$\frac{\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}\right)^3 - 12\,e^{(2\,bx+2\,a)} - 12\,e^{(-2\,bx-2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/384*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 12*e^(2*b*x + 2*a) - 12*e^(
-2*b*x - 2*a))/b
```

3.330 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{32}\sinh(2a)\text{Chi}(2bx)+\frac{1}{32}\sinh(6a)\text{Chi}(6bx)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

```
[Out] (-3*CoshIntegral[2*b*x]*Sinh[2*a])/32 + (CoshIntegral[6*b*x]*Sinh[6*a])/32
- (3*Cosh[2*a]*SinhIntegral[2*b*x])/32 + (Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Rubi [A] time = 0.15705, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{3}{32}\sinh(2a)\text{Chi}(2bx)+\frac{1}{32}\sinh(6a)\text{Chi}(6bx)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-3*CoshIntegral[2*b*x]*Sinh[2*a])/32 + (CoshIntegral[6*b*x]*Sinh[6*a])/32
- (3*Cosh[2*a]*SinhIntegral[2*b*x])/32 + (Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x}+\frac{\sinh(6a+6bx)}{32x}\right)dx\\ &= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= -\left(\frac{1}{32}(3\cosh(2a))\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{32}\cosh(6a)\int \frac{\sinh(6bx)}{x}\,dx-\frac{1}{32}(3\sinh(\\ &= -\frac{3}{32}\mathbf{Chi}(2bx)\sinh(2a)+\frac{1}{32}\mathbf{Chi}(6bx)\sinh(6a)-\frac{3}{32}\cosh(2a)\mathbf{Shi}(2bx)+\frac{1}{32}\cosh\end{aligned}$$

Mathematica [A] time = 0.180148, size = 47, normalized size = 0.89

$$\frac{1}{32}(\sinh(6a)\mathbf{Chi}(6bx)-6\sinh(a)\cosh(a)\mathbf{Chi}(2bx)-3\cosh(2a)\mathbf{Shi}(2bx)+\cosh(6a)\mathbf{Shi}(6bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-6*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + CoshIntegral[6*b*x]*Sinh[6*a] - 3
*Cosh[2*a]*SinhIntegral[2*b*x] + Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Maple [A] time = 0.088, size = 50, normalized size = 0.9

$$\frac{e^{-6a}\mathrm{Ei}(1,6bx)}{64}-\frac{3e^{-2a}\mathrm{Ei}(1,2bx)}{64}-\frac{e^{6a}\mathrm{Ei}(1,-6bx)}{64}+\frac{3e^{2a}\mathrm{Ei}(1,-2bx)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/64*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*a)*Ei(1,2*b*x)-1/64*exp(6*a)*Ei(1,-6
*b*x)+3/64*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.22843, size = 61, normalized size = 1.15

$$\frac{1}{64}\,\mathrm{Ei}\left(6\,bx\right)e^{(6\,a)} - \frac{3}{64}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} + \frac{3}{64}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} - \frac{1}{64}\,\mathrm{Ei}\left(-6\,bx\right)e^{(-6\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

Fricas [A] time = 1.80573, size = 225, normalized size = 4.25

$$\frac{1}{64}\left(\mathrm{Ei}\left(6\,bx\right) - \mathrm{Ei}\left(-6\,bx\right)\right)\cosh\left(6\,a\right) - \frac{3}{64}\left(\mathrm{Ei}\left(2\,bx\right) - \mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right) + \frac{1}{64}\left(\mathrm{Ei}\left(6\,bx\right) + \mathrm{Ei}\left(-6\,bx\right)\right)\sinh\left(6\,a\right) - \frac{3}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/64*(Ei(6*b*x) - Ei(-6*b*x))*cosh(6*a) - 3/64*(Ei(2*b*x) - Ei(-2*b*x))*cos
h(2*a) + 1/64*(Ei(6*b*x) + Ei(-6*b*x))*sinh(6*a) - 3/64*(Ei(2*b*x) + Ei(-2*
b*x))*sinh(2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{3}\left(a + bx\right)\cosh^{3}\left(a + bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x, x)
```

Giac [A] time = 1.18582, size = 61, normalized size = 1.15

$$\frac{1}{64}\,\text{Ei}\left(6\,bx\right)e^{(6\,a)}-\frac{3}{64}\,\text{Ei}\left(2\,bx\right)e^{(2\,a)}+\frac{3}{64}\,\text{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{64}\,\text{Ei}\left(-6\,bx\right)e^{(-6\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

3.331 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}$$

```
[Out] (-3*b*Cosh[2*a]*CoshIntegral[2*b*x])/16 + (3*b*Cosh[6*a]*CoshIntegral[6*b*x
])/16 + (3*Sinh[2*a + 2*b*x])/(32*x) - Sinh[6*a + 6*b*x]/(32*x) - (3*b*Sinh
[2*a]*SinhIntegral[2*b*x])/16 + (3*b*Sinh[6*a]*SinhIntegral[6*b*x])/16
```

Rubi [A] time = 0.19576, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-3*b*Cosh[2*a]*CoshIntegral[2*b*x])/16 + (3*b*Cosh[6*a]*CoshIntegral[6*b*x
])/16 + (3*Sinh[2*a + 2*b*x])/(32*x) - Sinh[6*a + 6*b*x]/(32*x) - (3*b*Sinh
[2*a]*SinhIntegral[2*b*x])/16 + (3*b*Sinh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^2}+\frac{\sinh(6a+6bx)}{32x^2}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^2}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= \frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{1}{16}(3b)\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{16}(3b)\int \frac{c
\end{aligned}$$

$$\begin{aligned}
&= \frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{1}{16}(3b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{16}(3b\,c \\
&= -\frac{3}{16}b\cosh(2a)\mathbf{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\mathbf{Chi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a}{32
\end{aligned}$$

Mathematica [A] time = 0.242686, size = 78, normalized size = 0.88

$$-\frac{6bx\cosh(2a)\mathbf{Chi}(2bx)-6bx\cosh(6a)\mathbf{Chi}(6bx)+6bx\sinh(2a)\mathbf{Shi}(2bx)-6bx\sinh(6a)\mathbf{Shi}(6bx)-3\sinh(2(a+bx))}{32x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -(6*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 6*b*x*Cosh[6*a]*CoshIntegral[6*b*x]
 - 3*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 6*b*x*Sinh[2*a]*SinhIntegral[2
```

```
*b*x] - 6*b*x*Sinh[6*a]*SinhIntegral[6*b*x])/(32*x)
```

Maple [A] time = 0.095, size = 110, normalized size = 1.2

$$\frac{e^{-6bx-6a}}{64x} - \frac{3be^{-6a}\mathrm{Ei}(1,6bx)}{32} - \frac{3e^{-2bx-2a}}{64x} + \frac{3be^{-2a}\mathrm{Ei}(1,2bx)}{32} - \frac{e^{6bx+6a}}{64x} - \frac{3be^{6a}\mathrm{Ei}(1,-6bx)}{32} + \frac{3e^{2bx+2a}}{64x} + \frac{3be^{2a}\mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/64*exp(-6*b*x-6*a)/x-3/32*b*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*b*x-2*a)/x+
3/32*b*exp(-2*a)*Ei(1,2*b*x)-1/64/x*exp(6*b*x+6*a)-3/32*b*exp(6*a)*Ei(1,-6*
b*x)+3/64*exp(2*b*x+2*a)/x+3/32*b*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.24542, size = 72, normalized size = 0.81

$$\frac{3}{32}be^{(-6a)}\Gamma(-1,6bx) - \frac{3}{32}be^{(-2a)}\Gamma(-1,2bx) - \frac{3}{32}be^{(2a)}\Gamma(-1,-2bx) + \frac{3}{32}be^{(6a)}\Gamma(-1,-6bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/32*b*e^(-6*a)*gamma(-1, 6*b*x) - 3/32*b*e^(-2*a)*gamma(-1, 2*b*x) - 3/32*
b*e^(2*a)*gamma(-1, -2*b*x) + 3/32*b*e^(6*a)*gamma(-1, -6*b*x)
```

Fricas [B] time = 1.72144, size = 432, normalized size = 4.85

$$-\frac{20\cosh(bx+a)^3\sinh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^5 - 3(bx\mathrm{Ei}(6bx) + bx\mathrm{Ei}(-6bx))\cosh(6a) + 3(bx\mathrm{Ei}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^5
 - 3*(b*x*Ei(6*b*x) + b*x*Ei(-6*b*x))*cosh(6*a) + 3*(b*x*Ei(2*b*x) + b*x*Ei
```

```
(-2*b*x))*cosh(2*a) + 6*(cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a) - 3
*(b*x*Ei(6*b*x) - b*x*Ei(-6*b*x))*sinh(6*a) + 3*(b*x*Ei(2*b*x) - b*x*Ei(-2*
b*x))*sinh(2*a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**2, x)
```

Giac [A] time = 1.18815, size = 135, normalized size = 1.52

$$\frac{6\,bx\mathrm{Ei}\,(6\,bx)\,e^{(6\,a)} - 6\,bx\mathrm{Ei}\,(2\,bx)\,e^{(2\,a)} - 6\,bx\mathrm{Ei}\,(-2\,bx)\,e^{(-2\,a)} + 6\,bx\mathrm{Ei}\,(-6\,bx)\,e^{(-6\,a)} - e^{(6\,bx+6\,a)} + 3\,e^{(2\,bx+2\,a)} - 3\,e^{(-2\,bx-2}}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/64*(6*b*x*Ei(6*b*x)*e^(6*a) - 6*b*x*Ei(2*b*x)*e^(2*a) - 6*b*x*Ei(-2*b*x)*
e^(-2*a) + 6*b*x*Ei(-6*b*x)*e^(-6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a)
- 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x
```

3.332 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=131

$$-\frac{3}{16}b^2\sinh(2a)\mathrm{Chi}(2bx)+\frac{9}{16}b^2\sinh(6a)\mathrm{Chi}(6bx)-\frac{3}{16}b^2\cosh(2a)\mathrm{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\mathrm{Shi}(6bx)+\frac{3\sinh(2a+2b$$

```
[Out] (3*b*Cosh[2*a + 2*b*x])/(32*x) - (3*b*Cosh[6*a + 6*b*x])/(32*x) - (3*b^2*Co
shIntegral[2*b*x]*Sinh[2*a])/16 + (9*b^2*CoshIntegral[6*b*x]*Sinh[6*a])/16
+ (3*Sinh[2*a + 2*b*x])/(64*x^2) - Sinh[6*a + 6*b*x]/(64*x^2) - (3*b^2*Cosh
[2*a]*SinhIntegral[2*b*x])/16 + (9*b^2*Cosh[6*a]*SinhIntegral[6*b*x])/16
```

Rubi [A] time = 0.25697, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{3}{16}b^2\sinh(2a)\mathrm{Chi}(2bx)+\frac{9}{16}b^2\sinh(6a)\mathrm{Chi}(6bx)-\frac{3}{16}b^2\cosh(2a)\mathrm{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\mathrm{Shi}(6bx)+\frac{3\sinh(2a+2b$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (3*b*Cosh[2*a + 2*b*x])/(32*x) - (3*b*Cosh[6*a + 6*b*x])/(32*x) - (3*b^2*Co
shIntegral[2*b*x]*Sinh[2*a])/16 + (9*b^2*CoshIntegral[6*b*x]*Sinh[6*a])/16
+ (3*Sinh[2*a + 2*b*x])/(64*x^2) - Sinh[6*a + 6*b*x]/(64*x^2) - (3*b^2*Cosh
[2*a]*SinhIntegral[2*b*x])/16 + (9*b^2*Cosh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^3}+\frac{\sinh(6a+6bx)}{32x^3}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^3}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^3}\,dx \\
&= \frac{3\sinh(2a+2bx)}{64x^2}-\frac{\sinh(6a+6bx)}{64x^2}-\frac{1}{32}(3b)\int \frac{\cosh(2a+2bx)}{x^2}\,dx+\frac{1}{32}(3b)\int \\
&= \frac{3b\cosh(2a+2bx)}{32x}-\frac{3b\cosh(6a+6bx)}{32x}+\frac{3\sinh(2a+2bx)}{64x^2}-\frac{\sinh(6a+6bx)}{64x^2}- \\
&= \frac{3b\cosh(2a+2bx)}{32x}-\frac{3b\cosh(6a+6bx)}{32x}+\frac{3\sinh(2a+2bx)}{64x^2}-\frac{\sinh(6a+6bx)}{64x^2}- \\
&= \frac{3b\cosh(2a+2bx)}{32x}-\frac{3b\cosh(6a+6bx)}{32x}-\frac{3}{16}b^2\text{Chi}(2bx)\sinh(2a)+\frac{9}{16}b^2\text{Chi}(6b
\end{aligned}$$

Mathematica [A] time = 0.247007, size = 118, normalized size = 0.9

$$-\frac{12b^2x^2\sinh(2a)\text{Chi}(2bx)-36b^2x^2\sinh(6a)\text{Chi}(6bx)+12b^2x^2\cosh(2a)\text{Shi}(2bx)-36b^2x^2\cosh(6a)\text{Shi}(6bx)-3\sinh}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -(-6*b*x*Cosh[2*(a + b*x)] + 6*b*x*Cosh[6*(a + b*x)] + 12*b^2*x^2*CoshInteg
ral[2*b*x]*Sinh[2*a] - 36*b^2*x^2*CoshIntegral[6*b*x]*Sinh[6*a] - 3*Sinh[2*
(a + b*x)] + Sinh[6*(a + b*x)] + 12*b^2*x^2*Cosh[2*a]*SinhIntegral[2*b*x] -
 36*b^2*x^2*Cosh[6*a]*SinhIntegral[6*b*x])/(64*x^2)
```

Maple [A] time = 0.092, size = 178, normalized size = 1.4

$$-\frac{3\,b\mathrm{e}^{-6\,bx-6\,a}}{64\,x}+\frac{\mathrm{e}^{-6\,bx-6\,a}}{128\,x^2}+\frac{9\,b^2\mathrm{e}^{-6\,a}\mathrm{Ei}\,(1,6\,bx)}{32}+\frac{3\,b\mathrm{e}^{-2\,bx-2\,a}}{64\,x}-\frac{3\,\mathrm{e}^{-2\,bx-2\,a}}{128\,x^2}-\frac{3\,b^2\mathrm{e}^{-2\,a}\mathrm{Ei}\,(1,2\,bx)}{32}-\frac{\mathrm{e}^{6\,bx+6\,a}}{128\,x^2}-\frac{3\,b\mathrm{e}^{6\,bx+6\,a}}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -3/64*b*exp(-6*b*x-6*a)/x+1/128*exp(-6*b*x-6*a)/x^2+9/32*b^2*exp(-6*a)*Ei(1
,6*b*x)+3/64*b*exp(-2*b*x-2*a)/x-3/128*exp(-2*b*x-2*a)/x^2-3/32*b^2*exp(-2*
a)*Ei(1,2*b*x)-1/128/x^2*exp(6*b*x+6*a)-3/64*b/x*exp(6*b*x+6*a)-9/32*b^2*ex
p(6*a)*Ei(1,-6*b*x)+3/128*exp(2*b*x+2*a)/x^2+3/64*b*exp(2*b*x+2*a)/x+3/32*b
^2*exp(2*a)*Ei(1,-2*b*x)
```

Maxima [A] time = 1.26362, size = 82, normalized size = 0.63

$$\frac{9}{16}\,b^2e^{(-6\,a)}\Gamma\left(-2,6\,bx\right)-\frac{3}{16}\,b^2e^{(-2\,a)}\Gamma\left(-2,2\,bx\right)+\frac{3}{16}\,b^2e^{(2\,a)}\Gamma\left(-2,-2\,bx\right)-\frac{9}{16}\,b^2e^{(6\,a)}\Gamma\left(-2,-6\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 9/16*b^2*e^(-6*a)*gamma(-2, 6*b*x) - 3/16*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 3
/16*b^2*e^(2*a)*gamma(-2, -2*b*x) - 9/16*b^2*e^(6*a)*gamma(-2, -6*b*x)
```

Fricas [B] time = 1.89599, size = 699, normalized size = 5.34

$$-\frac{3\,bx\cosh\left(bx+a\right)^6+45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+3\,bx\sinh\left(bx+a\right)^6+10\,\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+3\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/32*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 + 3*(15*b*x*cosh(b*x + a)^4 - b*x)*s
inh(b*x + a)^2 - 9*(b^2*x^2*Ei(6*b*x) - b^2*x^2*Ei(-6*b*x))*cosh(6*a) + 3*(
b^2*x^2*Ei(2*b*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + 3*(cosh(b*x + a)^5 - co
sh(b*x + a))*sinh(b*x + a) - 9*(b^2*x^2*Ei(6*b*x) + b^2*x^2*Ei(-6*b*x))*sin
h(6*a) + 3*(b^2*x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**3, x)
```

Giac [A] time = 1.16758, size = 227, normalized size = 1.73

$$\frac{36\,b^2x^2\mathrm{Ei}\,(6\,bx)\,e^{(6\,a)}-12\,b^2x^2\mathrm{Ei}\,(2\,bx)\,e^{(2\,a)}+12\,b^2x^2\mathrm{Ei}\,(-2\,bx)\,e^{(-2\,a)}-36\,b^2x^2\mathrm{Ei}\,(-6\,bx)\,e^{(-6\,a)}-6\,bxe^{(6\,bx+6\,a)}+6\,bxe}{128\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/128*(36*b^2*x^2*Ei(6*b*x)*e^(6*a) - 12*b^2*x^2*Ei(2*b*x)*e^(2*a) + 12*b^2
*x^2*Ei(-2*b*x)*e^(-2*a) - 36*b^2*x^2*Ei(-6*b*x)*e^(-6*a) - 6*b*x*e^(6*b*x
+ 6*a) + 6*b*x*e^(2*b*x + 2*a) + 6*b*x*e^(-2*b*x - 2*a) - 6*b*x*e^(-6*b*x -
 6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*
x - 6*a))/x^2
```

3.333 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx)-\frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)+\frac{b^2\sinh(2a+2bx)}{16x}$$

```
[Out] (b*Cosh[2*a + 2*b*x])/(32*x^2) - (b*Cosh[6*a + 6*b*x])/(32*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/8 + (9*b^3*Cosh[6*a]*CoshIntegral[6*b*x])/8 + Si
nh[2*a + 2*b*x]/(32*x^3) + (b^2*Sinh[2*a + 2*b*x])/(16*x) - Sinh[6*a + 6*b*
x]/(96*x^3) - (3*b^2*Sinh[6*a + 6*b*x])/(16*x) - (b^3*Sinh[2*a]*SinhIntegra
l[2*b*x])/8 + (9*b^3*Sinh[6*a]*SinhIntegral[6*b*x])/8
```

Rubi [A] time = 0.315137, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx)-\frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)+\frac{b^2\sinh(2a+2bx)}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(32*x^2) - (b*Cosh[6*a + 6*b*x])/(32*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/8 + (9*b^3*Cosh[6*a]*CoshIntegral[6*b*x])/8 + Si
nh[2*a + 2*b*x]/(32*x^3) + (b^2*Sinh[2*a + 2*b*x])/(16*x) - Sinh[6*a + 6*b*
x]/(96*x^3) - (3*b^2*Sinh[6*a + 6*b*x])/(16*x) - (b^3*Sinh[2*a]*SinhIntegra
l[2*b*x])/8 + (9*b^3*Sinh[6*a]*SinhIntegral[6*b*x])/8
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^4}+\frac{\sinh(6a+6bx)}{32x^4}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^4}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= \frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3}-\frac{1}{16}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{16}b\int \frac{\cosh(6a}{x^3} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3}-\frac{1}{16}b^2 \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2bx)}{16x}-\text{si} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2bx)}{16x}-\text{si} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\text{Cl}
\end{aligned}$$

Mathematica [A] time = 0.3308, size = 150, normalized size = 0.89

$$-\frac{12b^3x^3\cosh(2a)\text{Chi}(2bx)-108b^3x^3\cosh(6a)\text{Chi}(6bx)+12b^3x^3\sinh(2a)\text{Shi}(2bx)-108b^3x^3\sinh(6a)\text{Shi}(6bx)-6b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -(-3*b*x*Cosh[2*(a + b*x)] + 3*b*x*Cosh[6*(a + b*x)] + 12*b^3*x^3*Cosh[2*a]
*CoshIntegral[2*b*x] - 108*b^3*x^3*Cosh[6*a]*CoshIntegral[6*b*x] - 3*Sinh[2
*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 18*b^2*x^2*
Sinh[6*(a + b*x)] + 12*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 108*b^3*x^3*
Sinh[6*a]*SinhIntegral[6*b*x])/(96*x^3)
```

Maple [A] time = 0.097, size = 246, normalized size = 1.5

$$\frac{3\,b^{2}\mathrm{e}^{-6\,bx-6\,a}}{32\,x}-\frac{b\mathrm{e}^{-6\,bx-6\,a}}{64\,x^{2}}+\frac{\mathrm{e}^{-6\,bx-6\,a}}{192\,x^{3}}-\frac{9\,b^{3}\mathrm{e}^{-6\,a}\mathrm{Ei}\left(1,6\,bx\right)}{16}-\frac{b^{2}\mathrm{e}^{-2\,bx-2\,a}}{32\,x}+\frac{b\mathrm{e}^{-2\,bx-2\,a}}{64\,x^{2}}-\frac{\mathrm{e}^{-2\,bx-2\,a}}{64\,x^{3}}+\frac{b^{3}\mathrm{e}^{-2\,a}\mathrm{Ei}\left(1,2\,bx\right)}{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 3/32*b^2*exp(-6*b*x-6*a)/x-1/64*b*exp(-6*b*x-6*a)/x^2+1/192*exp(-6*b*x-6*a)
/x^3-9/16*b^3*exp(-6*a)*Ei(1,6*b*x)-1/32*b^2*exp(-2*b*x-2*a)/x+1/64*b*exp(-
2*b*x-2*a)/x^2-1/64*exp(-2*b*x-2*a)/x^3+1/16*b^3*exp(-2*a)*Ei(1,2*b*x)+1/64
*exp(2*b*x+2*a)/x^3+1/64*b*exp(2*b*x+2*a)/x^2+1/32*b^2*exp(2*b*x+2*a)/x+1/1
6*b^3*exp(2*a)*Ei(1,-2*b*x)-1/192/x^3*exp(6*b*x+6*a)-1/64*b/x^2*exp(6*b*x+6
*a)-3/32*b^2/x*exp(6*b*x+6*a)-9/16*b^3*exp(6*a)*Ei(1,-6*b*x)
```

Maxima [A] time = 1.37818, size = 82, normalized size = 0.49

$$\frac{27}{8}\,b^{3}e^{(-6\,a)}\Gamma\left(-3,6\,bx\right)-\frac{3}{8}\,b^{3}e^{(-2\,a)}\Gamma\left(-3,2\,bx\right)-\frac{3}{8}\,b^{3}e^{(2\,a)}\Gamma\left(-3,-2\,bx\right)+\frac{27}{8}\,b^{3}e^{(6\,a)}\Gamma\left(-3,-6\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 27/8*b^3*e^(-6*a)*gamma(-3, 6*b*x) - 3/8*b^3*e^(-2*a)*gamma(-3, 2*b*x) - 3/
8*b^3*e^(2*a)*gamma(-3, -2*b*x) + 27/8*b^3*e^(6*a)*gamma(-3, -6*b*x)
```

Fricas [B] time = 1.79641, size = 792, normalized size = 4.69

$$-\frac{3\,bx\cosh\left(bx+a\right)^{6}+45\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+3\,bx\sinh\left(bx+a\right)^{6}+20\left(18\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{3}\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 20*(18*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3 +
6*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 +
3*(15*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^2 - 54*(b^3*x^3*Ei(6*b*x) +
b^3*x^3*Ei(-6*b*x))*cosh(6*a) + 6*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*
cosh(2*a) + 6*((18*b^2*x^2 + 1)*cosh(b*x + a)^5 - (2*b^2*x^2 + 1)*cosh(b*x
+ a))*sinh(b*x + a) - 54*(b^3*x^3*Ei(6*b*x) - b^3*x^3*Ei(-6*b*x))*sinh(6*a)
 + 6*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**4, x)
```

Giac [A] time = 1.19145, size = 319, normalized size = 1.89

$$\frac{108\,b^{3}x^{3}\mathrm{Ei}\left(6\,bx\right)e^{(6\,a)}-12\,b^{3}x^{3}\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-12\,b^{3}x^{3}\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}+108\,b^{3}x^{3}\mathrm{Ei}\left(-6\,bx\right)e^{(-6\,a)}-18\,b^{2}x^{2}e^{(6\,bx+6\,a)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(108*b^3*x^3*Ei(6*b*x)*e^(6*a) - 12*b^3*x^3*Ei(2*b*x)*e^(2*a) - 12*b^
3*x^3*Ei(-2*b*x)*e^(-2*a) + 108*b^3*x^3*Ei(-6*b*x)*e^(-6*a) - 18*b^2*x^2*e^
```

```
(6*b*x + 6*a) + 6*b^2*x^2*e^(2*b*x + 2*a) - 6*b^2*x^2*e^(-2*b*x - 2*a) + 18
*b^2*x^2*e^(-6*b*x - 6*a) - 3*b*x*e^(6*b*x + 6*a) + 3*b*x*e^(2*b*x + 2*a) +
 3*b*x*e^(-2*b*x - 2*a) - 3*b*x*e^(-6*b*x - 6*a) - e^(6*b*x + 6*a) + 3*e^(2
*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x^3
```

3.334 $\int x^m \tanh(a+bx)\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(x^m \tanh(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Tanh[a + b*x], x]
```

Rubi [A] time = 0.0169331, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x],x]

[Out] Defer[Int][x^m*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

Mathematica [A] time = 0.459173, size = 0, normalized size = 0.

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x],x]

[Out] Integrate[x^m*Tanh[a + b*x], x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int x^{m}\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{\left(2\,bx+m\log\left(x\right)+2\,a\right)}}{(m+1)e^{\left(2\,bx+2\,a\right)}+m+1}-\int\frac{\left(\left(2\,bxe^{\left(2\,a\right)}+(m+1)e^{\left(2\,a\right)}\right)e^{\left(2\,bx\right)}+m+1\right)x^{m}}{(m+1)e^{\left(4\,bx+4\,a\right)}+2\,(m+1)e^{\left(2\,bx+2\,a\right)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m}\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

3.335 $\int x^3 \tanh(a + bx)\,dx$

Optimal. Leaf size=91

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^4}{4}$$

```
[Out] -x^4/4 + (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, -E^(2*(a + b*
x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4,
-E^(2*(a + b*x))])/(4*b^4)
```

Rubi [A] time = 0.159999, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tanh[a + b*x],x]
```

```
[Out] -x^4/4 + (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, -E^(2*(a + b*
x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4,
-E^(2*(a + b*x))])/(4*b^4)
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\tanh(a+bx)\,dx = -\frac{x^4}{4}+2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx$$
$$= -\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)dx}{b}$$
$$= -\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3\int x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)dx}{b^2}$$
$$= -\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3\int\mathrm{Li}_3\left(-e^{2(a+bx)}\right)dx}{2b^3}$$
$$= -\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_3(-x)}{x}dx,x,e\right.}{4b^4}$$
$$= -\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3\mathrm{Li}_4\left(-e^{2(a+bx)}\right)}{4b^4}$$

Mathematica [A] time = 2.23159, size = 88, normalized size = 0.97

$$\frac{-6b^2x^2\mathrm{PolyLog}\left(2,-e^{-2(a+bx)}\right)-6bx\mathrm{PolyLog}\left(3,-e^{-2(a+bx)}\right)-3\mathrm{PolyLog}\left(4,-e^{-2(a+bx)}\right)+4b^3x^3\log\left(e^{-2(a+bx)}+1\right)+b^4x}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x],x]
```

```
[Out] (b^4*x^4 + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*x^2*PolyLog[2, -E^(-
2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*PolyLog[4, -E^(-2*(
a + b*x))])/(4*b^4)
```

Maple [A] time = 0.098, size = 116, normalized size = 1.3

$$-\frac{x^4}{4}-2\frac{a^3x}{b^3}-\frac{3a^4}{2b^4}+\frac{x^3\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{3x^2\mathrm{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}-\frac{3x\mathrm{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3}+\frac{3\mathrm{polylog}\left(4,-e^{2bx}\right.}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*x^4-2/b^3*a^3*x-3/2/b^4*a^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog
(2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,
```

```
-exp(2*b*x+2*a))/b^4+2/b^4*a^3*ln(exp(b*x+a))
```

Maxima [A] time = 1.20261, size = 113, normalized size = 1.24

$$-\frac{1}{4}x^4 + \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\mathrm{Li}_3(-e^{(2bx+2a)}) + 3\mathrm{Li}_4(-e^{(2bx+2a)})}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*
b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x +
 2*a)))/b^4
```

Fricas [C] time = 2.08174, size = 761, normalized size = 8.36

$$-\frac{b^4x^4 - 12b^2x^2\mathrm{Li}_2\left(i\cosh\left(bx+a\right) + i\sinh\left(bx+a\right)\right) - 12b^2x^2\mathrm{Li}_2\left(-i\cosh\left(bx+a\right) - i\sinh\left(bx+a\right)\right) + 4a^3\log\left(\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 12*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*b^
2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) +
 sinh(b*x + a) + I) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) + 24*b*x
*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 24*b*x*polylog(3, -I*cosh(
b*x + a) - I*sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sin
h(b*x + a) + 1) - 4*(b^3*x^3 + a^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a)
+ 1) - 24*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*polylog(4, -I*
cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a), x)
```

3.336 $\int x^2 \tanh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3}{3}$$

```
[Out] -x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + (x*PolyLog[2, -E^(2*(a + b*x))]
)/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2*b^3)
```

Rubi [A] time = 0.135225, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3718, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x],x]
```

```
[Out] -x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + (x*PolyLog[2, -E^(2*(a + b*x))]
)/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2*b^3)
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh(a+bx)\,dx &= -\frac{x^3}{3} + 2\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{2\int x\log\left(1+e^{2(a+bx)}\right)dx}{b} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} - \frac{\int \mathrm{Li}_2\left(-e^{2(a+bx)}\right)dx}{b^2} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(-x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^3} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 2.05807, size = 66, normalized size = 1.02

$$
\frac{-6bx\,\mathrm{PolyLog}\left(2,-e^{-2(a+bx)}\right) - 3\mathrm{PolyLog}\left(3,-e^{-2(a+bx)}\right) + 2b^2x^2\left(3\log\left(e^{-2(a+bx)}+1\right)+bx\right)}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + b*x],x]
```

```
[Out] (2*b^2*x^2*(b*x + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(a
 + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(6*b^3)
```

Maple [A] time = 0.026, size = 94, normalized size = 1.5

$$-\frac{x^3}{3}-2\,\frac{a^2\ln\left(e^{bx+a}\right)}{b^3}+2\,\frac{a^2x}{b^2}+\frac{4\,a^3}{3\,b^3}+\frac{x^2\ln\left(1+e^{2\,bx+2\,a}\right)}{b}+\frac{x\mathrm{polylog}\left(2,-e^{2\,bx+2\,a}\right)}{b^2}-\frac{\mathrm{polylog}\left(3,-e^{2\,bx+2\,a}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/3*x^3-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*
x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b
^3
```

Maxima [A] time = 1.20669, size = 85, normalized size = 1.31

$$-\frac{1}{3}\,x^3+\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x
+ 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3
```

Fricas [C] time = 1.96507, size = 595, normalized size = 9.15

$$-\frac{b^3x^3-6\,bx\mathrm{Li}_2\left(i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right)\right)-6\,bx\mathrm{Li}_2\left(-i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right)\right)-3\,a^2\log\left(\cosh\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 6*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*b*x*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(b^2*x^2 - a^2
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*(b^2*x^2 - a^2)*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + 6*polylog(3, I*cosh(b*x + a) + I*sinh(b
*x + a)) + 6*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a) \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a), x)
```

3.337 $\int x \tanh(a + bx)\, dx$

Optimal. Leaf size=45

$$\frac{\text{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} + \frac{x \log\left(e^{2(a+bx)} + 1\right)}{b} - \frac{x^2}{2}$$

```
[Out] -x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*x))]/(2*b
^2)
```

Rubi [A] time = 0.0824533, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} + \frac{x \log\left(e^{2(a+bx)} + 1\right)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x],x]
```

```
[Out] -x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*x))]/(2*b
^2)
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\tanh(a+bx)\,dx &= -\frac{x^2}{2}+2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx\\
&= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\
&= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^2}\\
&= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [C] time = 3.52623, size = 149, normalized size = 3.31

$$
\frac{1}{2}\left(x^2\tanh(a)+\frac{-\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}(\coth(a))+bx\right)}\right)-b^2x^2\tanh(a)\sqrt{-\text{csch}^2(a)}e^{-\tanh^{-1}(\coth(a))}+2bx\log\left(1-e^{-2\left(\tanh^{-1}\right.}\right.}{}\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Tanh[a + b*x],x]
```

```
[Out] (x^2*Tanh[a] + (I*b*Pi*x - I*Pi*Log[1 + E^(2*b*x)] + 2*b*x*Log[1 - E^(-2*(b
*x + ArcTanh[Coth[a]]))] + I*Pi*Log[Cosh[b*x]] + 2*ArcTanh[Coth[a]]*(b*x +
Log[1 - E^(-2*(b*x + ArcTanh[Coth[a]]))] - Log[I*Sinh[b*x + ArcTanh[Coth[a]
]]]) - PolyLog[2, E^(-2*(b*x + ArcTanh[Coth[a]]))] - (b^2*x^2*Sqrt[-Csch[a]
^2]*Tanh[a])/E^ArcTanh[Coth[a]])/b^2)/2
```

Maple [A] time = 0.026, size = 70, normalized size = 1.6

$$-\frac{x^2}{2}-2\frac{ax}{b}-\frac{a^2}{b^2}+\frac{x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^2}+2\frac{a\ln\left(\mathrm{e}^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+
2*a))/b^2+2/b^2*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.1975, size = 54, normalized size = 1.2

$$-\frac{1}{2}x^2+\frac{2bx\log\left(e^{(2bx+2a)}+1\right)+\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b
^2
```

Fricas [C] time = 1.91707, size = 427, normalized size = 9.49

$$-\frac{b^2x^2+2a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+2a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-i\right)-2\left(bx+a\right)\log\left(i\cosh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 2*a*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*a*log(cosh(b
*x + a) + sinh(b*x + a) - I) - 2*(b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - 2*(b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*dilog(-I*cosh(b*x + a) - I*sinh(
b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a), x)
```

3.338 $\int \tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] Log[Cosh[a + b*x]]/b
```

Rubi [A] time = 0.0062648, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x],x]
```

```
[Out] Log[Cosh[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.0076157, size = 11, normalized size = 1.

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x],x]
```

```
[Out] Log[Cosh[a + b*x]]/b
```

Maple [A] time = 0.005, size = 13, normalized size = 1.2

$$-\frac{\ln\left(\operatorname{sech}\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/b*ln(sech(b*x+a))
```

Maxima [A] time = 1.00375, size = 28, normalized size = 2.55

$$\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + e^(-b*x - a))/b
```

Fricas [B] time = 1.86847, size = 88, normalized size = 8.

$$-\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x), x)
```

Giac [B] time = 1.26725, size = 36, normalized size = 3.27

$$-\frac{bx+a}{b}+\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(e^(2*b*x + 2*a) + 1)/b
```

3.339 $\int \frac{\tanh(a+bx)}{x} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\tanh(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]/x, x]
```

Rubi [A] time = 0.0181615, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]/x,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\tanh(a+bx)}{x} dx = \int \frac{\tanh(a+bx)}{x} dx$$

Mathematica [A] time = 11.7877, size = 0, normalized size = 0.

$$\int \frac{\tanh(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]/x,x]
```

```
[Out] Integrate[Tanh[a + b*x]/x, x]
```

Maple [A] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\int \frac{1}{xe^{(2\,bx+2\,a)}+x}\,dx+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -2*integrate(1/(x*e^(2*b*x + 2*a) + x), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x, x)
```

3.340 $\int \frac{\tanh(a+bx)}{x^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\tanh(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]/x^2, x]
```

Rubi [A] time = 0.0172607, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\tanh(a+bx)}{x^2} dx = \int \frac{\tanh(a+bx)}{x^2} dx$$

Mathematica [A] time = 18.5566, size = 0, normalized size = 0.

$$\int \frac{\tanh(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]/x^2,x]
```

```
[Out] Integrate[Tanh[a + b*x]/x^2, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - 2\int \frac{1}{x^2 e^{(2\,bx+2\,a)} + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - 2*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

3.341 $\int x^m \mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(x^m\tanh(a+bx)\text{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi [A] time = 0.394424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx$$

Mathematica [A] time = 3.16931, size = 0, normalized size = 0.

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{sech}\left(bx+a\right)\right)^{2} \sinh\left(bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m} \operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.342 $\int x^3\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}$$

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^
3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a +
 b*x)])/b^4 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (x^3*Sech[a + b*x])/b
```

Rubi [A] time = 0.105284, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5418, 4180, 2531, 2282, 6589}

$$-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```


```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^
3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a +
 b*x)])/b^4 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (x^3*Sech[a + b*x])/b
```


Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\int x^2\text{sech}(a+bx)\,dx}{b}$$
$$=\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{sech}(a+bx)}{b}-\frac{(6i)\int x\log\left(1-ie^{a+bx}\right)dx}{b^2}+\frac{(6i)\int x\log\left(1\right.}{b^2}$$
$$=\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^3\text{sech}(a+bx)}{b}+\frac{(6i)\int\text{I}}{}$$
$$=\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^3\text{sech}(a+bx)}{b}+\frac{(6i)\,\text{Su}}{}$$
$$=\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{6i\text{Li}_3\left(-ie^{a+bx}\right)}{b^4}-\frac{6i\text{Li}_3\left(i\right.}{b^4}$$

Mathematica [A] time = 2.19236, size = 130, normalized size = 1.15

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3i\left(-2bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\text{PolyLog}\left(2,ie^{a+bx}\right)+2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2\text{PolyLog}\left(3,ie^{a\cdots}\right.\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2
*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Pol
yLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (x^3*Sech[a
 + b*x])/b
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x^3 (\operatorname{sech}(bx+a))^2 \sinh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^3 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+6\int \frac{x^2 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 6*integrate(x^2*e^(b*x + a)/(b
*e^(2*b*x + 2*a) + b), x)
```

Fricas [C] time = 2.02914, size = 1890, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - (6*I*b*x*cosh(b*x + a
)^2 + 12*I*b*x*cosh(b*x + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 + 6*I*
b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^2 -
 12*I*b*x*cosh(b*x + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (3*I*a^2*cosh(b*x + a)^2 + 6*I*
a^2*cosh(b*x + a)*sinh(b*x + a) + 3*I*a^2*sinh(b*x + a)^2 + 3*I*a^2)*log(co
sh(b*x + a) + sinh(b*x + a) + I) - (-3*I*a^2*cosh(b*x + a)^2 - 6*I*a^2*cosh
(b*x + a)*sinh(b*x + a) - 3*I*a^2*sinh(b*x + a)^2 - 3*I*a^2)*log(cosh(b*x +
 a) + sinh(b*x + a) - I) - (-3*I*b^2*x^2 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*
x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-3*I*b^2
*x^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - (3*I*b^2*x^2 + (3*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + (6*I*
b^2*x^2 - 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (3*I*b^2*x^2 - 3*I*a^2)*si
nh(b*x + a)^2 - 3*I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (-6*
I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh(b*x + a) - 6*I*sinh(b*x + a)^2
- 6*I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*cosh(b*x + a)^2
 + 12*I*cosh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x + a)^2 + 6*I)*polylog(3,
 -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x
 + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.343 $\int x^2\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}$$

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 +
((2*I)*PolyLog[2, I*E^(a + b*x)])/b^3 - (x^2*Sech[a + b*x])/b
```

Rubi [A] time = 0.0614134, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5418, 4180, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 +
((2*I)*PolyLog[2, I*E^(a + b*x)])/b^3 - (x^2*Sech[a + b*x])/b
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{x^2\operatorname{sech}(a+bx)}{b}+\frac{2\int x\operatorname{sech}(a+bx)\,dx}{b}\\ &= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{sech}(a+bx)}{b}-\frac{(2i)\int\log\left(1-ie^{a+bx}\right)dx}{b^2}+\frac{(2i)\int\log\left(1+i\right.}{b^2}\\ &= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{sech}(a+bx)}{b}-\frac{(2i)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+\frac{(2i)\operatorname{Su}}{}\\ &= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2i\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\operatorname{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^2\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.435018, size = 125, normalized size = 1.81

$$-\frac{2i\left(\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)-\operatorname{PolyLog}\left(2,ie^{a+bx}\right)\right)+b^2x^2\operatorname{sech}(a+bx)+(-2ia-2ibx+\pi)\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie\right.\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(((((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b
*x)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(Pol
yLog[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) + b^2*x^2*Sech[a + b
*x])/b^3)
```

Maple [B] time = 0.054, size = 154, normalized size = 2.2

$$-2\frac{x^2 e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}-\frac{2i\ln\left(1+ie^{bx+a}\right)x}{b^2}-\frac{2i\ln\left(1+ie^{bx+a}\right)a}{b^3}+\frac{2i\ln\left(1-ie^{bx+a}\right)x}{b^2}+\frac{2i\ln\left(1-ie^{bx+a}\right)a}{b^3}-\frac{2i\operatorname{dilog}\left(1+\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2*I/b^2*ln(1+I*exp(b*x+a))*x-2*I/b^3
*ln(1+I*exp(b*x+a))*a+2*I/b^2*ln(1-I*exp(b*x+a))*x+2*I/b^3*ln(1-I*exp(b*x+a
))*a-2*I/b^3*dilog(1+I*exp(b*x+a))+2*I/b^3*dilog(1-I*exp(b*x+a))-4/b^3*a*ar
ctan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^2 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+4\int\frac{xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 4*integrate(x*e^(b*x + a)/(b*e
^(2*b*x + 2*a) + b), x)
```

Fricas [B] time = 2.06587, size = 1395, normalized size = 20.22

$$-\frac{2\,b^2x^2\cosh\left(bx+a\right)+2\,b^2x^2\sinh\left(bx+a\right)-\left(2i\,\cosh\left(bx+a\right)^2+4i\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2i\,\sinh\left(bx+a\right)^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - (2*I*cosh(b*x + a)^2
+ 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b*x + a)^2 + 2*I)*dilog(I*cosh
(b*x + a) + I*sinh(b*x + a)) - (-2*I*cosh(b*x + a)^2 - 4*I*cosh(b*x + a)*si
```

```
nh(b*x + a) - 2*I*sinh(b*x + a)^2 - 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*
x + a)) - (-2*I*a*cosh(b*x + a)^2 - 4*I*a*cosh(b*x + a)*sinh(b*x + a) - 2*I
*a*sinh(b*x + a)^2 - 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) - (2*I*a
*cosh(b*x + a)^2 + 4*I*a*cosh(b*x + a)*sinh(b*x + a) + 2*I*a*sinh(b*x + a)^
2 + 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-2*I*b*x - 2*I*a)*cos
h(b*x + a)^2 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)*sinh(b*x + a) + (-2*I*b*x -
 2*I*a)*sinh(b*x + a)^2 - 2*I*b*x - 2*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - ((2*I*b*x + 2*I*a)*cosh(b*x + a)^2 + (4*I*b*x + 4*I*a)*cosh(b*
x + a)*sinh(b*x + a) + (2*I*b*x + 2*I*a)*sinh(b*x + a)^2 + 2*I*b*x + 2*I*a)
*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*
cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.344 $\int x\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 - (x*Sech[a + b*x])/b
```

Rubi [A] time = 0.0190941, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5418, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 - (x*Sech[a + b*x])/b
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\text{sech}(a+bx)\tanh(a+bx)\,dx&=-\frac{x\text{sech}(a+bx)}{b}+\frac{\int \text{sech}(a+bx)\,dx}{b}\\&=\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0457742, size = 32, normalized size = 1.33

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (2*ArcTan[Tanh[a/2 + (b*x)/2]])/b^2 - (x*Sech[a + b*x])/b
```

Maple [C] time = 0.039, size = 59, normalized size = 2.5

$$-2\,\frac{x\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b^2}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)+I)-I/b^2*ln(exp(b*
x+a)-I)
```

Maxima [A] time = 1.78737, size = 50, normalized size = 2.08

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 2*arctan(e^(b*x + a))/b^2
```

Fricas [B] time = 1.77433, size = 327, normalized size = 13.62

$$-\frac{2\left(bx\cosh\left(bx+a\right)+bx\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\arctan\left(\right.\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(bx+a\right)^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*
x + a)^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Giac [B] time = 1.31225, size = 95, normalized size = 3.96

$$-\frac{2\left(\pi + bxe^{(bx+a)} + \pi e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)\right)}{b^2e^{(2\,bx+2\,a)} + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(pi + b*x*e^(b*x + a) + pi*e^(2*b*x + 2*a) - arctan(e^(b*x + a))*e^(2*b*
x + 2*a) - arctan(e^(b*x + a)))/(b^2*e^(2*b*x + 2*a) + b^2)
```

3.345 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

[Out] -(Sech[a + b*x]/b)

Rubi [A] time = 0.0149236, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]

[Out] -(Sech[a + b*x]/b)

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0050136, size = 11, normalized size = 1.

$$-\frac{\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Maple [A] time = 0.005, size = 12, normalized size = 1.1

$$-\frac{\text{sech}\,(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -sech(b*x+a)/b
```

Maxima [B] time = 0.99574, size = 31, normalized size = 2.82

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Fricas [B] time = 1.6971, size = 154, normalized size = 14.

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2, x)
```

Giac [B] time = 1.25078, size = 31, normalized size = 2.82

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.346 $\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Rubi [A] time = 0.176492, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [A] time = 6.74103, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}+bx}-2\int \frac{e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) - 2*integrate(e^(b*x + a)/(b*x^2
*e^(2*b*x + 2*a) + b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (a + bx) \operatorname{sech}^2 (a + bx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech} (bx + a)^2 \sinh (bx + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

3.347 $\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi [A] time = 0.196115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 7.77246, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2 \sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2} - 4\int \frac{e^{(bx+a)}}{bx^3e^{(2\,bx+2\,a)}+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) - 4*integrate(e^(b*x + a)/(b
*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh{(a+bx)}\operatorname{sech}^2{(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}{(bx+a)}^2\sinh{(bx+a)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

3.348 $\int x^m \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m\tanh(a+bx)\text{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Rubi [A] time = 0.469554, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m\text{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m\text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m\text{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Mathematica [A] time = 6.25282, size = 0, normalized size = 0.

$$\int x^m\text{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{sech}\left(bx+a\right)\right)^{3} \sinh\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{sech}\left(bx+a\right)^{3} \sinh\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \operatorname{sech}\left(bx+a\right)^{3} \sinh\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx + a)^3 \sinh(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.349 $\int x^3 \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2}{2b^2}$$

```
[Out] (3*x^2)/(2*b^2) - (3*x*Log[1 + E^(2*(a + b*x))])/b^3 - (3*PolyLog[2, -E^(2*
(a + b*x))])/(2*b^4) - (x^3*Sech[a + b*x]^2)/(2*b) + (3*x^2*Tanh[a + b*x])/
(2*b^2)
```

Rubi [A] time = 0.182871, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5418, 4184, 3718, 2190, 2279, 2391}

$$-\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*x^2)/(2*b^2) - (3*x*Log[1 + E^(2*(a + b*x))])/b^3 - (3*PolyLog[2, -E^(2*
(a + b*x))])/(2*b^4) - (x^3*Sech[a + b*x]^2)/(2*b) + (3*x^2*Tanh[a + b*x])/
(2*b^2)
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\int x^2\text{sech}^2(a+bx)\,dx}{2b}$$
$$= -\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3\int x\tanh(a+bx)\,dx}{b^2}$$
$$= \frac{3x^2}{2b^2}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{6\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b^2}$$
$$= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}+\frac{3\int\log\left(1+\right.}{b^3}$$
$$= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}+\frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{3\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+}{2b^2}$$

Mathematica [C] time = 6.12298, size = 228, normalized size = 2.75

$$\frac{3\operatorname{csch}(a)\operatorname{sech}(a)\left(-b^2x^2e^{-\tanh^{-1}(\coth(a))}+\frac{i\coth(a)\left(i\text{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}(\coth(a))+ibx\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\coth(a))\right)-2\left(i\tanh^{-1}(\coth(a))+i\cdots\right.\right.}{\cdots}\right.}{2b^4\sqrt{\operatorname{csch}^2(a)\left(\sinh^2(a)-\cos\cdots\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(x^3*Sech[a + b*x]^2)/(2*b) + (3*Csch[a]*(-((b^2*x^2)/E^ArcTanh[Coth[a]])
+ (I*Coth[a]*(-(b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1 + E^(2*b*x)]
 - 2*(I*b*x + I*ArcTanh[Coth[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Coth[
a]]))] + Pi*Log[Cosh[b*x]] + (2*I)*ArcTanh[Coth[a]]*Log[I*Sinh[b*x + ArcTan
h[Coth[a]]]] + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))]))/Sqrt[
1 - Coth[a]^2])*Sech[a])/(2*b^4*Sqrt[Csch[a]^2*(-Cosh[a]^2 + Sinh[a]^2)]) +
 (3*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] time = 0.028, size = 121, normalized size = 1.5

$$-\frac{x^2\left(2bx\mathrm{e}^{2bx+2a}+3\,\mathrm{e}^{2bx+2a}+3\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}+3\,\frac{x^2}{b^2}+6\,\frac{ax}{b^3}+3\,\frac{a^2}{b^4}-3\,\frac{x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3}-\frac{3\,\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2\,b^4}-6\,\frac{a\ln\left(\mathrm{e}^{bx+\cdots}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2+3*x
^2/b^2+6/b^3*a*x+3/b^4*a^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(
2*b*x+2*a))/b^4-6/b^4*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.5117, size = 149, normalized size = 1.8

$$-\frac{3\,x^2+\left(2\,bx^3e^{(2\,a)}+3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{3\,x^2}{b^2}-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
 + 2*b^2*e^(2*b*x + 2*a) + b^2) + 3*x^2/b^2 - 3/2*(2*b*x*log(e^(2*b*x + 2*a
) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

Fricas [C] time = 2.05874, size = 2966, normalized size = 35.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(
b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 + 6
*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
 + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(cosh(b*x + a)^4 +
4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(a*cosh(
b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*co
sh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x
 + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*si
nh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x
+ a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cos
h(b*x + a) + sinh(b*x + a) - I) - 3*((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*c
osh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2
+ b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x +
a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x +
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*
(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b
*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x
 + a)^3 + b^4*sinh(b*x + a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh
```

```
(b*x + a)^2 + b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x
+ a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.350 $\int x^2 \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{x\tanh(a+bx)}{b^2}-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\mathrm{sech}^2(a+bx)}{2b}$$

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Tanh[a + b*x])
/b^2
```

Rubi [A] time = 0.0610636, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5418, 4184, 3475}

$$\frac{x\tanh(a+bx)}{b^2}-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\mathrm{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Tanh[a + b*x])
/b^2
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\int x\text{sech}^2(a+bx)\,dx}{b}$$
$$= -\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}-\frac{\int \tanh(a+bx)\,dx}{b^2}$$
$$= -\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}$$

Mathematica [A] time = 0.117808, size = 55, normalized size = 1.31

$$\frac{x\tanh(a)}{b^2}-\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{b^2}-\frac{x^2\text{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Sech[a]*Sech[a
 + b*x]*Sinh[b*x])/b^2 + (x*Tanh[a])/b^2
```

Maple [A] time = 0.029, size = 73, normalized size = 1.7

$$2\,\frac{x}{b^2}+2\,\frac{a}{b^3}-2\,\frac{x\left(bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}+1\right)}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}-\frac{\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 2*x/b^2+2/b^3*a-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+
2*a))^2-1/b^3*ln(1+exp(2*b*x+2*a))
```

Maxima [B] time = 1.34775, size = 127, normalized size = 3.02

$$-\frac{2\left(\left(bx^2e^{(2\,a)}-xe^{(2\,a)}\right)e^{(2\,bx)}-xe^{(4\,bx+4\,a)}\right)}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) - x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - log((e^(2*b*x + 2*a) + 1)*e^(-2*a
))/b^3
```

Fricas [B] time = 1.82506, size = 977, normalized size = 23.26

$$\frac{2\,bx\cosh\left(bx+a\right)^4+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\,bx\sinh\left(bx+a\right)^4-2\left(b^2x^2-bx\right)\cosh\left(bx+a\right)^2-2\left(b^2x^2-6\,b\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(b
*x + a)^4 - 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 6*b*x*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a)
^3 - (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.225, size = 192, normalized size = 4.57

$$-\frac{2\,b^2x^2e^{(2\,bx+2\,a)} - 2\,bxe^{(4\,bx+4\,a)} - 2\,bxe^{(2\,bx+2\,a)} + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)} - 1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(-e^{(2\,bx+2\,a)} - 1\right) + \log\left(-\right.}{b^3e^{(4\,bx+4\,a)} + 2\,b^3e^{(2\,bx+2\,a)} + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 + e^(4*b*x + 4*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*
b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) + 2*b^3*e
^(2*b*x + 2*a) + b^3)
```

3.351 $\int x\operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\tanh(a+bx)}{2b^2}-\frac{x\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -(x*Sech[a + b*x]^2)/(2*b) + Tanh[a + b*x]/(2*b^2)
```

Rubi [A] time = 0.0304105, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5418, 3767, 8}

$$\frac{\tanh(a+bx)}{2b^2}-\frac{x\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(x*Sech[a + b*x]^2)/(2*b) + Tanh[a + b*x]/(2*b^2)
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x\mathrm{sech}^2(a+bx)}{2b} + \frac{\int \mathrm{sech}^2(a+bx)\,dx}{2b}$$
$$= -\frac{x\mathrm{sech}^2(a+bx)}{2b} + \frac{i\,\mathrm{Subst}(\int 1\,dx, x, -i\tanh(a+bx))}{2b^2}$$
$$= -\frac{x\mathrm{sech}^2(a+bx)}{2b} + \frac{\tanh(a+bx)}{2b^2}$$

Mathematica [A] time = 0.072022, size = 30, normalized size = 1.

$$\frac{\tanh(a+bx)}{2b^2} - \frac{x\mathrm{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(x*Sech[a + b*x]^2)/(2*b) + Tanh[a + b*x]/(2*b^2)
```

Maple [A] time = 0.028, size = 43, normalized size = 1.4

$$-\frac{2\,bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}+1}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
```

Maxima [B] time = 1.06165, size = 177, normalized size = 5.9

$$-\frac{2\,bxe^{(4\,bx+4\,a)}+\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}+\frac{2\,bxe^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}-1}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^(4*b*x + 4*a) + (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2
*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(2*b*x*e^(4*b*x + 4*a
) - e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2
)
```

Fricas [B] time = 1.75086, size = 270, normalized size = 9.

$$-\frac{2\left(bx\sinh\left(bx+a\right)+\left(bx+1\right)\cosh\left(bx+a\right)\right)}{b^{2}\cosh\left(bx+a\right)^{3}+3\,b^{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b^{2}\sinh\left(bx+a\right)^{3}+3\,b^{2}\cosh\left(bx+a\right)+\left(3\,b^{2}\cosh\left(bx+a\right)^{2}+b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*sinh(b*x + a) + (b*x + 1)*cosh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + 3*b^2*cosh(b*x + a
) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**3, x)
```

Giac [B] time = 1.25154, size = 248, normalized size = 8.27

$$-\frac{4\,bxe^{\left(2\,bx+2\,a\right)}-e^{\left(4\,bx+4\,a\right)}\log\left(e^{\left(2\,bx+2\,a\right)}+1\right)-2\,e^{\left(2\,bx+2\,a\right)}\log\left(e^{\left(2\,bx+2\,a\right)}+1\right)+e^{\left(4\,bx+4\,a\right)}\log\left(-e^{\left(2\,bx+2\,a\right)}-1\right)+2\,e^{\left(2\,bx+2\,a\right)}\,\mathrm{l}}{2\left(b^{2}e^{\left(4\,bx+4\,a\right)}+2\,b^{2}e^{\left(2\,bx+2\,a\right)}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) + 1) - 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) + 1) + log(-e^(2*b*x + 2*a) - 1) + 2)/(b^2*e^(4*b*x
+ 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2)
```

3.352 $\int \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0208823, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0087869, size = 15, normalized size = 1.

$$-\frac{\text{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$-\frac{(\text{sech}\,(bx+a))^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/2*sech(b*x+a)^2/b
```

Maxima [A] time = 1.01246, size = 31, normalized size = 2.07

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a))^2)
```

Fricas [B] time = 1.7522, size = 235, normalized size = 15.67

$$-\frac{2\,(\cosh\,(bx+a)+\sinh\,(bx+a))}{b\cosh\,(bx+a)^3+3\,b\cosh\,(bx+a)\sinh\,(bx+a)^2+b\sinh\,(bx+a)^3+3\,b\cosh\,(bx+a)+\left(3\,b\cosh\,(bx+a)^2+b\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3, x)
```

Giac [A] time = 1.16917, size = 36, normalized size = 2.4

$$-\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.353 $\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Rubi [A] time = 0.182005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [A] time = 23.7125, size = 0, normalized size = 0.

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^3 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2x^2e^{(4\,bx+4\,a)}+2\,b^2x^2e^{(2\,bx+2\,a)}+b^2x^2}+4\int \frac{1}{2\left(b^2x^3e^{(2\,bx+2\,a)}+b^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 4*integrate(1/2/(b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

3.354 $\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Rubi [A] time = 0.214894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]

[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 20.6187, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]

[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^3 \sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}-1\right)}{b^2x^3e^{(4bx+4a)}+2b^2x^3e^{(2bx+2a)}+b^2x^3}+12\int \frac{1}{2\left(b^2x^4e^{(2bx+2a)}+b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 12*integrate(1/2/(b^2*x^4*e^(2*b*x + 2*a)
 + b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh{\left(a + bx \right)}\operatorname{sech}^{3}{\left(a + bx \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(bx + a\right)^{3} \sinh\left(bx + a\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

3.355 $\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=73

$$-\text{Unintegrable}\left(x^m\text{sech}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Unintegrable[x^m*Sech[a + b*x], x]
```

Rubi [A] time = 0.106197, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh(a+bx)\tanh(a+bx)\,dx &= \int x^m\cosh(a+bx)\,dx-\int x^m\text{sech}(a+bx)\,dx\\&=\frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx-\int x^m\text{sech}(a+bx)\,dx\\&=\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}-\int x^m\text{sech}(a+bx)\,dx\end{aligned}$$

Mathematica [A] time = 13.5772, size = 0, normalized size = 0.

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x], x]
```

Maple [A] time = 0.096, size = 0, normalized size = 0.

$$\int x^{m}\mathrm{sech}\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m}\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.356 $\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=195

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyLog}}{b}$$

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a + b*
x])/b^2 + ((3*I)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog
[2, I*E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*
I)*x*PolyLog[3, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/
b^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^
3*Sinh[a + b*x])/b
```

Rubi [A] time = 0.197753, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5449, 3296, 2638, 4180, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyLog}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a + b*
x])/b^2 + ((3*I)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog
[2, I*E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*
I)*x*PolyLog[3, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/
b^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^
3*Sinh[a + b*x])/b
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^3 \cosh(a+bx)\,dx - \int x^3 \operatorname{sech}(a+bx)\,dx$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\sinh(a+bx)}{b} + \frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)dx}{b} - \frac{(3i)\int x^2\log}{}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{x}{}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} - \frac{6i}{}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix}{}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix}{}$$

Mathematica [A] time = 1.43698, size = 211, normalized size = 1.08

$$-\frac{i\left(-3b^2x^2\text{PolyLog}\left(2,-ie^{a+bx}\right)+3b^2x^2\text{PolyLog}\left(2,ie^{a+bx}\right)+6bx\text{PolyLog}\left(3,-ie^{a+bx}\right)-6bx\text{PolyLog}\left(3,ie^{a+bx}\right)-6\text{Pol}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-6*I)*Cosh[a + b*x] - (3*I)*b^2*x^2*Cosh[a + b*x] + b^3*x^3*Log[1 -
 I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (-I
)*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3, (-I
)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^(a +
 b*x)] + 6*PolyLog[4, I*E^(a + b*x)] + (6*I)*b*x*Sinh[a + b*x] + I*b^3*x^3*
Sinh[a + b*x]))/b^4
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int x^3\operatorname{sech}(bx+a)\left(\sinh(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(b^3x^3e^{(2\,a)}-3\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-6\,e^{(2\,a)}\right)e^{(bx)}-\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^4}-2\int\frac{x^3e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - 2*integrate(
x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.38427, size = 1704, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)
^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) - (b^3
*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x - (-6*I*b^2*x^2*cosh(
b*x + a) - 6*I*b^2*x^2*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x +
a)) - (6*I*b^2*x^2*cosh(b*x + a) + 6*I*b^2*x^2*sinh(b*x + a))*dilog(-I*cosh
(b*x + a) - I*sinh(b*x + a)) - (2*I*a^3*cosh(b*x + a) + 2*I*a^3*sinh(b*x +
a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - (-2*I*a^3*cosh(b*x + a) - 2*I*
a^3*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*I*b^3*x^3 +
 2*I*a^3)*cosh(b*x + a) + (2*I*b^3*x^3 + 2*I*a^3)*sinh(b*x + a))*log(I*cosh
(b*x + a) + I*sinh(b*x + a) + 1) - ((-2*I*b^3*x^3 - 2*I*a^3)*cosh(b*x + a)
+ (-2*I*b^3*x^3 - 2*I*a^3)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) - (-12*I*cosh(b*x + a) - 12*I*sinh(b*x + a))*polylog(4, I*cosh(b
```

```
*x + a) + I*sinh(b*x + a)) - (12*I*cosh(b*x + a) + 12*I*sinh(b*x + a))*poly
log(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) - (12*I*b*x*cosh(b*x + a) + 12*I
*b*x*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (-12*I*
b*x*cosh(b*x + a) - 12*I*b*x*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I
*sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.357 $\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=135

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}$$

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (2*x*Cosh[a + b*x])/b^2 + ((2*I)*x*PolyLog
[2, (-I)*E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 - ((2*
I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[3, I*E^(a + b*x)])/b^
3 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rubi [A] time = 0.129759, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5449, 3296, 2637, 4180, 2531, 2282, 6589}

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```


```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (2*x*Cosh[a + b*x])/b^2 + ((2*I)*x*PolyLog
[2, (-I)*E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 - ((2*
I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[3, I*E^(a + b*x)])/b^
3 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```


Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\sinh(a+bx)\tanh(a+bx)\,dx = \int x^2\cosh(a+bx)\,dx - \int x^2\mathbf{sech}(a+bx)\,dx$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\sinh(a+bx)}{b} + \frac{(2i)\int x\log\left(1-ie^{a+bx}\right)dx}{b} - \frac{(2i)\int x\log}{}$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{x^2}{}$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{2\,\text{s}}{}$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} - \frac{2i\text{L}}{}$$

Mathematica [A] time = 1.42879, size = 153, normalized size = 1.13

$$-\frac{i\left(-2bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\text{PolyLog}\left(2,ie^{a+bx}\right)+2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2\text{PolyLog}\left(3,ie^{a+bx}\right)+b^2x^2\log\left(1\right.\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-2*I)*b*x*Cosh[a + b*x] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*
Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog
[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a +
 b*x)] + (2*I)*Sinh[a + b*x] + I*b^2*x^2*Sinh[a + b*x]))/b^3
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x^2\text{sech}\,(bx+a)\,(\sinh\,(bx+a))^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + 2\,e^{(2\,a)}\right)e^{(bx)} - \left(b^2x^2 + 2\,bx + 2\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^3} - 2\int \frac{x^2e^{(bx+a)}}{e^{(2\,bx+2\,a)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) - (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - 2*integrate(x^2*e^(b*x + a)/(e^(2*b*x + 2*a)
 + 1), x)
```

Fricas [C] time = 2.35461, size = 1345, normalized size = 9.96

$$-\frac{b^2x^2 - \left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx + a\right)^2 - 2\left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right) - \left(b^2x^2 - 2\,bx + 2\right)\sinh\left(bx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 2*b*x
+ 2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 +
2*b*x - (-4*I*b*x*cosh(b*x + a) - 4*I*b*x*sinh(b*x + a))*dilog(I*cosh(b*x +
 a) + I*sinh(b*x + a)) - (4*I*b*x*cosh(b*x + a) + 4*I*b*x*sinh(b*x + a))*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (-2*I*a^2*cosh(b*x + a) - 2*I*a^2
*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - (2*I*a^2*cosh(b*x
+ a) + 2*I*a^2*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*
I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a) + (2*I*b^2*x^2 - 2*I*a^2)*sinh(b*x + a))
*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-2*I*b^2*x^2 + 2*I*a^2)*cos
h(b*x + a) + (-2*I*b^2*x^2 + 2*I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a) -
 I*sinh(b*x + a) + 1) - (4*I*cosh(b*x + a) + 4*I*sinh(b*x + a))*polylog(3,
I*cosh(b*x + a) + I*sinh(b*x + a)) - (-4*I*cosh(b*x + a) - 4*I*sinh(b*x + a
))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 2)/(b^3*cosh(b*x + a) +
 b^3*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh^2\left(a + bx\right)\operatorname{sech}\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a) \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.358 $\int x\sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\sinh(a+bx)}{b}$$

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - Cosh[a + b*x]/b^2 + (I*PolyLog[2, (-I)*E^(a
+ b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2 + (x*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0637488, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5449, 3296, 2638, 4180, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - Cosh[a + b*x]/b^2 + (I*PolyLog[2, (-I)*E^(a
+ b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2 + (x*Sinh[a + b*x])/b
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\cosh(a+bx)\,dx - \int x\text{sech}(a+bx)\,dx$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x\sinh(a+bx)}{b} + \frac{i\int\log\left(1-ie^{a+bx}\right)\,dx}{b} - \frac{i\int\log\left(1+ie^{a+bx}\right)}{b}$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{x\sinh(a+bx)}{b} + \frac{i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a}\right.}{b^2}$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{i\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{x\sinh(a+b}{b}$$

Mathematica [B] time = 0.109484, size = 212, normalized size = 2.75

$$\frac{i\left(\text{PolyLog}\left(2,-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\text{PolyLog}\left(2,e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)\right)+\left(-ia-ibx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-ia-ibx\right.}\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (((-I)*a + Pi/2 - I*b*x)*(Log[1 - E^(I*((-I)*a + Pi/2 - I*b*x))] - Log[1 +
E^(I*((-I)*a + Pi/2 - I*b*x))]) - ((-I)*a + Pi/2)*Log[Tan[((-I)*a + Pi/2 -
I*b*x)/2]] + I*(PolyLog[2, -E^(I*((-I)*a + Pi/2 - I*b*x))] - PolyLog[2, E^(
I*((-I)*a + Pi/2 - I*b*x))]))/b^2 + (Cosh[b*x]*(-Cosh[a] + b*x*Sinh[a]))/b^
2 + ((b*x*Cosh[a] - Sinh[a])*Sinh[b*x])/b^2
```

Maple [B] time = 0.057, size = 162, normalized size = 2.1

$$\frac{(bx-1)\,\mathrm{e}^{bx+a}}{2\,b^2}-\frac{(bx+1)\,\mathrm{e}^{-bx-a}}{2\,b^2}+\frac{i\ln\left(1+i\mathrm{e}^{bx+a}\right)x}{b}+\frac{i\ln\left(1+i\mathrm{e}^{bx+a}\right)a}{b^2}-\frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)x}{b}-\frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)a}{b^2}+\frac{i\mathrm{dilog}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)+I/b*ln(1+I*exp(b*x+a
))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b
*x+a))*a+I/b^2*dilog(1+I*exp(b*x+a))-I/b^2*dilog(1-I*exp(b*x+a))+2/b^2*a*ar
ctan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(bx)}-(bx+1)e^{(-bx)}\right)e^{(-a)}}{2\,b^2}-2\int\frac{xe^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) - (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - 2*i
ntegrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [B] time = 2.23961, size = 973, normalized size = 12.64

$$\frac{(bx-1)\cosh(bx+a)^2+2\,(bx-1)\cosh(bx+a)\sinh(bx+a)+(bx-1)\sinh(bx+a)^2-bx+(-2i\,\cosh(bx+a)-2i\,\mathrm{si}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 - b*x + (-2*I*cosh(b*x + a) - 2*I*sinh(b*x + a))*
dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (2*I*cosh(b*x + a) + 2*I*sinh(b*
x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (2*I*a*cosh(b*x + a) +
2*I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-2*I*a*cosh(
b*x + a) - 2*I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((
2*I*b*x + 2*I*a)*cosh(b*x + a) + (2*I*b*x + 2*I*a)*sinh(b*x + a))*log(I*cos
h(b*x + a) + I*sinh(b*x + a) + 1) + ((-2*I*b*x - 2*I*a)*cosh(b*x + a) + (-2
*I*b*x - 2*I*a)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
- 1)/(b^2*cosh(b*x + a) + b^2*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.359 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rubi [A] time = 0.0154926, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 203}

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(a+bx)\right)}{b}\\ &= -\frac{\tan^{-1}(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0116213, size = 23, normalized size = 1.

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Maple [A] time = 0.013, size = 24, normalized size = 1.

$$\frac{\sinh(bx+a)}{b}-2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] sinh(b*x+a)/b-2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.6059, size = 55, normalized size = 2.39

$$\frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Fricas [B] time = 2.08583, size = 254, normalized size = 11.04

$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x), x)
```

Giac [A] time = 1.16673, size = 51, normalized size = 2.22

$$-\frac{2\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")

[Out] -2*arctan(e^(b*x + a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

3.360 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x} dx$

Optimal. Leaf size=29

$$-\text{Unintegrable}\left(\frac{\text{sech}(a+bx)}{x},x\right)+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] - Unintegrable[Sech[a
 + b*x]/x, x]
```

Rubi [A] time = 0.0859443, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] - Defer[Int][Sech[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh(a+bx)}{x} dx &= \int \frac{\cosh(a+bx)}{x} dx - \int \frac{\text{sech}(a+bx)}{x} dx\\ &= \cosh(a)\int \frac{\cosh(bx)}{x} dx + \sinh(a)\int \frac{\sinh(bx)}{x} dx - \int \frac{\text{sech}(a+bx)}{x} dx\\ &= \cosh(a)\text{Chi}(bx) + \sinh(a)\text{Shi}(bx) - \int \frac{\text{sech}(a+bx)}{x} dx\end{aligned}$$

Mathematica [A] time = 8.73769, size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)(\sinh(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

3.361 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=42

$$-\text{Unintegrable}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)+b\sinh(a)\text{Chi}(bx)+b\cosh(a)\text{Shi}(bx)-\frac{\cosh(a+bx)}{x}$$

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] - Unintegrable[Sech[a + b*x]/x^2, x]
```

Rubi [A] time = 0.106884, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] - Defer[Int][Sech[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx &= \int \frac{\cosh(a+bx)}{x^2}\,dx-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\\ &= -\frac{\cosh(a+bx)}{x}+b\int \frac{\sinh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\\ &= -\frac{\cosh(a+bx)}{x}+(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\\ &= -\frac{\cosh(a+bx)}{x}+b\text{Chi}(bx)\sinh(a)+b\cosh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\end{aligned}$$

Mathematica [A] time = 8.73568, size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\mathrm{sech}\,(bx+a)\,(\sinh\,(bx+a))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{sech}\,(bx+a)\sinh\,(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{sech}\,(bx+a)\sinh\,(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

3.362 $\int x^m \tanh^2(a+bx)\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \tanh^2(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Tanh[a + b*x]^2, x]
```

Rubi [A] time = 0.0318242, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \tanh^2(a+bx)\,dx=\int x^m \tanh^2(a+bx)\,dx$$

Mathematica [A] time = 0.584517, size = 0, normalized size = 0.

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x]^2, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{sech}\left(bx+a\right)\right)^{2} \left(\sinh\left(bx+a\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{\left(4\,bx+m\log\left(x\right)+4\,a\right)}}{(m+1)e^{\left(4\,bx+4\,a\right)}+2\,(m+1)e^{\left(2\,bx+2\,a\right)}+m+1} - \int \frac{\left(2\left(2\,bxe^{\left(4\,a\right)}+(m+1)e^{\left(4\,a\right)}\right)e^{\left(4\,bx\right)}+(m+1)e^{\left(2\,bx+2\,a\right)}-m-1\right)x^{m}}{(m+1)e^{\left(6\,bx+6\,a\right)}+3\,(m+1)e^{\left(4\,bx+4\,a\right)}+3\,(m+1)e^{\left(2\,bx+2\,a\right)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
+ (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{m} \operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.363 $\int x^3 \tanh^2(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[Out] -(x^3/b) + x^4/4 + (3*x^2*Log[1 + E^(2*(a + b*x))])/b^2 + (3*x*PolyLog[2, -
E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, -E^(2*(a + b*x))])/(2*b^4) - (x^3*Tan
h[a + b*x])/b
```

Rubi [A] time = 0.17831, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3720, 3718, 2190, 2531, 2282, 6589, 30}

$$\frac{3x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tanh[a + b*x]^2,x]
```


```
[Out] -(x^3/b) + x^4/4 + (3*x^2*Log[1 + E^(2*(a + b*x))])/b^2 + (3*x*PolyLog[2, -
E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, -E^(2*(a + b*x))])/(2*b^4) - (x^3*Tan
h[a + b*x])/b
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{x^3\tanh(a+bx)}{b} + \frac{3\int x^2\tanh(a+bx)\,dx}{b} + \int x^3\,dx$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3\tanh(a+bx)}{b} + \frac{6\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx}{b}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{x^3\tanh(a+bx)}{b} - \frac{6\int x\log\left(1+e^{2(a+bx)}\right)\,dx}{b^2}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{x^3\tanh(a+bx)}{b} - \frac{3\int \mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{x^3\tanh(a+bx)}{b} - \frac{3\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(-x)}{x}\right.}{2b^4}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{3\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^4} - \frac{x^3\tanh(a+bx)}{b}$$

Mathematica [A] time = 4.53327, size = 104, normalized size = 1.17

$$\frac{-6bx\mathrm{PolyLog}\left(2,-e^{-2(a+bx)}\right) - 3\mathrm{PolyLog}\left(3,-e^{-2(a+bx)}\right) + 2b^2x^2\left(\frac{2bx}{e^{2a}+1} + 3\log\left(e^{-2(a+bx)}+1\right)\right)}{2b^4} - \frac{x^3\mathrm{sech}(a)\sinh(bx)\mathrm{se}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x]^2,x]
```

```
[Out] x^4/4 + (2*b^2*x^2*((2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-2*(a + b*x))]) -
6*b*x*PolyLog[2, -E^(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(2*b
^4) - (x^3*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b
```

Maple [A] time = 0.056, size = 125, normalized size = 1.4

$$\frac{x^4}{4} + 2\,\frac{x^3}{b\left(1+\mathrm{e}^{2\,bx+2\,a}\right)} - 6\,\frac{a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^4} - 2\,\frac{x^3}{b} + 6\,\frac{a^2x}{b^3} + 4\,\frac{a^3}{b^4} + 3\,\frac{x^2\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b^2} + 3\,\frac{x\mathrm{polylog}\left(2,-\mathrm{e}^{2\,bx+2\,a}\right)}{b^3} - \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/4*x^4+2*x^3/b/(1+exp(2*b*x+2*a))-6/b^4*a^2*ln(exp(b*x+a))-2*x^3/b+6/b^3*a
^2*x+4/b^4*a^3+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a)
)/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4
```

Maxima [A] time = 1.34843, size = 146, normalized size = 1.64

$$-\frac{2\,x^3}{b}+\frac{bx^4e^{(2\,bx+2\,a)}+bx^4+8\,x^3}{4\left(be^{(2\,bx+2\,a)}+b\right)}+\frac{3\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) + b*x^4 + 8*x^3)/(b*e^(2*b*x + 2*a) +
 b) + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a
)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
```

Fricas [C] time = 2.31947, size = 1890, normalized size = 21.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^4 - 8*a^3 + (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 + 2*(b
^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^4*x^4 - 8*b^3*
x^3 - 8*a^3)*sinh(b*x + a)^2 + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a
)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(I*cosh(b*x + a) + I*sinh
(b*x + a)) + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) +
b*x*sinh(b*x + a)^2 + b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 12*(
a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)
^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 12*(a^2*cosh(b*x + a)^2
+ 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 + a^2)*log(cosh(b
*x + a) + sinh(b*x + a) - I) + 12*(b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^
2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*sinh(b*
x + a)^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 12*(b^2*x^2 +
(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-I*cosh(b*x + a) - I*sinh
```

```
(b*x + a) + 1) - 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3,
 -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x
 + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.364 $\int x^2 \tanh^2(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[Out] -(x^2/b) + x^3/3 + (2*x*Log[1 + E^(2*(a + b*x))])/b^2 + PolyLog[2, -E^(2*(a
 + b*x))]/b^3 - (x^2*Tanh[a + b*x])/b
```

Rubi [A] time = 0.118195, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3718, 2190, 2279, 2391, 30}

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x]^2,x]
```



```
[Out] -(x^2/b) + x^3/3 + (2*x*Log[1 + E^(2*(a + b*x))])/b^2 + PolyLog[2, -E^(2*(a
 + b*x))]/b^3 - (x^2*Tanh[a + b*x])/b
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tanh^2(a+bx)\,dx &= -\frac{x^2\tanh(a+bx)}{b}+\frac{2\int x\tanh(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\tanh(a+bx)}{b}+\frac{4\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{2\int \log\left(1+e^{2(a+bx)}\right)dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}-\frac{x^2\tanh(a+bx)}{b}
\end{aligned}$$

Mathematica [C] time = 3.20437, size = 168, normalized size = 2.58

$$\frac{-3\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}(\coth(a))+bx\right)}\right)-3b^2x^2\text{sech}(a)\sinh(bx)\text{sech}(a+bx)-3b^2x^2\tanh(a)\sqrt{-\text{csch}^2(a)}e^{-\tanh^{-1}(\coth(a)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Tanh[a + b*x]^2,x]
```

```
[Out] ((3*I)*b*Pi*x + b^3*x^3 - (3*I)*Pi*Log[1 + E^(2*b*x)] + 6*b*x*Log[1 - E^(-2
*(b*x + ArcTanh[Coth[a]]))] + (3*I)*Pi*Log[Cosh[b*x]] + 6*ArcTanh[Coth[a]]*
(b*x + Log[1 - E^(-2*(b*x + ArcTanh[Coth[a]]))] - Log[I*Sinh[b*x + ArcTanh[
Coth[a]]]]) - 3*PolyLog[2, E^(-2*(b*x + ArcTanh[Coth[a]]))] - 3*b^2*x^2*Sec
h[a]*Sech[a + b*x]*Sinh[b*x] - (3*b^2*x^2*Sqrt[-Csch[a]^2]*Tanh[a])/E^ArcTa
nh[Coth[a]])/(3*b^3)
```

Maple [A] time = 0.055, size = 99, normalized size = 1.5

$$\frac{x^3}{3}+2\,\frac{x^2}{b\left(1+e^{2\,bx+2\,a}\right)}-2\,\frac{x^2}{b}-4\,\frac{ax}{b^2}-2\,\frac{a^2}{b^3}+2\,\frac{x\ln\left(1+e^{2\,bx+2\,a}\right)}{b^2}+\frac{\mathrm{polylog}\left(2,-e^{2\,bx+2\,a}\right)}{b^3}+4\,\frac{a\ln\left(e^{bx+a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/3*x^3+2*x^2/b/(1+exp(2*b*x+2*a))-2*x^2/b-4/b^2*a*x-2/b^3*a^2+2*x*ln(1+exp
(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+4/b^3*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.3793, size = 113, normalized size = 1.74

$$-\frac{2\,x^2}{b}+\frac{bx^3e^{(2\,bx+2\,a)}+bx^3+6\,x^2}{3\left(be^{(2\,bx+2\,a)}+b\right)}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) + b*x^3 + 6*x^2)/(b*e^(2*b*x + 2*a) +
 b) + (2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^3
```

Fricas [C] time = 2.25351, size = 1424, normalized size = 21.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^3 + (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 + 2*(b^3*x^3 -
 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 - 6*b^2*x^2 + 6*
a^2)*sinh(b*x + a)^2 + 6*a^2 + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dil
og(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*
x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh
(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 6*((b*x + a)*cosh
(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
+ a)^2 + b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 6*((b*x + a)
*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh
(b*x + a)^2 + b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*co
sh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b
^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.365 $\int x \tanh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\log(\cosh(a+bx))}{b^2} - \frac{x\tanh(a+bx)}{b} + \frac{x^2}{2}$$

```
[Out] x^2/2 + Log[Cosh[a + b*x]]/b^2 - (x*Tanh[a + b*x])/b
```

Rubi [A] time = 0.026975, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3720, 3475, 30}

$$\frac{\log(\cosh(a+bx))}{b^2} - \frac{x\tanh(a+bx)}{b} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[a + b*x]]/b^2 - (x*Tanh[a + b*x])/b
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\tanh^2(a+bx)\,dx = -\frac{x\tanh(a+bx)}{b} + \frac{\int \tanh(a+bx)\,dx}{b} + \int x\,dx$$
$$= \frac{x^2}{2} + \frac{\log(\cosh(a+bx))}{b^2} - \frac{x\tanh(a+bx)}{b}$$

Mathematica [A] time = 0.140666, size = 46, normalized size = 1.48

$$\frac{-2bx\tanh(a) + 2\log(\cosh(a+bx)) - 2bx\text{sech}(a)\sinh(bx)\text{sech}(a+bx) + b^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[a + b*x]^2,x]
```

```
[Out] (b^2*x^2 + 2*Log[Cosh[a + b*x]] - 2*b*x*Sech[a]*Sech[a + b*x]*Sinh[b*x] - 2
*b*x*Tanh[a])/(2*b^2)
```

Maple [A] time = 0.055, size = 54, normalized size = 1.7

$$\frac{x^2}{2} - 2\frac{x}{b} - 2\frac{a}{b^2} + 2\frac{x}{b\left(1+e^{2bx+2a}\right)} + \frac{\ln\left(1+e^{2bx+2a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/2*x^2-2*x/b-2/b^2*a+2*x/b/(1+exp(2*b*x+2*a))+1/b^2*ln(1+exp(2*b*x+2*a))
```

Maxima [B] time = 1.18184, size = 128, normalized size = 4.13

$$-\frac{xe^{(2bx+2a)}}{be^{(2bx+2a)}+b} + \frac{bx^2 + \left(bx^2e^{(2a)} - 2xe^{(2a)}\right)e^{(2bx)}}{2\left(be^{(2bx+2a)}+b\right)} + \frac{\log\left(\left(e^{(2bx+2a)}+1\right)e^{(-2a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) + 1/2*(b*x^2 + (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) + b) + log((e^(2*b*x + 2*a) + 1)
*e^(-2*a))/b^2
```

Fricas [B] time = 2.04819, size = 475, normalized size = 15.32

$$\frac{b^2x^2 + \left(b^2x^2 - 4\,bx\right)\cosh\left(bx + a\right)^2 + 2\left(b^2x^2 - 4\,bx\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right) + \left(b^2x^2 - 4\,bx\right)\sinh\left(bx + a\right)^2 + 2\left(\cos\right.}{2\left(b^2\cosh\left(bx + a\right)^2 + 2\,b^2\cosh\left(bx + a\right)\sinh\left(bx + a\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 4*b*x)*cosh
(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 + 2*(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b
*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh^2\left(a + bx\right)\operatorname{sech}^2\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [B] time = 1.19563, size = 128, normalized size = 4.13

$$\frac{b^2x^2e^{(2\,bx+2\,a)} + b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} + 1\right) + 2\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{2\left(b^2e^{(2\,bx+2\,a)} + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) + 1) + 2*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*
x + 2*a) + b^2)
```

3.366 $\int \tanh^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\tanh(a+bx)}{b}$$

```
[Out] x - Tanh[a + b*x]/b
```

Rubi [A] time = 0.0093024, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$x-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^2,x]
```

```
[Out] x - Tanh[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^2(a+bx)\,dx &= -\frac{\tanh(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0068711, size = 23, normalized size = 1.77

$$\frac{\tanh^{-1}(\tanh(a+bx))}{b}-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

Maple [A] time = 0.013, size = 18, normalized size = 1.4

$$\frac{bx+a-\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(b*x+a-tanh(b*x+a))
```

Maxima [A] time = 1.04342, size = 34, normalized size = 2.62

$$x+\frac{a}{b}-\frac{2}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] time = 1.97179, size = 82, normalized size = 6.31

$$\frac{(bx+1)\cosh(bx+a)-\sinh(bx+a)}{b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [B] time = 1.20453, size = 38, normalized size = 2.92

$$\frac{bx+a}{b}+\frac{2}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/b + 2/(b*(e^(2*b*x + 2*a) + 1))
```

3.367 $\int \frac{\tanh^2(a+bx)}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\tanh^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]^2/x, x]
```

Rubi [A] time = 0.0296648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+bx)}{x} \, dx = \int \frac{\tanh^2(a+bx)}{x} \, dx$$

Mathematica [A] time = 18.2445, size = 0, normalized size = 0.

$$\int \frac{\tanh^2(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^2/x,x]
```

```
[Out] Integrate[Tanh[a + b*x]^2/x, x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2 (\sinh(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2}{bxe^{(2\,bx+2\,a)}+bx}+2\int \frac{1}{bx^2e^{(2\,bx+2\,a)}+bx^2}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 2/(b*x*e^(2*b*x + 2*a) + b*x) + 2*integrate(1/(b*x^2*e^(2*b*x + 2*a) + b*x^
2), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

3.368 $\int \frac{\tanh^2(a+bx)}{x^2} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\tanh^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]^2/x^2, x]
```

Rubi [A] time = 0.0303789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \int \frac{\tanh^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 11.6612, size = 0, normalized size = 0.

$$\int \frac{\tanh^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Tanh[a + b*x]^2/x^2, x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2 (\sinh(bx+a))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bxe^{(2\,bx+2\,a)} + bx - 2}{bx^2e^{(2\,bx+2\,a)} + bx^2} + 4\int \frac{1}{bx^3e^{(2\,bx+2\,a)} + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) + b*x - 2)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integr
ate(1/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

3.369 $\int x^m \operatorname{sech}(a+bx) \tanh^2(a+bx)\, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(x^m \operatorname{sech}(a+bx), x\right) - \text{Unintegrable}\left(x^m \operatorname{sech}^3(a+bx), x\right)$$

```
[Out] Unintegrable[x^m*Sech[a + b*x], x] - Unintegrable[x^m*Sech[a + b*x]^3, x]
```

Rubi [A] time = 0.0668012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \operatorname{sech}(a+bx) \tanh^2(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]

[Out] Defer[Int][x^m*Sech[a + b*x], x] - Defer[Int][x^m*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \operatorname{sech}(a+bx) \tanh^2(a+bx)\, dx = \int x^m \operatorname{sech}(a+bx)\, dx - \int x^m \operatorname{sech}^3(a+bx)\, dx$$

Mathematica [A] time = 15.3298, size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(a+bx) \tanh^2(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]

[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2, x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int x^m (\mathrm{sech}\,(bx+a))^3 (\sinh(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \,\mathrm{sech}\,(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \,\mathrm{sech}\,(bx+a)^3 \sinh(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.370 $\int x^3 \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=240

$$-\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{3ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3i\text{PolyL}}{}$$

```
[Out] (6*x*ArcTan[E^(a + b*x)])/b^3 + (x^3*ArcTan[E^(a + b*x)])/b - ((3*I)*PolyLo
g[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((3*I)*PolyLog[2, I*E^(a + b*x)])/b^4 + (((3*I)/2)*x^2*PolyLog[2, I*E
^(a + b*x)])/b^2 + ((3*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*x*Po
lyLog[3, I*E^(a + b*x)])/b^3 - ((3*I)*PolyLog[4, (-I)*E^(a + b*x)])/b^4 + (
(3*I)*PolyLog[4, I*E^(a + b*x)])/b^4 - (3*x^2*Sech[a + b*x])/(2*b^2) - (x^3
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.297574, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5455, 4180, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$-\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{3ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3i\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```


```
[Out] (6*x*ArcTan[E^(a + b*x)])/b^3 + (x^3*ArcTan[E^(a + b*x)])/b - ((3*I)*PolyLo
g[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((3*I)*PolyLog[2, I*E^(a + b*x)])/b^4 + (((3*I)/2)*x^2*PolyLog[2, I*E
^(a + b*x)])/b^2 + ((3*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*x*Po
lyLog[3, I*E^(a + b*x)])/b^3 - ((3*I)*PolyLog[4, (-I)*E^(a + b*x)])/b^4 + (
(3*I)*PolyLog[4, I*E^(a + b*x)])/b^4 - (3*x^2*Sech[a + b*x])/(2*b^2) - (x^3
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```


Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
```

```
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^3\text{sech}(a+bx)\,dx - \int x^3\text{sech}^3(a+bx)\,dx$$

$$= \frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x^3\text{s}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3ix^2\text{Li}_2\left(ie^{a+bx}\right)}{b^2} - 3x$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{3ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b^2} + 6i$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + 3i\text{L}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + 3i\text{L}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + 3i\text{L}$$

Mathematica [A] time = 3.29312, size = 245, normalized size = 1.02

$$-\frac{-i\left(-3\left(b^2x^2+2\right)\text{PolyLog}\left(2,-ie^{a+bx}\right)+3\left(b^2x^2+2\right)\text{PolyLog}\left(2,ie^{a+bx}\right)+6bx\text{PolyLog}\left(3,-ie^{a+bx}\right)-6bx\text{PolyLog}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -((-I)*(6*b*x*Log[1 - I*E^(a + b*x)] + b^3*x^3*Log[1 - I*E^(a + b*x)] - 6*b
*x*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*(2 + b^2*x^2
)*PolyLog[2, (-I)*E^(a + b*x)] + 3*(2 + b^2*x^2)*PolyLog[2, I*E^(a + b*x)]
+ 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*
PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyLog[4, I*E^(a + b*x)]) + b^3*x^3*Sech[
a]*Sech[a + b*x]^2*Sinh[b*x] + b^2*x^2*Sech[a + b*x]*(3 + b*x*Tanh[a]))/(2*
b^4)
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int x^{3} (\operatorname{sech}(bx+a))^{3} (\sinh(bx+a))^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bx^{3}e^{(3a)} + 3x^{2}e^{(3a)}\right)e^{(3bx)} - \left(bx^{3}e^{a} - 3x^{2}e^{a}\right)e^{(bx)}}{b^{2}e^{(4bx+4a)} + 2b^{2}e^{(2bx+2a)} + b^{2}} + 2\int \frac{\left(b^{2}x^{3}e^{a} + 6xe^{a}\right)e^{(bx)}}{2\left(b^{2}e^{(2bx+2a)} + b^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) - (b*x^3*e^a - 3*x^2*e^a)*e^(b*
x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*(
b^2*x^3*e^a + 6*x*e^a)*e^(b*x)/(b^2*e^(2*b*x + 2*a) + b^2), x)
```

Fricas [C] time = 2.73347, size = 5955, normalized size = 24.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cos
h(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 - 2*(b
^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - ((3*I*b^2*x^2 + 6*I)*cosh(b*x + a)^4 +
(12*I*b^2*x^2 + 24*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (3*I*b^2*x^2 + 6*I)*s
inh(b*x + a)^4 + 3*I*b^2*x^2 + (6*I*b^2*x^2 + 12*I)*cosh(b*x + a)^2 + (6*I*
b^2*x^2 + (18*I*b^2*x^2 + 36*I)*cosh(b*x + a)^2 + 12*I)*sinh(b*x + a)^2 + (
(12*I*b^2*x^2 + 24*I)*cosh(b*x + a)^3 + (12*I*b^2*x^2 + 24*I)*cosh(b*x + a)
)*sinh(b*x + a) + 6*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - ((-3*I*b^
2*x^2 - 6*I)*cosh(b*x + a)^4 + (-12*I*b^2*x^2 - 24*I)*cosh(b*x + a)*sinh(b*
x + a)^3 + (-3*I*b^2*x^2 - 6*I)*sinh(b*x + a)^4 - 3*I*b^2*x^2 + (-6*I*b^2*x
^2 - 12*I)*cosh(b*x + a)^2 + (-6*I*b^2*x^2 + (-18*I*b^2*x^2 - 36*I)*cosh(b*
x + a)^2 - 12*I)*sinh(b*x + a)^2 + ((-12*I*b^2*x^2 - 24*I)*cosh(b*x + a)^3
+ (-12*I*b^2*x^2 - 24*I)*cosh(b*x + a))*sinh(b*x + a) - 6*I)*dilog(-I*cosh(
b*x + a) - I*sinh(b*x + a)) - ((-I*a^3 - 6*I*a)*cosh(b*x + a)^4 + (-4*I*a^3
 - 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*a^3 - 6*I*a)*sinh(b*x + a)^4
 - I*a^3 + (-2*I*a^3 - 12*I*a)*cosh(b*x + a)^2 + (-2*I*a^3 + (-6*I*a^3 - 36
*I*a)*cosh(b*x + a)^2 - 12*I*a)*sinh(b*x + a)^2 + ((-4*I*a^3 - 24*I*a)*cosh
(b*x + a)^3 + (-4*I*a^3 - 24*I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log
(cosh(b*x + a) + sinh(b*x + a) + I) - ((I*a^3 + 6*I*a)*cosh(b*x + a)^4 + (4
*I*a^3 + 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*a^3 + 6*I*a)*sinh(b*x +
 a)^4 + I*a^3 + (2*I*a^3 + 12*I*a)*cosh(b*x + a)^2 + (2*I*a^3 + (6*I*a^3 +
36*I*a)*cosh(b*x + a)^2 + 12*I*a)*sinh(b*x + a)^2 + ((4*I*a^3 + 24*I*a)*cos
h(b*x + a)^3 + (4*I*a^3 + 24*I*a)*cosh(b*x + a))*sinh(b*x + a) + 6*I*a)*log
(cosh(b*x + a) + sinh(b*x + a) - I) - (-I*b^3*x^3 + (-I*b^3*x^3 - I*a^3 - 6
*I*b*x - 6*I*a)*cosh(b*x + a)^4 + (-4*I*b^3*x^3 - 4*I*a^3 - 24*I*b*x - 24*I
*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*
sinh(b*x + a)^4 - I*a^3 + (-2*I*b^3*x^3 - 2*I*a^3 - 12*I*b*x - 12*I*a)*cosh
(b*x + a)^2 + (-2*I*b^3*x^3 - 2*I*a^3 + (-6*I*b^3*x^3 - 6*I*a^3 - 36*I*b*x
- 36*I*a)*cosh(b*x + a)^2 - 12*I*b*x - 12*I*a)*sinh(b*x + a)^2 - 6*I*b*x +
((-4*I*b^3*x^3 - 4*I*a^3 - 24*I*b*x - 24*I*a)*cosh(b*x + a)^3 + (-4*I*b^3*x
^3 - 4*I*a^3 - 24*I*b*x - 24*I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log
(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (I*b^3*x^3 + (I*b^3*x^3 + I*a^3 +
 6*I*b*x + 6*I*a)*cosh(b*x + a)^4 + (4*I*b^3*x^3 + 4*I*a^3 + 24*I*b*x + 24*
I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^3*x^3 + I*a^3 + 6*I*b*x + 6*I*a)*
sinh(b*x + a)^4 + I*a^3 + (2*I*b^3*x^3 + 2*I*a^3 + 12*I*b*x + 12*I*a)*cosh(
b*x + a)^2 + (2*I*b^3*x^3 + 2*I*a^3 + (6*I*b^3*x^3 + 6*I*a^3 + 36*I*b*x + 3
6*I*a)*cosh(b*x + a)^2 + 12*I*b*x + 12*I*a)*sinh(b*x + a)^2 + 6*I*b*x + ((4
*I*b^3*x^3 + 4*I*a^3 + 24*I*b*x + 24*I*a)*cosh(b*x + a)^3 + (4*I*b^3*x^3 +
4*I*a^3 + 24*I*b*x + 24*I*a)*cosh(b*x + a))*sinh(b*x + a) + 6*I*a)*log(-I*c
osh(b*x + a) - I*sinh(b*x + a) + 1) - (6*I*cosh(b*x + a)^4 + 24*I*cosh(b*x
```

```
+ a)*sinh(b*x + a)^3 + 6*I*sinh(b*x + a)^4 + (36*I*cosh(b*x + a)^2 + 12*I)*
sinh(b*x + a)^2 + 12*I*cosh(b*x + a)^2 + (24*I*cosh(b*x + a)^3 + 24*I*cosh(
b*x + a))*sinh(b*x + a) + 6*I)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)
) - (-6*I*cosh(b*x + a)^4 - 24*I*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*sinh(b
*x + a)^4 + (-36*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a)^2 - 12*I*cosh(b*x
+ a)^2 + (-24*I*cosh(b*x + a)^3 - 24*I*cosh(b*x + a))*sinh(b*x + a) - 6*I)*
polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^4
- 24*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*b*x*sinh(b*x + a)^4 - 12*I*b
*x*cosh(b*x + a)^2 + (-36*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)^2
 - 6*I*b*x + (-24*I*b*x*cosh(b*x + a)^3 - 24*I*b*x*cosh(b*x + a))*sinh(b*x
+ a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*b*x*cosh(b*x + a
)^4 + 24*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 6*I*b*x*sinh(b*x + a)^4 + 12
*I*b*x*cosh(b*x + a)^2 + (36*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a
)^2 + 6*I*b*x + (24*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(b*x + a))*sinh(b*
x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b^3*x^3 - 3*b^2
*x^2 - 3*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(b*
x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 + 2*b^
4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a)^2 +
 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.371 $\int x^2 \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=143

$$-\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{x\text{sech}(a+bx)}{b^2}+$$

```
[Out] (x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (I*x*PolyLog[2, (
-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (I*PolyLog[3,
 (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a + b
*x])/b^2 - (x^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.198598, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5455, 4180, 2531, 2282, 6589, 4186, 3770}

$$-\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{x\text{sech}(a+bx)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```


```
[Out] (x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (I*x*PolyLog[2, (
-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (I*PolyLog[3,
 (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a + b
*x])/b^2 - (x^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```


Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^2\text{sech}(a+bx)\,dx - \int x^2\text{sech}^3(a+bx)\,dx$$
$$= \frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{sech}(a+bx)}{b^2} - \frac{x^2\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x^2\text{sec}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} -$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} - \frac{x\text{se}}{}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{2i\text{L}}{}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{i\text{Li}}{}$$

Mathematica [A] time = 1.6052, size = 180, normalized size = 1.26

$$\frac{i\left(-2bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\text{PolyLog}\left(2,ie^{a+bx}\right)+2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2\text{PolyLog}\left(3,ie^{a+bx}\right)+b^2x^2\log\left(1-\right.\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((I/2)*((-4*I)*ArcTan[E^(a + b*x)] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x
^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*Poly
Log[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(
a + b*x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(2*b^
2) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x])/(2*b)
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int x^2\left(\text{sech}\left(bx+a\right)\right)^3\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2 b^2 \int \frac{x^2 e^{(bx+a)}}{2 \left(b^2 e^{(2 bx+2 a)} + b^2\right)} dx - \frac{\left(bx^2 e^{(3 a)} + 2 x e^{(3 a)}\right) e^{(3 bx)} - \left(bx^2 e^a - 2 x e^a\right) e^{(bx)}}{b^2 e^{(4 bx+4 a)} + 2 b^2 e^{(2 bx+2 a)} + b^2} + \frac{2 \arctan\left(e^{(bx+a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*integrate(1/2*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - ((b*x
^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) - (b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e
^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*arctan(e^(b*x + a))/b^3
```

Fricas [C] time = 2.52738, size = 4242, normalized size = 29.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 - 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) - (2*I*b*x*cosh(b*x + a)^4 + 8*I*b*x*cosh(b*x + a)*sinh(b
*x + a)^3 + 2*I*b*x*sinh(b*x + a)^4 + 4*I*b*x*cosh(b*x + a)^2 + (12*I*b*x*c
osh(b*x + a)^2 + 4*I*b*x)*sinh(b*x + a)^2 + 2*I*b*x + (8*I*b*x*cosh(b*x + a
)^3 + 8*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(
b*x + a)) - (-2*I*b*x*cosh(b*x + a)^4 - 8*I*b*x*cosh(b*x + a)*sinh(b*x + a)
^3 - 2*I*b*x*sinh(b*x + a)^4 - 4*I*b*x*cosh(b*x + a)^2 + (-12*I*b*x*cosh(b*
x + a)^2 - 4*I*b*x)*sinh(b*x + a)^2 - 2*I*b*x + (-8*I*b*x*cosh(b*x + a)^3 -
 8*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x
+ a)) - ((I*a^2 + 2*I)*cosh(b*x + a)^4 + (4*I*a^2 + 8*I)*cosh(b*x + a)*sinh
(b*x + a)^3 + (I*a^2 + 2*I)*sinh(b*x + a)^4 + (2*I*a^2 + 4*I)*cosh(b*x + a)
^2 + ((6*I*a^2 + 12*I)*cosh(b*x + a)^2 + 2*I*a^2 + 4*I)*sinh(b*x + a)^2 + I
*a^2 + ((4*I*a^2 + 8*I)*cosh(b*x + a)^3 + (4*I*a^2 + 8*I)*cosh(b*x + a))*si
nh(b*x + a) + 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((-I*a^2 - 2*I)
*cosh(b*x + a)^4 + (-4*I*a^2 - 8*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*a^2
```

```
 - 2*I)*sinh(b*x + a)^4 + (-2*I*a^2 - 4*I)*cosh(b*x + a)^2 + ((-6*I*a^2 - 1
2*I)*cosh(b*x + a)^2 - 2*I*a^2 - 4*I)*sinh(b*x + a)^2 - I*a^2 + ((-4*I*a^2
 - 8*I)*cosh(b*x + a)^3 + (-4*I*a^2 - 8*I)*cosh(b*x + a))*sinh(b*x + a) - 2*
I)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b^2*x^2 + I*a^2)*cosh(b*x
 + a)^4 + (-4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*x
^2 + I*a^2)*sinh(b*x + a)^4 - I*b^2*x^2 + (-2*I*b^2*x^2 + 2*I*a^2)*cosh(b*x
 + a)^2 + (-2*I*b^2*x^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^2 + 2*I*a^
2)*sinh(b*x + a)^2 + I*a^2 + ((-4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a)^3 + (-
4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) +
I*sinh(b*x + a) + 1) - ((I*b^2*x^2 - I*a^2)*cosh(b*x + a)^4 + (4*I*b^2*x^2
 - 4*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*sinh(b*x + a
)^4 + I*b^2*x^2 + (2*I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a)^2 + (2*I*b^2*x^2 +
(6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^2 - 2*I*a^2)*sinh(b*x + a)^2 - I*a^2
 + ((4*I*b^2*x^2 - 4*I*a^2)*cosh(b*x + a)^3 + (4*I*b^2*x^2 - 4*I*a^2)*cosh(b
*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (-2*I
*cosh(b*x + a)^4 - 8*I*cosh(b*x + a)*sinh(b*x + a)^3 - 2*I*sinh(b*x + a)^4
 + (-12*I*cosh(b*x + a)^2 - 4*I)*sinh(b*x + a)^2 - 4*I*cosh(b*x + a)^2 + (-8
*I*cosh(b*x + a)^3 - 8*I*cosh(b*x + a))*sinh(b*x + a) - 2*I)*polylog(3, I*c
osh(b*x + a) + I*sinh(b*x + a)) - (2*I*cosh(b*x + a)^4 + 8*I*cosh(b*x + a)*
sinh(b*x + a)^3 + 2*I*sinh(b*x + a)^4 + (12*I*cosh(b*x + a)^2 + 4*I)*sinh(b
*x + a)^2 + 4*I*cosh(b*x + a)^2 + (8*I*cosh(b*x + a)^3 + 8*I*cosh(b*x + a))
*sinh(b*x + a) + 2*I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b
^2*x^2 - 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a))/(b^3*c
osh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4
 + 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x +
a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.372 $\int x\mathbf{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}+\frac{x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

```
[Out] (x*ArcTan[E^(a + b*x)])/b - ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + ((I/
2)*PolyLog[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) - (x*Sech[a + b*x
]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.101526, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5455, 4180, 2279, 2391, 4185}

$$-\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}+\frac{x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (x*ArcTan[E^(a + b*x)])/b - ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + ((I/
2)*PolyLog[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) - (x*Sech[a + b*x
]*Tanh[a + b*x])/(2*b)
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int x\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x\operatorname{sech}(a+bx)\,dx - \int x\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x\operatorname{sech}(a+b$$
$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{i\,\mathrm{Subst}\left(\int \frac{\log(1-ix}{x}\right.}{b^2}$$
$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{i\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\,\mathrm{ta}}{2b}$$
$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{i\mathrm{Li}_2\left(ie^{a+bx}\right)}{2b^2} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\,\mathrm{ta}}{2b}$$

Mathematica [A] time = 0.752659, size = 93, normalized size = 1.02

$$-\frac{i\mathrm{PolyLog}(2,-i(\sinh(a+bx)+\cosh(a+bx))) - i\mathrm{PolyLog}(2,i(\sinh(a+bx)+\cosh(a+bx))) + \operatorname{sech}(a+bx) + bx\tanh}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -(-2*b*x*ArcTan[Cosh[a + b*x] + Sinh[a + b*x]] + I*PolyLog[2, (-I)*(Cosh[a
+ b*x] + Sinh[a + b*x])] - I*PolyLog[2, I*(Cosh[a + b*x] + Sinh[a + b*x])]
+ Sech[a + b*x] + b*x*Sech[a + b*x]*Tanh[a + b*x])/(2*b^2)
```

Maple [B] time = 0.068, size = 178, normalized size = 2.

$$-\frac{e^{bx+a}\left(bx e^{2bx+2a}-bx+e^{2bx+2a}+1\right)}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{\frac{i}{2}\ln\left(1+ie^{bx+a}\right)x}{b}-\frac{\frac{i}{2}\ln\left(1+ie^{bx+a}\right)a}{b^2}+\frac{\frac{i}{2}\ln\left(1-ie^{bx+a}\right)x}{b}+\frac{\frac{i}{2}\ln\left(1-ie^{bx+a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -exp(b*x+a)*(b*x*exp(2*b*x+2*a)-b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a)
)^2-1/2*I/b*ln(1+I*exp(b*x+a))*x-1/2*I/b^2*ln(1+I*exp(b*x+a))*a+1/2*I/b*ln(
1-I*exp(b*x+a))*x+1/2*I/b^2*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*dilog(1+I*exp(b*
x+a))+1/2*I/b^2*dilog(1-I*exp(b*x+a))-1/b^2*a*arctan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bxe^{(3a)}+e^{(3a)}\right)e^{(3bx)}-\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}+2\int\frac{xe^{(bx+a)}}{2\left(e^{(2bx+2a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*x*e^(b*x + a)/(e^
(2*b*x + 2*a) + 1), x)
```

Fricas [B] time = 2.35389, size = 2963, normalized size = 32.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 - 2*(b*x - 1)*cosh(b*x + a) - (I*cosh(b*x
+ a)^4 + 4*I*cosh(b*x + a)*sinh(b*x + a)^3 + I*sinh(b*x + a)^4 + (6*I*cosh(
b*x + a)^2 + 2*I)*sinh(b*x + a)^2 + 2*I*cosh(b*x + a)^2 + (4*I*cosh(b*x + a
)^3 + 4*I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(I*cosh(b*x + a) + I*sinh(
b*x + a)) - (-I*cosh(b*x + a)^4 - 4*I*cosh(b*x + a)*sinh(b*x + a)^3 - I*sin
h(b*x + a)^4 + (-6*I*cosh(b*x + a)^2 - 2*I)*sinh(b*x + a)^2 - 2*I*cosh(b*x
+ a)^2 + (-4*I*cosh(b*x + a)^3 - 4*I*cosh(b*x + a))*sinh(b*x + a) - I)*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (-I*a*cosh(b*x + a)^4 - 4*I*a*cosh(
b*x + a)*sinh(b*x + a)^3 - I*a*sinh(b*x + a)^4 - 2*I*a*cosh(b*x + a)^2 + (-
6*I*a*cosh(b*x + a)^2 - 2*I*a)*sinh(b*x + a)^2 + (-4*I*a*cosh(b*x + a)^3 -
4*I*a*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b*x + a) + sinh(b*x + a)
 + I) - (I*a*cosh(b*x + a)^4 + 4*I*a*cosh(b*x + a)*sinh(b*x + a)^3 + I*a*si
nh(b*x + a)^4 + 2*I*a*cosh(b*x + a)^2 + (6*I*a*cosh(b*x + a)^2 + 2*I*a)*sin
h(b*x + a)^2 + (4*I*a*cosh(b*x + a)^3 + 4*I*a*cosh(b*x + a))*sinh(b*x + a)
+ I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b*x - I*a)*cosh(b*x +
a)^4 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*x - I*a)*si
nh(b*x + a)^4 + (-2*I*b*x - 2*I*a)*cosh(b*x + a)^2 + ((-6*I*b*x - 6*I*a)*co
sh(b*x + a)^2 - 2*I*b*x - 2*I*a)*sinh(b*x + a)^2 - I*b*x + ((-4*I*b*x - 4*I
*a)*cosh(b*x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a))*sinh(b*x + a) - I*a
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((I*b*x + I*a)*cosh(b*x + a)
^4 + (4*I*b*x + 4*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*x + I*a)*sinh(b
*x + a)^4 + (2*I*b*x + 2*I*a)*cosh(b*x + a)^2 + ((6*I*b*x + 6*I*a)*cosh(b*x
 + a)^2 + 2*I*b*x + 2*I*a)*sinh(b*x + a)^2 + I*b*x + ((4*I*b*x + 4*I*a)*cos
h(b*x + a)^3 + (4*I*b*x + 4*I*a)*cosh(b*x + a))*sinh(b*x + a) + I*a)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*(3*(b*x + 1)*cosh(b*x + a)^2 - b*
x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a
)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 + b^2*cosh(b*x +
a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.373 $\int \mathbf{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\mathbf{sech}(a+bx)}{2b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0237051, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\mathbf{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0158222, size = 34, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] time = 0.014, size = 49, normalized size = 1.4

$$-\frac{\sinh(bx+a)}{b(\cosh(bx+a))^2}+\frac{\text{sech}(bx+a)\tanh(bx+a)}{2b}+\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/b*sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x
+a))/b
```

Maxima [B] time = 1.618, size = 89, normalized size = 2.62

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 1.99816, size = 759, normalized size = 22.32

$$-\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \mathrm{si}\right.}{b\cosh(bx+a)^4 + 4b\cosh(bx+a)\sinh(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^2(a+bx)\,\mathrm{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [B] time = 1.16275, size = 108, normalized size = 3.18

$$\frac{\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{4b} - \frac{e^{(bx+a)} - e^{(-bx-a)}}{\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - (e^(b*x + a
) - e^(-b*x - a))/(((e^(b*x + a) - e^(-b*x - a))^2 + 4)*b)
```

3.374 $\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\operatorname{sech}(a+bx)}{x},x\right)-\text{Unintegrable}\left(\frac{\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]/x, x] - Unintegrable[Sech[a + b*x]^3/x, x]
```

Rubi [A] time = 0.0642935, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/x, x] - Defer[Int][Sech[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(a+bx)}{x}\,dx-\int \frac{\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 16.0396, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{sech}\left(bx+a\right)\right)^{3}\left(\sinh\left(bx+a\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}-\left(bxe^{a}+e^{a}\right)e^{(bx)}}{b^{2}x^{2}e^{(4\,bx+4\,a)}+2\,b^{2}x^{2}e^{(2\,bx+2\,a)}+b^{2}x^{2}}+2\int\frac{\left(b^{2}x^{2}e^{a}+2\,e^{a}\right)e^{(bx)}}{2\left(b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) - (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) + 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/2*(b^2*
x^2*e^a + 2*e^a)*e^(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{2}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

3.375 $\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\operatorname{sech}(a+bx)}{x^2},x\right)-\text{Unintegrable}\left(\frac{\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]/x^2, x] - Unintegrable[Sech[a + b*x]^3/x^2, x]
```

Rubi [A] time = 0.0685913, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/x^2, x] - Defer[Int][Sech[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx-\int \frac{\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 12.8311, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.195, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{sech}\,(bx+a))^3 (\sinh\,(bx+a))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}-(bxe^{a}+2\,e^{a})e^{(bx)}}{b^2x^3e^{(4\,bx+4\,a)}+2\,b^2x^3e^{(2\,bx+2\,a)}+b^2x^3}+2\int\frac{\left(b^2x^2e^{a}+6\,e^{a}\right)e^{(bx)}}{2\left(b^2x^4e^{(2\,bx+2\,a)}+b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) - (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/2*(
b^2*x^2*e^a + 6*e^a)*e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{sech}\,(bx+a)^3 \sinh\,(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

3.376 $\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=84

$$-\text{Unintegrable}\left(x^m\tanh(a+bx),x\right)+\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1$$

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) - Unintegrable[x^m*Tanh[a + b*
x], x]
```

Rubi [A] time = 0.13894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) - Defer[Int][x^m*Tanh[a + b*x]
, x]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx &= \int x^m\cosh(a+bx)\sinh(a+bx)\,dx-\int x^m\tanh(a+bx)\,dx\\ &= \int \frac{1}{2}x^m\sinh(2a+2bx)\,dx-\int x^m\tanh(a+bx)\,dx\\ &= \frac{1}{2}\int x^m\sinh(2a+2bx)\,dx-\int x^m\tanh(a+bx)\,dx\\ &= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx-\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx-\int x^m\tanh(a+bx)\,dx\\ &= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}-\int x^m\, \text{t}\end{aligned}$$

Mathematica [A] time = 21.8972, size = 0, normalized size = 0.

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x], x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int x^m \mathrm{sech}\,(bx+a)\,(\sinh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m\,\mathrm{sech}\,(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m\,\mathrm{sech}\,(bx+a)\sinh(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.377 $\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=185

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+$$

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) + x^4/4 - (x^3*Log[1 + E^(2*(a + b*x))])/b - (3*x
^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) + (3*x*PolyLog[3, -E^(2*(a + b*x))
])/(2*b^3) - (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Si
nh[a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*S
inh[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.246804, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5449, 5372, 3311, 30, 2635, 8, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```


```
[Out] (3*x)/(8*b^3) + x^3/(4*b) + x^4/4 - (x^3*Log[1 + E^(2*(a + b*x))])/b - (3*x
^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) + (3*x*PolyLog[3, -E^(2*(a + b*x))
])/(2*b^3) - (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Si
nh[a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*S
inh[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```


Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx=\int x^3\cosh(a+bx)\sinh(a+bx)\,dx-\int x^3\tanh(a+bx)\,dx$$
$$=\frac{x^4}{4}+\frac{x^3\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx-\frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}$$
$$=\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}+$$
$$=\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3\cosh(a+bx)\sinh(a+bx}{8b^4}$$
$$=\frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}-$$
$$=\frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}-$$
$$=\frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}-$$

Mathematica [A] time = 2.86173, size = 191, normalized size = 1.03

$$\frac{1}{16}\left(\frac{12\left(2b^2x^2\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+2bx\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)+\text{PolyLog}\left(4,-e^{-2(a+bx)}\right)\right)}{b^4}+\frac{\cosh(2bx)\left(2bx\cosh(2\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] ((-8*x^4)/(1 + E^(2*a)) - (16*x^3*Log[1 + E^(-2*(a + b*x))])/b + (12*(2*b^2
*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3, -E^(-2*(a + b*x))] +
PolyLog[4, -E^(-2*(a + b*x))]))/b^4 + (Cosh[2*b*x]*(2*b*x*(3 + 2*b^2*x^2)*C
osh[2*a] - 3*(1 + 2*b^2*x^2)*Sinh[2*a]))/b^4 + ((-3*(1 + 2*b^2*x^2)*Cosh[2*
a] + 2*b*x*(3 + 2*b^2*x^2)*Sinh[2*a])*Sinh[2*b*x])/b^4 - 4*x^4*Tanh[a])/16
```

Maple [A] time = 0.071, size = 189, normalized size = 1.

$$\frac{x^4}{4}+\frac{\left(4\,x^3b^3-6\,x^2b^2+6\,bx-3\right)\mathrm{e}^{2\,bx+2\,a}}{32\,b^4}+\frac{\left(4\,x^3b^3+6\,x^2b^2+6\,bx+3\right)\mathrm{e}^{-2\,bx-2\,a}}{32\,b^4}+2\,\frac{a^3x}{b^3}+\frac{3\,a^4}{2\,b^4}-\frac{x^3\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*x
^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)+2/b^3*a^3*x+3/2/b^4*a^4-x^3*ln(1+
exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-e
xp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/b^4-2/b^4*a^3*ln(exp(b*x+
a))
```

Maxima [A] time = 1.3262, size = 244, normalized size = 1.32

$$\frac{1}{2}\,x^4 - \frac{\left(8\,b^4x^4e^{(2\,a)} - \left(4\,b^3x^3e^{(4\,a)} - 6\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(2\,bx)} - \left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{32\,b^4} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 1/32*(8*b^4*x^4*e^(2*a) - (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
+ 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x))*e^(-2*a)/b^4 - 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^
2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*poly
log(4, -e^(2*b*x + 2*a)))/b^4
```

Fricas [C] time = 2.46974, size = 2568, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 + 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 + 2*(4*b^4*x^4 - 8*a^4 + 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*(b^2*x^2*cosh(b*x + a)
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x
^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-I*cosh(b*x
```

```
 + a) - I*sinh(b*x + a)) + 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*si
nh(b*x + a) + a^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) +
 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b*x
 + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 32*((b^3*x^3 + a^3)*cosh(
b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3
)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 32*((b^3*x^
3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) +
(b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) - 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 192*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, -I*cosh(b*x + a) -
 I*sinh(b*x + a)) + 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x
 + a) + b*x*sinh(b*x + a)^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a))
+ 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b
*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((4*b^3*x^3 -
 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^3 + 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a)
)*sinh(b*x + a) + 3)/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x +
a) + b^4*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.378 $\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=130

$$-\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{x^2\log\left(e^{2(a+bx)}+\right.}{b}$$

```
[Out] x^2/(4*b) + x^3/3 - (x^2*Log[1 + E^(2*(a + b*x))])/b - (x*PolyLog[2, -E^(2*
(a + b*x))])/b^2 + PolyLog[3, -E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2
*b)
```

Rubi [A] time = 0.18999, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5449, 5372, 3310, 30, 3718, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{x^2\log\left(e^{2(a+bx)}+\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```


```
[Out] x^2/(4*b) + x^3/3 - (x^2*Log[1 + E^(2*(a + b*x))])/b - (x*PolyLog[2, -E^(2*
(a + b*x))])/b^2 + PolyLog[3, -E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2
*b)
```


Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\sinh^2(a+bx)\tanh(a+bx)\,dx=\int x^2\cosh(a+bx)\sinh(a+bx)\,dx-\int x^2\tanh(a+bx)\,dx$$
$$=\frac{x^3}{3}+\frac{x^2\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx-\frac{\int x\sinh^2(a+bx)\,dx}{b}$$
$$=\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\text{si}}{}$$
$$=\frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}$$
$$=\frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}$$
$$=\frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}+\frac{\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}-\frac{x\cosh(a+}{}$$

Mathematica [A] time = 2.63895, size = 154, normalized size = 1.18

$$\frac{1}{24}\left(\frac{4\left(6bx\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+3\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)+2b^2x^2\left(-\frac{2bx}{e^{2a}+1}-3\log\left(e^{-2(a+bx)}+1\right)\right)\right)}{b^3}+\frac{3\cosh(2bx)(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] ((4*(2*b^2*x^2*((-2*b*x)/(1 + E^(2*a)) - 3*Log[1 + E^(-2*(a + b*x))]) + 6*b
*x*PolyLog[2, -E^(-2*(a + b*x))] + 3*PolyLog[3, -E^(-2*(a + b*x))]))/b^3 +
(3*Cosh[2*b*x]*((1 + 2*b^2*x^2)*Cosh[2*a] - 2*b*x*Sinh[2*a]))/b^3 + (3*(-2*
b*x*Cosh[2*a] + (1 + 2*b^2*x^2)*Sinh[2*a])*Sinh[2*b*x])/b^3 - 8*x^3*Tanh[a]
)/24
```

Maple [A] time = 0.069, size = 152, normalized size = 1.2

$$\frac{x^3}{3}+\frac{\left(2x^2b^2-2bx+1\right)e^{2bx+2a}}{16b^3}+\frac{\left(2x^2b^2+2bx+1\right)e^{-2bx-2a}}{16b^3}+2\frac{a^2\ln\left(e^{bx+a}\right)}{b^3}-2\frac{a^2x}{b^2}-\frac{4a^3}{3b^3}-\frac{x^2\ln\left(1+e^{2bx+2a}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+1
)/b^3*exp(-2*b*x-2*a)+2/b^3*a^2*ln(exp(b*x+a))-2/b^2*a^2*x-4/3/b^3*a^3-x^2*
ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(
2*b*x+2*a))/b^3
```

Maxima [A] time = 1.38972, size = 186, normalized size = 1.43

$$\frac{2}{3}x^3 - \frac{\left(16\,b^3x^3e^{(2\,a)} - 3\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} - 3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} - \frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/48*(16*b^3*x^3*e^(2*a) - 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) +
 e^(4*a))*e^(2*b*x) - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 -
1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - p
olylog(3, -e^(2*b*x + 2*a)))/b^3
```

Fricas [C] time = 2.31393, size = 2090, normalized size = 16.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
 + 6*b^2*x^2 + 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 + 2*(8*b^3*x^3 + 16*a^3
 + 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b*x*cosh(b*x + a)^2 + 2
*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh
```

```
(b*x + a) + a^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) - 4
8*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x +
 a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 48*((b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*
sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 48*((b^2*x^2
- a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b
^2*x^2 - a^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
+ 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*po
lylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 96*(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*s
inh(b*x + a)) + 4*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 + 8*(b^3*x^3 +
 2*a^3)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh
(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a) \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.379 $\int x \sinh^2(a+bx) \tanh(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}-\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}+\frac{x^2}{2}$$

```
[Out] x/(4*b) + x^2/2 - (x*Log[1 + E^(2*(a + b*x))])/b - PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/
(2*b)
```

Rubi [A] time = 0.116242, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5449, 5372, 2635, 8, 3718, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}-\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] x/(4*b) + x^2/2 - (x*Log[1 + E^(2*(a + b*x))])/b - PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/
(2*b)
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx=\int x\cosh(a+bx)\sinh(a+bx)\,dx-\int x\tanh(a+bx)\,dx$$
$$=\frac{x^2}{2}+\frac{x\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx-\frac{\int\sinh^2(a+bx)\,dx}{2b}$$
$$=\frac{x^2}{2}-\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{\int 1\,dx}{4b}$$
$$=\frac{x}{4b}+\frac{x^2}{2}-\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{\text{Su}}{}$$
$$=\frac{x}{4b}+\frac{x^2}{2}-\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\,\text{si}}{}$$

Mathematica [A] time = 0.221195, size = 102, normalized size = 1.15

$$-\frac{-4\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+4a^2+8abx+8a\log\left(e^{-2(a+bx)}+1\right)+8bx\log\left(e^{-2(a+bx)}+1\right)+\sinh(2(a+bx))-2bx\cosh(}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(4*a^2 + 8*a*b*x + 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] + 8*a*Log[1 + E^(-2
*(a + b*x))] + 8*b*x*Log[1 + E^(-2*(a + b*x))] - 8*a*Log[Cosh[a + b*x]] - 4
*PolyLog[2, -E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/(8*b^2)
```

Maple [A] time = 0.065, size = 110, normalized size = 1.2

$$\frac{x^2}{2}+\frac{(2bx-1)e^{2bx+2a}}{16b^2}+\frac{(2bx+1)e^{-2bx-2a}}{16b^2}+2\frac{ax}{b}+\frac{a^2}{b^2}-\frac{x\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{\text{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}-2\frac{a\ln\left(e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a
)+2/b*a*x+a^2/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b
^2-2/b^2*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.36182, size = 128, normalized size = 1.44

$$x^2 - \frac{\left(8\,b^2x^2e^{(2\,a)} - \left(2\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(2\,bx)} - (2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2} - \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x^2 - 1/16*(8*b^2*x^2*e^(2*a) - (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) - (2*b*
x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dil
og(-e^(2*b*x + 2*a)))/b^2
```

Fricas [C] time = 2.28533, size = 1561, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 + 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 +
2*(4*b^2*x^2 + 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b
*x - 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 16*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x
 + a)) + 16*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 16*(a*cosh(b*x + a)^2 +
 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) - I) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(
b*x + a) + 1) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*s
inh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x
+ a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*
x + a) + b^2*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh^3(a + bx) \operatorname{sech}(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx + a) \sinh(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.380 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rubi [A] time = 0.024513, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.0174879, size = 25, normalized size = 0.89

$$-\frac{\log(\cosh(a+bx)) - \frac{1}{2}\cosh^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -((-Cosh[a + b*x]^2/2 + Log[Cosh[a + b*x]])/b)
```

Maple [A] time = 0.016, size = 27, normalized size = 1.

$$\frac{(\sinh(bx+a))^2}{2b} - \frac{\ln(\cosh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*sinh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Maxima [B] time = 1.56552, size = 76, normalized size = 2.71

$$-\frac{bx+a}{b} + \frac{e^{(2bx+2a)}}{8b} + \frac{e^{(-2bx-2a)}}{8b} - \frac{\log\left(e^{(-2bx-2a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

Fricas [B] time = 2.077, size = 551, normalized size = 19.68

$$\frac{8\,bx\cosh\left(bx+a\right)^{2}+\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(4\,bx+3\,\cosh\left(bx+a\right)^{2}\right)\sin}{8\left(b\cosh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x), x)
```

Giac [B] time = 1.17456, size = 92, normalized size = 3.29

$$-\frac{\left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}}{8\,b}+\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a)/b + (b*x + a)/b + 1/8*e^(2*b*
x + 2*a)/b - log(e^(2*b*x + 2*a) + 1)/b
```

3.381 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=41

$$-\text{Unintegrable}\left(\frac{\tanh(a+bx)}{x},x\right)+\frac{1}{2}\sinh(2a)\text{Chi}(2bx)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 - Uni
ntegrable[Tanh[a + b*x]/x, x]
```

Rubi [A] time = 0.103409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 - Def
er[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx&=\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx-\int \frac{\tanh(a+bx)}{x}\,dx\\&=\int \frac{\sinh(2a+2bx)}{2x}\,dx-\int \frac{\tanh(a+bx)}{x}\,dx\\&=\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx-\int \frac{\tanh(a+bx)}{x}\,dx\\&=\frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx-\int \frac{\tanh(a+bx)}{x}\,dx\\&=\frac{1}{2}\text{Chi}(2bx)\sinh(2a)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)-\int \frac{\tanh(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 18.7739, size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\left(\sinh(bx+a)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\operatorname{Ei}(2\,bx)e^{(2\,a)} - \frac{1}{4}\operatorname{Ei}(-2\,bx)e^{(-2\,a)} + 2\int \frac{1}{xe^{(2\,bx+2\,a)}+x}\,dx - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) + 2*integrate(1/(x*e^(2*b*x
 + 2*a) + x), x) - log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

3.382 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2} dx$

Optimal. Leaf size=53

$$-\text{Unintegrable}\left(\frac{\tanh(a+bx)}{x^2},x\right)+b\cosh(2a)\text{Chi}(2bx)+b\sinh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] - Unintegrable[Tanh[a + b*x]/x^2, x]
```

Rubi [A] time = 0.131459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] - Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2} dx &= \int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2} dx - \int \frac{\tanh(a+bx)}{x^2} dx \\
&= \int \frac{\sinh(2a+2bx)}{2x^2} dx - \int \frac{\tanh(a+bx)}{x^2} dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2} dx - \int \frac{\tanh(a+bx)}{x^2} dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + b\int \frac{\cosh(2a+2bx)}{x} dx - \int \frac{\tanh(a+bx)}{x^2} dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + (b\cosh(2a))\int \frac{\cosh(2bx)}{x} dx + (b\sinh(2a))\int \frac{\sinh(2bx)}{x} dx - \int \frac{\tanh(a+bx)}{x^2} dx \\
&= b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) - \int \frac{\tanh(a+bx)}{x^2} dx
\end{aligned}$$

Mathematica [A] time = 19.689, size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Maple [A] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\left(\sinh(bx+a)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right)+\frac{1}{2}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right)+\frac{1}{x}+2\int\frac{1}{x^2e^{(2\,bx+2\,a)}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) + 1/x + 2
*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

3.383 $\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=79

$$-\text{CannotIntegrate}\left(x^m\tanh(a+bx)\text{sech}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m$$

```
[Out] -CannotIntegrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x] + (E^a*x^m*Gamma[1 + m
, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(2*b*E^a*(b*x)^m)
```

Rubi [A] time = 0.167236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx &= \int x^m\sinh(a+bx)\,dx-\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx-\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}-\int x^m\text{sech}(a+bx)\tan\end{aligned}$$

Mathematica [A] time = 16.8345, size = 0, normalized size = 0.

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2, x]
```

Maple [A] time = 0.08, size = 0, normalized size = 0.

$$\int x^m (\mathrm{sech}\,(bx+a))^2 (\sinh\,(bx+a))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \,\mathrm{sech}\,(bx+a)^2 \sinh\,(bx+a)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \,\mathrm{sech}\,(bx+a)^2 \sinh\,(bx+a)^3 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.384 $\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}-\frac{6x^2\tan^{-1}\left(e^{a+bx}\right.}{b^2}$$

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a +
b*x])/b + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((6*I)*x*PolyLog[2,
I*E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4 + ((6*I)*Pol
yLog[3, I*E^(a + b*x)])/b^4 + (x^3*Sech[a + b*x])/b - (6*Sinh[a + b*x])/b^4
 - (3*x^2*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.18291, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5449, 3296, 2637, 5418, 4180, 2531, 2282, 6589}

$$\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}-\frac{6x^2\tan^{-1}\left(e^{a+bx}\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```


```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a +
b*x])/b + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((6*I)*x*PolyLog[2,
I*E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4 + ((6*I)*Pol
yLog[3, I*E^(a + b*x)])/b^4 + (x^3*Sech[a + b*x])/b - (6*Sinh[a + b*x])/b^4
 - (3*x^2*Sinh[a + b*x])/b^2
```


Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3\sinh(a+bx)\,dx - \int x^3\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{x^3\cosh(a+bx)}{b} + \frac{x^3\mathbf{sech}(a+bx)}{b} - \frac{3\int x^2\cosh(a+bx)\,dx}{b} - \frac{3\int x^2\mathbf{sech}(a+bx)}{b}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^3\cosh(a+bx)}{b} + \frac{x^3\mathbf{sech}(a+bx)}{b} - \frac{3x^2\sinh(a+bx)}{b^2} + \frac{(6}{}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\mathbf{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{6i}{}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\mathbf{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{6i}{}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\mathbf{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{6i}{}$$

Mathematica [A] time = 3.25533, size = 196, normalized size = 1.21

$$\frac{-3i\left(-2bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\text{PolyLog}\left(2,ie^{a+bx}\right)+2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2\text{PolyLog}\left(3,ie^{a+bx}\right)+b^2x^2\log\left(1-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((-3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] -
2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Po
lyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + b^3*x^3*Sech[a
+ b*x] + Cosh[b*x]*(b*x*(6 + b^2*x^2)*Cosh[a] - 3*(2 + b^2*x^2)*Sinh[a]) +
(-3*(2 + b^2*x^2)*Cosh[a] + b*x*(6 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^4
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x^3 \left(\operatorname{sech}(bx+a)\right)^2 \left(\sinh(bx+a)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)

[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^3x^3e^{(4\,a)} - 3\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 6\,e^{(4\,a)}\right)e^{(3\,bx)} + 6\,\left(b^3x^3e^{(2\,a)} + 2\,bxe^{(2\,a)}\right)e^{(bx)} + \left(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6\right)e^{(-bx)}}{2\,\left(b^4e^{(2\,bx+3\,a)} + b^4e^{a}\right)} - 6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3*b*x) + 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) + b^4*e^a) - 6*integrate(x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)

Fricas [C] time = 2.50269, size = 3351, normalized size = 20.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 + 6*(b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 6*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2 + 2*b*x)*sinh(b*x + a)^2 + 6*b*x + (-12*I*b*x*cosh(b*x + a)^3 - 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - 12*I*b*x*sinh(b*x + a)^3 - 12*I*b*x*cosh(b*x + a) + (-36*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (12*I*b*x*cosh(b*x + a)^3 + 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 12*I*b*x*sinh(b*x + a)^3 + 12*I*b*x*cosh(b*x + a) + (36*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (-6*I*a^2*cosh(b*x + a)^3 - 18*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2 - 6*I*a^2*sinh(b*x + a)^3 - 6*I*a^2*cosh(b*x + a) + (-18*I*a^2*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x +

```
a) + I) + (6*I*a^2*cosh(b*x + a)^3 + 18*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2
 + 6*I*a^2*sinh(b*x + a)^3 + 6*I*a^2*cosh(b*x + a) + (18*I*a^2*cosh(b*x + a
)^2 + 6*I*a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((6*
I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^3 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x +
 a)*sinh(b*x + a)^2 + (6*I*b^2*x^2 - 6*I*a^2)*sinh(b*x + a)^3 + (6*I*b^2*x^
2 - 6*I*a^2)*cosh(b*x + a) + (6*I*b^2*x^2 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(
b*x + a)^2 - 6*I*a^2)*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a)
+ 1) + ((-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^3 + (-18*I*b^2*x^2 + 18*I*a^
2)*cosh(b*x + a)*sinh(b*x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*sinh(b*x + a)^3
 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a) + (-6*I*b^2*x^2 + (-18*I*b^2*x^2
+ 18*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a)
- I*sinh(b*x + a) + 1) + (12*I*cosh(b*x + a)^3 + 36*I*cosh(b*x + a)*sinh(b*
x + a)^2 + 12*I*sinh(b*x + a)^3 + (36*I*cosh(b*x + a)^2 + 12*I)*sinh(b*x +
a) + 12*I*cosh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + (-
12*I*cosh(b*x + a)^3 - 36*I*cosh(b*x + a)*sinh(b*x + a)^2 - 12*I*sinh(b*x +
 a)^3 + (-36*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a) - 12*I*cosh(b*x + a))*
polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^3*x^3 - 3*b^2*x^2 +
6*b*x - 6)*cosh(b*x + a)^3 + 3*(b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x +
a) + 6)/(b^4*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)*sinh(b*x + a)^2 + b^4*si
nh(b*x + a)^3 + b^4*cosh(b*x + a) + (3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x
+ a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.385 $\int x^2 \sinh(a+bx) \tanh^2(a+bx)\,dx$

Optimal. Leaf size=104

$$\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+b}{b}$$

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x]
)/b + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*I)*PolyLog[2, I*E^(a +
 b*x)])/b^3 + (x^2*Sech[a + b*x])/b - (2*x*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.117554, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5449, 3296, 2638, 5418, 4180, 2279, 2391}

$$\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```


```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x]
)/b + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*I)*PolyLog[2, I*E^(a +
 b*x)])/b^3 + (x^2*Sech[a + b*x])/b - (2*x*Sinh[a + b*x])/b^2
```


Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \sinh(a+bx)\,dx - \int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\operatorname{sech}(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\,dx}{b} - \frac{2\int x\operatorname{sech}(a+bx)\,d}{b}$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\operatorname{sech}(a+bx)}{b} - \frac{2x\sinh(a+bx)}{b^2} + \frac{(2i)}{}$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\operatorname{sech}(a+bx)}{b} - \frac{2x\sin}{}$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{2i\text{Li}_2}{}$$

Mathematica [A] time = 0.492211, size = 172, normalized size = 1.65

$$\frac{2i\left(\text{PolyLog}\left(2,-ie^{a+bx}\right)-\text{PolyLog}\left(2,ie^{a+bx}\right)\right)+b^2x^2\operatorname{sech}(a+bx)+\cosh(bx)\left(\cosh(a)\left(b^2x^2+2\right)-2bx\sinh(a)\right)+\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x
)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(PolyL
og[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) + b^2*x^2*Sech[a + b*x
] + Cosh[b*x]*((2 + b^2*x^2)*Cosh[a] - 2*b*x*Sinh[a]) + (-2*b*x*Cosh[a] + (
2 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^3
```

Maple [B] time = 0.074, size = 205, normalized size = 2.

$$\frac{\left(x^2b^2-2\,bx+2\right)e^{bx+a}}{2\,b^3} + \frac{\left(x^2b^2+2\,bx+2\right)e^{-bx-a}}{2\,b^3} + 2\,\frac{e^{bx+a}x^2}{b\left(1+e^{2\,bx+2\,a}\right)} + \frac{2\,i\ln\left(1+ie^{bx+a}\right)x}{b^2} + \frac{2\,i\ln\left(1+ie^{bx+a}\right)a}{b^3} - \frac{2\,i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)+
2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*I/b^2*ln(1+I*exp(b*x+a))*x+2*I/b^3*
```

```
ln(1+I*exp(b*x+a))*a-2*I/b^2*ln(1-I*exp(b*x+a))*x-2*I/b^3*ln(1-I*exp(b*x+a)
)*a+2*I/b^3*dilog(1+I*exp(b*x+a))-2*I/b^3*dilog(1-I*exp(b*x+a))+4/b^3*a*arc
tan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2x^2e^{(4a)} - 2bxe^{(4a)} + 2e^{(4a)}\right)e^{(3bx)} + 2\left(3b^2x^2e^{(2a)} + 2e^{(2a)}\right)e^{(bx)} + \left(b^2x^2 + 2bx + 2\right)e^{(-bx)}}{2\left(b^3e^{(2bx+3a)} + b^3e^{a}\right)} - 4\int \frac{xe^{(bx+a)}}{be^{(2bx+2a)} + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) + 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) + b^3*e^a) - 4*integrate(x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x
)
```

Fricas [B] time = 2.47479, size = 2500, normalized size = 24.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 +
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2*(3*b^2*x^2 + 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + (-4*I*cosh(b*x + a)^3 - 12*
I*cosh(b*x + a)*sinh(b*x + a)^2 - 4*I*sinh(b*x + a)^3 + (-12*I*cosh(b*x + a
)^2 - 4*I)*sinh(b*x + a) - 4*I*cosh(b*x + a))*dilog(I*cosh(b*x + a) + I*sin
h(b*x + a)) + (4*I*cosh(b*x + a)^3 + 12*I*cosh(b*x + a)*sinh(b*x + a)^2 + 4
*I*sinh(b*x + a)^3 + (12*I*cosh(b*x + a)^2 + 4*I)*sinh(b*x + a) + 4*I*cosh(
b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (4*I*a*cosh(b*x + a)^
3 + 12*I*a*cosh(b*x + a)*sinh(b*x + a)^2 + 4*I*a*sinh(b*x + a)^3 + 4*I*a*co
sh(b*x + a) + (12*I*a*cosh(b*x + a)^2 + 4*I*a)*sinh(b*x + a))*log(cosh(b*x
+ a) + sinh(b*x + a) + I) + (-4*I*a*cosh(b*x + a)^3 - 12*I*a*cosh(b*x + a)*
sinh(b*x + a)^2 - 4*I*a*sinh(b*x + a)^3 - 4*I*a*cosh(b*x + a) + (-12*I*a*co
```

```
sh(b*x + a)^2 - 4*I*a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I
) + ((4*I*b*x + 4*I*a)*cosh(b*x + a)^3 + (12*I*b*x + 12*I*a)*cosh(b*x + a)*
sinh(b*x + a)^2 + (4*I*b*x + 4*I*a)*sinh(b*x + a)^3 + (4*I*b*x + 4*I*a)*cos
h(b*x + a) + ((12*I*b*x + 12*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*sinh(b
*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((-4*I*b*x - 4*I*a)*c
osh(b*x + a)^3 + (-12*I*b*x - 12*I*a)*cosh(b*x + a)*sinh(b*x + a)^2 + (-4*I
*b*x - 4*I*a)*sinh(b*x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a) + ((-12*I*
b*x - 12*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*x + a))*log(-I*cosh
(b*x + a) - I*sinh(b*x + a) + 1) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3
 + (3*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 +
 3*b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3 + b^3*cosh(b*x +
 a) + (3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.386 $\int x \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sinh(a+bx)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x])/b -
Sinh[a + b*x]/b^2
```

Rubi [A] time = 0.0508757, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5449, 3296, 2637, 5418, 3770}

$$-\frac{\sinh(a+bx)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x])/b -
Sinh[a + b*x]/b^2
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh(a+bx)\tanh^2(a+bx)\,dx &= \int x\sinh(a+bx)\,dx - \int x\,\mathrm{sech}(a+bx)\tanh(a+bx)\,dx \\
&= \frac{x\cosh(a+bx)}{b} + \frac{x\,\mathrm{sech}(a+bx)}{b} - \frac{\int\cosh(a+bx)\,dx}{b} - \frac{\int\mathrm{sech}(a+bx)\,dx}{b} \\
&= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\,\mathrm{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.11489, size = 50, normalized size = 1.09

$$
-\frac{\sinh(a+bx)}{b^2} - \frac{2\tan^{-1}\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\,\mathrm{sech}(a+bx)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]])/b^2 + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x]
)/b - Sinh[a + b*x]/b^2
```

Maple [C] time = 0.082, size = 94, normalized size = 2.

$$
\frac{(bx-1)\,e^{bx+a}}{2\,b^2} + \frac{(bx+1)\,e^{-bx-a}}{2\,b^2} + 2\,\frac{e^{bx+a}x}{b\left(1+e^{2\,bx+2\,a}\right)} + \frac{i\ln\left(e^{bx+a}-i\right)}{b^2} - \frac{i\ln\left(e^{bx+a}+i\right)}{b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+2*x*exp(b*x+a)/b/(1+
exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)-I)-I/b^2*ln(exp(b*x+a)+I)
```

Maxima [A] time = 1.85291, size = 109, normalized size = 2.37

$$\frac{6\,bxe^{(bx+2\,a)}+\left(bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(3\,bx)}+(bx+1)e^{(-bx)}}{2\left(b^2e^{(2\,bx+3\,a)}+b^2e^{a}\right)}-\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b*x*e^(b*x + 2*a) + (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) + (b*x + 1)*e^
(-b*x))/(b^2*e^(2*b*x + 3*a) + b^2*e^a) - 2*arctan(e^(b*x + a))/b^2
```

Fricas [B] time = 2.13181, size = 779, normalized size = 16.93

$$\frac{(bx-1)\cosh\left(bx+a\right)^4+4\,(bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(bx-1)\sinh\left(bx+a\right)^4+6\,bx\cosh\left(bx+a\right)^2+6\left((bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
 + a)^2 + b*x)*sinh(b*x + a)^2 + b*x - 4*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
+ cosh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 4*((b*x - 1)*cosh(
b*x + a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^3 +
 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + b^2*cosh(b*x +
 a) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2364, size = 138, normalized size = 3.

$$\frac{bxe^{(4\,bx+4\,a)} + 6\,bxe^{(2\,bx+2\,a)} + bx - 4\,\arctan\left(e^{(bx+a)}\right)e^{(3\,bx+3\,a)} - 4\,\arctan\left(e^{(bx+a)}\right)e^{(bx+a)} - e^{(4\,bx+4\,a)} + 1}{2\left(b^2e^{(3\,bx+3\,a)} + b^2e^{(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x*e^(4*b*x + 4*a) + 6*b*x*e^(2*b*x + 2*a) + b*x - 4*arctan(e^(b*x +
a))*e^(3*b*x + 3*a) - 4*arctan(e^(b*x + a))*e^(b*x + a) - e^(4*b*x + 4*a) +
 1)/(b^2*e^(3*b*x + 3*a) + b^2*e^(b*x + a))
```

3.387 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rubi [A] time = 0.0242972, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.0285767, size = 21, normalized size = 1.

$$\frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Maple [A] time = 0.016, size = 32, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\sinh(bx+a))^2}{\cosh(bx+a)} + 2\cosh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(-sinh(b*x+a)^2/cosh(b*x+a)+2*cosh(b*x+a))
```

Maxima [B] time = 1.03865, size = 73, normalized size = 3.48

$$\frac{e^{(-bx-a)}}{2b} + \frac{5e^{(-2bx-2a)} + 1}{2b\left(e^{(-bx-a)} + e^{(-3bx-3a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

Fricas [A] time = 1.94858, size = 85, normalized size = 4.05

$$\frac{\cosh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{2}+3}{2\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{3}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [B] time = 1.20887, size = 61, normalized size = 2.9

$$\frac{e^{\left(bx+a\right)}+e^{\left(-bx-a\right)}}{2\,b}+\frac{2}{b\left(e^{\left(bx+a\right)}+e^{\left(-bx-a\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a))/b + 2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.388 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=35

$$-\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{x},x\right)+\sinh(a)\text{Chi}(bx)+\cosh(a)\text{Shi}(bx)$$

```
[Out] -CannotIntegrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x] + CoshIntegral[b*x]*Si
nh[a] + Cosh[a]*SinhIntegral[b*x]
```

Rubi [A] time = 0.128341, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] - Defer[Int][(Sech[a
+ b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx &= \int \frac{\sinh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\\ &=\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\sinh(a)\int \frac{\cosh(bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\\ &=\text{Chi}(bx)\sinh(a)+\cosh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 11.5038, size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2 (\sinh(bx+a))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\operatorname{Ei}(-bx)e^{(-a)} + \frac{1}{2}\operatorname{Ei}(bx)e^{a} + \frac{2e^{(bx+a)}}{bxe^{(2bx+2a)} + bx} + 2\int \frac{e^{(bx+a)}}{bx^2e^{(2bx+2a)} + bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a + 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
 + b*x) + 2*integrate(e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

3.389 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\text{CannotIntegrate}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{x^2},x\right)+b\cosh(a)\text{Chi}(bx)+b\sinh(a)\text{Shi}(bx)-\frac{\sinh(a+bx)}{x}$$

```
[Out] -CannotIntegrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x] + b*Cosh[a]*CoshInte
gral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
```

Rubi [A] time = 0.125964, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 - Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx &= \int \frac{\sinh(a+bx)}{x^2}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(a+bx)}{x}+b\int \frac{\cosh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(a+bx)}{x}+(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx-\int \frac{\text{sec}
\end{aligned}$$

$$= b\cosh(a)\text{Chi}(bx)-\frac{\sinh(a+bx)}{x}+b\sinh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)\tanh(a+}{x^2}$$

Mathematica [A] time = 9.81036, size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{sech}\,(bx+a))^2 (\sinh\,(bx+a))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,be^{(-a)}\Gamma\left(-1,bx\right)+\frac{1}{2}\,be^{a}\Gamma\left(-1,-bx\right)+\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}+4\int\frac{e^{(bx+a)}}{bx^3e^{(2\,bx+2\,a)}+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) + 2*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integrate(e^(b*x + a)/(b*x^3*e^(2*b*x + 2*
a) + b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{sech}\,(bx+a)^2\sinh\,(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

3.390 $\int x^m \tanh^3(a+bx)\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \tanh^3(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Tanh[a + b*x]^3, x]
```

Rubi [A] time = 0.0303883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \tanh^3(a+bx)\,dx=\int x^m \tanh^3(a+bx)\,dx$$

Mathematica [A] time = 5.2647, size = 0, normalized size = 0.

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x]^3, x]
```

Maple [A] time = 0.087, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{sech}\left(bx+a\right)\right)^{3} \left(\sinh\left(bx+a\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(6\,bx+m\log(x)+6\,a)}}{(m+1)e^{(6\,bx+6\,a)}+3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}+m+1}-\int\frac{\left(3\left(2\,bxe^{(6\,a)}+(m+1)e^{(6\,a)}\right)e^{(6\,bx)}-2}{(m+1)e^{(8\,bx+8\,a)}+4\,(m+1)e^{(6\,bx+6\,a)}+6\,(m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m +
1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
+ 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m}\operatorname{sech}\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.391 $\int x^3 \tanh^3(a+bx)\,dx$

Optimal. Leaf size=183

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\tan\ldots}{2\ldots}$$

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 + (3*x*Log[1 + E^(2*(a + b*x))])/b^3 +
 (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4
) + (3*x^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a
 + b*x))])/(2*b^3) + (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*x^2*Tanh
[a + b*x])/(2*b^2) - (x^3*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.325774, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3720, 3718, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\tan\ldots}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tanh[a + b*x]^3,x]
```


```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 + (3*x*Log[1 + E^(2*(a + b*x))])/b^3 +
 (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4
) + (3*x^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a
 + b*x))])/(2*b^3) + (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*x^2*Tanh
[a + b*x])/(2*b^2) - (x^3*Tanh[a + b*x]^2)/(2*b)
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\tanh^3(a+bx)\,dx = -\frac{x^3\tanh^2(a+bx)}{2b}+\frac{3\int x^2\tanh^2(a+bx)\,dx}{2b}+\int x^3\tanh(a+bx)\,dx$$
$$= -\frac{x^4}{4}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}+2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx+\frac{3\int x\tanh(a+bx)\,dx}{b^2}+$$
$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{6\int\frac{e^{2(a+bx)}}{1+e^{2(a+b}}}{b^2}$$
$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\tan}{}$$
$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3(}{}$$
$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\mathrm{Li}_2(-}{2b}$$
$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\mathrm{Li}_2(-}{2b}$$

Mathematica [A] time = 2.8208, size = 230, normalized size = 1.26

$$\frac{1}{4}\left(\frac{e^{2a}\left(-3e^{-2a}\left(e^{2a}+1\right)\left(2b^2x^2\mathrm{PolyLog}\left(2,-e^{-2(a+bx)}\right)+2bx\mathrm{PolyLog}\left(3,-e^{-2(a+bx)}\right)+\mathrm{PolyLog}\left(4,-e^{-2(a+bx)}\right)\right)-6\left(e^{-}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x]^3,x]
```

```
[Out] ((E^(2*a)*((12*b^2*x^2)/E^(2*a) + (2*b^4*x^4)/E^(2*a) + 12*b*(1 + E^(-2*a))
*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*(1 + E^(-2*a))*x^3*Log[1 + E^(-2*(a +
b*x))] - 6*(1 + E^(-2*a))*PolyLog[2, -E^(-2*(a + b*x))] - (3*(1 + E^(2*a))*
(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3, -E^(-2*(a + b*x
))] + PolyLog[4, -E^(-2*(a + b*x))]))/E^(2*a)))/(b^4*(1 + E^(2*a))) + (2*x^
3*Sech[a + b*x]^2)/b - (6*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + x^4*Ta
nh[a])/4
```

Maple [A] time = 0.108, size = 234, normalized size = 1.3

$$-\frac{x^4}{4}+\frac{x^2\left(2\,bx\mathrm{e}^{2\,bx+2\,a}+3\,\mathrm{e}^{2\,bx+2\,a}+3\right)}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}-3\,\frac{x^2}{b^2}-3\,\frac{a^2}{b^4}-\frac{3\,x\mathrm{polylog}\left(3,-\mathrm{e}^{2\,bx+2\,a}\right)}{2\,b^3}+3\,\frac{x\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b^3}-6\,\frac{ax}{b^3}+\frac{3\,\mathrm{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/4*x^4+x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a
))^2-3*x^2/b^2-3/b^4*a^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3*x*ln(1+exp(
2*b*x+2*a))/b^3-6/b^3*a*x+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/4*polylog(4,
-exp(2*b*x+2*a))/b^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*
x+2*a))/b^2-2/b^3*a^3*x-3/2/b^4*a^4+6/b^4*a*ln(exp(b*x+a))+2/b^4*a^3*ln(exp
(b*x+a))
```

Maxima [A] time = 1.25645, size = 319, normalized size = 1.74

$$\frac{b^2x^4e^{(4\,bx+4\,a)}+b^2x^4+12\,x^2+2\left(b^2x^4e^{(2\,a)}+4\,bx^3e^{(2\,a)}+6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}-\frac{b^4x^4+6\,b^2x^2}{2\,b^4}+\frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 + 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + 1/3*(4*b^3*x^3*log(e^(2*
b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(
2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4 + 3/2*(2*b*x*log(e^(2*b
```

```
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

Fricas [C] time = 2.66918, size = 5662, normalized size = 30.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4
*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 + 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 + 2*(b^4*x^4 - 4*
b^3*x^3 - 2*a^4 + 6*b^2*x^2 + 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cos
h(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*
x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*si
nh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*
(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(
b*x + a)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2
 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x
^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 + 1)*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 + (b^2*x^2
 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x
 + a)) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(
b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 + 3*a)*cosh(b*x + a
)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a
^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)
*log(cosh(b*x + a) + sinh(b*x + a) + I) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 +
4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 +
 a^3 + 2*(a^3 + 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)
^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*co
sh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
4*(b^3*x^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a
^3 + 3*b*x + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x +
3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^
2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*
b*x + 3*a)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(
b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3
*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 4*(b^3*x^3 + (b^3*x^3 + a^
3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x
```

```
 + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3
 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(
b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2
 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^
3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-I*cosh(b*x + a) -
 I*sinh(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)
^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b
*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(
4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x +
a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 24*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(
b*x + a) + I*sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)
*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*c
osh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*
cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)
) + 4*((b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 + (b^4*x^4 -
 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4
*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^
4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x
+ a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.392 $\int x^2 \tanh^3(a+bx)\,dx$

Optimal. Leaf size=116

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-x^2\tan$$

```
[Out] x^2/(2*b) - x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + Log[Cosh[a + b*x]]/b
^3 + (x*PolyLog[2, -E^(2*(a + b*x))])/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2
*b^3) - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.201578, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3720, 3475, 30, 3718, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-x^2\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + Log[Cosh[a + b*x]]/b
^3 + (x*PolyLog[2, -E^(2*(a + b*x))])/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2
*b^3) - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tanh^3(a+bx)\,dx = -\frac{x^2\tanh^2(a+bx)}{2b}+\frac{\int x\tanh^2(a+bx)\,dx}{b}+\int x^2\tanh(a+bx)\,dx$$
$$= -\frac{x^3}{3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}+2\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx+\frac{\int\tanh(a+bx)\,dx}{b^2}+\frac{\int x\,dx}{b}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}-\frac{2}{}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2}{}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2}{}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}-\frac{x\,\mathrm{ta}}{}$$

Mathematica [A] time = 2.24243, size = 185, normalized size = 1.59

$$\frac{1}{6}\left(\frac{e^{2a}\left(-3\left(e^{-2a}+1\right)\left(2x\mathrm{PolyLog}\left(2,-e^{-2(a+bx)}\right)+\frac{\mathrm{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{b}\right)+4e^{-2a}b^2x^3+6\left(e^{-2a}+1\right)bx^2\log\left(e^{-2(a+bx)}+1\right)-}{\left(e^{2a}+1\right)b^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] ((E^(2*a)*((12*x)/E^(2*a) + (4*b^2*x^3)/E^(2*a) + 6*b*(1 + E^(-2*a))*x^2*Lo
g[1 + E^(-2*(a + b*x))] - (6*(1 + E^(-2*a))*(2*b*x - Log[1 + E^(2*(a + b*x)
)]))/b - 3*(1 + E^(-2*a))*(2*x*PolyLog[2, -E^(-2*(a + b*x))] + PolyLog[3, -
E^(-2*(a + b*x))]/b)))/(b^2*(1 + E^(2*a))) + (3*x^2*Sech[a + b*x]^2)/b - (6
*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + 2*x^3*Tanh[a])/6
```

Maple [A] time = 0.082, size = 164, normalized size = 1.4

$$-\frac{x^3}{3}+2\,\frac{x\left(bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}+1\right)}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}-2\,\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b^3}-2\,\frac{a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+2\,\frac{a^2x}{b^2}+\frac{4\,a^3}{3\,b^3}+\frac{x^2\ln\left(1+\mathrm{e}^{2\,bx}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/3*x^3+2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
-2/b^3*ln(exp(b*x+a))+1/b^3*ln(1+exp(2*b*x+2*a))-2/b^3*a^2*ln(exp(b*x+a))+2
/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*
a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3
```

Maxima [A] time = 1.28775, size = 247, normalized size = 2.13

$$-\frac{2}{3}x^3+\frac{b^2x^3e^{(4bx+4a)}+b^2x^3+2\left(b^2x^3e^{(2a)}+3bx^2e^{(2a)}+3xe^{(2a)}\right)e^{(2bx)}+6x}{3\left(b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2\right)}-\frac{2x}{b^2}+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{I}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 + 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + log(e
^(2*b*x + 2*a) + 1)/b^3
```

Fricas [C] time = 2.51573, size = 4211, normalized size = 36.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 +
 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*
b*x + 6*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*(b^3*x^3 +
2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*
```

```
x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3
*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3
+ b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a
)) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 +
1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x
 + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + s
inh(b*x + a) + I) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2
 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^
2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(co
sh(b*x + a) + sinh(b*x + a) - I) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(
b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a
)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2
 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*c
osh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b
*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^
2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^
4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 -
 a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cos
h(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*
x + a) - I*sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*p
olylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a) + 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^3*x^3
+ 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 + (b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*b
*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b*x + a)^4 + 4*b^3*
cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2
 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x
+ a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.393 $\int x \tanh^3(a+bx)\,dx$

Optimal. Leaf size=82

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[Out] x/(2*b) - x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.117372, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3720, 3473, 8, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*Tanh[a + b*x]^2)/(2*b)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\tanh^3(a+bx)\,dx &= -\frac{x\tanh^2(a+bx)}{2b}+\frac{\int \tanh^2(a+bx)\,dx}{2b}+\int x\tanh(a+bx)\,dx\\
&= -\frac{x^2}{2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}+2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx+\frac{\int 1\,dx}{2b}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2(}\right.}{2b^2}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] time = 6.12343, size = 232, normalized size = 2.83

$$-\frac{\operatorname{csch}(a)\operatorname{sech}(a)\left(-b^2x^2e^{-\tanh^{-1}(\coth(a))}+\frac{i\coth(a)\left(i\text{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}(\coth(a))+ibx\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\coth(a))\right)-2\left(i\tanh^{-1}(\coth(a))+ibx\right)\cdots\right.}{\cdots}\right)}{2b^2\sqrt{\operatorname{csch}^2(a)\left(\sinh^2(a)-\cosh^2(a)\right)}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Tanh[a + b*x]^3,x]
```

```
[Out] (x*Sech[a + b*x]^2)/(2*b) - (Csch[a]*(-((b^2*x^2)/E^ArcTanh[Coth[a]]) + (I*
Coth[a]*(-(b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*
(I*b*x + I*ArcTanh[Coth[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))
] + Pi*Log[Cosh[b*x]] + (2*I)*ArcTanh[Coth[a]]*Log[I*Sinh[b*x + ArcTanh[Cot
h[a]]]] + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))]))/Sqrt[1 - C
oth[a]^2])*Sech[a])/(2*b^2*Sqrt[Csch[a]^2*(-Cosh[a]^2 + Sinh[a]^2)]) - (Sec
h[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2) + (x^2*Tanh[a])/2
```

Maple [A] time = 0.079, size = 111, normalized size = 1.4

$$-\frac{x^2}{2}+\frac{2\,bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}+1}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}-2\,\frac{ax}{b}-\frac{a^2}{b^2}+\frac{x\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b}+\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{2\,bx+2\,a}\right)}{2\,b^2}+2\,\frac{a\ln\left(\mathrm{e}^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/2*x^2+(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2
/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2+
2/b^2*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.42492, size = 177, normalized size = 2.16

$$-x^2+\frac{b^2x^2e^{(4\,bx+4\,a)}+b^2x^2+2\left(b^2x^2e^{(2\,a)}+2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2
*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))
/b^2
```

Fricas [C] time = 2.40422, size = 2992, normalized size = 36.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x +
 a)) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*
x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^
2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x
 + a) + sinh(b*x + a) + I) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(
b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)
^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x
+ a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((b*x + a)*cosh(b*x +
a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^
4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)
*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x
+ a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b
*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x
+ a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*
x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 +
(b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b
*x + a) + 1) + 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 + (b^2*x^2 - 2*a^2 - 2*
b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cos
```

```
h(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 +
2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a
)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.394 $\int \tanh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0180643, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^3(a+bx)\,dx &= -\frac{\tanh^2(a+bx)}{2b}+\int \tanh(a+bx)\,dx\\ &= \frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0120026, size = 27, normalized size = 1.

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Maple [A] time = 0.017, size = 26, normalized size = 1.

$$\frac{\ln(\cosh(bx+a))}{b}-\frac{(\tanh(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Maxima [B] time = 1.72886, size = 82, normalized size = 3.04

$$x+\frac{a}{b}+\frac{\log\left(e^{(-2bx-2a)}+1\right)}{b}+\frac{2e^{(-2bx-2a)}}{b\left(2e^{(-2bx-2a)}+e^{(-4bx-4a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
- 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 2.16612, size = 930, normalized size = 34.44

$$-\frac{bx\cosh(bx+a)^4+4bx\cosh(bx+a)\sinh(bx+a)^3+bx\sinh(bx+a)^4+2(bx-1)\cosh(bx+a)^2+2\left(3bx\cosh(bx+\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 + (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13556, size = 93, normalized size = 3.44

$$-\frac{bx+a}{b}+\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}-\frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+3}{2\,b\left(e^{(2\,bx+2\,a)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(e^(2*b*x + 2*a) + 1)/b - 1/2*(3*e^(4*b*x + 4*a) + 2*e^(2
*b*x + 2*a) + 3)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.395 $\int \frac{\tanh^3(a+bx)}{x} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\tanh^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]^3/x, x]
```

Rubi [A] time = 0.0279834, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^3/x,x]

[Out] Defer[Int][Tanh[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\tanh^3(a+bx)}{x} dx = \int \frac{\tanh^3(a+bx)}{x} dx$$

Mathematica [A] time = 13.3935, size = 0, normalized size = 0.

$$\int \frac{\tanh^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^3/x,x]

[Out] Integrate[Tanh[a + b*x]^3/x, x]
```

Maple [A] time = 0.218, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^3 (\sinh(bx+a))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)} - 1}{b^2x^2e^{(4\,bx+4\,a)} + 2\,b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} - \int \frac{2\left(b^2x^2+1\right)}{b^2x^3e^{(2\,bx+2\,a)} + b^2x^3} dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate(2*(b^2*x^2 + 1)/(b^2*x^3*e^(2*b*
x + 2*a) + b^2*x^3), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

3.396 $\int \frac{\tanh^3(a+bx)}{x^2} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\tanh^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Tanh[a + b*x]^3/x^2, x]
```

Rubi [A] time = 0.0284037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^3(a+bx)}{x^2} \, dx = \int \frac{\tanh^3(a+bx)}{x^2} \, dx$$

Mathematica [A] time = 7.95668, size = 0, normalized size = 0.

$$\int \frac{\tanh^3(a+bx)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Tanh[a + b*x]^3/x^2, x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{sech}\left(bx+a\right)\right)^{3}\left(\sinh\left(bx+a\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^{2}x^{2}e^{(4\,bx+4\,a)}+b^{2}x^{2}+2\left(b^{2}x^{2}e^{(2\,a)}-bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{b^{2}x^{3}e^{(4\,bx+4\,a)}+2\,b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}}-\int \frac{2\left(b^{2}x^{2}+3\right)}{b^{2}x^{4}e^{(2\,bx+2\,a)}+b^{2}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate(2*(b^2*x^2 + 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{3}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

3.397 $\int x^m \coth(a+bx)\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(x^m \coth(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Coth[a + b*x], x]
```

Rubi [A] time = 0.0157878, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x], x]
```

Rubi steps

$$\int x^m \coth(a+bx)\,dx=\int x^m \coth(a+bx)\,dx$$

Mathematica [A] time = 7.18078, size = 0, normalized size = 0.

$$\int x^m \coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x],x]
```

```
[Out] Integrate[x^m*Coth[a + b*x], x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int x^{m} \cosh \left(b x +a \right) \operatorname{csch}\left(b x +a \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{\left(2\,bx+m\log\left(x\right)+2\,a\right)}}{(m+1)e^{\left(2\,bx+2\,a\right)}-m-1}+\int\frac{\left(\left(2\,bxe^{\left(2\,a\right)}+(m+1)e^{\left(2\,a\right)}\right)e^{\left(2\,bx\right)}-m-1\right)x^{m}}{(m+1)e^{\left(4\,bx+4\,a\right)}-2\,(m+1)e^{\left(2\,bx+2\,a\right)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) - m - 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m}\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a) \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

3.398 $\int x^3 \coth(a+bx)\,dx$

Optimal. Leaf size=87

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^4}{4}$$

```
[Out] -x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, E^(2*(a + b*x
))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4, E^
(2*(a + b*x))])/(4*b^4)
```

Rubi [A] time = 0.152552, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x],x]
```

```
[Out] -x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, E^(2*(a + b*x
))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4, E^
(2*(a + b*x))])/(4*b^4)
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^3 \coth(a+bx)\,dx &= -\frac{x^4}{4}-2\int \frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx\\ &= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{3\int x^2\log\left(1-e^{2(a+bx)}\right)dx}{b}\\ &= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3\int x\text{Li}_2\left(e^{2(a+bx)}\right)dx}{b^2}\\ &= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\int \text{Li}_3\left(e^{2(a+bx)}\right)dx}{2b^3}\\ &= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{Subst}\left(\int \frac{\text{Li}_3(x)}{x}\,dx,x,e^{2(a}\right.}{4b^4}\\ &= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{Li}_4\left(e^{2(a+bx)}\right)}{4b^4}\end{aligned}$$

Mathematica [A] time = 0.0101752, size = 91, normalized size = 1.05

$$\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}+\frac{x^3\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x],x]
```

```
[Out] -x^4/4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2*b*
x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4, E^
(2*a + 2*b*x)])/(4*b^4)
```

Maple [B] time = 0.026, size = 200, normalized size = 2.3

$$-\frac{x^4}{4}-\frac{a^3\ln\left(e^{bx+a}-1\right)}{b^4}+2\frac{a^3\ln\left(e^{bx+a}\right)}{b^4}+\frac{\ln\left(1-e^{bx+a}\right)a^3}{b^4}+3\frac{\text{polylog}\left(2,-e^{bx+a}\right)x^2}{b^2}-6\frac{\text{polylog}\left(3,-e^{bx+a}\right)x}{b^3}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/4*x^4-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*a^3*ln(exp(b*x+a))+1/b^4*ln(1-exp
(b*x+a))*a^3+3/b^2*polylog(2,-exp(b*x+a))*x^2-6/b^3*polylog(3,-exp(b*x+a))*
```

```
x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,exp(b*x+a))*x^2-6/b^3*polylog(3,
exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^3+6/b^4*polylog(4,exp(b*x+a))+6/b^4*po
lylog(4,-exp(b*x+a))-2/b^3*a^3*x-3/2/b^4*a^4
```

Maxima [A] time = 1.30128, size = 176, normalized size = 2.02

$$-\frac{1}{4}x^4 + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3\,b^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) -
6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3
*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^
(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.07796, size = 644, normalized size = 7.4

$$-\frac{b^4x^4 - 4\,b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) - 12\,b^2x^2\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right) - 12\,b^2x^2\mathrm{Li}_2\left(-\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 4*b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*b^2*x
^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*b^2*x^2*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 24*b*x*pol
ylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*b*x*polylog(3, -cosh(b*x + a) -
 sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
 - 24*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*polylog(4, -cosh(b*x +
 a) - sinh(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a) \operatorname{csch}(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a), x)
```

3.399 $\int x^2 \coth(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3}{3}$$

```
[Out] -x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(a + b*x))])
/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rubi [A] time = 0.132667, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x],x]
```


```
[Out] -x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(a + b*x))])
/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```


Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2\coth(a+bx)\,dx &= -\frac{x^3}{3}-2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{2\int x\log\left(1-e^{2(a+bx)}\right)dx}{b}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{\int \text{Li}_2\left(e^{2(a+bx)}\right)dx}{b^2}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0098819, size = 66, normalized size = 1.05

$$\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{x^2\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x],x]
```

```
[Out] -x^3/3 + (x^2*Log[1 - E^(2*a + 2*b*x)])/b + (x*PolyLog[2, E^(2*a + 2*b*x)])
/b^2 - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Maple [B] time = 0.023, size = 166, normalized size = 2.6

$$-\frac{x^3}{3}+\frac{a^2\ln\left(e^{bx+a}-1\right)}{b^3}-2\frac{a^2\ln\left(e^{bx+a}\right)}{b^3}+2\frac{a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{\ln\left(1+e^{bx+a}\right)x^2}{b}+2\frac{x\mathrm{polylog}\left(2,-e^{bx+a}\right)}{b^2}-2\frac{\mathrm{polylog}\left(3,-e\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/3*x^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/
3/b^3*a^3+1/b*ln(1+exp(b*x+a))*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-2*polylog
(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2*x
*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3
```

Maxima [A] time = 1.29407, size = 130, normalized size = 2.06

$$-\frac{1}{3}x^3+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3}+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*po
lylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(
e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Fricas [C] time = 2.10502, size = 489, normalized size = 7.76

$$-\frac{b^3x^3-3\,b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-6\,bx\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-6\,bx\mathrm{Li}_2\left(-\cosh\left(bx+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)*lo
g(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx + a) \operatorname{csch}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a), x)
```

3.400 $\int x \coth(a + bx)\, dx$

Optimal. Leaf size=45

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} + \frac{x \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x^2}{2}$$

```
[Out] -x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^
2)
```

Rubi [A] time = 0.0806948, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} + \frac{x \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x],x]
```

```
[Out] -x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^
2)
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\coth(a+bx)\,dx &= -\frac{x^2}{2}-2\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx\\
&= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\int\log\left(1-e^{2(a+bx)}\right)dx}{b}\\
&= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^2}\\
&= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0061351, size = 47, normalized size = 1.04

$$\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{x\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x],x]
```

```
[Out] -x^2/2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^
2)
```

Maple [B] time = 0.023, size = 122, normalized size = 2.7

$$-\frac{x^2}{2}-2\frac{ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1+e^{bx+a}\right)x}{b}+\frac{\text{polylog}\left(2,-e^{bx+a}\right)}{b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{b}+\frac{\ln\left(1-e^{bx+a}\right)a}{b^2}+\frac{\text{polylog}\left(2,e^{bx+a}\right)}{b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a)
)+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a
))-1/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.29799, size = 78, normalized size = 1.73

$$-\frac{1}{2}x^2+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(
-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [B] time = 2.07054, size = 336, normalized size = 7.47

$$-\frac{b^2x^2-2\,bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-2\,(bx+a)\log\left(-\cosh\right.}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 2*b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*a*log(cosh
(b*x + a) + sinh(b*x + a) - 1) - 2*(b*x + a)*log(-cosh(b*x + a) - sinh(b*x
+ a) + 1) - 2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*dilog(-cosh(b*x + a)
 - sinh(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a), x)
```

3.401 $\int \coth(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] Log[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0058973, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(a+bx)\,dx = \frac{\log(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.0127889, size = 19, normalized size = 1.73

$$\frac{\log(\tanh(a+bx))+\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x],x]
```

```
[Out] (Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

Maple [A] time = 0.007, size = 13, normalized size = 1.2

$$-\frac{\ln(\operatorname{csch}(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/b*ln(csch(b*x+a))
```

Maxima [B] time = 1.09975, size = 31, normalized size = 2.82

$$\frac{\log\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) - e^(-b*x - a))/b
```

Fricas [B] time = 2.07281, size = 88, normalized size = 8.

$$-\frac{bx-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh (a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x), x)
```

Giac [B] time = 1.21458, size = 38, normalized size = 3.45

$$-\frac{bx+a}{b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.402 $\int \frac{\coth(a+bx)}{x} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]/x, x]
```

Rubi [A] time = 0.0159891, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)}{x} dx = \int \frac{\coth(a+bx)}{x} dx$$

Mathematica [A] time = 0.434758, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]/x,x]
```

```
[Out] Integrate[Coth[a + b*x]/x, x]
```

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\,\mathrm{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{xe^{(bx+a)}+x}\,dx+\int \frac{1}{xe^{(bx+a)}-x}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x)
+ log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(bx+a)\,\mathrm{csch}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\,\mathrm{csch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x, x)
```

3.403 $\int \frac{\coth(a+bx)}{x^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]/x^2, x]
```

Rubi [A] time = 0.0170219, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)}{x^2} dx$$

Mathematica [A] time = 0.832757, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]/x^2,x]
```

```
[Out] Integrate[Coth[a + b*x]/x^2, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\cosh (bx + a) \operatorname{csch} (bx + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{1}{x^2 e^{(bx+a)} + x^2} \, dx + \int \frac{1}{x^2 e^{(bx+a)} - x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x +
a) - x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh (bx + a) \operatorname{csch} (bx + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (a + bx) \operatorname{csch} (a + bx)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

3.404 $\int x^m \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=71

$$\text{Unintegrable}\left(x^m\text{csch}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Unintegrable[x^m*Csch[a + b*x], x]
```

Rubi [A] time = 0.0907661, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Csch[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\coth(a+bx)\,dx &= \int x^m\text{csch}(a+bx)\,dx+\int x^m\sinh(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx+\int x^m\text{csch}(a+bx)\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}+\int x^m\text{csch}(a+bx)\,dx\end{aligned}$$

Mathematica [A] time = 17.8769, size = 0, normalized size = 0.

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x], x]
```

Maple [A] time = 0.112, size = 0, normalized size = 0.

$$\int x^{m} \left(\cosh \left(bx+a\right)\right)^{2} \operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \cosh \left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{m} \cosh \left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh (bx+a)^2 \operatorname{csch}(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.405 $\int x^3 \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=165

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\text{PolyLog}\left(4,-\right.}{b^4}$$

```
[Out] (-2*x^3*ArcTanh[E^(a + b*x)])/b + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a + b
*x])/b - (3*x^2*PolyLog[2, -E^(a + b*x)])/b^2 + (3*x^2*PolyLog[2, E^(a + b*
x)])/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - (6*x*PolyLog[3, E^(a + b*x)
])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])/b^4
 - (6*Sinh[a + b*x])/b^4 - (3*x^2*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.177773, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5450, 3296, 2637, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\text{PolyLog}\left(4,-\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(a + b*x)])/b + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a + b
*x])/b - (3*x^2*PolyLog[2, -E^(a + b*x)])/b^2 + (3*x^2*PolyLog[2, E^(a + b*
x)])/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - (6*x*PolyLog[3, E^(a + b*x)
])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])/b^4
 - (6*Sinh[a + b*x])/b^4 - (3*x^2*Sinh[a + b*x])/b^2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx = \int x^3\mathbf{csch}(a+bx)\,dx + \int x^3\sinh(a+bx)\,dx$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\cosh(a+bx)}{b} - \frac{3\int x^2\cosh(a+bx)\,dx}{b} - \frac{3\int x^2\log\left(1-\right.}{b}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\mathbf{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3x^2\mathbf{Li}_2\left(e^{a+bx}\right)}{b^2} - \frac{3x^2}{}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\mathbf{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3x}{}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\mathbf{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3x}{}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\mathbf{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3x}{}$$

Mathematica [A] time = 3.90936, size = 202, normalized size = 1.22

$$\frac{-3b^2x^2\text{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx))+3b^2x^2\text{PolyLog}(2,\sinh(a+bx)+\cosh(a+bx))+6bx\text{PolyLog}(3,-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*b^3*x^3*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] + 6*b*x*Cosh[a + b*x] +
b^3*x^3*Cosh[a + b*x] - 3*b^2*x^2*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]
] + 3*b^2*x^2*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] + 6*b*x*PolyLog[3,
-Cosh[a + b*x] - Sinh[a + b*x]] - 6*b*x*PolyLog[3, Cosh[a + b*x] + Sinh[a +
 b*x]] - 6*PolyLog[4, -Cosh[a + b*x] - Sinh[a + b*x]] + 6*PolyLog[4, Cosh[a
 + b*x] + Sinh[a + b*x]] - 6*Sinh[a + b*x] - 3*b^2*x^2*Sinh[a + b*x])/b^4
```

Maple [A] time = 0.092, size = 246, normalized size = 1.5

$$\frac{\left(x^3b^3-3x^2b^2+6bx-6\right)e^{bx+a}}{2b^4} + \frac{\left(x^3b^3+3x^2b^2+6bx+6\right)e^{-bx-a}}{2b^4} + 6\frac{\text{polylog}\left(4,e^{bx+a}\right)}{b^4} - \frac{\ln\left(1+e^{bx+a}\right)x^3}{b} - \frac{a^3\ln\left(1\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)+1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)+6/b^4*polylog(4,exp(b*x+a))-1/b*ln(1+exp(b*x+a))*x^3-1/
b^4*ln(1+exp(b*x+a))*a^3+1/b^4*ln(1-exp(b*x+a))*a^3+2/b^4*a^3*arctanh(exp(b
*x+a))-6/b^4*polylog(4,-exp(b*x+a))-6/b^3*polylog(3,exp(b*x+a))*x-3/b^2*pol
ylog(2,-exp(b*x+a))*x^2+6/b^3*polylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))
*x^3+3/b^2*polylog(2,exp(b*x+a))*x^2
```

Maxima [A] time = 1.4724, size = 278, normalized size = 1.68

$$\frac{\left(\left(b^3x^3e^{(2a)} - 3\,b^2x^2e^{(2a)} + 6\,bxe^{(2a)} - 6\,e^{(2a)}\right)e^{(bx)} + \left(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^4} - \frac{b^3x^3\log\left(e^{(bx+a)} + 1\right) + 3\,b^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - (b^3*x^3*log
(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*
x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1)
+ 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(
4, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.18018, size = 1409, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 3*b^2*x^2 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^
2 + 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) + (b^3*
x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(b^2*x^2*cosh(b*x
+ a) + b^2*x^2*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b^2
*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 2*(b^3*x^3*cosh(b*x + a) + b^3*x^3*sinh(b*x + a))*log(cosh(b*x + a
```

```
) + sinh(b*x + a) + 1) - 2*(a^3*cosh(b*x + a) + a^3*sinh(b*x + a))*log(cosh
(b*x + a) + sinh(b*x + a) - 1) + 2*((b^3*x^3 + a^3)*cosh(b*x + a) + (b^3*x^
3 + a^3)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 12*(cosh(
b*x + a) + sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 12*(c
osh(b*x + a) + sinh(b*x + a))*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) -
12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(
b*x + a)) + 12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, -cosh(b*x
 + a) - sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.406 $\int x^2 \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=115

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}+$$

```
[Out] (-2*x^2*ArcTanh[E^(a + b*x)])/b + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x
])/b - (2*x*PolyLog[2, -E^(a + b*x)])/b^2 + (2*x*PolyLog[2, E^(a + b*x)])/b
^2 + (2*PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 - (
2*x*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.123183, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5450, 3296, 2638, 4182, 2531, 2282, 6589}

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(a + b*x)])/b + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x
])/b - (2*x*PolyLog[2, -E^(a + b*x)])/b^2 + (2*x*PolyLog[2, E^(a + b*x)])/b
^2 + (2*PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 - (
2*x*Sinh[a + b*x])/b^2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx = \int x^2\mathbf{csch}(a+bx)\,dx + \int x^2\sinh(a+bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\cosh(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\,dx}{b} - \frac{2\int x\log\left(1-\right.}{b}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2x\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2} - \frac{2x\,\mathrm{si}}{}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2x}{}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2x}{}$$

Mathematica [A] time = 3.82008, size = 138, normalized size = 1.2

$$\frac{-2bx\mathrm{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx)) + 2bx\mathrm{PolyLog}(2,\sinh(a+bx)+\cosh(a+bx)) + 2\mathrm{PolyLog}(3,-\sinh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*b^2*x^2*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] + 2*Cosh[a + b*x] + b^2*
x^2*Cosh[a + b*x] - 2*b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]] + 2*b*
x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] + 2*PolyLog[3, -Cosh[a + b*x] -
 Sinh[a + b*x]] - 2*PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]] - 2*b*x*Sinh[
a + b*x])/b^3
```

Maple [A] time = 0.059, size = 196, normalized size = 1.7

$$\frac{\left(x^2b^2-2\,bx+2\right)\mathrm{e}^{bx+a}}{2\,b^3} + \frac{\left(x^2b^2+2\,bx+2\right)\mathrm{e}^{-bx-a}}{2\,b^3} - 2\,\frac{a^2\mathrm{Artanh}\left(\mathrm{e}^{bx+a}\right)}{b^3} - \frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{b} + \frac{a^2\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^3} - 2\,\frac{x\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-
2/b^3*a^2*arctanh(exp(b*x+a))-1/b*ln(1+exp(b*x+a))*x^2+1/b^3*ln(1+exp(b*x+a
```

```
))*a^2-2*x*polylog(2,-exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1
-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2*x*polylog(2,exp(b*x+a))/b^2-2
*polylog(3,exp(b*x+a))/b^3
```

Maxima [A] time = 1.34865, size = 205, normalized size = 1.78

$$\frac{\left(\left(b^2x^2e^{(2a)} - 2\,bxe^{(2a)} + 2\,e^{(2a)}\right)e^{(bx)} + \left(b^2x^2 + 2\,bx + 2\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^3} - \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(
-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a)
 + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Fricas [C] time = 2.20668, size = 1084, normalized size = 9.43

$$\frac{b^2x^2 + \left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx + a\right)^2 + 2\left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right) + \left(b^2x^2 - 2\,bx + 2\right)\sinh\left(bx + a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 2*b*x +
 2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 + 2
*b*x + 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh
(b*x + a)) - 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) - 2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + 2*(a^2*cosh(b*x + a) + a^2*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a
) + (b^2*x^2 - a^2)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
- 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a
)) + 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x
 + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.407 $\int x \cosh(a+bx) \coth(a+bx) \, dx$

Optimal. Leaf size=66

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\sinh(a+bx)}{b^2}+\frac{x\cosh(a+bx)}{b}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (-2*x*ArcTanh[E^(a + b*x)])/b + (x*Cosh[a + b*x])/b - PolyLog[2, -E^(a + b*
x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 - Sinh[a + b*x]/b^2
```

Rubi [A] time = 0.0611381, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5450, 3296, 2637, 4182, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\sinh(a+bx)}{b^2}+\frac{x\cosh(a+bx)}{b}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(a + b*x)])/b + (x*Cosh[a + b*x])/b - PolyLog[2, -E^(a + b*
x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 - Sinh[a + b*x]/b^2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \int x\text{csch}(a+bx)\,dx + \int x\sinh(a+bx)\,dx$$
$$= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\int\cosh(a+bx)\,dx}{b} - \frac{\int\log\left(1-e^{a+bx}\right)\,dx}{b}$$
$$= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2} - \frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+}\right.}{b^2}$$
$$= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{\text{Li}_2\left(e^{a+bx}\right)}{b^2} - \frac{\sinh(a+bx)}{b^2}$$

Mathematica [A] time = 0.132542, size = 131, normalized size = 1.98

$$-\frac{-\text{PolyLog}\left(2,-e^{-a-bx}\right)+\text{PolyLog}\left(2,e^{-a-bx}\right)-a\log\left(1-e^{-a-bx}\right)-bx\log\left(1-e^{-a-bx}\right)+a\log\left(e^{-a-bx}+1\right)+bx\log\left(e\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -((-(b*x*Cosh[a + b*x]) - a*Log[1 - E^(-a - b*x)] - b*x*Log[1 - E^(-a - b*x
)] + a*Log[1 + E^(-a - b*x)] + b*x*Log[1 + E^(-a - b*x)] + a*Log[Tanh[(a +
```

```
b*x)/2]] - PolyLog[2, -E^(-a - b*x)] + PolyLog[2, E^(-a - b*x)] + Sinh[a +
b*x])/b^2)
```

Maple [B] time = 0.061, size = 139, normalized size = 2.1

$$\frac{(bx-1)\,\mathrm{e}^{bx+a}}{2\,b^2}+\frac{(bx+1)\,\mathrm{e}^{-bx-a}}{2\,b^2}-\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b}-\frac{a\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^2}-\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)-1/b*ln(1+exp(b*x+a))
*x-1/b^2*ln(1+exp(b*x+a))*a-1/b^2*polylog(2,-exp(b*x+a))+1/b*ln(1-exp(b*x+a
))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))+2/b^2*a*arctanh(e
xp(b*x+a))
```

Maxima [A] time = 1.29479, size = 127, normalized size = 1.92

$$\frac{\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(bx)}+(bx+1)e^{(-bx)}\right)e^{(-a)}}{2\,b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) + (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - (b*
x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) +
 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [B] time = 2.16253, size = 753, normalized size = 11.41

$$\frac{(bx-1)\cosh(bx+a)^2+2\,(bx-1)\cosh(bx+a)\sinh(bx+a)+(bx-1)\sinh(bx+a)^2+bx+2\,(\cosh(bx+a)+\sinh(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 + b*x + 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(c
osh(b*x + a) + sinh(b*x + a)) - 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(-co
sh(b*x + a) - sinh(b*x + a)) - 2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) - 2*(a*cosh(b*x + a) + a*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b*x + a)*cosh(b*x + a) + (b*
x + a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cos
h(b*x + a) + b^2*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^2 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.408 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rubi [A] time = 0.0173175, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh(a+bx)\coth(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}$$

Mathematica [A] time = 0.0199289, size = 26, normalized size = 1.13

$$\frac{\cosh(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b + Log[Tanh[(a + b*x)/2]]/b
```

Maple [A] time = 0.013, size = 21, normalized size = 0.9

$$\frac{\cosh(bx+a) - 2\,\text{Artanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.04865, size = 80, normalized size = 3.48

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

Fricas [B] time = 2.12089, size = 355, normalized size = 15.43

$$\frac{\cosh(bx+a)^2 - 2(\cosh(bx+a) + \sinh(bx+a))\log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2(\cosh(bx+a) + \sinh(bx+}{2(b\cosh(bx+a) + b\sinh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x), x)
```

Giac [B] time = 1.14878, size = 73, normalized size = 3.17

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(\left|e^{(bx+a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(b*x + a) + 1)/b + log(abs(e
^(b*x + a) - 1))/b
```

3.409 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\text{csch}(a+bx)}{x},x\right)+\sinh(a)\text{Chi}(bx)+\cosh(a)\text{Shi}(bx)$$

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] + Unintegrable[Csch[a
 + b*x]/x, x]
```

Rubi [A] time = 0.0801649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] + Defer[Int][Csch[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx &= \int \frac{\text{csch}(a+bx)}{x}\,dx+\int \frac{\sinh(a+bx)}{x}\,dx\\ &= \cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\sinh(a)\int \frac{\cosh(bx)}{x}\,dx+\int \frac{\text{csch}(a+bx)}{x}\,dx\\ &= \text{Chi}(bx)\sinh(a)+\cosh(a)\text{Shi}(bx)+\int \frac{\text{csch}(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 25.9794, size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x, x]
```

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^2 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

3.410 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal. Leaf size=40

$$\text{Unintegrable}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)+b\cosh(a)\text{Chi}(bx)+b\sinh(a)\text{Shi}(bx)-\frac{\sinh(a+bx)}{x}$$

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 + Unintegrable[Csch[a + b*x]/x^2, x]
```

Rubi [A] time = 0.0987742, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 + Defer[Int][Csch[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx &= \int \frac{\text{csch}(a+bx)}{x^2}\,dx+\int \frac{\sinh(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x}+b\int \frac{\cosh(a+bx)}{x}\,dx+\int \frac{\text{csch}(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x}+(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx+\int \frac{\text{csc}}{}\\ &= b\cosh(a)\text{Chi}(bx)-\frac{\sinh(a+bx)}{x}+b\sinh(a)\text{Shi}(bx)+\int \frac{\text{csch}(a+bx)}{x^2}\,dx\end{aligned}$$

Mathematica [A] time = 36.9795, size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2, x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^2 \operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

3.411 $\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=82

$$\text{Unintegrable}\left(x^m\coth(a+bx),x\right)+\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2}{b}$$

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + Unintegrable[x^m*Coth[a + b*
x], x]
```

Rubi [A] time = 0.131473, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]
, x]
```

Rubi steps

$$\begin{aligned}\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx &= \int x^m\coth(a+bx)\,dx+\int x^m\cosh(a+bx)\sinh(a+bx)\,dx\\ &= \int x^m\coth(a+bx)\,dx+\int \frac{1}{2}x^m\sinh(2a+2bx)\,dx\\ &= \frac{1}{2}\int x^m\sinh(2a+2bx)\,dx+\int x^m\coth(a+bx)\,dx\\ &= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx-\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx+\int x^m\coth(a+bx)\,dx\\ &= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}+\int x^m\,\mathrm{c}\end{aligned}$$

Mathematica [A] time = 22.3679, size = 0, normalized size = 0.

$$\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x], x]
```

Maple [A] time = 0.074, size = 0, normalized size = 0.

$$\int x^m (\cosh(bx+a))^3 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx + a)^3 \operatorname{csch}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.412 $\int x^3 \cosh^2(a+bx) \coth(a+bx) \, dx$

Optimal. Leaf size=180

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\sin}{}$$

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x
^2*PolyLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])
/(2*b^3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Sinh[
a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh
[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.236801, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5450, 5372, 3311, 30, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```


```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x
^2*PolyLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])
/(2*b^3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Sinh[
a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh
[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```


Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx = \int x^3\coth(a+bx)\,dx + \int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{x^4}{4} + \frac{x^3\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx - \frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3} +$$
$$= \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3} - 3\,\text{co}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3} - 3\,\text{co}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3} + 3\text{L}$$

Mathematica [A] time = 2.52679, size = 236, normalized size = 1.31

$$\frac{\sinh(a)(\sinh(a)+\cosh(a))\left(-48b^2x^2\text{PolyLog}\left(2,-e^{-a-bx}\right)-48b^2x^2\text{PolyLog}\left(2,e^{-a-bx}\right)-96bx\text{PolyLog}\left(3,-e^{-a-bx}\right)-9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 + 6*b*x*Cosh[2*(a + b*x)] + 4*b^3*x
^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 - E^(-a - b*x)] + 16*b^3*x^3*Log[1
+ E^(-a - b*x)] - 48*b^2*x^2*PolyLog[2, -E^(-a - b*x)] - 48*b^2*x^2*PolyLog
[2, E^(-a - b*x)] - 96*b*x*PolyLog[3, -E^(-a - b*x)] - 96*b*x*PolyLog[3, E^
(-a - b*x)] - 96*PolyLog[4, -E^(-a - b*x)] - 96*PolyLog[4, E^(-a - b*x)] -
3*Sinh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)]))/(8*b^4*(-1 + E^(2*a)))
```

Maple [A] time = 0.087, size = 272, normalized size = 1.5

$$-\frac{x^4}{4} + \frac{\left(4x^3b^3-6x^2b^2+6bx-3\right)e^{2bx+2a}}{32b^4} + \frac{\left(4x^3b^3+6x^2b^2+6bx+3\right)e^{-2bx-2a}}{32b^4} - 2\frac{a^3x}{b^3} - \frac{a^3\ln\left(e^{bx+a}-1\right)}{b^4} + 2\frac{a^3\ln(}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*
x^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)-2/b^3*a^3*x-1/b^4*a^3*ln(exp(b*x
+a)-1)+2/b^4*a^3*ln(exp(b*x+a))+6/b^4*polylog(4,-exp(b*x+a))+6/b^4*polylog(
4,exp(b*x+a))-3/2/b^4*a^4+1/b^4*ln(1-exp(b*x+a))*a^3+3/b^2*polylog(2,exp(b*
x+a))*x^2-6/b^3*polylog(3,exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^3+3/b^2*poly
log(2,-exp(b*x+a))*x^2-6/b^3*polylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*
x^3
```

Maxima [A] time = 1.45046, size = 304, normalized size = 1.69

$$-\frac{1}{2}x^4 + \frac{\left(8\,b^4x^4e^{(2\,a)} + \left(4\,b^3x^3e^{(4\,a)} - 6\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(2\,bx)} + \left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{32\,b^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + 1/32*(8*b^4*x^4*e^(2*a) + (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
 + 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x +
3)*e^(-2*b*x))*e^(-2*a)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dil
og(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x +
a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) -
6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.1776, size = 2325, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 - 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 - 2*(4*b^4*x^4 - 8*a^4 - 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*(b^2*x^2*cosh(b*x + a)
```

```
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(cosh(b*x + a) + sinh(b*x + a)) + 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*c
osh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-cosh(b*x + a)
- sinh(b*x + a)) + 32*(b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x + a)*si
nh(b*x + a) + b^3*x^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) +
1) - 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh
(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 32*((b^3*x^3 + a^3)*c
osh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 +
 a^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 192*(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, co
sh(b*x + a) + sinh(b*x + a)) + 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 19
2*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x +
 a)^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 192*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 4*((4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b
*x + a)^3 - 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^4*cosh
(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.413 $\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=126

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+$$

```
[Out] x^2/(4*b) - x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(
a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2*b
)
```

Rubi [A] time = 0.196835, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5450, 5372, 3310, 30, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```


```
[Out] x^2/(4*b) - x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(
a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2*b
)
```


Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^2 \coth(a+bx)\,dx + \int x^2 \cosh(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{x^3}{3} + \frac{x^2\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx - \frac{\int x\sinh^2(a+bx)\,dx}{b}$$
$$= -\frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2}{}$$
$$= \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}$$
$$= \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}$$
$$= \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{Li}_3\left(e^{2(a+bx)}\right)}{2b^3} - \frac{x\cosh(a+bx}{2}$$

Mathematica [A] time = 2.51488, size = 178, normalized size = 1.41

$$\frac{\sinh(a)(\sinh(a)+\cosh(a))\left(-48bx\text{PolyLog}\left(2,-e^{-a-bx}\right)-48bx\text{PolyLog}\left(2,e^{-a-bx}\right)-48\text{PolyLog}\left(3,-e^{-a-bx}\right)-48\text{Pol}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2*C
osh[2*(a + b*x)] + 24*b^2*x^2*Log[1 - E^(-a - b*x)] + 24*b^2*x^2*Log[1 + E^
(-a - b*x)] - 48*b*x*PolyLog[2, -E^(-a - b*x)] - 48*b*x*PolyLog[2, E^(-a -
b*x)] - 48*PolyLog[3, -E^(-a - b*x)] - 48*PolyLog[3, E^(-a - b*x)] - 6*b*x*
Sinh[2*(a + b*x)]))/(12*b^3*(-1 + E^(2*a)))
```

Maple [A] time = 0.072, size = 222, normalized size = 1.8

$$-\frac{x^3}{3} + \frac{\left(2x^2b^2-2bx+1\right)e^{2bx+2a}}{16b^3} + \frac{\left(2x^2b^2+2bx+1\right)e^{-2bx-2a}}{16b^3} + \frac{a^2\ln\left(e^{bx+a}-1\right)}{b^3} - 2\frac{a^2\ln\left(e^{bx+a}\right)}{b^3} + 2\frac{a^2x}{b^2} + \frac{4a^3}{3b^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+
1)/b^3*exp(-2*b*x-2*a)+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+
2/b^2*a^2*x+4/3/b^3*a^3+1/b*ln(1+exp(b*x+a))*x^2+2*x*polylog(2,-exp(b*x+a))
/b^2-2*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b
*x+a))*a^2+2*x*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3
```

Maxima [A] time = 1.41126, size = 231, normalized size = 1.83

$$-\frac{2}{3}x^3 + \frac{\left(16\,b^3x^3e^{(2\,a)} + 3\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} + 3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} + \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/48*(16*b^3*x^3*e^(2*a) + 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a)
+ e^(4*a))*e^(2*b*x) + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 +
 (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -
e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a
)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Fricas [C] time = 2.17355, size = 1858, normalized size = 14.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
 + 6*b^2*x^2 - 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 - 2*(8*b^3*x^3 + 16*a^3
 - 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 96*(b*x*cosh(b*x + a)^2 + 2*b*x
```

```
*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-cosh(b*x + a) -
sinh(b*x + a)) + 48*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh
(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1)
 + 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 48*((b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a
^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 96*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, cosh(
b*x + a) + sinh(b*x + a)) - 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*(3*(
2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 - 8*(b^3*x^3 + 2*a^3)*cosh(b*x + a))
*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a
) + b^3*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.414 $\int x\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}-\frac{x^2}{2}$$

```
[Out] x/(4*b) - x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x
))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(
2*b)
```

Rubi [A] time = 0.120577, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5450, 5372, 2635, 8, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] x/(4*b) - x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x
))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(
2*b)
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\coth(a+bx)\,dx + \int x\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{x^2}{2} + \frac{x\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx - \frac{\int \sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} + \frac{\int 1\,dx}{4b}$$
$$= \frac{x}{4b} - \frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} - \frac{\text{Su}}{}$$
$$= \frac{x}{4b} - \frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sin}{}$$

Mathematica [A] time = 0.233824, size = 82, normalized size = 0.93

$$-\frac{4\left(\text{PolyLog}\left(2,e^{-2(a+bx)}\right)-(a+bx)^2\right)-8(a+bx)\log\left(1-e^{-2(a+bx)}\right)+\sinh(2(a+bx))-2bx\cosh(2(a+bx))+8a\log(\text{s}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] -(-2*b*x*Cosh[2*(a + b*x)] - 8*(a + b*x)*Log[1 - E^(-2*(a + b*x))] + 8*a*Lo
g[Sinh[a + b*x]] + 4*(-(a + b*x)^2 + PolyLog[2, E^(-2*(a + b*x))]) + Sinh[2
*(a + b*x)])/(8*b^2)
```

Maple [B] time = 0.07, size = 162, normalized size = 1.8

$$-\frac{x^2}{2} + \frac{(2bx-1)\,e^{2bx+2a}}{16b^2} + \frac{(2bx+1)\,e^{-2bx-2a}}{16b^2} - 2\frac{ax}{b} - \frac{a^2}{b^2} + \frac{\ln\left(1+e^{bx+a}\right)x}{b} + \frac{\text{polylog}\left(2,-e^{bx+a}\right)}{b^2} + \frac{\ln\left(1-e^{bx+a}\right)x}{b} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*
a)-2/b*a*x-a^2/b^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))+1/b*
ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))-1/b
^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.36419, size = 153, normalized size = 1.74

$$-x^2 + \frac{\left(8\,b^2x^2e^{(2\,a)} + \left(2\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(2\,bx)} + (2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -x^2 + 1/16*(8*b^2*x^2*e^(2*a) + (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) + (2*b
*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*
x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [B] time = 2.16096, size = 1355, normalized size = 15.4

$$\frac{(2\,bx-1)\cosh\left(bx+a\right)^4 + 4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + (2\,bx-1)\sinh\left(bx+a\right)^4 - 8\left(b^2x^2-2\,a^2\right)\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 - 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 -
2*(4*b^2*x^2 - 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b
*x + 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*dilog(cosh(b*x + a) + sinh(b*x + a)) + 16*(cosh(b*x + a)^2 + 2*cosh(b*x +
a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
 16*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x
 + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 16*(a*cosh(b*x + a)^2 + 2
*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + sin
h(b*x + a) - 1) + 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)
*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x +
 a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 - 4*(b^2*x^2 - 2*a^2)*cosh(b*x +
a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x
+ a) + b^2*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)^3\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.415 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0232833, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0199793, size = 25, normalized size = 0.93

$$\frac{\sinh^2(a+bx)+2\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.015, size = 26, normalized size = 1.

$$\frac{(\cosh(bx+a))^2}{2\,b}+\frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Maxima [B] time = 1.03546, size = 95, normalized size = 3.52

$$\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
- a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 2.10463, size = 552, normalized size = 20.44

$$-\frac{8\,bx\cosh\left(bx+a\right)^{2}-\cosh\left(bx+a\right)^{4}-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-\sinh\left(bx+a\right)^{4}+2\left(4\,bx-3\,\cosh\left(bx+a\right)^{2}\right)\sin}{8\left(b\cosh\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15075, size = 93, normalized size = 3.44

$$\frac{\left(4\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-2\,bx-2\,a\right)}}{8\,b}-\frac{bx+a}{b}+\frac{e^{\left(2\,bx+2\,a\right)}}{8\,b}+\frac{\log\left(\left|e^{\left(2\,bx+2\,a\right)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a)/b - (b*x + a)/b + 1/8*e^(2*b*x
 + 2*a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.416 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x},x\right)+\frac{1}{2}\sinh(2a)\text{Chi}(2bx)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 + Uni
ntegrable[Coth[a + b*x]/x, x]
```

Rubi [A] time = 0.104078, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 + Def
er[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx &= \int \frac{\coth(a+bx)}{x}\,dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx\\ &= \int \frac{\coth(a+bx)}{x}\,dx+\int \frac{\sinh(2a+2bx)}{2x}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x}\,dx\\ &= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x}\,dx\\ &= \frac{1}{2}\text{Chi}(2bx)\sinh(2a)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)+\int \frac{\coth(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 10.9073, size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x, x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(\cosh (bx+a))^3 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} - \frac{1}{4}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} - \int \frac{1}{xe^{(bx+a)}+x}\,dx + \int \frac{1}{xe^{(bx+a)}-x}\,dx + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) - integrate(1/(x*e^(b*x + a
) + x), x) + integrate(1/(x*e^(b*x + a) - x), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

3.417 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal. Leaf size=51

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x^2},x\right)+b\cosh(2a)\text{Chi}(2bx)+b\sinh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] + Unintegrable[Coth[a + b*x]/x^2, x]
```

Rubi [A] time = 0.1278, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] + Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx &= \int \frac{\coth(a+bx)}{x^2}\,dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx\\
&= \int \frac{\coth(a+bx)}{x^2}\,dx+\int \frac{\sinh(2a+2bx)}{2x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx+\int \frac{\coth(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+b\int \frac{\cosh(2a+2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+(b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx+\\
&= b\cosh(2a)\text{Chi}(2bx)-\frac{\sinh(2a+2bx)}{2x}+b\sinh(2a)\text{Shi}(2bx)+\int \frac{\coth(a+bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [A] time = 16.4086, size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2, x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^3 \operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right)+\frac{1}{2}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right)-\frac{1}{x}-\int \frac{1}{x^2e^{(bx+a)}+x^2}\,dx+\int \frac{1}{x^2e^{(bx+a)}-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) - 1/x - i
ntegrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x + a) - x^2
), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

3.418 $\int x \cosh^2(x) \coth^2(x)\, dx$

Optimal. Leaf size=33

$$\frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\sinh(x)\cosh(x)$$

```
[Out] (3*x^2)/4 - Cosh[x]^2/4 - x*Coth[x] + Log[Sinh[x]] + (x*Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.0543393, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5450, 3310, 30, 3720, 3475}

$$\frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/4 - Cosh[x]^2/4 - x*Coth[x] + Log[Sinh[x]] + (x*Cosh[x]*Sinh[x])/2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh^2(x)\coth^2(x)\,dx &= \int x\cosh^2(x)\,dx+\int x\coth^2(x)\,dx\\ &= -\frac{1}{4}\cosh^2(x)-x\coth(x)+\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\int x\,dx}{2}+\int x\,dx+\int \coth(x)\,dx\\ &= \frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\log(\sinh(x))+\frac{1}{2}x\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0288381, size = 33, normalized size = 1.

$$\frac{3x^2}{4}+\frac{1}{4}x\sinh(2x)-\frac{1}{8}\cosh(2x)-x\coth(x)+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/4 - Cosh[2*x]/8 - x*Coth[x] + Log[Sinh[x]] + (x*Sinh[2*x])/4
```

Maple [A] time = 0.04, size = 48, normalized size = 1.5

$$\frac{3\,x^2}{4}+\left(-\frac{1}{16}+\frac{x}{8}\right)e^{2\,x}+\left(-\frac{1}{16}-\frac{x}{8}\right)e^{-2\,x}-2\,x-2\,\frac{x}{e^{2\,x}-1}+\ln\left(e^{2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 3/4*x^2+(-1/16+1/8*x)*exp(2*x)+(-1/16-1/8*x)*exp(-2*x)-2*x-2*x/(exp(2*x)-1)
+ln(exp(2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.15228, size = 1022, normalized size = 30.97

$$\frac{(2x-1)\cosh(x)^6 + 6(2x-1)\cosh(x)\sinh(x)^5 + (2x-1)\sinh(x)^6 + \left(12x^2-34x+1\right)\cosh(x)^4 + \left(15(2x-1)\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x - 1)*cosh(x)^6 + 6*(2*x - 1)*cosh(x)*sinh(x)^5 + (2*x - 1)*sinh(
x)^6 + (12*x^2 - 34*x + 1)*cosh(x)^4 + (15*(2*x - 1)*cosh(x)^2 + 12*x^2 - 3
4*x + 1)*sinh(x)^4 + 4*(5*(2*x - 1)*cosh(x)^3 + (12*x^2 - 34*x + 1)*cosh(x)
)*sinh(x)^3 - (12*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x - 1)*cosh(x)^4 + 6*(1
2*x^2 - 34*x + 1)*cosh(x)^2 - 12*x^2 - 2*x - 1)*sinh(x)^2 + 16*(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2
+ 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2
*(3*(2*x - 1)*cosh(x)^5 + 2*(12*x^2 - 34*x + 1)*cosh(x)^3 - (12*x^2 + 2*x +
 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh^2(x)\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)**2*coth(x)**2,x)
```

```
[Out] Integral(x*cosh(x)**2*coth(x)**2, x)
```

Giac [B] time = 1.19143, size = 136, normalized size = 4.12

$$\frac{12\,x^2e^{(4\,x)}-12\,x^2e^{(2\,x)}+2\,xe^{(6\,x)}-34\,xe^{(4\,x)}-2\,xe^{(2\,x)}+16\,e^{(4\,x)}\log\left(e^{(2\,x)}-1\right)-16\,e^{(2\,x)}\log\left(e^{(2\,x)}-1\right)+2\,x-e^{(6\,x)}+e^{(4\,x)}}{16\left(e^{(4\,x)}-e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*(12*x^2*e^(4*x) - 12*x^2*e^(2*x) + 2*x*e^(6*x) - 34*x*e^(4*x) - 2*x*e^
(2*x) + 16*e^(4*x)*log(e^(2*x) - 1) - 16*e^(2*x)*log(e^(2*x) - 1) + 2*x - e
^(6*x) + e^(4*x) - e^(2*x) + 1)/(e^(4*x) - e^(2*x))
```

3.419 $\int x^2 \cosh^2(x) \coth^2(x)\,dx$

Optimal. Leaf size=73

$$\text{PolyLog}\left(2,e^{2x}\right)+\frac{x^3}{2}-x^2-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)+\frac{x}{4}+2x\log\left(1-e^{2x}\right)-\frac{1}{2}x\cosh^2(x)+\frac{1}{4}\sinh(x)\cosh$$

```
[Out] x/4 - x^2 + x^3/2 - (x*Cosh[x]^2)/2 - x^2*Coth[x] + 2*x*Log[1 - E^(2*x)] +
PolyLog[2, E^(2*x)] + (Cosh[x]*Sinh[x])/4 + (x^2*Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.157967, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5450, 3311, 30, 2635, 8, 3720, 3716, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,e^{2x}\right)+\frac{x^3}{2}-x^2-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)+\frac{x}{4}+2x\log\left(1-e^{2x}\right)-\frac{1}{2}x\cosh^2(x)+\frac{1}{4}\sinh(x)\cosh$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] x/4 - x^2 + x^3/2 - (x*Cosh[x]^2)/2 - x^2*Coth[x] + 2*x*Log[1 - E^(2*x)] +
PolyLog[2, E^(2*x)] + (Cosh[x]*Sinh[x])/4 + (x^2*Cosh[x]*Sinh[x])/2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\cosh^2(x)\coth^2(x)\,dx = \int x^2\cosh^2(x)\,dx + \int x^2\coth^2(x)\,dx$$
$$= -\frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) + \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cosh^2(x)\,dx + 2\int x$$
$$= -x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) + \frac{\int 1\,d}{4}$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cos$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cos$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \text{Li}_2\left(e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh($$

Mathematica [A] time = 0.0928496, size = 64, normalized size = 0.88

$$\frac{1}{8}\left(-8\text{PolyLog}\left(2,e^{-2x}\right) + 4x^3 + 8x^2 + 2x^2\sinh(2x) - 8x^2\coth(x) + 16x\log\left(1-e^{-2x}\right) + \sinh(2x) - 2x\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (8*x^2 + 4*x^3 - 2*x*Cosh[2*x] - 8*x^2*Coth[x] + 16*x*Log[1 - E^(-2*x)] - 8
*PolyLog[2, E^(-2*x)] + Sinh[2*x] + 2*x^2*Sinh[2*x])/8
```

Maple [A] time = 0.045, size = 87, normalized size = 1.2

$$\frac{x^3}{2} + \left(\frac{1}{16} - \frac{x}{8} + \frac{x^2}{8}\right)e^{2x} + \left(-\frac{1}{16} - \frac{x}{8} - \frac{x^2}{8}\right)e^{-2x} - 2\frac{x^2}{e^{2x}-1} - 2x^2 + 2x\ln\left(e^x+1\right) + 2\,\text{polylog}\left(2,-e^x\right) + 2x\ln\left(1-e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 1/2*x^3+(1/16-1/8*x+1/8*x^2)*exp(2*x)+(-1/16-1/8*x-1/8*x^2)*exp(-2*x)-2*x^2
/(exp(2*x)-1)-2*x^2+2*x*ln(exp(x)+1)+2*polylog(2,-exp(x))+2*x*ln(1-exp(x))+
2*polylog(2,exp(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.17395, size = 1840, normalized size = 25.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x^2 - 2*x + 1)*cosh(x)^6 + 6*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^5 +
 (2*x^2 - 2*x + 1)*sinh(x)^6 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^4 + (8*x^
3 + 15*(2*x^2 - 2*x + 1)*cosh(x)^2 - 34*x^2 + 2*x - 1)*sinh(x)^4 + 4*(5*(2*
x^2 - 2*x + 1)*cosh(x)^3 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x))*sinh(x)^3 -
(8*x^3 + 2*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x^2 - 2*x + 1)*cosh(x)^4 - 8*x
^3 + 6*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^2 - 2*x^2 - 2*x - 1)*sinh(x)^2 +
2*x^2 + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)
*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(cosh(x) +
 sinh(x)) + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2
- 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(-cosh
(x) - sinh(x)) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*
cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*s
inh(x))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^
3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh
(x)^3 - x*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*(3*(2*x^2 - 2*x
 + 1)*cosh(x)^5 + 2*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^3 - (8*x^3 + 2*x^2 +
 2*x + 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x
```

```
))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x)**2*coth(x)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(x)^2 \coth(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="giac")

[Out] integrate(x^2*cosh(x)^2*coth(x)^2, x)
```

3.420 $\int x^3 \cosh^2(x) \coth^2(x)\, dx$

Optimal. Leaf size=102

$$3x\text{PolyLog}\left(2,e^{2x}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2x}\right)+\frac{3x^4}{8}-x^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2x}\right)-\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+\frac{1}{2}x^3\sinh(x$$

```
[Out] (3*x^2)/8 - x^3 + (3*x^4)/8 - (3*Cosh[x]^2)/8 - (3*x^2*Cosh[x]^2)/4 - x^3*C
oth[x] + 3*x^2*Log[1 - E^(2*x)] + 3*x*PolyLog[2, E^(2*x)] - (3*PolyLog[3, E
^(2*x)])/2 + (3*x*Cosh[x]*Sinh[x])/4 + (x^3*Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.182629, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5450, 3311, 30, 3310, 3720, 3716, 2190, 2531, 2282, 6589}

$$3x\text{PolyLog}\left(2,e^{2x}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2x}\right)+\frac{3x^4}{8}-x^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2x}\right)-\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+\frac{1}{2}x^3\sinh(x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^2,x]
```


```
[Out] (3*x^2)/8 - x^3 + (3*x^4)/8 - (3*Cosh[x]^2)/8 - (3*x^2*Cosh[x]^2)/4 - x^3*C
oth[x] + 3*x^2*Log[1 - E^(2*x)] + 3*x*PolyLog[2, E^(2*x)] - (3*PolyLog[3, E
^(2*x)])/2 + (3*x*Cosh[x]*Sinh[x])/4 + (x^3*Cosh[x]*Sinh[x])/2
```


Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cosh^2(x)\coth^2(x)\,dx = \int x^3\cosh^2(x)\,dx + \int x^3\coth^2(x)\,dx$$

$$= -\frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + \frac{1}{2}x^3\cosh(x)\sinh(x) + \frac{\int x^3\,dx}{2} + \frac{3}{2}\int x\cosh^2(x)\,dx + 3\int x$$

$$= -x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + \frac{3}{4}x\cosh(x)$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{Li}_2\left(e^{2x}\right)$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{Li}_2\left(e^{2x}\right)$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{Li}_2\left(e^{2x}\right)$$

Mathematica [C] time = 0.146842, size = 94, normalized size = 0.92

$$\frac{1}{16}\left(48x\mathrm{PolyLog}\left(2,e^{2x}\right) - 24\mathrm{PolyLog}\left(3,e^{2x}\right) + 6x^4 - 16x^3 + 48x^2\log\left(1-e^{2x}\right) + 4x^3\sinh(2x) - 6x^2\cosh(2x) - 16x^3\,\mathrm{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] ((2*I)*Pi^3 - 16*x^3 + 6*x^4 - 3*Cosh[2*x] - 6*x^2*Cosh[2*x] - 16*x^3*Coth[
x] + 48*x^2*Log[1 - E^(2*x)] + 48*x*PolyLog[2, E^(2*x)] - 24*PolyLog[3, E^(
```

```
2*x)] + 6*x*Sinh[2*x] + 4*x^3*Sinh[2*x])/16
```

Maple [A] time = 0.046, size = 117, normalized size = 1.2

$$\frac{3x^4}{8}+\left(-\frac{3}{32}+\frac{3x}{16}-\frac{3x^2}{16}+\frac{x^3}{8}\right)e^{2x}+\left(-\frac{3}{32}-\frac{3x}{16}-\frac{3x^2}{16}-\frac{x^3}{8}\right)e^{-2x}-2\frac{x^3}{e^{2x}-1}-2x^3+3x^2\ln\left(e^x+1\right)+6x\text{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 3/8*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(2*x)+(-3/32-3/16*x-3/16*x^2-1/8
*x^3)*exp(-2*x)-2*x^3/(exp(2*x)-1)-2*x^3+3*x^2*ln(exp(x)+1)+6*x*polylog(2,-
exp(x))-6*polylog(3,-exp(x))+3*x^2*ln(1-exp(x))+6*x*polylog(2,exp(x))-6*pol
ylog(3,exp(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 2.32171, size = 2541, normalized size = 24.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 + 6*(4*x^3 - 6*x^2 + 6*x - 3)*cos
h(x)*sinh(x)^5 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^6 + (12*x^4 - 68*x^3 + 6
*x^2 - 6*x + 3)*cosh(x)^4 + (12*x^4 - 68*x^3 + 15*(4*x^3 - 6*x^2 + 6*x - 3)
```

```
*cosh(x)^2 + 6*x^2 - 6*x + 3)*sinh(x)^4 + 4*(5*(4*x^3 - 6*x^2 + 6*x - 3)*co
sh(x)^3 + (12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x))*sinh(x)^3 + 4*x^3 -
(12*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + (15*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^4 - 12*x^4 - 4*x^3 + 6*(12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x
)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh(x)^4 + 4*x*cosh(x)*s
inh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2
*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 -
x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh
(x)) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*co
sh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x
)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*cosh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(
x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) +
1) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*s
inh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog(3, cosh(x
) + sinh(x)) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x
)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog
(3, -cosh(x) - sinh(x)) + 2*(3*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 + 2*(12*
x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x)^3 - (12*x^4 + 4*x^3 + 6*x^2 + 6*x +
 3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(x)**2*coth(x)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(x)^2 \coth(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(x)^2*coth(x)^2, x)
```

3.421 $\int x\cosh^2(x)\coth^3(x)\,dx$

Optimal. Leaf size=63

$$\text{PolyLog}\left(2,e^{2x}\right)-x^2+\frac{3x}{4}+2x\log\left(1-e^{2x}\right)+\frac{1}{2}x\sinh^2(x)-\frac{1}{2}x\coth^2(x)-\frac{\coth(x)}{2}-\frac{1}{4}\sinh(x)\cosh(x)$$

```
[Out] (3*x)/4 - x^2 - Coth[x]/2 - (x*Coth[x]^2)/2 + 2*x*Log[1 - E^(2*x)] + PolyLo
g[2, E^(2*x)] - (Cosh[x]*Sinh[x])/4 + (x*Sinh[x]^2)/2
```

Rubi [A] time = 0.162843, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5450, 5372, 2635, 8, 3716, 2190, 2279, 2391, 3720, 3473}

$$\text{PolyLog}\left(2,e^{2x}\right)-x^2+\frac{3x}{4}+2x\log\left(1-e^{2x}\right)+\frac{1}{2}x\sinh^2(x)-\frac{1}{2}x\coth^2(x)-\frac{\coth(x)}{2}-\frac{1}{4}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x)/4 - x^2 - Coth[x]/2 - (x*Coth[x]^2)/2 + 2*x*Log[1 - E^(2*x)] + PolyLo
g[2, E^(2*x)] - (Cosh[x]*Sinh[x])/4 + (x*Sinh[x]^2)/2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh^2(x)\coth(x)\,dx + \int x\coth^3(x)\,dx$$
$$= -\frac{1}{2}x\coth^2(x) + \frac{1}{2}\int\coth^2(x)\,dx + 2\int x\coth(x)\,dx + \int x\cosh(x)\sinh(x)\,dx$$
$$= -\frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + \frac{1}{2}x\sinh^2(x) + \frac{\int 1\,dx}{2} - \frac{1}{2}\int\sinh^2(x)\,dx + 2\left(-\frac{x^2}{2} - 2\int\frac{e^{2x}x}{1-e^{2x}}\right.$$
$$= \frac{x}{2} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) + \frac{\int 1\,dx}{4} + 2\left(-\frac{x^2}{2} + x\log\left(1\right.\right.$$
$$= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right)\right.$$
$$= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + \frac{\mathrm{Li}_2\left(e^{2x}\right)}{2}\right) - \frac{1}{4}\cosh(x)\sinh(x) +$$

Mathematica [A] time = 0.0765283, size = 56, normalized size = 0.89

$$\frac{1}{8}\left(-8\text{PolyLog}\left(2,e^{-2x}\right) + 8x^2 + 16x\log\left(1-e^{-2x}\right) - \sinh(2x) + 2x\cosh(2x) - 4\coth(x) - 4x\text{csch}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (8*x^2 + 2*x*Cosh[2*x] - 4*Coth[x] - 4*x*Csch[x]^2 + 16*x*Log[1 - E^(-2*x)]
 - 8*PolyLog[2, E^(-2*x)] - Sinh[2*x])/8
```

Maple [A] time = 0.047, size = 82, normalized size = 1.3

$$-x^2 + \left(-\frac{1}{16} + \frac{x}{8}\right)e^{2x} + \left(\frac{1}{16} + \frac{x}{8}\right)e^{-2x} - \frac{2xe^{2x} + e^{2x} - 1}{\left(e^{2x}-1\right)^2} + 2x\ln\left(e^x+1\right) + 2\,\text{polylog}\left(2,-e^x\right) + 2x\ln\left(1-e^x\right) + 2\,\text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -x^2+(-1/16+1/8*x)*exp(2*x)+(1/16+1/8*x)*exp(-2*x)-(2*x*exp(2*x)+exp(2*x)-1
)/(exp(2*x)-1)^2+2*x*ln(exp(x)+1)+2*polylog(2,-exp(x))+2*x*ln(1-exp(x))+2*p
olylog(2,exp(x))
```

Maxima [B] time = 1.36158, size = 197, normalized size = 3.13

$$-2x^2 + 2x\log(e^x + 1) + 2x\log(-e^x + 1) + \frac{5}{8}x + \frac{16x^2 + (2x-1)e^{(6x)} + 2(8x^2 - 2x + 1)e^{(4x)} - (32x^2 + 8x + 11)e^{(2x)}}{16(e^{(4x)} - 2e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -2*x^2 + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 5/8*x + 1/16*(16*x^2 + (2*x
 - 1)*e^(6*x) + 2*(8*x^2 - 2*x + 1)*e^(4*x) - (32*x^2 + 8*x + 11)*e^(2*x) +
 (2*x + 1)*e^(-2*x) - 14*x + 9)/(e^(4*x) - 2*e^(2*x) + 1) - 5/16*(2*x*e^(4*
x) + e^(2*x) - 1)/(e^(4*x) - 2*e^(2*x) + 1) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Fricas [B] time = 2.25806, size = 2881, normalized size = 45.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x - 1)*cosh(x)^8 + 8*(2*x - 1)*cosh(x)*sinh(x)^7 + (2*x - 1)*sinh(
x)^8 - 2*(8*x^2 + 2*x - 1)*cosh(x)^6 + 2*(14*(2*x - 1)*cosh(x)^2 - 8*x^2 -
2*x + 1)*sinh(x)^6 + 4*(14*(2*x - 1)*cosh(x)^3 - 3*(8*x^2 + 2*x - 1)*cosh(x
))*sinh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^4 + 2*(35*(2*x - 1)*cosh(x)^4 -
15*(8*x^2 + 2*x - 1)*cosh(x)^2 + 16*x^2 - 14*x - 8)*sinh(x)^4 + 8*(7*(2*x -
 1)*cosh(x)^5 - 5*(8*x^2 + 2*x - 1)*cosh(x)^3 + 2*(8*x^2 - 7*x - 4)*cosh(x)
)*sinh(x)^3 - 2*(8*x^2 + 2*x - 7)*cosh(x)^2 + 2*(14*(2*x - 1)*cosh(x)^6 - 1
5*(8*x^2 + 2*x - 1)*cosh(x)^4 + 12*(8*x^2 - 7*x - 4)*cosh(x)^2 - 8*x^2 - 2*
x + 7)*sinh(x)^2 + 32*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*co
sh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^
3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^
```

```
5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 32*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh
(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^
2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh
(x))*dilog(-cosh(x) - sinh(x)) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cos
h(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh
(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x
))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x
*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh
(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh(
x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x)
)*log(-cosh(x) - sinh(x) + 1) + 4*(2*(2*x - 1)*cosh(x)^7 - 3*(8*x^2 + 2*x -
 1)*cosh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^3 - (8*x^2 + 2*x - 7)*cosh(x))*
sinh(x) + 2*x + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(
x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 +
 (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 -
 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(x)^2\coth(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(x)^2*coth(x)^3, x)
```

3.422 $\int x^2 \cosh^2(x) \coth^3(x) \, dx$

Optimal. Leaf size=96

$$2x\text{PolyLog}\left(2,e^{2x}\right)-\text{PolyLog}\left(3,e^{2x}\right)-\frac{2x^3}{3}+\frac{3x^2}{4}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{2}x^2\sinh^2(x)-\frac{1}{2}x^2\coth^2(x)+\frac{\sinh^2(x)}{4}-x$$

```
[Out] (3*x^2)/4 - (2*x^3)/3 - x*Coth[x] - (x^2*Coth[x]^2)/2 + 2*x^2*Log[1 - E^(2*
x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(2*x)] - PolyLog[3, E^(2*x)] - (x*Cos
h[x]*Sinh[x])/2 + Sinh[x]^2/4 + (x^2*Sinh[x]^2)/2
```

Rubi [A] time = 0.280165, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5450, 5372, 3310, 30, 3716, 2190, 2531, 2282, 6589, 3720, 3475}

$$2x\text{PolyLog}\left(2,e^{2x}\right)-\text{PolyLog}\left(3,e^{2x}\right)-\frac{2x^3}{3}+\frac{3x^2}{4}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{2}x^2\sinh^2(x)-\frac{1}{2}x^2\coth^2(x)+\frac{\sinh^2(x)}{4}-x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x^2)/4 - (2*x^3)/3 - x*Coth[x] - (x^2*Coth[x]^2)/2 + 2*x^2*Log[1 - E^(2*
x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(2*x)] - PolyLog[3, E^(2*x)] - (x*Cos
h[x]*Sinh[x])/2 + Sinh[x]^2/4 + (x^2*Sinh[x]^2)/2
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2(x)\coth^3(x)\,dx &= \int x^2\cosh^2(x)\coth(x)\,dx+\int x^2\coth^3(x)\,dx\\ &= -\frac{1}{2}x^2\coth^2(x)+2\int x^2\coth(x)\,dx+\int x\coth^2(x)\,dx+\int x^2\cosh(x)\sinh(x)\,dx\\ &= -x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\frac{1}{2}x^2\sinh^2(x)+2\left(-\frac{x^3}{3}-2\int\frac{e^{2x}x^2}{1-e^{2x}}\,dx\right)+\int x\,dx+\int \cot\\ &= \frac{x^2}{2}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\frac{1}{2}x^2\sin\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\frac{1}{2}x^2\,\mathrm{s}\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\frac{1}{2}x^2\,\mathrm{s}\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))+2\left(-\frac{x^3}{3}+x^2\log\left(1-e^{2x}\right)+x\mathrm{Li}_2\left(e^{2x}\right)-\right.\end{aligned}$$

Mathematica [C] time = 0.277203, size = 98, normalized size = 1.02

$$2x\mathrm{PolyLog}\left(2,e^{2x}\right)-\mathrm{PolyLog}\left(3,e^{2x}\right)-\frac{2x^3}{3}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{4}x^2\cosh(2x)-\frac{1}{2}x^2\mathrm{csch}^2(x)-\frac{1}{4}x\sinh(2x)+\frac{1}{8}\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - (2*x^3)/3 + Cosh[2*x]/8 + (x^2*Cosh[2*x])/4 - x*Coth[x] - (x^
2*Csch[x]^2)/2 + 2*x^2*Log[1 - E^(2*x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(
2*x)] - PolyLog[3, E^(2*x)] - (x*Sinh[2*x])/4
```

Maple [A] time = 0.061, size = 127, normalized size = 1.3

$$-\frac{2x^3}{3}+\left(\frac{1}{16}-\frac{x}{8}+\frac{x^2}{8}\right)e^{2x}+\left(\frac{1}{16}+\frac{x}{8}+\frac{x^2}{8}\right)e^{-2x}-2\,\frac{x\left(xe^{2x}+e^{2x}-1\right)}{\left(e^{2x}-1\right)^2}+\ln\left(e^x+1\right)-2\,\ln\left(e^x\right)+\ln\left(e^x-1\right)+2\,x^2\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -2/3*x^3+(1/16-1/8*x+1/8*x^2)*exp(2*x)+(1/16+1/8*x+1/8*x^2)*exp(-2*x)-2*x*(
x*exp(2*x)+exp(2*x)-1)/(exp(2*x)-1)^2+ln(exp(x)+1)-2*ln(exp(x))+ln(exp(x)-1
)+2*x^2*ln(exp(x)+1)+4*x*polylog(2,-exp(x))-4*polylog(3,-exp(x))+2*x^2*ln(1
-exp(x))+4*x*polylog(2,exp(x))-4*polylog(3,exp(x))
```

Maxima [B] time = 1.33457, size = 235, normalized size = 2.45

$$-\frac{4}{3}\,x^3+2\,x^2\log\left(e^x+1\right)+2\,x^2\log\left(-e^x+1\right)+4\,x\mathrm{Li}_2\left(-e^x\right)+4\,x\mathrm{Li}_2\left(e^x\right)-2\,x+\frac{32\,x^3-12\,x^2+3\left(2\,x^2-2\,x+1\right)e^{(6\,x)}+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*x^3 + 2*x^2*log(e^x + 1) + 2*x^2*log(-e^x + 1) + 4*x*dilog(-e^x) + 4*x
*dilog(e^x) - 2*x + 1/48*(32*x^3 - 12*x^2 + 3*(2*x^2 - 2*x + 1)*e^(6*x) + 2
*(16*x^3 - 6*x^2 + 6*x - 3)*e^(4*x) - 2*(32*x^3 + 42*x^2 + 48*x - 3)*e^(2*x
) + 3*(2*x^2 + 2*x + 1)*e^(-2*x) + 84*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + lo
g(e^x + 1) + log(e^x - 1) - 4*polylog(3, -e^x) - 4*polylog(3, e^x)
```

Fricas [C] time = 2.3186, size = 4618, normalized size = 48.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*x^2 - 2*x + 1)*cosh(x)^8 + 24*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^
7 + 3*(2*x^2 - 2*x + 1)*sinh(x)^8 - 2*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^6
 - 2*(16*x^3 - 42*(2*x^2 - 2*x + 1)*cosh(x)^2 + 6*x^2 + 42*x + 3)*sinh(x)^6
 + 12*(14*(2*x^2 - 2*x + 1)*cosh(x)^3 - (16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)
)*sinh(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^4 + 2*(105*(2*x^2 - 2*
x + 1)*cosh(x)^4 + 32*x^3 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^2 - 42*x
^2 + 48*x + 3)*sinh(x)^4 + 8*(21*(2*x^2 - 2*x + 1)*cosh(x)^5 - 5*(16*x^3 +
6*x^2 + 42*x + 3)*cosh(x)^3 + (32*x^3 - 42*x^2 + 48*x + 3)*cosh(x))*sinh(x)
^3 - 2*(16*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + 2*(42*(2*x^2 - 2*x + 1)*cosh(
x)^6 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^4 - 16*x^3 + 6*(32*x^3 - 42*x
^2 + 48*x + 3)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh
(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)
^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)
^2 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4
*x*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2
 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2
 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x
*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh(x)) + 48*((2*x^2 + 1
)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(
2*x^2 + 1)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4
*(5*(2*x^2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*
cosh(x)^2 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 +
1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^
2 + 1)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 48*(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4
*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*si
nh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(
cosh(x) + sinh(x) - 1) + 96*(x^2*cosh(x)^6 + 6*x^2*cosh(x)*sinh(x)^5 + x^2*
sinh(x)^6 - 2*x^2*cosh(x)^4 + (15*x^2*cosh(x)^2 - 2*x^2)*sinh(x)^4 + x^2*co
sh(x)^2 + 4*(5*x^2*cosh(x)^3 - 2*x^2*cosh(x))*sinh(x)^3 + (15*x^2*cosh(x)^4
 - 12*x^2*cosh(x)^2 + x^2)*sinh(x)^2 + 2*(3*x^2*cosh(x)^5 - 4*x^2*cosh(x)^3
 + x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) - 192*(cosh(x)^6 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 +
 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*
sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*po
```

```
lylog(3, cosh(x) + sinh(x)) - 192*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(
x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2
*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*polylog(3, -cosh(x) - sinh(
x)) + 4*(6*(2*x^2 - 2*x + 1)*cosh(x)^7 - 3*(16*x^3 + 6*x^2 + 42*x + 3)*cosh
(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^3 - (16*x^3 + 6*x^2 + 6*x +
3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))
*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3
*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(x)^2*coth(x)^3, x)
```

3.423 $\int x^3 \cosh^2(x) \coth^3(x) \, dx$

Optimal. Leaf size=158

$$3x^2\text{PolyLog}\left(2,e^{2x}\right)-3x\text{PolyLog}\left(3,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(2,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(4,e^{2x}\right)-\frac{x^4}{2}+\frac{3x^3}{4}-\frac{3x^2}{2}+2x^3\log\left(1-\right.$$

```
[Out] (3*x)/8 - (3*x^2)/2 + (3*x^3)/4 - x^4/2 - (3*x^2*Coth[x])/2 - (x^3*Coth[x]^
2)/2 + 3*x*Log[1 - E^(2*x)] + 2*x^3*Log[1 - E^(2*x)] + (3*PolyLog[2, E^(2*x
)])/2 + 3*x^2*PolyLog[2, E^(2*x)] - 3*x*PolyLog[3, E^(2*x)] + (3*PolyLog[4,
 E^(2*x)])/2 - (3*Cosh[x]*Sinh[x])/8 - (3*x^2*Cosh[x]*Sinh[x])/4 + (3*x*Sin
h[x]^2)/4 + (x^3*Sinh[x]^2)/2
```

Rubi [A] time = 0.385664, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {5450, 5372, 3311, 30, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3x^2\text{PolyLog}\left(2,e^{2x}\right)-3x\text{PolyLog}\left(3,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(2,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(4,e^{2x}\right)-\frac{x^4}{2}+\frac{3x^3}{4}-\frac{3x^2}{2}+2x^3\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^3,x]
```


```
[Out] (3*x)/8 - (3*x^2)/2 + (3*x^3)/4 - x^4/2 - (3*x^2*Coth[x])/2 - (x^3*Coth[x]^
2)/2 + 3*x*Log[1 - E^(2*x)] + 2*x^3*Log[1 - E^(2*x)] + (3*PolyLog[2, E^(2*x
)])/2 + 3*x^2*PolyLog[2, E^(2*x)] - 3*x*PolyLog[3, E^(2*x)] + (3*PolyLog[4,
 E^(2*x)])/2 - (3*Cosh[x]*Sinh[x])/8 - (3*x^2*Cosh[x]*Sinh[x])/4 + (3*x*Sin
h[x]^2)/4 + (x^3*Sinh[x]^2)/2
```


Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
```

```
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \int x^3\cosh^2(x)\coth(x)\,dx + \int x^3\coth^3(x)\,dx$$

$$= -\frac{1}{2}x^3\coth^2(x) + \frac{3}{2}\int x^2\coth^2(x)\,dx + 2\int x^3\coth(x)\,dx + \int x^3\cosh(x)\sinh(x)\,dx$$

$$= -\frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + \frac{1}{2}x^3\sinh^2(x) + \frac{3\int x^2\,dx}{2} - \frac{3}{2}\int x^2\sinh^2(x)\,dx + 2\left(-\frac{x^4}{4}\right.$$

$$= -\frac{3x^2}{2} + \frac{x^3}{2} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) - \frac{3}{4}x^2\cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) + \frac{1}{2}x^3\sinh$$

$$= -\frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x^2\cosh$$

$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x$$

$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) + \frac{3\text{Li}_2\left(e^{2x}\right)}{2} - \frac{3}{8}\cosh(x)$$

$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) + \frac{3\text{Li}_2\left(e^{2x}\right)}{2} + 2\left(-\frac{x^4}{4} +\right.$$

Mathematica [A] time = 0.333773, size = 133, normalized size = 0.84

$$\frac{1}{32}\left(96x^2\text{PolyLog}\left(2,e^{2x}\right) - 96x\text{PolyLog}\left(3,e^{2x}\right) - 48\text{PolyLog}\left(2,e^{-2x}\right) + 48\text{PolyLog}\left(4,e^{2x}\right) - 16x^4 + 48x^2 + 64x^3\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (Pi^4 + 48*x^2 - 16*x^4 + 12*x*Cosh[2*x] + 8*x^3*Cosh[2*x] - 48*x^2*Coth[x]
 - 16*x^3*Csch[x]^2 + 96*x*Log[1 - E^(-2*x)] + 64*x^3*Log[1 - E^(2*x)] - 48
*PolyLog[2, E^(-2*x)] + 96*x^2*PolyLog[2, E^(2*x)] - 96*x*PolyLog[3, E^(2*x
)] + 48*PolyLog[4, E^(2*x)] - 6*Sinh[2*x] - 12*x^2*Sinh[2*x])/32
```

Maple [A] time = 0.061, size = 184, normalized size = 1.2

$$-\frac{x^4}{2}+\left(-\frac{3}{32}+\frac{3x}{16}-\frac{3x^2}{16}+\frac{x^3}{8}\right)e^{2x}+\left(\frac{3}{32}+\frac{3x}{16}+\frac{3x^2}{16}+\frac{x^3}{8}\right)e^{-2x}-\frac{x^2\left(2xe^{2x}+3e^{2x}-3\right)}{\left(e^{2x}-1\right)^2}-3x^2+3x\ln\left(e^x+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -1/2*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(2*x)+(3/32+3/16*x+3/16*x^2+1/8
*x^3)*exp(-2*x)-x^2*(2*x*exp(2*x)+3*exp(2*x)-3)/(exp(2*x)-1)^2-3*x^2+3*x*ln
(exp(x)+1)+3*polylog(2,-exp(x))+3*x*ln(1-exp(x))+3*polylog(2,exp(x))+2*x^3*
ln(exp(x)+1)+6*x^2*polylog(2,-exp(x))-12*x*polylog(3,-exp(x))+12*polylog(4,
-exp(x))+2*x^3*ln(1-exp(x))+6*x^2*polylog(2,exp(x))-12*x*polylog(3,exp(x))+
12*polylog(4,exp(x))
```

Maxima [A] time = 1.29036, size = 321, normalized size = 2.03

$$-x^4+2x^3\log\left(e^x+1\right)+2x^3\log\left(-e^x+1\right)+6x^2\mathrm{Li}_2\left(-e^x\right)+6x^2\mathrm{Li}_2\left(e^x\right)-3x^2+3x\log\left(e^x+1\right)+3x\log\left(-e^x+1\right)-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -x^4 + 2*x^3*log(e^x + 1) + 2*x^3*log(-e^x + 1) + 6*x^2*dilog(-e^x) + 6*x^2
*dilog(e^x) - 3*x^2 + 3*x*log(e^x + 1) + 3*x*log(-e^x + 1) - 12*x*polylog(3
, -e^x) - 12*x*polylog(3, e^x) + 1/32*(16*x^4 - 8*x^3 + 84*x^2 + (4*x^3 - 6
*x^2 + 6*x - 3)*e^(6*x) + 2*(8*x^4 - 4*x^3 + 6*x^2 - 6*x + 3)*e^(4*x) - 4*(
8*x^4 + 14*x^3 + 24*x^2 - 3*x)*e^(2*x) + (4*x^3 + 6*x^2 + 6*x + 3)*e^(-2*x)
 - 12*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + 3*dilog(-e^x) + 3*dilog(e^x) + 12*
polylog(4, -e^x) + 12*polylog(4, e^x)
```

Fricas [C] time = 2.38015, size = 5913, normalized size = 37.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="fricas")

[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^8 + 8*(4*x^3 - 6*x^2 + 6*x - 3)*cos h(x)*sinh(x)^7 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^8 - 2*(8*x^4 + 4*x^3 + 4 2*x^2 + 6*x - 3)*cosh(x)^6 - 2*(8*x^4 + 4*x^3 - 14*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^2 + 42*x^2 + 6*x - 3)*sinh(x)^6 + 4*(14*(4*x^3 - 6*x^2 + 6*x - 3) *cosh(x)^3 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x))*sinh(x)^5 + 4*(8 *x^4 - 14*x^3 + 24*x^2 + 3*x)*cosh(x)^4 + 2*(35*(4*x^3 - 6*x^2 + 6*x - 3)*c osh(x)^4 + 16*x^4 - 28*x^3 - 15*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^ 2 + 48*x^2 + 6*x)*sinh(x)^4 + 8*(7*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 - 5* (8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^3 + 2*(8*x^4 - 14*x^3 + 24*x^2 + 3*x)*cosh(x))*sinh(x)^3 + 4*x^3 - 2*(8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh (x)^2 + 2*(14*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 - 15*(8*x^4 + 4*x^3 + 42* x^2 + 6*x - 3)*cosh(x)^4 - 8*x^4 - 4*x^3 + 12*(8*x^4 - 14*x^3 + 24*x^2 + 3* x)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 96*((2*x^2 + 1)*cosh(x) ^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^ 2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*co sh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 96*((2*x^2 + 1)*cosh(x)^6 + 6*(2 *x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1)*cosh(x) ^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^2 + 1)*co sh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2 + (15*(2 *x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*cosh(x))*si nh(x))*dilog(-cosh(x) - sinh(x)) + 32*((2*x^3 + 3*x)*cosh(x)^6 + 6*(2*x^3 + 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 - 2*(2*x^3 + 3*x)*cosh(x) ^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh(x)^4 + 4*(5*(2*x^3 + 3 *x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 + (2*x^3 + 3*x)*cosh(x)^ 2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3 + 3*x)*cosh(x)^2 + 3*x) *sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3 + 3*x)*cosh(x)^3 + (2* x^3 + 3*x)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 32*((2*x^3 + 3*x) *cosh(x)^6 + 6*(2*x^3 + 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 - 2*(2*x^3 + 3*x)*cosh(x)^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh (x)^4 + 4*(5*(2*x^3 + 3*x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 + (2*x^3 + 3*x)*cosh(x)^2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3 + 3*x)*cosh(x)^2 + 3*x)*sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3 + 3*x)*cosh(x)^3 + (2*x^3 + 3*x)*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 384*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cos h(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*polylog(4, cosh(x) + sinh(x)) + 384*(cosh(x)^6 + 6 *cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4

```
 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1
)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*
polylog(4, -cosh(x) - sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
 x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*co
sh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cos
h(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(
x))*polylog(3, cosh(x) + sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^
5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x
*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*
cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*si
nh(x))*polylog(3, -cosh(x) - sinh(x)) + 4*(2*(4*x^3 - 6*x^2 + 6*x - 3)*cosh
(x)^7 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^5 + 4*(8*x^4 - 14*x^3
+ 24*x^2 + 3*x)*cosh(x)^3 - (8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x))*sinh
(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2
 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15
*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*c
osh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh{(x)}^2 \coth{(x)}^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(x)^2*coth(x)^3, x)
```

3.424 $\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(x^m \coth(a+bx)\text{csch}(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi [A] time = 0.379737, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx=\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

Mathematica [A] time = 3.82479, size = 0, normalized size = 0.

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int x^{m} \cosh \left(b x+a\right) \left(\operatorname{csch}\left(b x+a\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \cosh \left(b x+a\right) \operatorname{csch}\left(b x+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m} \cosh \left(b x+a\right) \operatorname{csch}\left(b x+a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.425 $\int x^3 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=93

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}$$

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2,
 -E^(a + b*x)])/b^3 + (6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^
(a + b*x)])/b^4 - (6*PolyLog[3, E^(a + b*x)])/b^4
```

Rubi [A] time = 0.104649, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5419, 4182, 2531, 2282, 6589}

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2,
 -E^(a + b*x)])/b^3 + (6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^
(a + b*x)])/b^4 - (6*PolyLog[3, E^(a + b*x)])/b^4
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{x^3\text{csch}(a+bx)}{b}+\frac{3\int x^2\text{csch}(a+bx)\,dx}{b}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}-\frac{6\int x\log\left(1-e^{a+bx}\right)dx}{b^2}+\frac{6\int x\log\left(1+e\right.}{b^2}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}+\frac{6\int \text{Li}_2}{}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}+\frac{6\,\text{Subs}}{}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}+\frac{6\text{Li}_3\left(-\right.}{b}$$

Mathematica [A] time = 6.86935, size = 167, normalized size = 1.8

$$-\frac{\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)\left(6bx\sinh(a+bx)\text{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx))-6bx\sinh(a+bx)\text{Pol}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Cosh[a +
 b*x] + Sinh[a + b*x]]*Sinh[a + b*x] + 6*b*x*PolyLog[2, -Cosh[a + b*x] - Si
nh[a + b*x]]*Sinh[a + b*x] - 6*b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]
]*Sinh[a + b*x] - 6*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]]*Sinh[a + b*x
] + 6*PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a + b*x]))/(2*b^4)
```

Maple [A] time = 0.028, size = 174, normalized size = 1.9

$$-2\,{\frac{{x}^{3}{{\rm e}^{bx+a}}}{b \left({{\rm e}^{2\,bx+2\,a}}-1 \right) }}-6\,{\frac{{a}^{2}{\rm Artanh} \left({{\rm e}^{bx+a}} \right) }{{b}^{4}}}-3\,{\frac{\ln \left(1+{{\rm e}^{bx+a}} \right) {x}^{2}}{{b}^{2}}}+3\,{\frac{{a}^{2}\ln \left(1+{{\rm e}^{bx+a}} \right) }{{b}^{4}}}-6\,{\frac{x{\it polylog} \left(2,-{{\rm e}^{bx+a}} \right) }{{b}^{3}}}+6\,{\frac{{\it polyl}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh(exp(b*x+a))-3/b^2*
ln(1+exp(b*x+a))*x^2+3/b^4*ln(1+exp(b*x+a))*a^2-6*x*polylog(2,-exp(b*x+a))/
b^3+6*polylog(3,-exp(b*x+a))/b^4+3/b^2*ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(
b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,exp(b*x+a))/b^4
```

Maxima [A] time = 1.38132, size = 163, normalized size = 1.75

$$-\frac{2\,x^{3}e^{\left(bx+a\right)}}{be^{\left(2\,bx+2\,a\right)}-b}-\frac{3\,\left(b^{2}x^{2}\log\left(e^{\left(bx+a\right)}+1\right)+2\,bx\mathrm{Li}_{2}\left(-e^{\left(bx+a\right)}\right)-2\,\mathrm{Li}_{3}(-e^{\left(bx+a\right)})\right)}{b^{4}}+\frac{3\,\left(b^{2}x^{2}\log\left(-e^{\left(bx+a\right)}+1\right)+2\,bx\mathrm{Li}_{2}\left(\right.\right.}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4
```

Fricas [C] time = 2.11485, size = 1461, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)
*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)
 + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1
) - 3*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*x^2 - (b^2
*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
 - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a
) + 1) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*
cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a
) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a
) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a) \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.426 $\int x^2 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}$$

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch[a + b*x])/b - (2*PolyLog[2, -E^
(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
```

Rubi [A] time = 0.0605293, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5419, 4182, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch[a + b*x])/b - (2*PolyLog[2, -E^
(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{x^2\operatorname{csch}(a+bx)}{b}+\frac{2\int x\operatorname{csch}(a+bx)\,dx}{b}\\ &= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\int\log\left(1-e^{a+bx}\right)dx}{b^2}+\frac{2\int\log\left(1+e^{a+b\ldots}\right.}{b^2}\\ &= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+\frac{2\operatorname{Subs}\ldots}{}\\ &= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\operatorname{Li}_2\left(e^{a+bx}\right)}{b^3}\end{aligned}$$

Mathematica [B] time = 0.830012, size = 133, normalized size = 2.25

$$-\frac{-2\operatorname{PolyLog}\left(2,-e^{-a-bx}\right)+2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)+b^2x^2\operatorname{csch}(a+bx)-2bx\log\left(1-e^{-a-bx}\right)+2bx\log\left(e^{-a-bx}+1\right)-2a\ldots}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -((b^2*x^2*Csch[a + b*x] - 2*a*Log[1 - E^(-a - b*x)] - 2*b*x*Log[1 - E^(-a
- b*x)] + 2*a*Log[1 + E^(-a - b*x)] + 2*b*x*Log[1 + E^(-a - b*x)] + 2*a*Log
[Tanh[(a + b*x)/2]] - 2*PolyLog[2, -E^(-a - b*x)] + 2*PolyLog[2, E^(-a - b*
x)])/b^3)
```

Maple [B] time = 0.027, size = 134, normalized size = 2.3

$$-2\,\frac{x^2\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2\,bx+2\,a}-1\right)}-2\,\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^2}-2\,\frac{a\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^3}-2\,\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^3}+2\,\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^2}+2\,\frac{\ln\left(1-\mathrm{e}^{bx+\ldots}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2*x^2*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)-2/b^2*ln(1+exp(b*x+a))*x-2/b^3*ln(1+
exp(b*x+a))*a-2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*l
n(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.36937, size = 112, normalized size = 1.9

$$-\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3
```

Fricas [B] time = 2.13463, size = 1022, normalized size = 17.32

$$-\frac{2\left(b^2x^2\cosh\left(bx+a\right)+b^2x^2\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\mathrm{Li}_2\left(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh
(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^2 + 2*
b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x
+ a) + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*
x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a
)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*
```

```
cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 -
 b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.427 $\int x \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - (x*Csch[a + b*x])/b
```

Rubi [A] time = 0.018915, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5419, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - (x*Csch[a + b*x])/b
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\coth(a+bx)\text{csch}(a+bx)\,dx &= -\frac{x\text{csch}(a+bx)}{b}+\frac{\int \text{csch}(a+bx)\,dx}{b}\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.0483079, size = 114, normalized size = 4.56

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{x\text{csch}(a)}{b}+\frac{x\text{csch}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b}+\frac{x\text{sech}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{sec}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -((x*Csch[a])/b) - Log[Cosh[a/2 + (b*x)/2]]/b^2 + Log[Sinh[a/2 + (b*x)/2]]/
b^2 + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x*Sech[a/2]*
Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b)
```

Maple [B] time = 0.029, size = 54, normalized size = 2.2

$$-2\,\frac{x\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2\,bx+2\,a}-1\right)}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}-\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2*x*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(1+exp(
b*x+a))
```

Maxima [B] time = 1.31727, size = 86, normalized size = 3.44

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Fricas [B] time = 2.05509, size = 483, normalized size = 19.32

$$-\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)+\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\log\left(\cosh\right.}{b^{2}\cosh\left(bx+a\right)^{2}+2\,b^{2}\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) + (cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a
) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*
cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27432, size = 126, normalized size = 5.04

$$-\frac{2\,bxe^{(bx+a)}+e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}+1\right)-e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}-1\right)-\log\left(e^{(bx+a)}+1\right)+\log\left(e^{(bx+a)}-1\right)}{b^{2}e^{(2\,bx+2\,a)}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(b*x + a) + e^(2*b*x + 2*a)*log(e^(b*x + a) + 1) - e^(2*b*x + 2*a
)*log(e^(b*x + a) - 1) - log(e^(b*x + a) + 1) + log(e^(b*x + a) - 1))/(b^2*
e^(2*b*x + 2*a) - b^2)
```

3.428 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -(Csch[a + b*x]/b)
```

Rubi [A] time = 0.0113572, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{i\,\operatorname{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0067578, size = 11, normalized size = 1.

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Maple [A] time = 0.006, size = 12, normalized size = 1.1

$$-\frac{\operatorname{csch}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -csch(b*x+a)/b
```

Maxima [B] time = 1.03352, size = 34, normalized size = 3.09

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Fricas [B] time = 2.01971, size = 154, normalized size = 14.

$$-\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2, x)
```

Giac [B] time = 1.28148, size = 34, normalized size = 3.09

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.429 $\int \frac{\coth(a+bx)\mathbf{csch}(a+bx)}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

Rubi [A] time = 0.141851, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx$$

Mathematica [A] time = 31.313, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x,x]
```

```
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a) (\operatorname{csch}(bx+a))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\, e^{(bx+a)}}{bxe^{(2\, bx+2\, a)} - bx} - 2 \int \frac{1}{2 \left(bx^2 e^{(bx+a)} + bx^2\right)}\, dx - 2 \int \frac{1}{2 \left(bx^2 e^{(bx+a)} - bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 2*integrate(1/2/(b*x^2*e^(b*x
+ a) + b*x^2), x) - 2*integrate(1/2/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh (bx+a) \operatorname{csch}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

3.430 $\int \frac{\coth(a+bx)\mathbf{csch}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi [A] time = 0.192723, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 37.3008, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)(\operatorname{csch}(bx+a))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}-bx^2}-2\int\frac{1}{bx^3e^{(bx+a)}+bx^3}\,dx-2\int\frac{1}{bx^3e^{(bx+a)}-bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*
x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

3.431 $\int x^m \coth^2(a+bx)\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \coth^2(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Coth[a + b*x]^2, x]
```

Rubi [A] time = 0.031893, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \coth^2(a+bx)\,dx=\int x^m \coth^2(a+bx)\,dx$$

Mathematica [A] time = 9.63949, size = 0, normalized size = 0.

$$\int x^m \coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]^2, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int x^{m} (\cosh (bx+a))^{2} (\operatorname{csch} (bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(4\,bx+m\log(x)+4\,a)}}{(m+1)e^{(4\,bx+4\,a)}-2\,(m+1)e^{(2\,bx+2\,a)}+m+1}+\int\frac{\left(2\left(2\,bxe^{(4\,a)}+(m+1)e^{(4\,a)}\right)e^{(4\,bx)}-(m+1)e^{(2\,bx+2\,a)}-m-1\right)x^{m}}{(m+1)e^{(6\,bx+6\,a)}-3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) - 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) + integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
- (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m}\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh (bx + a)^2 \operatorname{csch}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.432 $\int x^3 \coth^2(a+bx)\,dx$

Optimal. Leaf size=87

$$\frac{3x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^3\coth(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rubi [A] time = 0.190154, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3720, 3716, 2190, 2531, 2282, 6589, 30}

$$\frac{3x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^3\coth(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^2(a+bx)\,dx &= -\frac{x^3\coth(a+bx)}{b}+\frac{3\int x^2\coth(a+bx)\,dx}{b}+\int x^3\,dx\\ &= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}-\frac{6\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx}{b}\\ &= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{6\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b^2}\\ &= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\int\operatorname{Li}_2\left(e^{2(a+bx)}\right)\,dx}{b^3}\\ &= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2(x)}{x}\,dx\right.}{2b^4}\\ &= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\operatorname{Li}_3\left(e^{2(a+bx)}\right)}{2b^4}\end{aligned}$$

Mathematica [B] time = 0.60123, size = 204, normalized size = 2.34

$$-\frac{e^{2a}\left(6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,-e^{-a-bx}\right)+\text{PolyLog}\left(3,-e^{-a-bx}\right)\right)+6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,e^{-a-bx}\right)+\text{PolyLog}\left(\right.\right.\right.}{\left(e^{2a}-1\right)b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^2,x]
```

```
[Out] x^4/4 - (E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^
(-a - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2
*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 6*(1 - E
^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)])))/(b^4*(
-1 + E^(2*a))) + (x^3*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [B] time = 0.056, size = 198, normalized size = 2.3

$$\frac{x^4}{4}-2\,\frac{x^3}{b\left(e^{2\,bx+2\,a}-1\right)}+3\,\frac{a^2\ln\left(e^{bx+a}-1\right)}{b^4}-6\,\frac{a^2\ln\left(e^{bx+a}\right)}{b^4}-2\,\frac{x^3}{b}+6\,\frac{a^2x}{b^3}+4\,\frac{a^3}{b^4}+3\,\frac{\ln\left(1+e^{bx+a}\right)x^2}{b^2}+6\,\frac{x\text{polylog}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln(
exp(b*x+a))-2*x^3/b+6/b^3*a^2*x+4/b^4*a^3+3/b^2*ln(1+exp(b*x+a))*x^2+6*x*po
lylog(2,-exp(b*x+a))/b^3-6*polylog(3,-exp(b*x+a))/b^4+3/b^2*ln(1-exp(b*x+a)
)*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,
exp(b*x+a))/b^4
```

Maxima [A] time = 1.38485, size = 197, normalized size = 2.26

$$-\frac{2x^3}{b}+\frac{bx^4e^{(2bx+2a)}-bx^4-8x^3}{4\left(be^{(2bx+2a)}-b\right)}+\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4}+\frac{3\left(b^2x^2\log\left(-e^{(bx+a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a) -
 b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polyl
og(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e
^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.16178, size = 1643, normalized size = 18.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*(
b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b^3
*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x
*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b^2*x^2
*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x
 + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2*cosh(b*
x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 - a^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 - a^2)*cosh(
b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - a^2
```

```
)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 24*(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*
x + a)^2 - b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.433 $\int x^2 \coth^2(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^2\coth(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[Out] -(x^2/b) + x^3/3 - (x^2*Coth[a + b*x])/b + (2*x*Log[1 - E^(2*(a + b*x))])/b
^2 + PolyLog[2, E^(2*(a + b*x))]/b^3
```

Rubi [A] time = 0.124658, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3716, 2190, 2279, 2391, 30}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^2\coth(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^2,x]
```

```
[Out] -(x^2/b) + x^3/3 - (x^2*Coth[a + b*x])/b + (2*x*Log[1 - E^(2*(a + b*x))])/b
^2 + PolyLog[2, E^(2*(a + b*x))]/b^3
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^2(a+bx)\,dx &= -\frac{x^2\coth(a+bx)}{b}+\frac{2\int x\coth(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}-\frac{4\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2(a+bx)}\right)dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}
\end{aligned}$$

Mathematica [C] time = 4.88174, size = 163, normalized size = 2.51

$$\frac{-\operatorname{PolyLog}\left(2,e^{-2\left(\tanh^{-1}(\tanh(a))+bx\right)}\right)-b^2x^2e^{-\tanh^{-1}(\tanh(a))}\coth(a)\sqrt{\operatorname{sech}^2(a)}+2bx\log\left(1-e^{-2\left(\tanh^{-1}(\tanh(a))+bx\right)}\right)+2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[a + b*x]^2,x]
```

```
[Out] x^3/3 + (I*b*Pi*x - I*Pi*Log[1 + E^(2*b*x)] + 2*b*x*Log[1 - E^(-2*(b*x + Ar
cTanh[Tanh[a]]))] + I*Pi*Log[Cosh[b*x]] + 2*ArcTanh[Tanh[a]]*(b*x + Log[1 -
 E^(-2*(b*x + ArcTanh[Tanh[a]]))] - Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]]) -
PolyLog[2, E^(-2*(b*x + ArcTanh[Tanh[a]]))] - (b^2*x^2*Coth[a]*Sqrt[Sech[a]
^2])/E^ArcTanh[Tanh[a]])/b^3 + (x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [B] time = 0.056, size = 156, normalized size = 2.4

$$\frac{x^3}{3}-2\,\frac{x^2}{b\left(\mathrm{e}^{2\,bx+2\,a}-1\right)}-2\,\frac{x^2}{b}-4\,\frac{ax}{b^2}-2\,\frac{a^2}{b^3}+2\,\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^2}+2\,\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^3}+2\,\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^2}+2\,\frac{\ln\left(1-\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/3*x^3-2*x^2/b/(exp(2*b*x+2*a)-1)-2*x^2/b-4/b^2*a*x-2/b^3*a^2+2/b^2*ln(1+e
xp(b*x+a))*x+2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln
(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^3*a*ln(exp(b*x+a)-1)+4/b^3
*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.40041, size = 146, normalized size = 2.25

$$-\frac{2\,x^2}{b}+\frac{bx^3e^{(2\,bx+2\,a)}-bx^3-6\,x^2}{3\left(be^{(2\,bx+2\,a)}-b\right)}+\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) - b*x^3 - 6*x^2)/(b*e^(2*b*x + 2*a) -
 b) + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-
e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Fricas [B] time = 2.17313, size = 1211, normalized size = 18.63

$$-\frac{b^3x^3 - \left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx + a\right)^2 - 2\left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right) - \left(b^3x^3 - 6\,b^2x^2 + 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 - 2*(b^3*x^3
- 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 + 6
*a^2)*sinh(b*x + a)^2 + 6*a^2 - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(
-cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
 a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sin
h(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*((b*x + a)*cos
h(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
 + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x +
 a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh\left(bx + a\right)^2 \operatorname{csch}\left(bx + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.434 $\int x \coth^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\log(\sinh(a+bx))}{b^2}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

```
[Out] x^2/2 - (x*Coth[a + b*x])/b + Log[Sinh[a + b*x]]/b^2
```

Rubi [A] time = 0.0281605, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3720, 3475, 30}

$$\frac{\log(\sinh(a+bx))}{b^2}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^2,x]
```

```
[Out] x^2/2 - (x*Coth[a + b*x])/b + Log[Sinh[a + b*x]]/b^2
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\coth^2(a+bx)\,dx = -\frac{x\coth(a+bx)}{b} + \frac{\int \coth(a+bx)\,dx}{b} + \int x\,dx$$
$$= \frac{x^2}{2} - \frac{x\coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}$$

Mathematica [A] time = 0.152163, size = 46, normalized size = 1.48

$$\frac{-2bx\coth(a) + 2\log(\sinh(a+bx)) + 2bx\,\text{csch}(a)\sinh(bx)\text{csch}(a+bx) + b^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]^2,x]
```

```
[Out] (b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b*
x]*Sinh[b*x])/(2*b^2)
```

Maple [A] time = 0.054, size = 54, normalized size = 1.7

$$\frac{x^2}{2} - 2\frac{x}{b} - 2\frac{a}{b^2} - 2\frac{x}{b\left(e^{2bx+2a}-1\right)} + \frac{\ln\left(e^{2bx+2a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/2*x^2-2*x/b-2/b^2*a-2*x/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(2*b*x+2*a)-1)
```

Maxima [B] time = 1.19411, size = 155, normalized size = 5.

$$-\frac{xe^{(2bx+2a)}}{be^{(2bx+2a)} - b} - \frac{bx^2 - \left(bx^2e^{(2a)} - 2xe^{(2a)}\right)e^{(2bx)}}{2\left(be^{(2bx+2a)} - b\right)} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e^(
-a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Fricas [B] time = 2.1674, size = 477, normalized size = 15.39

$$-\frac{b^2x^2 - \left(b^2x^2 - 4\,bx\right)\cosh\left(bx+a\right)^2 - 2\left(b^2x^2 - 4\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) - \left(b^2x^2 - 4\,bx\right)\sinh\left(bx+a\right)^2 - 2\left(\mathrm{co}\right.}{2\left(b^2\cosh\left(bx+a\right)^2 + 2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*cos
h(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(
b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15281, size = 132, normalized size = 4.26

$$\frac{b^2x^2e^{(2\,bx+2\,a)} - b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - 2\,\log\left(e^{(2\,bx+2\,a)} - 1\right)}{2\left(b^2e^{(2\,bx+2\,a)} - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*
x + 2*a) - b^2)
```

3.435 $\int \coth^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\coth(a+bx)}{b}$$

```
[Out] x - Coth[a + b*x]/b
```

Rubi [A] time = 0.00946, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3473, 8}

$$x-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2,x]
```

```
[Out] x - Coth[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\,dx &= -\frac{\coth(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0131388, size = 27, normalized size = 2.08

$$-\frac{\coth(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2,x]
```

```
[Out] -((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

Maple [A] time = 0.013, size = 18, normalized size = 1.4

$$\frac{bx+a-\coth(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(b*x+a-coth(b*x+a))
```

Maxima [A] time = 1.09518, size = 34, normalized size = 2.62

$$x+\frac{a}{b}+\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 2.20794, size = 82, normalized size = 6.31

$$\frac{(bx+1)\sinh(bx+a)-\cosh(bx+a)}{b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15989, size = 38, normalized size = 2.92

$$\frac{bx+a}{b} - \frac{2}{b\left(e^{(2\,bx+2\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

3.436 $\int \frac{\coth^2(a+bx)}{x} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]^2/x, x]
```

Rubi [A] time = 0.0300103, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x} dx = \int \frac{\coth^2(a+bx)}{x} dx$$

Mathematica [A] time = 0.378467, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^2/x,x]
```

```
[Out] Integrate[Coth[a + b*x]^2/x, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^2 (\operatorname{csch}(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{bxe^{(2\,bx+2\,a)}-bx}+\int \frac{1}{bx^2e^{(bx+a)}+bx^2}\,dx-\int \frac{1}{bx^2e^{(bx+a)}-bx^2}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x
) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^2 \operatorname{csch} (bx+a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

3.437 $\int \frac{\coth^2(a+bx)}{x^2} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]^2/x^2, x]
```

Rubi [A] time = 0.0306246, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 0.890475, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Coth[a + b*x]^2/x^2, x]
```

Maple [A] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{\left(\cosh \left(bx+a\right)\right)^{2} \left(\operatorname{csch}\left(bx+a\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bxe^{(2\,bx+2\,a)} - bx + 2}{bx^{2}e^{(2\,bx+2\,a)} - bx^{2}} + 2\int \frac{1}{bx^{3}e^{(bx+a)} + bx^{3}}\,dx - 2\int \frac{1}{bx^{3}e^{(bx+a)} - bx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integr
ate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) -
b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^2 \operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

3.438 $\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=77

$$\text{CannotIntegrate}\left(x^m \coth(a+bx)\text{csch}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1$$

```
[Out] CannotIntegrate[x^m*Coth[a + b*x]*Csch[a + b*x], x] + (E^a*x^m*Gamma[1 + m,
 -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(2*b*E^a*(b*x)^m)
```

Rubi [A] time = 0.153311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx &= \int x^m\cosh(a+bx)\,dx+\int x^m\coth(a+bx)\text{csch}(a+bx)\,dx\\ &=\frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx+\int x^m\coth(a+bx)\text{csch}(a+bx)\,dx\\ &=\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}+\int x^m\coth(a+bx)\text{csch}\end{aligned}$$

Mathematica [A] time = 15.9238, size = 0, normalized size = 0.

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int x^m (\cosh (bx+a))^3 (\operatorname{csch}(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh (bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \cosh (bx+a)^3 \operatorname{csch}(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh (bx + a)^3 \operatorname{csch} (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.439 $\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=143

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}$$

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a +
 b*x])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + (
6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - (6*Po
lyLog[3, E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^3*Sinh[a + b*x])/
b
```

Rubi [A] time = 0.184215, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5450, 3296, 2638, 5419, 4182, 2531, 2282, 6589}

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```


```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a +
 b*x])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + (
6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - (6*Po
lyLog[3, E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^3*Sinh[a + b*x])/
b
```


Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx=\int x^3\cosh(a+bx)\,dx+\int x^3\coth(a+bx)\mathrm{csch}(a+bx)\,dx$$
$$=-\frac{x^3\mathrm{csch}(a+bx)}{b}+\frac{x^3\sinh(a+bx)}{b}+\frac{3\int x^2\mathrm{csch}(a+bx)\,dx}{b}-\frac{3\int x^2\sinh(a+bx)\,dx}{b}$$
$$=-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\mathrm{csch}(a+bx)}{b}+\frac{x^3\sinh(a+bx)}{b}+$$
$$=-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\mathrm{csch}(a+bx)}{b}-\frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3}+$$
$$=-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6\cosh(a+bx)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\mathrm{csch}(a+bx)}{b}-$$
$$=-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6\cosh(a+bx)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\mathrm{csch}(a+bx)}{b}-$$

Mathematica [A] time = 0.325346, size = 225, normalized size = 1.57

$$\frac{\mathrm{csch}\left(\frac{1}{2}(a+bx)\right)\mathrm{sech}\left(\frac{1}{2}(a+bx)\right)\left(-12bx\sinh(a+bx)\mathrm{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx))+12bx\sinh(a+bx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-6*b*x - 3*b^3*x^3 + 6*b*x*Cosh[2*(a
+ b*x)] + b^3*x^3*Cosh[2*(a + b*x)] - 12*b^2*x^2*ArcTanh[Cosh[a + b*x] + Si
nh[a + b*x]]*Sinh[a + b*x] - 12*b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*
x]]*Sinh[a + b*x] + 12*b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a
 + b*x] + 12*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]]*Sinh[a + b*x] - 12*
PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a + b*x] - 6*Sinh[2*(a + b*x
)] - 3*b^2*x^2*Sinh[2*(a + b*x)]))/(4*b^4)
```

Maple [A] time = 0.079, size = 241, normalized size = 1.7

$$\frac{\left(x^3b^3-3x^2b^2+6bx-6\right)e^{bx+a}}{2b^4}-\frac{\left(x^3b^3+3x^2b^2+6bx+6\right)e^{-bx-a}}{2b^4}-2\frac{e^{bx+a}x^3}{b\left(e^{2bx+2a}-1\right)}-6\frac{a^2\mathrm{Artanh}\left(e^{bx+a}\right)}{b^4}-3\frac{\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)-1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)-2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh
(exp(b*x+a))-3/b^2*ln(1+exp(b*x+a))*x^2+3/b^4*ln(1+exp(b*x+a))*a^2-6*x*poly
log(2,-exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4+3/b^2*ln(1-exp(b*x+a))*
x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,ex
p(b*x+a))/b^4
```

Maxima [A] time = 1.46294, size = 292, normalized size = 2.04

$$\frac{\left(b^3x^3e^{(4\,a)} - 3\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 6\,e^{(4\,a)}\right)e^{(3\,bx)} - 6\left(b^3x^3e^{(2\,a)} + 2\,bxe^{(2\,a)}\right)e^{(bx)} + \left(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6\right)e^{(-bx)}}{2\left(b^4e^{(2\,bx+3\,a)} - b^4e^a\right)} - \frac{3\,(b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3
*b*x) - 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2
+ 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) - b^4*e^a) - 3*(b^2*x^2*log(e^(
b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4
 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(
3, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.57744, size = 2685, normalized size = 18.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^
2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 - 6*(b^3*x^3 + 2*b*x)*cosh(b
*x + a)^2 - 6*(b^3*x^3 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2
+ 2*b*x)*sinh(b*x + a)^2 + 6*b*x + 12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x
 + a)*sinh(b*x + a)^2 + b*x*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*co
```

```
sh(b*x + a)^2 - b*x)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) -
12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*sinh(b*
x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a))
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b^2*x^2*cosh(b*x + a)^3 + 3*b^2
*x^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*x^2*sinh(b*x + a)^3 - b^2*x^2*cosh
(b*x + a) + (3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + 6*(a^2*cosh(b*x + a)^3 + 3*a^2*cosh(b*x + a)
*sinh(b*x + a)^2 + a^2*sinh(b*x + a)^3 - a^2*cosh(b*x + a) + (3*a^2*cosh(b*
x + a)^2 - a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(
(b^2*x^2 - a^2)*cosh(b*x + a)^3 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a)^2 + (b^2*x^2 - a^2)*sinh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a) -
(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a))*log(-cos
h(b*x + a) - sinh(b*x + a) + 1) - 12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sin
h(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - co
sh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 12*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^3 - 3*(b^3*x^3 + 2
*b*x)*cosh(b*x + a))*sinh(b*x + a) + 6)/(b^4*cosh(b*x + a)^3 + 3*b^4*cosh(b
*x + a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3 - b^4*cosh(b*x + a) + (3*b^4*
cosh(b*x + a)^2 - b^4)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.440 $\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx$

Optimal. Leaf size=95

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}-\frac{2x\cosh(a+bx)}{b^2}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\sinh(a+}{b}$$

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (2*x*Cosh[a + b*x])/b^2 - (x^2*Csch[a + b
*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rubi [A] time = 0.117816, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5450, 3296, 2637, 5419, 4182, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}-\frac{2x\cosh(a+bx)}{b^2}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\sinh(a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (2*x*Cosh[a + b*x])/b^2 - (x^2*Csch[a + b
*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^2\cosh(a+bx)\,dx + \int x^2\coth(a+bx)\mathrm{csch}(a+bx)\,dx$$
$$= -\frac{x^2\mathrm{csch}(a+bx)}{b} + \frac{x^2\sinh(a+bx)}{b} + \frac{2\int x\mathrm{csch}(a+bx)\,dx}{b} - \frac{2\int x\sinh(a+bx}{b}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\mathrm{csch}(a+bx)}{b} + \frac{x^2\sinh(a+bx)}{b} + \frac{2}{}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\mathrm{csch}(a+bx)}{b} + \frac{2\sinh(a+bx)}{b^3} + \frac{x^2}{}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\mathrm{csch}(a+bx)}{b} - \frac{2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{2\mathrm{Li}}{}$$

Mathematica [B] time = 0.438756, size = 230, normalized size = 2.42

$$\frac{\mathrm{csch}\left(\frac{1}{2}(a+bx)\right)\mathrm{sech}\left(\frac{1}{2}(a+bx)\right)\left(4\sinh(a+bx)\mathrm{PolyLog}\left(2,-e^{-a-bx}\right) - 4\sinh(a+bx)\mathrm{PolyLog}\left(2,e^{-a-bx}\right) + b^2x^2\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-2 - 3*b^2*x^2 + 2*Cosh[2*(a + b*x)]
+ b^2*x^2*Cosh[2*(a + b*x)] + 4*a*Log[1 - E^(-a - b*x)]*Sinh[a + b*x] + 4*b
*x*Log[1 - E^(-a - b*x)]*Sinh[a + b*x] - 4*a*Log[1 + E^(-a - b*x)]*Sinh[a +
 b*x] - 4*b*x*Log[1 + E^(-a - b*x)]*Sinh[a + b*x] - 4*a*Log[Tanh[(a + b*x)/
2]]*Sinh[a + b*x] + 4*PolyLog[2, -E^(-a - b*x)]*Sinh[a + b*x] - 4*PolyLog[2
, E^(-a - b*x)]*Sinh[a + b*x] - 2*b*x*Sinh[2*(a + b*x)]))/(4*b^3)
```

Maple [A] time = 0.075, size = 185, normalized size = 2.

$$\frac{\left(x^2b^2 - 2\,bx + 2\right)\mathrm{e}^{bx+a}}{2\,b^3} - \frac{\left(x^2b^2 + 2\,bx + 2\right)\mathrm{e}^{-bx-a}}{2\,b^3} - 2\,\frac{\mathrm{e}^{bx+a}x^2}{b\left(\mathrm{e}^{2\,bx+2\,a}-1\right)} - 2\,\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^2} - 2\,\frac{a\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^3} - 2\,\frac{\mathrm{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-
2*x^2*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)-2/b^2*ln(1+exp(b*x+a))*x-2/b^3*ln(1+e
xp(b*x+a))*a-2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln
(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.44217, size = 212, normalized size = 2.23

$$\frac{\left(b^2x^2e^{(4a)} - 2\,bxe^{(4a)} + 2\,e^{(4a)}\right)e^{(3\,bx)} - 2\left(3\,b^2x^2e^{(2a)} + 2\,e^{(2a)}\right)e^{(bx)} + \left(b^2x^2 + 2\,bx + 2\right)e^{(-bx)}}{2\left(b^3e^{(2\,bx+3\,a)} - b^3e^a\right)} - \frac{2\left(bx\log\left(e^{(bx+a)} + 1\right) + \mathrm{Li}_2\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) - 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) - b^3*e^a) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b
^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Fricas [B] time = 2.41477, size = 1952, normalized size = 20.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(3*b^2*x^2 - 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + 4*(cosh(b*x + a)^3 + 3*cosh
(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(
b*x + a) - cosh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(cosh(b*
x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 4*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*
sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 4*(a*cosh(b*x + a)^3 + 3*a
*cosh(b*x + a)*sinh(b*x + a)^2 + a*sinh(b*x + a)^3 - a*cosh(b*x + a) + (3*a
```

```
*cosh(b*x + a)^2 - a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 4*((b*x + a)*cosh(b*x + a)^3 + 3*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^2
 + (b*x + a)*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a) + (3*(b*x + a)*cosh(
b*x + a)^2 - b*x - a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3 - (3*b^2*x^2 + 2)*cosh(b*x + a
))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)*sinh(b*x +
 a)^2 + b^3*sinh(b*x + a)^3 - b^3*cosh(b*x + a) + (3*b^3*cosh(b*x + a)^2 -
b^3)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.441 $\int x \cosh(a+bx) \coth^2(a+bx) \, dx$

Optimal. Leaf size=47

$$-\frac{\cosh(a+bx)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - Cosh[a + b*x]/b^2 - (x*Csch[a + b*x])/b + (
x*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0541248, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5450, 3296, 2638, 5419, 3770}

$$-\frac{\cosh(a+bx)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - Cosh[a + b*x]/b^2 - (x*Csch[a + b*x])/b + (
x*Sinh[a + b*x])/b
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\coth^2(a+bx)\,dx &= \int x\cosh(a+bx)\,dx+\int x\coth(a+bx)\text{csch}(a+bx)\,dx\\ &= -\frac{x\text{csch}(a+bx)}{b}+\frac{x\sinh(a+bx)}{b}+\frac{\int \text{csch}(a+bx)\,dx}{b}-\frac{\int \sinh(a+bx)\,dx}{b}\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{x\text{csch}(a+bx)}{b}+\frac{x\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.218691, size = 66, normalized size = 1.4

$$\frac{2bx\sinh(a+bx)-2\cosh(a+bx)+bx\tanh\left(\frac{1}{2}(a+bx)\right)-bx\coth\left(\frac{1}{2}(a+bx)\right)+2\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x] - b*x*Coth[(a + b*x)/2] + 2*Log[Tanh[(a + b*x)/2]] + 2*b*
x*Sinh[a + b*x] + b*x*Tanh[(a + b*x)/2])/(2*b^2)
```

Maple [A] time = 0.077, size = 89, normalized size = 1.9

$$\frac{(bx-1)\,e^{bx+a}}{2\,b^2}-\frac{(bx+1)\,e^{-bx-a}}{2\,b^2}-2\,\frac{e^{bx+a}x}{b\left(e^{2\,bx+2\,a}-1\right)}-\frac{\ln\left(1+e^{bx+a}\right)}{b^2}+\frac{\ln\left(e^{bx+a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)-2*x*exp(b*x+a)/b/(ex
p(2*b*x+2*a)-1)-1/b^2*ln(1+exp(b*x+a))+1/b^2*ln(exp(b*x+a)-1)
```

Maxima [B] time = 1.36774, size = 147, normalized size = 3.13

$$-\frac{6\,bxe^{(bx+2\,a)} - \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} - (bx+1)e^{(-bx)}}{2\left(b^2e^{(2\,bx+3\,a)} - b^2e^a\right)} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b*x*e^(b*x + 2*a) - (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) - (b*x + 1)*e
^(-b*x))/(b^2*e^(2*b*x + 3*a) - b^2*e^a) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Fricas [B] time = 2.29503, size = 1008, normalized size = 21.45

$$\frac{(bx-1)\cosh(bx+a)^4 + 4\,(bx-1)\cosh(bx+a)\sinh(bx+a)^3 + (bx-1)\sinh(bx+a)^4 - 6\,bx\cosh(bx+a)^2 + 6\,\big((bx-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 + b*x - 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
- cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 4*((b*x - 1)*cosh(b*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b
^2*cosh(b*x + a)^3 + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a
```

```
)^3 - b^2*cosh(b*x + a) + (3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23585, size = 194, normalized size = 4.13

$$\frac{bxe^{(4\,bx+4\,a)}-6\,bxe^{(2\,bx+2\,a)}+bx-2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}+1\right)+2\,e^{(bx+a)}\log\left(e^{(bx+a)}+1\right)+2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}-1\right)-2}{2\left(b^2e^{(3\,bx+3\,a)}-b^2e^{(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x*e^(4*b*x + 4*a) - 6*b*x*e^(2*b*x + 2*a) + b*x - 2*e^(3*b*x + 3*a)*
log(e^(b*x + a) + 1) + 2*e^(b*x + a)*log(e^(b*x + a) + 1) + 2*e^(3*b*x + 3*
a)*log(e^(b*x + a) - 1) - 2*e^(b*x + a)*log(e^(b*x + a) - 1) - e^(4*b*x + 4
*a) + 1)/(b^2*e^(3*b*x + 3*a) - b^2*e^(b*x + a))
```

3.442 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b

Rubi [A] time = 0.0221304, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]

[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{i\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{i\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.0129803, size = 22, normalized size = 1.

$$\frac{\sinh(a+bx)}{b} - \frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Maple [A] time = 0.013, size = 32, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{\sinh(bx+a)} + 2\sinh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)*cosh(b*x+a)^2+2*sinh(b*x+a))
```

Maxima [B] time = 1.11023, size = 76, normalized size = 3.45

$$-\frac{e^{(-bx-a)}}{2\,b} - \frac{5\,e^{(-2\,bx-2\,a)} - 1}{2\,b\left(e^{(-bx-a)} - e^{(-3\,bx-3\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

Fricas [A] time = 2.19848, size = 85, normalized size = 3.86

$$\frac{\cosh(bx+a)^2 + \sinh(bx+a)^2 - 3}{2\,b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16081, size = 66, normalized size = 3.

$$\frac{e^{(bx+a)} - e^{(-bx-a)}}{2\,b} - \frac{2}{b\left(e^{(bx+a)} - e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) - e^(-b*x - a))/b - 2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.443 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

Optimal. Leaf size=33

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x},x\right)+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x])/x, x] + Cosh[a]*CoshIntegral[
b*x] + Sinh[a]*SinhIntegral[b*x]
```

Rubi [A] time = 0.104907, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] + Defer[Int][(Coth[a
+ b*x]*Csch[a + b*x])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx &= \int \frac{\cosh(a+bx)}{x}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx\\ &= \cosh(a)\int \frac{\cosh(bx)}{x}\,dx+\sinh(a)\int \frac{\sinh(bx)}{x}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx\\ &= \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 23.097, size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^3 (\mathrm{csch}(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{2}\,\mathrm{Ei}(bx)\,e^{a} - \frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)} - bx} - \int \frac{1}{bx^2e^{(bx+a)} + bx^2}\,dx - \int \frac{1}{bx^2e^{(bx+a)} - bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a - 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
- b*x) - integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x) - integrate(1/(b*x^2*e
^(b*x + a) - b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh(bx+a)^3\,\mathrm{csch}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

3.444 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=46

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x^2},x\right)+b\sinh(a)\text{Chi}(bx)+b\cosh(a)\text{Shi}(bx)-\frac{\cosh(a+bx)}{x}$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x] - Cosh[a + b*x]/x + b
*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b*x]
```

Rubi [A] time = 0.136694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] + Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx &= \int \frac{\cosh(a+bx)}{x^2}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx\\ &= -\frac{\cosh(a+bx)}{x}+b\int \frac{\sinh(a+bx)}{x}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx\\ &= -\frac{\cosh(a+bx)}{x}+(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx+\int \frac{\coth}{}\\ &= -\frac{\cosh(a+bx)}{x}+b\text{Chi}(bx)\sinh(a)+b\cosh(a)\text{Shi}(bx)+\int \frac{\coth(a+bx)\text{csch}(a+}{x^2}\end{aligned}$$

Mathematica [A] time = 20.4555, size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(\cosh (bx + a))^3 (\operatorname{csch} (bx + a))^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} be^{(-a)}\Gamma (-1, bx) + \frac{1}{2} be^{a}\Gamma (-1, -bx) - \frac{2 e^{(bx+a)}}{bx^2e^{(2 bx+2 a)} - bx^2} - 2 \int \frac{1}{bx^3e^{(bx+a)} + bx^3} \, dx - 2 \int \frac{1}{bx^3e^{(bx+a)} - bx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) - 2*e^(b*x + a)/(b
*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*x + a) + b*x^3),
x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh (bx + a)^3 \operatorname{csch} (bx + a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^3 \operatorname{csch} (bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

3.445 $\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m \coth(a+bx)\operatorname{csch}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Rubi [A] time = 0.470627, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Mathematica [A] time = 8.1496, size = 0, normalized size = 0.

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\left(\operatorname{csch}(bx+a)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.446 $\int x^3 \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}-\frac{3x^2\coth(a+bx)}{2b^2}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3x^2}{2b^2}$$

```
[Out] (-3*x^2)/(2*b^2) - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3*Csch[a + b*x]^2)/(2
*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/
(2*b^4)
```

Rubi [A] time = 0.163233, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5419, 4184, 3716, 2190, 2279, 2391}

$$\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}-\frac{3x^2\coth(a+bx)}{2b^2}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```


```
[Out] (-3*x^2)/(2*b^2) - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3*Csch[a + b*x]^2)/(2
*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/
(2*b^4)
```


Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3\int x^2\text{csch}^2(a+bx)\,dx}{2b}$$
$$= -\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3\int x\coth(a+bx)\,dx}{b^2}$$
$$= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{6\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b^2}$$
$$= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3\int \log\left(1\right.}{}$$
$$= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{Subst}\left(\right.}{}$$
$$= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3\text{Li}_2\left(e^{2(a+\right.}}{2b^4}$$

Mathematica [C] time = 6.1293, size = 228, normalized size = 2.75

$$\frac{3\mathrm{csch}(a)\mathrm{sech}(a)\left(-b^2x^2e^{-\tanh^{-1}(\tanh(a))}+\frac{i\tanh(a)\left(i\mathrm{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}(\tanh(a))+ibx\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\tanh(a))\right)-2\left(i\tanh^{-1}(\tanh(a))+ib\right.}{}\right)}{2b^4\sqrt{\mathrm{sech}^2(a)\left(\cosh^2(a)-\sin\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -(x^3*Csch[a + b*x]^2)/(2*b) + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b
^2) + (3*Csch[a]*Sech[a]*(-((b^2*x^2)/E^ArcTanh[Tanh[a]]) + (I*(-(b*x*(-Pi
+ (2*I)*ArcTanh[Tanh[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*b*x + I*ArcTanh[T
anh[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))] + Pi*Log[Cosh[b*x]
] + (2*I)*ArcTanh[Tanh[a]]*Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]] + I*PolyLog[
2, E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))])*Tanh[a])/Sqrt[1 - Tanh[a]^2]))/
(2*b^4*Sqrt[Sech[a]^2*(Cosh[a]^2 - Sinh[a]^2)])
```

Maple [B] time = 0.032, size = 177, normalized size = 2.1

$$-\frac{x^2\left(2\,bx\mathrm{e}^{2\,bx+2\,a}+3\,\mathrm{e}^{2\,bx+2\,a}-3\right)}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2}-3\,\frac{x^2}{b^2}-6\,\frac{ax}{b^3}-3\,\frac{a^2}{b^4}+3\,\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^3}+3\,\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^4}+3\,\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-3*x
^2/b^2-6/b^3*a*x-3/b^4*a^2+3/b^3*ln(1+exp(b*x+a))*x+3/b^4*polylog(2,-exp(b*
x+a))+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a+3/b^4*polylog(2,exp
(b*x+a))-3/b^4*a*ln(exp(b*x+a)-1)+6/b^4*a*ln(exp(b*x+a))
```

Maxima [A] time = 1.53725, size = 176, normalized size = 2.12

$$\frac{3\,x^2-\left(2\,bx^3e^{(2\,a)}+3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{3\,x^2}{b^2}+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+\right.}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
- 2*b^2*e^(2*b*x + 2*a) + b^2) - 3*x^2/b^2 + 3*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^4
```

Fricas [B] time = 2.35122, size = 2545, normalized size = 30.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*
x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*
x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^4 + 4*c
osh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x +
a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a
)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) +
 a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b*x + a)*cosh(b*x + a)^4 +
 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*
(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(
b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*
sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(6*(b^2*x^2
- a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sin
h(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^
4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2
- b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b^4*cosh(b*x + a))*sinh(b
```

```
*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)\operatorname{csch}(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.447 $\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -((x*Coth[a + b*x])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]
/b^3
```

Rubi [A] time = 0.0581759, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5419, 4184, 3475}

$$-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -((x*Coth[a + b*x])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]
/b^3
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{x^2\text{csch}^2(a+bx)}{2b} + \frac{\int x\text{csch}^2(a+bx)\,dx}{b}$$
$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\text{csch}^2(a+bx)}{2b} + \frac{\int \coth(a+bx)\,dx}{b^2}$$
$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\text{csch}^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3}$$

Mathematica [A] time = 0.114799, size = 55, normalized size = 1.31

$$-\frac{x\coth(a)}{b^2} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\text{csch}(a)\sinh(bx)\text{csch}(a+bx)}{b^2} - \frac{x^2\text{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -((x*Coth[a])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 +
 (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

Maple [A] time = 0.03, size = 72, normalized size = 1.7

$$-2\,\frac{x}{b^2} - 2\,\frac{a}{b^3} - 2\,\frac{x\left(bx\text{e}^{2\,bx+2\,a} + \text{e}^{2\,bx+2\,a} - 1\right)}{b^2\left(\text{e}^{2\,bx+2\,a} - 1\right)^2} + \frac{\ln\left(\text{e}^{2\,bx+2\,a} - 1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -2*x/b^2-2/b^3*a-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2
*a)-1)^2+1/b^3*ln(exp(2*b*x+2*a)-1)
```

Maxima [B] time = 1.52887, size = 144, normalized size = 3.43

$$-\frac{2\left(\left(bx^2e^{(2\,a)} - xe^{(2\,a)}\right)e^{(2\,bx)} + xe^{(4\,bx+4\,a)}\right)}{b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2} + \frac{\log\left(\left(e^{(bx+a)} + 1\right)e^{(-a)}\right)}{b^3} + \frac{\log\left(\left(e^{(bx+a)} - 1\right)e^{(-a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) + x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + log((e^(b*x + a) + 1)*e^(-a))/b^3
 + log((e^(b*x + a) - 1)*e^(-a))/b^3
```

Fricas [B] time = 2.46102, size = 979, normalized size = 23.31

$$-\frac{2\,bx\cosh\left(bx+a\right)^{4}+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+2\,bx\sinh\left(bx+a\right)^{4}+2\left(b^{2}x^{2}-bx\right)\cosh\left(bx+a\right)^{2}+2\left(b^{2}x^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(
b*x + a)^4 + 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 6*b*x*cosh(b*
x + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a
)^3 + (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 +
4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x
+ a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cos
h(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17487, size = 188, normalized size = 4.48

$$-\frac{2\,b^2x^2e^{(2\,bx+2\,a)} + 2\,bxe^{(4\,bx+4\,a)} - 2\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - \log\left(e^{(2\,bx+2\,a)} - 1\right)}{b^3e^{(4\,bx+4\,a)} - 2\,b^3e^{(2\,bx+2\,a)} + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) + 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(e^(2*b*
x + 2*a) - 1) - log(e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) - 2*b^3*e^(2
*b*x + 2*a) + b^3)
```

3.448 $\int x \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\mathrm{csch}^2(a+bx)}{2b}$$

```
[Out] -Coth[a + b*x]/(2*b^2) - (x*Csch[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0299658, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5419, 3767, 8}

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\mathrm{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]/(2*b^2) - (x*Csch[a + b*x]^2)/(2*b)
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{x\operatorname{csch}^2(a+bx)}{2b} + \frac{\int \operatorname{csch}^2(a+bx)\,dx}{2b}$$
$$= -\frac{x\operatorname{csch}^2(a+bx)}{2b} - \frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(a+bx)\right)}{2b^2}$$
$$= -\frac{\coth(a+bx)}{2b^2} - \frac{x\operatorname{csch}^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0726683, size = 30, normalized size = 1.

$$-\frac{\coth(a+bx)}{2b^2} - \frac{x\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]/(2*b^2) - (x*Csch[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.028, size = 43, normalized size = 1.4

$$-\frac{2\,bx e^{2\,bx+2\,a} + e^{2\,bx+2\,a} - 1}{b^2\left(e^{2\,bx+2\,a} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
```

Maxima [B] time = 1.18956, size = 176, normalized size = 5.87

$$\frac{2\,bxe^{(4\,bx+4\,a)} - \left(4\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 1}{2\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{2\,bxe^{(4\,bx+4\,a)} + e^{(2\,bx+2\,a)} - 1}{2\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*e^(4*b*x + 4*a) - (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*
e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(2*b*x*e^(4*b*x + 4*a)
 + e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

Fricas [B] time = 2.21284, size = 267, normalized size = 8.9

$$-\frac{2\left(bx\cosh\left(bx+a\right)+\left(bx+1\right)\sinh\left(bx+a\right)\right)}{b^2\cosh\left(bx+a\right)^3+3\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b^2\sinh\left(bx+a\right)^3-b^2\cosh\left(bx+a\right)+3\left(b^2\cosh\left(bx+a\right)^2-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + (b*x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 - b^2*cosh(b*x + a)
+ 3*(b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16929, size = 248, normalized size = 8.27

$$-\frac{4\,bxe^{(2\,bx+2\,a)}-e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}+1\right)-2\,e^{(2\,bx+2\,a)}}{2\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
2*a) + 1) - 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) + 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) + 1) - 2)/(b^2*e^(4*b*x
+ 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

3.449 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0210658, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0113638, size = 15, normalized size = 1.

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$-\frac{(\operatorname{csch}(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -1/2*csch(b*x+a)^2/b
```

Maxima [A] time = 1.13012, size = 34, normalized size = 2.27

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a))^2)
```

Fricas [B] time = 2.29264, size = 232, normalized size = 15.47

$$-\frac{2(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3-b\cosh(bx+a)+3\left(b\cosh(bx+a)^2-b\right)\sinh(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22733, size = 36, normalized size = 2.4

$$-\frac{2 e^{(2 bx+2 a)}}{b\left(e^{(2 bx+2 a)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.450 $\int \frac{\coth(a+bx)\mathbf{csch}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Rubi [A] time = 0.162428, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 15.0187, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{\cosh (bx + a) (\operatorname{csch} (bx + a))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)} + 1}{b^2x^2e^{(4\,bx+4\,a)} - 2\,b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} - 4\int \frac{1}{4\left(b^2x^3e^{(bx+a)} + b^2x^3\right)}\,dx + 4\int \frac{1}{4\left(b^2x^3e^{(bx+a)} - b^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - 4*integrate(1/4/(b^2*x^3*e^(b*x + a) + b^
2*x^3), x) + 4*integrate(1/4/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh (bx + a) \operatorname{csch} (bx + a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

3.451 $\int \frac{\coth(a+bx)\mathbf{csch}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Rubi [A] time = 0.236077, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 19.7865, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a) (\operatorname{csch} (bx+a))^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}+1\right)}{b^{2}x^{3}e^{(4\,bx+4\,a)}-2\,b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}}-12\int \frac{1}{4\left(b^{2}x^{4}e^{(bx+a)}+b^{2}x^{4}\right)}\,dx+12\int \frac{1}{4\left(b^{2}x^{4}e^{(bx+a)}-b^{2}x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 12*integrate(1/4/(b^2*x^4*e^(b*x + a) + b
^2*x^4), x) + 12*integrate(1/4/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh (bx+a)\operatorname{csch} (bx+a)^{3}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

3.452 $\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(x^m\operatorname{csch}^3(a+bx),x\right)+\text{Unintegrable}\left(x^m\operatorname{csch}(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Csch[a + b*x], x] + Unintegrable[x^m*Csch[a + b*x]^3, x]
```

Rubi [A] time = 0.0743735, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x], x] + Defer[Int][x^m*Csch[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\int x^m\operatorname{csch}(a+bx)\,dx+\int x^m\operatorname{csch}^3(a+bx)\,dx$$

Mathematica [A] time = 23.8803, size = 0, normalized size = 0.

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x], x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int x^m (\cosh(bx+a))^2 (\operatorname{csch}(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.453 $\int x^3 \coth^2(a + bx)\mathbf{csch}(a + bx)\,dx$

Optimal. Leaf size=201

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{3x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3\text{PolyLog}\left(2,\right.}{b^4}$$

```
[Out] (-6*x*ArcTanh[E^(a + b*x)])/b^3 - (x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*Csc
h[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (3*PolyLog[
2, -E^(a + b*x)])/b^4 - (3*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*PolyL
og[2, E^(a + b*x)])/b^4 + (3*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) + (3*x*Po
lyLog[3, -E^(a + b*x)])/b^3 - (3*x*PolyLog[3, E^(a + b*x)])/b^3 - (3*PolyLo
g[4, -E^(a + b*x)])/b^4 + (3*PolyLog[4, E^(a + b*x)])/b^4
```

Rubi [A] time = 0.35973, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5457, 4182, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{3x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3\text{PolyLog}\left(2,\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```


```
[Out] (-6*x*ArcTanh[E^(a + b*x)])/b^3 - (x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*Csc
h[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (3*PolyLog[
2, -E^(a + b*x)])/b^4 - (3*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*PolyL
og[2, E^(a + b*x)])/b^4 + (3*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) + (3*x*Po
lyLog[3, -E^(a + b*x)])/b^3 - (3*x*PolyLog[3, E^(a + b*x)])/b^3 - (3*PolyLo
g[4, -E^(a + b*x)])/b^4 + (3*PolyLog[4, E^(a + b*x)])/b^4
```


Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x^3\text{csch}(a+bx)\,dx+\int x^3\text{csch}^3(a+bx)\,dx$$
$$=-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int x$$
$$=-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{cs}}{2b}$$
$$=-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{cs}}{2b}$$
$$=-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{cs}}{2b}$$
$$=-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{cs}}{2b}$$
$$=-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{cs}}{2b}$$

Mathematica [A] time = 6.84958, size = 280, normalized size = 1.39

$$-\frac{12\left(b^2x^2+2\right)\text{PolyLog}\left(2,-e^{a+bx}\right)-12\left(b^2x^2+2\right)\text{PolyLog}\left(2,e^{a+bx}\right)-24bx\text{PolyLog}\left(3,-e^{a+bx}\right)+24bx\text{PolyLog}\left(3,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -(12*b^2*x^2*Csch[a] + b^3*x^3*Csch[(a + b*x)/2]^2 - 24*b*x*Log[1 - E^(a +
b*x)] - 4*b^3*x^3*Log[1 - E^(a + b*x)] + 24*b*x*Log[1 + E^(a + b*x)] + 4*b^
3*x^3*Log[1 + E^(a + b*x)] + 12*(2 + b^2*x^2)*PolyLog[2, -E^(a + b*x)] - 12
*(2 + b^2*x^2)*PolyLog[2, E^(a + b*x)] - 24*b*x*PolyLog[3, -E^(a + b*x)] +
24*b*x*PolyLog[3, E^(a + b*x)] + 24*PolyLog[4, -E^(a + b*x)] - 24*PolyLog[4
, E^(a + b*x)] + b^3*x^3*Sech[(a + b*x)/2]^2 - 6*b^2*x^2*Csch[a/2]*Csch[(a
+ b*x)/2]*Sinh[(b*x)/2] - 6*b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/
2])/(8*b^4)
```

Maple [A] time = 0.078, size = 340, normalized size = 1.7

$$-\frac{x^2 e^{bx+a}\left(bx e^{2bx+2a}+bx+3e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-3\frac{\mathrm{polylog}\left(2,-e^{bx+a}\right)}{b^4}-3\frac{\mathrm{polylog}\left(4,-e^{bx+a}\right)}{b^4}+6\frac{a\mathrm{Artanh}\left(e^{bx+a}\right)}{b^4}+\frac{a^3\mathrm{Ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -x^2*exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+
2*a)-1)^2-3/b^4*polylog(2,-exp(b*x+a))-3/b^4*polylog(4,-exp(b*x+a))+6/b^4*a
*arctanh(exp(b*x+a))+1/b^4*a^3*arctanh(exp(b*x+a))+3/b^4*polylog(4,exp(b*x+
a))+3/b^4*polylog(2,exp(b*x+a))-3/b^4*a*ln(1+exp(b*x+a))+3/b^4*ln(1-exp(b*x
+a))*a-1/2/b*ln(1+exp(b*x+a))*x^3-3/2/b^2*polylog(2,-exp(b*x+a))*x^2+3/b^3*
polylog(3,-exp(b*x+a))*x+1/2/b*ln(1-exp(b*x+a))*x^3+3/2/b^2*polylog(2,exp(b
*x+a))*x^2-3/b^3*polylog(3,exp(b*x+a))*x-3/b^3*ln(1+exp(b*x+a))*x+3/b^3*ln(
1-exp(b*x+a))*x-1/2/b^4*ln(1+exp(b*x+a))*a^3+1/2/b^4*ln(1-exp(b*x+a))*a^3
```

Maxima [A] time = 1.62837, size = 354, normalized size = 1.76

$$-\frac{\left(bx^3e^{(3a)}+3x^2e^{(3a)}\right)e^{(3bx)}+\left(bx^3e^a-3x^2e^a\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6bx\mathrm{Li}_3(-e^{(bx+a)})+6}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) + (b*x^3*e^a - 3*x^2*e^a)*e^(b*
x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^3*x^3*log(
e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x
```

```
 + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + 1/2*(b^3*x^3*log(-e^(b*x + a) +
1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polyl
og(4, e^(b*x + a)))/b^4 - 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 2.48115, size = 4602, normalized size = 22.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cos
h(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 + 2*(b
^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b
^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 +
 b^2*x^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*c
osh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b
^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*dilog(cosh(b*x + a) + sinh(b*
x + a)) + 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 2)
*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b
*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh(b*x + a))
*sinh(b*x + a) + 2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^3*x^3 + (b^3
*x^3 + 6*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 6*b*x)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + 6*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 6*b*x)*cosh(b*x +
 a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 6*b*x)*cosh(b*x + a)^2 + 6*b*x)*sinh(b*x
+ a)^2 + 6*b*x + 4*((b^3*x^3 + 6*b*x)*cosh(b*x + a)^3 - (b^3*x^3 + 6*b*x)*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + ((a^3
 + 6*a)*cosh(b*x + a)^4 + 4*(a^3 + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^
3 + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 6*a)*cosh(b*x + a)^2 - 2*(a^3 - 3
*(a^3 + 6*a)*cosh(b*x + a)^2 + 6*a)*sinh(b*x + a)^2 + 4*((a^3 + 6*a)*cosh(b
*x + a)^3 - (a^3 + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)*log(cosh(b*x +
a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a)^4 + 4*(b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (
b^3*x^3 + a^3 + 6*b*x + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3 + 6*b
*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 + 6*b*x + 6
*a)*cosh(b*x + a)^2 + 6*b*x + 6*a)*sinh(b*x + a)^2 + 6*b*x + 4*((b^3*x^3 +
a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a))*sinh(b*x + a) + 6*a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
```

```
sh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x +
a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
 sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a
)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^
2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a)
)*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*
x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*
x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x +
 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 2*(b^3*x^3 - 3*b^2*x^2 + 3*(b^3*x^3 + 3*b^2*x
^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x +
 a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2
*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b
^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.454 $\int x^2 \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{x\text{csch}(a+bx)}{b^2}-\text{tanh}^{-}$$

```
[Out] -((x^2*ArcTanh[E^(a + b*x)])/b) - ArcTanh[Cosh[a + b*x]]/b^3 - (x*Csch[a +
b*x])/b^2 - (x^2*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (x*PolyLog[2, -E^(a +
 b*x)])/b^2 + (x*PolyLog[2, E^(a + b*x)])/b^2 + PolyLog[3, -E^(a + b*x)]/b^
3 - PolyLog[3, E^(a + b*x)]/b^3
```

Rubi [A] time = 0.235502, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5457, 4182, 2531, 2282, 6589, 4186, 3770}

$$-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{x\text{csch}(a+bx)}{b^2}-\text{tanh}^{-}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```


```
[Out] -((x^2*ArcTanh[E^(a + b*x)])/b) - ArcTanh[Cosh[a + b*x]]/b^3 - (x*Csch[a +
b*x])/b^2 - (x^2*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (x*PolyLog[2, -E^(a +
 b*x)])/b^2 + (x*PolyLog[2, E^(a + b*x)])/b^2 + PolyLog[3, -E^(a + b*x)]/b^
3 - PolyLog[3, E^(a + b*x)]/b^3
```


Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^2\text{csch}(a+bx)\,dx + \int x^2\text{csch}^3(a+bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\int x^2\text{c}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)}{2b}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)}{2b}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)}{2b}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)}{2b}$$

Mathematica [A] time = 4.16194, size = 222, normalized size = 1.8

$$-\frac{8bx\text{PolyLog}\left(2,-e^{a+bx}\right) - 8bx\text{PolyLog}\left(2,e^{a+bx}\right) - 8\text{PolyLog}\left(3,-e^{a+bx}\right) + 8\text{PolyLog}\left(3,e^{a+bx}\right) - 4b^2x^2\log\left(1-e^{a+b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -(8*b*x*Csch[a] + b^2*x^2*Csch[(a + b*x)/2]^2 - 8*Log[1 - E^(a + b*x)] - 4*
b^2*x^2*Log[1 - E^(a + b*x)] + 8*Log[1 + E^(a + b*x)] + 4*b^2*x^2*Log[1 + E
^(a + b*x)] + 8*b*x*PolyLog[2, -E^(a + b*x)] - 8*b*x*PolyLog[2, E^(a + b*x)
] - 8*PolyLog[3, -E^(a + b*x)] + 8*PolyLog[3, E^(a + b*x)] + b^2*x^2*Sech[(
a + b*x)/2]^2 - 4*b*x*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] - 4*b*x*Sec
h[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/(8*b^3)
```

Maple [A] time = 0.072, size = 210, normalized size = 1.7

$$-\frac{x\mathrm{e}^{bx+a}\left(bx\mathrm{e}^{2\,bx+2\,a}+bx+2\,\mathrm{e}^{2\,bx+2\,a}-2\right)}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2} - \frac{a^2\text{Artanh}\left(\mathrm{e}^{bx+a}\right)}{b^3} - \frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{2\,b} + \frac{a^2\ln\left(1+\mathrm{e}^{bx+a}\right)}{2\,b^3} - \frac{x\text{polylog}\left(2,-\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -x*exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+2*exp(2*b*x+2*a)-2)/b^2/(exp(2*b*x+2*
a)-1)^2-1/b^3*a^2*arctanh(exp(b*x+a))-1/2/b*ln(1+exp(b*x+a))*x^2+1/2/b^3*ln
(1+exp(b*x+a))*a^2-x*polylog(2,-exp(b*x+a))/b^2+polylog(3,-exp(b*x+a))/b^3+
1/2/b*ln(1-exp(b*x+a))*x^2-1/2/b^3*ln(1-exp(b*x+a))*a^2+x*polylog(2,exp(b*x
+a))/b^2-polylog(3,exp(b*x+a))/b^3-2/b^3*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.59088, size = 266, normalized size = 2.16

$$-\frac{\left(bx^{2}e^{(3\,a)}+2\,xe^{(3\,a)}\right)e^{(3\,bx)}+\left(bx^{2}e^{a}-2\,xe^{a}\right)e^{(bx)}}{b^{2}e^{(4\,bx+4\,a)}-2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}}-\frac{b^{2}x^{2}\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_{2}\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_{3}(-e^{(bx+a)})}{2\,b^{3}}+\frac{b^{2}x^{2}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) + (b*x^2*e^a - 2*x*e^a)*e^(b*x))/
(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^2*x^2*log(e^(b
*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3
+ 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog
(3, e^(b*x + a)))/b^3 - log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Fricas [C] time = 2.57823, size = 3370, normalized size = 27.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 + 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a
)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x +
a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
```

```
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x +
a) - sinh(b*x + a)) + ((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh
(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^
2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 +
 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh
(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((a^
2 + 2)*cosh(b*x + a)^4 + 4*(a^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 +
 2)*sinh(b*x + a)^4 - 2*(a^2 + 2)*cosh(b*x + a)^2 + 2*(3*(a^2 + 2)*cosh(b*x
 + a)^2 - a^2 - 2)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 2)*cosh(b*x + a)^3 - (
a^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a
) - 1) - ((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 -
 a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^
2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 -
a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x
 + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh
(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*
x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 +
4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
 + a) - sinh(b*x + a)) + 2*(b^2*x^2 + 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 -
 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x
+ a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(
b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x +
 a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.455 $\int x\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=82

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -((x*ArcTanh[E^(a + b*x)])/b) - Csch[a + b*x]/(2*b^2) - (x*Coth[a + b*x]*Cs
ch[a + b*x])/(2*b) - PolyLog[2, -E^(a + b*x)]/(2*b^2) + PolyLog[2, E^(a + b
*x)]/(2*b^2)
```

Rubi [A] time = 0.124972, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5457, 4182, 2279, 2391, 4185}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -((x*ArcTanh[E^(a + b*x)])/b) - Csch[a + b*x]/(2*b^2) - (x*Coth[a + b*x]*Cs
ch[a + b*x])/(2*b) - PolyLog[2, -E^(a + b*x)]/(2*b^2) + PolyLog[2, E^(a + b
*x)]/(2*b^2)
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int x\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx=\int x\mathrm{csch}(a+bx)\,dx+\int x\mathrm{csch}^3(a+bx)\,dx$$
$$=-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\mathrm{csch}(a+bx)}{2b^2}-\frac{x\coth(a+bx)\mathrm{csch}(a+bx)}{2b}-\frac{1}{2}\int x\mathrm{csch}(a$$
$$=-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\mathrm{csch}(a+bx)}{2b^2}-\frac{x\coth(a+bx)\mathrm{csch}(a+bx)}{2b}-\frac{\mathrm{Subst}\left(\int\frac{\log(1-}{x}\right.}{b^2}$$
$$=-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\mathrm{csch}(a+bx)}{2b^2}-\frac{x\coth(a+bx)\mathrm{csch}(a+bx)}{2b}-\frac{\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+$$
$$=-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\mathrm{csch}(a+bx)}{2b^2}-\frac{x\coth(a+bx)\mathrm{csch}(a+bx)}{2b}-\frac{\mathrm{Li}_2\left(-e^{a+bx}\right)}{2b^2}+$$

Mathematica [A] time = 2.07128, size = 144, normalized size = 1.76

$$-\frac{-4\mathrm{PolyLog}\left(2,-e^{-a-bx}\right)+4\mathrm{PolyLog}\left(2,e^{-a-bx}\right)-4(a+bx)\left(\log\left(1-e^{-a-bx}\right)-\log\left(e^{-a-bx}+1\right)\right)-2\tanh\left(\frac{1}{2}(a+bx)\right)+}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -(2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/2]^2 - 4*(a + b*x)*(Log[1 - E^(-
a - b*x)] - Log[1 + E^(-a - b*x)]) + 4*a*Log[Tanh[(a + b*x)/2]] - 4*PolyLog
[2, -E^(-a - b*x)] + 4*PolyLog[2, E^(-a - b*x)] + b*x*Sech[(a + b*x)/2]^2 -
 2*Tanh[(a + b*x)/2])/(8*b^2)
```

Maple [B] time = 0.064, size = 156, normalized size = 1.9

$$-\frac{e^{bx+a}\left(bx e^{2bx+2a}+bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(1+e^{bx+a}\right)x}{2b}-\frac{a\ln\left(1+e^{bx+a}\right)}{2b^2}-\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{2b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{2b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1
)^2-1/2/b*ln(1+exp(b*x+a))*x-1/2/b^2*ln(1+exp(b*x+a))*a-1/2/b^2*polylog(2,-
exp(b*x+a))+1/2/b*ln(1-exp(b*x+a))*x+1/2/b^2*ln(1-exp(b*x+a))*a+1/2/b^2*pol
ylog(2,exp(b*x+a))+1/b^2*a*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.59869, size = 167, normalized size = 2.04

$$-\frac{\left(bxe^{(3a)}+e^{(3a)}\right)e^{(3bx)}+\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{2b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b*x*log(e^(b*x + a) + 1) + d
ilog(-e^(b*x + a)))/b^2 + 1/2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a
)))/b^2
```

Fricas [B] time = 2.37047, size = 2294, normalized size = 27.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 + 2*(b*x - 1)*cosh(b*x + a) - (cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - co
sh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(co
sh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*s
inh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x
+ a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(
b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x -
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(
b*x + 1)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4
*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x +
 a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh
(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx + a)^2 \operatorname{csch}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.456 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rubi [A] time = 0.0331728, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx &= -\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}+\frac{1}{2}\int \text{csch}(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.032511, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) + Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

Maple [A] time = 0.017, size = 45, normalized size = 1.3

$$\frac{1}{b}\left(-\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^2*cosh(b*x+a)+1/2*csch(b*x+a)*coth(b*x+a)-arctanh(exp(b
*x+a)))
```

Maxima [B] time = 1.23242, size = 113, normalized size = 3.32

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 2.34596, size = 1088, normalized size = 32.

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16513, size = 122, normalized size = 3.59

$$-\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{4\,b}+\frac{\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}-\frac{e^{(bx+a)}+e^{(-bx-a)}}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 1/4*log(e^(b*x + a) + e^(-b*x
- a) - 2)/b - (e^(b*x + a) + e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2
 - 4)*b)
```

3.457 $\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(a+bx)}{x},x\right)+\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]/x, x] + Unintegrable[Csch[a + b*x]^3/x, x]
```

Rubi [A] time = 0.0715894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

```
[Out] Defer[Int][Csch[a + b*x]/x, x] + Defer[Int][Csch[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)}{x}\,dx+\int \frac{\operatorname{csch}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 52.2792, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

```
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x, x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{\left(\cosh \left(bx+a\right)\right)^{2} \left(\operatorname{csch}\left(bx+a\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)} + \left(bxe^{a} + e^{a}\right)e^{(bx)}}{b^{2}x^{2}e^{(4\,bx+4\,a)} - 2\,b^{2}x^{2}e^{(2\,bx+2\,a)} + b^{2}x^{2}} + 2\int \frac{b^{2}x^{2} + 2}{4\left(b^{2}x^{3}e^{(bx+a)} + b^{2}x^{3}\right)}\,dx + 2\int \frac{b^{2}x^{2} + 2}{4\left(b^{2}x^{3}e^{(bx+a)} - b^{2}x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) + (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) - 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/4*(b^2*
x^2 + 2)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) + 2*integrate(1/4*(b^2*x^2 + 2
)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{3}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (bx+a)^2 \operatorname{csch}(bx+a)^3}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

3.458 $\int \frac{\coth^2(a+bx)\mathrm{csch}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\text{csch}^3(a+bx)}{x^2},x\right)+\text{Unintegrable}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]/x^2, x] + Unintegrable[Csch[a + b*x]^3/x^2, x]
```

Rubi [A] time = 0.0755131, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]/x^2, x] + Defer[Int][Csch[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx=\int \frac{\text{csch}(a+bx)}{x^2}\,dx+\int \frac{\text{csch}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 44.2932, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2, x]
```

Maple [A] time = 0.239, size = 0, normalized size = 0.

$$\int \frac{(\cosh(bx+a))^2 (\operatorname{csch}(bx+a))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bxe^{(3a)} - 2e^{(3a)}\right)e^{(3bx)} + \left(bxe^{a} + 2e^{a}\right)e^{(bx)}}{b^2x^3e^{(4bx+4a)} - 2b^2x^3e^{(2bx+2a)} + b^2x^3} + 2\int \frac{b^2x^2+6}{4\left(b^2x^4e^{(bx+a)} + b^2x^4\right)}\,dx + 2\int \frac{b^2x^2+6}{4\left(b^2x^4e^{(bx+a)} - b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) + (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/4*(
b^2*x^2 + 6)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) + 2*integrate(1/4*(b^2*x^2
 + 6)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

3.459 $\int x^m \coth^3(a+bx)\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \coth^3(a+bx),x\right)$$

```
[Out] Unintegrable[x^m*Coth[a + b*x]^3, x]
```

Rubi [A] time = 0.0300256, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \coth^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \coth^3(a+bx)\,dx$$

Mathematica [A] time = 13.9819, size = 0, normalized size = 0.

$$\int x^m \coth^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]^3, x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int x^{m} \left(\cosh \left(bx+a\right)\right)^{3} \left(\operatorname{csch}\left(bx+a\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(6\,bx+m\log(x)+6\,a)}}{(m+1)e^{(6\,bx+6\,a)} - 3\,(m+1)e^{(4\,bx+4\,a)} + 3\,(m+1)e^{(2\,bx+2\,a)} - m - 1} + \int \frac{\left(3\left(2\,bxe^{(6\,a)} + (m+1)e^{(6\,a)}\right)e^{(6\,bx)} - 2\,(}{(m+1)e^{(8\,bx+8\,a)} - 4\,(m+1)e^{(6\,bx+6\,a)} + 6\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m +
1)*e^(8*b*x + 8*a) - 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
- 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.460 $\int x^3 \coth^3(a+bx)\,dx$

Optimal. Leaf size=179

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\coth(a+}{2b^2}$$

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3
*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (x^3*Log[1 -
 E^(2*(a + b*x))])/b + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*Pol
yLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^
3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

Rubi [A] time = 0.336685, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3720, 3716, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\coth(a+}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^3,x]
```


```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3
*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (x^3*Log[1 -
 E^(2*(a + b*x))])/b + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*Pol
yLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^
3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\coth^3(a+bx)\,dx = -\frac{x^3\coth^2(a+bx)}{2b}+\frac{3\int x^2\coth^2(a+bx)\,dx}{2b}+\int x^3\coth(a+bx)\,dx$$

$$= -\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx+\frac{3\int x\coth(a+bx)\,dx}{b^2}+$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{6\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}}{b^2}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-\right.}{b}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-\right.}{b}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-\right.}{b}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-\right.}{b}$$

Mathematica [B] time = 3.33511, size = 390, normalized size = 2.18

$$\frac{1}{4}\left(-\frac{2e^{2a}\left(6\left(1-e^{-2a}\right)\left(b^2x^2\text{PolyLog}\left(2,-e^{-a-bx}\right)+2\left(bx\text{PolyLog}\left(3,-e^{-a-bx}\right)+\text{PolyLog}\left(4,-e^{-a-bx}\right)\right)\right)+6\left(1-e^{-2a}\right)\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^3,x]
```

```
[Out] (x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a)
+ (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(
-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log
[1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*
a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[
2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*
(b*x*PolyLog[3, -E^(-a - b*x)] + PolyLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2
*a))*(b^2*x^2*PolyLog[2, E^(-a - b*x)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] +
PolyLog[4, E^(-a - b*x)]))))/(b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a +
 b*x]*Sinh[b*x])/b^2)/4
```

Maple [B] time = 0.091, size = 375, normalized size = 2.1

$$3\,\frac{\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^4}+3\,\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^4}-\frac{x^2\left(2\,bx\mathrm{e}^{2\,bx+2\,a}+3\,\mathrm{e}^{2\,bx+2\,a}-3\right)}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2}+3\,\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^3}+3\,\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 3/b^4*polylog(2,exp(b*x+a))+3/b^4*polylog(2,-exp(b*x+a))-x^2*(2*b*x*exp(2*b
*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2+3/b^3*ln(1+exp(b*x+a))
*x+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a-3/b^4*a*ln(exp(b*x+a)-
1)-3/2/b^4*a^4+6/b^4*polylog(4,exp(b*x+a))+6/b^4*polylog(4,-exp(b*x+a))-2/b
^3*a^3*x-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*a^3*ln(exp(b*x+a))+1/b^4*ln(1-exp
(b*x+a))*a^3+3/b^2*polylog(2,-exp(b*x+a))*x^2-6/b^3*polylog(3,-exp(b*x+a))*
x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,exp(b*x+a))*x^2-6/b^3*polylog(3,
exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^3-6/b^3*a*x+6/b^4*a*ln(exp(b*x+a))-3*x
^2/b^2-3/b^4*a^2-1/4*x^4
```

Maxima [A] time = 1.42995, size = 408, normalized size = 2.28

$$\frac{b^2x^4e^{(4\,bx+4\,a)}+b^2x^4+12\,x^2-2\left(b^2x^4e^{(2\,a)}+4\,bx^3e^{(2\,a)}+6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}-\frac{b^4x^4+6\,b^2x^2}{2\,b^4}+\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x + a)
 + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*
polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*
dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x +
a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 2.70712, size = 5023, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4
*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4 - 4*
b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cos
h(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*
x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*si
nh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*
(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(
b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x
 + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1
)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) -
 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 3*b*x)*cosh(
b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 +
 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2
+ 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*cosh(b*x + a)^3 - (
b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*s
inh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 3*a)*cosh(b*x
 + a)^2 - 2*(a^3 - 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4
*((a^3 + 3*a)*cosh(b*x + a)^3 - (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) +
3*a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 4*(b^3*x^3 + (b^3*x^3 + a^3 +
```

```
3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x +
a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 -
2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3
*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 +
3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 +
 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-cosh(b*x + a) - sinh
(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + si
nh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)
^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh
(b*x + a) + sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
 + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*c
osh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*pol
ylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 +
b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*((b^4*x^4 - 2*a^4
 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 - (b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*
b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^
4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)
^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*
x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.461 $\int x^2 \coth^3(a+bx)\,dx$

Optimal. Leaf size=114

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2\coth^2(}{2b}$$

```
[Out] x^2/(2*b) - x^3/3 - (x*Coth[a + b*x])/b^2 - (x^2*Coth[a + b*x]^2)/(2*b) + (
x^2*Log[1 - E^(2*(a + b*x))])/b + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, E^
(2*(a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rubi [A] time = 0.211503, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3720, 3475, 30, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2\coth^2(}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 - (x*Coth[a + b*x])/b^2 - (x^2*Coth[a + b*x]^2)/(2*b) + (
x^2*Log[1 - E^(2*(a + b*x))])/b + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, E^
(2*(a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^3(a+bx)\,dx = -\frac{x^2\coth^2(a+bx)}{2b} + \frac{\int x\coth^2(a+bx)\,dx}{b} + \int x^2\coth(a+bx)\,dx$$
$$= -\frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx + \frac{\int \coth(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} - 2\int$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\mathrm{L}}{}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\mathrm{L}}{}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\mathrm{L}}{}$$

Mathematica [B] time = 2.22533, size = 295, normalized size = 2.59

$$-\frac{e^{2a}\left(6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,-e^{-a-bx}\right)+\text{PolyLog}\left(3,-e^{-a-bx}\right)\right)+6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,e^{-a-bx}\right)+\text{PolyLog}\left(3,\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]^3,x]
```

```
[Out] (x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a)*((6*b*x)/E^(2*a) +
 (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*x)])/E^(
2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3*(1 - E^
(-2*a))*(b*x - Log[1 - E^(a + b*x)]) + 3*(1 - E^(-2*a))*(b*x - Log[1 + E^(a
 + b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E
^(-a - b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3,
 E^(-a - b*x)])))/(3*b^3*(-1 + E^(2*a))) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*
x])/b^2
```

Maple [B] time = 0.087, size = 246, normalized size = 2.2

$$-\frac{x^3}{3} - 2\,\frac{x\left(bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}-1\right)}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2} + \frac{4\,a^3}{3\,b^3} - \frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^3} - 2\,\frac{\mathrm{polylog}\left(3,-\mathrm{e}^{bx+a}\right)}{b^3} - 2\,\frac{\mathrm{polylog}\left(3,\mathrm{e}^{bx+a}\right)}{b^3} + \frac{\ln\left(\mathrm{e}^{bx+}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/3*x^3-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
+4/3/b^3*a^3-1/b^3*ln(1-exp(b*x+a))*a^2-2*polylog(3,-exp(b*x+a))/b^3-2*poly
log(3,exp(b*x+a))/b^3+1/b^3*ln(exp(b*x+a)-1)-2/b^3*ln(exp(b*x+a))+1/b^3*ln(
1+exp(b*x+a))+2/b^2*a^2*x+1/b*ln(1+exp(b*x+a))*x^2+2*x*polylog(2,-exp(b*x+a
))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp(b*x+a))/b^2+1/b^3*a^2*ln(
exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))
```

Maxima [B] time = 1.3763, size = 305, normalized size = 2.68

$$-\frac{2}{3}x^3 + \frac{b^2x^3e^{(4\,bx+4\,a)} + b^2x^3 - 2\left(b^2x^3e^{(2\,a)} + 3\,bx^2e^{(2\,a)} + 3\,xe^{(2\,a)}\right)e^{(2\,bx)} + 6\,x}{3\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{2\,x}{b^2} + \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 - 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) - 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*di
log(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
+ a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 + log
(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Fricas [C] time = 2.82254, size = 3729, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 +
 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*
b*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^3 +
2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
```

```
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(
(b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4
*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a
) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 + 1)*cosh(b*x + a)^
4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 -
 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 - a^2 - 1)*si
nh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 - (a^2 + 1)*cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2
- a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)
^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
- a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))
*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 - 3*b^2
*x^2 + 2*a^3 + 3*b*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b
*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b
^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2
+ 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.462 $\int x \coth^3(a+bx)\,dx$

Optimal. Leaf size=82

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[Out] x/(2*b) - x^2/2 - Coth[a + b*x]/(2*b^2) - (x*Coth[a + b*x]^2)/(2*b) + (x*Lo
g[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^2)
```

Rubi [A] time = 0.12246, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3720, 3473, 8, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 - Coth[a + b*x]/(2*b^2) - (x*Coth[a + b*x]^2)/(2*b) + (x*Lo
g[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^2)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\coth^3(a+bx)\,dx &= -\frac{x\coth^2(a+bx)}{2b}+\frac{\int \coth^2(a+bx)\,dx}{2b}+\int x\coth(a+bx)\,dx\\
&= -\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}-2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx+\frac{\int 1\,dx}{2b}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\int \log\left(1-e^{2(a+bx)}\right)dx}{b}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2(a}\right.}{2b^2}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}
\end{aligned}$$

Mathematica [C] time = 6.1231, size = 232, normalized size = 2.83

$$\frac{\operatorname{csch}(a)\operatorname{sech}(a)\left(-b^2x^2e^{-\tanh^{-1}(\tanh(a))}+\frac{i\tanh(a)\left(i\text{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}(\tanh(a))+ibx\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\tanh(a))\right)-2\left(i\tanh^{-1}(\tanh(a))+ibx\right.}{\cdots}\right.}{2b^2\sqrt{\operatorname{sech}^2(a)\left(\cosh^2(a)-\sinh\cdots\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Coth[a + b*x]^3,x]
```

```
[Out] (x^2*Coth[a])/2 - (x*Csch[a + b*x]^2)/(2*b) + (Csch[a]*Csch[a + b*x]*Sinh[b
*x])/(2*b^2) + (Csch[a]*Sech[a]*(-((b^2*x^2)/E^ArcTanh[Tanh[a]]) + (I*(-(b*
x*(-Pi + (2*I)*ArcTanh[Tanh[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*b*x + I*Ar
cTanh[Tanh[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))] + Pi*Log[Co
sh[b*x]] + (2*I)*ArcTanh[Tanh[a]]*Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]] + I*P
olyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))])*Tanh[a])/Sqrt[1 - Tanh[a
]^2]))/(2*b^2*Sqrt[Sech[a]^2*(Cosh[a]^2 - Sinh[a]^2)])
```

Maple [B] time = 0.083, size = 164, normalized size = 2.

$$-\frac{x^2}{2}-\frac{2\,bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}-1}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2}-2\,\frac{ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b}+\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\cdots\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/2*x^2-(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-2
/b*a*x-a^2/b^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))+1/b*ln(1
-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))-1/b^2*a
*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Maxima [B] time = 1.35998, size = 201, normalized size = 2.45

$$-x^2+\frac{b^2x^2e^{(4\,bx+4\,a)}+b^2x^2-2\left(b^2x^2e^{(2\,a)}+2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{2\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2
*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*lo
g(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [B] time = 2.52685, size = 2569, normalized size = 31.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 2
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
 a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x
+ a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cos
h(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))
*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b*x + a)*c
osh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh
(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2
- b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)
*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 - (b^2*x^2 - 2*a^2 - 2*b*x - 1)*cosh
(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*si
nh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh
(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^2*cos
```

```
h(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.463 $\int \coth^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) + Log[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0183687, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) + Log[Sinh[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\,dx &= -\frac{\coth^2(a+bx)}{2b}+\int \coth(a+bx)\,dx\\ &= -\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0710364, size = 34, normalized size = 1.26

$$-\frac{\coth^2(a+bx)-2\log(\tanh(a+bx))-2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3,x]
```

```
[Out] -(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/(2*b)
```

Maple [A] time = 0.019, size = 26, normalized size = 1.

$$-\frac{(\coth(bx+a))^2}{2b}+\frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Maxima [B] time = 1.07929, size = 107, normalized size = 3.96

$$x+\frac{a}{b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x -
 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 2.53651, size = 930, normalized size = 34.44

$$-\frac{bx\cosh(bx+a)^4+4\,bx\cosh(bx+a)\sinh(bx+a)^3+bx\sinh(bx+a)^4-2\,(bx-1)\cosh(bx+a)^2+2\left(3\,bx\cosh(bx+\right.}{b\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 - (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16715, size = 95, normalized size = 3.52

$$-\frac{bx+a}{b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}-\frac{3\,e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+3}{2\,b\left(e^{(2\,bx+2\,a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b - 1/2*(3*e^(4*b*x + 4*a) - 2
*e^(2*b*x + 2*a) + 3)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.464 $\int \frac{\coth^3(a+bx)}{x}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]^3/x, x]
```

Rubi [A] time = 0.0291413, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^3/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.7273, size = 0, normalized size = 0.

$$\int \frac{\coth^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^3/x,x]
```

```
[Out] Integrate[Coth[a + b*x]^3/x, x]
```

Maple [A] time = 0.228, size = 0, normalized size = 0.

$$\int \frac{(\cosh (bx+a))^3 (\operatorname{csch} (bx+a))^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^2x^2e^{(4\,bx+4\,a)}-2\,b^2x^2e^{(2\,bx+2\,a)}+b^2x^2}-\int \frac{b^2x^2+1}{b^2x^3e^{(bx+a)}+b^2x^3}\, dx+\int \frac{b^2x^2+1}{b^2x^3e^{(bx+a)}-b^2x^3}\, dx+\log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x +
 a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3
), x) + log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh (bx+a)^3 \operatorname{csch} (bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

3.465 $\int \frac{\coth^3(a+bx)}{x^2} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[Coth[a + b*x]^3/x^2, x]
```

Rubi [A] time = 0.0307226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x^2} dx = \int \frac{\coth^3(a+bx)}{x^2} dx$$

Mathematica [A] time = 0.422331, size = 0, normalized size = 0.

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Coth[a + b*x]^3/x^2, x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{\left(\cosh \left(bx+a\right)\right)^{3} \left(\operatorname{csch}\left(bx+a\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^{2}x^{2}e^{(4\,bx+4\,a)}+b^{2}x^{2}-2\left(b^{2}x^{2}e^{(2\,a)}-bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{b^{2}x^{3}e^{(4\,bx+4\,a)}-2\,b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}}-\int \frac{b^{2}x^{2}+3}{b^{2}x^{4}e^{(bx+a)}+b^{2}x^{4}}\,dx+\int \frac{b^{2}x^{2}+3}{b^{2}x^{4}e^{(bx+a)}-b^{2}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) +
integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

3.466 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

Rubi [A] time = 0.212319, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

Rubi steps

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Mathematica [A] time = 9.31969, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

Maple [A] time = 0.024, size = 0, normalized size = 0.

$$\int x^{m}\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m}\,\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\mathrm{csch}\left(a+bx\right)\mathrm{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

3.467 $\int x^3 \mathbf{csch}(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\text{PolyI}}{}$$

```
[Out] (-2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/
(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2) + (3*x*PolyLog[3, -E^
(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) - (3*Po
lyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^
4)
```

Rubi [A] time = 0.14994, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5461, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/
(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2) + (3*x*PolyLog[3, -E^
(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) - (3*Po
lyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^
4)
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = 2\int x^3\text{csch}(2a+2bx)\,dx$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3\int x^2\log\left(1-e^{2a+2bx}\right)dx}{b} + \frac{3\int x^2\log\left(1+e^{2a+2bx}\right)dx}{b}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3\int x\text{Li}_2\left(-e^{2a+2l}\right.}{b^2}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}$$

Mathematica [A] time = 4.00892, size = 150, normalized size = 1.01

$$\frac{-6b^2x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+6b^2x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)+6bx\text{PolyLog}\left(3,-e^{2(a+bx)}\right)-6bx\text{PolyLog}\left(3,e^{2(a+bx)}\right)-3}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (4*b^3*x^3*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Log[1 + E^(2*(a + b*x))] -
6*b^2*x^2*PolyLog[2, -E^(2*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^(2*(a + b*x
))] + 6*b*x*PolyLog[3, -E^(2*(a + b*x))] - 6*b*x*PolyLog[3, E^(2*(a + b*x))
] - 3*PolyLog[4, -E^(2*(a + b*x))] + 3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

Maple [A] time = 0.082, size = 241, normalized size = 1.6

$$\frac{\ln\left(1+e^{bx+a}\right)x^3}{b} - \frac{x^3\ln\left(1+e^{2bx+2a}\right)}{b} - \frac{3x^2\text{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2} + \frac{3x\text{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3} + 3\frac{\text{polylog}\left(2,-e^{bx+a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] 1/b*ln(1+exp(b*x+a))*x^3-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(
2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/b^2*polylog(2,-exp(b
```

```
*x+a))*x^2-6/b^3*polylog(3,-exp(b*x+a))*x+1/b^4*ln(1-exp(b*x+a))*a^3+6/b^4*
polylog(4,exp(b*x+a))+6/b^4*polylog(4,-exp(b*x+a))-3/4*polylog(4,-exp(2*b*x
+2*a))/b^4+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,exp(b*x+a))*x^2-6/b^3*p
olylog(3,exp(b*x+a))*x-1/b^4*a^3*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.16308, size = 274, normalized size = 1.85

$$-\frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4}+\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.63732, size = 1323, normalized size = 8.94

$$\frac{b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+3\,b^2x^2\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-3\,b^2x^2\mathrm{Li}_2\left(i\,\cosh\left(bx+a\right)+i\,s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2*dilog(cosh(b*x
+ a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
- 3*b^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*b^2*x^2*dilog(-co
sh(b*x + a) - sinh(b*x + a)) + a^3*log(cosh(b*x + a) + sinh(b*x + a) + I) +
 a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) - a^3*log(cosh(b*x + a) + sinh(
b*x + a) - 1) - 6*b*x*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*pol
ylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*b*x*polylog(3, -I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) -
 (b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + a^
3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3)*log(-cosh(
```

```
b*x + a) - sinh(b*x + a) + 1) + 6*polylog(4, cosh(b*x + a) + sinh(b*x + a))
 - 6*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*polylog(4, -I*cosh(b
*x + a) - I*sinh(b*x + a)) + 6*polylog(4, -cosh(b*x + a) - sinh(b*x + a)))/
b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(a+bx) \operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a), x)
```

3.468 $\int x^2 \mathbf{csch}(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{2x^2\tanh^{-1}\left(e^{2a+2\ldots}\right.}{b}$$

```
[Out] (-2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2
+ (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 + PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3
) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rubi [A] time = 0.105626, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5461, 4182, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{2x^2\tanh^{-1}\left(e^{2a+2\ldots}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2
+ (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 + PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3
) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = 2\int x^2\text{csch}(2a+2bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{2\int x\log\left(1-e^{2a+2bx}\right)dx}{b} + \frac{2\int x\log\left(1+e^{2a+2bx}\right)dx}{b}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\int\text{Li}_2\left(-e^{2a+2bx}\right)dx}{b^2}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}dx, x\right.}{2b^3}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3} - \text{Li}_3$$

Mathematica [A] time = 4.11854, size = 108, normalized size = 1.11

$$\frac{-2bx\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,e^{2(a+bx)}\right)+\text{PolyLog}\left(3,-e^{2(a+bx)}\right)-\text{PolyLog}\left(3,e^{2(a+bx)}\right)+2b^2x^2\log\left(1\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (2*b^2*x^2*Log[1 - E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + E^(2*(a + b*x))] -
2*b*x*PolyLog[2, -E^(2*(a + b*x))] + 2*b*x*PolyLog[2, E^(2*(a + b*x))] + Po
lyLog[3, -E^(2*(a + b*x))] - PolyLog[3, E^(2*(a + b*x))])/(2*b^3)
```

Maple [B] time = 0.043, size = 186, normalized size = 1.9

$$-2\,\frac{\mathrm{polylog}\left(3,-\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x^2}{b}+2\,\frac{x\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{b}+2\,\frac{x\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}-\frac{x^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] -2*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp(b*
x+a))/b^2+1/b*ln(1+exp(b*x+a))*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-x^2*ln(1+
exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2-2*polylog(3,exp(b*x+a))/
b^3+1/2*polylog(3,-exp(2*b*x+2*a))/b^3+1/b^3*a^2*ln(exp(b*x+a)-1)-1/b^3*ln(
1-exp(b*x+a))*a^2
```

Maxima [A] time = 1.1186, size = 200, normalized size = 2.06

$$-\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})}{2\,b^3}+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) -
polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*d
ilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Fricas [C] time = 2.52657, size = 1018, normalized size = 10.49

$$\frac{b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-2\,bx\mathrm{Li}_2\left(i\,\cosh\left(bx+a\right)+i\,\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dilog(cosh(b*x + a)
 + sinh(b*x + a)) - 2*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*b*x*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*b*x*dilog(-cosh(b*x + a) - si
nh(b*x + a)) - a^2*log(cosh(b*x + a) + sinh(b*x + a) + I) - a^2*log(cosh(b*
x + a) + sinh(b*x + a) - I) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) -
(b^2*x^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^2*x^2 - a^2
)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 - a^2)*log(-cosh(b
*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)) - 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b
^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(a + bx) \operatorname{sech}(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx + a) \operatorname{sech}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a), x)
```

3.469 $\int x\mathbf{csch}(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}$$

```
[Out] (-2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) +
PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2)
```

Rubi [A] time = 0.0557309, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5461, 4182, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) +
PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2)
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int x\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx &= 2\int x\operatorname{csch}(2a+2bx)\,dx\\ &= -\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\int\log\left(1-e^{2a+2bx}\right)dx}{b}+\frac{\int\log\left(1+e^{2a+2bx}\right)dx}{b}\\ &= -\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^2}+\frac{\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e\right.}{2b^2}\\ &= -\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\operatorname{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}+\frac{\operatorname{Li}_2\left(e^{2a+2bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.079429, size = 110, normalized size = 1.9

$$\frac{\operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right)-\operatorname{PolyLog}\left(2,e^{-2(a+bx)}\right)+2a\log\left(1-e^{-2(a+bx)}\right)+2bx\log\left(1-e^{-2(a+bx)}\right)-2a\log\left(e^{-2(a+bx)}+1\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (2*a*Log[1 - E^(-2*(a + b*x))] + 2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[
1 + E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] - 2*a*Log[Tanh[a +
b*x]] + PolyLog[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))])/(2*b^
2)
```

Maple [B] time = 0.039, size = 125, normalized size = 2.2

$$\frac{\ln\left(1+e^{bx+a}\right)x}{b}+\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}-\frac{x\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{b}+\frac{\ln\left(1-e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] 1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))-x*ln(1+exp(2*b*x+2*a))/
b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(
b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.1433, size = 117, normalized size = 2.02

$$-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 + (b*x*
log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1
) + dilog(e^(b*x + a)))/b^2
```

Fricas [C] time = 2.4579, size = 686, normalized size = 11.83

$$bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + a*log(cosh(b*x + a) + sinh(b*
x + a) + I) + a*log(cosh(b*x + a) + sinh(b*x + a) - I) - a*log(cosh(b*x + a
) + sinh(b*x + a) - 1) - (b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) +
1) - (b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b*x + a)*log(
-cosh(b*x + a) - sinh(b*x + a) + 1) + dilog(cosh(b*x + a) + sinh(b*x + a))
- dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - dilog(-I*cosh(b*x + a) - I*sin
h(b*x + a)) + dilog(-cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a), x)
```

3.470 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] Log[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0134658, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.0129567, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sinh(a+bx))}{2b}-\frac{\log(\cosh(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] 2*(-Log[Cosh[a + b*x]]/(2*b) + Log[Sinh[a + b*x]]/(2*b))
```

Maple [A] time = 0., size = 12, normalized size = 1.1

$$\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.63962, size = 68, normalized size = 6.18

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

Fricas [B] time = 2.28999, size = 154, normalized size = 14.

$$-\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

Giac [B] time = 1.19867, size = 47, normalized size = 4.27

$$-\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -log(e^(2*b*x + 2*a) + 1)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.471 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal. Leaf size=17

$$2\text{Unintegrable}\left(\frac{\operatorname{csch}(2a+2bx)}{x},x\right)$$

```
[Out] 2*Unintegrable[Csch[2*a + 2*b*x]/x, x]
```

Rubi [A] time = 0.0345511, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x,x]

[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = 2\int \frac{\operatorname{csch}(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 17.1623, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x,x]

[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x, x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)/x,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

3.472 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=17

$$2\text{Unintegrable}\left(\frac{\operatorname{csch}(2a+2bx)}{x^2},x\right)$$

```
[Out] 2*Unintegrable[Csch[2*a + 2*b*x]/x^2, x]
```

Rubi [A] time = 0.0372738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
```

```
[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = 2\int \frac{\operatorname{csch}(2a+2bx)}{x^2}\,dx$$

Mathematica [A] time = 15.6595, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2, x]
```

Maple [A] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

3.473 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Rubi [A] time = 0.391452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 21.0868, size = 0, normalized size = 0.

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int x^{m}\mathrm{csch}\left(bx+a\right)\left(\mathrm{sech}\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m}\,\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\mathrm{csch}\left(a+bx\right)\mathrm{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.474 $\int x^3\mathbf{csch}(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=226

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}(}{b^3}$$

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 - (2*x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*
PolyLog[2, -E^(a + b*x)])/b^2 + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3
- ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*x^2*PolyLog[2, E^(a + b*x)])
/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b
*x)])/b^4 + ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (6*x*PolyLog[3, E^(a +
b*x)])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])
/b^4 + (x^3*Sech[a + b*x])/b
```

Rubi [A] time = 0.337148, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2622, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 6609, 2282, 6589, 4180}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}(}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```


```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 - (2*x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*
PolyLog[2, -E^(a + b*x)])/b^2 + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3
- ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*x^2*PolyLog[2, E^(a + b*x)])
/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b
*x)])/b^4 + ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (6*x*PolyLog[3, E^(a +
b*x)])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])
/b^4 + (x^3*Sech[a + b*x])/b
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
```

```
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx = -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^3\mathrm{sech}(a+bx)}{b}-3\int x^2\left(-\frac{\tanh^{-1}(\cosh(a+bx))}{b}\right.$$

$$= -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^3\mathrm{sech}(a+bx)}{b}-3\int\left(-\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b}\right.$$

$$= -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^3\mathrm{sech}(a+bx)}{b}+\frac{3\int x^2\tanh^{-1}(\cosh(a+bx))\,dx}{b}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^3\mathrm{sech}(a+bx)}{b}+\frac{(6i)\int x\log\left(1-ie^{a+bx}\right)dx}{b^2}-\frac{(6i)\int x\log}{}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{6ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{x^3\mathrm{sech}(a+bx)}{b}-\frac{(6i)}{}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{6ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^3}+$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x^2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{6ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$

Mathematica [A] time = 0.377735, size = 282, normalized size = 1.25

$$-3i\left(-2bx\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\mathrm{PolyLog}\left(2,ie^{a+bx}\right)+2\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)-2\mathrm{PolyLog}\left(3,ie^{a+bx}\right)+b^2x^2\log\left(1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-2*b^3*x^3*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] - 3*b^2*x^2*PolyLog[2, -
Cosh[a + b*x] - Sinh[a + b*x]] + 3*b^2*x^2*PolyLog[2, Cosh[a + b*x] + Sinh[
a + b*x]] - (3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a
+ b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*
x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + 6*b*x
*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]] - 6*b*x*PolyLog[3, Cosh[a + b*x
```

```
] + Sinh[a + b*x]] - 6*PolyLog[4, -Cosh[a + b*x] - Sinh[a + b*x]] + 6*PolyL
og[4, Cosh[a + b*x] + Sinh[a + b*x]] + b^3*x^3*Sech[a + b*x])/b^4
```

Maple [F] time = 0.521, size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a) (\operatorname{sech}(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^3e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b} - \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^3*x^3*log(e^(b*x + a) + 1) +
 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog
(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e
^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^
4 - 24*integrate(1/4*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Fricas [C] time = 2.89573, size = 3553, normalized size = 15.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 3*(b^2*x^2*cosh(b*x +
a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^
2*x^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (-6*I*b*x*cosh(b*x + a)^2 - 1
2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) + (6*I*b*x*cosh(b*x + a)^2 + 12*I*b*
x*cosh(b*x + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 + 6*I*b*x)*dilog(-I
*cosh(b*x + a) - I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*dilog(-cos
h(b*x + a) - sinh(b*x + a)) - (b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x
 + a)*sinh(b*x + a) + b^3*x^3*sinh(b*x + a)^2 + b^3*x^3)*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + (-3*I*a^2*cosh(b*x + a)^2 - 6*I*a^2*cosh(b*x + a)*si
nh(b*x + a) - 3*I*a^2*sinh(b*x + a)^2 - 3*I*a^2)*log(cosh(b*x + a) + sinh(b
*x + a) + I) + (3*I*a^2*cosh(b*x + a)^2 + 6*I*a^2*cosh(b*x + a)*sinh(b*x +
a) + 3*I*a^2*sinh(b*x + a)^2 + 3*I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) -
 I) - (a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b
*x + a)^2 + a^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (3*I*b^2*x^2 + (3
*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x +
a)*sinh(b*x + a) + (3*I*b^2*x^2 - 3*I*a^2)*sinh(b*x + a)^2 - 3*I*a^2)*log(I
*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (-3*I*b^2*x^2 + (-3*I*b^2*x^2 + 3*I
*a^2)*cosh(b*x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a
) + (-3*I*b^2*x^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2)*log(-I*cosh(b*x + a
) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3 + (b^3*x^3 + a^3)*cosh(b*x + a)^2
 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3)*sinh(b*x
 + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(4, cosh(b*x + a)
+ sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(
b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)
*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + (6*I*cosh(b*x + a)^2 + 12*I*co
sh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x + a)^2 + 6*I)*polylog(3, I*cosh(b*
x + a) + I*sinh(b*x + a)) + (-6*I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh
(b*x + a) - 6*I*sinh(b*x + a)^2 - 6*I)*polylog(3, -I*cosh(b*x + a) - I*sinh
(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b
*x*sinh(b*x + a)^2 + b*x)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(b^4*
cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 +
 b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.475 $\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=146

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(3,-\right.}{b^3}$$

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 - (2*x^2*ArcTanh[E^(a + b*x)])/b - (2*x*Poly
Log[2, -E^(a + b*x)])/b^2 + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*
I)*PolyLog[2, I*E^(a + b*x)])/b^3 + (2*x*PolyLog[2, E^(a + b*x)])/b^2 + (2*
PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 + (x^2*Sech
[a + b*x])/b
```

Rubi [A] time = 0.227086, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {2622, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 2282, 6589, 4180, 2279, 2391}

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(3,-\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 - (2*x^2*ArcTanh[E^(a + b*x)])/b - (2*x*Poly
Log[2, -E^(a + b*x)])/b^2 + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*
I)*PolyLog[2, I*E^(a + b*x)])/b^3 + (2*x*PolyLog[2, E^(a + b*x)])/b^2 + (2*
PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 + (x^2*Sech
[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b}-2\int x\left(-\frac{\tanh^{-1}(\cosh(a+bx))}{b}+\right.$$
$$= -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b}-2\int\left(-\frac{x\tanh^{-1}(\cosh(a+bx))}{b}+\right.$$
$$= -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\int x\tanh^{-1}(\cosh(a+bx))\,dx}{b}-$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\text{sech}(a+bx)}{b}+\frac{(2i)\int\log\left(1-ie^{a+bx}\right)dx}{b^2}-\frac{(2i)\int\log\left(1+\right.}{b^2}$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\text{sech}(a+bx)}{b}+\frac{(2i)\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}-$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$
$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-$$

Mathematica [A] time = 0.641676, size = 225, normalized size = 1.54

$$\frac{-2\left(bx\text{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx))-bx\text{PolyLog}(2,\sinh(a+bx)+\cosh(a+bx))-\text{PolyLog}(3,-\sinh(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x
)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(PolyL
og[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) - 2*(b^2*x^2*ArcTanh[C
osh[a + b*x] + Sinh[a + b*x]] + b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*
x]] - b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] - PolyLog[3, -Cosh[a +
b*x] - Sinh[a + b*x]] + PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]) + b^2*x^
2*Sech[a + b*x])/b^3
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a) (\operatorname{sech}(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b} - \frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^2*x^2*log(e^(b*x + a) + 1) +
 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log
(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/
b^3 - 8*integrate(1/2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Fricas [C] time = 2.72131, size = 2657, normalized size = 18.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) + 2*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(cos
h(b*x + a) + sinh(b*x + a)) + (-2*I*cosh(b*x + a)^2 - 4*I*cosh(b*x + a)*sin
h(b*x + a) - 2*I*sinh(b*x + a)^2 - 2*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x
```

```
+ a)) + (2*I*cosh(b*x + a)^2 + 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b
*x + a)^2 + 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*
x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2
*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*log(cosh(
b*x + a) + sinh(b*x + a) + 1) + (2*I*a*cosh(b*x + a)^2 + 4*I*a*cosh(b*x + a
)*sinh(b*x + a) + 2*I*a*sinh(b*x + a)^2 + 2*I*a)*log(cosh(b*x + a) + sinh(b
*x + a) + I) + (-2*I*a*cosh(b*x + a)^2 - 4*I*a*cosh(b*x + a)*sinh(b*x + a)
- 2*I*a*sinh(b*x + a)^2 - 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (
a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)
^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((2*I*b*x + 2*I*a)*cosh(
b*x + a)^2 + (4*I*b*x + 4*I*a)*cosh(b*x + a)*sinh(b*x + a) + (2*I*b*x + 2*I
*a)*sinh(b*x + a)^2 + 2*I*b*x + 2*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a
) + 1) + ((-2*I*b*x - 2*I*a)*cosh(b*x + a)^2 + (-4*I*b*x - 4*I*a)*cosh(b*x
+ a)*sinh(b*x + a) + (-2*I*b*x - 2*I*a)*sinh(b*x + a)^2 - 2*I*b*x - 2*I*a)*
log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 + (b^2*x^2 - a^2)*co
sh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 -
a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylo
g(3, cosh(b*x + a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
 a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b
*x + a)^2 + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(a+bx) \operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.476 $\int x\mathbf{csch}(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) - (2*x*ArcTanh[E^(a + b*x)])/b - PolyLog[2, -E
^(a + b*x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 + (x*Sech[a + b*x])/b
```

Rubi [A] time = 0.114413, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) - (2*x*ArcTanh[E^(a + b*x)])/b - PolyLog[2, -E
^(a + b*x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 + (x*Sech[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b}-\int\left(-\frac{\tanh^{-1}(\cosh(a+bx))}{b}+\frac{\text{sech}(a}{b}\right.$$

$$= -\frac{x\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b}+\frac{\int\tanh^{-1}(\cosh(a+bx))\,dx}{b}-\frac{\int\text{sech}}{}$$

$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\text{sech}(a+bx)}{b}+\frac{\int bx\text{csch}(a+bx)\,dx}{b}$$

$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\text{sech}(a+bx)}{b}+\int x\text{csch}(a+bx)\,dx$$

$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}-\frac{\int\log\left(1-e^{a+bx}\right)\,dx}{b}$$

$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}-\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx\right.}{b^2}$$

$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{b^2}+\frac{x\text{sech}(a}{b}$$

Mathematica [A] time = 0.282995, size = 106, normalized size = 1.58

$$\frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)-\text{PolyLog}\left(2,e^{-a-bx}\right)+(a+bx)\left(\log\left(1-e^{-a-bx}\right)-\log\left(e^{-a-bx}+1\right)\right)+bx\text{sech}(a+bx)-a\log\left(\tan\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]] + (a + b*x)*(Log[1 - E^(-a - b*x)] - Log[1 +
E^(-a - b*x)]) - a*Log[Tanh[(a + b*x)/2]] + PolyLog[2, -E^(-a - b*x)] - Pol
yLog[2, E^(-a - b*x)] + b*x*Sech[a + b*x])/b^2
```

Maple [A] time = 0.085, size = 95, normalized size = 1.4

$$2\,\frac{x e^{bx+a}}{b\left(1+e^{2\,bx+2\,a}\right)}-2\,\frac{\arctan\left(e^{bx+a}\right)}{b^2}-\frac{\text{dilog}\left(e^{bx+a}\right)}{b^2}-\frac{\text{dilog}\left(1+e^{bx+a}\right)}{b^2}-\frac{\ln\left(1+e^{bx+a}\right)x}{b}-\frac{a\ln\left(e^{bx+a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2/b^2*arctan(exp(b*x+a))-1/b^2*dilog(ex
p(b*x+a))-1/b^2*dilog(1+exp(b*x+a))-1/b*ln(1+exp(b*x+a))*x-1/b^2*a*ln(exp(b
*x+a)-1)
```

Maxima [A] time = 1.72526, size = 122, normalized size = 1.82

$$\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}-\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b*x*log(e^(b*x + a) + 1) + dilog
(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
- 2*arctan(e^(b*x + a))/b^2
```

Fricas [B] time = 2.50582, size = 1166, normalized size = 17.4

$$\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)-2\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - 2*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x
 + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x
+ a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(
b*x + a)^2 + b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a*cosh(b*x + a)
^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x
+ a) + sinh(b*x + a) - 1) + ((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b
*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2 + b*x + a)*log(-cosh(b*x
+ a) - sinh(b*x + a) + 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(
```

```
b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.477 $\int \mathbf{csch}(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rubi [A] time = 0.0264866, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 321, 207}

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.0287162, size = 26, normalized size = 1.13

$$\frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

Maple [A] time = 0.013, size = 23, normalized size = 1.

$$\frac{(\cosh(bx+a))^{-1} - 2\operatorname{Artanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.11271, size = 82, normalized size = 3.57

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

Fricas [B] time = 2.31909, size = 462, normalized size = 20.09

$$-\frac{\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\left(\cosh\left(\right.\right.}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

Giac [B] time = 1.19063, size = 95, normalized size = 4.13

$$-\frac{\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right)}{2\,b} + \frac{\log\left(e^{(bx+a)} + e^{(-bx-a)} - 2\right)}{2\,b} + \frac{2}{b\left(e^{(bx+a)} + e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 1/2*log(e^(b*x + a) + e^(-b*x
- a) - 2)/b + 2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.478 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Rubi [A] time = 0.213599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 33.6089, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\,(\operatorname{sech}(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}+bx}+8\int \frac{e^{(bx+a)}}{4\,\left(bx^2e^{(2\,bx+2\,a)}+bx^2\right)}\,dx+8\int \frac{1}{8\,\left(xe^{(bx+a)}+x\right)}\,dx+8\int \frac{1}{8\,\left(xe^{(bx+a)}-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) + 8*integrate(1/4*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2), x) + 8*integrate(1/8/(x*e^(b*x + a) + x), x)
+ 8*integrate(1/8/(x*e^(b*x + a) - x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

3.479 $\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Rubi [A] time = 0.254192, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 24.7013, size = 0, normalized size = 0.

$$\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.314, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\left(\operatorname{sech}(bx+a)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(bx+a)}}{bx^{2}e^{(2\,bx+2\,a)}+bx^{2}}+8\int \frac{e^{(bx+a)}}{2\left(bx^{3}e^{(2\,bx+2\,a)}+bx^{3}\right)}\,dx+8\int \frac{1}{8\left(x^{2}e^{(bx+a)}+x^{2}\right)}\,dx+8\int \frac{1}{8\left(x^{2}e^{(bx+a)}-x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 8*integrate(1/2*e^(b*x + a)
/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x) + 8*integrate(1/8/(x^2*e^(b*x + a) + x
^2), x) + 8*integrate(1/8/(x^2*e^(b*x + a) - x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^{2}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

3.480 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Rubi [A] time = 0.475069, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 18.3369, size = 0, normalized size = 0.

$$\int x^m\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Maple [A] time = 0.041, size = 0, normalized size = 0.

$$\int x^{m}\operatorname{csch}\left(bx+a\right)\left(\operatorname{sech}\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m}\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\operatorname{csch}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.481 $\int x^3 \mathbf{csch}(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=240

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyL}}{}$$

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b + (3*x*Lo
g[1 + E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4) - (3
*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b
*x)])/(2*b^2) + (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3
, E^(2*a + 2*b*x)])/(2*b^3) - (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3
*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^4) - (3*x^2*Tanh[a + b*x])/(2*b^2) - (x^
3*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.43124, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2620, 14, 5462, 2551, 12, 4182, 2531, 6609, 2282, 6589, 3720, 3718, 2190, 2279, 2391, 30}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```


```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b + (3*x*Lo
g[1 + E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4) - (3
*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b
*x)])/(2*b^2) + (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3
, E^(2*a + 2*b*x)])/(2*b^3) - (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3
*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^4) - (3*x^2*Tanh[a + b*x])/(2*b^2) - (x^
3*Tanh[a + b*x]^2)/(2*b)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
```

```
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\mathbf{csch}(a+bx)\mathbf{sech}^3(a+bx)\,dx = \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int x^2\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(\right.$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int\left(\frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh\right.$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x^2\tanh^2(a+bx)\,dx}{2b} - \frac{3\int x^2\log}{}$$

$$= -\frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x\tanh(a+bx)\,dx}{b^2} + \frac{\int 2bx^3\mathbf{csch}(2}{b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + 2\int x^3\mathbf{csch}(2a+2bx)\,dx +$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\tanh(a+bx)}{2b^2} - $$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\mathbf{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + $$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\mathbf{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4} - \frac{3x}{}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\mathbf{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4} - \frac{3x}{}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\mathbf{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4} - \frac{3x}{}$$

Mathematica [A] time = 7.3167, size = 462, normalized size = 1.92

$$-\frac{e^{2a}\left(6\left(1-e^{-2a}\right)\left(b^2x^2\text{PolyLog}\left(2,-e^{-a-bx}\right)+2\left(bx\text{PolyLog}\left(3,-e^{-a-bx}\right)+\text{PolyLog}\left(4,-e^{-a-bx}\right)\right)\right)+6\left(1-e^{-2a}\right)\left(b^2x^2\text{P}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -(E^(2*a)*((b^4*x^4)/E^(2*a) - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 - E^(-a - b*x
)] - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2*a))*(b^2
*x^2*PolyLog[2, -E^(-a - b*x)] + 2*(b*x*PolyLog[3, -E^(-a - b*x)] + PolyLog
[4, -E^(-a - b*x)])) + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, E^(-a - b*x)] +
 2*(b*x*PolyLog[3, E^(-a - b*x)] + PolyLog[4, E^(-a - b*x)]))))/(2*b^4*(-1
+ E^(2*a))) - (E^(2*a)*((-12*b^2*x^2)/E^(2*a) + (2*b^4*x^4)/E^(2*a) - 12*b*
(1 + E^(-2*a))*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*(1 + E^(-2*a))*x^3*Log[1
 + E^(-2*(a + b*x))] + 6*(1 + E^(-2*a))*PolyLog[2, -E^(-2*(a + b*x))] - (3*
(1 + E^(2*a))*(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3, -
E^(-2*(a + b*x))] + PolyLog[4, -E^(-2*(a + b*x))]))/E^(2*a)))/(4*b^4*(1 + E
^(2*a))) + (x^4*Csch[a]*Sech[a])/4 + (x^3*Sech[a + b*x]^2)/(2*b) - (3*x^2*S
ech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] time = 0.088, size = 359, normalized size = 1.5

$$\frac{x^2\left(2\,bx\mathrm{e}^{2\,bx+2\,a}+3\,\mathrm{e}^{2\,bx+2\,a}+3\right)}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2}-3\,\frac{x^2}{b^2}-3\,\frac{a^2}{b^4}+3\,\frac{x\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b^3}-6\,\frac{ax}{b^3}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^3}{b}-\frac{x^3\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2-3*x^
2/b^2-3/b^4*a^2+3*x*ln(1+exp(2*b*x+2*a))/b^3-6/b^3*a*x+1/b*ln(1+exp(b*x+a))
*x^3-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*
x*polylog(3,-exp(2*b*x+2*a))/b^3+3/b^2*polylog(2,-exp(b*x+a))*x^2-6/b^3*pol
ylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,exp(b*x+a))*
x^2-6/b^3*polylog(3,exp(b*x+a))*x+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+6/b^4*
polylog(4,exp(b*x+a))+6/b^4*polylog(4,-exp(b*x+a))-3/4*polylog(4,-exp(2*b*x
+2*a))/b^4-1/b^4*a^3*ln(exp(b*x+a)-1)+6/b^4*a*ln(exp(b*x+a))+1/b^4*ln(1-exp
(b*x+a))*a^3
```

Maxima [A] time = 1.20206, size = 444, normalized size = 1.85

$$-\frac{1}{2}x^4 + \frac{3x^2 + \left(2bx^3e^{(2a)} + 3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)} + 2b^2e^{(2bx+2a)} + b^2} + \frac{b^4x^4 - 6b^2x^2}{2b^4} - \frac{4b^3x^3\log\left(e^{(2bx+2a)} + 1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + (3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 -
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^4
```

Fricas [C] time = 3.18665, size = 8806, normalized size = 36.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(b^2*x^2*cosh(b*x + a)^4 + 4
*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^
2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(
b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^2*x^2*cosh(b*x + a))*sinh(b*x +
 a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^
4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
+ a)^4 + b^2*x^2 + 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^
2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a
)^3 + (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*co
```

sh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(
b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 1)*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 + (b^2*x^2 - 1)*co
sh(b*x + a))*sinh(b*x + a) - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) -
 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2
*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*c
osh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^
2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) -
 (b^3*x^3*cosh(b*x + a)^4 + 4*b^3*x^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*x
^3*sinh(b*x + a)^4 + 2*b^3*x^3*cosh(b*x + a)^2 + b^3*x^3 + 2*(3*b^3*x^3*cos
h(b*x + a)^2 + b^3*x^3)*sinh(b*x + a)^2 + 4*(b^3*x^3*cosh(b*x + a)^3 + b^3*
x^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 - 3*a)*cosh(b*x + a)^2 + 2*(a^
3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((a^3 - 3*a)*c
osh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)
*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3
 - 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sin
h(b*x + a)^2 + 4*((a^3 - 3*a)*cosh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*
sinh(b*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (a^3*cosh(b*x
 + a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 + 2*a^3
*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 + a^3)*sinh(b*x + a)^2 +
4*(a^3*cosh(b*x + a)^3 + a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*
x - 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*
a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a
^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a))*sinh(b*x + a) - 3*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (b^
3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 -
3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*
sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 + 2
*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x -
 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x +
 a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*l
og(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cos
h(b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 +
 a^3)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^
3 + a^3 + 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3
+ a^3)*cosh(b*x + a)^3 + (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(
-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)

```
+ 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a
)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)
^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2
 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))
*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x
 + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x
*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x +
4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cos
h(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a
)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*
cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x
*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a
)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh
(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x +
a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(
b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/
(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x +
 a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(
b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**3, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.482 $\int x^2 \mathbf{csch}(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}$$

```
[Out] x^2/(2*b) - (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b + Log[Cosh[a + b*x]]/b^3 - (
x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 + (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 +
 PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
 - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.250863, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2620, 14, 5462, 2551, 12, 4182, 2531, 2282, 6589, 3720, 3475, 30}

$$-\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```


```
[Out] x^2/(2*b) - (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b + Log[Cosh[a + b*x]]/b^3 - (
x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 + (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 +
 PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
 - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} - 2\int x\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+\right.$$
$$= \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} - 2\int\left(\frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a\right.$$
$$= \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{\int x\tanh^2(a+bx)\,dx}{b} - \frac{2\int x\log(\tanh}{b}$$
$$= -\frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{\int\tanh(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b} + \frac{\int 2bx^2\text{csch}}{}$$
$$= \frac{x^2}{2b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b} + 2\int x^2\text{csch}(2a+$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+}{2b}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2}$$

Mathematica [B] time = 3.87705, size = 362, normalized size = 2.45

$$\frac{1}{6}\left(-\frac{2e^{2a}\left(6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,-e^{-a-bx}\right)+\text{PolyLog}\left(3,-e^{-a-bx}\right)\right)+6\left(1-e^{-2a}\right)\left(bx\text{PolyLog}\left(2,e^{-a-bx}\right)+\text{PolyLo}\right.\right.}{\left(e^{2a}-1\right)b^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] ((-2*E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^(-a
- b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2*a))
*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 6*(1 - E^(-2
*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)])))/(b^3*(-1 +
 E^(2*a))) + (-12*b*E^(2*a)*x - 4*b^3*x^3 - 6*b^2*(1 + E^(2*a))*x^2*Log[1 +
 E^(-2*(a + b*x))] + 6*Log[1 + E^(2*(a + b*x))] + 6*E^(2*a)*Log[1 + E^(2*(a
 + b*x))] + 6*b*(1 + E^(2*a))*x*PolyLog[2, -E^(-2*(a + b*x))] + 3*(1 + E^(2
*a))*PolyLog[3, -E^(-2*(a + b*x))])/(b^3*(1 + E^(2*a))) + 2*x^3*Csch[a]*Sec
h[a] + (3*x^2*Sech[a + b*x]^2)/b - (6*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^
```

```
2)/6
```

Maple [A] time = 0.051, size = 256, normalized size = 1.7

$$2\frac{x\left(bx\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}+1\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-2\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3}-2\frac{\mathrm{polylog}\left(3,\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{b}+2\frac{x\mathrm{polylo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2/b^3*ln
(exp(b*x+a))+1/b^3*ln(1+exp(2*b*x+2*a))-2*polylog(3,exp(b*x+a))/b^3+1/b*ln(
1+exp(b*x+a))*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-x^2*ln(1+exp(2*b*x+2*a))/b
-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,ex
p(b*x+a))/b^2-2*polylog(3,-exp(b*x+a))/b^3+1/2*polylog(3,-exp(2*b*x+2*a))/b
^3+1/b^3*a^2*ln(exp(b*x+a)-1)-1/b^3*ln(1-exp(b*x+a))*a^2
```

Maxima [A] time = 1.19151, size = 309, normalized size = 2.09

$$\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}+x\right)}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2}-\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3(-e^{(2bx+2a)})}{2b^3}+\frac{b^2x^2\log\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) + x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 - 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*
x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x
 + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2
*polylog(3, e^(b*x + a)))/b^3 + log(e^(2*b*x + 2*a) + 1)/b^3
```

Fricas [C] time = 2.89095, size = 6546, normalized size = 44.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 +
 2*(b*x + a)*sinh(b*x + a)^4 - 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 - 2*
(b^2*x^2 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 - 2*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) +
2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 +
 b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x
^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3
+ b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^
3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1
)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x
+ a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + I) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*s
inh(b*x + a)^3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 +
2*(3*(a^2 - 1)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 -
 1)*cosh(b*x + a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(
b*x + a) + sinh(b*x + a) - I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*
sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 + 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*co
sh(b*x + a)^2 + a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 + a^2*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x
 + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a
```

```
)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 - (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(a+bx) \operatorname{sech}^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.483 $\int x\,\mathbf{csch}(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}+\frac{x}{2b}$$

```
[Out] x/(2*b) - (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(
2*b^2) + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*T
anh[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.11963, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2620, 14, 5462, 2548, 12, 4182, 2279, 2391, 3473, 8}

$$-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] x/(2*b) - (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(
2*b^2) + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*T
anh[a + b*x]^2)/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
```

```
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} - \int\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\right.$$
$$= \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int\tanh^2(a+bx)\,dx}{2b} - \frac{\int\log(\tanh(a+b}{b}$$
$$= -\frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int 1\,dx}{2b} + \frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b}$$
$$= \frac{x}{2b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + 2\int x\text{csch}(2a+2bx)\,dx$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\int\log\left(1-e^{2a+2bx}\right)}{b}$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\right.}{2b^2}$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tan}{}$$

Mathematica [A] time = 0.41402, size = 139, normalized size = 1.46

$$\frac{\text{PolyLog}\left(2,-e^{-2(a+bx)}\right) - \text{PolyLog}\left(2,e^{-2(a+bx)}\right) + 2a\log\left(1-e^{-2(a+bx)}\right) + 2bx\log\left(1-e^{-2(a+bx)}\right) - 2a\log\left(e^{-2(a+bx)}+1\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (2*a*Log[1 - E^(-2*(a + b*x))] + 2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[
1 + E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] + 2*a*Log[Cosh[a +
b*x]] - 2*a*Log[Sinh[a + b*x]] + PolyLog[2, -E^(-2*(a + b*x))] - PolyLog[2,
 E^(-2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b*x])/(2*b^2)
```

Maple [B] time = 0.04, size = 166, normalized size = 1.8

$$\frac{2\,bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}+1}{b^2\left(1+\mathrm{e}^{2\,bx+2\,a}\right)^2} + \frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b} + \frac{\text{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2} - \frac{x\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b} - \frac{\text{polylog}\left(2,-\mathrm{e}^{2\,bx+2\,a}\right)}{2\,b^2} + \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] (2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2+1/b*ln(1+e
xp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))-x*ln(1+exp(2*b*x+2*a))/b-1/2*poly
log(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+
1/b^2*polylog(2,exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.28162, size = 192, normalized size = 2.02

$$\frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
+ 2*a)))/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [C] time = 2.70782, size = 4325, normalized size = 45.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
(2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
 a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog
(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
```

```
) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^
4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(
b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x
*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 +
 a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh
(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*lo
g(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(
b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*s
inh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a)
^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x +
 a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
+ a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*
cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*
sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2
+ 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x
 + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a
)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)
^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x
 + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cosh(b*x + a)^
4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(
b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2
*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.484 $\int \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0261388, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0311195, size = 36, normalized size = 1.33

$$-\frac{-\operatorname{sech}^2(a+bx) - 2\log(\sinh(a+bx)) + 2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/(2*b)
```

Maple [A] time = 0., size = 26, normalized size = 1.

$$\frac{1}{2\,b\,(\cosh(bx+a))^2} + \frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.60789, size = 119, normalized size = 4.41

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 2.32279, size = 1035, normalized size = 38.33

$$\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2+1\right)\sinh(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

Giac [B] time = 1.17346, size = 135, normalized size = 5.

$$-\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}{2b}+\frac{\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}+\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}+6}{2b\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b + 1/2*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2)/b + 1/2*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/
(b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2))
```

3.485 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Rubi [A] time = 0.213803, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 54.7984, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Maple [A] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\left(\operatorname{sech}(bx+a)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2x^2e^{(4\,bx+4\,a)}+2\,b^2x^2e^{(2\,bx+2\,a)}+b^2x^2}+16\int\frac{b^2x^2-1}{8\left(b^2x^3e^{(2\,bx+2\,a)}+b^2x^3\right)}\,dx-16\int\frac{1}{16\left(xe^{(bx+a)}+x\right)}\,dx+16\int\frac{1}{16\left(xe^{(bx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/8*(b^2*x^2 - 1)/(b^2*x^3*e^
(2*b*x + 2*a) + b^2*x^3), x) - 16*integrate(1/16/(x*e^(b*x + a) + x), x) +
16*integrate(1/16/(x*e^(b*x + a) - x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

3.486 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Rubi [A] time = 0.254342, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 28.0959, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Maple [A] time = 0.53, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\left(\operatorname{sech}(bx+a)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}-1\right)}{b^{2}x^{3}e^{(4bx+4a)}+2b^{2}x^{3}e^{(2bx+2a)}+b^{2}x^{3}}+16\int\frac{b^{2}x^{2}-3}{8\left(b^{2}x^{4}e^{(2bx+2a)}+b^{2}x^{4}\right)}\,dx-16\int\frac{1}{16\left(x^{2}e^{(bx+a)}+x^{2}\right)}\,dx+16\int\frac{}{16\left(x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*
x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/8*(b^2*x^2 - 3)/(b^2*x^4*e^
(2*b*x + 2*a) + b^2*x^4), x) - 16*integrate(1/16/(x^2*e^(b*x + a) + x^2), x
) + 16*integrate(1/16/(x^2*e^(b*x + a) - x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^{3}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

3.487 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

Rubi [A] time = 0.399438, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

Rubi steps

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Mathematica [A] time = 21.0348, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int x^{m} (\operatorname{csch}(bx+a))^{2} \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}(bx+a)^{2} \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \operatorname{csch}(bx+a)^{2} \operatorname{sech}(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{2}(a+bx) \operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

3.488 $\int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(3\right.}{b^3}$$

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Cs
ch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + ((3*I)*x^2*PolyLog[2,
 (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog[2, I*E^(a + b*x)])/b^2 + (6*x*
PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - ((6*I)*x*
PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[3, I*E^(a + b*x)])/b^3
 - (6*PolyLog[3, E^(a + b*x)])/b^4 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/b
^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4
```

Rubi [A] time = 0.343965, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2621, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 6609, 2282, 6589, 4182}

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```


```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Cs
ch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + ((3*I)*x^2*PolyLog[2,
 (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog[2, I*E^(a + b*x)])/b^2 + (6*x*
PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - ((6*I)*x*
PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[3, I*E^(a + b*x)])/b^3
 - (6*PolyLog[3, E^(a + b*x)])/b^4 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/b
^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
```

```
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx = -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\mathrm{csch}(a+bx)}{b} - 3\int x^2\left(-\frac{\tan^{-1}(\sinh(a+bx))}{b} - \right.$$

$$= -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\mathrm{csch}(a+bx)}{b} - 3\int\left(-\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \right.$$

$$= -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\mathrm{csch}(a+bx)}{b} + \frac{3\int x^2\tan^{-1}(\sinh(a+bx))\,dx}{b} + $$

$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6\int x\log\left(1-e^{a+bx}\right)dx}{b^2} + \frac{6\int x\log\left(1+\right.}{b^2}$$

$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{6x\mathrm{Li}_2\left(e^{a+bx}\right)}{b^3} + \frac{6\int \mathrm{I}}{}$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + $$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + $$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + $$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + $$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\mathrm{csch}(a+bx)}{b} - \frac{6x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3} + $$

Mathematica [A] time = 1.76773, size = 333, normalized size = 1.41

$$\frac{-2i\left(-3b^2x^2\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)+3b^2x^2\mathrm{PolyLog}\left(2,ie^{a+bx}\right)+6bx\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)-6bx\mathrm{PolyLog}\left(3,ie^{a+bx}\right)-6\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^3*x^3*Csch[a] - 12*(b^2*x^2*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] +
b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]] - b*x*PolyLog[2, Cosh[a + b*
x] + Sinh[a + b*x]] - PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]] + PolyLog[
3, Cosh[a + b*x] + Sinh[a + b*x]]) - (2*I)*(b^3*x^3*Log[1 - I*E^(a + b*x)]
- b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (-I)*E^(a + b*x)] +
 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] -
```

```
 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyL
og[4, I*E^(a + b*x)]) + b^3*x^3*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] +
 b^3*x^3*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^4)
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int x^{3} \left(\operatorname{csch}\left(bx+a\right)\right)^{2} \operatorname{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^{3}e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{3\left(b^{2}x^{2}\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_{2}\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_{3}(-e^{(bx+a)})\right)}{b^{4}}+\frac{3\left(b^{2}x^{2}\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_{2}\left(e^{(bx+a)}\right.\right.}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4 - 8*integrate(1/4*x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.80188, size = 3559, normalized size = 15.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) - (-3*I*b^2*x^2*cosh(b*x + a)^2 - 6*I*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a) - 3*I*b^2*x^2*sinh(b*x + a)^2 + 3*I*b^2*x^2)*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) - (3*I*b^2*x^2*cosh(b*x + a)^2 + 6*I
*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + 3*I*b^2*x^2*sinh(b*x + a)^2 - 3*I*b^
2*x^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 +
 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*
x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) - (I*a^3*cosh(b*x + a)^2 + 2*I*a^3*cosh(b*x + a)*sinh
(b*x + a) + I*a^3*sinh(b*x + a)^2 - I*a^3)*log(cosh(b*x + a) + sinh(b*x + a
) + I) - (-I*a^3*cosh(b*x + a)^2 - 2*I*a^3*cosh(b*x + a)*sinh(b*x + a) - I*
a^3*sinh(b*x + a)^2 + I*a^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(a^
2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2
 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^3*x^3 - I*a^3 + (I*b
^3*x^3 + I*a^3)*cosh(b*x + a)^2 + (2*I*b^3*x^3 + 2*I*a^3)*cosh(b*x + a)*sin
h(b*x + a) + (I*b^3*x^3 + I*a^3)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*s
inh(b*x + a) + 1) - (I*b^3*x^3 + I*a^3 + (-I*b^3*x^3 - I*a^3)*cosh(b*x + a)
^2 + (-2*I*b^3*x^3 - 2*I*a^3)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^3*x^3 - I
*a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 3*(b^2
*x^2 - (b^2*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*si
nh(b*x + a) - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - s
inh(b*x + a) + 1) - (-6*I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh(b*x + a
) - 6*I*sinh(b*x + a)^2 + 6*I)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)
) - (6*I*cosh(b*x + a)^2 + 12*I*cosh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x
+ a)^2 - 6*I)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, co
sh(b*x + a) + sinh(b*x + a)) - (6*I*b*x*cosh(b*x + a)^2 + 12*I*b*x*cosh(b*x
 + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*polylog(3, I*cosh(
b*x + a) + I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^2 - 12*I*b*x*cosh(b*x
 + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 + 6*I*b*x)*polylog(3, -I*cosh
(b*x + a) - I*sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(
b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)
^2 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a), x)
```

3.489 $\int x^2\textbf{csch}^2(a+bx)\textbf{sech}(a+bx)\,dx$

Optimal. Leaf size=157

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(3,-i\right.}{b^3}$$

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch
[a + b*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + ((2*I)*x*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (2*PolyLog[2,
 E^(a + b*x)])/b^3 - ((2*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*Poly
Log[3, I*E^(a + b*x)])/b^3
```

Rubi [A] time = 0.233922, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {2621, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 2282, 6589, 4182, 2279, 2391}

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(3,-i\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch
[a + b*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + ((2*I)*x*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (2*PolyLog[2,
 E^(a + b*x)])/b^3 - ((2*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*Poly
Log[3, I*E^(a + b*x)])/b^3
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} - 2\int x\left(-\frac{\tan^{-1}(\sinh(a+bx))}{b} - \text{csch}\right.$$

$$= -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} - 2\int\left(-\frac{x\tan^{-1}(\sinh(a+bx))}{b} - \frac{x\text{csc}}{}\right.$$

$$= -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} + \frac{2\int x\tan^{-1}(\sinh(a+bx))\,dx}{b} + \frac{2\int x}{}$$

$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\int\log\left(1-e^{a+bx}\right)dx}{b^2} + \frac{2\int\log\left(1+e^{a+bx}\right.}{b^2}$$

$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{a+bx}\right)}{b^3} + \frac{2\,\text{Subst}}{}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{2\text{Li}_2}{}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{2ix\text{L}}{}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{2ix\text{L}}{}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{2ix\text{L}}{}$$

Mathematica [A] time = 1.85044, size = 312, normalized size = 1.99

$$\frac{4ibx\text{PolyLog}\left(2,-ie^{a+bx}\right) - 4ibx\text{PolyLog}\left(2,ie^{a+bx}\right) + 4\text{PolyLog}\left(2,-e^{-a-bx}\right) - 4\text{PolyLog}\left(2,e^{-a-bx}\right) - 4i\text{PolyLog}\left(3,-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^2*x^2*Csch[a] + 4*a*Log[1 - E^(-a - b*x)] + 4*b*x*Log[1 - E^(-a - b*x
)] - 4*a*Log[1 + E^(-a - b*x)] - 4*b*x*Log[1 + E^(-a - b*x)] - (2*I)*b^2*x^
2*Log[1 - I*E^(a + b*x)] + (2*I)*b^2*x^2*Log[1 + I*E^(a + b*x)] - 4*a*Log[T
anh[(a + b*x)/2]] + 4*PolyLog[2, -E^(-a - b*x)] - 4*PolyLog[2, E^(-a - b*x)
] + (4*I)*b*x*PolyLog[2, (-I)*E^(a + b*x)] - (4*I)*b*x*PolyLog[2, I*E^(a +
b*x)] - (4*I)*PolyLog[3, (-I)*E^(a + b*x)] + (4*I)*PolyLog[3, I*E^(a + b*x)
] + b^2*x^2*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^2*x^2*Sech[a/2]*S
ech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^3)
```

Maple [F] time = 0.371, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{csch}(bx+a))^2 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}-8\int\frac{x^2e^{(bx+a)}}{4\left(e^{(2\,bx+2\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3 - 8*integrate(1/4*x^2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.76475, size = 2685, normalized size = 17.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - 2*(cosh(b*x + a)^2 +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) +
sinh(b*x + a)) - (-2*I*b*x*cosh(b*x + a)^2 - 4*I*b*x*cosh(b*x + a)*sinh(b*x
 + a) - 2*I*b*x*sinh(b*x + a)^2 + 2*I*b*x)*dilog(I*cosh(b*x + a) + I*sinh(b
*x + a)) - (2*I*b*x*cosh(b*x + a)^2 + 4*I*b*x*cosh(b*x + a)*sinh(b*x + a) +
```

```
 2*I*b*x*sinh(b*x + a)^2 - 2*I*b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^2 + 2*b*x*
cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) - (-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*sinh
(b*x + a) - I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x + a
) + I) - (I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) + I*a
^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*(a*c
osh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 - a)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*c
osh(b*x + a)^2 + (2*I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b
^2*x^2 - I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x +
 a) + 1) - (I*b^2*x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + (-2*I*b^2*x^
2 + 2*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x +
a)^2 - I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*((b*x + a)*co
sh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*
x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - (2*I*cosh(b*x
 + a)^2 + 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b*x + a)^2 - 2*I)*poly
log(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (-2*I*cosh(b*x + a)^2 - 4*I*cos
h(b*x + a)*sinh(b*x + a) - 2*I*sinh(b*x + a)^2 + 2*I)*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(
b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.490 $\int x\mathbf{csch}^2(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (x*Csch[a + b*x
])/b + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])
/b^2
```

Rubi [A] time = 0.11245, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2621, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (x*Csch[a + b*x
])/b + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])
/b^2
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{x\tan^{-1}(\sinh(a+bx))}{b} - \frac{x\text{csch}(a+bx)}{b} - \int\left(-\frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\text{csch}(a+b}{b}\right.$$
$$= -\frac{x\tan^{-1}(\sinh(a+bx))}{b} - \frac{x\text{csch}(a+bx)}{b} + \frac{\int\tan^{-1}(\sinh(a+bx))\,dx}{b} + \frac{\int\text{csch}(a+}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} - \frac{\int bx\text{sech}(a+bx)\,dx}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} - \int x\text{sech}(a+bx)\,dx$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\int\log\left(1-ie^{a+bx}\right)d}{b}$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}d\right.}{b^2}$$
$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{i\text{Li}}{}$$

Mathematica [A] time = 0.805017, size = 112, normalized size = 1.42

$$\frac{2i\text{PolyLog}(2,-i(\sinh(a+bx)+\cosh(a+bx))) - 2i\text{PolyLog}(2,i(\sinh(a+bx)+\cosh(a+bx))) + bx\tanh\left(\frac{1}{2}(a+bx)\right) -}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-4*b*x*ArcTan[Cosh[a + b*x] + Sinh[a + b*x]] - b*x*Coth[(a + b*x)/2] + 2*L
og[Tanh[(a + b*x)/2]] + (2*I)*PolyLog[2, (-I)*(Cosh[a + b*x] + Sinh[a + b*x
])] - (2*I)*PolyLog[2, I*(Cosh[a + b*x] + Sinh[a + b*x])] + b*x*Tanh[(a + b
*x)/2])/(2*b^2)
```

Maple [B] time = 0.041, size = 179, normalized size = 2.3

$$-2\frac{xe^{bx+a}}{b\left(e^{2bx+2a}-1\right)} + 2\frac{a\arctan\left(e^{bx+a}\right)}{b^2} - \frac{i\text{dilog}\left(1-ie^{bx+a}\right)}{b^2} + \frac{i\text{dilog}\left(1+ie^{bx+a}\right)}{b^2} + \frac{i\ln\left(1+ie^{bx+a}\right)x}{b} + \frac{i\ln\left(1+ie^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -2*x*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)+2/b^2*a*arctan(exp(b*x+a))-I/b^2*dilog
(1-I*exp(b*x+a))+I/b^2*dilog(1+I*exp(b*x+a))+I/b*ln(1+I*exp(b*x+a))*x+I/b^2
*ln(1+I*exp(b*x+a))*a-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b*x+a))*a+1
/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(1+exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}-8\int\frac{xe^{(bx+a)}}{4\left(e^{(2\,bx+2\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 - 8*integrate(1/4*x*e^(b*x + a)/(e^(2
*b*x + 2*a) + 1), x)
```

Fricas [B] time = 2.67801, size = 1624, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - (-I*cosh(b*x + a)^2 - 2*I*cos
h(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) + I
*sinh(b*x + a)) - (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) + I*
sinh(b*x + a)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x
+ a) + sinh(b*x + a) + 1) - (I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh
(b*x + a) + I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b*x + a) +
I) - (-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x + a)*sinh(b*x + a) - I*a*sinh(b
*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (cosh(b*x + a)^2
```

```
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) - ((I*b*x + I*a)*cosh(b*x + a)^2 + (2*I*b*x + 2*I*a)*cos
h(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x - I*a)*log
(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-I*b*x - I*a)*cosh(b*x + a)^2 +
 (-2*I*b*x - 2*I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*sinh(b*x +
 a)^2 + I*b*x + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^2*cosh
(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{csch}^2\,(a+bx)\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{csch}\,(bx+a)^2\,\mathrm{sech}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a), x)
```

3.491 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=24

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rubi [A] time = 0.0248662, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2621, 321, 207}

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{i\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} - \frac{i\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}$$

Mathematica [C] time = 0.0155382, size = 29, normalized size = 1.21

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] time = 0.015, size = 27, normalized size = 1.1

$$-\frac{1}{b\sinh(bx+a)} - 2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -1/b/sinh(b*x+a)-2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.71663, size = 58, normalized size = 2.42

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 2.34735, size = 305, normalized size = 12.71

$$-\frac{2\left(\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+\cosh}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

Giac [B] time = 1.19237, size = 78, normalized size = 3.25

$$-\frac{\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{2b}-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - 2/(b*(e^(b
*x + a) - e^(-b*x - a)))
```

3.492 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Rubi [A] time = 0.161535, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Mathematica [A] time = 21.9544, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2 \operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}-bx}-8\int \frac{e^{(bx+a)}}{4\left(xe^{(2\,bx+2\,a)}+x\right)}\,dx-8\int \frac{1}{8\left(bx^2e^{(bx+a)}+bx^2\right)}\,dx-8\int \frac{1}{8\left(bx^2e^{(bx+a)}-bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 8*integrate(1/4*e^(b*x + a)/(x
*e^(2*b*x + 2*a) + x), x) - 8*integrate(1/8/(b*x^2*e^(b*x + a) + b*x^2), x)
 - 8*integrate(1/8/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

3.493 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Rubi [A] time = 0.210548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 24.5053, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Maple [A] time = 0.322, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{csch}\left(bx+a\right)\right)^{2} \operatorname{sech}\left(bx+a\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^{(bx+a)}}{bx^{2}e^{(2\,bx+2\,a)}-bx^{2}}-8\int \frac{e^{(bx+a)}}{4\left(x^{2}e^{(2\,bx+2\,a)}+x^{2}\right)}\,dx-8\int \frac{1}{4\left(bx^{3}e^{(bx+a)}+bx^{3}\right)}\,dx-8\int \frac{1}{4\left(bx^{3}e^{(bx+a)}-bx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 8*integrate(1/4*e^(b*x + a
)/(x^2*e^(2*b*x + 2*a) + x^2), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) + b*
x^3), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{csch}\left(bx+a\right)^{2} \operatorname{sech}\left(bx+a\right)}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)/x^2, x)
```

3.494 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Rubi [A] time = 0.488521, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 7.17241, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Maple [A] time = 0.032, size = 0, normalized size = 0.

$$\int x^{m} (\operatorname{csch}(bx+a))^{2} (\operatorname{sech}(bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}(bx+a)^{2} \operatorname{sech}(bx+a)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{m} \operatorname{csch}(bx+a)^{2} \operatorname{sech}(bx+a)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{2}(a+bx) \operatorname{sech}^{2}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.495 $\int x^3 \mathbf{csch}^2(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{3x\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{2x^3}{b}$$

```
[Out] (-2*x^3)/b - (2*x^3*Coth[2*a + 2*b*x])/b + (3*x^2*Log[1 - E^(4*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(4*(a + b*x))])/(2*b^3) - (3*PolyLog[3, E^(4*(a +
b*x))])/(8*b^4)
```

Rubi [A] time = 0.238234, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5461, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{3x\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{2x^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^3)/b - (2*x^3*Coth[2*a + 2*b*x])/b + (3*x^2*Log[1 - E^(4*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(4*(a + b*x))])/(2*b^3) - (3*PolyLog[3, E^(4*(a +
b*x))])/(8*b^4)
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = 4\int x^3\text{csch}^2(2a+2bx)\,dx$$
$$= -\frac{2x^3\coth(2a+2bx)}{b} + \frac{6\int x^2\coth(2a+2bx)\,dx}{b}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} - \frac{12\int \frac{e^{2(2a+2bx)}x^2}{1-e^{2(2a+2bx)}}\,dx}{b}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} - \frac{6\int x\log\left(1-e^{2(2a+2bx)}\right)\,dx}{b^2}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\int \text{Li}_2\left(\right.}{}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\,\text{Subst}}{}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\text{Li}_3\left(e^{4(\right.}}{8b^4}$$

Mathematica [B] time = 5.73757, size = 284, normalized size = 3.34

$$4\left(\frac{x^3\text{csch}(2a)\sinh(2bx)\text{csch}(2a+2bx)}{2b} - \frac{e^{4a}\left(12\left(1-e^{-4a}\right)\left(bx\text{PolyLog}\left(2,-e^{-a-bx}\right)+\text{PolyLog}\left(3,-e^{-a-bx}\right)\right)+12\left(1-\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-(E^(4*a)*((8*b^3*x^3)/E^(4*a) - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 - E^(-a
- b*x)] - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 + E^(-a - b*x)] - 6*b^2*(1 - E^(-4
*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 12*(1 - E^(-4*a))*(b*x*PolyLog[2, -E^(
-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 12*(1 - E^(-4*a))*(b*x*PolyLog[2,
 E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)]) + (3*(-1 + E^(4*a))*(2*b*x*PolyL
og[2, -E^(-2*(a + b*x))] + PolyLog[3, -E^(-2*(a + b*x))]))/E^(4*a)))/(8*b^4
*(-1 + E^(4*a))) + (x^3*Csch[2*a]*Csch[2*a + 2*b*x]*Sinh[2*b*x])/(2*b))
```

Maple [B] time = 0.052, size = 263, normalized size = 3.1

$$-4\,\frac{x^3}{b\left(e^{2\,bx+2\,a}-1\right)\left(1+e^{2\,bx+2\,a}\right)} + 8\,\frac{a^3}{b^4} - 6\,\frac{\text{polylog}\left(3,e^{bx+a}\right)}{b^4} - \frac{3\,\text{polylog}\left(3,-e^{2\,bx+2\,a}\right)}{2\,b^4} + 12\,\frac{a^2x}{b^3} + 3\,\frac{x^2\ln\left(1+e^{2\,bx+2\,a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x^3/b/(exp(2*b*x+2*a)-1)/(1+exp(2*b*x+2*a))+8/b^4*a^3-6*polylog(3,exp(b*
x+a))/b^4-3/2*polylog(3,-exp(2*b*x+2*a))/b^4+12/b^3*a^2*x+3*x^2*ln(1+exp(2*
b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a))/b^3+6*x*polylog(2,exp(b*x+a))/
b^3+3/b^2*ln(1+exp(b*x+a))*x^2+6*x*polylog(2,-exp(b*x+a))/b^3+3/b^2*ln(1-ex
p(b*x+a))*x^2-6*polylog(3,-exp(b*x+a))/b^4-4*x^3/b+3/b^4*a^2*ln(exp(b*x+a)-
1)-12/b^4*a^2*ln(exp(b*x+a))-3/b^4*ln(1-exp(b*x+a))*a^2
```

Maxima [B] time = 1.17352, size = 243, normalized size = 2.86

$$-\frac{4\,x^3}{be^{(4\,bx+4\,a)}-b}-\frac{4\,x^3}{b}+\frac{3\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}+\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x^3/(b*e^(4*b*x + 4*a) - b) - 4*x^3/b + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*
a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
 + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(
3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b
*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.82888, size = 5019, normalized size = 59.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*(b^3*x^3 + a^3)*cosh(b*x + a)^4 + 16*(b^3*x^3 + a^3)*cosh(b*x + a)^3*si
nh(b*x + a) + 24*(b^3*x^3 + a^3)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 16*(b^3*
x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + 4*(b^3*x^3 + a^3)*sinh(b*x + a)^
4 - 4*a^3 - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) +
6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3
```

```
 + b*x*sinh(b*x + a)^4 - b*x)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x
*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a
)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x +
a)^4 - b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x +
 a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*co
sh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x
*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*dilog(-cosh(b*x
 + a) - sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*b^2*x^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - b^2*x^2)*log(cosh
(b*x + a) + sinh(b*x + a) + 1) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x
 + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh
(b*x + a) + I) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x +
a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x + a) + 6*a^2*cosh(
b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh
(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2 - a
^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(
b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x
 + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*lo
g(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-I*cosh(b*x + a
) - I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2
- a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*cosh(b*x + a)^2*si
nh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
- a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-cosh(b*x + a) - sinh(b*x + a)
+ 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 -
1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x +
 a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, I*cosh(b*x + a) + I*s
inh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 - 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, -cosh(b
*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)^3*sinh
(b*x + a) + 6*b^4*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^4*cosh(b*x + a)*sin
h(b*x + a)^3 + b^4*sinh(b*x + a)^4 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.496 $\int x^2 \mathbf{csch}^2(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{2x^2}{b}$$

```
[Out] (-2*x^2)/b - (2*x^2*Coth[2*a + 2*b*x])/b + (2*x*Log[1 - E^(4*(a + b*x))])/b
^2 + PolyLog[2, E^(4*(a + b*x))]/(2*b^3)
```

Rubi [A] time = 0.167392, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5461, 4184, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{2x^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^2)/b - (2*x^2*Coth[2*a + 2*b*x])/b + (2*x*Log[1 - E^(4*(a + b*x))])/b
^2 + PolyLog[2, E^(4*(a + b*x))]/(2*b^3)
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx &= 4\int x^2\operatorname{csch}^2(2a+2bx)\,dx\\
&= -\frac{2x^2\coth(2a+2bx)}{b}+\frac{4\int x\coth(2a+2bx)\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{8\int\frac{e^{2(2a+2bx)}x}{1-e^{2(2a+2bx)}}\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2(2a+2bx)}\right)dx}{b^2}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(2}\right.}{2b^3}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}+\frac{\operatorname{Li}_2\left(e^{4(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [B] time = 4.06738, size = 216, normalized size = 3.38

$$4\left(\frac{x^2\operatorname{csch}(2a)\sinh(2bx)\operatorname{csch}(2a+2bx)}{2b}-\frac{e^{4a}\left(2\left(1-e^{-4a}\right)\operatorname{PolyLog}\left(2,-e^{-a-bx}\right)+2\left(1-e^{-4a}\right)\operatorname{PolyLog}\left(2,e^{-a-bx}\right)+(1\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-(E^(4*a)*((4*b^2*x^2)/E^(4*a) - 2*b*(1 - E^(-4*a))*x*Log[1 - E^(-a - b*
x)] - 2*b*(1 - E^(-4*a))*x*Log[1 + E^(-a - b*x)] - 2*b*(1 - E^(-4*a))*x*Log
[1 + E^(-2*(a + b*x))] + 2*(1 - E^(-4*a))*PolyLog[2, -E^(-a - b*x)] + 2*(1
- E^(-4*a))*PolyLog[2, E^(-a - b*x)] + (1 - E^(-4*a))*PolyLog[2, -E^(-2*(a
+ b*x))]))/(4*b^3*(-1 + E^(4*a))) + (x^2*Csch[2*a]*Csch[2*a + 2*b*x]*Sinh[2
*b*x])/(2*b))
```

Maple [B] time = 0.041, size = 199, normalized size = 3.1

$$-4\,{\frac {{x}^{2}}{b \left({{\rm e}^{2\,bx+2\,a}}-1 \right) \left(1+{{\rm e}^{2\,bx+2\,a}} \right) }}-4\,{\frac {{x}^{2}}{b}}-8\,{\frac {ax}{{b}^{2}}}-4\,{\frac {{a}^{2}}{{b}^{3}}}+2\,{\frac {\ln \left(1-{{\rm e}^{bx+a}} \right) x}{{b}^{2}}}+2\,{\frac {\ln \left(1-{{\rm e}^{bx+a}} \right) a}{{b}^{3}}}+2\,{\frac {{\rm polylog} \left(2,{{\rm e}^{bx+a}}\right)}{{b}^{3}}}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x^2/b/(exp(2*b*x+2*a)-1)/(1+exp(2*b*x+2*a))-4*x^2/b-8/b^2*a*x-4/b^3*a^2+
2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b
^3+2*x*ln(1+exp(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+2/b^2*ln(1+e
xp(b*x+a))*x+2*polylog(2,-exp(b*x+a))/b^3-2/b^3*a*ln(exp(b*x+a)-1)+8/b^3*a*
ln(exp(b*x+a))
```

Maxima [A] time = 1.22338, size = 159, normalized size = 2.48

$$-\frac{4\,x^{2}}{b e^{\left(4\,b x + 4\,a\right)} - b} - \frac{4\,x^{2}}{b} + \frac{2\,b x \log\left(e^{\left(2\,b x + 2\,a\right)} + 1\right) + \mathrm{Li}_2\left(-e^{\left(2\,b x + 2\,a\right)}\right)}{b^{3}} + \frac{2\,\left(b x \log\left(e^{\left(b x + a\right)} + 1\right) + \mathrm{Li}_2\left(-e^{\left(b x + a\right)}\right)\right)}{b^{3}} + \frac{2\,\left(b x \log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x^2/(b*e^(4*b*x + 4*a) - b) - 4*x^2/b + (2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^3 + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*
x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Fricas [C] time = 2.82204, size = 3605, normalized size = 56.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(2*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)^3*s
inh(b*x + a) + 12*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 8*(b^2*
x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + 2*(b^2*x^2 - a^2)*sinh(b*x + a)^
4 + 2*a^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x
 + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x +
 a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)
*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)
) - (b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh
(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 - b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x +
a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a
)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sin
h(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b
*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b*x + a) + sinh(b*x + a) - I) +
 (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4
- a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x + a)^4 +
4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sin
h(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b
*x + a)^4 - b*x - a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a
)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)
*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-I*cosh(b*x + a) - I*sinh(b*x
+ a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b
*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-cosh(b*x
 + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)^3*si
nh(b*x + a) + 6*b^3*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^3*cosh(b*x + a)*s
```

```
inh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.497 $\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\sinh(2a+2bx))}{b^2}-\frac{2x\coth(2a+2bx)}{b}$$

```
[Out] (-2*x*Coth[2*a + 2*b*x])/b + Log[Sinh[2*a + 2*b*x]]/b^2
```

Rubi [A] time = 0.0568041, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5461, 4184, 3475}

$$\frac{\log(\sinh(2a+2bx))}{b^2}-\frac{2x\coth(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x*Coth[2*a + 2*b*x])/b + Log[Sinh[2*a + 2*b*x]]/b^2
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = 4\int x\mathrm{csch}^2(2a+2bx)\,dx$$
$$= -\frac{2x\coth(2a+2bx)}{b} + \frac{2\int \coth(2a+2bx)\,dx}{b}$$
$$= -\frac{2x\coth(2a+2bx)}{b} + \frac{\log(\sinh(2a+2bx))}{b^2}$$

Mathematica [A] time = 0.145231, size = 26, normalized size = 0.87

$$\frac{\log(\sinh(2(a+bx))) - 2bx\coth(2(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*b*x*Coth[2*(a + b*x)] + Log[Sinh[2*(a + b*x)]])/b^2
```

Maple [B] time = 0.037, size = 62, normalized size = 2.1

$$-4\,\frac{x}{b} - 4\,\frac{a}{b^2} - 4\,\frac{x}{b\left(1+e^{2\,bx+2\,a}\right)\left(e^{2\,bx+2\,a}-1\right)} + \frac{\ln\left(e^{4\,bx+4\,a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x/b-4/b^2*a-4*x/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(4*b
*x+4*a)-1)
```

Maxima [B] time = 1.11937, size = 117, normalized size = 3.9

$$-\frac{4\,xe^{(4\,bx+4\,a)}}{be^{(4\,bx+4\,a)}-b} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x*e^(4*b*x + 4*a)/(b*e^(4*b*x + 4*a) - b) + log((e^(b*x + a) + 1)*e^(-a)
)/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 + log((e^(2*b*x + 2*a) + 1)*e^(-2
*a))/b^2
```

Fricas [B] time = 2.2761, size = 788, normalized size = 26.27

$$-\frac{4\,bx\cosh\left(bx+a\right)^{4}+16\,bx\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+24\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+16\,bx\cosh\left(bx+a\right)\sin}{b^{2}\cosh\left(bx+a\right)^{4}+4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*b*x*cosh(b*x + a)^4 + 16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 24*b*x*cos
h(b*x + a)^2*sinh(b*x + a)^2 + 16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 4*b*x
*sinh(b*x + a)^4 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*c
osh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
 + a)^4 - 1)*log(4*cosh(b*x + a)*sinh(b*x + a)/(cosh(b*x + a)^2 - 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh
(b*x + a)^3*sinh(b*x + a) + 6*b^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*c
osh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [B] time = 1.16235, size = 97, normalized size = 3.23

$$-\frac{4\,bxe^{(4\,bx+4\,a)}-e^{(4\,bx+4\,a)}\log\left(e^{(4\,bx+4\,a)}-1\right)+\log\left(e^{(4\,bx+4\,a)}-1\right)}{b^2e^{(4\,bx+4\,a)}-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b*x*e^(4*b*x + 4*a) - e^(4*b*x + 4*a)*log(e^(4*b*x + 4*a) - 1) + log(e^
(4*b*x + 4*a) - 1))/(b^2*e^(4*b*x + 4*a) - b^2)
```

3.498 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rubi [A] time = 0.033186, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}$$

Mathematica [A] time = 0.0097339, size = 13, normalized size = 0.57

$$-\frac{2\coth(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*Coth[2*(a + b*x)])/b
```

Maple [A] time = 0., size = 32, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{1}{\cosh(bx+a)\sinh(bx+a)} - 2\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

Maxima [A] time = 1.07723, size = 24, normalized size = 1.04

$$\frac{4}{b\left(e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 2.25071, size = 213, normalized size = 9.26

$$-\frac{4}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Giac [A] time = 1.17213, size = 24, normalized size = 1.04

$$-\frac{4}{b\left(e^{\left(4\,bx+4\,a\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

3.499 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$4\text{Unintegrable}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x},x\right)$$

```
[Out] 4*Unintegrable[Csch[2*a + 2*b*x]^2/x, x]
```

Rubi [A] time = 0.0661142, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]

[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 23.5733, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]

[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x, x]
```

Maple [A] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2 (\operatorname{sech}(bx+a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4}{bxe^{(4\,bx+4\,a)} - bx} + 16 \int \frac{1}{8\left(bx^2e^{(2\,bx+2\,a)} + bx^2\right)} dx + 16 \int \frac{1}{16\left(bx^2e^{(bx+a)} + bx^2\right)} dx - 16 \int \frac{1}{16\left(bx^2e^{(bx+a)} - bx^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -4/(b*x*e^(4*b*x + 4*a) - b*x) + 16*integrate(1/8/(b*x^2*e^(2*b*x + 2*a) +
b*x^2), x) + 16*integrate(1/16/(b*x^2*e^(b*x + a) + b*x^2), x) - 16*integra
te(1/16/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

3.500 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=19

$$4\text{Unintegrable}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x^2},x\right)$$

```
[Out] 4*Unintegrable[Csch[2*a + 2*b*x]^2/x^2, x]
```

Rubi [A] time = 0.0706101, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x^2}\,dx$$

Mathematica [A] time = 20.4321, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2 (\operatorname{sech}(bx+a))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4}{bx^2e^{(4\,bx+4\,a)}-bx^2}+16\int \frac{1}{4\left(bx^3e^{(2\,bx+2\,a)}+bx^3\right)}\,dx+16\int \frac{1}{8\left(bx^3e^{(bx+a)}+bx^3\right)}\,dx-16\int \frac{1}{8\left(bx^3e^{(bx+a)}-bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4/(b*x^2*e^(4*b*x + 4*a) - b*x^2) + 16*integrate(1/4/(b*x^3*e^(2*b*x + 2*a
) + b*x^3), x) + 16*integrate(1/8/(b*x^3*e^(b*x + a) + b*x^3), x) - 16*inte
grate(1/8/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

3.501 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Rubi [A] time = 0.57759, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 23.0535, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{csch}\left(bx+a\right)\right)^{2} \left(\operatorname{sech}\left(bx+a\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}\left(bx+a\right)^{2} \operatorname{sech}\left(bx+a\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m} \operatorname{csch}\left(bx+a\right)^{2} \operatorname{sech}\left(bx+a\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{2}\left(a+bx\right) \operatorname{sech}^{3}\left(a+bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.502 $\int x^2\mathbf{csch}^2(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=206

$$\frac{3ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{3i\text{PolyLog}\left(3,-i\right.}{b^3}$$

```
[Out] (-3*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (4*x*ArcTanh[E
^(a + b*x)])/b^2 - (3*x^2*Csch[a + b*x])/(2*b) - (2*PolyLog[2, -E^(a + b*x)
])/b^3 + ((3*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x*PolyLog[2, I
*E^(a + b*x)])/b^2 + (2*PolyLog[2, E^(a + b*x)])/b^3 - ((3*I)*PolyLog[3, (-
I)*E^(a + b*x)])/b^3 + ((3*I)*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a +
b*x])/b^2 + (x^2*Csch[a + b*x]*Sech[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.434258, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2621, 288, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 2282, 6589, 4182, 2279, 2391, 2622, 6271, 3770}

$$\frac{3ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{3i\text{PolyLog}\left(3,-i\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (4*x*ArcTanh[E
^(a + b*x)])/b^2 - (3*x^2*Csch[a + b*x])/(2*b) - (2*PolyLog[2, -E^(a + b*x)
])/b^3 + ((3*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x*PolyLog[2, I
*E^(a + b*x)])/b^2 + (2*PolyLog[2, E^(a + b*x)])/b^3 - ((3*I)*PolyLog[3, (-
I)*E^(a + b*x)])/b^3 + ((3*I)*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a +
b*x])/b^2 + (x^2*Csch[a + b*x]*Sech[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} -$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} -$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} -$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} + \frac{x\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sec}$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} + \frac{x\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sec}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+bx)}{b^2} +$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+bx)}{b^2} +$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+b}{2b}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+b}{2b}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+b}{2b}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+b}{2b}$$

Mathematica [A] time = 7.50837, size = 397, normalized size = 1.93

$$-\frac{i\left(-6bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+6bx\text{PolyLog}\left(2,ie^{a+bx}\right)+6\text{PolyLog}\left(3,-ie^{a+bx}\right)-6\text{PolyLog}\left(3,ie^{a+bx}\right)+3b^2x^2\log\left(1\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((x^2*Csch[a])/b) + (2*(-(a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*b*x)*(Lo
g[1 - E^(I*(I*a + I*b*x))] - Log[1 + E^(I*(I*a + I*b*x))]) + I*(PolyLog[2,
-E^(I*(I*a + I*b*x))] - PolyLog[2, E^(I*(I*a + I*b*x))]))))/b^3 - ((I/2)*((
```

```
4*I)*ArcTan[E^(a + b*x)] + 3*b^2*x^2*Log[1 - I*E^(a + b*x)] - 3*b^2*x^2*Log
[1 + I*E^(a + b*x)] - 6*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 6*b*x*PolyLog[2,
 I*E^(a + b*x)] + 6*PolyLog[3, (-I)*E^(a + b*x)] - 6*PolyLog[3, I*E^(a + b*
x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(2*b^2) + (
x^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x^2*Sech[a/2]*Sec
h[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b
*x])/(2*b)
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{csch}(bx+a))^2 (\operatorname{sech}(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-96\,b^2 \int \frac{x^2 e^{(bx+a)}}{32\left(b^2 e^{(2\,bx+2\,a)} + b^2\right)}\,dx - \frac{2\,bx^2 e^{(3\,bx+3\,a)} + \left(3\,bx^2 e^{(5\,a)} + 2\,xe^{(5\,a)}\right)e^{(5\,bx)} + \left(3\,bx^2 e^{a} - 2\,xe^{a}\right)e^{(bx)}}{b^2 e^{(6\,bx+6\,a)} + b^2 e^{(4\,bx+4\,a)} - b^2 e^{(2\,bx+2\,a)} - b^2} - \frac{2\left(bx\log\left(e^{(bx+\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -96*b^2*integrate(1/32*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - (2
*b*x^2*e^(3*b*x + 3*a) + (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) + (3*b*x
^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2
*e^(2*b*x + 2*a) - b^2) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3 + 2*arctan(e
^(b*x + a))/b^3
```

Fricas [C] time = 3.08021, size = 10452, normalized size = 50.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^2*x^2*cosh(b*x + a)^3 + 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 + 1
0*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(3*b^2*x^2 + 2*b*x)
*sinh(b*x + a)^5 + 4*(b^2*x^2 + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh
(b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
 a)^3)*sinh(b*x + a)^2 + 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 4*(cosh(b*x
+ a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) -
(-6*I*b*x*cosh(b*x + a)^6 - 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 - 6*I*b*
x*sinh(b*x + a)^6 - 6*I*b*x*cosh(b*x + a)^4 + (-90*I*b*x*cosh(b*x + a)^2 -
6*I*b*x)*sinh(b*x + a)^4 + 6*I*b*x*cosh(b*x + a)^2 + (-120*I*b*x*cosh(b*x +
 a)^3 - 24*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (-90*I*b*x*cosh(b*x + a)^
4 - 36*I*b*x*cosh(b*x + a)^2 + 6*I*b*x)*sinh(b*x + a)^2 + 6*I*b*x + (-36*I*
b*x*cosh(b*x + a)^5 - 24*I*b*x*cosh(b*x + a)^3 + 12*I*b*x*cosh(b*x + a))*si
nh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*b*x*cosh(b*x +
 a)^6 + 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + 6*I*b*x*sinh(b*x + a)^6 +
6*I*b*x*cosh(b*x + a)^4 + (90*I*b*x*cosh(b*x + a)^2 + 6*I*b*x)*sinh(b*x + a
)^4 - 6*I*b*x*cosh(b*x + a)^2 + (120*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(
b*x + a))*sinh(b*x + a)^3 + (90*I*b*x*cosh(b*x + a)^4 + 36*I*b*x*cosh(b*x +
 a)^2 - 6*I*b*x)*sinh(b*x + a)^2 - 6*I*b*x + (36*I*b*x*cosh(b*x + a)^5 + 24
*I*b*x*cosh(b*x + a)^3 - 12*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*co
sh(b*x + a) - I*sinh(b*x + a)) + 4*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(
b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + c
osh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (1
5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^
2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 4*(b*x*cosh(b*x + a)^6 + 6*b*
x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 + b*x*cosh(b*x + a)^4
 + (15*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4
*(5*b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh
(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 - b*x + 2*(3*b*x
*cosh(b*x + a)^5 + 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((-3*I*a^2 + 2*I)*cosh(b*x + a)^
6 + (-18*I*a^2 + 12*I)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*a^2 + 2*I)*sin
h(b*x + a)^6 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^4 + ((-45*I*a^2 + 30*I)*cosh(
b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)^4 + ((-60*I*a^2 + 40*I)*cosh(b*x
+ a)^3 + (-12*I*a^2 + 8*I)*cosh(b*x + a))*sinh(b*x + a)^3 + (3*I*a^2 - 2*I)
*cosh(b*x + a)^2 + ((-45*I*a^2 + 30*I)*cosh(b*x + a)^4 + (-18*I*a^2 + 12*I)
```

```
*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^2 + 3*I*a^2 + ((-18*I*a^2 +
 12*I)*cosh(b*x + a)^5 + (-12*I*a^2 + 8*I)*cosh(b*x + a)^3 + (6*I*a^2 - 4*I
)*cosh(b*x + a))*sinh(b*x + a) - 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I
) - ((3*I*a^2 - 2*I)*cosh(b*x + a)^6 + (18*I*a^2 - 12*I)*cosh(b*x + a)*sinh
(b*x + a)^5 + (3*I*a^2 - 2*I)*sinh(b*x + a)^6 + (3*I*a^2 - 2*I)*cosh(b*x +
a)^4 + ((45*I*a^2 - 30*I)*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^4
+ ((60*I*a^2 - 40*I)*cosh(b*x + a)^3 + (12*I*a^2 - 8*I)*cosh(b*x + a))*sinh
(b*x + a)^3 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^2 + ((45*I*a^2 - 30*I)*cosh(b*
x + a)^4 + (18*I*a^2 - 12*I)*cosh(b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)
^2 - 3*I*a^2 + ((18*I*a^2 - 12*I)*cosh(b*x + a)^5 + (12*I*a^2 - 8*I)*cosh(b
*x + a)^3 + (-6*I*a^2 + 4*I)*cosh(b*x + a))*sinh(b*x + a) + 2*I)*log(cosh(b
*x + a) + sinh(b*x + a) - I) + 4*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x + a)*sin
h(b*x + a)^5 + a*sinh(b*x + a)^6 + a*cosh(b*x + a)^4 + (15*a*cosh(b*x + a)^
2 + a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x
 + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 + 6*a*cosh(b*x + a)^2 -
 a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 + 2*a*cosh(b*x + a)^3 - a*cosh
(b*x + a))*sinh(b*x + a) - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((3*
I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^6 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x +
 a)*sinh(b*x + a)^5 + (3*I*b^2*x^2 - 3*I*a^2)*sinh(b*x + a)^6 + (3*I*b^2*x^
2 - 3*I*a^2)*cosh(b*x + a)^4 + (3*I*b^2*x^2 + (45*I*b^2*x^2 - 45*I*a^2)*cos
h(b*x + a)^2 - 3*I*a^2)*sinh(b*x + a)^4 - 3*I*b^2*x^2 + ((60*I*b^2*x^2 - 60
*I*a^2)*cosh(b*x + a)^3 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a))*sinh(b*x
 + a)^3 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^2 + ((45*I*b^2*x^2 - 45*I*
a^2)*cosh(b*x + a)^4 - 3*I*b^2*x^2 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x + a
)^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2 + ((18*I*b^2*x^2 - 18*I*a^2)*cosh(
b*x + a)^5 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a)^3 + (-6*I*b^2*x^2 + 6*
I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a)
+ 1) - ((-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^6 + (-18*I*b^2*x^2 + 18*I*a^
2)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*b^2*x^2 + 3*I*a^2)*sinh(b*x + a)^6
 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^4 + (-3*I*b^2*x^2 + (-45*I*b^2*x^
2 + 45*I*a^2)*cosh(b*x + a)^2 + 3*I*a^2)*sinh(b*x + a)^4 + 3*I*b^2*x^2 + ((
-60*I*b^2*x^2 + 60*I*a^2)*cosh(b*x + a)^3 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh
(b*x + a))*sinh(b*x + a)^3 + (3*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + ((-4
5*I*b^2*x^2 + 45*I*a^2)*cosh(b*x + a)^4 + 3*I*b^2*x^2 + (-18*I*b^2*x^2 + 18
*I*a^2)*cosh(b*x + a)^2 - 3*I*a^2)*sinh(b*x + a)^2 - 3*I*a^2 + ((-18*I*b^2*
x^2 + 18*I*a^2)*cosh(b*x + a)^5 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh(b*x + a)^
3 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x +
 a) - I*sinh(b*x + a) + 1) - 4*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x + a)*cos
h(b*x + a)*sinh(b*x + a)^5 + (b*x + a)*sinh(b*x + a)^6 + (b*x + a)*cosh(b*x
 + a)^4 + (15*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^4 + 4*(5*(
b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 - (b*x
+ a)*cosh(b*x + a)^2 + (15*(b*x + a)*cosh(b*x + a)^4 + 6*(b*x + a)*cosh(b*x
 + a)^2 - b*x - a)*sinh(b*x + a)^2 - b*x + 2*(3*(b*x + a)*cosh(b*x + a)^5 +
 2*(b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) - a)*
log(-cosh(b*x + a) - sinh(b*x + a) + 1) - (6*I*cosh(b*x + a)^6 + 36*I*cosh(
```

```
b*x + a)*sinh(b*x + a)^5 + 6*I*sinh(b*x + a)^6 + (90*I*cosh(b*x + a)^2 + 6*
I)*sinh(b*x + a)^4 + 6*I*cosh(b*x + a)^4 + (120*I*cosh(b*x + a)^3 + 24*I*co
sh(b*x + a))*sinh(b*x + a)^3 + (90*I*cosh(b*x + a)^4 + 36*I*cosh(b*x + a)^2
 - 6*I)*sinh(b*x + a)^2 - 6*I*cosh(b*x + a)^2 + (36*I*cosh(b*x + a)^5 + 24*
I*cosh(b*x + a)^3 - 12*I*cosh(b*x + a))*sinh(b*x + a) - 6*I)*polylog(3, I*c
osh(b*x + a) + I*sinh(b*x + a)) - (-6*I*cosh(b*x + a)^6 - 36*I*cosh(b*x + a
)*sinh(b*x + a)^5 - 6*I*sinh(b*x + a)^6 + (-90*I*cosh(b*x + a)^2 - 6*I)*sin
h(b*x + a)^4 - 6*I*cosh(b*x + a)^4 + (-120*I*cosh(b*x + a)^3 - 24*I*cosh(b*
x + a))*sinh(b*x + a)^3 + (-90*I*cosh(b*x + a)^4 - 36*I*cosh(b*x + a)^2 + 6
*I)*sinh(b*x + a)^2 + 6*I*cosh(b*x + a)^2 + (-36*I*cosh(b*x + a)^5 - 24*I*c
osh(b*x + a)^3 + 12*I*cosh(b*x + a))*sinh(b*x + a) + 6*I)*polylog(3, -I*cos
h(b*x + a) - I*sinh(b*x + a)) + 2*(6*b^2*x^2*cosh(b*x + a)^2 + 5*(3*b^2*x^2
 + 2*b*x)*cosh(b*x + a)^4 + 3*b^2*x^2 - 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x
 + a)^6 + 6*b^3*cosh(b*x + a)*sinh(b*x + a)^5 + b^3*sinh(b*x + a)^6 + b^3*c
osh(b*x + a)^4 - b^3*cosh(b*x + a)^2 + (15*b^3*cosh(b*x + a)^2 + b^3)*sinh(
b*x + a)^4 + 4*(5*b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a)^3
- b^3 + (15*b^3*cosh(b*x + a)^4 + 6*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a
)^2 + 2*(3*b^3*cosh(b*x + a)^5 + 2*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))
*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.503 $\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal. Leaf size=120

$$\frac{3i\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}-\frac{3i\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\mathrm{csch}(a}{2b}$$

```
[Out] (-3*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (3*x*Csch[a + b
*x])/(2*b) + (((3*I)/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (((3*I)/2)*Poly
Log[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) + (x*Csch[a + b*x]*Sech[
a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.166424, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2621, 288, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 2622}

$$\frac{3i\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}-\frac{3i\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\mathrm{csch}(a}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```


```
[Out] (-3*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (3*x*Csch[a + b
*x])/(2*b) + (((3*I)/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (((3*I)/2)*Poly
Log[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) + (x*Csch[a + b*x]*Sech[
a + b*x]^2)/(2*b)
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3x\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \int\Big(-$$

$$= -\frac{3x\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\int \text{cs}}{}$$

$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\text{Sub}}{}$$

$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+bx)\text{sech}}{2b}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{c}}{}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{c}}{}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{3i\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} -$$

Mathematica [A] time = 1.19066, size = 133, normalized size = 1.11

$$-\frac{-3i\text{PolyLog}(2,-i(\sinh(a+bx)+\cosh(a+bx)))+3i\text{PolyLog}(2,i(\sinh(a+bx)+\cosh(a+bx)))-bx\tanh\left(\frac{1}{2}(a+b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -(6*b*x*ArcTan[Cosh[a + b*x] + Sinh[a + b*x]] + b*x*Coth[(a + b*x)/2] - 2*L
og[Tanh[(a + b*x)/2]] - (3*I)*PolyLog[2, (-I)*(Cosh[a + b*x] + Sinh[a + b*x
])] + (3*I)*PolyLog[2, I*(Cosh[a + b*x] + Sinh[a + b*x])] + Sech[a + b*x] -
 b*x*Tanh[(a + b*x)/2] + b*x*Sech[a + b*x]*Tanh[a + b*x])/(2*b^2)
```

Maple [B] time = 0.082, size = 232, normalized size = 1.9

$$-\frac{e^{bx+a}\left(3\,bxe^{4\,bx+4\,a}+2\,bxe^{2\,bx+2\,a}+e^{4\,bx+4\,a}+3\,bx-1\right)}{b^2\left(1+e^{2\,bx+2\,a}\right)^2\left(e^{2\,bx+2\,a}-1\right)}+\frac{\ln\left(e^{bx+a}-1\right)}{b^2}-\frac{\ln\left(1+e^{bx+a}\right)}{b^2}+3\,\frac{a\arctan\left(e^{bx+a}\right)}{b^2}+\frac{\frac{3\,i}{2}\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(3*b*x*exp(4*b*x+4*a)+2*b*x*exp(2*b*x+2*a)+exp(4*b*x+4*a)+3*b*x
-1)/b^2/(1+exp(2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^
2*ln(1+exp(b*x+a))+3/b^2*a*arctan(exp(b*x+a))+3/2*I/b*ln(1+I*exp(b*x+a))*x+
3/2*I/b^2*ln(1+I*exp(b*x+a))*a-3/2*I/b^2*dilog(1-I*exp(b*x+a))+3/2*I/b^2*di
log(1+I*exp(b*x+a))-3/2*I/b*ln(1-I*exp(b*x+a))*x-3/2*I/b^2*ln(1-I*exp(b*x+a
))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,bxe^{(3\,bx+3\,a)}+\left(3\,bxe^{(5\,a)}+e^{(5\,a)}\right)e^{(5\,bx)}+\left(3\,bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)}+b^2e^{(4\,bx+4\,a)}-b^2e^{(2\,bx+2\,a)}-b^2}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}-96\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) + (3*b*x*e^a
- e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
 2*a) - b^2) - log((e^(b*x + a) + 1)*e^(-a))/b^2 + log((e^(b*x + a) - 1)*e^
(-a))/b^2 - 96*integrate(1/32*x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [B] time = 2.74036, size = 6375, normalized size = 53.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x
 + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 + 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b
*x + 1)*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x
+ a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(3*b*x - 1)*cosh(b*x + a)
 - (-3*I*cosh(b*x + a)^6 - 18*I*cosh(b*x + a)*sinh(b*x + a)^5 - 3*I*sinh(b*
x + a)^6 + (-45*I*cosh(b*x + a)^2 - 3*I)*sinh(b*x + a)^4 - 3*I*cosh(b*x + a
)^4 + (-60*I*cosh(b*x + a)^3 - 12*I*cosh(b*x + a))*sinh(b*x + a)^3 + (-45*I
*cosh(b*x + a)^4 - 18*I*cosh(b*x + a)^2 + 3*I)*sinh(b*x + a)^2 + 3*I*cosh(b
*x + a)^2 + (-18*I*cosh(b*x + a)^5 - 12*I*cosh(b*x + a)^3 + 6*I*cosh(b*x +
a))*sinh(b*x + a) + 3*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (3*I*co
sh(b*x + a)^6 + 18*I*cosh(b*x + a)*sinh(b*x + a)^5 + 3*I*sinh(b*x + a)^6 +
(45*I*cosh(b*x + a)^2 + 3*I)*sinh(b*x + a)^4 + 3*I*cosh(b*x + a)^4 + (60*I*
cosh(b*x + a)^3 + 12*I*cosh(b*x + a))*sinh(b*x + a)^3 + (45*I*cosh(b*x + a)
^4 + 18*I*cosh(b*x + a)^2 - 3*I)*sinh(b*x + a)^2 - 3*I*cosh(b*x + a)^2 + (1
8*I*cosh(b*x + a)^5 + 12*I*cosh(b*x + a)^3 - 6*I*cosh(b*x + a))*sinh(b*x +
a) - 3*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))
*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (3*I*a*
cosh(b*x + a)^6 + 18*I*a*cosh(b*x + a)*sinh(b*x + a)^5 + 3*I*a*sinh(b*x + a
)^6 + 3*I*a*cosh(b*x + a)^4 + (45*I*a*cosh(b*x + a)^2 + 3*I*a)*sinh(b*x + a
)^4 + (60*I*a*cosh(b*x + a)^3 + 12*I*a*cosh(b*x + a))*sinh(b*x + a)^3 - 3*I
*a*cosh(b*x + a)^2 + (45*I*a*cosh(b*x + a)^4 + 18*I*a*cosh(b*x + a)^2 - 3*I
*a)*sinh(b*x + a)^2 + (18*I*a*cosh(b*x + a)^5 + 12*I*a*cosh(b*x + a)^3 - 6*
I*a*cosh(b*x + a))*sinh(b*x + a) - 3*I*a)*log(cosh(b*x + a) + sinh(b*x + a)
 + I) - (-3*I*a*cosh(b*x + a)^6 - 18*I*a*cosh(b*x + a)*sinh(b*x + a)^5 - 3*
I*a*sinh(b*x + a)^6 - 3*I*a*cosh(b*x + a)^4 + (-45*I*a*cosh(b*x + a)^2 - 3*
```

```
I*a)*sinh(b*x + a)^4 + (-60*I*a*cosh(b*x + a)^3 - 12*I*a*cosh(b*x + a))*sin
h(b*x + a)^3 + 3*I*a*cosh(b*x + a)^2 + (-45*I*a*cosh(b*x + a)^4 - 18*I*a*co
sh(b*x + a)^2 + 3*I*a)*sinh(b*x + a)^2 + (-18*I*a*cosh(b*x + a)^5 - 12*I*a*
cosh(b*x + a)^3 + 6*I*a*cosh(b*x + a))*sinh(b*x + a) + 3*I*a)*log(cosh(b*x
+ a) + sinh(b*x + a) - I) - 2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)
*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((3*I*b*x + 3*I*a)*cosh(b*x + a)^
6 + (18*I*b*x + 18*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*I*b*x + 3*I*a)*s
inh(b*x + a)^6 + (3*I*b*x + 3*I*a)*cosh(b*x + a)^4 + ((45*I*b*x + 45*I*a)*c
osh(b*x + a)^2 + 3*I*b*x + 3*I*a)*sinh(b*x + a)^4 + ((60*I*b*x + 60*I*a)*co
sh(b*x + a)^3 + (12*I*b*x + 12*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (-3*I*
b*x - 3*I*a)*cosh(b*x + a)^2 + ((45*I*b*x + 45*I*a)*cosh(b*x + a)^4 + (18*I
*b*x + 18*I*a)*cosh(b*x + a)^2 - 3*I*b*x - 3*I*a)*sinh(b*x + a)^2 - 3*I*b*x
 + ((18*I*b*x + 18*I*a)*cosh(b*x + a)^5 + (12*I*b*x + 12*I*a)*cosh(b*x + a)
^3 + (-6*I*b*x - 6*I*a)*cosh(b*x + a))*sinh(b*x + a) - 3*I*a)*log(I*cosh(b*
x + a) + I*sinh(b*x + a) + 1) - ((-3*I*b*x - 3*I*a)*cosh(b*x + a)^6 + (-18*
I*b*x - 18*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*b*x - 3*I*a)*sinh(b*x
 + a)^6 + (-3*I*b*x - 3*I*a)*cosh(b*x + a)^4 + ((-45*I*b*x - 45*I*a)*cosh(b
*x + a)^2 - 3*I*b*x - 3*I*a)*sinh(b*x + a)^4 + ((-60*I*b*x - 60*I*a)*cosh(b
*x + a)^3 + (-12*I*b*x - 12*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (3*I*b*x
+ 3*I*a)*cosh(b*x + a)^2 + ((-45*I*b*x - 45*I*a)*cosh(b*x + a)^4 + (-18*I*b
*x - 18*I*a)*cosh(b*x + a)^2 + 3*I*b*x + 3*I*a)*sinh(b*x + a)^2 + 3*I*b*x +
 ((-18*I*b*x - 18*I*a)*cosh(b*x + a)^5 + (-12*I*b*x - 12*I*a)*cosh(b*x + a)
^3 + (6*I*b*x + 6*I*a)*cosh(b*x + a))*sinh(b*x + a) + 3*I*a)*log(-I*cosh(b*
x + a) - I*sinh(b*x + a) + 1) + 2*(5*(3*b*x + 1)*cosh(b*x + a)^4 + 6*b*x*co
sh(b*x + a)^2 + 3*b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh
(b*x + a)*sinh(b*x + a)^5 + b^2*sinh(b*x + a)^6 + b^2*cosh(b*x + a)^4 + (15
*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^
2*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^2*cosh(b*x +
 a)^4 + 6*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 - b^2 + 2*(3*b^2*cosh(
b*x + a)^5 + 2*b^2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.504 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0416801, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$
$$= -\frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a\right.}{2b}$$
$$= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Mathematica [C] time = 0.016493, size = 29, normalized size = 0.59

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] time = 0., size = 52, normalized size = 1.1

$$-\frac{1}{b\sinh(bx+a)(\cosh(bx+a))^2}-\frac{3\,\mathrm{sech}\,(bx+a)\tanh(bx+a)}{2b}-3\,\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)/b-3*arctan(exp(b
*x+a))/b
```

Maxima [B] time = 1.67357, size = 122, normalized size = 2.49

$$\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{3\,e^{(-bx-a)}+2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

Fricas [B] time = 2.12227, size = 1401, normalized size = 28.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
```

```
cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Giac [B] time = 1.15774, size = 140, normalized size = 2.86

$$-\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{4b}-\frac{3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4e^{(bx+a)}-4e^{(-bx-a)}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b - (3*(e^(b*x
 + a) - e^(-b*x - a))^2 + 8)/(((e^(b*x + a) - e^(-b*x - a))^3 + 4*e^(b*x +
a) - 4*e^(-b*x - a))*b)
```

3.505 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Rubi [A] time = 0.239708, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 40.4613, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Maple [A] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2 (\operatorname{sech}(bx+a))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,bxe^{(3\,bx+3\,a)} + \left(3\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)} + \left(3\,bxe^{a} + e^{a}\right)e^{(bx)}}{b^2x^2e^{(6\,bx+6\,a)} + b^2x^2e^{(4\,bx+4\,a)} - b^2x^2e^{(2\,bx+2\,a)} - b^2x^2} - 32\int \frac{\left(3\,b^2x^2e^{a} - 2\,e^{a}\right)e^{(bx)}}{32\left(b^2x^3e^{(2\,bx+2\,a)} + b^2x^3\right)}\,dx - 32\int \frac{1}{32\left(bx^2e^{(bx+a)} +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) + (3*b*x*e^a
+ e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) + b^2*x^2*e^(4*b*x + 4*a) - b^2*x^
2*e^(2*b*x + 2*a) - b^2*x^2) - 32*integrate(1/32*(3*b^2*x^2*e^a - 2*e^a)*e^
(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) - 32*integrate(1/32/(b*x^2*e^
(b*x + a) + b*x^2), x) - 32*integrate(1/32/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

3.506 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Rubi [A] time = 0.30223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]

[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 33.5094, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]

[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Maple [A] time = 0.507, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2 (\operatorname{sech}(bx+a))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,bxe^{(3\,bx+3\,a)} + \left(3\,bxe^{(5\,a)} - 2\,e^{(5\,a)}\right)e^{(5\,bx)} + (3\,bxe^{a} + 2\,e^{a})e^{(bx)}}{b^2x^3e^{(6\,bx+6\,a)} + b^2x^3e^{(4\,bx+4\,a)} - b^2x^3e^{(2\,bx+2\,a)} - b^2x^3} - 32\int \frac{3\left(b^2x^2e^{a} - 2\,e^{a}\right)e^{(bx)}}{32\left(b^2x^4e^{(2\,bx+2\,a)} + b^2x^4\right)}\,dx - 32\int \frac{1}{16\left(bx^3e^{(bx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) + (3*b*x*e^
a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) + b^2*x^3*e^(4*b*x + 4*a) - b^
2*x^3*e^(2*b*x + 2*a) - b^2*x^3) - 32*integrate(3/32*(b^2*x^2*e^a - 2*e^a)*
e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) - 32*integrate(1/16/(b*x^3*
e^(b*x + a) + b*x^3), x) - 32*integrate(1/16/(b*x^3*e^(b*x + a) - b*x^3), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

3.507 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Rubi [A] time = 0.452598, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Rubi steps

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Mathematica [A] time = 19.4434, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Maple [A] time = 0.04, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{csch}\left(bx+a\right)\right)^{3} \operatorname{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{3}\left(a+bx\right) \operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

3.508 $\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=240

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyLog}}{}$$

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) + (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*
Coth[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a
 + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*PolyLog[2
, -E^(2*a + 2*b*x)])/(2*b^2) - (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2)
- (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) + (3*x*PolyLog[3, E^(2*a + 2*b
*x)])/(2*b^3) + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) - (3*PolyLog[4, E^
(2*a + 2*b*x)])/(4*b^4)
```

Rubi [A] time = 0.423705, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2620, 14, 5462, 3720, 3716, 2190, 2279, 2391, 30, 2551, 12, 4182, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```


```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) + (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*
Coth[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a
 + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*PolyLog[2
, -E^(2*a + 2*b*x)])/(2*b^2) - (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2)
- (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) + (3*x*PolyLog[3, E^(2*a + 2*b
*x)])/(2*b^3) + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) - (3*PolyLog[4, E^
(2*a + 2*b*x)])/(4*b^4)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\mathbf{csch}^3(a+bx)\mathbf{sech}(a+bx)\,dx = -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}-3\int x^2\left(-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh\right.$$

$$= -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}-3\int\left(-\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\mathrm{ta}\right.$$

$$= -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}+\frac{3\int x^2\coth^2(a+bx)\,dx}{2b}+\frac{3\int x^2\log\right.$$

$$= -\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3\int x\coth(a+bx)\,dx}{b^2}-\frac{\int 2bx^3\mathbf{csch}(2a}{b}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}-2\int x^3\mathbf{csch}(2a+2bx)\,dx-$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\,\mathrm{l}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\,\mathrm{l}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\,\mathrm{l}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\,\mathrm{l}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\,\mathrm{l}}{}$$

Mathematica [B] time = 7.48996, size = 490, normalized size = 2.04

$$-\frac{3\left(2b^2x^2\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+2bx\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)+\text{PolyLog}\left(4,-e^{-2(a+bx)}\right)\right)}{4b^4}+\frac{e^{2a}\left(6\left(1-e^{-2a}\right)\left(b^2x^2\text{Poly}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x^4/(2 + 2*E^(2*a)) - (x^3*Csch[a + b*x]^2)/(2*b) + (x^3*Log[1 + E^(-2*(a +
 b*x))])/b + (E^(2*a)*((-6*b^2*x^2)/E^(2*a) + (b^4*x^4)/E^(2*a) + 6*b*(1 -
E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 - E^(-a
 - b*x)])/E^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 + E^(-a - b*x)] - (2*b^3*(-1
 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*a) - 6*(1 - E^(-2*a))*PolyLog[2
, -E^(-a - b*x)] - 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2
*a))*(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*(b*x*PolyLog[3, -E^(-a - b*x)]
+ PolyLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, E^(-a
- b*x)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] + PolyLog[4, E^(-a - b*x)]))))/(2
*b^4*(-1 + E^(2*a))) - (3*(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*
PolyLog[3, -E^(-2*(a + b*x))] + PolyLog[4, -E^(-2*(a + b*x))]))/(4*b^4) - (
x^4*Csch[a]*Sech[a])/4 + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] time = 0.056, size = 417, normalized size = 1.7

$$3\,\frac{\text{polylog}\left(2,e^{bx+a}\right)}{b^4}+3\,\frac{\text{polylog}\left(2,-e^{bx+a}\right)}{b^4}-\frac{x^2\left(2\,bxe^{2\,bx+2\,a}+3\,e^{2\,bx+2\,a}-3\right)}{b^2\left(e^{2\,bx+2\,a}-1\right)^2}+3\,\frac{\ln\left(1+e^{bx+a}\right)x}{b^3}+3\,\frac{\ln\left(1-e^{bx+a}\right)x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] 3/b^4*polylog(2,exp(b*x+a))+3/b^4*polylog(2,-exp(b*x+a))-x^2*(2*b*x*exp(2*b
*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2+3/b^3*ln(1+exp(b*x+a))
*x+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a-3/b^4*a*ln(exp(b*x+a)-
1)+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-6/b^4*polylog(4,exp(b*x+a))-6/b^4*pol
ylog(4,-exp(b*x+a))+1/b^4*a^3*ln(exp(b*x+a)-1)-1/b^4*ln(1-exp(b*x+a))*a^3-3
/b^2*polylog(2,-exp(b*x+a))*x^2+6/b^3*polylog(3,-exp(b*x+a))*x-1/b*ln(1-exp
(b*x+a))*x^3-3/b^2*polylog(2,exp(b*x+a))*x^2+6/b^3*polylog(3,exp(b*x+a))*x-
1/b*ln(1+exp(b*x+a))*x^3+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(
2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-6/b^3*a*x+6/b^4*a*ln(e
```

```
xp(b*x+a))-3*x^2/b^2-3/b^4*a^2
```

Maxima [A] time = 1.22673, size = 475, normalized size = 1.98

$$-\frac{1}{2}x^4 + \frac{3x^2 - \left(2bx^3e^{(2a)} + 3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2} + \frac{b^4x^4 - 6b^2x^2}{2b^4} + \frac{4b^3x^3\log\left(e^{(2bx+2a)} + 1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 +
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(
-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 3.00788, size = 8660, normalized size = 36.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 + 3*((b^2*x^2 - 1)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
 + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x
^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x +
a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a)
+ sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sin
h(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^
2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*co
```

```
sh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*
sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2
*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2
*cosh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x
+ a) - I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1
)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
 (b^3*x^3 + (b^3*x^3 - 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 - 3*b*x)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 - 3
*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 - 3*b*x)*cosh(b*x + a)^2 -
3*b*x)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 - 3*b*x)*cosh(b*x + a)^3 - (b^
3*x^3 - 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (a^3*cosh(b*x + a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3
*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 -
 a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a^3*cosh(b*x + a)^4 + 4*a
^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a
)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 - a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b
*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
+ a) - I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh
(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 - 3*a)*cosh(b*x +
a)^2 - 2*(a^3 - 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((
a^3 - 3*a)*cosh(b*x + a)^3 - (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(
b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a
^3)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3
+ a^3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 +
a^3)*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(I*
cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3)*
sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^
3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 + a^3)
*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 +
 a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 -
 3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((
b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(-cosh(b*x + a) - sinh(b*x + a) +
1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(
```

```
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + s
inh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, I*cosh
(b*x + a) + I*sinh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*po
lylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a) + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x
+ a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*
cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4
*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b
*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sin
h(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(
b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh
(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
 a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*po
lylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b
*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a
)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b
*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - s
inh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x
^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh
(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b
^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)
^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a), x)
```

3.509 $\int x^2 \mathbf{csch}^3(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=148

$$\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+$$

```
[Out] x^2/(2*b) + (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*Coth[a + b*x])/b^2 - (x
^2*Coth[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, -E^(2*a
+ 2*b*x)])/b^2 - (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - PolyLog[3, -E^(2*a +
 2*b*x)]/(2*b^3) + PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rubi [A] time = 0.231857, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2620, 14, 5462, 3720, 3475, 30, 2551, 12, 4182, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x^2/(2*b) + (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*Coth[a + b*x])/b^2 - (x
^2*Coth[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, -E^(2*a
+ 2*b*x)])/b^2 - (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - PolyLog[3, -E^(2*a +
 2*b*x)]/(2*b^3) + PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx = -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} - 2\int x\left(-\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a\right.$$
$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} - 2\int\left(-\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh\right.$$
$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} + \frac{\int x\coth^2(a+bx)\,dx}{b} + \frac{2\int x\log(\tan$$
$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\int\coth(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b} - \frac{\int 2bx^2\mathrm{csch}(}{b}$$
$$= \frac{x^2}{2b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} - 2\int x^2\mathrm{csch}(2a+2$$
$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx}{b^3}$$
$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx}{b^3}$$
$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx}{b^3}$$
$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx}{b^3}$$

<u>Mathematica</u> [B] time = 4.50804, size = 369, normalized size = 2.49

$$\frac{1}{6}\left(-\frac{2e^{2a}\left(-6\left(1-e^{-2a}\right)\left(bx\mathrm{PolyLog}\left(2,-e^{-a-bx}\right)+\mathrm{PolyLog}\left(3,-e^{-a-bx}\right)\right)-6\left(1-e^{-2a}\right)\left(bx\mathrm{PolyLog}\left(2,e^{-a-bx}\right)+\mathrm{PolyL}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] ((-3*x^2*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b*x)/E^(2*a) - (2*b^3*x^3)/E^(
2*a) + (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*x)])/E^(2*a) + (3*b^2*(-
1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3*(1 - E^(-2*a))*(b*x - L
og[1 - E^(a + b*x)]) + 3*(1 - E^(-2*a))*(b*x - Log[1 + E^(a + b*x)]) - 6*(1
 - E^(-2*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) -
6*(1 - E^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)]))
)/(b^3*(-1 + E^(2*a))) + (2*b^2*x^2*((2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-
2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*
(a + b*x))])/b^3 - 2*x^3*Csch[a]*Sech[a] + (6*x*Csch[a]*Csch[a + b*x]*Sinh[
```

```
b*x])/b^2)/6
```

Maple [A] time = 0.05, size = 266, normalized size = 1.8

$$-2\,{\frac {x \left(bx{{\rm e}^{2\,bx+2\,a}}+{{\rm e}^{2\,bx+2\,a}}-1 \right) }{{b}^{2} \left({{\rm e}^{2\,bx+2\,a}}-1 \right) ^{2}}}-{\frac {\ln \left(1+{{\rm e}^{bx+a}} \right) {x}^{2}}{b}}-2\,{\frac {x{\it polylog} \left(2,-{{\rm e}^{bx+a}} \right) }{{b}^{2}}}+{\frac {{x}^{2}\ln \left(1+{{\rm e}^{2\,bx+2\,a}} \right) }{b}}+{\frac {x{\it polylog} \left(2,-{{\rm e}^{2\,b}} \right.}{{b}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b*ln(
1+exp(b*x+a))*x^2-2*x*polylog(2,-exp(b*x+a))/b^2+x^2*ln(1+exp(2*b*x+2*a))/b
+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(1-exp(b*x+a))*x^2-2*x*polylog(2,ex
p(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))/b^3-1/2*polylog(3,-exp(2*b*x+2*a))/b
^3+2*polylog(3,exp(b*x+a))/b^3-2/b^3*ln(exp(b*x+a))+1/b^3*ln(1+exp(b*x+a))+
1/b^3*ln(exp(b*x+a)-1)+1/b^3*ln(1-exp(b*x+a))*a^2-1/b^3*a^2*ln(exp(b*x+a)-1
)
```

Maxima [A] time = 1.17773, size = 328, normalized size = 2.22

$$-\frac{2\,{\left({\left(b x^{2} e^{\left(2\,a\right)} + x e^{\left(2\,a\right)}\right)} e^{\left(2\,b x\right)} - x\right)}}{b^{2} e^{\left(4\,b x + 4\,a\right)} - 2\,b^{2} e^{\left(2\,b x + 2\,a\right)} + b^{2}} - \frac{2\,x}{b^{2}} + \frac{2\,b^{2} x^{2} \log\left(e^{\left(2\,b x + 2\,a\right)} + 1\right) + 2\,b x {\rm Li}_2\left(-e^{\left(2\,b x + 2\,a\right)}\right) - {\rm Li}_{3}(-e^{\left(2\,b x + 2\,a\right)})}{2\,b^{3}} - \frac{b^{2} x^{2} \log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) - x)/(b^2*e^(4*b*x + 4*a) - 2*b^2
*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
 + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 - (b^2
*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*
x + a)))/b^3 - (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) -
2*polylog(3, e^(b*x + a)))/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a)
 - 1)/b^3
```

Fricas [C] time = 2.62374, size = 6546, normalized size = 44.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 +
 2*(b*x + a)*sinh(b*x + a)^4 + 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 + 2*
(b^2*x^2 + 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 + 2*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) -
2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) + ((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(
b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh
(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh
(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a^2*cosh(b*x + a)^
4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(
b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2
*cosh(b*x + a)^3 - a^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) + I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)
^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2
 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 - a^2*cosh(b*x + a))
*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((a^2 - 1)*cosh(b*
x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 - 1)*sinh(b*x +
 a)^4 - 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1)*cosh(b*x + a)^2 - a^2
+ 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x + a)^3 - (a^2 - 1)*cosh(
b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x
 + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
```

```
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 + (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^3(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a), x)
```

3.510 $\int x\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

```
[Out] x/(2*b) + (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - Coth[a + b*x]/(2*b^2) - (x*Cot
h[a + b*x]^2)/(2*b) + PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) - PolyLog[2, E^(
2*a + 2*b*x)]/(2*b^2)
```

Rubi [A] time = 0.120756, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2620, 14, 5462, 3473, 8, 2548, 12, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x/(2*b) + (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - Coth[a + b*x]/(2*b^2) - (x*Cot
h[a + b*x]^2)/(2*b) + PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) - PolyLog[2, E^(
2*a + 2*b*x)]/(2*b^2)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
```

```
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh(a+bx))}{b} - \int\Bigg(-\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx}{b}$$
$$= -\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh(a+bx))}{b} + \frac{\int\coth^2(a+bx)\,dx}{2b} + \frac{\int\log(\tanh(a+}{b}$$
$$= -\frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\int 1\,dx}{2b} - \frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b}$$
$$= \frac{x}{2b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} - 2\int x\text{csch}(2a+2bx)\,dx$$
$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\int\log\left(1-e^{2a+2bx}\right)}{b}$$
$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,d\right.}{2b^2}$$
$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} - \text{L}$$

Mathematica [A] time = 0.656768, size = 137, normalized size = 1.44

$$-\frac{\text{PolyLog}\left(2,-e^{-2(a+bx)}\right) - \text{PolyLog}\left(2,e^{-2(a+bx)}\right) + 2a\log\left(1-e^{-2(a+bx)}\right) + 2bx\log\left(1-e^{-2(a+bx)}\right) - 2a\log\left(e^{-2(a+bx)}+\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 2*a*Log[1 - E^(-2*(a + b*x))] + 2*b
*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[1 + E^(-2*(a + b*x))] - 2*b*x*Log[1
+ E^(-2*(a + b*x))] + 2*a*Log[Cosh[a + b*x]] - 2*a*Log[Sinh[a + b*x]] + Pol
yLog[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))])/(2*b^2)
```

Maple [B] time = 0.043, size = 170, normalized size = 1.8

$$-\frac{2\,bx\mathrm{e}^{2\,bx+2\,a}+\mathrm{e}^{2\,bx+2\,a}-1}{b^2\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^2} - \frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b} - \frac{\text{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2} + \frac{x\ln\left(1+\mathrm{e}^{2\,bx+2\,a}\right)}{b} + \frac{\text{polylog}\left(2,-\mathrm{e}^{2\,bx+2\,a}\right)}{2\,b^2} - \text{ln}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b*ln(1+
exp(b*x+a))*x-1/b^2*polylog(2,-exp(b*x+a))+x*ln(1+exp(2*b*x+2*a))/b+1/2*pol
ylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(1-exp(b*x+a))*x-1/b^2*ln(1-exp(b*x+a))*a
-1/b^2*polylog(2,exp(b*x+a))+1/b^2*a*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.12766, size = 196, normalized size = 2.06

$$-\frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}-\frac{bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(
2*b*x + 2*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
 + 2*a)))/b^2 - (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - (b*x
*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [C] time = 2.34645, size = 4327, normalized size = 45.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
 (2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
+ a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*co
sh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*
dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x +
a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
```

```
a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh
(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*
x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3
+ a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sin
h(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) + a)*l
og(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
 a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh
(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*
sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a
)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x
+ a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
 - a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) -
1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)
*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
 a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I
*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2
 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*
x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
 + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x +
a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a
)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*
x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 1)/(b^2*cosh(b*x + a)
^4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh
(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^
2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a), x)
```

3.511 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0274256, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.04275, size = 34, normalized size = 1.21

$$-\frac{\operatorname{csch}^2(a+bx) + 2\log(\sinh(a+bx)) - 2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/(2*b)
```

Maple [A] time = 0.018, size = 27, normalized size = 1.

$$-\frac{1}{2\,b\,(\sinh(bx+a))^2} - \frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2-ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.59034, size = 123, normalized size = 4.39

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.93696, size = 1037, normalized size = 37.04

$$-\frac{2\cosh\left(bx+a\right)^{2}-\left(\cosh\left(bx+a\right)^{4}+4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

Giac [B] time = 1.1694, size = 135, normalized size = 4.82

$$\frac{\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)}{2\,b}-\frac{\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}+\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-6}{2\,b\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b - 1/2*log(e^(2*b*x + 2*a)
 + e^(-2*b*x - 2*a) - 2)/b + 1/2*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(
b*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2))
```

3.512 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Rubi [A] time = 0.19599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Mathematica [A] time = 54.7689, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Maple [A] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3 \operatorname{sech}(bx+a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)} + 1}{b^2x^2e^{(4\,bx+4\,a)} - 2\,b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} + 16\int \frac{b^2x^2 - 1}{16\left(b^2x^3e^{(bx+a)} + b^2x^3\right)}\, dx - 16\int \frac{b^2x^2 - 1}{16\left(b^2x^3e^{(bx+a)} - b^2x^3\right)}\, dx - 16\int \frac{}{8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*
e^(b*x + a) + b^2*x^3), x) - 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 16*integrate(1/8/(x*e^(2*b*x + 2*a) + x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

3.513 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Rubi [A] time = 0.24371, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

Mathematica [A] time = 26.442, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3 \operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}+1\right)}{b^2x^3e^{(4bx+4a)}-2b^2x^3e^{(2bx+2a)}+b^2x^3}+16\int\frac{b^2x^2-3}{16\left(b^2x^4e^{(bx+a)}+b^2x^4\right)}\,dx-16\int\frac{b^2x^2-3}{16\left(b^2x^4e^{(bx+a)}-b^2x^4\right)}\,dx-16\int\frac{}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*
e^(b*x + a) + b^2*x^4), x) - 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*e^(b*
x + a) - b^2*x^4), x) - 16*integrate(1/8/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

3.514 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Rubi [A] time = 0.600951, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 21.15, size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{csch}\left(bx+a\right)\right)^{3} \left(\operatorname{sech}\left(bx+a\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{3}\left(a+bx\right) \operatorname{sech}^{2}\left(a+bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.515 $\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=317

$$\frac{9x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{9x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{9x\text{PolyLog}\left(3\right.}{b^3}$$

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - (6*x*ArcTanh[E^(a + b*x)])/b^3 + (3*x^3*A
rcTanh[E^(a + b*x)])/b - (3*x^2*Csch[a + b*x])/(2*b^2) - (3*PolyLog[2, -E^(
a + b*x)])/b^4 + (9*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) - ((6*I)*x*PolyLo
g[2, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*
PolyLog[2, E^(a + b*x)])/b^4 - (9*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (9
*x*PolyLog[3, -E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4
 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 + (9*x*PolyLog[3, E^(a + b*x)])/b^
3 + (9*PolyLog[4, -E^(a + b*x)])/b^4 - (9*PolyLog[4, E^(a + b*x)])/b^4 - (3
*x^3*Sech[a + b*x])/(2*b) - (x^3*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rubi [A] time = 1.17728, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.95, Rules used = {2622, 288, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 6609, 2282, 6589, 6742, 4180, 2621, 5205, 2279, 2391}

$$\frac{9x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{9x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{9x\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - (6*x*ArcTanh[E^(a + b*x)])/b^3 + (3*x^3*A
rcTanh[E^(a + b*x)])/b - (3*x^2*Csch[a + b*x])/(2*b^2) - (3*PolyLog[2, -E^(
a + b*x)])/b^4 + (9*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) - ((6*I)*x*PolyLo
g[2, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*
PolyLog[2, E^(a + b*x)])/b^4 - (9*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (9
*x*PolyLog[3, -E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4
 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 + (9*x*PolyLog[3, E^(a + b*x)])/b^
3 + (9*PolyLog[4, -E^(a + b*x)])/b^4 - (9*PolyLog[4, E^(a + b*x)])/b^4 - (3
*x^3*Sech[a + b*x])/(2*b) - (x^3*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
```

```
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - 3$$

$$= \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - 3$$

$$= \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + $$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\int bx^3\text{csch}(a+bx)\,dx}{2b} + \frac{3}{}$$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x^3\text{csch}(a+bx)\,dx + \frac{3}{}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} +$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} +$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} -$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} -$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} -$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} -$$

Mathematica [A] time = 8.26819, size = 433, normalized size = 1.37

$$\frac{3i\left(-2bx\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\text{PolyLog}\left(2,ie^{a+bx}\right)+2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2\text{PolyLog}\left(3,ie^{a+bx}\right)+b^2x^2\log\left(1-\right.\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (-3*x^2*Csch[a])/(2*b^2) - (x^3*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((3*I)*(b^2*
x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog
[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)
*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (3*(-2*b*x*Log[1 - E^(a
 + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] + 2*b*x*Log[1 + E^(a + b*x)] - b^3*
x^3*Log[1 + E^(a + b*x)] + (2 - 3*b^2*x^2)*PolyLog[2, -E^(a + b*x)] + (-2 +
 3*b^2*x^2)*PolyLog[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*
x*PolyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a
+ b*x)]))/(2*b^4) - (x^3*Sech[a/2 + (b*x)/2]^2)/(8*b) - (x^3*Sech[a + b*x])
/b + (3*x^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (3*x^2*S
ech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int x^{3} \left(\operatorname{csch}\left(bx+a\right)\right)^{3} \left(\operatorname{sech}\left(bx+a\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bx^{3}e^{(3\,bx+3\,a)} - 3\left(bx^{3}e^{(5\,a)} + x^{2}e^{(5\,a)}\right)e^{(5\,bx)} - 3\left(bx^{3}e^{a} - x^{2}e^{a}\right)e^{(bx)}}{b^{2}e^{(6\,bx+6\,a)} - b^{2}e^{(4\,bx+4\,a)} - b^{2}e^{(2\,bx+2\,a)} + b^{2}} + \frac{3\left(b^{3}x^{3}\log\left(e^{(bx+a)}+1\right) + 3\,b^{2}x^{2}\mathrm{Li}_{2}\left(-e^{(bx+a)}\right) - 6\,b}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x^3*e^(3*b*x + 3*a) - 3*(b*x^3*e^(5*a) + x^2*e^(5*a))*e^(5*b*x) - 3*(b
*x^3*e^a - x^2*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b
^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*d
```

```
ilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x
+ a)))/b^4 - 3/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x +
a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3*(b
*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a
) + 1) + dilog(e^(b*x + a)))/b^4 + 96*integrate(1/16*x^2*e^(b*x + a)/(b*e^(
2*b*x + 2*a) + b), x)
```

Fricas [C] time = 3.40904, size = 14033, normalized size = 44.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^3*x^3*cosh(b*x + a)^3 - 6*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^5 - 30
*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)*sinh(b*x + a)^4 - 6*(b^3*x^3 + b^2*x^2)*
sinh(b*x + a)^5 + 4*(b^3*x^3 - 15*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^2)*sinh
(b*x + a)^3 + 12*(b^3*x^3*cosh(b*x + a) - 5*(b^3*x^3 + b^2*x^2)*cosh(b*x +
a)^3)*sinh(b*x + a)^2 - 6*(b^3*x^3 - b^2*x^2)*cosh(b*x + a) - 3*((3*b^2*x^2
 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5 + (
3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (3*b^2*x
^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2*x^2 +
4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*cosh(b*x
 + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^
2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*x + a)^
3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*dilog(cosh(b*x + a) +
 sinh(b*x + a)) + (12*I*b*x*cosh(b*x + a)^6 + 72*I*b*x*cosh(b*x + a)*sinh(b
*x + a)^5 + 12*I*b*x*sinh(b*x + a)^6 - 12*I*b*x*cosh(b*x + a)^4 + (180*I*b*
x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)^4 - 12*I*b*x*cosh(b*x + a)^2 +
(240*I*b*x*cosh(b*x + a)^3 - 48*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (180
*I*b*x*cosh(b*x + a)^4 - 72*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)
^2 + 12*I*b*x + (72*I*b*x*cosh(b*x + a)^5 - 48*I*b*x*cosh(b*x + a)^3 - 24*I
*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
 + (-12*I*b*x*cosh(b*x + a)^6 - 72*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 - 12
*I*b*x*sinh(b*x + a)^6 + 12*I*b*x*cosh(b*x + a)^4 + (-180*I*b*x*cosh(b*x +
a)^2 + 12*I*b*x)*sinh(b*x + a)^4 + 12*I*b*x*cosh(b*x + a)^2 + (-240*I*b*x*c
osh(b*x + a)^3 + 48*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (-180*I*b*x*cosh
(b*x + a)^4 + 72*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a)^2 - 12*I*b
*x + (-72*I*b*x*cosh(b*x + a)^5 + 48*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(
b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*((3*
```

```
b^2*x^2 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a
)^5 + (3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (
3*b^2*x^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2
*x^2 + 4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)
)*sinh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*c
osh(b*x + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*
x + a)^2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*
x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*dilog(-cosh(b*
x + a) - sinh(b*x + a)) + 3*((b^3*x^3 - 2*b*x)*cosh(b*x + a)^6 + 6*(b^3*x^3
 - 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (b^3*x^3 - 2*b*x)*sinh(b*x + a)^6
 + b^3*x^3 - (b^3*x^3 - 2*b*x)*cosh(b*x + a)^4 - (b^3*x^3 - 15*(b^3*x^3 - 2
*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a)^4 + 4*(5*(b^3*x^3 - 2*b*x)*cos
h(b*x + a)^3 - (b^3*x^3 - 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^3 - (b^3*x^3
- 2*b*x)*cosh(b*x + a)^2 - (b^3*x^3 - 15*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^4
+ 6*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a)^2 - 2*b*x + 2*
(3*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^5 - 2*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^3
- (b^3*x^3 - 2*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(
b*x + a) + 1) + (6*I*a^2*cosh(b*x + a)^6 + 36*I*a^2*cosh(b*x + a)*sinh(b*x
+ a)^5 + 6*I*a^2*sinh(b*x + a)^6 - 6*I*a^2*cosh(b*x + a)^4 + (90*I*a^2*cosh
(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^4 - 6*I*a^2*cosh(b*x + a)^2 + (120*I*a
^2*cosh(b*x + a)^3 - 24*I*a^2*cosh(b*x + a))*sinh(b*x + a)^3 + (90*I*a^2*co
sh(b*x + a)^4 - 36*I*a^2*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^2 + 6*I*a
^2 + (36*I*a^2*cosh(b*x + a)^5 - 24*I*a^2*cosh(b*x + a)^3 - 12*I*a^2*cosh(b
*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-6*I*a^2*
cosh(b*x + a)^6 - 36*I*a^2*cosh(b*x + a)*sinh(b*x + a)^5 - 6*I*a^2*sinh(b*x
 + a)^6 + 6*I*a^2*cosh(b*x + a)^4 + (-90*I*a^2*cosh(b*x + a)^2 + 6*I*a^2)*s
inh(b*x + a)^4 + 6*I*a^2*cosh(b*x + a)^2 + (-120*I*a^2*cosh(b*x + a)^3 + 24
*I*a^2*cosh(b*x + a))*sinh(b*x + a)^3 + (-90*I*a^2*cosh(b*x + a)^4 + 36*I*a
^2*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a)^2 - 6*I*a^2 + (-36*I*a^2*cosh(b
*x + a)^5 + 24*I*a^2*cosh(b*x + a)^3 + 12*I*a^2*cosh(b*x + a))*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 3*((a^3 - 2*a)*cosh(b*x + a)^6
+ 6*(a^3 - 2*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (a^3 - 2*a)*sinh(b*x + a)^6
 - (a^3 - 2*a)*cosh(b*x + a)^4 - (a^3 - 15*(a^3 - 2*a)*cosh(b*x + a)^2 - 2*
a)*sinh(b*x + a)^4 + 4*(5*(a^3 - 2*a)*cosh(b*x + a)^3 - (a^3 - 2*a)*cosh(b*
x + a))*sinh(b*x + a)^3 + a^3 - (a^3 - 2*a)*cosh(b*x + a)^2 + (15*(a^3 - 2*
a)*cosh(b*x + a)^4 - a^3 - 6*(a^3 - 2*a)*cosh(b*x + a)^2 + 2*a)*sinh(b*x +
a)^2 + 2*(3*(a^3 - 2*a)*cosh(b*x + a)^5 - 2*(a^3 - 2*a)*cosh(b*x + a)^3 - (
a^3 - 2*a)*cosh(b*x + a))*sinh(b*x + a) - 2*a)*log(cosh(b*x + a) + sinh(b*x
 + a) - 1) + ((-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^6 + (-36*I*b^2*x^2 + 3
6*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (-6*I*b^2*x^2 + 6*I*a^2)*sinh(b*x
+ a)^6 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^4 + (6*I*b^2*x^2 + (-90*I*b^
2*x^2 + 90*I*a^2)*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^4 - 6*I*b^2*x^2
+ ((-120*I*b^2*x^2 + 120*I*a^2)*cosh(b*x + a)^3 + (24*I*b^2*x^2 - 24*I*a^2)
*cosh(b*x + a))*sinh(b*x + a)^3 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^2 +
 ((-90*I*b^2*x^2 + 90*I*a^2)*cosh(b*x + a)^4 + 6*I*b^2*x^2 + (36*I*b^2*x^2
```

- 36*I*a^2)*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^2 + 6*I*a^2 + ((-36*I*b^2*x^2 + 36*I*a^2)*cosh(b*x + a)^5 + (24*I*b^2*x^2 - 24*I*a^2)*cosh(b*x + a)^3 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^6 + (36*I*b^2*x^2 - 36*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (6*I*b^2*x^2 - 6*I*a^2)*sinh(b*x + a)^6 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^4 + (-6*I*b^2*x^2 + (90*I*b^2*x^2 - 90*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a)^4 + 6*I*b^2*x^2 + ((120*I*b^2*x^2 - 120*I*a^2)*cosh(b*x + a)^3 + (-24*I*b^2*x^2 + 24*I*a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^2 + ((90*I*b^2*x^2 - 90*I*a^2)*cosh(b*x + a)^4 - 6*I*b^2*x^2 + (-36*I*b^2*x^2 + 36*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a)^2 - 6*I*a^2 + ((36*I*b^2*x^2 - 36*I*a^2)*cosh(b*x + a)^5 + (-24*I*b^2*x^2 + 24*I*a^2)*cosh(b*x + a)^3 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 3*((b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^6 + 6*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (b^3*x^3 + a^3 - 2*b*x - 2*a)*sinh(b*x + a)^6 + b^3*x^3 - (b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^4 - (b^3*x^3 + a^3 - 15*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^2 - 2*b*x - 2*a)*sinh(b*x + a)^4 + 4*(5*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a)^3 + a^3 - (b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^2 - (b^3*x^3 - 15*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^4 + a^3 + 6*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^2 - 2*b*x - 2*a)*sinh(b*x + a)^2 - 2*b*x + 2*(3*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^5 - 2*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) - 2*a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 18*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) + 18*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 18*(b*x*cosh(b*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + (-12*I*cosh(b*x + a)^6 - 72*I*cosh(b*x + a)*sinh(b*x + a)^5 - 12*I*sinh(b*x + a)^6 + (-180*I*cosh(b*x + a)^2 + 12*I)*sinh(b*x + a)^4 + 12*I*cosh(b*x + a)^4 + (-240*I*cosh(b*x + a)^3 + 48*I*cosh(b*x + a))*sinh(b*x + a)^3 + (-180*I*cosh(b*x + a)^4 + 72

```
*I*cosh(b*x + a)^2 + 12*I)*sinh(b*x + a)^2 + 12*I*cosh(b*x + a)^2 + (-72*I*
cosh(b*x + a)^5 + 48*I*cosh(b*x + a)^3 + 24*I*cosh(b*x + a))*sinh(b*x + a)
- 12*I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + (12*I*cosh(b*x + a)
^6 + 72*I*cosh(b*x + a)*sinh(b*x + a)^5 + 12*I*sinh(b*x + a)^6 + (180*I*cos
h(b*x + a)^2 - 12*I)*sinh(b*x + a)^4 - 12*I*cosh(b*x + a)^4 + (240*I*cosh(b
*x + a)^3 - 48*I*cosh(b*x + a))*sinh(b*x + a)^3 + (180*I*cosh(b*x + a)^4 -
72*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a)^2 - 12*I*cosh(b*x + a)^2 + (72*I
*cosh(b*x + a)^5 - 48*I*cosh(b*x + a)^3 - 24*I*cosh(b*x + a))*sinh(b*x + a)
 + 12*I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 18*(b*x*cosh(b*x
+ a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*co
sh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b
*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 +
 (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b
*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*
sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 6*(2*b^3*x^3*co
sh(b*x + a)^2 - b^3*x^3 - 5*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^4 + b^2*x^2)*
sinh(b*x + a))/(b^4*cosh(b*x + a)^6 + 6*b^4*cosh(b*x + a)*sinh(b*x + a)^5 +
 b^4*sinh(b*x + a)^6 - b^4*cosh(b*x + a)^4 - b^4*cosh(b*x + a)^2 + (15*b^4*
cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^4 + b^4 + 4*(5*b^4*cosh(b*x + a)^3 - b
^4*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^4*cosh(b*x + a)^4 - 6*b^4*cosh(b*
x + a)^2 - b^4)*sinh(b*x + a)^2 + 2*(3*b^4*cosh(b*x + a)^5 - 2*b^4*cosh(b*x
 + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.516 $\int x^2 \mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=197

$$\frac{3x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{3x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,-e^{\cdots}\right.}{b^3}$$

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 + (3*x^2*ArcTanh[E^(a + b*x)])/b - ArcTanh[Co
sh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 + (3*x*PolyLog[2, -E^(a + b*x)])/b
^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[2, I*E^(a +
b*x)])/b^3 - (3*x*PolyLog[2, E^(a + b*x)])/b^2 - (3*PolyLog[3, -E^(a + b*x)
])/b^3 + (3*PolyLog[3, E^(a + b*x)])/b^3 - (3*x^2*Sech[a + b*x])/(2*b) - (x
^2*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rubi [A] time = 0.488723, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.95, Rules used = {2622, 288, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 2282, 6589, 6742, 4180, 2279, 2391, 2621, 5203, 3770}

$$\frac{3x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{3x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,-e^{\cdots}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```


```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 + (3*x^2*ArcTanh[E^(a + b*x)])/b - ArcTanh[Co
sh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 + (3*x*PolyLog[2, -E^(a + b*x)])/b
^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[2, I*E^(a +
b*x)])/b^3 - (3*x*PolyLog[2, E^(a + b*x)])/b^2 - (3*PolyLog[3, -E^(a + b*x)
])/b^3 + (3*PolyLog[3, E^(a + b*x)])/b^3 - (3*x^2*Sech[a + b*x])/(2*b) - (x
^2*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} -$$

$$= \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} -$$

$$= \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} +$$

$$= -\frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{\int\left(3x\text{sech}(a+bx) + x\text{csc}\right.}{}$$

$$= -\frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x^2\text{csch}(a+bx)\,dx +$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2}$$

Mathematica [B] time = 7.94988, size = 441, normalized size = 2.24

$$\frac{6bx\text{PolyLog}\left(2,-e^{a+bx}\right) - 6bx\text{PolyLog}\left(2,e^{a+bx}\right) - 6\text{PolyLog}\left(3,-e^{a+bx}\right) + 6\text{PolyLog}\left(3,e^{a+bx}\right) - 3b^2x^2\log\left(1-e^{a+bx}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -((x*Csch[a])/b^2) - (x^2*Csch[a/2 + (b*x)/2]^2)/(8*b) - (2*(((-I)*a + Pi/2
 - I*b*x)*(Log[1 - E^(I*((-I)*a + Pi/2 - I*b*x))] - Log[1 + E^(I*((-I)*a +
```

```
Pi/2 - I*b*x))]) - ((-I)*a + Pi/2)*Log[Tan[((-I)*a + Pi/2 - I*b*x)/2]] + I*
(PolyLog[2, -E^(I*((-I)*a + Pi/2 - I*b*x))] - PolyLog[2, E^(I*((-I)*a + Pi/
2 - I*b*x))])))/b^3 + (2*Log[1 - E^(a + b*x)] - 3*b^2*x^2*Log[1 - E^(a + b*
x)] - 2*Log[1 + E^(a + b*x)] + 3*b^2*x^2*Log[1 + E^(a + b*x)] + 6*b*x*PolyL
og[2, -E^(a + b*x)] - 6*b*x*PolyLog[2, E^(a + b*x)] - 6*PolyLog[3, -E^(a +
b*x)] + 6*PolyLog[3, E^(a + b*x)])/(2*b^3) - (x^2*Sech[a/2 + (b*x)/2]^2)/(8
*b) - (x^2*Sech[a + b*x])/b + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2
])/(2*b^2) + (x*Sech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b^2)
```

Maple [F] time = 0.291, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{csch}(bx+a))^3 (\operatorname{sech}(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bx^2e^{(3\,bx+3\,a)} - \left(3\,bx^2e^{(5\,a)} + 2\,xe^{(5\,a)}\right)e^{(5\,bx)} - \left(3\,bx^2e^{a} - 2\,xe^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)} - b^2e^{(4\,bx+4\,a)} - b^2e^{(2\,bx+2\,a)} + b^2} + \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x^2*e^(3*b*x + 3*a) - (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) - (3*b
*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b
^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog
(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 3/2*(b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - lo
g(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3 + 32*integrate(1/8*x*e^(b
*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Fricas [C] time = 2.5645, size = 10167, normalized size = 51.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*cosh(b*x + a)^3 - 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 - 10
*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 - 2*(3*b^2*x^2 + 2*b*x)*
sinh(b*x + a)^5 + 4*(b^2*x^2 - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh(
b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
a)^3)*sinh(b*x + a)^2 - 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 6*(b*x*cosh(b
*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x
*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cos
h(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^
3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2
+ b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a
))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + (4*I*cosh(b*x + a)
^6 + 24*I*cosh(b*x + a)*sinh(b*x + a)^5 + 4*I*sinh(b*x + a)^6 + (60*I*cosh(
b*x + a)^2 - 4*I)*sinh(b*x + a)^4 - 4*I*cosh(b*x + a)^4 + (80*I*cosh(b*x +
a)^3 - 16*I*cosh(b*x + a))*sinh(b*x + a)^3 + (60*I*cosh(b*x + a)^4 - 24*I*c
osh(b*x + a)^2 - 4*I)*sinh(b*x + a)^2 - 4*I*cosh(b*x + a)^2 + (24*I*cosh(b*
x + a)^5 - 16*I*cosh(b*x + a)^3 - 8*I*cosh(b*x + a))*sinh(b*x + a) + 4*I)*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-4*I*cosh(b*x + a)^6 - 24*I*cosh
(b*x + a)*sinh(b*x + a)^5 - 4*I*sinh(b*x + a)^6 + (-60*I*cosh(b*x + a)^2 +
4*I)*sinh(b*x + a)^4 + 4*I*cosh(b*x + a)^4 + (-80*I*cosh(b*x + a)^3 + 16*I*
cosh(b*x + a))*sinh(b*x + a)^3 + (-60*I*cosh(b*x + a)^4 + 24*I*cosh(b*x + a
)^2 + 4*I)*sinh(b*x + a)^2 + 4*I*cosh(b*x + a)^2 + (-24*I*cosh(b*x + a)^5 +
 16*I*cosh(b*x + a)^3 + 8*I*cosh(b*x + a))*sinh(b*x + a) - 4*I)*dilog(-I*co
sh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^6 + 6*b*x*cosh(b*x +
a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (15*b*x*co
sh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(
b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x + a)^4 -
 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh(b*x + a
)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + ((3*b^2*x^2 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^
2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3
*b^2*x^2 - 2)*cosh(b*x + a)^4 - (3*b^2*x^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x +
a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2*x^2 + 4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^
3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b
*x + a)^2 + (15*(3*b^2*x^2 - 2)*cosh(b*x + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2
- 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x +
 a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*
```

```
sinh(b*x + a) - 2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (-4*I*a*cosh(b*
x + a)^6 - 24*I*a*cosh(b*x + a)*sinh(b*x + a)^5 - 4*I*a*sinh(b*x + a)^6 + 4
*I*a*cosh(b*x + a)^4 + (-60*I*a*cosh(b*x + a)^2 + 4*I*a)*sinh(b*x + a)^4 +
(-80*I*a*cosh(b*x + a)^3 + 16*I*a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*I*a*co
sh(b*x + a)^2 + (-60*I*a*cosh(b*x + a)^4 + 24*I*a*cosh(b*x + a)^2 + 4*I*a)*
sinh(b*x + a)^2 + (-24*I*a*cosh(b*x + a)^5 + 16*I*a*cosh(b*x + a)^3 + 8*I*a
*cosh(b*x + a))*sinh(b*x + a) - 4*I*a)*log(cosh(b*x + a) + sinh(b*x + a) +
I) + (4*I*a*cosh(b*x + a)^6 + 24*I*a*cosh(b*x + a)*sinh(b*x + a)^5 + 4*I*a*
sinh(b*x + a)^6 - 4*I*a*cosh(b*x + a)^4 + (60*I*a*cosh(b*x + a)^2 - 4*I*a)*
sinh(b*x + a)^4 + (80*I*a*cosh(b*x + a)^3 - 16*I*a*cosh(b*x + a))*sinh(b*x
+ a)^3 - 4*I*a*cosh(b*x + a)^2 + (60*I*a*cosh(b*x + a)^4 - 24*I*a*cosh(b*x
+ a)^2 - 4*I*a)*sinh(b*x + a)^2 + (24*I*a*cosh(b*x + a)^5 - 16*I*a*cosh(b*x
 + a)^3 - 8*I*a*cosh(b*x + a))*sinh(b*x + a) + 4*I*a)*log(cosh(b*x + a) + s
inh(b*x + a) - I) - ((3*a^2 - 2)*cosh(b*x + a)^6 + 6*(3*a^2 - 2)*cosh(b*x +
 a)*sinh(b*x + a)^5 + (3*a^2 - 2)*sinh(b*x + a)^6 - (3*a^2 - 2)*cosh(b*x +
a)^4 + (15*(3*a^2 - 2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^4 + 4*(5*
(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a))*sinh(b*x + a)^3 -
(3*a^2 - 2)*cosh(b*x + a)^2 + (15*(3*a^2 - 2)*cosh(b*x + a)^4 - 6*(3*a^2 -
2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^2 + 3*a^2 + 2*(3*(3*a^2 - 2)*
cosh(b*x + a)^5 - 2*(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a)
)*sinh(b*x + a) - 2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((-4*I*b*x -
4*I*a)*cosh(b*x + a)^6 + (-24*I*b*x - 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^5
 + (-4*I*b*x - 4*I*a)*sinh(b*x + a)^6 + (4*I*b*x + 4*I*a)*cosh(b*x + a)^4 +
 ((-60*I*b*x - 60*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*sinh(b*x + a)^4 +
 ((-80*I*b*x - 80*I*a)*cosh(b*x + a)^3 + (16*I*b*x + 16*I*a)*cosh(b*x + a))
*sinh(b*x + a)^3 + (4*I*b*x + 4*I*a)*cosh(b*x + a)^2 + ((-60*I*b*x - 60*I*a
)*cosh(b*x + a)^4 + (24*I*b*x + 24*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*
sinh(b*x + a)^2 - 4*I*b*x + ((-24*I*b*x - 24*I*a)*cosh(b*x + a)^5 + (16*I*b
*x + 16*I*a)*cosh(b*x + a)^3 + (8*I*b*x + 8*I*a)*cosh(b*x + a))*sinh(b*x +
a) - 4*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((4*I*b*x + 4*I*a)
*cosh(b*x + a)^6 + (24*I*b*x + 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (4*I
*b*x + 4*I*a)*sinh(b*x + a)^6 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)^4 + ((60*I
*b*x + 60*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*x + a)^4 + ((80*I*
b*x + 80*I*a)*cosh(b*x + a)^3 + (-16*I*b*x - 16*I*a)*cosh(b*x + a))*sinh(b*
x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)^2 + ((60*I*b*x + 60*I*a)*cosh(b
*x + a)^4 + (-24*I*b*x - 24*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*
x + a)^2 + 4*I*b*x + ((24*I*b*x + 24*I*a)*cosh(b*x + a)^5 + (-16*I*b*x - 16
*I*a)*cosh(b*x + a)^3 + (-8*I*b*x - 8*I*a)*cosh(b*x + a))*sinh(b*x + a) + 4
*I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh
(b*x + a)^6 + 6*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (b^2*x^2 -
a^2)*sinh(b*x + a)^6 - (b^2*x^2 - a^2)*cosh(b*x + a)^4 - (b^2*x^2 - 15*(b^2
*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 4*(5*(b^2*x^
2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a)^3 -
 (b^2*x^2 - a^2)*cosh(b*x + a)^2 + (15*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^
2*x^2 - 6*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + a^2)*sinh(b*x + a)^2 - a^2 + 2*
```

```
(3*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b
^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x +
 a) + 1) + 6*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
+ a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)
^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, cosh(
b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)
*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(6*b^2*x^2*cosh(b*x + a)^2
- 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^4 - 3*b^2*x^2 + 2*b*x)*sinh(b*x + a))
/(b^3*cosh(b*x + a)^6 + 6*b^3*cosh(b*x + a)*sinh(b*x + a)^5 + b^3*sinh(b*x
+ a)^6 - b^3*cosh(b*x + a)^4 - b^3*cosh(b*x + a)^2 + (15*b^3*cosh(b*x + a)^
2 - b^3)*sinh(b*x + a)^4 + 4*(5*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*si
nh(b*x + a)^3 + b^3 + (15*b^3*cosh(b*x + a)^4 - 6*b^3*cosh(b*x + a)^2 - b^3
)*sinh(b*x + a)^2 + 2*(3*b^3*cosh(b*x + a)^5 - 2*b^3*cosh(b*x + a)^3 - b^3*
cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.517 $\int x\mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{3\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\text{sech}(a+}{2b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 + (3*x*ArcTanh[E^(a + b*x)])/b - Csch[a + b*x]/(2
*b^2) + (3*PolyLog[2, -E^(a + b*x)])/(2*b^2) - (3*PolyLog[2, E^(a + b*x)])/
(2*b^2) - (3*x*Sech[a + b*x])/(2*b) - (x*Csch[a + b*x]^2*Sech[a + b*x])/(2*
b)
```

Rubi [A] time = 0.165331, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2622, 288, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 2621}

$$\frac{3\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\text{sech}(a+}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 + (3*x*ArcTanh[E^(a + b*x)])/b - Csch[a + b*x]/(2
*b^2) + (3*PolyLog[2, -E^(a + b*x)])/(2*b^2) - (3*PolyLog[2, E^(a + b*x)])/
(2*b^2) - (3*x*Sech[a + b*x])/(2*b) - (x*Csch[a + b*x]^2*Sech[a + b*x])/(2*
b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \int\Big($$

$$= \frac{3x\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{\int c}{}$$

$$= \frac{3\tan^{-1}(\sinh(a+bx))}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{i\,\text{Subs}}{}$$

$$= \frac{3\tan^{-1}(\sinh(a+bx))}{2b^2} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(}{2b}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{xc}{}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{xc}{}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} + \frac{3\text{Li}_2\left(-e^{a+bx}\right)}{2b^2} - \frac{3\text{Li}}{}$$

Mathematica [A] time = 3.0529, size = 168, normalized size = 1.54

$$-\frac{12\left(\text{PolyLog}\left(2,-e^{-a-bx}\right)-\text{PolyLog}\left(2,e^{-a-bx}\right)\right)+12(a+bx)\left(\log\left(1-e^{-a-bx}\right)-\log\left(e^{-a-bx}+1\right)\right)-2\tanh\left(\frac{1}{2}(a+bx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -(-16*ArcTan[Tanh[(a + b*x)/2]] + 2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/
2]^2 + 12*(a + b*x)*(Log[1 - E^(-a - b*x)] - Log[1 + E^(-a - b*x)]) - 12*a*
Log[Tanh[(a + b*x)/2]] + 12*(PolyLog[2, -E^(-a - b*x)] - PolyLog[2, E^(-a -
 b*x)]) + b*x*Sech[(a + b*x)/2]^2 + 8*b*x*Sech[a + b*x] - 2*Tanh[(a + b*x)/
2])/(8*b^2)
```

Maple [A] time = 0.05, size = 148, normalized size = 1.4

$$-\frac{e^{bx+a}\left(3bxe^{4bx+4a}-2bxe^{2bx+2a}+e^{4bx+4a}+3bx-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)}+2\frac{\arctan\left(e^{bx+a}\right)}{b^2}+\frac{3a\ln\left(e^{bx+a}-1\right)}{2b^2}+\frac{3\,\text{dilog}\left(e^{bx+a}\right)}{2b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] -exp(b*x+a)*(3*b*x*exp(4*b*x+4*a)-2*b*x*exp(2*b*x+2*a)+exp(4*b*x+4*a)+3*b*x
-1)/b^2/(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))+2/b^2*arctan(exp(b*x+a))+3/
2/b^2*a*ln(exp(b*x+a)-1)+3/2/b^2*dilog(exp(b*x+a))+3/2/b^2*dilog(1+exp(b*x+
a))+3/2/b*ln(1+exp(b*x+a))*x
```

Maxima [A] time = 1.71509, size = 224, normalized size = 2.06

$$\frac{2bxe^{(3bx+3a)}-\left(3bxe^{(5a)}+e^{(5a)}\right)e^{(5bx)}-\left(3bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(6bx+6a)}-b^2e^{(4bx+4a)}-b^2e^{(2bx+2a)}+b^2}+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{2b^2}-\frac{3\left(bx\log\left(-e^{(bx+a)}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) - (3*b*x*e^a -
 e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
2*a) + b^2) + 3/2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - 3/
2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2 + 2*arctan(e^(b*x +
a))/b^2
```

Fricas [B] time = 2.04225, size = 4531, normalized size = 41.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x
 + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 - 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b
*x + 1)*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x
+ a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 4*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 2*(3*b*x - 1)*
cosh(b*x + a) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh
(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4
 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x
 + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(
b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x + a)^6 + 6*b*x*cos
h(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (1
5*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b
*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x
+ a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh
(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x +
 a)*sinh(b*x + a)^5 + a*sinh(b*x + a)^6 - a*cosh(b*x + a)^4 + (15*a*cosh(b*
x + a)^2 - a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 - a*cosh(b*x + a))*s
```

```
inh(b*x + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 - 6*a*cosh(b*x +
 a)^2 - a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 - 2*a*cosh(b*x + a)^3 -
 a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 3*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^5
 + (b*x + a)*sinh(b*x + a)^6 - (b*x + a)*cosh(b*x + a)^4 + (15*(b*x + a)*co
sh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^4 + 4*(5*(b*x + a)*cosh(b*x + a)^3 -
 (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a)^2 + (15
*(b*x + a)*cosh(b*x + a)^4 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*
x + a)^2 + b*x + 2*(3*(b*x + a)*cosh(b*x + a)^5 - 2*(b*x + a)*cosh(b*x + a)
^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1) + 2*(5*(3*b*x + 1)*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 +
3*b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh(b*x + a)*sinh(b
*x + a)^5 + b^2*sinh(b*x + a)^6 - b^2*cosh(b*x + a)^4 + (15*b^2*cosh(b*x +
a)^2 - b^2)*sinh(b*x + a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^2*cosh(b*x + a)^
3 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^2*cosh(b*x + a)^4 - 6*b^2*co
sh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 2*(3*b^2*cosh(b*x + a)^5 - 2*b
^2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.518 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rubi [A] time = 0.0483274, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= -\frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= \frac{3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0349659, size = 68, normalized size = 1.39

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b} - \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) - (3*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

Maple [A] time = 0.019, size = 43, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{1}{2\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{2}}-\frac{3}{2\cosh\left(bx+a\right)}+3\,\mathrm{Artanh}\left(\mathrm{e}^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.1438, size = 143, normalized size = 2.92

$$\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}-\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{3\,e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

Fricas [B] time = 1.87732, size = 1935, normalized size = 39.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
```

```
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Giac [B] time = 1.15404, size = 154, normalized size = 3.14

$$\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{4\,b}-\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}-\frac{3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}-4\,e^{(-bx-a)}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/4*log(e^(b*x + a) + e^(-b*x - a) + 2)/b - 3/4*log(e^(b*x + a) + e^(-b*x -
 a) - 2)/b - (3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/(((e^(b*x + a) + e^(-b*
x - a))^3 - 4*e^(b*x + a) - 4*e^(-b*x - a))*b)
```

3.519 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

Rubi [A] time = 0.276633, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]

[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 64.2115, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]

[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

Maple [A] time = 0.39, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3 (\operatorname{sech}(bx+a))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bxe^{(3\,bx+3\,a)} - \left(3\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)} - (3\,bxe^{a} + e^{a})e^{(bx)}}{b^2x^2e^{(6\,bx+6\,a)} - b^2x^2e^{(4\,bx+4\,a)} - b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} - 32\int \frac{3\,b^2x^2 - 2}{64\left(b^2x^3e^{(bx+a)} + b^2x^3\right)}\,dx - 32\int \frac{3\,b^2x^2 - 2}{64\left(b^2x^3e^{(bx+a)} - b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) - (3*b*x*e^a +
 e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) - b^2*x^2*e^(4*b*x + 4*a) - b^2*x^2
*e^(2*b*x + 2*a) + b^2*x^2) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^
(b*x + a) + b^2*x^3), x) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 32*integrate(1/16*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a
) + b*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

3.520 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Rubi [A] time = 0.332759, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]

[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 43.5843, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]

[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.498, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3 (\operatorname{sech}(bx+a))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bxe^{(3\,bx+3\,a)} - \left(3\,bxe^{(5\,a)} - 2\,e^{(5\,a)}\right)e^{(5\,bx)} - (3\,bxe^{a} + 2\,e^{a})e^{(bx)}}{b^2x^3e^{(6\,bx+6\,a)} - b^2x^3e^{(4\,bx+4\,a)} - b^2x^3e^{(2\,bx+2\,a)} + b^2x^3} - 32\int \frac{3\left(b^2x^2-2\right)}{64\left(b^2x^4e^{(bx+a)} + b^2x^4\right)}\,dx - 32\int \frac{3\left(b^2x^2-\right.}{64\left(b^2x^4e^{(bx+a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) - (3*b*x*e^a
 + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) - b^2*x^3*e^(4*b*x + 4*a) - b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*
e^(b*x + a) + b^2*x^4), x) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*e^(b*
x + a) - b^2*x^4), x) - 32*integrate(1/8*e^(b*x + a)/(b*x^3*e^(2*b*x + 2*a)
 + b*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^2/x^2, x)
```

3.521 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(x^m\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Rubi [A] time = 0.613247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 24.1459, size = 0, normalized size = 0.

$$\int x^m\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int x^{m} (\operatorname{csch}(bx+a))^{3} (\operatorname{sech}(bx+a))^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}(bx+a)^{3} \operatorname{sech}(bx+a)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{m} \operatorname{csch}(bx+a)^{3} \operatorname{sech}(bx+a)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \operatorname{csch}^{3}(a+bx) \operatorname{sech}^{3}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.522 $\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=240

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{3\text{PolyLog}}{}$$

```
[Out] (-6*x*ArcTanh[E^(2*a + 2*b*x)])/b^3 + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/b -
(3*x^2*Csch[2*a + 2*b*x])/b^2 - (2*x^3*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])
/b - (3*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^4) + (3*x^2*PolyLog[2, -E^(2*a +
 2*b*x)])/b^2 + (3*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^4) - (3*x^2*PolyLog[2,
 E^(2*a + 2*b*x)])/b^2 - (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/b^3 + (3*x*Poly
Log[3, E^(2*a + 2*b*x)])/b^3 + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(2*b^4) - (
3*PolyLog[4, E^(2*a + 2*b*x)])/(2*b^4)
```

Rubi [A] time = 0.304029, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5461, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```


```
[Out] (-6*x*ArcTanh[E^(2*a + 2*b*x)])/b^3 + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/b -
(3*x^2*Csch[2*a + 2*b*x])/b^2 - (2*x^3*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])
/b - (3*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^4) + (3*x^2*PolyLog[2, -E^(2*a +
 2*b*x)])/b^2 + (3*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^4) - (3*x^2*PolyLog[2,
 E^(2*a + 2*b*x)])/b^2 - (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/b^3 + (3*x*Poly
Log[3, E^(2*a + 2*b*x)])/b^3 + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(2*b^4) - (
3*PolyLog[4, E^(2*a + 2*b*x)])/(2*b^4)
```


Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = 8\int x^3\text{csch}^3(2a+2bx)\,dx$$

$$= -\frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - 4\int x^3\text{csch}(2a+2b$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a}{}$$

Mathematica [A] time = 7.51133, size = 274, normalized size = 1.14

$$\frac{\left(6b^2x^2-3\right)\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+\left(3-6b^2x^2\right)\text{PolyLog}\left(2,e^{2(a+bx)}\right)-6bx\text{PolyLog}\left(3,-e^{2(a+bx)}\right)+6bx\text{PolyLog}\left(3,e^{2(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (-(b^3*x^3*Csch[a + b*x]^2) + 6*b*x*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Lo
g[1 - E^(2*(a + b*x))] - 6*b*x*Log[1 + E^(2*(a + b*x))] + 4*b^3*x^3*Log[1 +
```

```
 E^(2*(a + b*x))] + (-3 + 6*b^2*x^2)*PolyLog[2, -E^(2*(a + b*x))] + (3 - 6*
b^2*x^2)*PolyLog[2, E^(2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(2*(a + b*x))] +
 6*b*x*PolyLog[3, E^(2*(a + b*x))] + 3*PolyLog[4, -E^(2*(a + b*x))] - 3*Pol
yLog[4, E^(2*(a + b*x))] - 3*b^2*x^2*Csch[a]*Sech[a] - b^3*x^3*Sech[a + b*x
]^2 + 3*b^2*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x] + 3*b^2*x^2*Sech[a]*Sech[a
+ b*x]*Sinh[b*x])/(2*b^4)
```

Maple [A] time = 0.064, size = 445, normalized size = 1.9

$$-2\,\frac{x^2e^{2\,bx+2\,a}\left(2\,bxe^{4\,bx+4\,a}+3\,e^{4\,bx+4\,a}+2\,bx-3\right)}{b^2\left(1+e^{2\,bx+2\,a}\right)^2\left(e^{2\,bx+2\,a}-1\right)^2}-3\,\frac{a\ln\left(e^{bx+a}-1\right)}{b^4}+2\,\frac{x^3\ln\left(1+e^{2\,bx+2\,a}\right)}{b}+3\,\frac{x^2\mathrm{polylog}\left(2,-e^{2\,bx+\cdots}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -2*x^2*exp(2*b*x+2*a)*(2*b*x*exp(4*b*x+4*a)+3*exp(4*b*x+4*a)+2*b*x-3)/b^2/(
1+exp(2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)^2-3/b^4*a*ln(exp(b*x+a)-1)+2*x^3*ln(
1+exp(2*b*x+2*a))/b+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3*x*polylog(3,-exp
(2*b*x+2*a))/b^3+12/b^3*polylog(3,-exp(b*x+a))*x-2/b*ln(1-exp(b*x+a))*x^3-6
/b^2*polylog(2,exp(b*x+a))*x^2+12/b^3*polylog(3,exp(b*x+a))*x-6/b^2*polylog
(2,-exp(b*x+a))*x^2-3*x*ln(1+exp(2*b*x+2*a))/b^3+3/b^4*polylog(2,-exp(b*x+a
))+2/b^4*a^3*ln(exp(b*x+a)-1)-3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/b^4*poly
log(2,exp(b*x+a))-12/b^4*polylog(4,-exp(b*x+a))+3/2*polylog(4,-exp(2*b*x+2*
a))/b^4-2/b*ln(1+exp(b*x+a))*x^3+3/b^3*ln(1+exp(b*x+a))*x+3/b^3*ln(1-exp(b*
x+a))*x-2/b^4*ln(1-exp(b*x+a))*a^3-12/b^4*polylog(4,exp(b*x+a))+3/b^4*ln(1-
exp(b*x+a))*a
```

Maxima [A] time = 1.16782, size = 514, normalized size = 2.14

$$-\frac{2\left(\left(2\,bx^3e^{(6\,a)}+3\,x^2e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bx^3e^{(2\,a)}-3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2}+\frac{2\left(4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\cdots\right.}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x^3*e^(6*a) + 3*x^2*e^(6*a))*e^(6*b*x) + (2*b*x^3*e^(2*a) - 3*x^2*
e^(2*a))*e^(2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + 2
```

```
/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4
- 2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - 2*(b^3*x^3*log(
-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x
+ a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1
) + dilog(-e^(2*b*x + 2*a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(
b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 2.98907, size = 17071, normalized size = 71.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^6 + 40*(2*b^3*x^3 + 3*b^2*x^2)*co
sh(b*x + a)^3*sinh(b*x + a)^3 + 30*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^2*
sinh(b*x + a)^4 + 12*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)*sinh(b*x + a)^5
+ 2*(2*b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^6 + 2*(2*b^3*x^3 - 3*b^2*x^2)*cos
h(b*x + a)^2 + 2*(2*b^3*x^3 + 15*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^4 -
3*b^2*x^2)*sinh(b*x + a)^2 + 3*((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2
*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*
b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2
*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2
+ 8*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sin
h(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cos
h(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2
*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 3*((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x
+ a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6
 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b
*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(
b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 -
 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7
*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x
+ a)^3)*sinh(b*x + a) - 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*((2
*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x
 + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2
```

```
 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(
2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*
b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 - 1)*cosh(b*x +
a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1
)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*
((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*
x + a) - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*((2*b^2*x^2 - 1)*
cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(
2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x
 + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)
*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*s
inh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*
x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a
)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 -
1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*x + a) - 1)*di
log(-cosh(b*x + a) - sinh(b*x + a)) + ((2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^8
+ 56*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 -
3*b*x)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a
)*sinh(b*x + a)^7 + (2*b^3*x^3 - 3*b*x)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*
b^3*x^3 - 3*b*x)*cosh(b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 - 3*b*x)*co
sh(b*x + a)^4 - 3*b*x)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 - 3*b*x)*cosh(b*x
+ a)^5 - (2*b^3*x^3 - 3*b*x)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^3*x
^3 - 3*b*x)*cosh(b*x + a)^6 - 3*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 - 3*b*x + 8*((2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^7 - (2*b^3*x^3 - 3
*b*x)*cosh(b*x + a)^3)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1
) + ((2*a^3 - 3*a)*cosh(b*x + a)^8 + 56*(2*a^3 - 3*a)*cosh(b*x + a)^3*sinh(
b*x + a)^5 + 28*(2*a^3 - 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^3 -
3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*a^3 - 3*a)*sinh(b*x + a)^8 - 2*(2*a
^3 - 3*a)*cosh(b*x + a)^4 + 2*(35*(2*a^3 - 3*a)*cosh(b*x + a)^4 - 2*a^3 + 3
*a)*sinh(b*x + a)^4 + 8*(7*(2*a^3 - 3*a)*cosh(b*x + a)^5 - (2*a^3 - 3*a)*co
sh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*a^3 - 3*a)*cosh(b*x + a)^6 -
 3*(2*a^3 - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*a^3 - 3*a)*cosh(b
*x + a)^7 - (2*a^3 - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(cosh(b*
x + a) + sinh(b*x + a) + I) + ((2*a^3 - 3*a)*cosh(b*x + a)^8 + 56*(2*a^3 -
3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*a^3 - 3*a)*cosh(b*x + a)^2*sin
h(b*x + a)^6 + 8*(2*a^3 - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*a^3 - 3*a
)*sinh(b*x + a)^8 - 2*(2*a^3 - 3*a)*cosh(b*x + a)^4 + 2*(35*(2*a^3 - 3*a)*c
osh(b*x + a)^4 - 2*a^3 + 3*a)*sinh(b*x + a)^4 + 8*(7*(2*a^3 - 3*a)*cosh(b*x
 + a)^5 - (2*a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*a^
3 - 3*a)*cosh(b*x + a)^6 - 3*(2*a^3 - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2
 + 8*((2*a^3 - 3*a)*cosh(b*x + a)^7 - (2*a^3 - 3*a)*cosh(b*x + a)^3)*sinh(b
*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*a^3 - 3*a)*cosh
(b*x + a)^8 + 56*(2*a^3 - 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*a^3
- 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^3 - 3*a)*cosh(b*x + a)*sinh
(b*x + a)^7 + (2*a^3 - 3*a)*sinh(b*x + a)^8 - 2*(2*a^3 - 3*a)*cosh(b*x + a)
```

^4 + 2*(35*(2*a^3 - 3*a)*cosh(b*x + a)^4 - 2*a^3 + 3*a)*sinh(b*x + a)^4 + 8
(7(2*a^3 - 3*a)*cosh(b*x + a)^5 - (2*a^3 - 3*a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 2*a^3 + 4*(7*(2*a^3 - 3*a)*cosh(b*x + a)^6 - 3*(2*a^3 - 3*a)*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 8*((2*a^3 - 3*a)*cosh(b*x + a)^7 - (2*a^3 - 3*
a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a)
- 1) - ((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^8 + 56*(2*b^3*x^3 +
2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 + 2*a
^3 - 3*b*x - 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 + 2*a^3 -
3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^3*x^3 + 2*a^3 - 3*b*x - 3
*a)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(
b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x +
a)^4 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 + 2*a^3 - 3*
b*x - 3*a)*cosh(b*x + a)^5 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a
))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*
x + a)^6 - 3*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2)*sinh(b*x +
a)^2 - 3*b*x + 8*((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^7 - (2*b^
3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(I*co
sh(b*x + a) + I*sinh(b*x + a) + 1) - ((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cos
h(b*x + a)^8 + 56*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3*sinh(b*
x + a)^5 + 28*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2*sinh(b*x +
a)^6 + 8*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 +
(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^
3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2
*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4
+ 8*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^5 - (2*b^3*x^3 + 2*a
^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3
+ 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^6 - 3*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a
)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 3*b*x + 8*((2*b^3*x^3 + 2*a^3 - 3*b*x
- 3*a)*cosh(b*x + a)^7 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3)
*sinh(b*x + a) - 3*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((2*b^3
*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^8 + 56*(2*b^3*x^3 + 2*a^3 - 3*b*x
- 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3
*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*c
osh(b*x + a)*sinh(b*x + a)^7 + (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x +
a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 - 2
*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 2*a^3
- 3*b*x - 3*a)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cos
h(b*x + a)^5 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^6 - 3*(
2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 3*b*x +
8*((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^7 - (2*b^3*x^3 + 2*a^3
- 3*b*x - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(-cosh(b*x + a) - s
inh(b*x + a) + 1) + 12*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^
5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 +
sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x +

```
 a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b
*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(
b*x + a)^3)*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) -
12*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)
^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*
(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*
x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x
 + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*(cosh(b*x + a
)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a
)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(
b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polyl
og(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 12*(cosh(b*x + a)^8 + 56*cosh(b
*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x
 + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b
*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(
b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x
 + a) - sinh(b*x + a)) - 12*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*s
inh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x +
a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*
x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*c
osh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x +
 a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3
)*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 12*(b*x*cosh(b
*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^
8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^
4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*
x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*c
osh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*polylog(3, I*cosh(b*x
+ a) + I*sinh(b*x + a)) + 12*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*
sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x +
 a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b
*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*
cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x
+ a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^
3)*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 12*(b*x*
cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x
+ a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
 + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x
 + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*
```

```
(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*polylog(3, -cosh
(b*x + a) - sinh(b*x + a)) + 4*(3*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^5 +
 (2*b^3*x^3 - 3*b^2*x^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^8
 + 56*b^4*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b^4*cosh(b*x + a)^2*sinh(b*x
 + a)^6 + 8*b^4*cosh(b*x + a)*sinh(b*x + a)^7 + b^4*sinh(b*x + a)^8 - 2*b^4
*cosh(b*x + a)^4 + 2*(35*b^4*cosh(b*x + a)^4 - b^4)*sinh(b*x + a)^4 + b^4 +
 8*(7*b^4*cosh(b*x + a)^5 - b^4*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b^4*c
osh(b*x + a)^6 - 3*b^4*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b^4*cosh(b*x +
 a)^7 - b^4*cosh(b*x + a)^3)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.523 $\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=149

$$\frac{2x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{2x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{2x\text{csch}(2a+2b}{b^2}$$

```
[Out] (4*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - ArcTanh[Cosh[2*a + 2*b*x]]/b^3 - (2*x*
Csch[2*a + 2*b*x])/b^2 - (2*x^2*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])/b + (2
*x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 - (2*x*PolyLog[2, E^(2*a + 2*b*x)])/b^
2 - PolyLog[3, -E^(2*a + 2*b*x)]/b^3 + PolyLog[3, E^(2*a + 2*b*x)]/b^3
```

Rubi [A] time = 0.198246, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5461, 4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{2x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{2x\text{csch}(2a+2b}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```


```
[Out] (4*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - ArcTanh[Cosh[2*a + 2*b*x]]/b^3 - (2*x*
Csch[2*a + 2*b*x])/b^2 - (2*x^2*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])/b + (2
*x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 - (2*x*PolyLog[2, E^(2*a + 2*b*x)])/b^
2 - PolyLog[3, -E^(2*a + 2*b*x)]/b^3 + PolyLog[3, E^(2*a + 2*b*x)]/b^3
```


Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = 8\int x^2\operatorname{csch}^3(2a+2bx)\,dx$$
$$= -\frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} - 4\int x^2\operatorname{csch}(2a+2b$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\operatorname{cot}}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\operatorname{cot}}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\operatorname{cot}}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\operatorname{cot}}{}$$

Mathematica [A] time = 6.36937, size = 192, normalized size = 1.29

$$-\frac{-4bx\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)+4bx\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)+2\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)-2\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)+4b^2x^2\operatorname{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -(4*ArcTanh[E^(2*(a + b*x))] + b^2*x^2*Csch[a + b*x]^2 + 4*b^2*x^2*Log[1 -
E^(2*(a + b*x))] - 4*b^2*x^2*Log[1 + E^(2*(a + b*x))] - 4*b*x*PolyLog[2, -E
^(2*(a + b*x))] + 4*b*x*PolyLog[2, E^(2*(a + b*x))] + 2*PolyLog[3, -E^(2*(a
 + b*x))] - 2*PolyLog[3, E^(2*(a + b*x))] + 2*b*x*Csch[a]*Sech[a] + b^2*x^2
*Sech[a + b*x]^2 - 2*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x] - 2*b*x*Sech[a]*Se
ch[a + b*x]*Sinh[b*x])/(2*b^3)
```

Maple [B] time = 0.057, size = 299, normalized size = 2.

$$-4\,\frac{x{e}^{2\,bx+2\,a}\left(bx{e}^{4\,bx+4\,a}+{e}^{4\,bx+4\,a}+bx-1\right)}{b^2\left({e}^{2\,bx+2\,a}-1\right)^2\left(1+{e}^{2\,bx+2\,a}\right)^2}-2\,\frac{\ln\left(1+{e}^{bx+a}\right)x^2}{b}-4\,\frac{x\operatorname{polylog}\left(2,-{e}^{bx+a}\right)}{b^2}+2\,\frac{x^2\ln\left(1+{e}^{2\,bx+2\,a}\right)}{b}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -4*x*exp(2*b*x+2*a)*(b*x*exp(4*b*x+4*a)+exp(4*b*x+4*a)+b*x-1)/b^2/(exp(2*b*
x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-2/b*ln(1+exp(b*x+a))*x^2-4*x*polylog(2,-ex
p(b*x+a))/b^2+2*x^2*ln(1+exp(2*b*x+2*a))/b+2*x*polylog(2,-exp(2*b*x+2*a))/b
^2-2/b*ln(1-exp(b*x+a))*x^2-4*x*polylog(2,exp(b*x+a))/b^2-polylog(3,-exp(2*
b*x+2*a))/b^3+4*polylog(3,exp(b*x+a))/b^3+4*polylog(3,-exp(b*x+a))/b^3+1/b^
3*ln(exp(b*x+a)-1)-1/b^3*ln(1+exp(2*b*x+2*a))+1/b^3*ln(1+exp(b*x+a))+2/b^3*
ln(1-exp(b*x+a))*a^2-2/b^3*a^2*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.18679, size = 369, normalized size = 2.48

$$-\frac{4\left(\left(bx^2e^{(6a)}+xe^{(6a)}\right)e^{(6bx)}+\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}\right)}{b^2e^{(8bx+8a)}-2b^2e^{(4bx+4a)}+b^2}+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3(-e^{(2bx+2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*((b*x^2*e^(6*a) + x*e^(6*a))*e^(6*b*x) + (b*x^2*e^(2*a) - x*e^(2*a))*e^(
2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b^2*x^2*lo
g(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b
*x + 2*a)))/b^3 - 2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a
)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2
*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - log(e^(2*b*x + 2
*a) + 1)/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Fricas [C] time = 2.63012, size = 12081, normalized size = 81.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(4*(b^2*x^2 + b*x)*cosh(b*x + a)^6 + 80*(b^2*x^2 + b*x)*cosh(b*x + a)^3*si
nh(b*x + a)^3 + 60*(b^2*x^2 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 24*(b^
2*x^2 + b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + 4*(b^2*x^2 + b*x)*sinh(b*x + a
)^6 + 4*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 4*(15*(b^2*x^2 + b*x)*cosh(b*x +
a)^4 + b^2*x^2 - b*x)*sinh(b*x + a)^2 + 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cos
h(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b
*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a
```

)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x
+ a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*
b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*c
osh(b*x + a)^3)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(b*
x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*
x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b
*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b
x + a)^4 + 8(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 +
 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x +
8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^
3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x
 + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35
*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*
x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*
x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a
)^3)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*(b*x*cosh
(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)
^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a
)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a
)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*
b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x
*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) + ((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*c
osh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1
)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 -
1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b
^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^
2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*c
osh(b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 ((2*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b
*x + a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(
b*x + a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4
 + 8*(7*(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x +
 a)^3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*s
inh(b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(
b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((2
*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5
+ 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x +
 a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x
+ a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8
(7(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^

3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x
+ a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((2*a^2
 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28
*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x + a)*
sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x + a)
^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8*(7*
(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 +
4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(b*x
+ a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x + a)
^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b^2*x^2
 - a^2)*cosh(b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^
5 + 28*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*
cosh(b*x + a)*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^
2 - a^2)*cosh(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2
+ a^2)*sinh(b*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (
b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b
x + a)^6 - 3(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((
b^2*x^2 - a^2)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x
+ a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b^2*x^2 - a^2)*cosh(
b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x
^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh
(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b
*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2
)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3
*(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2
)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a)^8 +
 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^4
+ 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b*x + a)^4 +
b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2)*cosh(b*x +
 a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3*(b^2*x^2 -
a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2)*cosh(b*x +
 a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-cosh(b*x + a)
- sinh(b*x + a) + 1) - 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7
+ sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x
 + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cos
h(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2

```
*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b
*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*
x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 4*(cosh(b*x + a
)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a
)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(
b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polyl
og(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(cosh(b*x + a)^8 + 56*cosh(b*
x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x
+ a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*
x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b
*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(
cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
+ a) - sinh(b*x + a)) + 8*(3*(b^2*x^2 + b*x)*cosh(b*x + a)^5 + (b^2*x^2 - b
*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^8 + 56*b^3*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b^3*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b^3*cosh(
b*x + a)*sinh(b*x + a)^7 + b^3*sinh(b*x + a)^8 - 2*b^3*cosh(b*x + a)^4 + 2*
(35*b^3*cosh(b*x + a)^4 - b^3)*sinh(b*x + a)^4 + 8*(7*b^3*cosh(b*x + a)^5 -
 b^3*cosh(b*x + a))*sinh(b*x + a)^3 + b^3 + 4*(7*b^3*cosh(b*x + a)^6 - 3*b^
3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b^3*cosh(b*x + a)^7 - b^3*cosh(b*x
+ a)^3)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.524 $\int x\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\operatorname{csch}(2a+2bx)}{b^2}+\frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{2x\coth(2a+2bx)\operatorname{csch}(2a+$$

```
[Out] (4*x*ArcTanh[E^(2*a + 2*b*x)])/b - Csch[2*a + 2*b*x]/b^2 - (2*x*Coth[2*a +
2*b*x]*Csch[2*a + 2*b*x])/b + PolyLog[2, -E^(2*a + 2*b*x)]/b^2 - PolyLog[2,
 E^(2*a + 2*b*x)]/b^2
```

Rubi [A] time = 0.110237, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5461, 4185, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\operatorname{csch}(2a+2bx)}{b^2}+\frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{2x\coth(2a+2bx)\operatorname{csch}(2a+$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (4*x*ArcTanh[E^(2*a + 2*b*x)])/b - Csch[2*a + 2*b*x]/b^2 - (2*x*Coth[2*a +
2*b*x]*Csch[2*a + 2*b*x])/b + PolyLog[2, -E^(2*a + 2*b*x)]/b^2 - PolyLog[2,
 E^(2*a + 2*b*x)]/b^2
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = 8\int x\text{csch}^3(2a+2bx)\,dx$$

$$= -\frac{\text{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - 4\int x\text{csch}(2a+2bx)\,dx$$

$$= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\text{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b} + \frac{2\int \text{l}}{}$$

$$= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\text{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b} + \frac{\text{Sub}}{}$$

$$= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\text{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b} + \frac{\text{Li}_2\big(}{}$$

Mathematica [A] time = 1.38667, size = 148, normalized size = 1.63

$$-\frac{2\text{PolyLog}\left(2,-e^{-2(a+bx)}\right) - 2\text{PolyLog}\left(2,e^{-2(a+bx)}\right) + 4a\log\left(1-e^{-2(a+bx)}\right) + 4bx\log\left(1-e^{-2(a+bx)}\right) - 4a\log\left(e^{-2(a+bx)} + \right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 4*a*Log[1 - E^(-2*(a + b*x))] + 4*b
*x*Log[1 - E^(-2*(a + b*x))] - 4*a*Log[1 + E^(-2*(a + b*x))] - 4*b*x*Log[1
```

```
+ E^(-2*(a + b*x))] - 4*a*Log[Tanh[a + b*x]] + 2*PolyLog[2, -E^(-2*(a + b*x
))] - 2*PolyLog[2, E^(-2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b*x])
/(2*b^2)
```

Maple [B] time = 0.056, size = 197, normalized size = 2.2

$$-2\,\frac{e^{2\,bx+2\,a}\left(2\,bxe^{4\,bx+4\,a}+e^{4\,bx+4\,a}+2\,bx-1\right)}{b^2\left(1+e^{2\,bx+2\,a}\right)^2\left(e^{2\,bx+2\,a}-1\right)^2}-2\,\frac{\ln\left(1+e^{bx+a}\right)x}{b}-2\,\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}+2\,\frac{x\ln\left(1+e^{2\,bx+2\,a}\right)}{b}+\frac{\mathrm{pol}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -2*exp(2*b*x+2*a)*(2*b*x*exp(4*b*x+4*a)+exp(4*b*x+4*a)+2*b*x-1)/b^2/(1+exp(
2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)^2-2/b*ln(1+exp(b*x+a))*x-2/b^2*polylog(2,-
exp(b*x+a))+2*x*ln(1+exp(2*b*x+2*a))/b+polylog(2,-exp(2*b*x+2*a))/b^2-2/b*l
n(1-exp(b*x+a))*x-2/b^2*ln(1-exp(b*x+a))*a-2/b^2*polylog(2,exp(b*x+a))+2/b^
2*a*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.15926, size = 221, normalized size = 2.43

$$-\frac{2\left(\left(2\,bxe^{(6\,a)}+e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^2}-\frac{2\left(bx\log\left(e^{(bx+a)}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x*e^(6*a) + e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*
x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b*x*log(e^(2*b
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 - 2*(b*x*log(e^(b*x + a) + 1)
 + dilog(-e^(b*x + a)))/b^2 - 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
 a)))/b^2
```

Fricas [C] time = 2.31796, size = 8173, normalized size = 89.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")

[Out] -2*((2*b*x + 1)*cosh(b*x + a)^6 + 20*(2*b*x + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3 + 15*(2*b*x + 1)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 6*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b*x + 1)*sinh(b*x + a)^6 + (2*b*x - 1)*cosh(b*x + a)^2 + (15*(2*b*x + 1)*cosh(b*x + a)^4 + 2*b*x - 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x + a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 - a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b

```
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I
) - (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)
^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^
4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x
+ a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x
 + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x
+ 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) +
 a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^8
 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)
*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x +
 a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x +
a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x
 + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7
 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*s
inh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)
^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a
)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a
)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)
*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(2*b*x +
 1)*cosh(b*x + a)^5 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a))/(b^2*cosh(b
*x + a)^8 + 56*b^2*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b^2*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*b^2*cosh(b*x + a)*sinh(b*x + a)^7 + b^2*sinh(b*x + a)^
8 - 2*b^2*cosh(b*x + a)^4 + 2*(35*b^2*cosh(b*x + a)^4 - b^2)*sinh(b*x + a)^
4 + 8*(7*b^2*cosh(b*x + a)^5 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b^
2*cosh(b*x + a)^6 - 3*b^2*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b^2 + 8*(b^2*c
osh(b*x + a)^7 - b^2*cosh(b*x + a)^3)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.525 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0443224, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.011343, size = 47, normalized size = 1.09

$$8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} - \frac{\operatorname{sech}^2(a+bx)}{16b} - \frac{\log(\tanh(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] 8*(-Csch[a + b*x]^2/(16*b) - Log[Tanh[a + b*x]]/(4*b) - Sech[a + b*x]^2/(16
*b))
```

Maple [A] time = 0.023, size = 48, normalized size = 1.1

$$-\frac{1}{2b(\sinh(bx+a))^2(\cosh(bx+a))^2} - \frac{1}{b(\cosh(bx+a))^2} - 2\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^2-1/b/cosh(b*x+a)^2-2*ln(tanh(b*x+a))/b
```

Maxima [B] time = 1.56957, size = 138, normalized size = 3.21

$$-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

Fricas [B] time = 1.84967, size = 2102, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
```

```
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a + bx)\operatorname{sech}^3(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Giac [B] time = 1.13687, size = 136, normalized size = 3.16

$$\frac{\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} + 2\right)}{b} - \frac{\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} - 2\right)}{b} - \frac{4\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}\right)}{\left(\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}\right)^2 - 4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)/b - log(e^(2*b*x + 2*a) + e^(-2
*b*x - 2*a) - 2)/b - 4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/(((e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a))^2 - 4)*b)
```

3.526 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$8\text{Unintegrable}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x},x\right)$$

```
[Out] 8*Unintegrable[Csch[2*a + 2*b*x]^3/x, x]
```

Rubi [A] time = 0.0682176, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

```
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = 8\int \frac{\operatorname{csch}^3(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 58.8337, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x, x]
```

Maple [A] time = 0.42, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{csch}\left(bx+a\right)\right)^{3}\left(\operatorname{sech}\left(bx+a\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(2\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^{2}x^{2}e^{(8\,bx+8\,a)}-2\,b^{2}x^{2}e^{(4\,bx+4\,a)}+b^{2}x^{2}}-64\int\frac{2\,b^{2}x^{2}-1}{32\left(b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}\right)}\,dx+64\int\frac{2\,b^{2}x^{2}-1}{64\left(b^{2}x^{3}e^{(bx+a)}+b^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) + e^(2*a))*e^(2*b*
x))/(b^2*x^2*e^(8*b*x + 8*a) - 2*b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2) - 64*in
tegrate(1/32*(2*b^2*x^2 - 1)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) + 64*i
ntegrate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) - 64*inte
grate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

3.527 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=19

$$8\text{Unintegrable}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x^2},x\right)$$

```
[Out] 8*Unintegrable[Csch[2*a + 2*b*x]^3/x^2, x]
```

Rubi [A] time = 0.0729499, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = 8\int \frac{\operatorname{csch}^3(2a+2bx)}{x^2}\,dx$$

Mathematica [A] time = 42.1611, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2, x]
```

Maple [A] time = 0.521, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3 (\operatorname{sech}(bx+a))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(\left(bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}+\left(bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2x^3e^{(8\,bx+8\,a)}-2\,b^2x^3e^{(4\,bx+4\,a)}+b^2x^3}-64\int\frac{2\,b^2x^2-3}{32\left(b^2x^4e^{(2\,bx+2\,a)}+b^2x^4\right)}\,dx+64\int\frac{2\,b^2x^2-3}{64\left(b^2x^4e^{(bx+a)}+b^2x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -4*((b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (b*x*e^(2*a) + e^(2*a))*e^(2*b*x))/
(b^2*x^3*e^(8*b*x + 8*a) - 2*b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3) - 64*integr
ate(1/32*(2*b^2*x^2 - 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) + 64*integ
rate(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) - 64*integrat
e(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

3.528 $\int x \cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=87

$$\frac{20i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{49b^2}-\frac{20\sinh(a+bx)\sqrt{\cosh(a+bx)}}{147b^2}+\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] (2*x*Cosh[a + b*x]^(7/2))/(7*b) + (((20*I)/147)*EllipticF[(I/2)*(a + b*x),
2])/b^2 - (20*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(147*b^2) - (4*Cosh[a + b*
x]^(5/2)*Sinh[a + b*x])/(49*b^2)
```

Rubi [A] time = 0.0578226, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2641}

$$\frac{20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{49b^2}-\frac{20\sinh(a+bx)\sqrt{\cosh(a+bx)}}{147b^2}+\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(7/2))/(7*b) + (((20*I)/147)*EllipticF[(I/2)*(a + b*x),
2])/b^2 - (20*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(147*b^2) - (4*Cosh[a + b*
x]^(5/2)*Sinh[a + b*x])/(49*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int\cosh^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2} - \frac{10\int\cosh^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2} - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}$$
$$= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} + \frac{20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2} - \frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2} - \frac{4\cos}{}$$

Mathematica [A] time = 0.343983, size = 77, normalized size = 0.89

$$\frac{\sqrt{\cosh(a+bx)}(-46\sinh(a+bx)-6\sinh(3(a+bx))+63bx\cosh(a+bx)+21bx\cosh(3(a+bx)))+40i\text{EllipticF}\left(\frac{1}{2}i(a+\right.}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] ((40*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(63*b*x*Cosh[a
+ b*x] + 21*b*x*Cosh[3*(a + b*x)] - 46*Sinh[a + b*x] - 6*Sinh[3*(a + b*x)])
)/(294*b^2)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x\left(\cosh\left(bx+a\right)\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)
```

[Out] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^{\frac{5}{2}} \sinh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")

[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(b*x+a)**(5/2)*sinh(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^{\frac{5}{2}} \sinh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.529 $\int x \cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (2*x*Cosh[a + b*x]^(5/2))/(5*b) + (((12*I)/25)*EllipticE[(I/2)*(a + b*x), 2
])/b^2 - (4*Cosh[a + b*x]^(3/2)*Sinh[a + b*x])/(25*b^2)
```

Rubi [A] time = 0.0426493, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2639}

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(5/2))/(5*b) + (((12*I)/25)*EllipticE[(I/2)*(a + b*x), 2
])/b^2 - (4*Cosh[a + b*x]^(3/2)*Sinh[a + b*x])/(25*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int\cosh^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2} - \frac{6\int\sqrt{\cosh(a+bx)}\,dx}{25b}$$
$$= \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}$$

Mathematica [C] time = 1.84358, size = 142, normalized size = 2.22

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{e^{2(a+bx)}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+\left(e^{2(a+bx)}+1\right)\left(2(5bx-12)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5bx+2\right.\right.}{50\sqrt{2}b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] ((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4*
(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a +
 b*x))*Sqrt[E^(-a - b*x) + E^(a + b*x)])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\left(\cosh\left(bx+a\right)\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

```
[Out] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^{\frac{3}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**(3/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)^{\frac{3}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.530 $\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}-\frac{4\sinh(a+bx)\sqrt{\cosh(a+bx)}}{9b^2}+\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (2*x*Cosh[a + b*x]^(3/2))/(3*b) + (((4*I)/9)*EllipticF[(I/2)*(a + b*x), 2])
/b^2 - (4*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(9*b^2)
```

Rubi [A] time = 0.0422691, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2641}

$$\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}-\frac{4\sinh(a+bx)\sqrt{\cosh(a+bx)}}{9b^2}+\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(3/2))/(3*b) + (((4*I)/9)*EllipticF[(I/2)*(a + b*x), 2])
/b^2 - (4*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(9*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\cosh^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2}-\frac{2\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{9b}\\ &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}+\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}-\frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.159411, size = 56, normalized size = 0.88

$$\frac{2\sqrt{\cosh(a+bx)}(3bx\cosh(a+bx)-2\sinh(a+bx))+4i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] ((4*I)*EllipticF[(I/2)*(a + b*x), 2] + 2*Sqrt[Cosh[a + b*x]]*(3*b*x*Cosh[a
+ b*x] - 2*Sinh[a + b*x]))/(9*b^2)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\sinh(bx+a)\sqrt{\cosh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

```
[Out] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh(bx+a)}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh(bx+a)}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

3.531 $\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}} \, dx$

Optimal. Leaf size=37

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] (2*x*Sqrt[Cosh[a + b*x]])/b + ((4*I)*EllipticE[(I/2)*(a + b*x), 2])/b^2
```

Rubi [A] time = 0.0295638, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5373, 2639}

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (2*x*Sqrt[Cosh[a + b*x]])/b + ((4*I)*EllipticE[(I/2)*(a + b*x), 2])/b^2
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2x\sqrt{\cosh(a+bx)}}{b} - \frac{2\int \sqrt{\cosh(a+bx)}\,dx}{b}$$
$$= \frac{2x\sqrt{\cosh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Mathematica [C] time = 1.46792, size = 190, normalized size = 5.14

$$\frac{\sqrt{2}e^{-a-bx}\sqrt{e^{2(a+bx)}+1}\left(18\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{3}{4},\frac{3}{4}\right\},-e^{2(a+bx)}\right)-2e^{2(a+bx)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\right.\right.\right.}{9b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[1 + E^(2*(a + b*x))]*(3*b*x*(3*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^(2*(a + b*x))] + E^(2*(a + b*x))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^(2*(a + b*x))]) + 18*HypergeometricPFQ[{-1/4, -1/4, 1/2}, {3/
4, 3/4}, -E^(2*(a + b*x))] - 2*E^(2*(a + b*x))*HypergeometricPFQ[{1/2, 3/4,
 3/4}, {7/4, 7/4}, -E^(2*(a + b*x))]))/(9*b^2*Sqrt[E^(-a - b*x) + E^(a + b*
x)])
```

Maple [B] time = 0.066, size = 250, normalized size = 6.8

$$\frac{(bx-2)\left(\left(e^{bx+a}\right)^2+1\right)\sqrt{2}}{b^2e^{bx+a}}\frac{1}{\sqrt{\frac{\left(e^{bx+a}\right)^2+1}{e^{bx+a}}}}-2\frac{\sqrt{2}\sqrt{\left(\left(e^{bx+a}\right)^2+1\right)e^{bx+a}}}{b^2e^{bx+a}}\left(-2\frac{\left(e^{bx+a}\right)^2+1}{\sqrt{\left(\left(e^{bx+a}\right)^2+1\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\sqrt{2}\sqrt{i}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
```

```
)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(bx+a)}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\sqrt{\cosh(bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

3.532 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} dx$

Optimal. Leaf size=37

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}}-\frac{4i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Rubi [A] time = 0.0293948, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5373, 2641}

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}}-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Mathematica [A] time = 0.162898, size = 37, normalized size = 1.

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\sinh(bx+a)\,(\cosh(bx+a))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

```
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

3.533 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=64

$$\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[Out] (-2*x)/(3*b*Cosh[a + b*x]^(3/2)) + (((4*I)/3)*EllipticE[(I/2)*(a + b*x), 2]
)/b^2 + (4*Sinh[a + b*x])/(3*b^2*Sqrt[Cosh[a + b*x]])
```

Rubi [A] time = 0.0424529, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2639}

$$\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Cosh[a + b*x]^(3/2)) + (((4*I)/3)*EllipticE[(I/2)*(a + b*x), 2]
)/b^2 + (4*Sinh[a + b*x])/(3*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$
$$= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}} - \frac{2\int \sqrt{\cosh(a+bx)}\,dx}{3b}$$
$$= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2} + \frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}$$

Mathematica [A] time = 0.190923, size = 57, normalized size = 0.89

$$\frac{2\left(\sinh(2(a+bx)) + 2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) - bx\right)}{3b^2\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (2*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(3*b^2*Cosh[a + b*x]^(3/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\sinh(bx+a)(\cosh(bx+a))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

```
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

3.534 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=64

$$-\frac{4i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

```
[Out] (-2*x)/(5*b*Cosh[a + b*x]^(5/2)) - (((4*I)/15)*EllipticF[(I/2)*(a + b*x), 2
])/b^2 + (4*Sinh[a + b*x])/(15*b^2*Cosh[a + b*x]^(3/2))
```

Rubi [A] time = 0.042516, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2641}

$$-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (-2*x)/(5*b*Cosh[a + b*x]^(5/2)) - (((4*I)/15)*EllipticF[(I/2)*(a + b*x), 2
])/b^2 + (4*Sinh[a + b*x])/(15*b^2*Cosh[a + b*x]^(3/2))
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{15b}\\ &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.234105, size = 57, normalized size = 0.89

$$\frac{2\left(-2i\cosh^{\frac{5}{2}}(a+bx)\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(2(a+bx))-3bx\right)}{15b^2\cosh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (2*(-3*b*x - (2*I)*Cosh[a + b*x]^(5/2)*EllipticF[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(15*b^2*Cosh[a + b*x]^(5/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\sinh(bx+a)(\cosh(bx+a))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
```

[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="maxima")

[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

3.535 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)} dx$

Optimal. Leaf size=87

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

```
[Out] (-2*x)/(7*b*Cosh[a + b*x]^(7/2)) + (((12*I)/35)*EllipticE[(I/2)*(a + b*x),
2])/b^2 + (4*Sinh[a + b*x])/(35*b^2*Cosh[a + b*x]^(5/2)) + (12*Sinh[a + b*x
])/(35*b^2*Sqrt[Cosh[a + b*x]])
```

Rubi [A] time = 0.0534982, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2639}

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Cosh[a + b*x]^(7/2)) + (((12*I)/35)*EllipticE[(I/2)*(a + b*x),
2])/b^2 + (4*Sinh[a + b*x])/(35*b^2*Cosh[a + b*x]^(5/2)) + (12*Sinh[a + b*x
])/(35*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx &= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{35b} \\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}} - \frac{6\int \sqrt{\cosh(a+bx)}\,dx}{35b} \\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.279004, size = 69, normalized size = 0.79

$$\frac{10\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + 24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) - 20bx}{70b^2\cosh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

```
[Out] (-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*(a + b*x), 2] + 10*Si
nh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(70*b^2*Cosh[a + b*x]^(7/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\sinh(bx+a)\left(\cosh(bx+a)\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

```
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

3.536 $\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=107

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{35b^2}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}-\frac{2x}{}$$

```
[Out] (((12*I)/35)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(7/2))/(7*b) + (12*Sqrt[Sech[a + b*x]]*Si
nh[a + b*x])/(35*b^2) + (4*Sech[a + b*x]^(5/2)*Sinh[a + b*x])/(35*b^2)
```

Rubi [A] time = 0.0649697, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2639}

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{35b^2}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}-\frac{2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```



```
[Out] (((12*I)/35)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(7/2))/(7*b) + (12*Sqrt[Sech[a + b*x]]*Si
nh[a + b*x])/(35*b^2) + (4*Sech[a + b*x]^(5/2)*Sinh[a + b*x])/(35*b^2)
```


Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int \operatorname{sech}^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}+\frac{6\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+b}{35b^2}$$
$$= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+b}{35b^2}$$
$$= \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{35b^2}-\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\operatorname{sec}}}{}$$

Mathematica [A] time = 0.326297, size = 69, normalized size = 0.64

$$\frac{\operatorname{sech}^{\frac{7}{2}}(a+bx)\left(10\sinh(2(a+bx))+3\sinh(4(a+bx))+24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)-20bx\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

```
[Out] (Sech[a + b*x]^(7/2)*(-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*
(a + b*x), 2] + 10*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)]))/(70*b^2)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x\left(\operatorname{sech}\left(bx+a\right)\right)^{\frac{9}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{sech}\left(bx+a\right)^{\frac{9}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(9/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{9}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

3.537 $\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=84

$$-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2}+\frac{4\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (((-4*I)/15)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(5/2))/(5*b) + (4*Sech[a + b*x]^(3/2)*Sin
h[a + b*x])/(15*b^2)
```

Rubi [A] time = 0.0522645, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2641}

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```


```
[Out] (((-4*I)/15)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(5/2))/(5*b) + (4*Sech[a + b*x]^(3/2)*Sin
h[a + b*x])/(15*b^2)
```


Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\mathrm{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx &= -\frac{2x\mathrm{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\mathrm{sech}^{\frac{5}{2}}(a+bx)\,dx}{5b}\\ &= -\frac{2x\mathrm{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}+\frac{2\int\sqrt{\mathrm{sech}(a+bx)}\,dx}{15b}\\ &= -\frac{2x\mathrm{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\mathrm{sech}(a+\right.}{15b}\\ &= -\frac{4i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\mathrm{sech}(a+bx)}}{15b^2}-\frac{2x\mathrm{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\mathrm{sech}^{\frac{3}{2}}}{}\end{aligned}$$

Mathematica [A] time = 0.240138, size = 65, normalized size = 0.77

$$-\frac{2\sqrt{\mathrm{sech}(a+bx)}\left(2i\sqrt{\cosh(a+bx)}\mathrm{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)-2\tanh(a+bx)+3bx\mathrm{sech}^2(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*Sqrt[Sech[a + b*x]]*((2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x
), 2] + 3*b*x*Sech[a + b*x]^2 - 2*Tanh[a + b*x]))/(15*b^2)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x\left(\mathrm{sech}\left(bx+a\right)\right)^{\frac{7}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{7}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(7/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

3.538 $\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=84

$$\frac{4\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{3b^2}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (((4*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
b*x]])/b^2 - (2*x*Sech[a + b*x]^(3/2))/(3*b) + (4*Sqrt[Sech[a + b*x]]*Sinh[
a + b*x])/(3*b^2)
```

Rubi [A] time = 0.0517026, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2639}

$$\frac{4\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{3b^2}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (((4*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
b*x]])/b^2 - (2*x*Sech[a + b*x]^(3/2))/(3*b) + (4*Sqrt[Sech[a + b*x]]*Sinh[
a + b*x])/(3*b^2)
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}-\frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{3b}\\ &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}-\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+\right.}{3b}\\ &= \frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b^2}-\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+4\sqrt{\operatorname{sech}(}\end{aligned}$$

Mathematica [A] time = 0.195932, size = 57, normalized size = 0.68

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\left(\sinh(2(a+bx))+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)-bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*
(a + b*x), 2] + Sinh[2*(a + b*x)]))/(3*b^2)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int x\left(\operatorname{sech}(bx+a)\right)^{\frac{5}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{5}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(5/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{5}{2}} \sinh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.539 $\int x\mathbf{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[Out] (-2*x*Sqrt[Sech[a + b*x]])/b - ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(
a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rubi [A] time = 0.0386573, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5444, 3771, 2641}

$$-\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*x*Sqrt[Sech[a + b*x]])/b - ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(
a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx &= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2\int\sqrt{\operatorname{sech}(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.137988, size = 46, normalized size = 0.81

$$-\frac{2\sqrt{\operatorname{sech}(a+bx)}\left(bx+2i\sqrt{\cosh(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*(b*x + (2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2])*Sqrt[Se
ch[a + b*x]])/b^2
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x\left(\operatorname{sech}(bx+a)\right)^{\frac{3}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{3}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(3/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{sech}(bx+a)^{\frac{3}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.540 $\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] (2*x)/(b*Sqrt[Sech[a + b*x]]) + ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*
(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rubi [A] time = 0.039545, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5444, 3771, 2639}

$$\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x)/(b*Sqrt[Sech[a + b*x]]) + ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*
(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} + \frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}$$

Mathematica [C] time = 1.07534, size = 100, normalized size = 1.75

$$\frac{\sqrt{2}e^{-a-bx}\sqrt{\frac{e^{a+bx}}{e^{2(a+bx)}+1}}\left(4\sqrt{e^{2(a+bx)}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+(bx-2)\left(e^{2(a+bx)}+1\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(1 + E^(2*(a + b*x)))]*((1 + E^(2*(a
 + b*x)))*(-2 + b*x) + 4*Sqrt[1 + E^(2*(a + b*x))]*Hypergeometric2F1[-1/4,
1/2, 3/4, -E^(2*(a + b*x))]))/b^2
```

Maple [B] time = 0.053, size = 250, normalized size = 4.4

$$\frac{(bx-2)\left(\left(e^{bx+a}\right)^2+1\right)\sqrt{2}}{b^2e^{bx+a}}\sqrt{\frac{e^{bx+a}}{\left(e^{bx+a}\right)^2+1}} - 2\frac{\sqrt{2}\sqrt{\left(\left(e^{bx+a}\right)^2+1\right)e^{bx+a}}}{b^2e^{bx+a}}\left(-2\frac{\left(e^{bx+a}\right)^2+1}{\sqrt{\left(\left(e^{bx+a}\right)^2+1\right)e^{bx+a}}} + \frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{sech}(bx+a)}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(1/2)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sqrt(sech(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{sech}(bx+a)}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

3.541 $\int \frac{x \sinh(a+bx)}{\sqrt{\mathbf{sech}(a+bx)}}\,dx$

Optimal. Leaf size=84

$$\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\mathrm{sech}(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}-\frac{4\sinh(a+bx)}{9b^2\sqrt{\mathrm{sech}(a+bx)}}+\frac{2x}{3b\,\mathrm{sech}^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*x)/(3*b*Sech[a + b*x]^(3/2)) + (((4*I)/9)*Sqrt[Cosh[a + b*x]]*EllipticF[
(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(9*b^2*Sqr
t[Sech[a + b*x]])
```

Rubi [A] time = 0.0520962, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2641}

$$-\frac{4\sinh(a+bx)}{9b^2\sqrt{\mathrm{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\mathrm{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}+\frac{2x}{3b\,\mathrm{sech}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Sech[a + b*x]^(3/2)) + (((4*I)/9)*Sqrt[Cosh[a + b*x]]*EllipticF[
(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(9*b^2*Sqr
t[Sech[a + b*x]])
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx &= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}-\frac{2\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{9b}\\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}-\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{9b}\\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{4i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{9b^2}-\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.159412, size = 71, normalized size = 0.85

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(4i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)-2\sinh(2(a+bx))+3bx\cosh(2(a+bx))+3bx\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*(3*b*x + 3*b*x*Cosh[2*(a + b*x)] + (4*I)*Sqrt[Cosh[a +
 b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 2*Sinh[2*(a + b*x)]))/(9*b^2)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x \sinh (bx + a) \frac{1}{\sqrt{\operatorname{sech} (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh (bx + a)}{\sqrt{\operatorname{sech} (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh (a + bx)}{\sqrt{\operatorname{sech} (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)/sqrt(sech(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

3.542 $\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}+\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}$$

```
[Out] (2*x)/(5*b*Sech[a + b*x]^(5/2)) + (((12*I)/25)*Sqrt[Cosh[a + b*x]]*Elliptic
E[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(25*b^2*
Sech[a + b*x]^(3/2))
```

Rubi [A] time = 0.0534484, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2639}

$$-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}+\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Sech[a + b*x]^(5/2)) + (((12*I)/25)*Sqrt[Cosh[a + b*x]]*Elliptic
E[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(25*b^2*
Sech[a + b*x]^(3/2))
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{25b}\\ &= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{\left(6\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{25b}\\ &= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}+\frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{25b^2}-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [C] time = 2.13397, size = 125, normalized size = 1.49

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{e^{2(a+bx)}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+\left(e^{2(a+bx)}+1\right)\left(2(5bx-12)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5bx+\right.\right.}{100b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4
*(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hy
```

```
pergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])*Sqrt[Sech[a + b*x]])/(10
0*b^2*E^(3*(a + b*x)))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x \sinh(bx+a) (\operatorname{sech}(bx+a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)/sech(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

3.543 $\int \frac{x\sinh(a+bx)}{\mathbf{sech}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=107

$$\frac{20i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2}-\frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)}$$

```
[Out] (2*x)/(7*b*Sech[a + b*x]^(7/2)) + (((20*I)/147)*Sqrt[Cosh[a + b*x]]*Ellipti
cF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(49*b^2
*Sech[a + b*x]^(5/2)) - (20*Sinh[a + b*x])/(147*b^2*Sqrt[Sech[a + b*x]])
```

Rubi [A] time = 0.069102, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2641}

$$-\frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{20i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}+\frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Sech[a + b*x]^(7/2)) + (((20*I)/147)*Sqrt[Cosh[a + b*x]]*Ellipti
cF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(49*b^2
*Sech[a + b*x]^(5/2)) - (20*Sinh[a + b*x])/(147*b^2*Sqrt[Sech[a + b*x]])
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{10\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(10\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+\right.}{147b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} + \frac{20i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{147b^2} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} -$$

Mathematica [A] time = 0.273311, size = 93, normalized size = 0.87

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(80i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right) - 52\sinh(2(a+bx)) - 6\sinh(4(a+bx)) + 84bx\cosh(2(a+\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*(63*b*x + 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] + (80*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 52*Sin
h[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int x \sinh (bx + a) (\operatorname{sech} (bx + a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh (bx + a)}{\operatorname{sech} (bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

3.544 $\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=121

$$\frac{20i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{147b^2\sqrt{\sinh(a+bx)}}-\frac{4\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{49b^2}+\frac{20\sqrt{\sinh(a+bx)}\cosh(a+bx)}{147b^2}+\frac{2...}{...}$$

```
[Out] (((20*I)/147)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b^2*Sqrt[Sinh[a + b*x]]) + (20*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(147*b^2)
 - (4*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(49*b^2) + (2*x*Sinh[a + b*x]^(7/2
))/(7*b)
```

Rubi [A] time = 0.0702647, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2642, 2641}

$$-\frac{4\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{49b^2}+\frac{20\sqrt{\sinh(a+bx)}\cosh(a+bx)}{147b^2}+\frac{20i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{7}{2}}...}{7...}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((20*I)/147)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b^2*Sqrt[Sinh[a + b*x]]) + (20*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(147*b^2)
 - (4*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(49*b^2) + (2*x*Sinh[a + b*x]^(7/2
))/(7*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx &= \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sinh^{\frac{7}{2}}(a+bx)\,dx}{7b}\\ &= -\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}+\frac{10\int\sinh^{\frac{3}{2}}(a+bx)\,dx}{49b}\\ &= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2}-\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sinh^{\frac{7}{2}}(a+b}{7b}\\ &= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2}-\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sinh^{\frac{7}{2}}(a+b}{7b}\\ &= \frac{20iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{147b^2\sqrt{\sinh(a+bx)}}+\frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2}-\frac{4c}{}\end{aligned}$$

Mathematica [A] time = 0.310207, size = 103, normalized size = 0.85

$$\frac{-80i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)+52\sinh(2(a+bx))-6\sinh(4(a+bx))-84bx\cosh(2(a+bx))}{588b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

```
[Out] (63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a + b*x)] - (80*I)*Elli
pticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + 52*Sinh[2*(
```

```
a + b*x)] - 6*Sinh[4*(a + b*x)])/(588*b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)

[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(5/2), x)
```

3.545 $\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{4\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{25b^2}-\frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}}+\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (((-12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^
2*Sqrt[I*Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(25*b^2) +
 (2*x*Sinh[a + b*x]^(5/2))/(5*b)
```

Rubi [A] time = 0.0524617, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2640, 2639}

$$-\frac{4\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{25b^2}-\frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}}+\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((-12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^
2*Sqrt[I*Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(25*b^2) +
 (2*x*Sinh[a + b*x]^(5/2))/(5*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int\sinh^{\frac{5}{2}}(a+bx)\,dx}{5b}$$

$$= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} + \frac{6\int\sqrt{\sinh(a+bx)}\,dx}{25b}$$

$$= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} + \frac{\left(6\sqrt{\sinh(a+bx)}\right)\int\sqrt{i\sin}}{25b\sqrt{i\sinh(a+b}}$$

$$= -\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{25b^2\sqrt{i\sinh(a+bx)}} - \frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\,\mathrm{s}}{}$$

Mathematica [C] time = 2.1644, size = 143, normalized size = 1.46

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{1-e^{2(a+bx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2(a+bx)}\right)+\left(e^{2(a+bx)}-1\right)\left((24-10bx)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5bx+2\right.\right.}{50\sqrt{2}b^2\sqrt{e^{a+bx}-e^{-a-bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

```
[Out] ((-1 + E^(2*(a + b*x)))*(2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(
a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 - E^(2*(a + b*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a + b
*x))*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

3.546 $\int x \cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{9b^2\sqrt{\sinh(a+bx)}}-\frac{4\sqrt{\sinh(a+bx)}\cosh(a+bx)}{9b^2}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (((-4*I)/9)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b^
2*Sqrt[Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(9*b^2) + (2
*x*Sinh[a + b*x]^(3/2))/(3*b)
```

Rubi [A] time = 0.0530854, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2642, 2641}

$$-\frac{4\sqrt{\sinh(a+bx)}\cosh(a+bx)}{9b^2}-\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (((-4*I)/9)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b^
2*Sqrt[Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(9*b^2) + (2
*x*Sinh[a + b*x]^(3/2))/(3*b)
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx &= \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sinh^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{9b}\\ &= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}+\frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sin}}}{9b\sqrt{\sinh(a+bx)}}\\ &= -\frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{9b^2\sqrt{\sinh(a+bx)}}-\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2}+\frac{2x}{}\end{aligned}$$

Mathematica [A] time = 0.199701, size = 77, normalized size = 0.79

$$\frac{2\left(2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)+3bx\sinh^2(a+bx)-\sinh(2(a+bx))\right)}{9b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*((2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]
 + 3*b*x*Sinh[a + b*x]^2 - Sinh[2*(a + b*x)]))/(9*b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\sqrt{\sinh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

```
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sinh\left(a+bx\right)}\cosh\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(sinh(a + b*x))*cosh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh (bx + a) \sqrt{\sinh (bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

3.547 $\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=71

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

```
[Out] (2*x*Sqrt[Sinh[a + b*x]])/b + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*S
qrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.03856, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 2640, 2639}

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```


```
[Out] (2*x*Sqrt[Sinh[a + b*x]])/b + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*S
qrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a + b*x]])
```


Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{2\int\sqrt{\sinh(a+bx)}\,dx}{b}$$
$$= \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{\left(2\sqrt{\sinh(a+bx)}\right)\int\sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{i\sinh(a+bx)}}$$
$$= \frac{2x\sqrt{\sinh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b^2\sqrt{i\sinh(a+bx)}}$$

Mathematica [C] time = 1.69049, size = 182, normalized size = 2.56

$$\frac{e^{-a-bx}\sqrt{2-2e^{2(a+bx)}}\left(-18\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{3}{4},\frac{3}{4}\right\},e^{2(a+bx)}\right)-2e^{2(a+bx)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\right.\right.\right.}{9b^2\sqrt{e^{a+bx}-e^{-a-bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (E^(-a - b*x)*Sqrt[2 - 2*E^(2*(a + b*x))]*(-3*b*x*(3*Hypergeometric2F1[-1/4
, 1/2, 3/4, E^(2*(a + b*x))] - E^(2*(a + b*x))*Hypergeometric2F1[1/2, 3/4,
7/4, E^(2*(a + b*x))]) - 18*HypergeometricPFQ[{-1/4, -1/4, 1/2}, {3/4, 3/4}
, E^(2*(a + b*x))] - 2*E^(2*(a + b*x))*HypergeometricPFQ[{1/2, 3/4, 3/4}, {
7/4, 7/4}, E^(2*(a + b*x))]))/(9*b^2*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

Maple [B] time = 0.062, size = 229, normalized size = 3.2

$$\frac{(bx-2)\left(\left(e^{bx+a}\right)^2-1\right)\sqrt{2}}{b^2e^{bx+a}}\frac{1}{\sqrt{\frac{\left(e^{bx+a}\right)^2-1}{e^{bx+a}}}}+2\frac{\sqrt{2}\sqrt{\left(\left(e^{bx+a}\right)^2-1\right)e^{bx+a}}}{b^2e^{bx+a}}\left(2\frac{\left(e^{bx+a}\right)^2-1}{\sqrt{\left(\left(e^{bx+a}\right)^2-1\right)e^{bx+a}}}-\frac{\sqrt{1+e^{bx+a}}\sqrt{2-2\,e^{bx+}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)-(1+e
xp(b*x+a))^(1/2)*(2-2*exp(b*x+a))^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-e
xp(b*x+a))^(1/2)*(-2*EllipticE((1+exp(b*x+a))^(1/2),1/2*2^(1/2))+EllipticF(
(1+exp(b*x+a))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(
1/2)*((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)/exp(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sqrt(sinh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh (bx + a)}{\sqrt{\sinh (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

3.548 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=71

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

```
[Out] (-2*x)/(b*Sqrt[Sinh[a + b*x]]) - ((4*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2
]*Sqrt[I*Sinh[a + b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rubi [A] time = 0.0395053, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 2642, 2641}

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Sinh[a + b*x]]) - ((4*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2
]*Sqrt[I*Sinh[a + b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\sinh(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sinh(a+bx)}} + \frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2x}{b\sqrt{\sinh(a+bx)}} - \frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

Mathematica [A] time = 0.164629, size = 56, normalized size = 0.79

$$\frac{-2bx+4i\sqrt{i\sinh(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*b*x + (4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a +
 b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\left(\sinh(bx+a)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sinh(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

3.549 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[Out] (-2*x)/(3*b*Sinh[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(3*b^2*Sqrt[Sinh[a + b
*x]]) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]]
)/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0516689, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2640, 2639}

$$-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Sinh[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(3*b^2*Sqrt[Sinh[a + b
*x]]) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]]
)/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{3b} \\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} + \frac{2\int \sqrt{\sinh(a+bx)}\,dx}{3b} \\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} + \frac{\left(2\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{3b\sqrt{i\sinh(a+bx)}} \\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} - \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{3b^2\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.203667, size = 66, normalized size = 0.67

$$-\frac{2\left(\sinh(2(a+bx))+2i(i\sinh(a+bx))^{3/2}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)+bx\right)}{3b^2\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*(b*x + (2*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a + b*
x])^(3/2) + Sinh[2*(a + b*x)]))/(3*b^2*Sinh[a + b*x]^(3/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

3.550 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$\frac{4i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{15b^2\sqrt{\sinh(a+bx)}}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

```
[Out] (-2*x)/(5*b*Sinh[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(15*b^2*Sinh[a + b*x]^
(3/2)) + (((4*I)/15)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b
*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rubi [A] time = 0.0530723, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2642, 2641}

$$-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}+\frac{4i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2\sqrt{\sinh(a+bx)}}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```


```
[Out] (-2*x)/(5*b*Sinh[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(15*b^2*Sinh[a + b*x]^
(3/2)) + (((4*I)/15)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b
*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```


Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{15b}\\ &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}-\frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{15b\sqrt{\sinh(a+bx)}}\\ &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}+\frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{15b^2\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.270612, size = 67, normalized size = 0.68

$$-\frac{2\left(-2i(i\sinh(a+bx))^{5/2}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)+\sinh(2(a+bx))+3bx\right)}{15b^2\sinh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*(3*b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a +
b*x])^(5/2) + Sinh[2*(a + b*x)]))/(15*b^2*Sinh[a + b*x]^(5/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x \cosh (bx + a) (\sinh (bx + a))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh (bx + a)}{\sinh (bx + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

3.551 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=121

$$-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

```
[Out] (-2*x)/(7*b*Sinh[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(35*b^2*Sinh[a + b*x]^
(5/2)) + (12*Cosh[a + b*x])/(35*b^2*Sqrt[Sinh[a + b*x]]) + (((12*I)/35)*Ell
ipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a +
 b*x]])
```

Rubi [A] time = 0.065694, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2640, 2639}

$$-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Sinh[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(35*b^2*Sinh[a + b*x]^
(5/2)) + (12*Cosh[a + b*x])/(35*b^2*Sqrt[Sinh[a + b*x]]) + (((12*I)/35)*Ell
ipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a +
 b*x]])
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx &= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx}{7b}\\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}-\frac{6\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{35b}\\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}-\frac{6\int \sqrt{\sinh(a+bx)}\,dx}{35b}\\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}-\frac{\left(6\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(}}{35b\sqrt{i\sinh(a+bx}}\\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{}}{35b^2\sqrt{i\sinh(a+b}}
\end{aligned}
$$

Mathematica [A] time = 0.272986, size = 89, normalized size = 0.74

$$
-\frac{2\left(\sinh(2(a+bx))-6\sinh^3(a+bx)\cosh(a+bx)+6\sqrt{i\sinh(a+bx)}\sinh^3(a+bx)E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)+5bx\right)}{35b^2\sinh^{\frac{7}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

```
[Out] (-2*(5*b*x - 6*Cosh[a + b*x]*Sinh[a + b*x]^3 + 6*EllipticE[((-2*I)*a + Pi -
 (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]*Sinh[a + b*x]^3 + Sinh[2*(a + b*x)]
))/(35*b^2*Sinh[a + b*x]^(7/2))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\left(\sinh\left(bx+a\right)\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

3.552 $\int x \cosh(a+bx)\mathbf{csch}^{\frac{9}{2}}(a+bx)\,dx$

Optimal. Leaf size=121

$$-\frac{4\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{35b^2}+\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}-\frac{2x\text{csch}^{\frac{7}{2}}(a+b}{7b}$$

```
[Out] (12*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(35*b^2) - (4*Cosh[a + b*x]*Csch[a +
 b*x]^(5/2))/(35*b^2) - (2*x*Csch[a + b*x]^(7/2))/(7*b) + (((12*I)/35)*Elli
pticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a +
b*x]])
```

Rubi [A] time = 0.0661242, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2639}

$$-\frac{4\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{35b^2}+\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}-\frac{2x\text{csch}^{\frac{7}{2}}(a+b}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (12*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(35*b^2) - (4*Cosh[a + b*x]*Csch[a +
 b*x]^(5/2))/(35*b^2) - (2*x*Csch[a + b*x]^(7/2))/(7*b) + (((12*I)/35)*Elli
pticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a +
b*x]])
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx &= -\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx}{7b}\\
&= -\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}-\frac{6\int\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx}{35b}\\
&= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}\\
&= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+b\\
&= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+b
\end{aligned}
$$

Mathematica [A] time = 0.514715, size = 83, normalized size = 0.69

$$
-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(-6\cosh(a+bx)+(\sinh(2(a+bx))+5bx)\operatorname{csch}^3(a+bx)+6\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\right|}{35b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(-6*Cosh[a + b*x] + 6*EllipticE[((-2*I)*a + Pi - (2
*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Csch[a + b*x]^3*(5*b*x + Sinh[2*(a +
```

```
b*x)]))))/(35*b^2)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\left(\operatorname{csch}(bx+a)\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

3.553 $\int x\cosh(a+bx)\mathbf{csch}^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{15b^2}-\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-4*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(15*b^2) - (2*x*Csch[a + b*x]^(5/2))
/(5*b) + (((4*I)/15)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2,
2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rubi [A] time = 0.0532232, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2641}

$$-\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(15*b^2) - (2*x*Csch[a + b*x]^(5/2))
/(5*b) + (((4*I)/15)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2,
2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx &= -\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx}{5b}\\ &= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sqrt{\text{csch}(a+bx)}\,dx}{15b}\\ &= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{\left(2\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a\cdots}{15b}\\ &= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4i\sqrt{\text{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\cdots}{15\cdots}\end{aligned}$$

Mathematica [A] time = 0.297586, size = 75, normalized size = 0.77

$$-\frac{2\sqrt{\text{csch}(a+bx)}\left(2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)+2\coth(a+bx)+3bx\text{csch}^2(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Coth[a + b*x] + 3*b*x*Csch[a + b*x]^2 + (2*I)*El
lipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(15*b^2)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)(\text{csch}(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

3.554 $\int x\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (-4*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(3*b^2) - (2*x*Csch[a + b*x]^(3/2))/
(3*b) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a +
 b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0507373, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2639}

$$-\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(3*b^2) - (2*x*Csch[a + b*x]^(3/2))/
(3*b) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a +
 b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \text{csch}^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx}{3b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \sqrt{i\sinh(a+bx)}\,dx}{3b\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{3b^2\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [A] time = 0.207189, size = 70, normalized size = 0.71

$$-\frac{2\sqrt{\text{csch}(a+bx)}\left(2\cosh(a+bx)+bx\text{csch}(a+bx)-2\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Cosh[a + b*x] + b*x*Csch[a + b*x] - 2*EllipticE[
((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b^2)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\left(\operatorname{csch}\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

3.555 $\int x\cosh(a+bx)\mathbf{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=71

$$-\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2}$$

```
[Out] (-2*x*Sqrt[Csch[a + b*x]])/b - ((4*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rubi [A] time = 0.0416293, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5445, 3771, 2641}

$$-\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csch[a + b*x]])/b - ((4*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{2\int\sqrt{\operatorname{csch}(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\left(2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int\frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.145454, size = 56, normalized size = 0.79

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(bx-2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/
4, 2]*Sqrt[I*Sinh[a + b*x]]))/b^2
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\left(\operatorname{csch}(bx+a)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

3.556 $\int x \cosh(a+bx)\sqrt{\mathbf{csch}(a+bx)}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] (2*x)/(b*Sqrt[Csch[a + b*x]]) + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]
)/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0405722, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5445, 3771, 2639}

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csch[a + b*x]]) + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]
)/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} - \frac{2\int\frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} - \frac{2\int\sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$
$$= \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [C] time = 1.01549, size = 183, normalized size = 2.58

$$\frac{e^{-a-bx}\sqrt{2-2e^{2(a+bx)}}\sqrt{\frac{e^{a+bx}}{e^{2(a+bx)}-1}}\left(-18\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{3}{4},\frac{3}{4}\right\},e^{2(a+bx)}\right)-2e^{2(a+bx)}\text{Hypergeometric}\right.}{9b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

```
[Out] (E^(-a - b*x)*Sqrt[2 - 2*E^(2*(a + b*x))]*Sqrt[E^(a + b*x)/(-1 + E^(2*(a +
b*x)))]*(-3*b*x*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))] - E^(
2*(a + b*x))*Hypergeometric2F1[1/2, 3/4, 7/4, E^(2*(a + b*x))]) - 18*Hyperg
eometricPFQ[{-1/4, -1/4, 1/2}, {3/4, 3/4}, E^(2*(a + b*x))] - 2*E^(2*(a + b
*x))*HypergeometricPFQ[{1/2, 3/4, 3/4}, {7/4, 7/4}, E^(2*(a + b*x))]))/(9*b
^2)
```

Maple [B] time = 0.062, size = 229, normalized size = 3.2

$$\frac{(bx-2)\left(\left(e^{bx+a}\right)^2-1\right)\sqrt{2}}{b^2e^{bx+a}}\sqrt{\frac{e^{bx+a}}{\left(e^{bx+a}\right)^2-1}}+2\frac{\sqrt{2}\sqrt{\left(\left(e^{bx+a}\right)^2-1\right)e^{bx+a}}}{b^2e^{bx+a}}\left(2\frac{\left(e^{bx+a}\right)^2-1}{\sqrt{\left(\left(e^{bx+a}\right)^2-1\right)e^{bx+a}}}-\frac{\sqrt{1+e^{bx+a}}\sqrt{2-2e}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(1/2)/ex
p(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)-(1+e
xp(b*x+a))^(1/2)*(2-2*exp(b*x+a))^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-e
xp(b*x+a))^(1/2)*(-2*EllipticE((1+exp(b*x+a))^(1/2),1/2*2^(1/2))+EllipticF(
(1+exp(b*x+a))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(
1/2)*((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)/exp(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

3.557 $\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{9b^2}-\frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}}+\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*x)/(3*b*Csch[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(9*b^2*Sqrt[Csch[a + b*
x]]) - (((4*I)/9)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*
Sqrt[I*Sinh[a + b*x]])/b^2
```

Rubi [A] time = 0.0543573, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2641}

$$-\frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```


```
[Out] (2*x)/(3*b*Csch[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(9*b^2*Sqrt[Csch[a + b*
x]]) - (((4*I)/9)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*
Sqrt[I*Sinh[a + b*x]])/b^2
```


Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx &= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\ &= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{2\int \sqrt{\operatorname{csch}(a+bx)}\,dx}{9b}\\ &= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{\left(2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{9b}\\ &= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{4i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+b}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.413625, size = 67, normalized size = 0.68

$$\frac{-\frac{4i\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)}{(i\sinh(a+bx))^{3/2}} - 4\coth(a+bx) + 6bx}{9b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (6*b*x - 4*Coth[a + b*x] - ((4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4,
2])/(I*Sinh[a + b*x])^(3/2))/(9*b^2*Csch[a + b*x]^(3/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x\cosh\left(bx+a\right)\frac{1}{\sqrt{\operatorname{csch}\left(bx+a\right)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(bx+a\right)}{\sqrt{\operatorname{csch}\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(a+bx\right)}{\sqrt{\operatorname{csch}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sqrt(csch(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh (bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

3.558 $\int \frac{x\cosh(a+bx)}{\mathrm{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{25b^2\mathrm{csch}^{\frac{3}{2}}(a+bx)}-\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}\sqrt{\mathrm{csch}(a+bx)}}+\frac{2x}{5b\mathrm{csch}^{\frac{5}{2}}(a+bx)}$$

```
[Out] (2*x)/(5*b*Csch[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(25*b^2*Csch[a + b*x]^(
3/2)) - (((12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a
 + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0527631, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2639}

$$-\frac{4\cosh(a+bx)}{25b^2\mathrm{csch}^{\frac{3}{2}}(a+bx)}-\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}\sqrt{\mathrm{csch}(a+bx)}}+\frac{2x}{5b\mathrm{csch}^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csch[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(25*b^2*Csch[a + b*x]^(
3/2)) - (((12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a
 + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx}{5b} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{25b} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \sqrt{i\sinh(a+bx)}\,dx}{25b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{25b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 1.91286, size = 111, normalized size = 1.13

$$\frac{e^{-2(a+bx)}\left(-\frac{48e^{2(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2(a+bx)}\right)}{\sqrt{1-e^{2(a+bx)}}}+(24-10bx)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5bx+2\right)}{50b^2\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

```
[Out] (2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(a + b*x))*(-2 + 5*b*x) -
 (48*E^(2*(a + b*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/Sq
```

```
rt[1 - E^(2*(a + b*x))])/(50*b^2*E^(2*(a + b*x))*Sqrt[Csch[a + b*x]])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\cosh(bx+a)\left(\operatorname{csch}(bx+a)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

3.559 $\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=121

$$\frac{20i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{147b^2}-\frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)}+\frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}}+\frac{}{7b\operatorname{csch}}$$

```
[Out] (2*x)/(7*b*Csch[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(49*b^2*Csch[a + b*x]^(
5/2)) + (20*Cosh[a + b*x])/(147*b^2*Sqrt[Csch[a + b*x]]) + (((20*I)/147)*Sq
rt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]
])/b^2
```

Rubi [A] time = 0.0688958, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2641}

$$-\frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)}+\frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}}+\frac{20i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csch[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(49*b^2*Csch[a + b*x]^(
5/2)) + (20*Cosh[a + b*x])/(147*b^2*Sqrt[Csch[a + b*x]]) + (((20*I)/147)*Sq
rt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]
])/b^2
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{10\int \sqrt{\operatorname{csch}(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{\left(10\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a}\right.}{147b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{20i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\right.\right.}{147}$$

Mathematica [A] time = 0.396626, size = 103, normalized size = 0.85

$$\frac{\sqrt{\operatorname{csch}(a+bx)}\left(-80i\sqrt{i\sinh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)+52\sinh(2(a+bx))-6\sinh(4(a+bx))-84bx\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*(63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] - (80*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a
+ b*x]] + 52*Sinh[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x \cosh(bx+a) (\operatorname{csch}(bx+a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

3.560 $\int \sqrt{\sinh(x)\tanh(x)}\,dx$

Optimal. Leaf size=13

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Rubi [A] time = 0.0504159, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4398, 4400, 2589}

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[x]*Tanh[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sinh(x)\tanh(x)}\,dx &= \frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{-\sinh(x)\tanh(x)}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\ &= \frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= 2\coth(x)\sqrt{\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.0618208, size = 13, normalized size = 1.

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[x]*Tanh[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Maple [B] time = 0.145, size = 42, normalized size = 3.2

$$\frac{\sqrt{2}\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{\frac{\left(e^{2x}-1\right)^2 e^{-x}}{e^{2x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((exp(2*x)-1)^2*exp(-x)/(exp(2*x)+1))^(1/2)/(exp(2*x)-1)*(exp(2*x)+
1)
```

Maxima [B] time = 1.65304, size = 47, normalized size = 3.62

$$-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-2x)}+1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-2x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*e^(1/2*x)/sqrt(e^(-2*x) + 1) - sqrt(2)*e^(-3/2*x)/sqrt(e^(-2*x) +
1)
```

Fricas [B] time = 2.22741, size = 201, normalized size = 15.46

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)}{\sqrt{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)+\cosh\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh\left(x\right)\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)*tanh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh\left(x\right)\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)*tanh(x)), x)
```

3.561 $\int (\sinh(x)\tanh(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{8}{3}\mathrm{csch}(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] (8*Csch[x]*Sqrt[Sinh[x]*Tanh[x]])/3 + (2*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Rubi [A] time = 0.08793, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4398, 4400, 2598, 2589}

$$\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{8}{3}\mathrm{csch}(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]*Tanh[x])^(3/2),x]
```

```
[Out] (8*Csch[x]*Sqrt[Sinh[x]*Tanh[x]])/3 + (2*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
```

```
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int (\sinh(x)\tanh(x))^{3/2}\,dx &= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{3/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\ &= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{\left(4\sqrt{\sinh(x)\tanh(x)}\right)\int\frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= \frac{8}{3}\mathbf{csch}(x)\sqrt{\sinh(x)\tanh(x)}+\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.0673715, size = 23, normalized size = 0.74

$$\frac{2}{3}\sinh(x)\left(4\mathbf{csch}^2(x)+1\right)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]*Tanh[x])^(3/2),x]
```

```
[Out] (2*(1 + 4*Csch[x]^2)*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int(\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(3/2),x)
```

```
[Out] int((sinh(x)*tanh(x))^(3/2),x)
```

Maxima [B] time = 1.62995, size = 93, normalized size = 3.

$$-\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*e^(3/2*x)/(e^(-2*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(e^(-2
*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(e^(-2*x) + 1)^(3/2) - 1/6*sqrt(2)*
e^(-9/2*x)/(e^(-2*x) + 1)^(3/2)
```

Fricas [B] time = 2.30302, size = 348, normalized size = 11.23

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+7\right)\sinh(x)^2+14\cosh(x)^2+4\left(\cosh(x)^3+7\cosh(\right.\right.}{3\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2+1\right)\sinh(x)+\cosh(x)}(\cosh(x)+\sinh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x)
+ cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sinh(x)\tanh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)*tanh(x))^(3/2), x)
```

3.562 $\int (\sinh(x)\tanh(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] (-64*Coth[x]*Sqrt[Sinh[x]*Tanh[x]])/15 + (16*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])
/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/5
```

Rubi [A] time = 0.117818, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]*Tanh[x])^(5/2),x]
```

```
[Out] (-64*Coth[x]*Sqrt[Sinh[x]*Tanh[x]])/15 + (16*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])
/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/5
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int (\sinh(x)\tanh(x))^{5/2}\,dx &= \frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{5/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\
&= \frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{\left(8\sqrt{\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}(i\tanh(x))^{5/2}\,dx}{5\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)} - \frac{\left(32\sqrt{\sinh(x)\tanh\right.}{15\sqrt{i}}\\
&= -\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sin
\end{aligned}
$$

Mathematica [A] time = 0.113306, size = 29, normalized size = 0.58

$$
-\frac{2}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}\left(-3\cosh^2(x)+32\coth^2(x)-5\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]*Tanh[x])^(5/2),x]
```

```
[Out] (-2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/15
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (\sinh (x) \tanh (x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(5/2),x)
```

```
[Out] int((sinh(x)*tanh(x))^(5/2),x)
```

Maxima [B] time = 1.75192, size = 139, normalized size = 2.78

$$-\frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(2)*e^(5/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(e^(-2
*x) + 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(e^(-2*x) + 1)^(5/2) + 41/6*sqrt(2
)*e^(-7/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(e^(-2*x) + 1)^
(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(e^(-2*x) + 1)^(5/2)
```

Fricas [B] time = 2.39637, size = 883, normalized size = 17.66

$$\frac{\sqrt{\frac{1}{2}}\left(3\,\cosh (x)^{8}+24\,\cosh (x)\sinh (x)^{7}+3\,\sinh (x)^{8}+12\left(7\,\cosh (x)^{2}-9\right)\sinh (x)^{6}-108\,\cosh (x)^{6}+24\left(7\,\cosh (x)^{3}\right.}{30\left(\cosh (x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^
2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sinh(x)\tanh(x))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)*tanh(x))^(5/2), x)
```

3.563 $\int \sqrt{\cosh(x)\coth(x)}\,dx$

Optimal. Leaf size=13

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rubi [A] time = 0.0525148, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4398, 4400, 2589}

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[x]*Coth[x]],x]
```

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}$$
$$= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$
$$= 2\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

Mathematica [B] time = 0.0805042, size = 35, normalized size = 2.69

$$\frac{2\left(\sqrt[4]{-\sinh^2(x)}-1\right)\tanh(x)\sqrt{\cosh(x)\coth(x)}}{\sqrt[4]{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[x]*Coth[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

Maple [B] time = 0.142, size = 42, normalized size = 3.2

$$\frac{\sqrt{2}\left(e^{2x}-1\right)}{e^{2x}+1}\sqrt{\frac{\left(e^{2x}+1\right)^2 e^{-x}}{e^{2x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((exp(2*x)+1)^2*exp(-x)/(exp(2*x)-1))^(1/2)/(exp(2*x)+1)*(exp(2*x)-
1)
```

Maxima [B] time = 1.63417, size = 73, normalized size = 5.62

$$\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

Fricas [B] time = 2.4171, size = 201, normalized size = 15.46

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}{\sqrt{\cosh\left(x\right)^{3}+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\cosh\left(x\right)\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(x)*coth(x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh(x)\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(x)*coth(x)), x)
```

3.564 $\int (\cosh(x)\coth(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rubi [A] time = 0.0969951, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4398, 4400, 2598, 2589}

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Coth[x])^(3/2),x]
```

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (\cosh(x)\coth(x))^{3/2}\,dx &= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int(-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}+\frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\,\mathrm{sech}(x)
\end{aligned}$$

Mathematica [A] time = 0.0493426, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cosh^2(x)-4\right)\mathrm{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Coth[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int(\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(3/2),x)

[Out] int((cosh(x)*coth(x))^(3/2),x)
```

Maxima [B] time = 1.66442, size = 147, normalized size = 4.74

$$\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

Fricas [B] time = 2.32693, size = 348, normalized size = 11.23

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-7\right)\sinh(x)^2-14\cosh(x)^2+4\left(\cosh(x)^3-7\cosh(\right.\right.}{3\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}(\cosh(x)+\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="fricas")

[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cosh(x)*coth(x))^(3/2), x)
```

3.565 $\int (\cosh(x)\coth(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rubi [A] time = 0.131647, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Coth[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = -\frac{\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$= -\frac{\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}(-i\coth(x))^{5/2}\,dx}{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(32\sqrt{\cosh(x)\mathrm{cot}\ldots}\right.}{15\sqrt{\ldots}}$$

$$= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\mathrm{cot}\ldots}$$

Mathematica [A] time = 0.311567, size = 44, normalized size = 0.88

$$\frac{1}{15}\sqrt{\cosh(x)\coth(x)}\left(64\tanh(x) - 10\coth(x) + 6\sinh(x)\cosh(x) + 57\left(-\sinh^2(x)\right)^{3/4}\mathbf{csch}(x)\mathbf{sech}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Coth[x])^(5/2),x]
```

```
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int (\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(5/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(5/2),x)
```

Maxima [B] time = 1.69129, size = 220, normalized size = 4.4

$$\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}-\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}-\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

Fricas [B] time = 2.46197, size = 883, normalized size = 17.66

$$\frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^{8}+24\cosh(x)\sinh(x)^{7}+3\sinh(x)^{8}+12\left(7\cosh(x)^{2}+9\right)\sinh(x)^{6}+108\cosh(x)^{6}+24\left(7\cosh(x)^{3}\right.\right.}{30\left(\cosh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((cosh(x)*coth(x))^(5/2), x)
```

3.566 $\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{\log(a+b\sinh(x))}{b}-\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] (-2*(b + c)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + L
og[a + b*Sinh[x]]/b
```

Rubi [A] time = 0.132436, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}-\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + L
og[a + b*Sinh[x]]/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{\left(1+\frac{b}{c}\right)c}{a+b\sinh(x)}+\frac{\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a+b\sinh(x)}\,dx+\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}+(2(b+c))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{\log(a+b\sinh(x))}{b}-(4(b+c))\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.106482, size = 60, normalized size = 1.15

$$
\frac{2(b+c)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(x))}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*(b + c)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + L
og[a + b*Sinh[x]]/b
```

Maple [B] time = 0.025, size = 120, normalized size = 2.3

$$-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)\,b-a\right)+2\frac{b}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2a\tanh(x/2)-2b}{\sqrt{a^2+b^2}}\right)+2\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2*b/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*c-1/b*ln(tanh(1/2*x)
-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62594, size = 458, normalized size = 8.81

$$\frac{\sqrt{a^2+b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2+b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)-b}\right)-\left(a^2+b^2\right)x}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*co
sh(x) + a)*sinh(x) - b)) - (a^2 + b^2)*x + (a^2 + b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [A] time = 94.0366, size = 770, normalized size = 14.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), ((b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*log
(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), (-2*I*b/(-b*tanh(x/2) + I*b
) - 2*I*c/(-b*tanh(x/2) + I*b) - x*tanh(x/2)/(-b*tanh(x/2) + I*b) + I*x/(-b
*tanh(x/2) + I*b) + 2*log(tanh(x/2) + 1)*tanh(x/2)/(-b*tanh(x/2) + I*b) - 2
*I*log(tanh(x/2) + 1)/(-b*tanh(x/2) + I*b) - 2*log(tanh(x/2) - I)*tanh(x/2)
/(-b*tanh(x/2) + I*b) + 2*I*log(tanh(x/2) - I)/(-b*tanh(x/2) + I*b), Eq(a,
-I*b)), (-2*I*b/(b*tanh(x/2) + I*b) - 2*I*c/(b*tanh(x/2) + I*b) + x*tanh(x/
2)/(b*tanh(x/2) + I*b) + I*x/(b*tanh(x/2) + I*b) - 2*log(tanh(x/2) + 1)*tan
h(x/2)/(b*tanh(x/2) + I*b) - 2*I*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) + 2
*log(tanh(x/2) + I)*tanh(x/2)/(b*tanh(x/2) + I*b) + 2*I*log(tanh(x/2) + I)/
(b*tanh(x/2) + I*b), Eq(a, I*b)), ((c*x + sinh(x))/a, Eq(b, 0)), (a**2*x/(a
**2*b + b**3) - 2*a**2*log(tanh(x/2) + 1)/(a**2*b + b**3) + a**2*log(tanh(x
/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + a**2*log(tanh(x/2) - b/a
 + sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + b**2*x/(a**2*b + b**3) - b**2*sqr
t(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) +
 b**2*sqrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*b
+ b**3) - 2*b**2*log(tanh(x/2) + 1)/(a**2*b + b**3) + b**2*log(tanh(x/2) -
b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + b**2*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/(a**2*b + b**3) - b*c*sqrt(a**2 + b**2)*log(tanh(x/2) - b
/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + b*c*sqrt(a**2 + b**2)*log(tanh(
x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*b + b**3), True))
```

Giac [A] time = 1.15197, size = 117, normalized size = 2.25

$$\frac{(b+c)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}-\frac{x}{b}+\frac{\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (b + c)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/sqrt(a^2 + b^2) - x/b + log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

3.567 $\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{2(b+c)\tanh^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}$$

```
[Out] (2*(b + c)*ArcTanh[(b + a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Lo
g[a - b*Sinh[x]]/b
```

Rubi [A] time = 0.132723, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(b + a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Lo
g[a - b*Sinh[x]]/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx &= \int \left(\frac{\left(1+\frac{b}{c}\right)c}{a-b\sinh(x)}+\frac{\cosh(x)}{a-b\sinh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a-b\sinh(x)}\,dx+\int \frac{\cosh(x)}{a-b\sinh(x)}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, -b\sinh(x)\right)}{b}+(2(b+c))\,\text{Subst}\left(\int \frac{1}{a-2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{\log(a-b\sinh(x))}{b}-(4(b+c))\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, -2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{2(b+c)\tanh^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.105536, size = 62, normalized size = 1.17

$$
-\frac{2(b+c)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{\log(b\sinh(x)-a)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[(b + a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] -
Log[-a + b*Sinh[x]]/b
```

Maple [B] time = 0.023, size = 119, normalized size = 2.3

$$\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh\left(x/2\right)b-a\right)+2\frac{b}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\,\mathrm{t}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+cosh(x))/(a-b*sinh(x)),x)
```

```
[Out] 1/b*ln(tanh(1/2*x)+1)+1/b*ln(tanh(1/2*x)-1)-1/b*ln(a*tanh(1/2*x)^2+2*tanh(1
/2*x)*b-a)+2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2+b^2)^
(1/2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2+b^2)^(1/2))
*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56922, size = 458, normalized size = 8.64

$$\frac{\sqrt{a^2+b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)-ab\right)\sinh(x)+2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)-b}\right)+\left(a^2+b^2\right.}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*co
sh(x) - a)*sinh(x) - b)) + (a^2 + b^2)*x - (a^2 + b^2)*log(2*(b*sinh(x) - a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [A] time = 94.6613, size = 772, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), (-(b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*lo
g(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), (2*I*b/(b*tanh(x/2) + I*b)
 + 2*I*c/(b*tanh(x/2) + I*b) - x*tanh(x/2)/(b*tanh(x/2) + I*b) - I*x/(b*tan
h(x/2) + I*b) + 2*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) + I*b) + 2*I*lo
g(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) - 2*log(tanh(x/2) + I)*tanh(x/2)/(b*ta
nh(x/2) + I*b) - 2*I*log(tanh(x/2) + I)/(b*tanh(x/2) + I*b), Eq(a, -I*b)),
(2*I*b/(-b*tanh(x/2) + I*b) + 2*I*c/(-b*tanh(x/2) + I*b) + x*tanh(x/2)/(-b*
tanh(x/2) + I*b) - I*x/(-b*tanh(x/2) + I*b) - 2*log(tanh(x/2) + 1)*tanh(x/2
)/(-b*tanh(x/2) + I*b) + 2*I*log(tanh(x/2) + 1)/(-b*tanh(x/2) + I*b) + 2*lo
g(tanh(x/2) - I)*tanh(x/2)/(-b*tanh(x/2) + I*b) - 2*I*log(tanh(x/2) - I)/(-
b*tanh(x/2) + I*b), Eq(a, I*b)), ((c*x + sinh(x))/a, Eq(b, 0)), (-a**2*x/(a
**2*b + b**3) + 2*a**2*log(tanh(x/2) + 1)/(a**2*b + b**3) - a**2*log(tanh(x
/2) + b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) - a**2*log(tanh(x/2) + b/a
 + sqrt(a**2 + b**2)/a)/(a**2*b + b**3) - b**2*x/(a**2*b + b**3) - b**2*sqr
t(a**2 + b**2)*log(tanh(x/2) + b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) +
 b**2*sqrt(a**2 + b**2)*log(tanh(x/2) + b/a + sqrt(a**2 + b**2)/a)/(a**2*b
+ b**3) + 2*b**2*log(tanh(x/2) + 1)/(a**2*b + b**3) - b**2*log(tanh(x/2) +
b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) - b**2*log(tanh(x/2) + b/a + sqr
t(a**2 + b**2)/a)/(a**2*b + b**3) - b*c*sqrt(a**2 + b**2)*log(tanh(x/2) + b
/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + b*c*sqrt(a**2 + b**2)*log(tanh(
x/2) + b/a + sqrt(a**2 + b**2)/a)/(a**2*b + b**3), True))
```

Giac [A] time = 1.16626, size = 119, normalized size = 2.25

$$-\frac{(b+c)\log\left(\frac{\left|2\,be^{x}-2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}-2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}+\frac{x}{b}-\frac{\log\left(\left|be^{(2\,x)}-2\,ae^{x}-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -(b + c)*log(abs(2*b*e^x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x - 2*a + 2*s
qrt(a^2 + b^2)))/sqrt(a^2 + b^2) + x/b - log(abs(b*e^(2*x) - 2*a*e^x - b))/
b
```

3.568 $\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]) + Log[a + b*Cosh[x]]/b
```

Rubi [A] time = 0.13272, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]) + Log[a + b*Cosh[x]]/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{b+c}{a+b\cosh(x)}+\frac{\sinh(x)}{a+b\cosh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{b}+(2(b+c))\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0940014, size = 56, normalized size = 0.98

$$
\frac{\log(a+b\cosh(x))}{b}-\frac{2(b+c)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
+ Log[a + b*Cosh[x]]/b
```

Maple [B] time = 0.018, size = 127, normalized size = 2.2

$$-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+2\frac{b}{\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2*b/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*c-1/b*ln(tanh(1/2*
x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52639, size = 717, normalized size = 12.58

$$\left[\frac{\sqrt{a^2-b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(a^2-b^2\right)}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*
```

```
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) + b)) - (a^2 - b^2)*x + (a^2 - b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*(b^2 + b*c)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^
2)*x - (a^2 - b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17219, size = 81, normalized size = 1.42

$$\frac{2\,(b+c)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}-\frac{x}{b}+\frac{\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*(b + c)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - x/b + log
(b*e^(2*x) + 2*a*e^x + b)/b
```

3.569 $\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$

Optimal. Leaf size=59

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a + b]*Tanh[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[
a + b]) - Log[a - b*Cosh[x]]/b
```

Rubi [A] time = 0.13998, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a + b]*Tanh[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[
a + b]) - Log[a - b*Cosh[x]]/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx &= \int \left(\frac{-b-c}{-a+b\cosh(x)}+\frac{\sinh(x)}{a-b\cosh(x)}\right)dx\\
&= (-b-c)\int \frac{1}{-a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a-b\cosh(x)}\,dx\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,-b\cosh(x)\right)}{b}-(2(b+c))\operatorname{Subst}\left(\int \frac{1}{-a+b-(-a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}
\end{aligned}$$

Mathematica [A] time = 0.0947193, size = 56, normalized size = 0.95

$$-\frac{2(b+c)\tan^{-1}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-\frac{\log(a-b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[((a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
- Log[a - b*Cosh[x]]/b
```

Maple [B] time = 0.021, size = 154, normalized size = 2.6

$$\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{a}{b(a+b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a+b\right)-\frac{1}{a+b}\ln\left(a\left(\tanh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+sinh(x))/(a-b*cosh(x)),x)
```

```
[Out] 1/b*ln(tanh(1/2*x)+1)+1/b*ln(tanh(1/2*x)-1)-1/b/(a+b)*ln(a*tanh(1/2*x)^2+ta
nh(1/2*x)^2*b-a+b)*a-1/(a+b)*ln(a*tanh(1/2*x)^2+tanh(1/2*x)^2*b-a+b)+2*b/((
a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/((a+b)*(
a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49708, size = 716, normalized size = 12.14

$$\left[\frac{\sqrt{a^2-b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)-ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)+b}\right)+\left(a^2-b^2\right)\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*
```

```
cosh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*c
osh(x) - a)*sinh(x) + b)) + (a^2 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x) -
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*(b^2 + b*c)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) - a)/(a^2 - b^2)) + (a^2 - b^2
)*x - (a^2 - b^2)*log(2*(b*cosh(x) - a)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15017, size = 84, normalized size = 1.42

$$-\frac{2\,(b+c)\arctan\left(\frac{be^{x}-a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{x}{b}-\frac{\log\left(be^{(2\,x)}-2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*(b + c)*arctan((b*e^x - a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) + x/b - lo
g(b*e^(2*x) - 2*a*e^x + b)/b
```

3.570 $\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Rubi [A] time = 0.057565, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5636, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Rule 5636

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sinh[(c_.) + (d_.)*(x_)]))/((a_) +
(b_.)*Sinh[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(B*(e + f*x)*Cosh[c +
d*x])/(a*d*(a + b*Sinh[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Cosh[c + d*x]
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ
[a*A + b*B, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx &= -\frac{x\cosh(x)}{a+b\sinh(x)}+\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\ &= -\frac{x\cosh(x)}{a+b\sinh(x)}+\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.168827, size = 25, normalized size = 1.

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Maple [B] time = 0.175, size = 58, normalized size = 2.3

$$-2\,\frac{x}{b}+2\,\frac{x\left(a{e^{x}}-b\right)}{b\left(b{e^{2\,x}}+2\,a{e^{x}}-b\right)}+\frac{1}{b}\ln\left({e^{2\,x}}+2\,\frac{a{e^{x}}}{b}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x)
```

```
[Out] -2*x/b+2*x*(a*exp(x)-b)/b/(b*exp(2*x)+2*a*exp(x)-b)+1/b*ln(exp(2*x)+2/b*a*e
xp(x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4049, size = 396, normalized size = 15.84

$$-\frac{2\,bx\cosh\left(x\right)^{2}+2\,bx\sinh\left(x\right)^{2}+2\,ax\cosh\left(x\right)-\left(b\cosh\left(x\right)^{2}+b\sinh\left(x\right)^{2}+2\,a\cosh\left(x\right)+2\,\left(b\cosh\left(x\right)+a\right)\sinh\left(x\right)-b}{b^{2}\cosh\left(x\right)^{2}+b^{2}\sinh\left(x\right)^{2}+2\,ab\cosh\left(x\right)-b^{2}+2\,\left(b^{2}\cosh\left(x\right)+ab\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*(b*sinh(x) + a)/
(cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^
2*sinh(x)^2 + 2*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19389, size = 130, normalized size = 5.2

$$-\frac{2\,bxe^{(2\,x)}-be^{(2\,x)}\log\left(-be^{(2\,x)}-2\,ae^{x}+b\right)-2\,ae^{x}\log\left(-be^{(2\,x)}-2\,ae^{x}+b\right)+2\,bx+b\log\left(-be^{(2\,x)}-2\,ae^{x}+b\right)}{b^{2}e^{(2\,x)}+2\,abe^{x}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x + b) - 2*a*e^x*log(-b*
e^(2*x) - 2*a*e^x + b) + 2*b*x + b*log(-b*e^(2*x) - 2*a*e^x + b))/(b^2*e^(2
*x) + 2*a*b*e^x - b^2)
```

3.571 $\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2} \, dx$

Optimal. Leaf size=25

$$\frac{x\sinh(x)}{a+b\cosh(x)} - \frac{\log(a+b\cosh(x))}{b}$$

```
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Rubi [A] time = 0.0579455, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5637, 2668, 31}

$$\frac{x\sinh(x)}{a+b\cosh(x)} - \frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]

[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Rule 5637

```
Int[((Cosh[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cosh[(c
_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[(B*(e + f*x)*Sinh[c +
d*x])/(a*d*(a + b*Cosh[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Sinh[c + d*x]
/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ
[a*A - b*B, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx &= \frac{x\sinh(x)}{a+b\cosh(x)}-\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\ &= \frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{b}\\ &= -\frac{\log(a+b\cosh(x))}{b}+\frac{x\sinh(x)}{a+b\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.122942, size = 25, normalized size = 1.

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
```

```
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Maple [B] time = 0.062, size = 55, normalized size = 2.2

$$2\,\frac{x}{b}-2\,\frac{x\,(a{\rm e}^{x}+b)}{b\left(b{\rm e}^{2\,x}+2\,a{\rm e}^{x}+b\right)}-\frac{1}{b}\ln\left({\rm e}^{2\,x}+2\,\frac{a{\rm e}^{x}}{b}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x)
```

```
[Out] 2*x/b-2*x*(a*exp(x)+b)/b/(b*exp(2*x)+2*a*exp(x)+b)-1/b*ln(exp(2*x)+2/b*a*ex
p(x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43145, size = 394, normalized size = 15.76

$$\frac{2\,bx\cosh(x)^2+2\,bx\sinh(x)^2+2\,ax\cosh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b\right)}{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+b^2+2\left(b^2\cosh(x)+ab\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*log(2*(b*cosh(x) + a)/(
cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21412, size = 135, normalized size = 5.4

$$\frac{2\,bxe^{(2\,x)}-be^{(2\,x)}\log\left(-be^{(2\,x)}-2\,ae^{x}-b\right)-2\,ae^{x}\log\left(-be^{(2\,x)}-2\,ae^{x}-b\right)-2\,bx-b\log\left(-be^{(2\,x)}-2\,ae^{x}-b\right)}{b^2e^{(2\,x)}+2\,abe^{x}+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] (2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x - b) - 2*a*e^x*log(-b*e
^(2*x) - 2*a*e^x - b) - 2*b*x - b*log(-b*e^(2*x) - 2*a*e^x - b))/(b^2*e^(2*
x) + 2*a*b*e^x + b^2)
```

3.572 $\int \frac{a+b\,\mathbf{sech}(x)}{c+d\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{b\tan^{-1}(\sinh(x))}{c}$$

```
[Out] (b*ArcTan[Sinh[x]])/c + (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tanh[x/2])/Sqrt
[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d])
```

Rubi [A] time = 0.156827, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2828, 3001, 3770, 2659, 208}

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{b\tan^{-1}(\sinh(x))}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/c + (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tanh[x/2])/Sqrt
[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d])
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\,\mathrm{sech}(x)}{c+d\cosh(x)}\,dx &= \int \frac{(b+a\cosh(x))\mathrm{sech}(x)}{c+d\cosh(x)}\,dx\\ &= \frac{b\int \mathrm{sech}(x)\,dx}{c}+\frac{(ac-bd)\int \frac{1}{c+d\cosh(x)}\,dx}{c}\\ &= \frac{b\tan^{-1}(\sinh(x))}{c}+\frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{c+d-(c-d)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}\\ &= \frac{b\tan^{-1}(\sinh(x))}{c}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}\end{aligned}$$

Mathematica [A] time = 0.124403, size = 63, normalized size = 1.02

$$\frac{2\left(\frac{(bd-ac)\tan^{-1}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}}+b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

```
[Out] (2*(b*ArcTan[Tanh[x/2]] + ((-(a*c) + b*d)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-
c^2 + d^2]])/Sqrt[-c^2 + d^2]))/c
```

Maple [A] time = 0.04, size = 89, normalized size = 1.4

$$2\,\frac{b\arctan\left(\tanh\left(x/2\right)\right)}{c}+2\,\frac{a}{\sqrt{\left(c+d\right)\left(c-d\right)}}\mathrm{Artanh}\left(\frac{\left(c-d\right)\tanh\left(x/2\right)}{\sqrt{\left(c+d\right)\left(c-d\right)}}\right)-2\,\frac{bd}{c\sqrt{\left(c+d\right)\left(c-d\right)}}\mathrm{Artanh}\left(\frac{\left(c-d\right)\tanh\left(x/2\right.}{\sqrt{\left(c+d\right)\left(c-d\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x))/(c+d*cosh(x)),x)
```

```
[Out] 2*b/c*arctan(tanh(1/2*x))+2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/(
(c+d)*(c-d))^(1/2))*a-2/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c
+d)*(c-d))^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.78853, size = 643, normalized size = 10.37

$$\left[-\frac{(ac-bd)\sqrt{c^2-d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2-d^2+2\left(d^2\cosh(x)+cd\right)\sinh(x)+2\,\sqrt{c^2-d^2}(d\cosh(x)+d\sinh(x)+c)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)+d}\right)-2\left(bc^2-\right.}{c^3-cd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((a*c - b*d)*sqrt(c^2 - d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*c
osh(x) + 2*c^2 - d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 - d^2)*(d
*cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*
cosh(x) + c)*sinh(x) + d)) - 2*(b*c^2 - b*d^2)*arctan(cosh(x) + sinh(x)))/(
c^3 - c*d^2), -2*((a*c - b*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c)/(c^2 - d^2)) - (b*c^2 - b*d^2)*arctan(cosh(x) + si
nh(x)))/(c^3 - c*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x))/(c + d*cosh(x)), x)
```

Giac [A] time = 1.15093, size = 72, normalized size = 1.16

$$\frac{2\,b\arctan\left(e^{x}\right)}{c}+\frac{2\,(ac-bd)\arctan\left(\frac{de^{x}+c}{\sqrt{-c^{2}+d^{2}}}\right)}{\sqrt{-c^{2}+d^{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/c + 2*(a*c - b*d)*arctan((d*e^x + c)/sqrt(-c^2 + d^2))/(sqr
t(-c^2 + d^2)*c)
```

3.573 $\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}-\frac{b\tanh^{-1}(\cosh(x))}{c}$$

```
[Out] -((b*ArcTanh[Cosh[x]])/c) - (2*(a*c - b*d)*ArcTanh[(d - c*Tanh[x/2])/Sqrt[c
^2 + d^2]])/(c*Sqrt[c^2 + d^2])
```

Rubi [A] time = 0.167788, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2828, 3001, 3770, 2660, 618, 206}

$$-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}-\frac{b\tanh^{-1}(\cosh(x))}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

```
[Out] -((b*ArcTanh[Cosh[x]])/c) - (2*(a*c - b*d)*ArcTanh[(d - c*Tanh[x/2])/Sqrt[c
^2 + d^2]])/(c*Sqrt[c^2 + d^2])
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\,\mathbf{csch}(x)}{c+d\sinh(x)}\,dx &= -\left(i\int \frac{\mathbf{csch}(x)(ib+ia\sinh(x))}{c+d\sinh(x)}\,dx\right)\\
&= \frac{b\int \mathbf{csch}(x)\,dx}{c}+\frac{(ac-bd)\int \frac{1}{c+d\sinh(x)}\,dx}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}+\frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}-\frac{(4(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{4\left(c^2+d^2\right)-x^2}\,dx,x,2d-2c\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}
\end{aligned}
$$

Mathematica [A] time = 0.136297, size = 67, normalized size = 1.16

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^2
] + b*Log[Tanh[x/2]])/c
```

Maple [A] time = 0.032, size = 86, normalized size = 1.5

$$\frac{b}{c}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\,\frac{a}{\sqrt{c^2+d^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,c\tanh(x/2)-2\,d}{\sqrt{c^2+d^2}}\right)-2\,\frac{bd}{c\sqrt{c^2+d^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,c\tanh(x/2)-2\,d}{\sqrt{c^2+d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(x))/(c+d*sinh(x)),x)
```

```
[Out] b/c*ln(tanh(1/2*x))+2/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^
2+d^2)^(1/2))*a-2/c/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+
d^2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.79355, size = 482, normalized size = 8.31

$$-\frac{(ac-bd)\sqrt{c^2+d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2+d^2+2\left(d^2\cosh(x)+cd\right)\sinh(x)+2\,\sqrt{c^2+d^2}(d\cosh(x)+d\sinh(x)+c)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)-d}\right)+\left(bc^2+\right.}{c^3+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a*c - b*d)*sqrt(c^2 + d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*co
sh(x) + 2*c^2 + d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*c
osh(x) + c)*sinh(x) - d)) + (b*c^2 + b*d^2)*log(cosh(x) + sinh(x) + 1) - (b
*c^2 + b*d^2)*log(cosh(x) + sinh(x) - 1))/(c^3 + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x)
```

```
[Out] Integral((a + b*csch(x))/(c + d*sinh(x)), x)
```

Giac [A] time = 1.17117, size = 122, normalized size = 2.1

$$-\frac{b\log\left(e^x+1\right)}{c}+\frac{b\log\left(|e^x-1|\right)}{c}+\frac{(ac-bd)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(e^x + 1)/c + b*log(abs(e^x - 1))/c + (a*c - b*d)*log(abs(2*d*e^x + 2
*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/(sqrt(c^2 +
 d^2)*c)
```

3.574 $\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=19

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rubi [A] time = 0.0415577, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 206}

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -x+2\int \frac{1}{1-\sinh^2(x)}\,dx$$
$$= -x+2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)$$
$$= -x+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)$$

Mathematica [A] time = 0.031268, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

Maple [B] time = 0.02, size = 54, normalized size = 2.8

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)+2\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)/(1-sinh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+ln(tanh(1
/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))
```

Maxima [B] time = 1.61228, size = 86, normalized size = 4.53

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Fricas [B] time = 2.4234, size = 220, normalized size = 11.58

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^{2}-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^{2}-2\sqrt{2}+3}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)/(1-sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15389, size = 55, normalized size = 2.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

3.575 $\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=8

$$2\tanh(x) - x$$

```
[Out] -x + 2*Tanh[x]
```

Rubi [A] time = 0.0418972, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\tanh(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

```
[Out] -x + 2*Tanh[x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx &= -x+2\int \frac{1}{1+\sinh^2(x)}\,dx\\ &= -x+2\int \operatorname{sech}^2(x)\,dx\\ &= -x+2i\,\text{Subst}\Big(\int 1\,dx, x, -i\tanh(x)\Big)\\ &= -x+2\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0149371, size = 8, normalized size = 1.

$$2\tanh(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

```
[Out] -x + 2*Tanh[x]
```

Maple [B] time = 0.024, size = 34, normalized size = 4.3

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+4\,\frac{\tanh(x/2)}{(\tanh(x/2))^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)/(1+sinh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)-1)+4*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.06353, size = 19, normalized size = 2.38

$$-x+\frac{4}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -x + 4/(e^(-2*x) + 1)
```

Fricas [B] time = 2.17497, size = 54, normalized size = 6.75

$$-\frac{(x+2)\cosh(x) - 2\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cosh(x) - 2*sinh(x))/cosh(x)
```

Sympy [B] time = 1.71308, size = 41, normalized size = 5.12

$$-\frac{x\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} - \frac{x}{\tanh^2\left(\frac{x}{2}\right)+1} + \frac{4\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)/(1+sinh(x)**2),x)
```

```
[Out] -x*tanh(x/2)**2/(tanh(x/2)**2 + 1) - x/(tanh(x/2)**2 + 1) + 4*tanh(x/2)/(ta
nh(x/2)**2 + 1)
```

Giac [A] time = 1.16762, size = 19, normalized size = 2.38

$$-x - \frac{4}{e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="giac")
```

```
[Out] -x - 4/(e^(2*x) + 1)
```

3.576 $\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)} \, dx$

Optimal. Leaf size=8

$$2\coth(x) - x$$

```
[Out] -x + 2*Coth[x]
```

Rubi [A] time = 0.0433169, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\coth(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

```
[Out] -x + 2*Coth[x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx &= -x+2\int \frac{1}{1-\cosh^2(x)}\,dx\\ &= -x-2\int \operatorname{csch}^2(x)\,dx\\ &= -x+2i\,\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)\\ &= -x+2\coth(x)\end{aligned}$$

Mathematica [A] time = 0.0061044, size = 8, normalized size = 1.

$$2\coth(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

```
[Out] -x + 2*Coth[x]
```

Maple [B] time = 0.022, size = 28, normalized size = 3.5

$$\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)/(1-cosh(x)^2),x)
```

```
[Out] tanh(1/2*x)-ln(tanh(1/2*x)+1)+1/tanh(1/2*x)+ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.07999, size = 19, normalized size = 2.38

$$-x-\frac{4}{e^{(-2\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -x - 4/(e^(-2*x) - 1)
```

Fricas [B] time = 2.37461, size = 54, normalized size = 6.75

$$-\frac{(x+2)\sinh(x) - 2\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -((x + 2)*sinh(x) - 2*cosh(x))/sinh(x)
```

Sympy [B] time = 1.58885, size = 12, normalized size = 1.5

$$-x + \tanh\left(\frac{x}{2}\right) + \frac{1}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)/(1-cosh(x)**2),x)
```

```
[Out] -x + tanh(x/2) + 1/tanh(x/2)
```

Giac [A] time = 1.16469, size = 19, normalized size = 2.38

$$-x + \frac{4}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="giac")
```

```
[Out] -x + 4/(e^(2*x) - 1)
```

3.577 $\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx$

Optimal. Leaf size=19

$$\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)-x$$

```
[Out] -x + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rubi [A] time = 0.0394022, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx &= -x+2\int \frac{1}{1+\cosh^2(x)}\,dx\\ &= -x+2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\coth(x)\right)\\ &= -x+\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0302714, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2])
```

Maple [B] time = 0.014, size = 102, normalized size = 5.4

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{\sqrt{2}}{4}\ln\left(\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\sqrt{2}\tanh\left(\frac{x}{2}\right)+1\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2-\sqrt{2}\tanh\left(\frac{x}{2}\right)+1\right)^{-1}\right)-\frac{\sqrt{2}}{4}\ln\left(\left(\left(\tanh\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cosh(x)^2)/(1+cosh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+1/4*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(ta
nh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-1/4*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*t
anh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))+ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.6124, size = 138, normalized size = 7.26

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)-\frac{5}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-2x+\frac{1}{4}\log\left(e^{(4x)}+6e^{(2x)}+1\right)-\frac{1}{4}\log\left(6e^{(-2x)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - 5/
16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) - 2*
x + 1/4*log(e^(4*x) + 6*e^(2*x) + 1) - 1/4*log(6*e^(-2*x) + e^(-4*x) + 1)
```

Fricas [B] time = 2.27978, size = 220, normalized size = 11.58

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^{2}-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^{2}+2\sqrt{2}-3}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}+3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^
2 + 3)) - x
```

Sympy [B] time = 5.58405, size = 61, normalized size = 3.21

$$-x-\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2}+\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)**2)/(1+cosh(x)**2),x)
```

```
[Out] -x - sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/2 + sqrt(2)*log(
4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/2
```

Giac [B] time = 1.15681, size = 51, normalized size = 2.68

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - x
```

3.578 $\int \frac{a+b\,\mathbf{sech}^2(x)}{c+d\cosh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}-\frac{bd\tan^{-1}(\sinh(x))}{c^2}+\frac{b\tanh(x)}{c}$$

```
[Out] -((b*d*ArcTan[Sinh[x]])/c^2) + (2*(a*c^2 + b*d^2)*ArcTanh[(Sqrt[c - d]*Tanh
[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c - d]*Sqrt[c + d]) + (b*Tanh[x])/c
```

Rubi [A] time = 0.245407, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4234, 3056, 3001, 3770, 2659, 208}

$$\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}-\frac{bd\tan^{-1}(\sinh(x))}{c^2}+\frac{b\tanh(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] -((b*d*ArcTan[Sinh[x]])/c^2) + (2*(a*c^2 + b*d^2)*ArcTanh[(Sqrt[c - d]*Tanh
[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c - d]*Sqrt[c + d]) + (b*Tanh[x])/c
```

Rule 4234

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Cos[a + b*x]^2))/Cos[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\,\text{sech}^2(x)}{c+d\cosh(x)}\,dx = \int \frac{\left(b+a\cosh^2(x)\right)\text{sech}^2(x)}{c+d\cosh(x)}\,dx$$
$$= \frac{b\tanh(x)}{c} + \frac{\int \frac{(-bd+ac\cosh(x))\text{sech}(x)}{c+d\cosh(x)}\,dx}{c}$$
$$= \frac{b\tanh(x)}{c} - \frac{(bd)\int \text{sech}(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\cosh(x)}\,dx$$
$$= -\frac{bd\tan^{-1}(\sinh(x))}{c^2} + \frac{b\tanh(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+d-(c-d)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= -\frac{bd\tan^{-1}(\sinh(x))}{c^2} + \frac{2\left(a+\frac{bd^2}{c^2}\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}} + \frac{b\tanh(x)}{c}$$

Mathematica [A] time = 0.217917, size = 127, normalized size = 1.72

$$-\frac{2\text{sech}(x)\left(a\cosh^2(x)+b\right)\left(2\cosh(x)\left(\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)+bd\sqrt{d^2-c^2}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)-bc\sqrt{d^2-c^2}\,s\right.}{c^2\sqrt{d^2-c^2}(a\cosh(2x)+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*Sech[x]*(2*(b*d*Sqrt[-c^2 + d^2]*ArcTan[Tanh[x/2]] +
(a*c^2 + b*d^2)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-c^2 + d^2]])*Cosh[x] - b*c
*Sqrt[-c^2 + d^2]*Sinh[x]))/(c^2*Sqrt[-c^2 + d^2]*(a + 2*b + a*Cosh[2*x]))
```

Maple [A] time = 0.04, size = 112, normalized size = 1.5

$$2\,\frac{\tanh(x/2)\,b}{c\left((\tanh(x/2))^2+1\right)} - 2\,\frac{bd\arctan(\tanh(x/2))}{c^2} + 2\,\frac{a}{\sqrt{(c+d)(c-d)}}\text{Artanh}\left(\frac{(c-d)\tanh(x/2)}{\sqrt{(c+d)(c-d)}}\right) + 2\,\frac{bd^2}{c^2\sqrt{(c+d)(c-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)/(c+d*cosh(x)),x)
```

```
[Out] 2*b/c*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*b/c^2*d*arctan(tanh(1/2*x))+2/((c+d)*
(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*a+2/c^2/((c+d)*
```

```
(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.2375, size = 1480, normalized size = 20.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*b*c^3 - 2*b*c*d^2 - (a*c^2 + b*d^2 + (a*c^2 + b*d^2)*cosh(x)^2 + 2*(a*
c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 - d^2)*l
og((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 - d^2 + 2*(d^2*co
sh(x) + c*d)*sinh(x) - 2*sqrt(c^2 - d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*co
sh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) + d)) + 2*(
b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d - b*d^3)*cosh(x)
*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(c^4 - c
^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cosh(x)*sinh(x) + (c
^4 - c^2*d^2)*sinh(x)^2), -2*(b*c^3 - b*c*d^2 + (a*c^2 + b*d^2 + (a*c^2 + b
*d^2)*cosh(x)^2 + 2*(a*c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(
x)^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c)
/(c^2 - d^2)) + (b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d
 - b*d^3)*cosh(x)*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + s
inh(x)))/(c^4 - c^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cos
h(x)*sinh(x) + (c^4 - c^2*d^2)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{sech}^2(x)}{c + d\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x)**2)/(c + d*cosh(x)), x)
```

Giac [A] time = 1.13716, size = 96, normalized size = 1.3

$$-\frac{2\, bd\arctan\left(e^{x}\right)}{c^{2}} + \frac{2\left(ac^{2} + bd^{2}\right)\arctan\left(\frac{de^{x}+c}{\sqrt{-c^{2}+d^{2}}}\right)}{\sqrt{-c^{2} + d^{2}}c^{2}} - \frac{2\, b}{c\left(e^{(2\, x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*d*arctan(e^x)/c^2 + 2*(a*c^2 + b*d^2)*arctan((d*e^x + c)/sqrt(-c^2 + d
^2))/(sqrt(-c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) + 1))
```

3.579 $\int \frac{a+b\,\mathbf{csch}^2(x)}{c+d\sinh(x)}\,dx$

Optimal. Leaf size=69

$$-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}+\frac{bd\tanh^{-1}(\cosh(x))}{c^2}-\frac{b\coth(x)}{c}$$

```
[Out] (b*d*ArcTanh[Cosh[x]])/c^2 - (2*(a*c^2 + b*d^2)*ArcTanh[(d - c*Tanh[x/2])/S
qrt[c^2 + d^2]])/(c^2*Sqrt[c^2 + d^2]) - (b*Coth[x])/c
```

Rubi [A] time = 0.26344, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4233, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}+\frac{bd\tanh^{-1}(\cosh(x))}{c^2}-\frac{b\coth(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

```
[Out] (b*d*ArcTanh[Cosh[x]])/c^2 - (2*(a*c^2 + b*d^2)*ArcTanh[(d - c*Tanh[x/2])/S
qrt[c^2 + d^2]])/(c^2*Sqrt[c^2 + d^2]) - (b*Coth[x])/c
```

Rule 4233

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Sin[a + b*x]^2))/Sin[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx = -\int \frac{\operatorname{csch}^2(x)\left(-b-a\sinh^2(x)\right)}{c+d\sinh(x)}\,dx$$
$$= -\frac{b\coth(x)}{c} - \frac{i\int \frac{\operatorname{csch}(x)(-ibd+iac\sinh(x))}{c+d\sinh(x)}\,dx}{c}$$
$$= -\frac{b\coth(x)}{c} - \frac{(bd)\int \operatorname{csch}(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sinh(x)}\,dx$$
$$= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} - \left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(c^2+d^2\right)-x^2}\,dx, x, 2d-2c\tanh\left(\right.\right.$$
$$= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{2\left(a+\frac{bd^2}{c^2}\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}} - \frac{b\coth(x)}{c}$$

Mathematica [A] time = 0.410293, size = 125, normalized size = 1.81

$$-\frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(\sinh(x)\left(bd\sqrt{-c^2-d^2}\log\left(\tanh\left(\frac{x}{2}\right)\right)-2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)\right)+bc\sqrt{-c^2-d^2}\cosh(x)\right)}{2c^2\sqrt{-c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

```
[Out] -(Csch[x/2]*Sech[x/2]*(b*c*Sqrt[-c^2 - d^2]*Cosh[x] + (-2*(a*c^2 + b*d^2)*A
rcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]] + b*d*Sqrt[-c^2 - d^2]*Log[Tanh[x
/2]])*Sinh[x]))/(2*c^2*Sqrt[-c^2 - d^2])
```

Maple [A] time = 0.036, size = 112, normalized size = 1.6

$$-\frac{b}{2c}\tanh\left(\frac{x}{2}\right)-\frac{b}{2c}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{bd}{c^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\,\frac{a}{\sqrt{c^2+d^2}}\text{Artanh}\left(1/2\,\frac{2\,c\tanh(x/2)-2\,d}{\sqrt{c^2+d^2}}\right)+2\,\frac{bd^2}{c^2\sqrt{c^2+d^2}}\text{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(x)^2)/(c+d*sinh(x)),x)
```

```
[Out] -1/2*b/c*tanh(1/2*x)-1/2*b/c/tanh(1/2*x)-1/c^2*b*d*ln(tanh(1/2*x))+2/(c^2+d
^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))*a+2/c^2/(c^2+d
^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.54043, size = 1037, normalized size = 15.03

$$\frac{2\,bc^3 + 2\,bcd^2 + \left(ac^2 + bd^2 - \left(ac^2 + bd^2\right)\cosh\left(x\right)^2 - 2\left(ac^2 + bd^2\right)\cosh\left(x\right)\sinh\left(x\right) - \left(ac^2 + bd^2\right)\sinh\left(x\right)^2\right)\sqrt{c^2 + d^2}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*c^3 + 2*b*c*d^2 + (a*c^2 + b*d^2 - (a*c^2 + b*d^2)*cosh(x)^2 - 2*(a*c^
2 + b*d^2)*cosh(x)*sinh(x) - (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 + d^2)*log
((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 + d^2 + 2*(d^2*cosh
(x) + c*d)*sinh(x) - 2*sqrt(c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*cosh
(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) - d)) + (b*c^
2*d + b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sin
h(x) - (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (b*c^2*d +
 b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sinh(x)
- (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(c^4 + c^2*d^2 -
 (c^4 + c^2*d^2)*cosh(x)^2 - 2*(c^4 + c^2*d^2)*cosh(x)*sinh(x) - (c^4 + c^2
*d^2)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{csch}^2(x)}{c+d\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)**2)/(c+d*sinh(x)),x)
```

```
[Out] Integral((a + b*csch(x)**2)/(c + d*sinh(x)), x)
```

Giac [A] time = 1.17998, size = 147, normalized size = 2.13

$$\frac{bd\log\left(e^x+1\right)}{c^2}-\frac{bd\log\left(|e^x-1|\right)}{c^2}+\frac{\left(ac^2+bd^2\right)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c^2}-\frac{2\,b}{c\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="giac")
```

```
[Out] b*d*log(e^x + 1)/c^2 - b*d*log(abs(e^x - 1))/c^2 + (a*c^2 + b*d^2)*log(abs(
2*d*e^x + 2*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/
(sqrt(c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) - 1))
```

3.580 $\int (a\cosh(x)+b\sinh(x))\,dx$

Optimal. Leaf size=9

$$a\sinh(x)+b\cosh(x)$$

```
[Out] b*Cosh[x] + a*Sinh[x]
```

Rubi [A] time = 0.0087967, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2637, 2638}

$$a\sinh(x)+b\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[x] + b*Sinh[x],x]
```

```
[Out] b*Cosh[x] + a*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))\,dx &= a\int \cosh(x)\,dx+b\int \sinh(x)\,dx\\ &= b\cosh(x)+a\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0037957, size = 9, normalized size = 1.

$$a\sinh(x)+b\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[x] + b*Sinh[x],x]
```

```
[Out] b*Cosh[x] + a*Sinh[x]
```

Maple [A] time = 0.001, size = 10, normalized size = 1.1

$$b\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(x)+b*sinh(x),x)
```

```
[Out] b*cosh(x)+a*sinh(x)
```

Maxima [A] time = 1.02031, size = 12, normalized size = 1.33

$$b\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="maxima")
```

```
[Out] b*cosh(x) + a*sinh(x)
```

Fricas [A] time = 2.43486, size = 31, normalized size = 3.44

$$b\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="fricas")
```

```
[Out] b*cosh(x) + a*sinh(x)
```

Sympy [A] time = 0.129453, size = 8, normalized size = 0.89

$$a\sinh(x) + b\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x)
```

```
[Out] a*sinh(x) + b*cosh(x)
```

Giac [B] time = 1.12386, size = 31, normalized size = 3.44

$$\frac{1}{2}\,b\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}\,a\left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*b*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

3.581 $\int (a\cosh(x)+b\sinh(x))^2\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x\left(a^2-b^2\right)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

```
[Out] ((a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2
```

Rubi [A] time = 0.01757, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2-b^2\right)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^2\,dx &= \frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{2}\left(a^2-b^2\right)\int 1\,dx\\ &= \frac{1}{2}\left(a^2-b^2\right)x+\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0541152, size = 36, normalized size = 0.97

$$\frac{1}{4}\left(\left(a^2+b^2\right)\sinh(2x)+2x(a-b)(a+b)+2ab\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (2*(a - b)*(a + b)*x + 2*a*b*Cosh[2*x] + (a^2 + b^2)*Sinh[2*x])/4
```

Maple [A] time = 0.02, size = 37, normalized size = 1.

$$b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+ab\left(\cosh(x)\right)^2+a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] b^2*(1/2*cosh(x)*sinh(x)-1/2*x)+a*b*cosh(x)^2+a^2*(1/2*cosh(x)*sinh(x)+1/2*
x)
```

Maxima [A] time = 1.04819, size = 62, normalized size = 1.68

$$ab\cosh(x)^2+\frac{1}{8}a^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-\frac{1}{8}b^2\left(4x-e^{(2x)}+e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] a*b*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*b^2*(4*x - e^(2*x)
 + e^(-2*x))
```

Fricas [A] time = 2.18391, size = 126, normalized size = 3.41

$$\frac{1}{2}ab\cosh(x)^2+\frac{1}{2}ab\sinh(x)^2+\frac{1}{2}\left(a^2+b^2\right)\cosh(x)\sinh(x)+\frac{1}{2}\left(a^2-b^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*cosh(x)^2 + 1/2*a*b*sinh(x)^2 + 1/2*(a^2 + b^2)*cosh(x)*sinh(x) + 1
/2*(a^2 - b^2)*x
```

Sympy [B] time = 0.243936, size = 78, normalized size = 2.11

$$-\frac{a^2x\sinh^2(x)}{2}+\frac{a^2x\cosh^2(x)}{2}+\frac{a^2\sinh(x)\cosh(x)}{2}+ab\sinh^2(x)+\frac{b^2x\sinh^2(x)}{2}-\frac{b^2x\cosh^2(x)}{2}+\frac{b^2\sinh(x)\cosh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*sinh(x)*cosh(x)/2 + a*b*s
inh(x)**2 + b**2*x*sinh(x)**2/2 - b**2*x*cosh(x)**2/2 + b**2*sinh(x)*cosh(x
)/2
```

Giac [B] time = 1.1521, size = 100, normalized size = 2.7

$$\frac{1}{8}a^2e^{(2x)}+\frac{1}{4}abe^{(2x)}+\frac{1}{8}b^2e^{(2x)}+\frac{1}{2}\left(a^2-b^2\right)x-\frac{1}{8}\left(2a^2e^{(2x)}-2b^2e^{(2x)}+a^2-2ab+b^2\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*b*e^(2*x) + 1/8*b^2*e^(2*x) + 1/2*(a^2 - b^2)*x - 1
/8*(2*a^2*e^(2*x) - 2*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-2*x)
```

3.582 $\int (a\cosh(x)+b\sinh(x))^3\,dx$

Optimal. Leaf size=35

$$\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))+\frac{1}{3}(a\sinh(x)+b\cosh(x))^3$$

```
[Out] (a^2 - b^2)*(b*Cosh[x] + a*Sinh[x]) + (b*Cosh[x] + a*Sinh[x])^3/3
```

Rubi [A] time = 0.0248492, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3072}

$$\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))+\frac{1}{3}(a\sinh(x)+b\cosh(x))^3$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a^2 - b^2)*(b*Cosh[x] + a*Sinh[x]) + (b*Cosh[x] + a*Sinh[x])^3/3
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^3\,dx&=i\,\mathrm{Subst}\left(\int\left(a^2-b^2-x^2\right)\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)\\&=\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))+\frac{1}{3}(b\cosh(x)+a\sinh(x))^3\end{aligned}$$

Mathematica [A] time = 0.126669, size = 63, normalized size = 1.8

$$\frac{1}{12}\left(9a\left(a^2-b^2\right)\sinh(x)+a\left(a^2+3b^2\right)\sinh(3x)+9b\left(a^2-b^2\right)\cosh(x)+b\left(3a^2+b^2\right)\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (9*b*(a^2 - b^2)*Cosh[x] + b*(3*a^2 + b^2)*Cosh[3*x] + 9*a*(a^2 - b^2)*Sinh
[x] + a*(a^2 + 3*b^2)*Sinh[3*x])/12
```

Maple [B] time = 0.025, size = 68, normalized size = 1.9

$$b^3\left(-\frac{2}{3}+\frac{(\sinh(x))^2}{3}\right)\cosh(x)+3ab^2\left(1/3\sinh(x)(\cosh(x))^2-1/3\sinh(x)\right)+3a^2b\left(1/3\cosh(x)(\sinh(x))^2+1/3\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] b^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+3*a*b^2*(1/3*sinh(x)*cosh(x)^2-1/3*sinh(x)
)+3*a^2*b*(1/3*cosh(x)*sinh(x)^2+1/3*cosh(x))+a^3*(2/3+1/3*cosh(x)^2)*sinh(
x)
```

Maxima [B] time = 1.01723, size = 93, normalized size = 2.66

$$a^2b\cosh(x)^3+ab^2\sinh(x)^3+\frac{1}{24}b^3\left(e^{(3x)}-9e^{(-x)}+e^{(-3x)}-9e^x\right)+\frac{1}{24}a^3\left(e^{(3x)}-9e^{(-x)}-e^{(-3x)}+9e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*b*cosh(x)^3 + a*b^2*sinh(x)^3 + 1/24*b^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x)
```

Fricas [B] time = 2.38148, size = 261, normalized size = 7.46

$$\frac{1}{12}\left(3a^2b+b^3\right)\cosh(x)^3+\frac{1}{4}\left(3a^2b+b^3\right)\cosh(x)\sinh(x)^2+\frac{1}{12}\left(a^3+3ab^2\right)\sinh(x)^3+\frac{3}{4}\left(a^2b-b^3\right)\cosh(x)+\frac{1}{4}\left(3a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*b + b^3)*cosh(x)^3 + 1/4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^2 + 1/
12*(a^3 + 3*a*b^2)*sinh(x)^3 + 3/4*(a^2*b - b^3)*cosh(x) + 1/4*(3*a^3 - 3*a
*b^2 + (a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)
```

Sympy [B] time = 0.43813, size = 66, normalized size = 1.89

$$-\frac{2a^3\sinh^3(x)}{3}+a^3\sinh(x)\cosh^2(x)+a^2b\cosh^3(x)+ab^2\sinh^3(x)+b^3\sinh^2(x)\cosh(x)-\frac{2b^3\cosh^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] -2*a**3*sinh(x)**3/3 + a**3*sinh(x)*cosh(x)**2 + a**2*b*cosh(x)**3 + a*b**2
*sinh(x)**3 + b**3*sinh(x)**2*cosh(x) - 2*b**3*cosh(x)**3/3
```

Giac [B] time = 1.14039, size = 181, normalized size = 5.17

$$\frac{1}{24}a^3e^{(3x)}+\frac{1}{8}a^2be^{(3x)}+\frac{1}{8}ab^2e^{(3x)}+\frac{1}{24}b^3e^{(3x)}+\frac{3}{8}a^3e^x+\frac{3}{8}a^2be^x-\frac{3}{8}ab^2e^x-\frac{3}{8}b^3e^x-\frac{1}{24}\left(9a^3e^{(2x)}-9a^2be^{(2x)}-9a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*b*e^(3*x) + 1/8*a*b^2*e^(3*x) + 1/24*b^3*e^(3*x)
 + 3/8*a^3*e^x + 3/8*a^2*b*e^x - 3/8*a*b^2*e^x - 3/8*b^3*e^x - 1/24*(9*a^3*
e^(2*x) - 9*a^2*b*e^(2*x) - 9*a*b^2*e^(2*x) + 9*b^3*e^(2*x) + a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*e^(-3*x)
```

3.583 $\int (a\cosh(x)+b\sinh(x))^4\,dx$

Optimal. Leaf size=72

$$\frac{3}{8}x\left(a^2-b^2\right)^2+\frac{3}{8}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh$$

```
[Out] (3*(a^2 - b^2)^2*x)/8 + (3*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] +
 b*Sinh[x]))/8 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^3)/4
```

Rubi [A] time = 0.037078, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3073, 8}

$$\frac{3}{8}x\left(a^2-b^2\right)^2+\frac{3}{8}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^4,x]
```


```
[Out] (3*(a^2 - b^2)^2*x)/8 + (3*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] +
 b*Sinh[x]))/8 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^3)/4
```


Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = \frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3+\frac{1}{4}\left(3\left(a^2-b^2\right)\right)\int (a\cosh(x)+b\sin$$
$$= \frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(b\cosh(x)+a\sinh(x))($$
$$= \frac{3}{8}\left(a^2-b^2\right)^2x+\frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(b\cosh($$

Mathematica [A] time = 0.14543, size = 87, normalized size = 1.21

$$\frac{1}{32}\left(8\left(a^4-b^4\right)\sinh(2x)+\left(6a^2b^2+a^4+b^4\right)\sinh(4x)+16ab\left(a^2-b^2\right)\cosh(2x)+4ab\left(a^2+b^2\right)\cosh(4x)+12x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^4,x]
```

```
[Out] (12*(a - b)^2*(a + b)^2*x + 16*a*b*(a^2 - b^2)*Cosh[2*x] + 4*a*b*(a^2 + b^2
)*Cosh[4*x] + 8*(a^4 - b^4)*Sinh[2*x] + (a^4 + 6*a^2*b^2 + b^4)*Sinh[4*x])/
32
```

Maple [A] time = 0.027, size = 118, normalized size = 1.6

$$b^4\left(\left(\frac{(\sinh(x))^3}{4}-\frac{3\sinh(x)}{8}\right)\cosh(x)+\frac{3x}{8}\right)+4ab^3\left(1/4\,(\sinh(x))^2(\cosh(x))^2-1/4\,(\cosh(x))^2\right)+6a^2b^2\left(1/4\sinh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^4,x)
```

```
[Out] b^4*((1/4*sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)+4*a*b^3*(1/4*sinh(x)^2*cosh
(x)^2-1/4*cosh(x)^2)+6*a^2*b^2*(1/4*sinh(x)*cosh(x)^3-1/8*cosh(x)*sinh(x)-1
/8*x)+4*a^3*b*(1/4*sinh(x)^2*cosh(x)^2+1/4*cosh(x)^2)+a^4*((1/4*cosh(x)^3+3
/8*cosh(x))*sinh(x)+3/8*x)
```

Maxima [A] time = 1.07815, size = 139, normalized size = 1.93

$$a^3b\cosh(x)^4+ab^3\sinh(x)^4+\frac{1}{64}a^4\left(24x+e^{(4x)}+8e^{(2x)}-8e^{(-2x)}-e^{(-4x)}\right)+\frac{1}{64}b^4\left(24x+e^{(4x)}-8e^{(2x)}+8e^{(-2x)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] a^3*b*cosh(x)^4 + a*b^3*sinh(x)^4 + 1/64*a^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*b^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*a^2*b^2*(8*x - e^(4*x) + e^(-4*x))
```

Fricas [B] time = 2.35033, size = 425, normalized size = 5.9

$$\frac{1}{8}\left(a^3b+ab^3\right)\cosh(x)^4+\frac{1}{8}\left(a^4+6a^2b^2+b^4\right)\cosh(x)\sinh(x)^3+\frac{1}{8}\left(a^3b+ab^3\right)\sinh(x)^4+\frac{1}{2}\left(a^3b-ab^3\right)\cosh(x)^2+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/8*(a^3*b + a*b^3)*cosh(x)^4 + 1/8*(a^4 + 6*a^2*b^2 + b^4)*cosh(x)*sinh(x)
^3 + 1/8*(a^3*b + a*b^3)*sinh(x)^4 + 1/2*(a^3*b - a*b^3)*cosh(x)^2 + 1/4*(2
*a^3*b - 2*a*b^3 + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x)^2 + 3/8*(a^4 - 2*a^
2*b^2 + b^4)*x + 1/8*((a^4 + 6*a^2*b^2 + b^4)*cosh(x)^3 + 4*(a^4 - b^4)*cos
h(x))*sinh(x)
```

Sympy [B] time = 0.946163, size = 265, normalized size = 3.68

$$\frac{3a^4x\sinh^4(x)}{8}-\frac{3a^4x\sinh^2(x)\cosh^2(x)}{4}+\frac{3a^4x\cosh^4(x)}{8}-\frac{3a^4\sinh^3(x)\cosh(x)}{8}+\frac{5a^4\sinh(x)\cosh^3(x)}{8}+a^3b\cosh^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] 3*a**4*x*sinh(x)**4/8 - 3*a**4*x*sinh(x)**2*cosh(x)**2/4 + 3*a**4*x*cosh(x)
**4/8 - 3*a**4*sinh(x)**3*cosh(x)/8 + 5*a**4*sinh(x)*cosh(x)**3/8 + a**3*b*
cosh(x)**4 - 3*a**2*b**2*x*sinh(x)**4/4 + 3*a**2*b**2*x*sinh(x)**2*cosh(x)*
*2/2 - 3*a**2*b**2*x*cosh(x)**4/4 + 3*a**2*b**2*sinh(x)**3*cosh(x)/4 + 3*a*
*2*b**2*sinh(x)*cosh(x)**3/4 + a*b**3*sinh(x)**4 + 3*b**4*x*sinh(x)**4/8 -
3*b**4*x*sinh(x)**2*cosh(x)**2/4 + 3*b**4*x*cosh(x)**4/8 + 5*b**4*sinh(x)**
3*cosh(x)/8 - 3*b**4*sinh(x)*cosh(x)**3/8
```

Giac [B] time = 1.14752, size = 281, normalized size = 3.9

$$\frac{1}{64}a^4e^{(4x)} + \frac{1}{16}a^3be^{(4x)} + \frac{3}{32}a^2b^2e^{(4x)} + \frac{1}{16}ab^3e^{(4x)} + \frac{1}{64}b^4e^{(4x)} + \frac{1}{8}a^4e^{(2x)} + \frac{1}{4}a^3be^{(2x)} - \frac{1}{4}ab^3e^{(2x)} - \frac{1}{8}b^4e^{(2x)} + \frac{3}{8}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/64*a^4*e^(4*x) + 1/16*a^3*b*e^(4*x) + 3/32*a^2*b^2*e^(4*x) + 1/16*a*b^3*e
^(4*x) + 1/64*b^4*e^(4*x) + 1/8*a^4*e^(2*x) + 1/4*a^3*b*e^(2*x) - 1/4*a*b^3
*e^(2*x) - 1/8*b^4*e^(2*x) + 3/8*(a^4 - 2*a^2*b^2 + b^4)*x - 1/64*(18*a^4*e
^(4*x) - 36*a^2*b^2*e^(4*x) + 18*b^4*e^(4*x) + 8*a^4*e^(2*x) - 16*a^3*b*e^(
2*x) + 16*a*b^3*e^(2*x) - 8*b^4*e^(2*x) + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4)*e^(-4*x)
```

3.584 $\int (a\cosh(x)+b\sinh(x))^5\,dx$

Optimal. Leaf size=61

$$\frac{2}{3}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))^3+\left(a^2-b^2\right)^2(a\sinh(x)+b\cosh(x))+\frac{1}{5}(a\sinh(x)+b\cosh(x))^5$$

```
[Out] (a^2 - b^2)^2*(b*Cosh[x] + a*Sinh[x]) + (2*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[
x])^3)/3 + (b*Cosh[x] + a*Sinh[x])^5/5
```

Rubi [A] time = 0.0450399, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3072, 194}

$$\frac{2}{3}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))^3+\left(a^2-b^2\right)^2(a\sinh(x)+b\cosh(x))+\frac{1}{5}(a\sinh(x)+b\cosh(x))^5$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (a^2 - b^2)^2*(b*Cosh[x] + a*Sinh[x]) + (2*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[
x])^3)/3 + (b*Cosh[x] + a*Sinh[x])^5/5
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = i\,\mathrm{Subst}\left(\int \left(a^2-b^2-x^2\right)^2\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)$$
$$= i\,\mathrm{Subst}\left(\int \left(a^4\left(1+\frac{-2a^2b^2+b^4}{a^4}\right)-2a^2\left(1-\frac{b^2}{a^2}\right)x^2+x^4\right)dx, x, -ib\cosh(x)-ia\sinh($$
$$= \left(a^2-b^2\right)^2(b\cosh(x)+a\sinh(x))+\frac{2}{3}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))^3+\frac{1}{5}(b\cosh(x)$$

Mathematica [B] time = 0.225237, size = 133, normalized size = 2.18

$$\frac{1}{240}\left(150a\left(a^2-b^2\right)^2\sinh(x)+25a\left(2a^2b^2+a^4-3b^4\right)\sinh(3x)+3a\left(10a^2b^2+a^4+5b^4\right)\sinh(5x)+150b\left(a^2-b^2\right)^2\mathrm{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (150*b*(a^2 - b^2)^2*Cosh[x] - 25*b*(-3*a^4 + 2*a^2*b^2 + b^4)*Cosh[3*x] +
3*b*(5*a^4 + 10*a^2*b^2 + b^4)*Cosh[5*x] + 150*a*(a^2 - b^2)^2*Sinh[x] + 25
*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sinh[3*x] + 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Sin
h[5*x])/240
```

Maple [B] time = 0.056, size = 160, normalized size = 2.6

$$b^5\left(\frac{8}{15}+\frac{(\sinh(x))^4}{5}-\frac{4\,(\sinh(x))^2}{15}\right)\cosh(x)+5\,ab^4\left(1/5\,(\sinh(x))^3(\cosh(x))^2-1/5\,\sinh(x)(\cosh(x))^2+1/5\,\sinh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^5,x)
```

```
[Out] b^5*(8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)+5*a*b^4*(1/5*sinh(x)^3*cosh
(x)^2-1/5*sinh(x)*cosh(x)^2+1/5*sinh(x))+10*a^2*b^3*(1/5*sinh(x)^2*cosh(x)^
3-2/15*cosh(x)*sinh(x)^2-2/15*cosh(x))+10*a^3*b^2*(1/5*cosh(x)^4*sinh(x)-1/
5*(2/3+1/3*cosh(x)^2)*sinh(x))+5*a^4*b*(1/5*sinh(x)^2*cosh(x)^3+1/5*cosh(x)
*sinh(x)^2+1/5*cosh(x))+a^5*(8/15+1/5*cosh(x)^4+4/15*cosh(x)^2)*sinh(x)
```

Maxima [B] time = 1.16099, size = 258, normalized size = 4.23

$$a^4b\cosh(x)^5 + ab^4\sinh(x)^5 + \frac{1}{48}\left(\left(5e^{(-2x)} - 30e^{(-4x)} + 3\right)e^{(5x)} + 30e^{(-x)} - 5e^{(-3x)} - 3e^{(-5x)}\right)a^3b^2 - \frac{1}{48}\left(\left(5e^{(-2x)} + 30e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

```
[Out] a^4*b*cosh(x)^5 + a*b^4*sinh(x)^5 + 1/48*((5*e^(-2*x) - 30*e^(-4*x) + 3)*e^
(5*x) + 30*e^(-x) - 5*e^(-3*x) - 3*e^(-5*x))*a^3*b^2 - 1/48*((5*e^(-2*x) +
30*e^(-4*x) - 3)*e^(5*x) + 30*e^(-x) + 5*e^(-3*x) - 3*e^(-5*x))*a^2*b^3 + 1
/480*a^5*(3*e^(5*x) + 25*e^(3*x) - 150*e^(-x) - 25*e^(-3*x) - 3*e^(-5*x) +
150*e^x) + 1/480*b^5*(3*e^(5*x) - 25*e^(3*x) + 150*e^(-x) - 25*e^(-3*x) + 3
*e^(-5*x) + 150*e^x)
```

Fricas [B] time = 2.46245, size = 747, normalized size = 12.25

$$\frac{1}{80}\left(5a^4b + 10a^2b^3 + b^5\right)\cosh(x)^5 + \frac{1}{16}\left(5a^4b + 10a^2b^3 + b^5\right)\cosh(x)\sinh(x)^4 + \frac{1}{80}\left(a^5 + 10a^3b^2 + 5ab^4\right)\sinh(x)^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

```
[Out] 1/80*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(x)^5 + 1/16*(5*a^4*b + 10*a^2*b^3 +
b^5)*cosh(x)*sinh(x)^4 + 1/80*(a^5 + 10*a^3*b^2 + 5*a*b^4)*sinh(x)^5 + 5/48
*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)^3 + 1/48*(5*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 6*(a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^2)*sinh(x)^3 + 1/16*(2*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(x)^3 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh
(x)^2 + 5/8*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x) + 1/16*(10*a^5 - 20*a^3*b^2 +
 10*a*b^4 + (a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^4 + 5*(a^5 + 2*a^3*b^2 - 3
*a*b^4)*cosh(x)^2)*sinh(x)
```

Sympy [B] time = 1.65456, size = 172, normalized size = 2.82

$$\frac{8a^5\sinh^5(x)}{15} - \frac{4a^5\sinh^3(x)\cosh^2(x)}{3} + a^5\sinh(x)\cosh^4(x) + a^4b\cosh^5(x) - \frac{4a^3b^2\sinh^5(x)}{3} + \frac{10a^3b^2\sinh^3(x)\cosh}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**5,x)
```

```
[Out] 8*a**5*sinh(x)**5/15 - 4*a**5*sinh(x)**3*cosh(x)**2/3 + a**5*sinh(x)*cosh(x
)**4 + a**4*b*cosh(x)**5 - 4*a**3*b**2*sinh(x)**5/3 + 10*a**3*b**2*sinh(x)*
*3*cosh(x)**2/3 + 10*a**2*b**3*sinh(x)**2*cosh(x)**3/3 - 4*a**2*b**3*cosh(x
)**5/3 + a*b**4*sinh(x)**5 + b**5*sinh(x)**4*cosh(x) - 4*b**5*sinh(x)**2*co
sh(x)**3/3 + 8*b**5*cosh(x)**5/15
```

Giac [B] time = 1.16068, size = 464, normalized size = 7.61

$$\frac{1}{160}a^5e^{(5x)} + \frac{1}{32}a^4be^{(5x)} + \frac{1}{16}a^3b^2e^{(5x)} + \frac{1}{16}a^2b^3e^{(5x)} + \frac{1}{32}ab^4e^{(5x)} + \frac{1}{160}b^5e^{(5x)} + \frac{5}{96}a^5e^{(3x)} + \frac{5}{32}a^4be^{(3x)} + \frac{5}{48}a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 1/160*a^5*e^(5*x) + 1/32*a^4*b*e^(5*x) + 1/16*a^3*b^2*e^(5*x) + 1/16*a^2*b^
3*e^(5*x) + 1/32*a*b^4*e^(5*x) + 1/160*b^5*e^(5*x) + 5/96*a^5*e^(3*x) + 5/3
2*a^4*b*e^(3*x) + 5/48*a^3*b^2*e^(3*x) - 5/48*a^2*b^3*e^(3*x) - 5/32*a*b^4*
e^(3*x) - 5/96*b^5*e^(3*x) + 5/16*a^5*e^x + 5/16*a^4*b*e^x - 5/8*a^3*b^2*e^
x - 5/8*a^2*b^3*e^x + 5/16*a*b^4*e^x + 5/16*b^5*e^x - 1/480*(150*a^5*e^(4*x
) - 150*a^4*b*e^(4*x) - 300*a^3*b^2*e^(4*x) + 300*a^2*b^3*e^(4*x) + 150*a*b
^4*e^(4*x) - 150*b^5*e^(4*x) + 25*a^5*e^(2*x) - 75*a^4*b*e^(2*x) + 50*a^3*b
^2*e^(2*x) + 50*a^2*b^3*e^(2*x) - 75*a*b^4*e^(2*x) + 25*b^5*e^(2*x) + 3*a^5
 - 15*a^4*b + 30*a^3*b^2 - 30*a^2*b^3 + 15*a*b^4 - 3*b^5)*e^(-5*x)
```

3.585 $\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0292113, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx = i\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)$$
$$= \frac{\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [A] time = 0.0436594, size = 46, normalized size = 1.21

$$\frac{2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[a
 + b])
```

Maple [A] time = 0.036, size = 39, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38284, size = 423, normalized size = 11.13

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)}{a^2-b^2}, -\frac{2\arctan\left(\frac{\sqrt{a^2-b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2 - b
^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqrt(a^
2 - b^2)]
```

Sympy [A] time = 6.54172, size = 119, normalized size = 3.13

$$\begin{cases} \tilde{\infty}\log\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ -\frac{1}{-b\sinh(x)+b\cosh(x)} & \text{for } a=-b \\ -\frac{1}{b\sinh(x)+b\cosh(x)} & \text{for } a=b \\ -\frac{\sqrt{-a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2}+\frac{\sqrt{-a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*log(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tanh(x/2))/b, Eq(
a, 0)), (-1/(-b*sinh(x) + b*cosh(x)), Eq(a, -b)), (-1/(b*sinh(x) + b*cosh(x
)), Eq(a, b)), (-sqrt(-a**2 + b**2)*log(tanh(x/2) + b/a - sqrt(-a**2 + b**2
```

```
)/a)/(a**2 - b**2) + sqrt(-a**2 + b**2)*log(tanh(x/2) + b/a + sqrt(-a**2 +
b**2)/a)/(a**2 - b**2), True))
```

Giac [A] time = 1.18965, size = 47, normalized size = 1.24

$$\frac{2\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

3.586 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))

Rubi [A] time = 0.0145181, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cosh[x] + b*Sinh[x])^(-2),x]

[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.0240173, size = 17, normalized size = 1.

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-2),x]
```

```
[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] time = 0.056, size = 29, normalized size = 1.7

$$2\,\frac{\tanh\left(x/2\right)}{a\left(a+2\,\tanh\left(x/2\right)b+a\left(\tanh\left(x/2\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 2/a*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)
```

Maxima [A] time = 1.23529, size = 39, normalized size = 2.29

$$\frac{2}{a^{2}-b^{2}+\left(a^{2}-2\,ab+b^{2}\right)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/(a^2 - b^2 + (a^2 - 2*a*b + b^2)*e^(-2*x))
```

Fricas [B] time = 2.36344, size = 162, normalized size = 9.53

$$-\frac{2}{\left(a^{2}+2\,ab+b^{2}\right)\cosh\left(x\right)^{2}+2\left(a^{2}+2\,ab+b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)+\left(a^{2}+2\,ab+b^{2}\right)\sinh\left(x\right)^{2}+a^{2}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/((a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x) +
 (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2)
```

Sympy [A] time = 151.74, size = 206, normalized size = 12.12

$$\begin{cases} \tilde{\infty}\left(-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}\right) \\ \frac{-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}}{b^2} \\ \frac{x\tanh^2(x)}{2a^2\sinh^2(x) - 4a^2\sinh(x)\cosh(x)\tanh(x) + 2a^2\cosh^2(x)\tanh^2(x)} - \frac{x}{2a^2\sinh^2(x) - 4a^2\sinh(x)\cosh(x)\tanh(x) + 2a^2\cosh^2(x)\tanh^2(x)} + \frac{\tanh(x)}{2a^2\sinh^2(x) - 4a^2\sinh(x)\cosh(x)\tanh(x) + 2a^2\cosh^2(x)\tanh^2(x)} \\ \frac{2\tanh\left(\frac{x}{2}\right)}{a^2\tanh^2\left(\frac{x}{2}\right) + a^2 + 2ab\tanh\left(\frac{x}{2}\right)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(-tanh(x/2)/2 - 1/(2*tanh(x/2))), Eq(a, 0) & Eq(b, 0)), ((-t
anh(x/2)/2 - 1/(2*tanh(x/2)))/b**2, Eq(a, 0)), (x*tanh(x)**2/(2*a**2*sinh(x
)**2 - 4*a**2*sinh(x)*cosh(x)*tanh(x) + 2*a**2*cosh(x)**2*tanh(x)**2) - x/(
2*a**2*sinh(x)**2 - 4*a**2*sinh(x)*cosh(x)*tanh(x) + 2*a**2*cosh(x)**2*tanh
(x)**2) + tanh(x)/(2*a**2*sinh(x)**2 - 4*a**2*sinh(x)*cosh(x)*tanh(x) + 2*a
**2*cosh(x)**2*tanh(x)**2), Eq(b, -a/tanh(x))), (2*tanh(x/2)/(a**2*tanh(x/2
)**2 + a**2 + 2*a*b*tanh(x/2)), True))
```

Giac [A] time = 1.17418, size = 35, normalized size = 2.06

$$-\frac{2}{\left(ae^{(2x)} + be^{(2x)} + a - b\right)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/((a*e^(2*x) + b*e^(2*x) + a - b)*(a + b))
```

3.587 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=77

$$\frac{a\sinh(x)+b\cosh(x)}{2\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)^2}+\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/(2*(a^2 - b^2)^(3/2)) + (b*
Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2)
```

Rubi [A] time = 0.0466193, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$\frac{a\sinh(x)+b\cosh(x)}{2\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)^2}+\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/(2*(a^2 - b^2)^(3/2)) + (b*
Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx &= \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{2\left(a^2-b^2\right)}\\ &= \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{i\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x\right.}{2\left(a^2-b^2\right)}\\ &= \frac{\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}+\frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.478082, size = 96, normalized size = 1.25

$$\frac{1}{2}\left(\frac{2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{b}{a(a-b)(a+b)(a\cosh(x)+b\sinh(x))}+\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ((2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a
+ b)^(3/2)) + Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x])^2) + b/(a*(a - b)*(a + b)*
(a*Cosh[x] + b*Sinh[x])))/2
```

Maple [B] time = 0.066, size = 167, normalized size = 2.2

$$2\,\frac{1}{\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)^2}\left(-1/2\,\frac{\left(a^2-2\,b^2\right)(\tanh(x/2))^3}{\left(a^2-b^2\right)a}+1/2\,\frac{b\left(a^2+2\,b^2\right)(\tanh(x/2))^2}{\left(a^2-b^2\right)a^2}+1/2\,\frac{\left(a^2+2\,b^2\right.}{\left(a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(a^2-2*b^2)/(a^2-b^2)/a*tanh(1/2*x)^3+1/2*b*(a^2+2*b^2)/(a^2-b^2)/a
^2*tanh(1/2*x)^2+1/2*(a^2+2*b^2)/(a^2-b^2)/a*tanh(1/2*x)+1/2*b/(a^2-b^2))/(
a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^2+1/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tanh
(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6482, size = 3501, normalized size = 45.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 6*(a^3 + a^2*b - a*b^2 - b^
3)*cosh(x)*sinh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3 + ((a^2 + 2*
a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a
*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*co
sh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)) - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x) -
2*(a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2)*sin
h(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 + (a^6
 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^4 + 4*(
a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)*sinh
(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*sin
```

```
h(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 2*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2
*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^
3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x))*sinh(x)), ((a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 3*(a^3 + a^2*
b - a*b^2 - b^3)*cosh(x)*sinh(x)^2 + (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3
- ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3
+ (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 +
2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arct
an(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x) - (a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 -
b^3)*cosh(x)^2)*sinh(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2
*a*b^5 + b^6 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b
^6)*cosh(x)^4 + 4*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5
+ b^6)*cosh(x)*sinh(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 +
 2*a*b^5 + b^6)*sinh(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2
 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^
3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b -
 a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.147, size = 119, normalized size = 1.55

$$\frac{\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{ae^{(3x)}+be^{(3x)}-ae^{x}+be^{x}}{\left(a^{2}-b^{2}\right)\left(ae^{(2x)}+be^{(2x)}+a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + (a*e^(3*x) + b*
e^(3*x) - a*e^x + b*e^x)/((a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.588 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh(x)}{3a\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{a\sinh(x)+b\cosh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3}$$

```
[Out] (b*Cosh[x] + a*Sinh[x])/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^3) + (2*Sinh
[x])/(3*a*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.0315738, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3076, 3075}

$$\frac{2\sinh(x)}{3a\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{a\sinh(x)+b\cosh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

```
[Out] (b*Cosh[x] + a*Sinh[x])/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^3) + (2*Sinh
[x])/(3*a*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \frac{b\cosh(x)+a\sinh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3} + \frac{2\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{3\left(a^2-b^2\right)}$$
$$= \frac{b\cosh(x)+a\sinh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3} + \frac{2\sinh(x)}{3a\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.132782, size = 64, normalized size = 0.96

$$\frac{\sinh(x)\left(\left(a^2+b^2\right)\cosh(2x)+2a^2-b^2\right)+ab\cosh(3x)}{3a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

```
[Out] (a*b*Cosh[3*x] + (2*a^2 - b^2 + (a^2 + b^2)*Cosh[2*x])*Sinh[x])/(3*a*(a - b
)*(a + b)*(a*Cosh[x] + b*Sinh[x])^3)
```

Maple [A] time = 0.081, size = 87, normalized size = 1.3

$$-2\,\frac{1}{\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)^3}\left(-\frac{(\tanh(x/2))^5}{a}-2\,\frac{b\,(\tanh(x/2))^4}{a^2}-2/3\,\frac{\left(a^2+2\,b^2\right)(\tanh(x/2))^3}{a^3}-2\,\frac{(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^4,x)
```

```
[Out] -2*(-1/a*tanh(1/2*x)^5-2/a^2*b*tanh(1/2*x)^4-2/3/a^3*(a^2+2*b^2)*tanh(1/2*x
)^3-2/a^2*b*tanh(1/2*x)^2-1/a*tanh(1/2*x))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)
^2)^3
```

Maxima [B] time = 1.12521, size = 672, normalized size = 10.03

$$\frac{4\,(a-b)e^{(-2\,x)}}{a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\left(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5\right)e^{(-2\,x)}+3\left(a^5-3\,a^4b+2\,a^3b^2+2\,a^2b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4*(a - b)*e^(-2*x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10
*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x)) + 4/3*a/(a^5 + a^4*b - 2*a
^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
 a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4
 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*e^(-6*x)) + 4/3*b/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*
(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a
^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 1
0*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x))
```

Fricas [B] time = 2.30457, size = 1234, normalized size = 18.42

$$-\frac{}{3\left(\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\cosh(x)^5+5\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\cosh(x)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -8/3*((2*a + b)*cosh(x) + (a + 2*b)*sinh(x))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^4 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^5 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 6*a^2*b^
3 - 9*a*b^4 - 3*b^5 + 10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*cosh(x)^2)*sinh(x)^3 + (10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*cosh(x)^3 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*cosh(x))*sinh(x)^2 + 2*(2*a^5 + a^4*b - 4*a^3*b^2 - 2*a^2*b^3
+ 2*a*b^4 + b^5)*cosh(x) + (2*a^5 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 2*a*b
^4 + 4*b^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^4 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)
^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14393, size = 72, normalized size = 1.07

$$-\frac{4\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}+a-b\right)}{3\left(a^2+2\,ab+b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*a*e^(2*x) + 3*b*e^(2*x) + a - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) +
b*e^(2*x) + a - b)^3)
```

3.589 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx$

Optimal. Leaf size=112

$$\frac{3(a\sinh(x)+b\cosh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}$$

```
[Out] (3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(5/2)) +
 (b*Cosh[x] + a*Sinh[x])/(4*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^4) + (3*(b*
Cosh[x] + a*Sinh[x]))/(8*(a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x])^2)
```

Rubi [A] time = 0.0686614, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$\frac{3(a\sinh(x)+b\cosh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] (3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(5/2)) +
 (b*Cosh[x] + a*Sinh[x])/(4*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^4) + (3*(b*
Cosh[x] + a*Sinh[x]))/(8*(a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4} + \frac{3\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx}{4\left(a^2-b^2\right)}$$

$$= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4} + \frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2} + \frac{3\int \frac{}{a\cos}}{8(}$$

$$= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4} + \frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2} + \frac{(3i)\,\mathrm{Sub}}{}$$

$$= \frac{3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}} + \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4} + \frac{3(b\cosh(x)}{8\left(a^2-b^2\right)^2(a\cos}$$

Mathematica [A] time = 0.984947, size = 147, normalized size = 1.31

$$\frac{1}{8}\left(\frac{6\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{b\left(3(a\cosh(x)+b\sinh(x))^2+2(a-b)(a+b)\right)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^3} + \frac{\sinh(x)\left(\frac{3(a\cosh(x)+b\sinh(x))^2}{(a-b)(a+b)}+2\right)}{a(a\cosh(x)+b\sinh(x))^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] ((6*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(5/2)*(a
+ b)^(5/2)) + (b*(2*(a - b)*(a + b) + 3*(a*Cosh[x] + b*Sinh[x])^2))/(a*(a -
 b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])^3) + (Sinh[x]*(2 + (3*(a*Cosh[x] +
b*Sinh[x])^2)/((a - b)*(a + b))))/(a*(a*Cosh[x] + b*Sinh[x])^4))/8
```

Maple [B] time = 0.135, size = 462, normalized size = 4.1

$$2\frac{1}{\left(a+2\,\tanh\left(x/2\right)b+a\left(\tanh\left(x/2\right)\right)^{2}\right)^{4}}\left(-1/8\,\frac{\left(5\,a^{4}-16\,a^{2}b^{2}+8\,b^{4}\right)\left(\tanh\left(x/2\right)\right)^{7}}{a\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)}-3/8\,\frac{b\left(a^{4}-16\,a^{2}b^{2}+8\,b^{4}\right)\left(\tanh\left(x\right.\right.}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^5,x)
```

```
[Out] 2*(-1/8*(5*a^4-16*a^2*b^2+8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^7-3/8*b*
(a^4-16*a^2*b^2+8*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^6+1/8/a^3*(3*a^6
+36*a^4*b^2+56*a^2*b^4-32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^5+1/8/a^4*b*
(15*a^6+114*a^4*b^2-8*a^2*b^4-16*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^4-1/8
/a^3*(3*a^6-84*a^4*b^2-56*a^2*b^4+32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^3
+1/8*b*(23*a^4+64*a^2*b^2-24*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/8
*(5*a^4+24*a^2*b^2-8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)+1/8*(5*a^2-2*b^
2)*b/(a^4-2*a^2*b^2+b^4))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^4+3/4/(a^4-2*
a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.00787, size = 15709, normalized size = 140.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

```
[Out] [1/8*(6*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^7 +
 42*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)*sinh(x)
^6 + 6*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*sinh(x)^7 +
22*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^5 + 2*(11*a^
5 + 11*a^4*b - 22*a^3*b^2 - 22*a^2*b^3 + 11*a*b^4 + 11*b^5 + 63*(a^5 + 3*a^
4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^5 + 10*(21*
(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + 11*(a^5
 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x))*sinh(x)^4 - 22*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 2*(11*a^5 - 11
*a^4*b - 22*a^3*b^2 + 22*a^2*b^3 + 11*a*b^4 - 11*b^5 - 105*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 - 110*(a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2)*sinh(x)^3 + 2*(63*(a^5 + 3*a^4*
b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + 110*(a^5 + a^4*b - 2
*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^3 - 33*(a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^2 - 3*((a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*cosh(x)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*cosh(x)*sinh(x)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)
^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^6 + 4*(a^4 + 2*a^3*b - 2*a*b
^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*sinh(x)
^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^3 + 3*(a^4 +
2*a^3*b - 2*a*b^3 - b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^4 + 3*a^
4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^2)*sinh(
x)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos
h(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^3*b +
2*a*b^3 - b^4)*cosh(x)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*cosh(x)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^4 + a^4 - 2*a^3*b +
 2*a*b^3 - b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^
3 - b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^3*b +
 2*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a - b)) - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^
4 + b^5)*cosh(x) + 2*(21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
 b^5)*cosh(x)^6 - 3*a^5 + 9*a^4*b - 6*a^3*b^2 - 6*a^2*b^3 + 9*a*b^4 - 3*b^5
 + 55*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 - 33*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x))/(a^10
 - 4*a^9*b + 3*a^8*b^2 + 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 -
3*a^2*b^8 + 4*a*b^9 - b^10 + (a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a
^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^8 + 8
*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*
b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)*sinh(x)^7 + (a^10 + 4*a^9*b + 3*a
^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*
```

b^9 - b^10)*sinh(x)^8 + 4*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b
^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(
x)^6 + 4*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 +
 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3
*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*
a*b^9 - b^10)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*
cosh(x)^3 + 3*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*
b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x))*sinh(x)^
5 + 6*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(
x)^4 + 2*(3*a^10 - 15*a^8*b^2 + 30*a^6*b^4 - 30*a^4*b^6 + 15*a^2*b^8 - 3*b^
10 + 35*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 +
 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^4 + 30*(a^10 + 2*a^9*b - 3
*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a
^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^10 + 4*a^9*b + 3*a^
8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b
^9 - b^10)*cosh(x)^5 + 10*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b
^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(
x)^3 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*co
sh(x))*sinh(x)^3 + 4*(a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 -
12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x)^2
+ 4*(a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^
4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*
b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9
 - b^10)*cosh(x)^6 + 15*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4
 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)
^4 + 9*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh
(x)^2)*sinh(x)^2 + 8*((a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4
+ 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^7 + 3*(a^10
+ 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*
a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^5 + 3*(a^10 - 5*a^8*b^2 + 10*
a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^3 + (a^10 - 2*a^9*b - 3*a^
8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*
b^8 - 2*a*b^9 + b^10)*cosh(x))*sinh(x)), 1/4*(3*(a^5 + 3*a^4*b + 2*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^7 + 21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*
a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)*sinh(x)^6 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*sinh(x)^7 + 11*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2
*b^3 + a*b^4 + b^5)*cosh(x)^5 + (11*a^5 + 11*a^4*b - 22*a^3*b^2 - 22*a^2*b^
3 + 11*a*b^4 + 11*b^5 + 63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*cosh(x)^2)*sinh(x)^5 + 5*(21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
 - 3*a*b^4 - b^5)*cosh(x)^3 + 11*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b
^4 + b^5)*cosh(x))*sinh(x)^4 - 11*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*
b^4 - b^5)*cosh(x)^3 - (11*a^5 - 11*a^4*b - 22*a^3*b^2 + 22*a^2*b^3 + 11*a*
b^4 - 11*b^5 - 105*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(x)^4 - 110*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)

^2)*sinh(x)^3 + (63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)
cosh(x)^5 + 110(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x
)^3 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(
x)^2 - 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^8 + 8*(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos
h(x)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4)*cosh(x)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x))*sinh(x)^
5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 + 2*a^3*b
- 2*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^5 + 10*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 4*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x)^2 + 4*(7*(a^4 + 4*a
^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 -
 b^4)*cosh(x)^4 + a^4 - 2*a^3*b + 2*a*b^3 - b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)
*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh
(x)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(a^2
 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 3*(a^
5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x) + (21*(a^5 + 3
*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 - 3*a^5 + 9*a^4*b
 - 6*a^3*b^2 - 6*a^2*b^3 + 9*a*b^4 - 3*b^5 + 55*(a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x))/(a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a^7*b^
3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10 + (a^1
0 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 -
 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^8 + 8*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*
a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)
*cosh(x)*sinh(x)^7 + (a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 +
 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*sinh(x)^8 + 4*(a^10 +
 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a
^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^6 + 4*(a^10 + 2*a^9*b - 3*a^8*
b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^
8 + 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4
 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^2)*sinh(x)^
6 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6
+ 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^3 + 3*(a^10 + 2*a^9*b - 3
*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a
^2*b^8 + 2*a*b^9 + b^10)*cosh(x))*sinh(x)^5 + 6*(a^10 - 5*a^8*b^2 + 10*a^6*
b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^4 + 2*(3*a^10 - 15*a^8*b^2 + 3
0*a^6*b^4 - 30*a^4*b^6 + 15*a^2*b^8 - 3*b^10 + 35*(a^10 + 4*a^9*b + 3*a^8*b
^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9
- b^10)*cosh(x)^4 + 30*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4

```
+ 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^
2)*sinh(x)^4 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 +
14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^5 + 10*(a^10 +
 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a
^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^3 + 3*(a^10 - 5*a^8*b^2 + 10*a
^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x))*sinh(x)^3 + 4*(a^10 - 2*a^
9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^
7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x)^2 + 4*(a^10 - 2*a^9*b - 3*a^8*b^2 +
 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2
*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14
*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^6 + 15*(a^10 + 2
*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3
*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^4 + 9*(a^10 - 5*a^8*b^2 + 10*a^6
*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^2)*sinh(x)^2 + 8*((a^10 + 4*a
^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*
b^8 - 4*a*b^9 - b^10)*cosh(x)^7 + 3*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3
 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b
^10)*cosh(x)^5 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8
- b^10)*cosh(x)^3 + (a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 1
2*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x))*si
nh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16447, size = 319, normalized size = 2.85

$$\frac{3\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{4\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{3a^{3}e^{(7x)}+9a^{2}be^{(7x)}+9ab^{2}e^{(7x)}+3b^{3}e^{(7x)}+11a^{3}e^{(5x)}+11a^{2}be^{(5x)}-11ab^{2}e^{(5x)}-11b^{3}}{4\left(a^{4}-2a^{2}b^{2}+b^{4}\right)(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 3/4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a
^2 - b^2)) + 1/4*(3*a^3*e^(7*x) + 9*a^2*b*e^(7*x) + 9*a*b^2*e^(7*x) + 3*b^3
*e^(7*x) + 11*a^3*e^(5*x) + 11*a^2*b*e^(5*x) - 11*a*b^2*e^(5*x) - 11*b^3*e^
(5*x) - 11*a^3*e^(3*x) + 11*a^2*b*e^(3*x) + 11*a*b^2*e^(3*x) - 11*b^3*e^(3*
x) - 3*a^3*e^x + 9*a^2*b*e^x - 9*a*b^2*e^x + 3*b^3*e^x)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*e^(2*x) + b*e^(2*x) + a - b)^4)
```

3.590 $\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=65

$$-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x
]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rubi [A] time = 0.0273524, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3078, 2639}

$$-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x
]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx = \frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}\,dx}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Mathematica [C] time = 0.702785, size = 206, normalized size = 3.17

$$\frac{b\left(b^2-a^2\right)\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cosh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\right)+\sqrt{-\sinh^2\left(\tanh\right.}}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] (b*(-a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a
]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a^3*Sqrt[
1 - b^2/a^2]*Cosh[x] - 2*a*(a^2 - b^2)*Cosh[x + ArcTanh[b/a]] + 2*a^2*b*Sqr
t[1 - b^2/a^2]*Sinh[x] + a^2*b*Sinh[x + ArcTanh[b/a]] - b^3*Sinh[x + ArcTan
h[b/a]]))/(a*b*Sqrt[1 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x +
 ArcTanh[b/a]]^2])
```

Maple [A] time = 0.129, size = 33, normalized size = 0.5

$$-\cosh(x)\sqrt{a^2-b^2}\frac{1}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^(1/2),x)
```

```
[Out] -(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cosh(x) + b\sinh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.591 $\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3 - (((2*I)/3)*(a^2
 - b^2)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rubi [A] time = 0.0532752, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3073, 3078, 2641}

$$\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3 - (((2*I)/3)*(a^2
 - b^2)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
```

```
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)}+\frac{1}{3}\left(a^2-b^2\right)\int\frac{1}{\sqrt{a\cosh(x)+b\sinh}}$$

$$=\frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)}+\frac{\left(\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\right)\int}{3\sqrt{a\cosh(x)+b\sin}}$$

$$=\frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i\left(a^2-b^2\right)F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib\right.\right.}{3\sqrt{a\cosh(x)+b}}$$

Mathematica [C] time = 0.573857, size = 92, normalized size = 0.89

$$\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}\left(-b\sqrt{1-\frac{a^2}{b^2}}\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\text{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] - Sqrt[1 - a^2/b^2]*b*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/
b]] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3
```

Maple [A] time = 0.238, size = 171, normalized size = 1.7

$$-\frac{1}{2\left(\sinh(x)\right)^2}\sqrt{-\sqrt{a^2-b^2}\left(\sinh(x)\right)^3}\left(\cosh(x)\sqrt{\sinh(x)\sqrt{a^2-b^2}}\sqrt{-\sqrt{a^2-b^2}\left(\sinh(x)\right)^3}\left(a^2-b^2\right)+\sinh(x)\,\text{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^(3/2),x)
```

```
[Out] -1/2*(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)*(cosh(x)*(sinh(x)*(a^2-b^2)^(1/2))^
(1/2)*(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)*(a^2-b^2)+sinh(x)*arctan((sinh(x)*
(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2))*(a^2-b^2
)^(3/2))/sinh(x)^2/(a^2-b^2)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)/(-sinh(x
)*(a^2-b^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x) + b\sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\cosh(x) + b\sinh(x))^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cosh(x) + b*sinh(x))^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x) + b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.592 $\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^(3/2))/5 - (((6*I)/5)*(a
^2 - b^2)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh
[x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rubi [A] time = 0.0501036, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3073, 3078, 2639}

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^(3/2))/5 - (((6*I)/5)*(a
^2 - b^2)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh
[x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
```

```
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{1}{5}\left(3\left(a^2-b^2\right)\right)\int \sqrt{a\cosh(x)+$$

$$=\frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{\left(3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh}\right.}{5\sqrt{\frac{a\cos}{}}}$$

$$=\frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,\right.\right.}{5\sqrt{\frac{a\cosh(}{\sqrt{}}}}$$

Mathematica [C] time = 0.813822, size = 193, normalized size = 1.87

$$\frac{(a\cosh(x)+b\sinh(x))\left(b\left(a^2+b^2\right)\sinh(2x)+6a\left(a^2-b^2\right)+2ab^2\cosh(2x)\right)-\frac{3(a-b)^2(a+b)^2\left(b\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\text{Hypergeome}\right.}{}}{5b\sqrt{a\cosh(x)+b\sinh(x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

```
[Out] ((a*Cosh[x] + b*Sinh[x])*(6*a*(a^2 - b^2) + 2*a*b^2*Cosh[2*x] + b*(a^2 + b^
2)*Sinh[2*x]) - (3*(a - b)^2*(a + b)^2*(b*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + Arc
Tanh[b/a]]^2]*(2*a*Cosh[x + ArcTanh[b/a]] - b*Sinh[x + ArcTanh[b/a]])))/(a*
Sqrt[1 - b^2/a^2]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2]))/(5*b*Sqrt[a*Cosh[x] + b
*Sinh[x]])
```

Maple [A] time = 0.152, size = 51, normalized size = 0.5

$$\left(-\frac{(\cosh(x))^3}{3}\left(a^2-b^2\right)^{\frac{3}{2}}+\left(a^2-b^2\right)^{\frac{3}{2}}\cosh(x)\right)\frac{1}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^(5/2),x)
```

```
[Out] 1/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*(-1/3*(a^2-b^2)^(3/2)*cosh(x)^3+(a^2-b^2
)^(3/2)*cosh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(a\cosh(x)+b\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cosh(x)^2+2\,ab\cosh(x)\sinh(x)+b^2\sinh(x)^2\right)\sqrt{a\cosh(x)+b\sinh(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + b^2*sinh(x)^2)*sqrt(a*cos
h(x) + b*sinh(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x) + b\sinh(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.593 $\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rubi [A] time = 0.0268969, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3078, 2641}

$$-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}}\,dx}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$= -\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

Mathematica [C] time = 0.0926322, size = 81, normalized size = 1.25

$$\frac{2\sqrt{a\cosh(x)+b\sinh(x)}\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\operatorname{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},-\sinh^2\right.}{b\sqrt{1-\frac{a^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sin
h[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/b]]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
(Sqrt[1 - a^2/b^2]*b)
```

Maple [A] time = 0.19, size = 97, normalized size = 1.5

$$\frac{1}{\sinh(x)}\sqrt{-\sqrt{a^2-b^2}(\sinh(x))^3}\arctan\left(\cosh(x)\sqrt{\sinh(x)\sqrt{a^2-b^2}}\frac{1}{\sqrt{-\sqrt{a^2-b^2}(\sinh(x))^3}}\right)\frac{1}{\sqrt{\sinh(x)\sqrt{a^2-b^2}}}\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(1/2),x)
```

```
[Out] (-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctan((
sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2))/
sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.594 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]]) + ((2
*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
((a^2 - b^2)*Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])
```

Rubi [A] time = 0.0505367, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3076, 3078, 2639}

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]]) + ((2
*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
((a^2 - b^2)*Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
```

```
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}} + \frac{\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx}{-a^2+b^2}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}} + \frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\tan^{-1}(a,\right.}}{\left(-a^2+b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}} + \frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{a\cosh(x)+b\,\mathrm{si}}}{\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Mathematica [C] time = 0.446826, size = 148, normalized size = 1.32

$$\frac{b\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\mathrm{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cosh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\right)-\sqrt{-\sinh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+\right.}}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{-\sinh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)}\sqrt{a\cosh(x)\ -}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x
+ ArcTanh[b/a]] - Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a*Sqrt[1 - b^2/a^2]*Co
sh[x] - 2*a*Cosh[x + ArcTanh[b/a]] + b*Sinh[x + ArcTanh[b/a]]))/(a*b*Sqrt[1
 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2])
```

Maple [A] time = 0.12, size = 33, normalized size = 0.3

$$\operatorname{Artanh}\left(\cosh\left(x\right)\right)\frac{1}{\sqrt{a^{2}-b^{2}}}\frac{1}{\sqrt{-\sinh\left(x\right)\sqrt{a^{2}-b^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(3/2),x)
```

```
[Out] 1/(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctanh(cosh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cosh\left(x\right)+b\sinh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a\cosh\left(x\right)+b\sinh\left(x\right)}}{a^{2}\cosh\left(x\right)^{2}+2\,ab\cosh\left(x\right)\sinh\left(x\right)+b^{2}\sinh\left(x\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x))/(a^2*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x)
 + b^2*sinh(x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cosh\left(x\right)+b\sinh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x) + b*sinh(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

3.595 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2(a\sinh(x)+b\cosh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^(3/2)) -
 (((2*I)/3)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*S
inh[x])/Sqrt[a^2 - b^2]])/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]])
```

Rubi [A] time = 0.0481128, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3076, 3078, 2641}

$$\frac{2(a\sinh(x)+b\cosh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```


```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^(3/2)) -
 (((2*I)/3)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*S
inh[x])/Sqrt[a^2 - b^2]])/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]])
```


Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
```

```
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}}\,dx}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{\frac{a\cosh(x)+b\sin}{\sqrt{a^2-b^2}}}}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

Mathematica [C] time = 0.55822, size = 133, normalized size = 1.15

$$-\frac{2\left((a\cosh(x)+b\sinh(x))^2\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\text{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},-\text{si}\right.\right.}{3b\sqrt{1-\frac{a^2}{b^2}}(b-a)(a+b)(a\cosh(x)+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(Sqrt[1 - a^2/b^2]*b*(b*Cosh[x] + a*Sinh[x]) + Sqrt[Cosh[x + ArcTanh[a/
b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech
[x + ArcTanh[a/b]]*(a*Cosh[x] + b*Sinh[x])^2))/(3*Sqrt[1 - a^2/b^2]*b*(-a +
 b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^(3/2))
```

Maple [A] time = 0.144, size = 37, normalized size = 0.3

$$-\frac{\cosh(x)}{(a^2-b^2)\sinh(x)}\frac{1}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(5/2),x)
```

```
[Out] -cosh(x)/(a^2-b^2)/sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)+b\sinh(x)}}{a^3\cosh(x)^3+3\,a^2b\cosh(x)^2\sinh(x)+3\,ab^2\cosh(x)\sinh(x)^2+b^3\sinh(x)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x))/(a^3*cosh(x)^3 + 3*a^2*b*cosh(x)^2*sin
h(x) + 3*a*b^2*cosh(x)*sinh(x)^2 + b^3*sinh(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

3.596 $\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

```
[Out] (a*Cosh[c + d*x])/d + (a*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0152554, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c + d*x])/d + (a*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx &= a\int \cosh(c+dx)\,dx+a\int \sinh(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0169584, size = 45, normalized size = 1.96

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d
 + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.009, size = 19, normalized size = 0.8

$$\frac{a\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(d*x+c)+a*sinh(d*x+c),x)
```

```
[Out] a*(cosh(d*x+c)+sinh(d*x+c))/d
```

Maxima [A] time = 1.06774, size = 31, normalized size = 1.35

$$\frac{a\cosh\left(dx+c\right)}{d}+\frac{a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

Fricas [A] time = 1.97798, size = 53, normalized size = 2.3

$$\frac{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

Sympy [A] time = 0.254128, size = 29, normalized size = 1.26

$$a\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\ x\cosh\left(c\right) & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{\cosh\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\ x\sinh\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x)
```

```
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Giac [B] time = 1.14229, size = 76, normalized size = 3.3

$$\frac{1}{2}\,a\left(\frac{e^{\left(dx+c\right)}}{d}+\frac{e^{\left(-dx-c\right)}}{d}\right)+\frac{1}{2}\,a\left(\frac{e^{\left(dx+c\right)}}{d}-\frac{e^{\left(-dx-c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x - c
)/d)
```

3.597 $\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^2/(2*d)
```

Rubi [A] time = 0.0162527, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^2/(2*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^2}{2d}$$

Mathematica [A] time = 0.0507779, size = 25, normalized size = 0.96

$$\frac{a^2(\sinh(c+dx)+\cosh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

```
[Out] (a^2*(Cosh[c + d*x] + Sinh[c + d*x])^2)/(2*d)
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\frac{a^2 (\cosh(dx+c)+\sinh(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*(cosh(d*x+c)+sinh(d*x+c))^2/d
```

Maxima [B] time = 1.06098, size = 119, normalized size = 4.58

$$\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{a^2\cosh(dx+c)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*cosh(d*x + c)^2/d
```

Fricas [A] time = 1.98951, size = 109, normalized size = 4.19

$$\frac{a^2\cosh(dx+c)+a^2\sinh(dx+c)}{2(d\cosh(dx+c)-d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cosh(d*x + c) + a^2*sinh(d*x + c))/(d*cosh(d*x + c) - d*sinh(d*x +
 c))
```

Sympy [A] time = 0.485086, size = 44, normalized size = 1.69

$$\begin{cases} \frac{a^2 \sinh(c+dx)\cosh(c+dx)}{d} + \frac{a^2 \cosh^2(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\sinh(c) + a\cosh(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)*cosh(c + d*x)/d + a**2*cosh(c + d*x)**2/d, Ne
(d, 0)), (x*(a*sinh(c) + a*cosh(c))**2, True))
```

Giac [A] time = 1.13193, size = 23, normalized size = 0.88

$$\frac{a^2 e^{(2\,dx+2\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*e^(2*d*x + 2*c)/d
```

3.598 $\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^3}{3d}$$

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^3/(3*d)
```

Rubi [A] time = 0.0151791, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^3/(3*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^3}{3d}$$

Mathematica [A] time = 0.0763161, size = 25, normalized size = 0.96

$$\frac{a^3(\sinh(c+dx)+\cosh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

```
[Out] (a^3*(Cosh[c + d*x] + Sinh[c + d*x])^3)/(3*d)
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\frac{a^3(\cosh(dx+c)+\sinh(dx+c))^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x)
```

```
[Out] 1/3*a^3*(cosh(d*x+c)+sinh(d*x+c))^3/d
```

Maxima [B] time = 1.06453, size = 197, normalized size = 7.58

$$\frac{a^3\cosh(dx+c)^3}{d}+\frac{a^3\sinh(dx+c)^3}{d}+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/d
 + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 1/24*a^3*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.95203, size = 163, normalized size = 6.27

$$\frac{a^3\cosh(dx+c)^2+2\,a^3\cosh(dx+c)\sinh(dx+c)+a^3\sinh(dx+c)^2}{3\,(d\cosh(dx+c)-d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x
 + c)^2)/(d*cosh(d*x + c) - d*sinh(d*x + c))
```

Sympy [A] time = 0.877147, size = 83, normalized size = 3.19

$$\begin{cases} \frac{a^3 \sinh^3(c+dx)}{3d} + \frac{a^3 \sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3 \sinh(c+dx)\cosh^2(c+dx)}{d} + \frac{a^3 \cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x\,(a \sinh(c) + a \cosh(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d + a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(a*sinh(c) + a*cosh(c))**3, True))
```

Giac [A] time = 1.15384, size = 23, normalized size = 0.88

$$\frac{a^3 e^{(3\,dx+3\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*a^3*e^(3*d*x + 3*c)/d
```

3.599 $\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^n/(d*n)
```

Rubi [A] time = 0.0151521, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^n/(d*n)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^n}{dn}$$

Mathematica [A] time = 0.0741302, size = 24, normalized size = 0.92

$$\frac{(a(\sinh(c+dx)+\cosh(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

```
[Out] (a*(Cosh[c + d*x] + Sinh[c + d*x]))^n/(d*n)
```

Maple [A] time = 0.009, size = 27, normalized size = 1.

$$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x)
```

```
[Out] (a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

Maxima [A] time = 1.02406, size = 24, normalized size = 0.92

$$\frac{a^n e^{((dx+c)n)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] a^n*e^((d*x + c)*n)/(d*n)
```

Fricas [A] time = 2.0289, size = 93, normalized size = 3.58

$$\frac{\cosh(dnx+cn+n\log(a))+\sinh(dnx+cn+n\log(a))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (cosh(d*n*x + c*n + n*log(a)) + sinh(d*n*x + c*n + n*log(a)))/(d*n)
```

Sympy [A] time = 0.283659, size = 36, normalized size = 1.38

$$\begin{cases} x & \text{for } d = 0 \wedge n = 0 \\ x\,(a\sinh(c) + a\cosh(c))^n & \text{for } d = 0 \\ x & \text{for } n = 0 \\ \frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(a*sinh(c) + a*cosh(c))**n, Eq(d, 0)
), (x, Eq(n, 0)), ((a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(dx + c) + a\sinh(dx + c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(d*x + c) + a*sinh(d*x + c))^n, x)
```

3.600 $\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

```
[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

Rubi [A] time = 0.0164745, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
```

```
[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=-\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

Mathematica [A] time = 0.0382466, size = 24, normalized size = 1.

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
```

```
[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$-\frac{1}{da\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
```

```
[Out] -1/d/a/(cosh(d*x+c)+sinh(d*x+c))
```

Maxima [A] time = 1.08815, size = 23, normalized size = 0.96

$$-\frac{e^{(-dx-c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^(-d*x - c)/(a*d)
```

Fricas [A] time = 1.9821, size = 59, normalized size = 2.46

$$-\frac{1}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/(a*d*cosh(d*x + c) + a*d*sinh(d*x + c))
```

Sympy [A] time = 0.624938, size = 34, normalized size = 1.42

$$\begin{cases} -\frac{1}{ad\sinh{(c+dx)}+ad\cosh{(c+dx)}} & \text{for } d \neq 0 \\ \frac{x}{a\sinh{(c)}+a\cosh{(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((-1/(a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(a*sin
h(c) + a*cosh(c)), True))
```

Giac [A] time = 1.12213, size = 23, normalized size = 0.96

$$-\frac{e^{(-dx-c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -e^(-d*x - c)/(a*d)
```

3.601 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2} dx$

Optimal. Leaf size=26

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

```
[Out] -1/(2*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Rubi [A] time = 0.0158443, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/(2*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2} dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

Mathematica [A] time = 0.0425167, size = 26, normalized size = 1.

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/(2*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$-\frac{1}{2\,da^{2}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2/(cosh(d*x+c)+sinh(d*x+c))^2
```

Maxima [A] time = 1.15344, size = 23, normalized size = 0.88

$$-\frac{e^{\left(-2\,dx-2\,c\right)}}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

Fricas [B] time = 1.98545, size = 124, normalized size = 4.77

$$-\frac{1}{2\left(a^{2}d\cosh\left(dx+c\right)^{2}+2\,a^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^{2}d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*s
inh(d*x + c)^2)
```

Sympy [A] time = 1.2183, size = 66, normalized size = 2.54

$$\begin{cases} -\frac{1}{2a^2d\sinh^2(c+dx)+4a^2d\sinh(c+dx)\cosh(c+dx)+2a^2d\cosh^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sinh(c + d*x)**2 + 4*a**2*d*sinh(c + d*x)*cosh(c +
d*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**2
, True))
```

Giac [A] time = 1.13492, size = 23, normalized size = 0.88

$$-\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

3.602 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

```
[Out] -1/(3*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Rubi [A] time = 0.0158623, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

```
[Out] -1/(3*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

Mathematica [A] time = 0.0456126, size = 26, normalized size = 1.

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

```
[Out] -1/(3*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Maple [A] time = 0., size = 24, normalized size = 0.9

$$-\frac{1}{3\,da^{3}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3/(cosh(d*x+c)+sinh(d*x+c))^3
```

Maxima [A] time = 1.07047, size = 23, normalized size = 0.88

$$-\frac{e^{\left(-3\,dx-3\,c\right)}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

Fricas [B] time = 1.94219, size = 181, normalized size = 6.96

$$-\frac{1}{3\left(a^{3}d\cosh\left(dx+c\right)^{3}+3\,a^{3}d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a^{3}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a^{3}d\sinh\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3
*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*sinh(d*x + c)^3)
```

Sympy [A] time = 2.20831, size = 90, normalized size = 3.46

$$\begin{cases} -\frac{1}{3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)+9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((-1/(3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) + 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**3, True))
```

Giac [A] time = 1.17145, size = 23, normalized size = 0.88

$$-\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

3.603 $\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

```
[Out] (2*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0161512, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3071}

$$\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}{d}$$

Mathematica [A] time = 0.0223419, size = 24, normalized size = 0.92

$$\frac{2\sqrt{a(\sinh(c+dx)+\cosh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])/d
```

Maple [A] time = 0.002, size = 25, normalized size = 1.

$$2\,\frac{\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x)
```

```
[Out] 2*(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)/d
```

Maxima [A] time = 1.01627, size = 23, normalized size = 0.88

$$\frac{2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

Fricas [A] time = 2.00343, size = 61, normalized size = 2.35

$$\frac{2\,\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sinh{\left(c + dx \right)} + a \cosh{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Giac [A] time = 1.16006, size = 23, normalized size = 0.88

$$\frac{2 \sqrt{a} e^{\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

3.604 $\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx$

Optimal. Leaf size=26

$$-\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

```
[Out] -2/(d*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])
```

Rubi [A] time = 0.0170826, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3071}

$$-\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] -2/(d*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx=-\frac{2}{d\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}$$

Mathematica [A] time = 0.0344189, size = 24, normalized size = 0.92

$$-\frac{2}{d\sqrt{a(\sinh(c+dx)+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] -2/(d*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])
```

Maple [A] time = 0.003, size = 25, normalized size = 1.

$$-2\frac{1}{d\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x)
```

```
[Out] -2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)
```

Maxima [A] time = 1.07291, size = 23, normalized size = 0.88

$$-\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

Fricas [A] time = 1.95519, size = 113, normalized size = 4.35

$$-\frac{2\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/(a*d*cosh(d*x + c) + a*d*sinh(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Giac [A] time = 1.14864, size = 23, normalized size = 0.88

$$-\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

3.605 $\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

```
[Out] -((a*Cosh[c + d*x])/d) + (a*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0148298, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637, 2638}

$$\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d) + (a*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx &= a\int \cosh(c+dx)\,dx-a\int \sinh(c+dx)\,dx\\ &= -\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0185646, size = 47, normalized size = 1.96

$$-\frac{a\sinh(c)\sinh(dx)}{d}-\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

```
[Out] -((a*Cosh[c]*Cosh[d*x])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x]
)/d - (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.002, size = 21, normalized size = 0.9

$$\frac{a(\sinh(dx+c)-\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(d*x+c)-a*sinh(d*x+c),x)
```

```
[Out] a*(sinh(d*x+c)-cosh(d*x+c))/d
```

Maxima [A] time = 0.987686, size = 32, normalized size = 1.33

$$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] -a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

Fricas [A] time = 1.84096, size = 54, normalized size = 2.25

$$-\frac{a}{d\cosh(dx+c)+d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -a/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [A] time = 0.198325, size = 29, normalized size = 1.21

$$a\left(\begin{cases}\frac{\sinh(c+dx)}{d} & \text{for } d \neq 0\\ x\cosh(c) & \text{otherwise}\end{cases}\right) - a\left(\begin{cases}\frac{\cosh(c+dx)}{d} & \text{for } d \neq 0\\ x\sinh(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x)
```

```
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) - a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Giac [B] time = 1.13443, size = 76, normalized size = 3.17

$$-\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x -
c)/d)
```

3.606 $\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx$

Optimal. Leaf size=27

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/(2*d)
```

Rubi [A] time = 0.0150284, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/(2*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

Mathematica [A] time = 0.0280605, size = 27, normalized size = 1.

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/(2*d)
```

Maple [A] time = 0., size = 26, normalized size = 1.

$$-\frac{a^{2}\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^{2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*(cosh(d*x+c)-sinh(d*x+c))^2/d
```

Maxima [B] time = 1.07296, size = 120, normalized size = 4.44

$$\frac{1}{8}\,a^{2}\left(4\,x+\frac{e^{\left(2\,dx+2\,c\right)}}{d}-\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-\frac{1}{8}\,a^{2}\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-\frac{a^{2}\cosh\left(dx+c\right)^{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - a^2*cosh(d*x + c)^2/d
```

Fricas [A] time = 1.95543, size = 113, normalized size = 4.19

$$-\frac{a^{2}}{2\left(d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*a^2/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2)
```

Sympy [A] time = 0.267267, size = 44, normalized size = 1.63

$$\begin{cases} \frac{a^2 \sinh(c+dx)\cosh(c+dx)}{d} - \frac{a^2\cosh^2(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(-a\sinh(c) + a\cosh(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)*cosh(c + d*x)/d - a**2*cosh(c + d*x)**2/d, Ne
(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**2, True))
```

Giac [A] time = 1.12694, size = 23, normalized size = 0.85

$$-\frac{a^2 e^{(-2\,dx-2\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*a^2*e^(-2*d*x - 2*c)/d
```

3.607 $\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/(3*d)
```

Rubi [A] time = 0.0148465, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/(3*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Mathematica [A] time = 0.0231849, size = 27, normalized size = 1.

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/(3*d)
```

Maple [A] time = 0., size = 26, normalized size = 1.

$$-\frac{a^3 (\cosh(dx+c) - \sinh(dx+c))^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*(cosh(d*x+c)-sinh(d*x+c))^3/d
```

Maxima [B] time = 1.06504, size = 198, normalized size = 7.33

$$-\frac{a^3 \cosh(dx+c)^3}{d} + \frac{a^3 \sinh(dx+c)^3}{d} + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) - \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx-}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/
d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) - 1/24*a^3*(e^
(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 2.01943, size = 165, normalized size = 6.11

$$-\frac{a^3}{3\left(d\cosh(dx+c)^3 + 3\,d\cosh(dx+c)^2\sinh(dx+c) + 3\,d\cosh(dx+c)\sinh(dx+c)^2 + d\sinh(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*a^3/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [A] time = 0.676045, size = 83, normalized size = 3.07

$$\begin{cases} \frac{a^3 \sinh^3(c+dx)}{3d} - \frac{a^3 \sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3 \sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{a^3 \cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(-a\sinh(c) + a\cosh(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) - a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d - a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**3, True))
```

Giac [A] time = 1.15167, size = 23, normalized size = 0.85

$$-\frac{a^3 e^{(-3\,dx-3\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*a^3*e^(-3*d*x - 3*c)/d
```

3.608 $\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx$

Optimal. Leaf size=28

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

```
[Out] -((a*Cosh[c + d*x] - a*Sinh[c + d*x])^n/(d*n))
```

Rubi [A] time = 0.0158082, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

```
[Out] -((a*Cosh[c + d*x] - a*Sinh[c + d*x])^n/(d*n))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

Mathematica [A] time = 0.0449479, size = 27, normalized size = 0.96

$$-\frac{(a(\cosh(c+dx)-\sinh(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]

[Out] -((a*(Cosh[c + d*x] - Sinh[c + d*x]))^n/(d*n))
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x)

[Out] -(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

Maxima [A] time = 1.05378, size = 27, normalized size = 0.96

$$-\frac{a^n e^{(-(dx+c)n)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="maxima")

[Out] -a^n*e^(-(d*x + c)*n)/(d*n)
```

Fricas [A] time = 2.16816, size = 97, normalized size = 3.46

$$-\frac{\cosh(-dnx-cn+n\log(a))+\sinh(-dnx-cn+n\log(a))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="fricas")

[Out] -(cosh(-d*n*x - c*n + n*log(a)) + sinh(-d*n*x - c*n + n*log(a)))/(d*n)
```

Sympy [A] time = 0.238199, size = 37, normalized size = 1.32

$$\begin{cases} x & \text{for } d = 0 \wedge n = 0 \\ x\left(-a\sinh\left(c\right) + a\cosh\left(c\right)\right)^{n} & \text{for } d = 0 \\ x & \text{for } n = 0 \\ -\frac{\left(-a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)\right)^{n}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(-a*sinh(c) + a*cosh(c))**n, Eq(d, 0
)), (x, Eq(n, 0)), (-(-a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cosh\left(dx + c\right) - a\sinh\left(dx + c\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(d*x + c) - a*sinh(d*x + c))^n, x)
```

3.609 $\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

```
[Out] 1/(d*(a*Cosh[c + d*x] - a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0155538, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]
```

```
[Out] 1/(d*(a*Cosh[c + d*x] - a*Sinh[c + d*x]))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

Mathematica [A] time = 0.0074059, size = 22, normalized size = 0.92

$$\frac{1}{ad\cosh(c+dx)-ad\sinh(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]
```

```
[Out] (a*d*Cosh[c + d*x] - a*d*Sinh[c + d*x])^(-1)
```

Maple [A] time = 0., size = 25, normalized size = 1.

$$\frac{1}{da\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
```

```
[Out] 1/d/a/(cosh(d*x+c)-sinh(d*x+c))
```

Maxima [A] time = 1.03732, size = 18, normalized size = 0.75

$$\frac{e^{(dx+c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(d*x + c)/(a*d)
```

Fricas [A] time = 2.2483, size = 53, normalized size = 2.21

$$\frac{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (cosh(d*x + c) + sinh(d*x + c))/(a*d)
```

Sympy [A] time = 0.475574, size = 32, normalized size = 1.33

$$\begin{cases} \frac{1}{-ad\sinh(c+dx)+ad\cosh(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{-a\sinh(c)+a\cosh(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((1/(-a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(-a*si
nh(c) + a*cosh(c)), True))
```

Giac [A] time = 1.12332, size = 18, normalized size = 0.75

$$\frac{e^{(dx+c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] e^(d*x + c)/(a*d)
```

3.610 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Rubi [A] time = 0.0153704, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Mathematica [A] time = 0.044446, size = 27, normalized size = 1.

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Maple [A] time = 0.001, size = 26, normalized size = 1.

$$\frac{1}{2\,da^{2}\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2/(cosh(d*x+c)-sinh(d*x+c))^2
```

Maxima [A] time = 1.06942, size = 23, normalized size = 0.85

$$\frac{e^{\left(2\,dx+2\,c\right)}}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

Fricas [A] time = 2.17896, size = 109, normalized size = 4.04

$$\frac{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}{2\left(a^{2}d\cosh\left(dx+c\right)-a^{2}d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(d*x + c) + sinh(d*x + c))/(a^2*d*cosh(d*x + c) - a^2*d*sinh(d*x +
 c))
```

Sympy [A] time = 0.923225, size = 65, normalized size = 2.41

$$\begin{cases} \frac{1}{2a^2d\sinh^2(c+dx)-4a^2d\sinh(c+dx)\cosh(c+dx)+2a^2d\cosh^2(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((1/(2*a**2*d*sinh(c + d*x)**2 - 4*a**2*d*sinh(c + d*x)*cosh(c + d
*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**2
, True))
```

Giac [A] time = 1.11292, size = 23, normalized size = 0.85

$$\frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

3.611 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Rubi [A] time = 0.0155465, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3071}

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Mathematica [A] time = 0.0683504, size = 27, normalized size = 1.

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Maple [A] time = 0.001, size = 26, normalized size = 1.

$$\frac{1}{3\,da^{3}\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x)
```

```
[Out] 1/3/d/a^3/(cosh(d*x+c)-sinh(d*x+c))^3
```

Maxima [A] time = 1.08383, size = 23, normalized size = 0.85

$$\frac{e^{(3\,dx+3\,c)}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

Fricas [B] time = 2.36608, size = 158, normalized size = 5.85

$$\frac{\cosh\left(dx+c\right)^{2}+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}}{3\left(a^{3}d\cosh\left(dx+c\right)-a^{3}d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)/(a^
3*d*cosh(d*x + c) - a^3*d*sinh(d*x + c))
```

Sympy [A] time = 1.74038, size = 88, normalized size = 3.26

$$\begin{cases} \frac{1}{-3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)-9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((1/(-3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) - 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**3, True))
```

Giac [A] time = 1.13889, size = 23, normalized size = 0.85

$$\frac{e^{(3\,dx+3\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

3.612 $\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

```
[Out] (-2*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0158502, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

Mathematica [A] time = 0.0220003, size = 26, normalized size = 0.96

$$-\frac{2\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])/d
```

Maple [A] time = 0., size = 26, normalized size = 1.

$$-2\,{\frac {\sqrt {a\cosh \left(dx+c \right) -a\sinh \left(dx+c \right) }}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x)
```

```
[Out] -2*(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)/d
```

Maxima [A] time = 1.03767, size = 23, normalized size = 0.85

$$-\frac{2\,\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

Fricas [A] time = 2.26464, size = 62, normalized size = 2.3

$$-\frac{2\,\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a \sinh \left(c+d x\right)+a \cosh \left(c+d x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Giac [A] time = 1.15519, size = 23, normalized size = 0.85

$$-\frac{2\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

3.613 $\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

```
[Out] 2/(d*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])
```

Rubi [A] time = 0.016832, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] 2/(d*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

Mathematica [A] time = 0.0289077, size = 26, normalized size = 0.96

$$\frac{2}{d\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] 2/(d*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])
```

Maple [A] time = 0.001, size = 26, normalized size = 1.

$$2\frac{1}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)
```

Maxima [A] time = 1.02596, size = 23, normalized size = 0.85

$$\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

Fricas [A] time = 2.34977, size = 109, normalized size = 4.04

$$\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}(\cosh(dx+c)+\sinh(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c))/(
a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Giac [A] time = 1.15991, size = 23, normalized size = 0.85

$$\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

3.614 $\int (a\,\mathbf{sech}(x) + b\tanh(x))^5\,dx$

Optimal. Leaf size=124

$$-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)+\frac{1}{8}a\left(10a^2b^2+3a^4+15b^4\right)\tan^{-1}(\sinh(x))-\frac{1}{8}\mathbf{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3\right.\right.$$

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/8 + b^5*Log[Cosh[x]] - (a
*b^2*(3*a^2 + 7*b^2)*Sinh[x])/8 - (Sech[x]^4*(b - a*Sinh[x])*(a + b*Sinh[x]
)^4)/4 - (Sech[x]^2*(a + b*Sinh[x])^2*(2*b*(a^2 + 2*b^2) - a*(3*a^2 + 5*b^2
)*Sinh[x]))/8
```

Rubi [A] time = 0.185464, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2668, 739, 819, 774, 635, 204, 260}

$$-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)+\frac{1}{8}a\left(10a^2b^2+3a^4+15b^4\right)\tan^{-1}(\sinh(x))-\frac{1}{8}\mathbf{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^5,x]
```


```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/8 + b^5*Log[Cosh[x]] - (a
*b^2*(3*a^2 + 7*b^2)*Sinh[x])/8 - (Sech[x]^4*(b - a*Sinh[x])*(a + b*Sinh[x]
)^4)/4 - (Sech[x]^2*(a + b*Sinh[x])^2*(2*b*(a^2 + 2*b^2) - a*(3*a^2 + 5*b^2
)*Sinh[x]))/8
```


Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (a\text{sech}(x)+b\tanh(x))^5\,dx = \int \text{sech}^5(x)(a+b\sinh(x))^5\,dx$$
$$= -\left(b^5\,\text{Subst}\left(\int \frac{(a+x)^5}{\left(-b^2-x^2\right)^3}\,dx, x, b\sinh(x)\right)\right)$$
$$= -\frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 - \frac{1}{4}b^3\,\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2-4b^2+ax\right)}{\left(-b^2-x^2\right)^2}\right.$$
$$= -\frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-\right.$$
$$= -\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x) - \frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a+$$
$$= -\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x) - \frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a+$$
$$= \frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)\tan^{-1}(\sinh(x)) + b^5\log(\cosh(x)) - \frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)$$

<u>Mathematica</u> [B] time = 1.87455, size = 355, normalized size = 2.86

$$\frac{b\left(2ab^5\left(5b^2-3a^2\right)\sinh^5(x)+4b^4\left(12a^2b^2-9a^4+b^4\right)\sinh^4(x)+10ab^3\left(8a^2b^2-9a^4+b^4\right)\sinh^3(x)-8b^2\left(-4a^4b^2+2a^2b^4+15a^6+b^6\right)\sinh^2(x)-10ab\left(6a^4b^2+8a^2b^4+9a^6+3b\right.\right.}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^5,x]
```

```
[Out] (4*Sech[x]^4*(b + a*Sinh[x])*(a + b*Sinh[x])^6 + (2*Sech[x]^2*(a + b*Sinh[x
])^6*(6*a^2*b - 2*b^3 + a*(3*a^2 - 5*b^2)*Sinh[x]))/(a^2 + b^2) + (b*(((a^2
 + b^2)^2*((3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*b^4*Sqrt[-b^2])*Log[Sqrt[-b^2
] - b*Sinh[x]] + (-3*a^5 - 10*a^3*b^2 - 15*a*b^4 + 8*(-b^2)^(5/2))*Log[Sqrt
[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] - 10*a*b*(9*a^6 + 6*a^4*b^2 + 8*a^2*b^4 +
3*b^6)*Sinh[x] - 8*b^2*(15*a^6 - 4*a^4*b^2 + 2*a^2*b^4 + b^6)*Sinh[x]^2 + 1
0*a*b^3*(-9*a^4 + 8*a^2*b^2 + b^4)*Sinh[x]^3 + 4*b^4*(-9*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[x]^4 + 2*a*b^5*(-3*a^2 + 5*b^2)*Sinh[x]^5))/(a^2 + b^2))/(16*(a^
2 + b^2))
```

Maple [A] time = 0.046, size = 223, normalized size = 1.8

$$\frac{a^5\tanh(x)(\operatorname{sech}(x))^3}{4}+\frac{3a^5\operatorname{sech}(x)\tanh(x)}{8}+\frac{3a^5\arctan(e^x)}{4}+\frac{5a^4b(\sinh(x))^2}{4(\cosh(x))^4}+\frac{5a^4b(\sinh(x))^2}{4(\cosh(x))^2}-\frac{10a^3b^2\sinh}{3(\cosh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^5,x)
```

```
[Out] 1/4*a^5*tanh(x)*sech(x)^3+3/8*a^5*sech(x)*tanh(x)+3/4*a^5*arctan(exp(x))+5/
4*a^4*b*sinh(x)^2/cosh(x)^4+5/4*a^4*b*sinh(x)^2/cosh(x)^2-10/3*a^3*b^2*sinh
(x)/cosh(x)^4+5/6*a^3*b^2*tanh(x)*sech(x)^3+5/4*a^3*b^2*sech(x)*tanh(x)+5/2
*a^3*b^2*arctan(exp(x))-5/2*a^2*b^3*sinh(x)^2/cosh(x)^4+5/2*a^2*b^3*sinh(x)
^2/cosh(x)^2-5*b^4*a*sinh(x)^3/cosh(x)^4-5*b^4*a*sinh(x)/cosh(x)^4+5/4*b^4*
a*tanh(x)*sech(x)^3+15/8*b^4*a*sech(x)*tanh(x)+15/4*b^4*a*arctan(exp(x))+b^
5*ln(cosh(x))-1/2*b^5*tanh(x)^2-1/4*b^5*tanh(x)^4
```

Maxima [B] time = 1.6891, size = 377, normalized size = 3.04

$$\frac{5}{2}a^2b^3\tanh(x)^4+b^5\left(x+\frac{4\left(e^{(-2x)}+e^{(-4x)}+e^{(-6x)}\right)}{4e^{(-2x)}+6e^{(-4x)}+4e^{(-6x)}+e^{(-8x)}+1}+\log\left(e^{(-2x)}+1\right)\right)-\frac{5}{4}ab^4\left(\frac{5e^{(-x)}-3e^{(-3x)}+3e^{(-}}{4e^{(-2x)}+6e^{(-4x)}+4e^{(-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tanh(x)^4 + b^5*(x + 4*(e^(-2*x) + e^(-4*x) + e^(-6*x))/(4*e^(-
2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + log(e^(-2*x) + 1)) - 5/4*a
*b^4*((5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 3*arctan(e^(-x))) + 1/4*a^5*((3*e^(-x)
 + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(
-6*x) + e^(-8*x) + 1) - 3*arctan(e^(-x))) + 5/2*a^3*b^2*((e^(-x) - 7*e^(-3*
x) + 7*e^(-5*x) - e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x
) + 1) - arctan(e^(-x))) - 20*a^4*b/(e^(-x) + e^x)^4
```

Fricas [B] time = 2.81096, size = 5216, normalized size = 42.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^5*x*cosh(x)^8 + 4*b^5*x*sinh(x)^8 - (3*a^5 + 10*a^3*b^2 - 25*a*b^
4)*cosh(x)^7 + (32*b^5*x*cosh(x) - 3*a^5 - 10*a^3*b^2 + 25*a*b^4)*sinh(x)^7
 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^6 + (112*b^5*x*cosh(x)^2 + 16*b^5*x
 + 80*a^2*b^3 - 16*b^5 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x))*sinh(x)
^6 + 4*b^5*x - (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + (224*b^5*x*cosh
(x)^3 - 11*a^5 + 70*a^3*b^2 - 15*a*b^4 - 21*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)
*cosh(x)^2 + 96*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 8*(3*b^5*x +
 10*a^4*b - 2*b^5)*cosh(x)^4 + (280*b^5*x*cosh(x)^4 + 24*b^5*x + 80*a^4*b -
 16*b^5 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^3 + 240*(b^5*x + 5*a^2
*b^3 - b^5)*cosh(x)^2 - 5*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)
^4 + (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (224*b^5*x*cosh(x)^5 + 11
*a^5 - 70*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^4
 + 320*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^3 - 10*(11*a^5 - 70*a^3*b^2 + 15*a
*b^4)*cosh(x)^2 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x))*sinh(x)^3 + 16*(
b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^2 + (112*b^5*x*cosh(x)^6 + 16*b^5*x - 21*(
3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^5 + 80*a^2*b^3 - 16*b^5 + 240*(b^5*x
 + 5*a^2*b^3 - b^5)*cosh(x)^4 - 10*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)
^3 + 48*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^2 + 3*(11*a^5 - 70*a^3*b^2 + 1
5*a*b^4)*cosh(x))*sinh(x)^2 - ((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^8 +
8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 4*(3*a
^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*s
inh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a
^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 6*(
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^5 + 30*a^3*b^2 + 45*a*b^4
 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(3*a^5 + 10*a^3*b^2 +
15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(
x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)
^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 3*a^5 + 10*a^3*b^2 +
15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 10*a^3
*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*
cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(3*a^5 + 10*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(
x))*arctan(cosh(x) + sinh(x)) + (3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x) - 4
*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 + 4*b^5*cosh(x)^6
 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 + b^5)*sinh(x)^6
+ 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4
 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 + 10*b^5*cosh(x
)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6 + 15*b^5*cosh(x)^4 + 9*
```

```
b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 + 3*b^5*cosh(x)^5 + 3*b^5
*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (32
*b^5*x*cosh(x)^7 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^6 + 96*(b^5*x
+ 5*a^2*b^3 - b^5)*cosh(x)^5 + 3*a^5 + 10*a^3*b^2 - 25*a*b^4 - 5*(11*a^5 -
70*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^
3 + 3*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 32*(b^5*x + 5*a^2*b^3 -
b^5)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*
cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x
)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)
^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
+ 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,\mathrm{sech}\,(x) + b\tanh\,(x))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**5,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**5, x)
```

Giac [B] time = 1.13656, size = 324, normalized size = 2.61

$$\frac{1}{2}\,b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)+\frac{1}{16}\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)-\frac{3\,b^5\left(e^{(-x)}-e^x\right)^4+3\,a^5\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log((e^(-x) - e^x)^2 + 4) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e
^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b^4) - 1/4*(3*b^5*(e^(-x) - e^x)^4 + 3*a
^5*(e^(-x) - e^x)^3 + 10*a^3*b^2*(e^(-x) - e^x)^3 - 25*a*b^4*(e^(-x) - e^x)
^3 + 80*a^2*b^3*(e^(-x) - e^x)^2 + 8*b^5*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
- e^x) - 40*a^3*b^2*(e^(-x) - e^x) - 60*a*b^4*(e^(-x) - e^x) + 80*a^4*b + 1
60*a^2*b^3)/((e^(-x) - e^x)^2 + 4)^2
```

3.615 $\int (a\,\mathbf{sech}(x) + b\tanh(x))^4\,dx$

Optimal. Leaf size=100

$$-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(x)+\frac{1}{3}\mathbf{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-\frac{1}{3}\mathrm{se}$$

```
[Out] b^4*x - (4*a*b*(a^2 + 2*b^2)*Cosh[x])/3 - (b^2*(2*a^2 + 3*b^2)*Cosh[x]*Sinh
[x])/3 - (Sech[x]^3*(b - a*Sinh[x])*(a + b*Sinh[x])^3)/3 + (Sech[x]*(a + b*
Sinh[x])^2*(a*b + (2*a^2 + 3*b^2)*Sinh[x]))/3
```

Rubi [A] time = 0.208518, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2691, 2861, 2734}

$$-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(x)+\frac{1}{3}\mathbf{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-\frac{1}{3}\mathrm{se}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] b^4*x - (4*a*b*(a^2 + 2*b^2)*Cosh[x])/3 - (b^2*(2*a^2 + 3*b^2)*Cosh[x]*Sinh
[x])/3 - (Sech[x]^3*(b - a*Sinh[x])*(a + b*Sinh[x])^3)/3 + (Sech[x]*(a + b*
Sinh[x])^2*(a*b + (2*a^2 + 3*b^2)*Sinh[x]))/3
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = \int \text{sech}^4(x)(a + b\sinh(x))^4\,dx$$
$$= -\frac{1}{3}\text{sech}^3(x)(b - a\sinh(x))(a + b\sinh(x))^3 - \frac{1}{3}\int \text{sech}^2(x)(a + b\sinh(x))^2\left(-2a^2 - 3b^2\right.$$
$$= -\frac{1}{3}\text{sech}^3(x)(b - a\sinh(x))(a + b\sinh(x))^3 + \frac{1}{3}\text{sech}(x)(a + b\sinh(x))^2\left(ab + \left(2a^2 + 3b\right.\right.$$
$$= b^4x - \frac{4}{3}ab\left(a^2 + 2b^2\right)\cosh(x) - \frac{1}{3}b^2\left(2a^2 + 3b^2\right)\cosh(x)\sinh(x) - \frac{1}{3}\text{sech}^3(x)(b - a\sin$$

Mathematica [A] time = 0.176357, size = 79, normalized size = 0.79

$$\frac{1}{3}\left(2\left(3a^2b^2 + a^4 - 2b^4\right)\tanh(x) - 4ab\left(a^2 - b^2\right)\text{sech}^3(x) + \left(-6a^2b^2 + a^4 + b^4\right)\tanh(x)\text{sech}^2(x) - 12ab^3\text{sech}(x) + 3b^4x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] (3*b^4*x - 12*a*b^3*Sech[x] - 4*a*b*(a^2 - b^2)*Sech[x]^3 + 2*(a^4 + 3*a^2*
b^2 - 2*b^4)*Tanh[x] + (a^4 - 6*a^2*b^2 + b^4)*Sech[x]^2*Tanh[x])/3
```

Maple [A] time = 0.032, size = 123, normalized size = 1.2

$$a^4\left(\frac{2}{3}+\frac{(\operatorname{sech}(x))^2}{3}\right)\tanh(x)+4a^3b\left(1/3\,\frac{(\sinh(x))^2}{(\cosh(x))^3}+1/3\,\frac{(\sinh(x))^2}{\cosh(x)}-1/3\,\cosh(x)\right)+6a^2b^2\left(-1/2\,\frac{\sinh(x)}{(\cosh(x))^3}+1/2\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^4,x)
```

```
[Out] a^4*(2/3+1/3*sech(x)^2)*tanh(x)+4*a^3*b*(1/3*sinh(x)^2/cosh(x)^3+1/3*sinh(x
)^2/cosh(x)-1/3*cosh(x))+6*a^2*b^2*(-1/2*sinh(x)/cosh(x)^3+1/2*(2/3+1/3*sec
h(x)^2)*tanh(x))+4*a*b^3*(-1/3*sinh(x)^2/cosh(x)^3+2/3*sinh(x)^2/cosh(x)-2/
3*cosh(x))+b^4*(x-tanh(x)-1/3*tanh(x)^3)
```

Maxima [B] time = 1.05966, size = 284, normalized size = 2.84

$$2a^2b^2\tanh(x)^3+\frac{1}{3}b^4\left(3x-\frac{4\left(3e^{(-2x)}+3e^{(-4x)}+2\right)}{3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1}\right)-\frac{8}{3}ab^3\left(\frac{3e^{(-x)}}{3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1}+\frac{2e^{(-3x)}}{3e^{(-2x)}+3e^{(-4x)}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tanh(x)^3 + 1/3*b^4*(3*x - 4*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(
-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) - 8/3*a*b^3*(3*e^(-x)/(3*e^(-2*x) + 3*e
^(-4*x) + e^(-6*x) + 1) + 2*e^(-3*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
1) + 3*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) + 4/3*a^4*(3*e^(-
2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 1/(3*e^(-2*x) + 3*e^(-4*x)
+ e^(-6*x) + 1)) - 32/3*a^3*b/(e^(-x) + e^x)^3
```

Fricas [B] time = 2.30987, size = 505, normalized size = 5.05

$$-\frac{24ab^3\cosh(x)^2+16a^3b+8ab^3-\left(3b^4x-2a^4-6a^2b^2+4b^4\right)\cosh(x)^3-2\left(a^4+3a^2b^2-2b^4\right)\sinh(x)^3+3\left(8ab^3-\cdots\right.}{3\left(\cosh(x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*a*b^3*cosh(x)^2 + 16*a^3*b + 8*a*b^3 - (3*b^4*x - 2*a^4 - 6*a^2*b^
2 + 4*b^4)*cosh(x)^3 - 2*(a^4 + 3*a^2*b^2 - 2*b^4)*sinh(x)^3 + 3*(8*a*b^3 -
 (3*b^4*x - 2*a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^2 - 3*(3*b^4*x - 2*
a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x) - 6*(a^4 - 3*a^2*b^2 + (a^4 + 3*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,\mathrm{sech}\,(x) + b\tanh\,(x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**4,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**4, x)
```

Giac [A] time = 1.13681, size = 149, normalized size = 1.49

$$b^4x - \frac{4\left(6\,ab^3e^{(5\,x)} + 9\,a^2b^2e^{(4\,x)} - 3\,b^4e^{(4\,x)} + 8\,a^3be^{(3\,x)} + 4\,ab^3e^{(3\,x)} + 3\,a^4e^{(2\,x)} - 3\,b^4e^{(2\,x)} + 6\,ab^3e^{x} + a^4 + 3\,a^2b^2 - 2\,b^4\right)}{3\left(e^{(2\,x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 4/3*(6*a*b^3*e^(5*x) + 9*a^2*b^2*e^(4*x) - 3*b^4*e^(4*x) + 8*a^3*b*
e^(3*x) + 4*a*b^3*e^(3*x) + 3*a^4*e^(2*x) - 3*b^4*e^(2*x) + 6*a*b^3*e^x + a
^4 + 3*a^2*b^2 - 2*b^4)/(e^(2*x) + 1)^3
```

3.616 $\int (a\,\textbf{sech}(x) + b\tanh(x))^3\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\textbf{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+b^3\log(\cosh(x))$$

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/2 + b^3*Log[Cosh[x]] - (a*b^2*Sinh[x])/2
- (Sech[x]^2*(b - a*Sinh[x])*(a + b*Sinh[x])^2)/2
```

Rubi [A] time = 0.106695, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4391, 2668, 739, 774, 635, 204, 260}

$$\frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\textbf{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+b^3\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/2 + b^3*Log[Cosh[x]] - (a*b^2*Sinh[x])/2
- (Sech[x]^2*(b - a*Sinh[x])*(a + b*Sinh[x])^2)/2
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (a\text{sech}(x)+b\tanh(x))^3\,dx=\int \text{sech}^3(x)(a+b\sinh(x))^3\,dx$$
$$=b^3\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(-b^2-x^2\right)^2}\,dx,x,b\sinh(x)\right)$$
$$=-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+\frac{1}{2}b\,\text{Subst}\left(\int \frac{(a+x)\left(-a^2-2b^2+ax\right)}{-b^2-x^2}\,dx,\right.$$
$$=-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2-\frac{1}{2}b\,\text{Subst}\left(\int \frac{ab^2-a\left(-a^2\right.}{-b^2}\right.$$
$$=-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2-b^3\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx,x\right.$$
$$=\frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))+b^3\log(\cosh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh($$

Mathematica [B] time = 1.86863, size = 194, normalized size = 3.34

$$\frac{1}{4}\left(\frac{2a^4b\text{sech}^2(x)}{a^2+b^2}+\frac{b\left(\left(a^3+3ab^2-2\left(-b^2\right)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-\left(a^3+3ab^2+2\left(-b^2\right)^{3/2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right.}{\sqrt{-b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] ((b*((a^3 + 3*a*b^2 - 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[x]] - (a^3 +
3*a*b^2 + 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (2*a^4
*b*Sech[x]^2)/(a^2 + b^2) + (a*(2*a^4 - 4*a^2*b^2 - 7*b^4 + b^4*Cosh[2*x])*
Sech[x]*Tanh[x])/(a^2 + b^2) - (2*b*(-4*a^4 - 2*a^2*b^2 + b^4 + a*b^3*Sinh[
x])*Tanh[x]^2)/(a^2 + b^2))/4
```

Maple [A] time = 0.03, size = 79, normalized size = 1.4

$$\frac{a^3\text{sech}\,(x)\tanh\,(x)}{2}+a^3\arctan\,(\mathrm{e}^x)+\frac{3\,a^2b\,(\sinh\,(x))^2}{2\,(\cosh\,(x))^2}-3\,\frac{ab^2\sinh\,(x)}{(\cosh\,(x))^2}+\frac{3\,ab^2\text{sech}\,(x)\tanh\,(x)}{2}+3\,ab^2\arctan\,(\mathrm{e}^x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^3,x)
```

```
[Out] 1/2*a^3*sech(x)*tanh(x)+a^3*arctan(exp(x))+3/2*a^2*b*sinh(x)^2/cosh(x)^2-3*
a*b^2/cosh(x)^2*sinh(x)+3/2*a*b^2*sech(x)*tanh(x)+3*a*b^2*arctan(exp(x))+b^
3*ln(cosh(x))-1/2*b^3*tanh(x)^2
```

Maxima [B] time = 1.57354, size = 162, normalized size = 2.79

$$\frac{3}{2}a^2b\tanh(x)^2 + b^3\left(x + \frac{2e^{(-2x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + \log\left(e^{(-2x)} + 1\right)\right) - 3ab^2\left(\frac{e^{(-x)} - e^{(-3x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + \arctan\left(e^{(-x)}\right)\right) + a^3\left(\frac{}{2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tanh(x)^2 + b^3*(x + 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + log
(e^(-2*x) + 1)) - 3*a*b^2*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ arctan(e^(-x))) + a^3*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
arctan(e^(-x)))
```

Fricas [B] time = 2.47664, size = 1359, normalized size = 23.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + b^3*x - (a^3 - 3*a*b^2)*cosh(x)^3 + (
4*b^3*x*cosh(x) - a^3 + 3*a*b^2)*sinh(x)^3 + 2*(b^3*x + 3*a^2*b - b^3)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 6*a^2*b - 2*b^3 - 3*(a^3 - 3*a*b^2)*
cosh(x))*sinh(x)^2 - ((a^3 + 3*a*b^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)
*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*
cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 + 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x
) + sinh(x)) + (a^3 - 3*a*b^2)*cosh(x) - (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sin
h(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*
sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
 - sinh(x))) + (4*b^3*x*cosh(x)^3 + a^3 - 3*a*b^2 - 3*(a^3 - 3*a*b^2)*cosh(
```

```
x)^2 + 4*(b^3*x + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cos
h(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,\mathrm{sech}\,(x) + b\tanh\,(x))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**3,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**3, x)
```

Giac [B] time = 1.14454, size = 158, normalized size = 2.72

$$\frac{1}{2}\,b^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right) + \frac{1}{4}\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3\,ab^2\right) - \frac{b^3\left(e^{(-x)} - e^x\right)^2 + 2\,a^3\left(e^{(-x)} - e^x\right) - 6\,ab^2}{2\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log((e^(-x) - e^x)^2 + 4) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^
(-x)))*(a^3 + 3*a*b^2) - 1/2*(b^3*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) -
 6*a*b^2*(e^(-x) - e^x) + 12*a^2*b)/((e^(-x) - e^x)^2 + 4)
```

3.617 $\int (a\,\mathbf{sech}(x) + b\tanh(x))^2\,dx$

Optimal. Leaf size=29

$$-ab\cosh(x) - \mathrm{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) + b^2x$$

```
[Out] b^2*x - a*b*Cosh[x] - Sech[x]*(b - a*Sinh[x])*(a + b*Sinh[x])
```

Rubi [A] time = 0.0627443, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2691, 2638}

$$-ab\cosh(x) - \mathrm{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^2,x]
```

```
[Out] b^2*x - a*b*Cosh[x] - Sech[x]*(b - a*Sinh[x])*(a + b*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a\text{sech}(x)+b\tanh(x))^2\,dx=\int \text{sech}^2(x)(a+b\sinh(x))^2\,dx$$
$$=-\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))-\int \left(-b^2+ab\sinh(x)\right)\,dx$$
$$=b^2x-\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))-(ab)\int \sinh(x)\,dx$$
$$=b^2x-ab\cosh(x)-\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))$$

Mathematica [A] time = 0.0476505, size = 26, normalized size = 0.9

$$\left(a^2-b^2\right)\tanh(x)-2ab\text{sech}(x)+b^2\tanh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^2,x]
```

```
[Out] b^2*ArcTanh[Tanh[x]] - 2*a*b*Sech[x] + (a^2 - b^2)*Tanh[x]
```

Maple [A] time = 0.014, size = 36, normalized size = 1.2

$$a^2\tanh(x)+2\,ab\left(\frac{(\sinh(x))^2}{\cosh(x)}-\cosh(x)\right)+b^2\,(x-\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^2,x)
```

```
[Out] a^2*tanh(x)+2*a*b*(sinh(x)^2/cosh(x)-cosh(x))+b^2*(x-tanh(x))
```

Maxima [A] time = 1.05664, size = 58, normalized size = 2.

$$b^2\left(x-\frac{2}{e^{(-2\,x)}+1}\right)-\frac{4\,ab}{e^{(-x)}+e^x}+\frac{2\,a^2}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x - 2/(e^(-2*x) + 1)) - 4*a*b/(e^(-x) + e^x) + 2*a^2/(e^(-2*x) + 1)
```

Fricas [A] time = 2.26537, size = 95, normalized size = 3.28

$$-\frac{2\,ab - \left(b^2x - a^2 + b^2\right)\cosh\left(x\right) - \left(a^2 - b^2\right)\sinh\left(x\right)}{\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b - (b^2*x - a^2 + b^2)*cosh(x) - (a^2 - b^2)*sinh(x))/cosh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\operatorname{sech}\left(x\right) + b\tanh\left(x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**2,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**2, x)
```

Giac [A] time = 1.13688, size = 42, normalized size = 1.45

$$b^2x - \frac{2\left(2\,abe^x + a^2 - b^2\right)}{e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
```

```
[Out] b^2*x - 2*(2*a*b*e^x + a^2 - b^2)/(e^(2*x) + 1)
```

3.618 $\int (a\,\mathbf{sech}(x) + b\tanh(x))\,dx$

Optimal. Leaf size=11

$$a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))$$

```
[Out] a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

Rubi [A] time = 0.0099126, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Sech[x] + b*Tanh[x],x]

[Out] a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\,\text{sech}(x) + b\tanh(x))\,dx &= a\int \text{sech}(x)\,dx + b\int \tanh(x)\,dx\\ &= a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0040934, size = 16, normalized size = 1.45

$$2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + b\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sech[x] + b*Tanh[x],x]
```

```
[Out] 2*a*ArcTan[Tanh[x/2]] + b*Log[Cosh[x]]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$a \arctan(\sinh(x)) + b \ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sech(x)+b*tanh(x),x)
```

```
[Out] a*arctan(sinh(x))+b*ln(cosh(x))
```

Maxima [A] time = 1.0436, size = 15, normalized size = 1.36

$$a \arctan(\sinh(x)) + b \log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="maxima")
```

```
[Out] a*arctan(sinh(x)) + b*log(cosh(x))
```

Fricas [B] time = 2.41303, size = 104, normalized size = 9.45

$$-bx + 2\,a \arctan(\cosh(x) + \sinh(x)) + b \log\left(\frac{2\,\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="fricas")
```

```
[Out] -b*x + 2*a*arctan(cosh(x) + sinh(x)) + b*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a \operatorname{sech}(x) + b \tanh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sech(x)+b*tanh(x),x)
```

```
[Out] Integral(a*sech(x) + b*tanh(x), x)
```

Giac [A] time = 1.1132, size = 28, normalized size = 2.55

$$-b\left(x - \log\left(e^{(2x)} + 1\right)\right) + 2\, a \arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="giac")
```

```
[Out] -b*(x - log(e^(2*x) + 1)) + 2*a*arctan(e^x)
```

3.619 $\int \frac{1}{a\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sinh(x))}{b}$$

```
[Out] Log[a + b*Sinh[x]]/b
```

Rubi [A] time = 0.0385105, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a\mathrm{sech}(x) + b\tanh(x)}\,dx = \int \frac{\cosh(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}$$
$$= \frac{\log(a + b\sinh(x))}{b}$$

Mathematica [A] time = 0.0058458, size = 11, normalized size = 1.

$$\frac{\log(a + b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Maple [B] time = 0.046, size = 50, normalized size = 4.6

$$-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-2\,\tanh(x/2)\,b-a\right)-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-1/b*ln(tan
h(1/2*x)-1)
```

Maxima [B] time = 1.02182, size = 38, normalized size = 3.45

$$\frac{x}{b} + \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] x/b + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b
```

Fricas [B] time = 2.28765, size = 72, normalized size = 6.55

$$-\frac{x-\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] time = 0.654301, size = 32, normalized size = 2.91

$$\begin{cases} \frac{x}{b}+\frac{\log\left(\frac{a\operatorname{sech}(x)}{b}+\tanh(x)\right)}{b}-\frac{\log(\tanh(x)+1)}{b} & \text{for } b\neq 0 \\ \frac{\tanh(x)}{a\operatorname{sech}(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x)
```

```
[Out] Piecewise((x/b + log(a*sech(x)/b + tanh(x))/b - log(tanh(x) + 1)/b, Ne(b, 0
)), (tanh(x)/(a*sech(x)), True))
```

Giac [A] time = 1.15566, size = 30, normalized size = 2.73

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

3.620 $\int \frac{1}{(a\mathbf{sech}(x)+b\tanh(x))^2}\,dx$

Optimal. Leaf size=62

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rubi [A] time = 0.126822, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2693, 2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx = \int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$$

$$= -\frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

$$= \frac{x}{b^2} + \frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\cosh(x)}{b(a+b\sinh(x))}$$

<u>Mathematica</u> [C] time = 3.68398, size = 659, normalized size = 10.63

$$\frac{\cosh(x)\left(\sqrt{a+ib}\left(\sqrt{b^2}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(2\sqrt[4]{-1}a\sqrt{b}(b+ia)\sin^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{b}}\right)-\sqrt{a-ib}\left(a^2+b^2\right)\sqrt{1+i\sinh(x}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] (Cosh[x]*((2*I)*a*Sqrt[b^2]*(I*a + b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
 Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))
])]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*((2*I)*a^2*Sqrt[a -
 I*b]*b*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[I*
b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + 2*Sinh[x]*
(I*a*Sqrt[a - I*b]*b^2*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I
*b))])/(Sqrt[I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x
]] + (-1)^(1/4)*b^(3/2)*Sqrt[b^2]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*
b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sqrt[-((b*(-I + Sinh[x]))
/(a + I*b))]) + Sqrt[b^2]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(2*(-1)^(1/
4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sin
h[x]))/(a - I*b))])/Sqrt[b]] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]
```

```
]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]]))))/((a - I*b)^(3/2)*(a + I*b)^(3/2)
*b*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt
[-((b*(I + Sinh[x]))/(a - I*b))]*(a + b*Sinh[x]))
```

Maple [B] time = 0.072, size = 119, normalized size = 1.9

$$\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2\,\frac{\tanh(x/2)}{\left(a\left(\tanh(x/2)\right)^2-2\,\tanh(x/2)\,b-a\right)a}+2\,\frac{1}{b\left(a\left(\tanh(x/2)\right)^2-2\,\tanh(x/2)\,b-a\right)}-2\,\frac{a}{b^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^2,x)
```

```
[Out] 1/b^2*ln(tanh(1/2*x)+1)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)
+2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2/b^2*a/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40107, size = 933, normalized size = 15.05

$$-\frac{\left(a^2b+b^3\right)x\cosh(x)^2+\left(a^2b+b^3\right)x\sinh(x)^2-2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)-ab+2\left(ab\,c\right.\right.}{a^2b^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,\mathrm{sech}\,(x)+b\tanh\,(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**2,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**(-2), x)
```

Giac [A] time = 1.19518, size = 131, normalized size = 2.11

$$-\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{x}{b^2}+\frac{2\,(ae^x-b)}{\left(be^{(2\,x)}+2\,ae^x-b\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

3.621 $\int \frac{1}{(a\mathbf{sech}(x)+b\tanh(x))^3}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^3}$$

```
[Out] Log[a + b*Sinh[x]]/b^3 - (a^2 + b^2)/(2*b^3*(a + b*Sinh[x])^2) + (2*a)/(b^3
*(a + b*Sinh[x]))
```

Rubi [A] time = 0.0798414, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^3 - (a^2 + b^2)/(2*b^3*(a + b*Sinh[x])^2) + (2*a)/(b^3
*(a + b*Sinh[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx = \int \frac{\cosh^3(x)}{(a+b\sinh(x))^3}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^3}\,dx, x, b\sinh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2-b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx, x, b\sinh(x)\right)}{b^3}$$
$$= \frac{\log(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}$$

Mathematica [A] time = 0.133074, size = 42, normalized size = 0.88

$$-\frac{\frac{-3a^2-4ab\sinh(x)+b^2}{2(a+b\sinh(x))^2}-\log(a+b\sinh(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

```
[Out] -((-Log[a + b*Sinh[x]] + (-3*a^2 + b^2 - 4*a*b*Sinh[x])/(2*(a + b*Sinh[x])^
2))/b^3)
```

Maple [B] time = 0.088, size = 241, normalized size = 5.

$$-\frac{1}{b^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2\frac{a(\tanh(x/2))^3}{b^2\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)^2}-2\frac{(\tanh(x/2))^3}{\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)^2a}-6-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^3,x)
```

```
[Out] -1/b^3*ln(tanh(1/2*x)+1)+2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2*a*tanh
(1/2*x)^3-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2/a*tanh(1/2*x)^3-6/b/(a*ta
```

```
nh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2*tanh(1/2*x)^2+2*b/(a*tanh(1/2*x)^2-2*tanh(
1/2*x)*b-a)^2/a^2*tanh(1/2*x)^2-2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2
*a*tanh(1/2*x)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2/a*tanh(1/2*x)+1/b^3*
ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-1/b^3*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.08794, size = 158, normalized size = 3.29

$$\frac{2\left(2\,abe^{(-x)}-2\,abe^{(-3\,x)}+\left(3\,a^2-b^2\right)e^{(-2\,x)}\right)}{4\,ab^4e^{(-x)}-4\,ab^4e^{(-3\,x)}+b^5e^{(-4\,x)}+b^5+2\left(2\,a^2b^3-b^5\right)e^{(-2\,x)}}+\frac{x}{b^3}+\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*e^(-x) - 2*a*b*e^(-3*x) + (3*a^2 - b^2)*e^(-2*x))/(4*a*b^4*e^(-x)
- 4*a*b^4*e^(-3*x) + b^5*e^(-4*x) + b^5 + 2*(2*a^2*b^3 - b^5)*e^(-2*x)) + x
/b^3 + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Fricas [B] time = 2.56839, size = 1362, normalized size = 28.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^4 + b^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(b^2*x*
cosh(x) + a*b*x - a*b)*sinh(x)^3 + b^2*x - 2*(3*a^2 - b^2 - (2*a^2 - b^2)*x
)*cosh(x)^2 + 2*(3*b^2*x*cosh(x)^2 - 3*a^2 + b^2 + (2*a^2 - b^2)*x + 6*(a*b
*x - a*b)*cosh(x))*sinh(x)^2 - 4*(a*b*x - a*b)*cosh(x) - (b^2*cosh(x)^4 + b
^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(b^2*cosh(x) + a*b)*sinh(x)^3 - 4*a*b*co
sh(x) + 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 6*a*b*cosh(x) + 2*
a^2 - b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + 3*a*b*cosh(x)^2 - a*b + (2*
a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 4
*(b^2*x*cosh(x)^3 - a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 + a*b - (3*a^2 - b^2
- (2*a^2 - b^2)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 + 4*a*b
^4*cosh(x)^3 - 4*a*b^4*cosh(x) + b^5 + 4*(b^5*cosh(x) + a*b^4)*sinh(x)^3 +
2*(2*a^2*b^3 - b^5)*cosh(x)^2 + 2*(3*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) + 2*a^
2*b^3 - b^5)*sinh(x)^2 + 4*(b^5*cosh(x)^3 + 3*a*b^4*cosh(x)^2 - a*b^4 + (2*
```

```
a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15932, size = 101, normalized size = 2.1

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b^{3}}-\frac{3\,b\left(e^{(-x)}-e^{x}\right)^{2}-4\,a\left(e^{(-x)}-e^{x}\right)+4\,b}{2\,\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 - 1/2*(3*b*(e^(-x) - e^x)^2 - 4*a*(e^
(-x) - e^x) + 4*b)/((b*(e^(-x) - e^x) - 2*a)^2*b^2)
```

3.622 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^4}\,dx$

Optimal. Leaf size=146

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}$$

```
[Out] x/b^4 + (a*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^4
*(a^2 + b^2)^(3/2)) - Cosh[x]^3/(3*b*(a + b*Sinh[x])^3) + (a*Cosh[x]^3)/(2*
b*(a^2 + b^2)*(a + b*Sinh[x])^2) - (Cosh[x]*(2*(a^2 + b^2) + a*b*Sinh[x]))/
(2*b^3*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.365036, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2693, 2864, 2863, 2735, 2660, 618, 206}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

```
[Out] x/b^4 + (a*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^4
*(a^2 + b^2)^(3/2)) - Cosh[x]^3/(3*b*(a + b*Sinh[x])^3) + (a*Cosh[x]^3)/(2*
b*(a^2 + b^2)*(a + b*Sinh[x])^2) - (Cosh[x]*(2*(a^2 + b^2) + a*b*Sinh[x]))/
(2*b^3*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\,\mathrm{sech}(x)+b\tanh(x))^4}\,dx = \int \frac{\cosh^4(x)}{(a+b\sinh(x))^4}\,dx$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{\int \frac{\cosh^2(x)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{b}$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} + \frac{i\int \frac{\cosh^2(x)(-2ib+ia\sinh(x))}{(a+b\sinh(x))^2}\,dx}{2b\left(a^2+b^2\right)}$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x\right.}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sin\right.}{2b^3\left(a^2+b^2\right)(a+b\sinh(x\right.}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sin\right.}{2b^3\left(a^2+b^2\right)(a+b\sinh(x\right.}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sin\right.}{2b^3\left(a^2+b^2\right)(a+b\sinh(x\right.}$$

$$= \frac{x}{b^4} + \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh\right.}$$

Mathematica [C] time = 6.45567, size = 3458, normalized size = 23.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

[Out] ((-I)*Sech[x]*(a + b*Sinh[x])^4*(((I/3)*b*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x])^3) - (((I/2)*a*b^3*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((a^2 + b^2)*(((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x])^2) - (-(((((3*I)*a^2*b^5)/(a^2 + b^2)^2 - ((2*I)*b^5*(3*a^2 + 2*b^2))/(a^2 + b^2)^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x]))) - ((16*Sqrt[2]*(a - I*b)*b^6*(3*a^2 + 4*b^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((5*(1/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1)))/8 + (((15*I)/32)*b^3*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b + ((a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2)/(3*b^2) + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b])))/((a - I*b)^3*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^3*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2)))/(5*(a + I*b)*(a^2 + b^2)^3*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) + (I*(((4*I)*a*b^7*(3*a^2 + 4*b^2))/(a^2 + b^2)^3 - (I*a*b^7*(6*a^2 + 7*b^2))/(a^2 + b^2)^3)*((-4*Sqrt[2]*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(3/2)*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((3/(4*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1))/2 - (3*b^2*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b])))/(8*(a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2)))/(3*(a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) - (I*((I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*((I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*((I*a*b)/(a + I*b) - b^2/(a + I*b))*(((-2*I)*((I*a*b)/(a + I*b) - b^2/(a + I*b))*ArcTan[(Sqrt[((-I)*a*b)/(a + I*b) + b^2/(a + I*b)]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[(I*a*b)/(a - I*b) + b^2/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)])]])/(b*Sqrt[(I*a*b)/(a - I*b) + b^2/(a - I*b)]*Sqrt[((-I)*a*b)/(a + I*b) + b^2/(a + I*b)]) + ((2*I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[a + I*b]*Sqrt[(I*b)/(a

```
 + I*b) - (b*Sinh[x])/(a + I*b)])])/(Sqrt[a + I*b]*b)))/b + ((2*I)*Sqrt[2]*
(a - I*b)*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*Sqrt[(I*b)/(a +
I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (
b*Sinh[x])/(a - I*b)))/b)^(3/2)*(-(((-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*S
qrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqr
t[b])])/(Sqrt[2]*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I
*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)
^(3/2))) + 1/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a
- I*b)))/b))))/((a + I*b)*b*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh
[x])/(a + I*b)))/b])))/b - (4*Sqrt[2]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])
/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a -
I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((-3*(-1)^(3/4)
*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh
[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - I*b]*Sqrt[((-I)*b)
/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*
b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)) + (3/(2*(1 - ((I/2)*(a - I*b)*(((-I)
*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I
)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1))/4))/((a + I*b)*Sqrt[((-I)
*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b])))/b))/b)/((((-I)*
a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))))/(2
*(((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*
b))))/(3*(((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2
/(a + I*b)))))/((1 - (a + b*Sinh[x])/(a - I*b))^(3/2)*(1 - (a + b*Sinh[x])/
(a + I*b))^(3/2)*(a*Sech[x] + b*Tanh[x])^4)
```

Maple [B] time = 0.109, size = 972, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^4,x)
```

```
[Out] 2/3*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)+1/b^2/(a*tanh(1/2*x)^
2-2*tanh(1/2*x)*b-a)^3*a^3/(a^2+b^2)*tanh(1/2*x)^5+2*b^2/(a*tanh(1/2*x)^2-2
*tanh(1/2*x)*b-a)^3/a/(a^2+b^2)*tanh(1/2*x)^5+2/b^3/(a*tanh(1/2*x)^2-2*tanh
(1/2*x)*b-a)^3/(a^2+b^2)*a^4*tanh(1/2*x)^4-3/b/(a*tanh(1/2*x)^2-2*tanh(1/2*
x)*b-a)^3/(a^2+b^2)*a^2*tanh(1/2*x)^4-4*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*
b-a)^3/(a^2+b^2)/a^2*tanh(1/2*x)^4-12/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-
a)^3*a^3/(a^2+b^2)*tanh(1/2*x)^3+8/3*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a
)^3/a/(a^2+b^2)*tanh(1/2*x)^3+8/3*b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3
/a^3/(a^2+b^2)*tanh(1/2*x)^3-4/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^
```

```
4/(a^2+b^2)*tanh(1/2*x)^2+16/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^2/(a
^2+b^2)*tanh(1/2*x)^2+4*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a^2/(a^2+
b^2)*tanh(1/2*x)^2+11/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^3/(a^2+b^
2)*tanh(1/2*x)+2*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a/(a^2+b^2)*tanh
(1/2*x)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a/(a^2+b^2)*tanh(1/2*x)^5-4
*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*tanh(1/2*x)^4-2/(a*tanh(
1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a/(a^2+b^2)*tanh(1/2*x)^3+14*b/(a*tanh(1/2*x)
^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*tanh(1/2*x)^2+8/(a*tanh(1/2*x)^2-2*tanh(1
/2*x)*b-a)^3*a/(a^2+b^2)*tanh(1/2*x)+2/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b
-a)^3/(a^2+b^2)*a^4+5/3/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*a
^2-2/b^4*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1
/2))-3/b^2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1
/2))+1/b^4*ln(tanh(1/2*x)+1)-1/b^4*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.08197, size = 6724, normalized size = 46.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^6 + 6*(a^4*b^3 + 2*a^2*b^5 +
b^7)*x*sinh(x)^6 - 22*a^4*b^3 - 38*a^2*b^5 - 16*b^7 + 6*(6*a^5*b^2 + 11*a^3
*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^5 + 6*(6*a^5*b^
2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x) + 6*(a^5
*b^2 + 2*a^3*b^4 + a*b^6)*x)*sinh(x)^5 + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b
^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^4 + 6*(18
*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^7 + 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*
cosh(x)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x + 5*(6*a^5*b^2 + 11
```

```
*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x))*sinh(x)^4
+ 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^
3*b^4 - 3*a*b^6)*x)*cosh(x)^3 + 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b
^6 + 30*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^3 + 15*(6*a^5*b^2 + 11*a^3*b^
4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^2 + 6*(2*a^7 + a^5
*b^2 - 4*a^3*b^4 - 3*a*b^6)*x + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^
7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - 6*(26
*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^
5 - b^7)*x)*cosh(x)^2 - 6*(26*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 - 15*(
a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^4 - 10*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b
^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^3 - 6*(18*a^6*b + 27*a^4*b^
3 + 5*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x
)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x - 2*(22*a^7 + 5*a^5*b^2 -
 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3*a*b^6)*x)*cosh(
x))*sinh(x)^2 + 3*((2*a^3*b^3 + 3*a*b^5)*cosh(x)^6 + (2*a^3*b^3 + 3*a*b^5)*
sinh(x)^6 - 2*a^3*b^3 - 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 6*(
2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^5 + 3*(8*a^5
*b + 10*a^3*b^3 - 3*a*b^5)*cosh(x)^4 + 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 +
5*(2*a^3*b^3 + 3*a*b^5)*cosh(x)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sin
h(x)^4 + 4*(4*a^6 - 9*a^2*b^4)*cosh(x)^3 + 4*(4*a^6 - 9*a^2*b^4 + 5*(2*a^3*
b^3 + 3*a*b^5)*cosh(x)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 3*(8*a^5*
b + 10*a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x)^3 - 3*(8*a^5*b + 10*a^3*b^3 - 3*
a*b^5)*cosh(x)^2 - 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 - 5*(2*a^3*b^3 + 3*a*b
^5)*cosh(x)^4 - 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - 6*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^2 - 4*(4*a^6 - 9*a^2*b^4)*cosh(x))*sinh(x)^2 + 6*(2*
a^4*b^2 + 3*a^2*b^4)*cosh(x) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*
b^5)*cosh(x)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^3 + 2*(4*a^6 - 9*a^2*b^4)*cosh(x)^2 - (8*a^5*b + 10*
a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b^3 + 2*a^2*b^5 +
 b^7)*x + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*x)*cosh(x) + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^4*b^3 + 2*
a^2*b^5 + b^7)*x*cosh(x)^5 + 5*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b
^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^4 + 4*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5
 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^3 + 2*(22*a
^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3
*a*b^6)*x)*cosh(x)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x - 2*(26*a^6*b + 38
*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)
*cosh(x))*sinh(x))/(a^4*b^7 + 2*a^2*b^9 + b^11 - (a^4*b^7 + 2*a^2*b^9 + b^1
1)*cosh(x)^6 - (a^4*b^7 + 2*a^2*b^9 + b^11)*sinh(x)^6 - 6*(a^5*b^6 + 2*a^3*
b^8 + a*b^10)*cosh(x)^5 - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7 + 2*a^
2*b^9 + b^11)*cosh(x))*sinh(x)^5 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b
^11)*cosh(x)^4 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 + 5*(a^4*b^7 +
```

```
 2*a^2*b^9 + b^11)*cosh(x)^2 + 10*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x))*s
inh(x)^4 - 4*(2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^3 - 4*(2*
a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10 + 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*c
osh(x)^3 + 15*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a
^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x))*sinh(x)^3 + 3*(4*a^6*b^5 + 7*a^4*b^7 +
2*a^2*b^9 - b^11)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 -
 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*cosh(x)^4 - 20*(a^5*b^6 + 2*a^3*b^8 + a*b^1
0)*cosh(x)^3 - 6*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^2 - 4*(
2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x))*sinh(x)^2 - 6*(a^5*b^6
 + 2*a^3*b^8 + a*b^10)*cosh(x) - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7
 + 2*a^2*b^9 + b^11)*cosh(x)^5 + 5*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^4
 + 2*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^3 + 2*(2*a^7*b^4 +
a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^2 - (4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*
b^9 - b^11)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**4,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**(-4), x)
```

Giac [A] time = 1.18791, size = 360, normalized size = 2.47

$$-\frac{\left(2\,a^3+3\,ab^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{18\,a^3b^2e^{(5\,x)}+15\,ab^4e^{(5\,x)}+54\,a^4be^{(4\,x)}+27\,a^2b^3e^{(4\,x)}-12\,b^5e^{(4\,x)}+44\,a^5e^{(3\,x)}}{3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 + 3*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e
^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(18*
a^3*b^2*e^(5*x) + 15*a*b^4*e^(5*x) + 54*a^4*b*e^(4*x) + 27*a^2*b^3*e^(4*x)
```

```
- 12*b^5*e^(4*x) + 44*a^5*e^(3*x) - 34*a^3*b^2*e^(3*x) - 48*a*b^4*e^(3*x) -
 78*a^4*b*e^(2*x) - 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) + 48*a^3*b^2*e^x +
33*a*b^4*e^x - 11*a^2*b^3 - 8*b^5)/((a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x -
b)^3) + x/b^4
```

3.623 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^5}\,dx$

Optimal. Leaf size=95

$$-\frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+\frac{4a}{b^5(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^5}$$

```
[Out] Log[a + b*Sinh[x]]/b^5 - (a^2 + b^2)^2/(4*b^5*(a + b*Sinh[x])^4) + (4*a*(a^
2 + b^2))/(3*b^5*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(b^5*(a + b*Sinh[x])^2)
 + (4*a)/(b^5*(a + b*Sinh[x]))
```

Rubi [A] time = 0.115842, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+\frac{4a}{b^5(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^5 - (a^2 + b^2)^2/(4*b^5*(a + b*Sinh[x])^4) + (4*a*(a^
2 + b^2))/(3*b^5*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(b^5*(a + b*Sinh[x])^2)
 + (4*a)/(b^5*(a + b*Sinh[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx=\int \frac{\cosh^5(x)}{(a+b\sinh(x))^5}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{(a+x)^5}\,dx,x,b\sinh(x)\right)}{b^5}$$
$$=\frac{\text{Subst}\left(\int \left(\frac{\left(a^2+b^2\right)^2}{(a+x)^5}-\frac{4a\left(a^2+b^2\right)}{(a+x)^4}+\frac{2\left(3a^2+b^2\right)}{(a+x)^3}-\frac{4a}{(a+x)^2}+\frac{1}{a+x}\right)dx,x,b\sinh(x)\right)}{b^5}$$
$$=\frac{\log(a+b\sinh(x))}{b^5}-\frac{\left(a^2+b^2\right)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a\left(a^2+b^2\right)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+$$

Mathematica [A] time = 0.229112, size = 83, normalized size = 0.87

$$\frac{-\frac{\left(a^2+b^2\right)^2}{4(a+b\sinh(x))^4}+\frac{4a\left(a^2+b^2\right)}{3(a+b\sinh(x))^3}-\frac{3a^2+b^2}{(a+b\sinh(x))^2}+\frac{4a}{a+b\sinh(x)}+\log(a+b\sinh(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] (Log[a + b*Sinh[x]] - (a^2 + b^2)^2/(4*(a + b*Sinh[x])^4) + (4*a*(a^2 + b^2
))/(3*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(a + b*Sinh[x])^2 + (4*a)/(a + b*S
inh[x]))/b^5
```

Maple [B] time = 0.128, size = 721, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^5,x)
```

```
[Out] -1/b^5*ln(tanh(1/2*x)+1)+2/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^3*ta
nh(1/2*x)^7-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)^7-14/b^3/
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^2*tanh(1/2*x)^6+6*b/(a*tanh(1/2*x)^
2-2*tanh(1/2*x)*b-a)^4/a^2*tanh(1/2*x)^6-6/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*
x)*b-a)^4*a^3*tanh(1/2*x)^5+104/3/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4
*a*tanh(1/2*x)^5+2/3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)^5-
8*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^3*tanh(1/2*x)^5+28/b^3/(a*tan
h(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^2*tanh(1/2*x)^4-100/3/b/(a*tanh(1/2*x)^2-
2*tanh(1/2*x)*b-a)^4*tanh(1/2*x)^4-28/3*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-
a)^4/a^2*tanh(1/2*x)^4+4*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^4*tanh
(1/2*x)^4+6/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^3*tanh(1/2*x)^3-104
/3/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a*tanh(1/2*x)^3-2/3/(a*tanh(1/
2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)^3+8*b^2/(a*tanh(1/2*x)^2-2*tanh(1
/2*x)*b-a)^4/a^3*tanh(1/2*x)^3-14/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4
*a^2*tanh(1/2*x)^2+6*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^2*tanh(1/2*x
)^2-2/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^3*tanh(1/2*x)+2/(a*tanh(1
/2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)+1/b^5*ln(a*tanh(1/2*x)^2-2*tanh(
1/2*x)*b-a)-1/b^5*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.2044, size = 401, normalized size = 4.22

$$\frac{4\left(6\,ab^{3}e^{(-x)}-6\,ab^{3}e^{(-7\,x)}+3\left(9\,a^{2}b^{2}-b^{4}\right)e^{(-2\,x)}+22\left(2\,a^{3}b-ab^{3}\right)e^{(-3\,x)}+\left(25\,a^{4}-56\,a^{2}b^{2}+3\,b^{4}\right)e^{(-4\,x)}-22\cdots}{3\left(8\,ab^{8}e^{(-x)}-8\,ab^{8}e^{(-7\,x)}+b^{9}e^{(-8\,x)}+b^{9}+4\left(6\,a^{2}b^{7}-b^{9}\right)e^{(-2\,x)}+8\left(4\,a^{3}b^{6}-3\,ab^{8}\right)e^{(-3\,x)}+2\left(8\,a^{4}b^{5}-24\,a^{2}b^{7}+3\,b^{9}\right)e^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a*b^3*e^(-x) - 6*a*b^3*e^(-7*x) + 3*(9*a^2*b^2 - b^4)*e^(-2*x) + 22*
(2*a^3*b - a*b^3)*e^(-3*x) + (25*a^4 - 56*a^2*b^2 + 3*b^4)*e^(-4*x) - 22*(2
*a^3*b - a*b^3)*e^(-5*x) + 3*(9*a^2*b^2 - b^4)*e^(-6*x))/(8*a*b^8*e^(-x) -
8*a*b^8*e^(-7*x) + b^9*e^(-8*x) + b^9 + 4*(6*a^2*b^7 - b^9)*e^(-2*x) + 8*(4
*a^3*b^6 - 3*a*b^8)*e^(-3*x) + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*e^(-4*x)
- 8*(4*a^3*b^6 - 3*a*b^8)*e^(-5*x) + 4*(6*a^2*b^7 - b^9)*e^(-6*x)) + x/b^5
+ log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^5
```

Fricas [B] time = 2.91015, size = 6251, normalized size = 65.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^8 + 3*b^4*x*sinh(x)^8 + 24*(a*b^3*x - a*b^3)*cosh(x)^
7 + 24*(b^4*x*cosh(x) + a*b^3*x - a*b^3)*sinh(x)^7 - 12*(9*a^2*b^2 - b^4 -
(6*a^2*b^2 - b^4)*x)*cosh(x)^6 + 12*(7*b^4*x*cosh(x)^2 - 9*a^2*b^2 + b^4 +
(6*a^2*b^2 - b^4)*x + 14*(a*b^3*x - a*b^3)*cosh(x))*sinh(x)^6 - 8*(22*a^3*b
 - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^5 + 8*(21*b^4*x*cosh(x)^3 -
22*a^3*b + 11*a*b^3 + 63*(a*b^3*x - a*b^3)*cosh(x)^2 + 3*(4*a^3*b - 3*a*b^3
)*x - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x)^5 + 3*b^4*
x - 2*(50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cos
h(x)^4 + 2*(105*b^4*x*cosh(x)^4 - 50*a^4 + 112*a^2*b^2 - 6*b^4 + 420*(a*b^3
*x - a*b^3)*cosh(x)^3 - 90*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^
2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x - 20*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b
 - 3*a*b^3)*x)*cosh(x))*sinh(x)^4 + 8*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3
*a*b^3)*x)*cosh(x)^3 + 8*(21*b^4*x*cosh(x)^5 + 105*(a*b^3*x - a*b^3)*cosh(x
)^4 + 22*a^3*b - 11*a*b^3 - 30*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh
(x)^3 - 10*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 - 3*(4
*a^3*b - 3*a*b^3)*x - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2
 + 3*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x
)*cosh(x)^2 + 4*(21*b^4*x*cosh(x)^6 + 126*(a*b^3*x - a*b^3)*cosh(x)^5 - 45*
(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^4 - 27*a^2*b^2 + 3*b^4 - 20
*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^3 - 3*(50*a^4 - 11
2*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cosh(x)^2 + 3*(6*a^2*
b^2 - b^4)*x + 6*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x))*s
inh(x)^2 - 24*(a*b^3*x - a*b^3)*cosh(x) - 3*(b^4*cosh(x)^8 + b^4*sinh(x)^8
+ 8*a*b^3*cosh(x)^7 + 8*(b^4*cosh(x) + a*b^3)*sinh(x)^7 + 4*(6*a^2*b^2 - b^
4)*cosh(x)^6 + 4*(7*b^4*cosh(x)^2 + 14*a*b^3*cosh(x) + 6*a^2*b^2 - b^4)*sin
h(x)^6 + 8*(4*a^3*b - 3*a*b^3)*cosh(x)^5 + 8*(7*b^4*cosh(x)^3 + 21*a*b^3*co
sh(x)^2 + 4*a^3*b - 3*a*b^3 + 3*(6*a^2*b^2 - b^4)*cosh(x))*sinh(x)^5 - 8*a*
b^3*cosh(x) + 2*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^4 + 2*(35*b^4*cosh(x)^
4 + 140*a*b^3*cosh(x)^3 + 8*a^4 - 24*a^2*b^2 + 3*b^4 + 30*(6*a^2*b^2 - b^4)
*cosh(x)^2 + 20*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^4 + b^4 - 8*(4*a^3*b -
 3*a*b^3)*cosh(x)^3 + 8*(7*b^4*cosh(x)^5 + 35*a*b^3*cosh(x)^4 - 4*a^3*b + 3
*a*b^3 + 10*(6*a^2*b^2 - b^4)*cosh(x)^3 + 10*(4*a^3*b - 3*a*b^3)*cosh(x)^2
+ (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x)^3 + 4*(6*a^2*b^2 - b^4)*cos
h(x)^2 + 4*(7*b^4*cosh(x)^6 + 42*a*b^3*cosh(x)^5 + 15*(6*a^2*b^2 - b^4)*cos
h(x)^4 + 6*a^2*b^2 - b^4 + 20*(4*a^3*b - 3*a*b^3)*cosh(x)^3 + 3*(8*a^4 - 24
*a^2*b^2 + 3*b^4)*cosh(x)^2 - 6*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^2 + 8*
(b^4*cosh(x)^7 + 7*a*b^3*cosh(x)^6 + 3*(6*a^2*b^2 - b^4)*cosh(x)^5 + 5*(4*a
^3*b - 3*a*b^3)*cosh(x)^4 - a*b^3 + (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^3
- 3*(4*a^3*b - 3*a*b^3)*cosh(x)^2 + (6*a^2*b^2 - b^4)*cosh(x))*sinh(x))*log
(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*x*cosh(x)^7 + 21*(a*b^3*
```

```
x - a*b^3)*cosh(x)^6 - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^5
- 3*a*b^3*x - 5*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^4 +
 3*a*b^3 - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x
)*cosh(x)^3 + 3*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 -
 3*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x))/(b^9*cosh(x)^8
 + b^9*sinh(x)^8 + 8*a*b^8*cosh(x)^7 - 8*a*b^8*cosh(x) + b^9 + 8*(b^9*cosh(
x) + a*b^8)*sinh(x)^7 + 4*(6*a^2*b^7 - b^9)*cosh(x)^6 + 4*(7*b^9*cosh(x)^2
+ 14*a*b^8*cosh(x) + 6*a^2*b^7 - b^9)*sinh(x)^6 + 8*(4*a^3*b^6 - 3*a*b^8)*c
osh(x)^5 + 8*(7*b^9*cosh(x)^3 + 21*a*b^8*cosh(x)^2 + 4*a^3*b^6 - 3*a*b^8 +
3*(6*a^2*b^7 - b^9)*cosh(x))*sinh(x)^5 + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)
*cosh(x)^4 + 2*(35*b^9*cosh(x)^4 + 140*a*b^8*cosh(x)^3 + 8*a^4*b^5 - 24*a^2
*b^7 + 3*b^9 + 30*(6*a^2*b^7 - b^9)*cosh(x)^2 + 20*(4*a^3*b^6 - 3*a*b^8)*co
sh(x))*sinh(x)^4 - 8*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^3 + 8*(7*b^9*cosh(x)^5 +
 35*a*b^8*cosh(x)^4 - 4*a^3*b^6 + 3*a*b^8 + 10*(6*a^2*b^7 - b^9)*cosh(x)^3
+ 10*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^2 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cos
h(x))*sinh(x)^3 + 4*(6*a^2*b^7 - b^9)*cosh(x)^2 + 4*(7*b^9*cosh(x)^6 + 42*a
*b^8*cosh(x)^5 + 6*a^2*b^7 - b^9 + 15*(6*a^2*b^7 - b^9)*cosh(x)^4 + 20*(4*a
^3*b^6 - 3*a*b^8)*cosh(x)^3 + 3*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^2
- 6*(4*a^3*b^6 - 3*a*b^8)*cosh(x))*sinh(x)^2 + 8*(b^9*cosh(x)^7 + 7*a*b^8*c
osh(x)^6 - a*b^8 + 3*(6*a^2*b^7 - b^9)*cosh(x)^5 + 5*(4*a^3*b^6 - 3*a*b^8)*
cosh(x)^4 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^3 - 3*(4*a^3*b^6 - 3*a
*b^8)*cosh(x)^2 + (6*a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

Sympy [A] time = 28.2562, size = 2166, normalized size = 22.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**5,x)
```

```
[Out] Piecewise((12*a**4*x*sech(x)**4/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tan
h(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3
*sech(x) + 12*b**9*tanh(x)**4) + 12*a**4*log(a*sech(x)/b + tanh(x))*sech(x)
**4/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b*
*7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**
4) - 12*a**4*log(tanh(x) + 1)*sech(x)**4/(12*a**4*b**5*sech(x)**4 + 48*a**3
*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*t
anh(x)**3*sech(x) + 12*b**9*tanh(x)**4) + 11*a**4*sech(x)**4/(12*a**4*b**5*
sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech
(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) + 48*a**3*b*x*t
anh(x)*sech(x)**3/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**
```

```
3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*
b**9*tanh(x)**4) + 48*a**3*b*log(a*sech(x)/b + tanh(x))*tanh(x)*sech(x)**3/
(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*t
anh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) -
 48*a**3*b*log(tanh(x) + 1)*tanh(x)*sech(x)**3/(12*a**4*b**5*sech(x)**4 + 4
8*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*
b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) + 32*a**3*b*tanh(x)*sech(x)**
3/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7
*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4)
 + 72*a**2*b**2*x*tanh(x)**2*sech(x)**2/(12*a**4*b**5*sech(x)**4 + 48*a**3*
b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*ta
nh(x)**3*sech(x) + 12*b**9*tanh(x)**4) + 72*a**2*b**2*log(a*sech(x)/b + tan
h(x))*tanh(x)**2*sech(x)**2/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)
*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sec
h(x) + 12*b**9*tanh(x)**4) - 72*a**2*b**2*log(tanh(x) + 1)*tanh(x)**2*sech(
x)**2/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*
b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)
**4) + 26*a**2*b**2*tanh(x)**2*sech(x)**2/(12*a**4*b**5*sech(x)**4 + 48*a**
3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*
tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) - 2*a**2*b**2*sech(x)**2/(12*a**4*
b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2
*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) + 48*a*b**
3*x*tanh(x)**3*sech(x)/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech
(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x)
+ 12*b**9*tanh(x)**4) + 48*a*b**3*log(a*sech(x)/b + tanh(x))*tanh(x)**3*sec
h(x)/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b
**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)*
*4) - 48*a*b**3*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(12*a**4*b**5*sech(x)**
4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 +
48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) - 8*a*b**3*tanh(x)*sech(
x)/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**
7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4
) + 12*b**4*x*tanh(x)**4/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*se
ch(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x
) + 12*b**9*tanh(x)**4) + 12*b**4*log(a*sech(x)/b + tanh(x))*tanh(x)**4/(12
*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh
(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) - 12
*b**4*log(tanh(x) + 1)*tanh(x)**4/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*t
anh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)*
*3*sech(x) + 12*b**9*tanh(x)**4) - 5*b**4*tanh(x)**4/(12*a**4*b**5*sech(x)*
*4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh(x)**2*sech(x)**2 +
 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) - 6*b**4*tanh(x)**2/(12
*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b**7*tanh
(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)**4) - 3*
b**4/(12*a**4*b**5*sech(x)**4 + 48*a**3*b**6*tanh(x)*sech(x)**3 + 72*a**2*b
```

```
**7*tanh(x)**2*sech(x)**2 + 48*a*b**8*tanh(x)**3*sech(x) + 12*b**9*tanh(x)*
*4), Ne(b, 0)), ((8*tanh(x)**5/(15*sech(x)**5) - 4*tanh(x)**3/(3*sech(x)**5
) + tanh(x)/sech(x)**5)/a**5, True))
```

Giac [A] time = 1.18885, size = 205, normalized size = 2.16

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b^{5}}-\frac{25\,b^{3}\left(e^{(-x)}-e^{x}\right)^{4}-104\,ab^{2}\left(e^{(-x)}-e^{x}\right)^{3}+168\,a^{2}b\left(e^{(-x)}-e^{x}\right)^{2}+48\,b^{3}\left(e^{(-x)}-e^{x}\right)^{2}-96\,a^{3}\left(\cdots\right.}{12\,\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)^{4}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^5 - 1/12*(25*b^3*(e^(-x) - e^x)^4 - 104
*a*b^2*(e^(-x) - e^x)^3 + 168*a^2*b*(e^(-x) - e^x)^2 + 48*b^3*(e^(-x) - e^x
)^2 - 96*a^3*(e^(-x) - e^x) - 64*a*b^2*(e^(-x) - e^x) + 32*a^2*b + 48*b^3)/
((b*(e^(-x) - e^x) - 2*a)^4*b^4)
```

3.624 $\int (\text{sech}(x) + i \tanh(x))^5 \, dx$

Optimal. Leaf size=40

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0610507, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^5,x]
```

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx = \int \operatorname{sech}^5(x)(1+i\sinh(x))^5\,dx$$
$$= -\left(i\operatorname{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, i\sinh(x)\right)\right)$$
$$= -\left(i\operatorname{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, i\sinh(x)\right)\right)$$
$$= i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}$$

Mathematica [A] time = 0.111476, size = 62, normalized size = 1.55

$$-\frac{11}{4}i\tanh^4(x)-\frac{1}{2}i\tanh^2(x)-\frac{5}{4}i\operatorname{sech}^4(x)+\tan^{-1}(\sinh(x))+i\log(\cosh(x))-\tanh(x)\operatorname{sech}^3(x)-5\tanh^3(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]] - ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] - (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 - ((11*I)/4)*Tanh
[x]^4
```

Maple [B] time = 0.046, size = 82, normalized size = 2.1

$$\frac{8\tanh(x)}{3}\left(\frac{(\operatorname{sech}(x))^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)+2\arctan(e^x)+\frac{\frac{15i}{4}(\sinh(x))^2}{(\cosh(x))^4}-\frac{\frac{5i}{4}(\sinh(x))^2}{(\cosh(x))^2}-\frac{5\sinh(x)}{3(\cosh(x))^4}-5\frac{(\sinh(x))^3}{(\cosh(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))+15/4*I*sinh(x)^2/c
osh(x)^4-5/4*I*sinh(x)^2/cosh(x)^2-5/3*sinh(x)/cosh(x)^4-5*sinh(x)^3/cosh(x
)^4+I*ln(cosh(x))-1/2*I*tanh(x)^2-1/4*I*tanh(x)^4
```

Maxima [B] time = 1.60041, size = 317, normalized size = 7.92

$$-\frac{5}{2}i\,\tanh(x)^4+i\,x-\frac{5\left(5\,e^{(-x)}-3\,e^{(-3\,x)}+3\,e^{(-5\,x)}-5\,e^{(-7\,x)}\right)}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}+\frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}-\frac{5\,(}{2\left(4\,e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*I*tanh(x)^4 + I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*
x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6
*x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*
e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4*I*(e^(-2*x) + e^(-4*
x) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 20*I
/(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) + I*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.44246, size = 273, normalized size = 6.82

$$\frac{-i\,xe^{(4\,x)}+4\,(x-2)e^{(3\,x)}+(6i\,x-8i)e^{(2\,x)}-4\,(x-2)e^{x}+\left(2i\,e^{(4\,x)}-8\,e^{(3\,x)}-12i\,e^{(2\,x)}+8\,e^{x}+2i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (6*I*x - 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (2*I*e^(4*x) - 8*e^(3*x) - 12*I*e^(2*x) + 8*e^x + 2*I)*log(e^x + I) - I*x)
/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\tanh(x)+\operatorname{sech}(x)\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**5,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**5, x)
```

Giac [A] time = 1.15558, size = 49, normalized size = 1.22

$$-i\,x - \frac{8\,e^{(3\,x)} + 8i\,e^{(2\,x)} - 8\,e^{x}}{(e^{x} + i)^{4}} + 2i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -I*x - (8*e^(3*x) + 8*I*e^(2*x) - 8*e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

3.625 $\int (\mathbf{sech}(x) + i\tanh(x))^4 \, dx$

Optimal. Leaf size=38

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rubi [A] time = 0.112979, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^4,x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)+i\tanh(x))^4\,dx &= \int \operatorname{sech}^4(x)(1+i\sinh(x))^4\,dx\\ &= \int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}\,dx\\ &= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\ &= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}+\int 1\,dx\\ &= x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.126771, size = 74, normalized size = 1.95

$$\frac{3(3x-8i)\cosh\left(\frac{x}{2}\right)+(-3x+16i)\cosh\left(\frac{3x}{2}\right)-6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)-4i)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^4,x]
```

```
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [B] time = 0.054, size = 89, normalized size = 2.3

$$-2\left(2/3+1/3\,(\operatorname{sech}(x))^2\right)\tanh(x)+4\,i\left(\frac{(\sinh(x))^2}{3\,(\cosh(x))^3}+\frac{(\sinh(x))^2}{3\,\cosh(x)}-\frac{\cosh(x)}{3}\right)+3\,\frac{\sinh(x)}{(\cosh(x))^3}-4\,i\left(-\frac{(\sinh(x))^2}{3\,(\cosh(x))^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^4,x)
```

```
[Out] -2*(2/3+1/3*sech(x)^2)*tanh(x)+4*I*(1/3*sinh(x)^2/cosh(x)^3+1/3*sinh(x)^2/c
osh(x)-1/3*cosh(x))+3*sinh(x)/cosh(x)^3-4*I*(-1/3*sinh(x)^2/cosh(x)^3+2/3*s
inh(x)^2/cosh(x)-2/3*cosh(x))+x-tanh(x)-1/3*tanh(x)^3
```

Maxima [B] time = 1.056, size = 244, normalized size = 6.42

$$-2\tanh(x)^3 + x - \frac{4\left(3e^{(-2x)} + 3e^{(-4x)} + 2\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} + \frac{8ie^{(-x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{4e^{(-2x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) + 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 32/3*I/(e
^(-x) + e^x)^3
```

Fricas [A] time = 2.32624, size = 153, normalized size = 4.03

$$\frac{3xe^{(3x)} + (9ix + 24i)e^{(2x)} - 3(3x + 8)e^{x} - 3ix - 16i}{3e^{(3x)} + 9ie^{(2x)} - 9e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 24*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*I)/(3*
e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (i\tanh(x) + \operatorname{sech}(x))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**4,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**4, x)
```

Giac [A] time = 1.14611, size = 30, normalized size = 0.79

$$x - \frac{-24i\, e^{(2\,x)} + 24\, e^{x} + 16i}{3\, (e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(-24*I*e^(2*x) + 24*e^x + 16*I)/(e^x + I)^3
```

3.626 $\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx$

Optimal. Leaf size=26

$$-\frac{2i}{1 - i\sinh(x)} - i\log(\sinh(x) + i)$$

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0535539, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{2i}{1 - i\sinh(x)} - i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^3,x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (\mathrm{sech}(x)+i\tanh(x))^3\,dx &= \int \mathrm{sech}^3(x)(1+i\sinh(x))^3\,dx\\ &= -\left(i\,\mathrm{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, i\sinh(x)\right)\right)\\ &= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, i\sinh(x)\right)\right)\\ &= -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0270304, size = 39, normalized size = 1.5

$$\frac{1}{2}i\tanh^2(x)-\frac{3}{2}i\mathrm{sech}^2(x)-\tan^{-1}(\sinh(x))-i\log(\cosh(x))+2\tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^3,x]
```

```
[Out] -ArcTan[Sinh[x]] - I*Log[Cosh[x]] - ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 + (I/2)*Tanh[x]^2
```

Maple [A] time = 0.029, size = 45, normalized size = 1.7

$$-\mathrm{sech}(x)\tanh(x)-2\arctan(\mathrm{e}^x)+\frac{\frac{3i}{2}(\sinh(x))^2}{(\cosh(x))^2}+3\frac{\sinh(x)}{(\cosh(x))^2}-i\ln(\cosh(x))+\frac{i}{2}(\tanh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^3,x)
```

```
[Out] -sech(x)*tanh(x)-2*arctan(exp(x))+3/2*I*sinh(x)^2/cosh(x)^2+3/cosh(x)^2*sin
h(x)-I*ln(cosh(x))+1/2*I*tanh(x)^2
```

Maxima [B] time = 1.60457, size = 99, normalized size = 3.81

$$\frac{3}{2}i\tanh(x)^2-ix+\frac{4\left(e^{(-x)}-e^{(-3x)}\right)}{2e^{(-2x)}+e^{(-4x)}+1}-\frac{2ie^{(-2x)}}{2e^{(-2x)}+e^{(-4x)}+1}+2\arctan\left(e^{(-x)}\right)-i\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*I*tanh(x)^2 - I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) - I*log(e^(-2*
x) + 1)
```

Fricas [B] time = 2.47527, size = 142, normalized size = 5.46

$$\frac{i\,xe^{(2\,x)} - 2\,(x-2)e^{x} + \left(-2i\,e^{(2\,x)} + 4\,e^{x} + 2i\right)\log\left(e^{x} + i\right) - i\,x}{e^{(2\,x)} + 2i\,e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**3,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**3, x)
```

Giac [A] time = 1.1199, size = 28, normalized size = 1.08

$$i\,x + \frac{4\,e^{x}}{\left(e^{x} + i\right)^{2}} - 2i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

3.627 $\int (\mathrm{sech}(x) + i \tanh(x))^2 \, dx$

Optimal. Leaf size=20

$$-x - \frac{2i \cosh(x)}{1 - i \sinh(x)}$$

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0751763, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$-x - \frac{2i \cosh(x)}{1 - i \sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^2,x]
```

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)+i\tanh(x))^2\,dx &= \int \operatorname{sech}^2(x)(1+i\sinh(x))^2\,dx\\ &= \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\ &= -\frac{2i\cosh(x)}{1-i\sinh(x)}-\int 1\,dx\\ &= -x-\frac{2i\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0048836, size = 14, normalized size = 0.7

$$-x+2\tanh(x)-2i\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^2,x]

[Out] -x - (2*I)*Sech[x] + 2*Tanh[x]
```

Maple [A] time = 0.015, size = 26, normalized size = 1.3

$$2\tanh(x)+2i\left(\frac{(\sinh(x))^2}{\cosh(x)}-\cosh(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^2,x)

[Out] 2*tanh(x)+2*I*(sinh(x)^2/cosh(x)-cosh(x))-x
```

Maxima [A] time = 1.04959, size = 34, normalized size = 1.7

$$-x - \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

Fricas [A] time = 2.29544, size = 43, normalized size = 2.15

$$-\frac{xe^x + ix + 4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (i\tanh(x) + \operatorname{sech}(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**2,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**2, x)
```

Giac [A] time = 1.11273, size = 16, normalized size = 0.8

$$-x - \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4*I/(e^x + I)
```

3.628 $\int (\text{sech}(x) + i\tanh(x))\,dx$

Optimal. Leaf size=11

$$\tan^{-1}(\sinh(x)) + i\log(\cosh(x))$$

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

Rubi [A] time = 0.008228, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$\tan^{-1}(\sinh(x)) + i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x] + I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\text{sech}(x) + i\tanh(x))\,dx &= i\int \tanh(x)\,dx + \int \text{sech}(x)\,dx\\ &= \tan^{-1}(\sinh(x)) + i\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0044426, size = 17, normalized size = 1.55

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x] + I*Tanh[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

Maple [A] time = 0., size = 11, normalized size = 1.

$$\arctan(\sinh(x)) + i\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)+I*tanh(x),x)
```

```
[Out] arctan(sinh(x))+I*ln(cosh(x))
```

Maxima [A] time = 1.01438, size = 12, normalized size = 1.09

$$\arctan(\sinh(x)) + i\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x)) + I*log(cosh(x))
```

Fricas [A] time = 2.40346, size = 34, normalized size = 3.09

$$-i\,x + 2i\log(e^x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="fricas")
```

```
[Out] -I*x + 2*I*log(e^x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (i\tanh(x) + \operatorname{sech}(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x)
```

```
[Out] Integral(I*tanh(x) + sech(x), x)
```

Giac [A] time = 1.15644, size = 24, normalized size = 2.18

$$-i\,x + 2\,\arctan(e^{x}) + i\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="giac")
```

```
[Out] -I*x + 2*arctan(e^x) + I*log(e^(2*x) + 1)
```

3.629 $\int \frac{1}{\text{sech}(x)+i\tanh(x)}\,dx$

Optimal. Leaf size=13

$$-i\log(-\sinh(x)+i)$$

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rubi [A] time = 0.029448, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2667, 31}

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-1),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx=\int \frac{\cosh(x)}{1+i\sinh(x)}\,dx$$
$$=-\left(i\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,i\sinh(x)\right)\right)$$
$$=-i\log(i-\sinh(x))$$

Mathematica [A] time = 0.0167274, size = 17, normalized size = 1.31

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-1),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

Maple [B] time = 0.044, size = 33, normalized size = 2.5

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)+1)-2*I*ln(tanh(1/2*x)-I)+I*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05101, size = 20, normalized size = 1.54

$$-i\,x-2i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

Fricas [A] time = 2.48037, size = 32, normalized size = 2.46

$$ix - 2i\log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="fricas")
```

```
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [B] time = 0.667332, size = 22, normalized size = 1.69

$$-ix - i\log(i\tanh(x) + \operatorname{sech}(x)) + i\log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x)
```

```
[Out] -I*x - I*log(I*tanh(x) + sech(x)) + I*log(tanh(x) + 1)
```

Giac [A] time = 1.13534, size = 18, normalized size = 1.38

$$ix - 2i\log(ie^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="giac")
```

```
[Out] I*x - 2*I*log(I*e^x + 1)
```

3.630 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^2}\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0443195, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-2),x]

[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx &= \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh(x)}{1+i\sinh(x)} - \int 1\,dx\\ &= -x+\frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0272138, size = 31, normalized size = 1.55

$$-x+\frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-2),x]
```

```
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] time = 0.051, size = 29, normalized size = 1.5

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+4\left(\tanh(x/2)-i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^2,x)
```

```
[Out] -ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)-1)+4/(tanh(1/2*x)-I)
```

Maxima [A] time = 1.06524, size = 19, normalized size = 0.95

$$-x+\frac{4i}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 4*I/(e^(-x) + I)
```

Fricas [A] time = 2.58902, size = 43, normalized size = 2.15

$$-\frac{xe^{x} - ix - 4i}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\tanh(x) + \operatorname{sech}(x))^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**2,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**(-2), x)
```

Giac [A] time = 1.14637, size = 16, normalized size = 0.8

$$-x + \frac{4i}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -x + 4*I/(e^x - I)
```

3.631 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^3}\,dx$

Optimal. Leaf size=28

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0521178, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-3),x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx &= \int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx\\ &= -\left(i\,\mathrm{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx,x,i\sinh(x)\right)\right)\\ &= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx,x,i\sinh(x)\right)\right)\\ &= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0381816, size = 40, normalized size = 1.43

$$-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))+\frac{2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-3),x]

[Out] -2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I)/(Cosh[x/2] + I*Sinh[x/2])^2
```

Maple [B] time = 0.072, size = 56, normalized size = 2.

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-4\,i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-4\,\left(\tanh(x/2)-i\right)^{-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^3,x)

[Out] -I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-I)-4*I/(tanh(1/2*x)-I)^2-4/(tanh(1/
2*x)-I)-I*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05399, size = 45, normalized size = 1.61

$$ix - \frac{4 e^{(-x)}}{2i e^{(-x)} + e^{(-2x)} - 1} + 2i \log\left(e^{(-x)} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Fricas [B] time = 2.2852, size = 142, normalized size = 5.07

$$\frac{-i\,xe^{(2\,x)} - 2\,(x-2)e^{x} + \left(2i\,e^{(2\,x)} + 4\,e^{x} - 2i\right)\log\left(e^{x} - i\right) + i\,x}{e^{(2\,x)} - 2i\,e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [B] time = 30.0508, size = 513, normalized size = 18.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**3,x)
```

```
[Out] -6*I*x*tanh(x)**2/(-6*tanh(x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) - 1
2*x*tanh(x)*sech(x)/(-6*tanh(x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) +
 6*I*x*sech(x)**2/(-6*tanh(x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) + 6
*I*log(tanh(x) + 1)*tanh(x)**2/(-6*tanh(x)**2 + 12*I*tanh(x)*sech(x) + 6*se
ch(x)**2) + 12*log(tanh(x) + 1)*tanh(x)*sech(x)/(-6*tanh(x)**2 + 12*I*tanh(
x)*sech(x) + 6*sech(x)**2) - 6*I*log(tanh(x) + 1)*sech(x)**2/(-6*tanh(x)**2
 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) - 2*I*log(-I*tanh(x)**3 - 3*tanh(x)
**2*sech(x) + 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*tanh(x)**2/(-6*tanh(x)**
```

```
2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) - 4*log(-I*tanh(x)**3 - 3*tanh(x)*
*2*sech(x) + 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*tanh(x)*sech(x)/(-6*tanh(
x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) + 2*I*log(-I*tanh(x)**3 - 3*ta
nh(x)**2*sech(x) + 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*sech(x)**2/(-6*tanh
(x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) - 6*tanh(x)*sech(x)/(-6*tanh(
x)**2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) + 6*I*sech(x)**2/(-6*tanh(x)**
2 + 12*I*tanh(x)*sech(x) + 6*sech(x)**2) + 3*I/(-6*tanh(x)**2 + 12*I*tanh(x
)*sech(x) + 6*sech(x)**2)
```

Giac [A] time = 1.15608, size = 36, normalized size = 1.29

$$\frac{4\,e^{x}}{\left(e^{x}-i\right)^{2}}-i\,\log\left(i\,e^{x}\right)+2i\,\log\left(-i\,e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x - I)^2 - I*log(I*e^x) + 2*I*log(-I*e^x - 1)
```

3.632 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^4}\,dx$

Optimal. Leaf size=38

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rubi [A] time = 0.0774291, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx &= \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx\\ &= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}+\int 1\,dx\\ &= x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0745308, size = 75, normalized size = 1.97

$$\frac{3(3x+8i)\cosh\left(\frac{x}{2}\right)-(3x+16i)\cosh\left(\frac{3x}{2}\right)+6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)+4i)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [A] time = 0.116, size = 41, normalized size = 1.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+8\,i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-\frac{16}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^4,x)
```

```
[Out] ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)-I)^2-16/3/(tanh(1/2*x)-I)^3-ln(tanh(1/2*
x)-1)
```

Maxima [A] time = 1.0918, size = 54, normalized size = 1.42

$$x - \frac{24\,e^{(-x)} - 24i\,e^{(-2\,x)} + 16i}{9\,e^{(-x)} - 9i\,e^{(-2\,x)} - 3\,e^{(-3\,x)} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] x - (24*e^(-x) - 24*I*e^(-2*x) + 16*I)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x
) + 3*I)
```

Fricas [A] time = 2.05434, size = 154, normalized size = 4.05

$$\frac{3\,xe^{(3\,x)} + (-9i\,x - 24i)e^{(2\,x)} - 3\,(3\,x + 8)e^{x} + 3i\,x + 16i}{3\,e^{(3\,x)} - 9i\,e^{(2\,x)} - 9\,e^{x} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (-9*I*x - 24*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I)/(3
*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13871, size = 30, normalized size = 0.79

$$x - \frac{24i\,e^{(2\,x)} + 24\,e^{x} - 16i}{3\,(e^{x} - i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(24*I*e^(2*x) + 24*e^x - 16*I)/(e^x - I)^3
```

3.633 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^5}\,dx$

Optimal. Leaf size=42

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0554388, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-5),x]
```

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx &= \int \frac{\cosh^5(x)}{(1+i\sinh(x))^5}\,dx\\ &= -\left(i\,\mathrm{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx,x,i\sinh(x)\right)\right)\\ &= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx,x,i\sinh(x)\right)\right)\\ &= -i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0986096, size = 45, normalized size = 1.07

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))+\frac{4\sinh(x)-2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-5),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + (-2*I + 4*Sinh[x])/(Cosh[x/2] + I*Si
nh[x/2])^4
```

Maple [A] time = 0.106, size = 68, normalized size = 1.6

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+8\,i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}-2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-8\,i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+16\,\left(\tanh\left(x/2\right)-i\right)^{-3}+i\ln\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^5,x)
```

```
[Out] I*ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)-I)^4-2*I*ln(tanh(1/2*x)-I)-8*I/(tanh(1
/2*x)-I)^2+16/(tanh(1/2*x)-I)^3+I*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.0906, size = 81, normalized size = 1.93

$$-i\,x - \frac{8\,e^{(-x)} - 8i\,e^{(-2\,x)} - 8\,e^{(-3\,x)}}{-4i\,e^{(-x)} - 6\,e^{(-2\,x)} + 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} - 2i\,\log\left(e^{(-x)} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] -I*x - (8*e^(-x) - 8*I*e^(-2*x) - 8*e^(-3*x))/(-4*I*e^(-x) - 6*e^(-2*x) + 4
*I*e^(-3*x) + e^(-4*x) + 1) - 2*I*log(e^(-x) + I)
```

Fricas [B] time = 2.14075, size = 274, normalized size = 6.52

$$\frac{i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} + (-6i\,x + 8i)e^{(2\,x)} - 4\,(x-2)e^{x} + \left(-2i\,e^{(4\,x)} - 8\,e^{(3\,x)} + 12i\,e^{(2\,x)} + 8\,e^{x} - 2i\right)\log\left(e^{x} - i\right) + i\,x}{e^{(4\,x)} - 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} + 4i\,e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (-6*I*x + 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (-2*I*e^(4*x) - 8*e^(3*x) + 12*I*e^(2*x) + 8*e^x - 2*I)*log(e^x - I) + I*x
)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16191, size = 54, normalized size = 1.29

$$-\frac{8\,e^{(3\,x)} - 8i\,e^{(2\,x)} - 8\,e^{x}}{\left(e^{x} - i\right)^{4}} + i\,\log\left(i\,e^{x}\right) - 2i\,\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -(8*e^(3*x) - 8*I*e^(2*x) - 8*e^x)/(e^x - I)^4 + I*log(I*e^x) - 2*I*log(e^x
 - I)
```

3.634 $\int (\text{sech}(x) - i\tanh(x))^5\,dx$

Optimal. Leaf size=42

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0591995, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^5,x]
```

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int(\mathrm{sech}(x)-i\tanh(x))^5\,dx &= \int \mathrm{sech}^5(x)(1-i\sinh(x))^5\,dx\\ &= i\,\mathrm{Subst}\left(\int\frac{(1+x)^2}{(1-x)^3}\,dx,x,-i\sinh(x)\right)\\ &= i\,\mathrm{Subst}\left(\int\left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx,x,-i\sinh(x)\right)\\ &= -i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0578758, size = 62, normalized size = 1.48

$$\frac{11}{4}i\tanh^4(x)+\frac{1}{2}i\tanh^2(x)+\frac{5}{4}i\mathbf{sech}^4(x)+\tan^{-1}(\sinh(x))-i\log(\cosh(x))-\tanh(x)\mathbf{sech}^3(x)-5\tanh^3(x)\mathbf{sech}(x)+\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]] + ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] + (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 + ((11*I)/4)*Tanh
[x]^4
```

Maple [B] time = 0.048, size = 82, normalized size = 2.

$$\frac{8\tanh(x)}{3}\left(\frac{(\mathrm{sech}(x))^3}{4}+\frac{3\,\mathrm{sech}(x)}{8}\right)+2\arctan(\mathrm{e}^x)-\frac{\frac{15\,i}{4}(\sinh(x))^2}{(\cosh(x))^4}+\frac{\frac{5\,i}{4}(\sinh(x))^2}{(\cosh(x))^2}-\frac{5\sinh(x)}{3(\cosh(x))^4}-5\frac{(\sinh(x))^3}{(\cosh(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))-15/4*I*sinh(x)^2/c
osh(x)^4+5/4*I*sinh(x)^2/cosh(x)^2-5/3*sinh(x)/cosh(x)^4-5*sinh(x)^3/cosh(x
)^4-I*ln(cosh(x))+1/2*I*tanh(x)^2+1/4*I*tanh(x)^4
```

Maxima [B] time = 1.64778, size = 317, normalized size = 7.55

$$\frac{5}{2}i\tanh(x)^4 - ix - \frac{5\left(5e^{(-x)} - 3e^{(-3x)} + 3e^{(-5x)} - 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} + \frac{3e^{(-x)} + 11e^{(-3x)} - 11e^{(-5x)} - 3e^{(-7x)}}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} - \frac{5(\cdots}{2(4e^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*I*tanh(x)^4 - I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x
))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*
x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*e
^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 4*I*(e^(-2*x) + e^(-4*x
) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 20*I/
(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) - I*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.09173, size = 274, normalized size = 6.52

$$\frac{ixe^{(4x)} + 4(x-2)e^{(3x)} + (-6ix + 8i)e^{(2x)} - 4(x-2)e^{x} + \left(-2ie^{(4x)} - 8e^{(3x)} + 12ie^{(2x)} + 8e^{x} - 2i\right)\log\left(e^{x} - i\right) + ix}{e^{(4x)} - 4ie^{(3x)} - 6e^{(2x)} + 4ie^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (-6*I*x + 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (-2*I*e^(4*x) - 8*e^(3*x) + 12*I*e^(2*x) + 8*e^x - 2*I)*log(e^x - I) + I*x
)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int i\tanh^5(x)\,dx - \int 10\tanh^2(x)\operatorname{sech}^3(x)\,dx - \int -5\tanh^4(x)\operatorname{sech}(x)\,dx - \int 5i\tanh(x)\operatorname{sech}^4(x)\,dx - \int -10i\,\mathrm{t}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**5,x)
```

```
[Out] -Integral(I*tanh(x)**5, x) - Integral(10*tanh(x)**2*sech(x)**3, x) - Integr
al(-5*tanh(x)**4*sech(x), x) - Integral(5*I*tanh(x)*sech(x)**4, x) - Integr
al(-10*I*tanh(x)**3*sech(x)**2, x) - Integral(-sech(x)**5, x)
```

Giac [A] time = 1.13774, size = 49, normalized size = 1.17

$$ix - \frac{8\,e^{(3\,x)} - 8i\,e^{(2\,x)} - 8\,e^{x}}{(e^{x} - i)^{4}} - 2i\,\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] I*x - (8*e^(3*x) - 8*I*e^(2*x) - 8*e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

3.635 $\int (\text{sech}(x) - i\tanh(x))^4 \, dx$

Optimal. Leaf size=38

$$x + \frac{2i\cosh^3(x)}{3(1 + i\sinh(x))^3} - \frac{2i\cosh(x)}{1 + i\sinh(x)}$$

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rubi [A] time = 0.108817, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$x + \frac{2i\cosh^3(x)}{3(1 + i\sinh(x))^3} - \frac{2i\cosh(x)}{1 + i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^4,x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (\operatorname{sech}(x)-i\tanh(x))^4\,dx &= \int \operatorname{sech}^4(x)(1-i\sinh(x))^4\,dx \\
&= \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx \\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx \\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}+\int 1\,dx \\
&= x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0535444, size = 75, normalized size = 1.97

$$
\frac{3(3x+8i)\cosh\left(\frac{x}{2}\right)-(3x+16i)\cosh\left(\frac{3x}{2}\right)+6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)+4i)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^4,x]
```

```
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] time = 0.054, size = 89, normalized size = 2.3

$$
-2\left(2/3+1/3\,(\operatorname{sech}(x))^2\right)\tanh(x)-4\,i\left(\frac{(\sinh(x))^2}{3\,(\cosh(x))^3}+\frac{(\sinh(x))^2}{3\,\cosh(x)}-\frac{\cosh(x)}{3}\right)+3\,\frac{\sinh(x)}{(\cosh(x))^3}+4\,i\left(-\frac{(\sinh(x))^2}{3\,(\cosh(x))^3}\right.+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^4,x)
```

```
[Out] -2*(2/3+1/3*sech(x)^2)*tanh(x)-4*I*(1/3*sinh(x)^2/cosh(x)^3+1/3*sinh(x)^2/c
osh(x)-1/3*cosh(x))+3*sinh(x)/cosh(x)^3+4*I*(-1/3*sinh(x)^2/cosh(x)^3+2/3*s
inh(x)^2/cosh(x)-2/3*cosh(x))+x-tanh(x)-1/3*tanh(x)^3
```

Maxima [B] time = 1.06726, size = 244, normalized size = 6.42

$$-2\tanh(x)^3 + x - \frac{4\left(3e^{(-2x)} + 3e^{(-4x)} + 2\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} - \frac{8i\,e^{(-x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{4e^{(-2x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) - 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 32/3*I/(e
^(-x) + e^x)^3
```

Fricas [A] time = 2.09844, size = 154, normalized size = 4.05

$$\frac{3\,xe^{(3\,x)} + (-9i\,x - 24i)e^{(2\,x)} - 3\,(3\,x + 8)e^{x} + 3i\,x + 16i}{3\,e^{(3\,x)} - 9i\,e^{(2\,x)} - 9\,e^{x} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (-9*I*x - 24*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I)/(3
*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\tanh (x) + \operatorname{sech}(x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**4,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**4, x)
```

Giac [A] time = 1.13222, size = 30, normalized size = 0.79

$$x - \frac{24i\,e^{(2\,x)} + 24\,e^{x} - 16i}{3\,(e^{x} - i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(24*I*e^(2*x) + 24*e^x - 16*I)/(e^x - I)^3
```

3.636 $\int (\mathbf{sech}(x) - i\tanh(x))^3 \, dx$

Optimal. Leaf size=28

$$\frac{2i}{1 + i\sinh(x)} + i\log(-\sinh(x) + i)$$

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0599885, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{2i}{1 + i\sinh(x)} + i\log(-\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx &= \int \operatorname{sech}^3(x)(1 - i\sinh(x))^3\,dx\\ &= i\,\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, -i\sinh(x)\right)\\ &= i\,\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2} + \frac{1}{-1+x}\right) dx, x, -i\sinh(x)\right)\\ &= i\log(i - \sinh(x)) + \frac{2i}{1 + i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0284044, size = 39, normalized size = 1.39

$$-\frac{1}{2}i\tanh^2(x) + \frac{3}{2}i\operatorname{sech}^2(x) - \tan^{-1}(\sinh(x)) + i\log(\cosh(x)) + 2\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] -ArcTan[Sinh[x]] + I*Log[Cosh[x]] + ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 - (I/2)*Tanh[x]^2
```

Maple [A] time = 0.029, size = 45, normalized size = 1.6

$$-\operatorname{sech}(x)\tanh(x) - 2\arctan(e^x) - \frac{\frac{3i}{2}(\sinh(x))^2}{(\cosh(x))^2} + 3\frac{\sinh(x)}{(\cosh(x))^2} + i\ln(\cosh(x)) - \frac{i}{2}(\tanh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^3,x)
```

```
[Out] -sech(x)*tanh(x)-2*arctan(exp(x))-3/2*I*sinh(x)^2/cosh(x)^2+3/cosh(x)^2*sin
h(x)+I*ln(cosh(x))-1/2*I*tanh(x)^2
```

Maxima [B] time = 1.57972, size = 99, normalized size = 3.54

$$-\frac{3}{2}i\tanh(x)^2 + ix + \frac{4\left(e^{(-x)} - e^{(-3x)}\right)}{2e^{(-2x)} + e^{(-4x)} + 1} + \frac{2ie^{(-2x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + 2\arctan\left(e^{(-x)}\right) + i\log\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*I*tanh(x)^2 + I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) + I*log(e^(-2
*x) + 1)
```

Fricas [B] time = 2.09119, size = 142, normalized size = 5.07

$$\frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(2i\,e^{(2\,x)}+4\,e^{x}-2i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -i\tanh^{3}(x)\,dx-\int 3\tanh^{2}(x)\operatorname{sech}(x)\,dx-\int 3i\tanh(x)\operatorname{sech}^{2}(x)\,dx-\int -\operatorname{sech}^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**3,x)
```

```
[Out] -Integral(-I*tanh(x)**3, x) - Integral(3*tanh(x)**2*sech(x), x) - Integral(
3*I*tanh(x)*sech(x)**2, x) - Integral(-sech(x)**3, x)
```

Giac [A] time = 1.15122, size = 28, normalized size = 1.

$$-i\,x+\frac{4\,e^{x}}{\left(e^{x}-i\right)^{2}}+2i\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] -I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

3.637 $\int (\mathbf{sech}(x) - i\tanh(x))^2\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0743424, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^2,x]
```

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)-i\tanh(x))^2\,dx &= \int \operatorname{sech}^2(x)(1-i\sinh(x))^2\,dx\\ &= \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh(x)}{1+i\sinh(x)}-\int 1\,dx\\ &= -x+\frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0048979, size = 14, normalized size = 0.7

$$-x+2\tanh(x)+2i\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^2,x]
```

```
[Out] -x + (2*I)*Sech[x] + 2*Tanh[x]
```

Maple [A] time = 0.017, size = 26, normalized size = 1.3

$$2\tanh(x)-2i\left(\frac{(\sinh(x))^2}{\cosh(x)}-\cosh(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^2,x)
```

```
[Out] 2*tanh(x)-2*I*(sinh(x)^2/cosh(x)-cosh(x))-x
```

Maxima [A] time = 0.997069, size = 34, normalized size = 1.7

$$-x + \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

Fricas [A] time = 2.01498, size = 43, normalized size = 2.15

$$-\frac{xe^x - ix - 4i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\tanh(x) + \operatorname{sech}(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**2,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**2, x)
```

Giac [A] time = 1.12289, size = 16, normalized size = 0.8

$$-x + \frac{4i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -x + 4*I/(e^x - I)
```

3.638 $\int (\mathbf{sech}(x) - i\tanh(x))\,dx$

Optimal. Leaf size=11

$$\tan^{-1}(\sinh(x)) - i\log(\cosh(x))$$

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

Rubi [A] time = 0.008347, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$\tan^{-1}(\sinh(x)) - i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x] - I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\text{sech}(x) - i\tanh(x))\,dx &= -(i\int \tanh(x)\,dx) + \int \text{sech}(x)\,dx\\ &= \tan^{-1}(\sinh(x)) - i\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0040074, size = 17, normalized size = 1.55

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x] - I*Tanh[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

Maple [A] time = 0.002, size = 11, normalized size = 1.

$$\arctan(\sinh(x)) - i\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)-I*tanh(x),x)
```

```
[Out] arctan(sinh(x))-I*ln(cosh(x))
```

Maxima [A] time = 0.990642, size = 12, normalized size = 1.09

$$\arctan(\sinh(x)) - i\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x)) - I*log(cosh(x))
```

Fricas [A] time = 2.08936, size = 32, normalized size = 2.91

$$ix - 2i\log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="fricas")
```

```
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\tanh(x) + \operatorname{sech}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x)
```

```
[Out] Integral(-I*tanh(x) + sech(x), x)
```

Giac [A] time = 1.13755, size = 24, normalized size = 2.18

$$ix + 2\arctan(e^x) - i\log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="giac")
```

```
[Out] I*x + 2*arctan(e^x) - I*log(e^(2*x) + 1)
```

3.639 $\int \frac{1}{\mathbf{sech}(x)-i\tanh(x)}\,dx$

Optimal. Leaf size=11

$$i\log(\sinh(x)+i)$$

[Out] I*Log[I + Sinh[x]]

Rubi [A] time = 0.0301713, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2667, 31}

$$i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] - I*Tanh[x])^(-1),x]

[Out] I*Log[I + Sinh[x]]

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = \int \frac{\cosh(x)}{1 - i\sinh(x)}\,dx$$
$$= i\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -i\sinh(x)\right)$$
$$= i\log(i + \sinh(x))$$

Mathematica [A] time = 0.0173788, size = 17, normalized size = 1.55

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-1),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

Maple [B] time = 0.045, size = 33, normalized size = 3.

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2\,i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)-I*ln(tanh(1/2*x)-1)+2*I*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.04957, size = 20, normalized size = 1.82

$$i\,x + 2i\,\log\left(i\,e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="maxima")
```

```
[Out] I*x + 2*I*log(I*e^(-x) + 1)
```

Fricas [A] time = 2.05302, size = 34, normalized size = 3.09

$$-i\,x + 2i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="fricas")
```

```
[Out] -I*x + 2*I*log(e^x + I)
```

Sympy [B] time = 0.663409, size = 22, normalized size = 2.

$$ix + i\log\left(-i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right) - i\log\left(\tanh\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x)
```

```
[Out] I*x + I*log(-I*tanh(x) + sech(x)) - I*log(tanh(x) + 1)
```

Giac [A] time = 1.17548, size = 18, normalized size = 1.64

$$-i\,x + 2i\,\log\left(-i\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="giac")
```

```
[Out] -I*x + 2*I*log(-I*e^x + 1)
```

3.640 $\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx$

Optimal. Leaf size=20

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0468274, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-2),x]
```

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = \int \frac{\cosh^2(x)}{(1 - i\sinh(x))^2}\,dx$$
$$= -\frac{2i\cosh(x)}{1 - i\sinh(x)} - \int 1\,dx$$
$$= -x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Mathematica [A] time = 0.0285395, size = 31, normalized size = 1.55

$$-x + \frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-2),x]
```

```
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
```

Maple [A] time = 0.054, size = 29, normalized size = 1.5

$$-\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + 4\,(\tanh(x/2) + i)^{-1} + \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^2,x)
```

```
[Out] -ln(tanh(1/2*x)+1)+4/(tanh(1/2*x)+I)+ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.03978, size = 19, normalized size = 0.95

$$-x - \frac{4i}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 4*I/(e^(-x) - I)
```

Fricas [A] time = 2.02849, size = 43, normalized size = 2.15

$$-\frac{xe^x + ix + 4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-i\tanh(x) + \operatorname{sech}(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**2,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**(-2), x)
```

Giac [A] time = 1.13175, size = 16, normalized size = 0.8

$$-x - \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4*I/(e^x + I)
```

3.641 $\int \frac{1}{(\mathbf{sech}(x)-i\tanh(x))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0517354, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-3),x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^3}\,dx &= \int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx\\ &= i\,\mathrm{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx,x,-i\sinh(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx,x,-i\sinh(x)\right)\\ &= -i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.036104, size = 27, normalized size = 1.04

$$\frac{2}{\sinh(x)+i}-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-3),x]
```

```
[Out] -2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + 2/(I + Sinh[x])
```

Maple [B] time = 0.078, size = 56, normalized size = 2.2

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+4i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-4\,(\tanh(x/2)+i)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^3,x)
```

```
[Out] I*ln(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)+4*I/(tanh(1/2*x)+I)^2-2*I*ln(tanh(1
/2*x)+I)-4/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.0343, size = 45, normalized size = 1.73

$$-i\,x - \frac{4\,e^{(-x)}}{-2i\,e^{(-x)} + e^{(-2\,x)} - 1} - 2i\,\log\left(e^{(-x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Fricas [B] time = 2.07498, size = 142, normalized size = 5.46

$$\frac{i\,xe^{(2\,x)} - 2\,(x-2)e^{x} + \left(-2i\,e^{(2\,x)} + 4\,e^{x} + 2i\right)\log\left(e^{x} + i\right) - i\,x}{e^{(2\,x)} + 2i\,e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] time = 29.5032, size = 513, normalized size = 19.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**3,x)
```

```
[Out] -6*I*x*tanh(x)**2/(6*tanh(x)**2 + 12*I*tanh(x)*sech(x) - 6*sech(x)**2) + 12
*x*tanh(x)*sech(x)/(6*tanh(x)**2 + 12*I*tanh(x)*sech(x) - 6*sech(x)**2) + 6
*I*x*sech(x)**2/(6*tanh(x)**2 + 12*I*tanh(x)*sech(x) - 6*sech(x)**2) + 6*I*
log(tanh(x) + 1)*tanh(x)**2/(6*tanh(x)**2 + 12*I*tanh(x)*sech(x) - 6*sech(x
)**2) - 12*log(tanh(x) + 1)*tanh(x)*sech(x)/(6*tanh(x)**2 + 12*I*tanh(x)*se
ch(x) - 6*sech(x)**2) - 6*I*log(tanh(x) + 1)*sech(x)**2/(6*tanh(x)**2 + 12*
I*tanh(x)*sech(x) - 6*sech(x)**2) - 2*I*log(I*tanh(x)**3 - 3*tanh(x)**2*sec
h(x) - 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*tanh(x)**2/(6*tanh(x)**2 + 12*I
```

```
*tanh(x)*sech(x) - 6*sech(x)**2) + 4*log(I*tanh(x)**3 - 3*tanh(x)**2*sech(x
) - 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*tanh(x)*sech(x)/(6*tanh(x)**2 + 12
*I*tanh(x)*sech(x) - 6*sech(x)**2) + 2*I*log(I*tanh(x)**3 - 3*tanh(x)**2*se
ch(x) - 3*I*tanh(x)*sech(x)**2 + sech(x)**3)*sech(x)**2/(6*tanh(x)**2 + 12*
I*tanh(x)*sech(x) - 6*sech(x)**2) + 6*tanh(x)*sech(x)/(6*tanh(x)**2 + 12*I*
tanh(x)*sech(x) - 6*sech(x)**2) + 6*I*sech(x)**2/(6*tanh(x)**2 + 12*I*tanh(
x)*sech(x) - 6*sech(x)**2) + 3*I/(6*tanh(x)**2 + 12*I*tanh(x)*sech(x) - 6*s
ech(x)**2)
```

Giac [A] time = 1.14519, size = 36, normalized size = 1.38

$$\frac{4 e^{x}}{\left(e^{x}+i\right)^{2}}+i \log\left(-i e^{x}\right)-2 i \log\left(i e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x + I)^2 + I*log(-I*e^x) - 2*I*log(I*e^x - 1)
```

3.642 $\int \frac{1}{(\mathbf{sech}(x)-i\tanh(x))^4}\,dx$

Optimal. Leaf size=38

$$x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rubi [A] time = 0.0767709, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-4),x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}\,dx &= \int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}\,dx \\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx \\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)} + \int 1\,dx \\
&= x - \frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.0639656, size = 74, normalized size = 1.95

$$\frac{3(3x-8i)\cosh\left(\frac{x}{2}\right)+(-3x+16i)\cosh\left(\frac{3x}{2}\right)-6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)-4i)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-4),x]
```

```
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [A] time = 0.118, size = 41, normalized size = 1.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-8\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-\frac{16}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^4,x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)-8*I/(tanh(1/2*x)+I)^2-16/3/(tanh(1/2*x)
+I)^3
```

Maxima [A] time = 1.07945, size = 54, normalized size = 1.42

$$x - \frac{24\,e^{(-x)} + 24i\,e^{(-2\,x)} - 16i}{9\,e^{(-x)} + 9i\,e^{(-2\,x)} - 3\,e^{(-3\,x)} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] x - (24*e^(-x) + 24*I*e^(-2*x) - 16*I)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x
) - 3*I)
```

Fricas [A] time = 2.10535, size = 153, normalized size = 4.03

$$\frac{3\,xe^{(3\,x)} + (9i\,x + 24i)e^{(2\,x)} - 3\,(3\,x + 8)e^{x} - 3i\,x - 16i}{3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 24*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*I)/(3*
e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**4,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.14992, size = 30, normalized size = 0.79

$$x - \frac{-24i\, e^{(2\, x)} + 24\, e^{x} + 16i}{3\,(e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(-24*I*e^(2*x) + 24*e^x + 16*I)/(e^x + I)^3
```

3.643 $\int \frac{1}{(\mathbf{sech}(x)-i\tanh(x))^5}\,dx$

Optimal. Leaf size=40

$$\frac{4i}{1-i\sinh(x)}-\frac{2i}{(1-i\sinh(x))^2}+i\log(\sinh(x)+i)$$

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0551645, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{4i}{1-i\sinh(x)}-\frac{2i}{(1-i\sinh(x))^2}+i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-5),x]
```

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx &= \int \frac{\cosh^5(x)}{(1-i\sinh(x))^5}\,dx\\ &= i\,\mathrm{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, -i\sinh(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, -i\sinh(x)\right)\\ &= i\log(i+\sinh(x))-\frac{2i}{(1-i\sinh(x))^2}+\frac{4i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0983803, size = 45, normalized size = 1.12

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))+\frac{4\sinh(x)+2i}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-5),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I + 4*Sinh[x])/(Cosh[x/2] - I*Sin
h[x/2])^4
```

Maple [A] time = 0.098, size = 68, normalized size = 1.7

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+8\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}+2\,i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-8\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}+16\,(\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^5,x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)-I*ln(tanh(1/2*x)-1)+8*I/(tanh(1/2*x)+I)^2+2*I*ln(tanh(
1/2*x)+I)-8*I/(tanh(1/2*x)+I)^4+16/(tanh(1/2*x)+I)^3
```

Maxima [B] time = 1.08159, size = 81, normalized size = 2.02

$$ix - \frac{8\,e^{(-x)} + 8i\,e^{(-2\,x)} - 8\,e^{(-3\,x)}}{4i\,e^{(-x)} - 6\,e^{(-2\,x)} - 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} + 2i\,\log\left(e^{(-x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] I*x - (8*e^(-x) + 8*I*e^(-2*x) - 8*e^(-3*x))/(4*I*e^(-x) - 6*e^(-2*x) - 4*I
*e^(-3*x) + e^(-4*x) + 1) + 2*I*log(e^(-x) - I)
```

Fricas [B] time = 2.16657, size = 273, normalized size = 6.82

$$\frac{-i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} + (6i\,x - 8i)e^{(2\,x)} - 4\,(x-2)e^{x} + \left(2i\,e^{(4\,x)} - 8\,e^{(3\,x)} - 12i\,e^{(2\,x)} + 8\,e^{x} + 2i\right)\log\left(e^{x} + i\right) - i\,x}{e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (6*I*x - 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (2*I*e^(4*x) - 8*e^(3*x) - 12*I*e^(2*x) + 8*e^x + 2*I)*log(e^x + I) - I*x)
/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15358, size = 54, normalized size = 1.35

$$-\frac{8\,e^{(3\,x)}+8i\,e^{(2\,x)}-8\,e^{x}}{(e^{x}+i)^{4}}-i\,\log\left(-i\,e^{x}\right)+2i\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -(8*e^(3*x) + 8*I*e^(2*x) - 8*e^x)/(e^x + I)^4 - I*log(-I*e^x) + 2*I*log(e^
x + I)
```

3.644 $\int (a\coth(x)+b\text{csch}(x))^5\,dx$

Optimal. Leaf size=124

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x)-\frac{1}{8}b\left(-10a^2b^2+15a^4+3b^4\right)\tanh^{-1}(\cosh(x))-\frac{1}{8}\text{csch}^2(x)(a\cosh(x)+b)^2\left(b\left(5a^2-3b^2\right)\cosh\right.$$

```
[Out] -(b*(15*a^4 - 10*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/8 + (a^2*b*(7*a^2 - 3*b
^2)*Cosh[x])/8 - ((b + a*Cosh[x])^2*(2*a*(2*a^2 - b^2) + b*(5*a^2 - 3*b^2)*
Cosh[x])*Csch[x]^2)/8 - ((b + a*Cosh[x])^4*(a + b*Cosh[x])*Csch[x]^4)/4 + a
^5*Log[Sinh[x]]
```

Rubi [A] time = 0.243005, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4392, 2668, 739, 819, 774, 635, 204, 260}

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x)-\frac{1}{8}b\left(-10a^2b^2+15a^4+3b^4\right)\tanh^{-1}(\cosh(x))-\frac{1}{8}\text{csch}^2(x)(a\cosh(x)+b)^2\left(b\left(5a^2-3b^2\right)\cosh\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] -(b*(15*a^4 - 10*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/8 + (a^2*b*(7*a^2 - 3*b
^2)*Cosh[x])/8 - ((b + a*Cosh[x])^2*(2*a*(2*a^2 - b^2) + b*(5*a^2 - 3*b^2)*
Cosh[x])*Csch[x]^2)/8 - ((b + a*Cosh[x])^4*(a + b*Cosh[x])*Csch[x]^4)/4 + a
^5*Log[Sinh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (a\coth(x) + b\mathbf{csch}(x))^5\,dx = -\left(i\int (ib + ia\cosh(x))^5\mathbf{csch}^5(x)\,dx\right)$$
$$= -\left(a^5\,\text{Subst}\left(\int \frac{(ib+x)^5}{\left(-a^2-x^2\right)^3}\,dx, x, ia\cosh(x)\right)\right)$$
$$= -\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\mathbf{csch}^4(x) - \frac{1}{4}a^3\,\text{Subst}\left(\int \frac{(ib+x)^3\left(-4a^2+3b^2+ibx\right.}{\left(-a^2-x^2\right)^2}\right.$$
$$= -\frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\mathbf{csch}^2(x) - \frac{1}{4}(b+a\cosh(x))^4$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\mathbf{cs}$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\mathbf{cs}$$
$$= -\frac{1}{8}b\left(15a^4-10a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x)) + \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh($$

Mathematica [A] time = 0.469392, size = 244, normalized size = 1.97

$$-\frac{1}{64}\mathbf{csch}^4(x)\left(20a^2b^3\cosh(3x) + 2b\left(70a^2b^2+15a^4+11b^4\right)\cosh(x) + 30a^2b^3\log\left(\tanh\left(\frac{x}{2}\right)\right) + 10a^2b^3\cosh(4x)\log\left(\tan\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] -(Csch[x]^4*(-16*a^5 + 80*a*b^4 + 2*b*(15*a^4 + 70*a^2*b^2 + 11*b^4)*Cosh[x
] + 50*a^4*b*Cosh[3*x] + 20*a^2*b^3*Cosh[3*x] - 6*b^5*Cosh[3*x] - 24*a^5*Lo
g[Sinh[x]] - 8*a^5*Cosh[4*x]*Log[Sinh[x]] - 45*a^4*b*Log[Tanh[x/2]] + 30*a^
2*b^3*Log[Tanh[x/2]] - 9*b^5*Log[Tanh[x/2]] - 15*a^4*b*Cosh[4*x]*Log[Tanh[x
/2]] + 10*a^2*b^3*Cosh[4*x]*Log[Tanh[x/2]] - 3*b^5*Cosh[4*x]*Log[Tanh[x/2]]
 + 4*Cosh[2*x]*(8*(a^5 + 5*a^3*b^2) + 8*a^5*Log[Sinh[x]] + b*(15*a^4 - 10*a
^2*b^2 + 3*b^4)*Log[Tanh[x/2]])))/64
```

Maple [A] time = 0.028, size = 223, normalized size = 1.8

$$a^5\ln(\sinh(x)) - \frac{a^5(\coth(x))^2}{2} - \frac{a^5(\coth(x))^4}{4} - 5\frac{a^4b(\cosh(x))^3}{(\sinh(x))^4} + 5\frac{a^4b\cosh(x)}{(\sinh(x))^4} - \frac{5a^4b\coth(x)(\mathrm{csch}(x))^3}{4} + \frac{15a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^5,x)
```

```
[Out] a^5*ln(sinh(x))-1/2*a^5*coth(x)^2-1/4*a^5*coth(x)^4-5*a^4*b/sinh(x)^4*cosh(
x)^3+5*a^4*b/sinh(x)^4*cosh(x)-5/4*a^4*b*coth(x)*csch(x)^3+15/8*a^4*b*csch(
x)*coth(x)-15/4*a^4*b*arctanh(exp(x))-5/2*a^3*b^2/sinh(x)^4*cosh(x)^2-5/2*a
^3*b^2*cosh(x)^2/sinh(x)^2-10/3*a^2*b^3/sinh(x)^4*cosh(x)+5/6*a^2*b^3*coth(
x)*csch(x)^3-5/4*a^2*b^3*csch(x)*coth(x)+5/2*a^2*b^3*arctanh(exp(x))-5/4*b^
4*a/sinh(x)^4*cosh(x)^2+5/4*b^4*a*cosh(x)^2/sinh(x)^2-1/4*b^5*coth(x)*csch(
x)^3+3/8*b^5*csch(x)*coth(x)-3/4*b^5*arctanh(exp(x))
```

Maxima [B] time = 1.07308, size = 446, normalized size = 3.6

$$-\frac{5}{2}a^3b^2\coth(x)^4 + a^5\left(x + \frac{4\left(e^{(-2x)} - e^{(-4x)} + e^{(-6x)}\right)}{4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)\right) + \frac{5}{8}a^4b\left(\frac{2\left(5e^{(-x}\right.}{4e^{(-2x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*a^3*b^2*coth(x)^4 + a^5*(x + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(
-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + log(e^(-x) + 1) + log(e^(
-x) - 1)) + 5/8*a^4*b*(2*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 3*log(e^(-x) + 1) +
 3*log(e^(-x) - 1)) - 1/8*b^5*(2*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*
e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3*log(e^(
-x) + 1) - 3*log(e^(-x) - 1)) + 5/4*a^2*b^3*(2*(e^(-x) + 7*e^(-3*x) + 7*e^(
-5*x) + e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + l
og(e^(-x) + 1) - log(e^(-x) - 1)) - 20*a*b^4/(e^(-x) - e^x)^4
```

Fricas [B] time = 2.4722, size = 6710, normalized size = 54.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

[Out] -1/8*(8*a^5*x*cosh(x)^8 + 8*a^5*x*sinh(x)^8 + 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^7 + 2*(32*a^5*x*cosh(x) + 25*a^4*b + 10*a^2*b^3 - 3*b^5)*sinh(x)^7 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^6 + 2*(112*a^5*x*cosh(x)^2 - 16*a^5*x + 16*a^5 + 80*a^3*b^2 + 7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^6 + 8*a^5*x + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^5 + 2*(224*a^5*x*cosh(x)^3 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 21*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2 - 96*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(x)^5 + 16*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^4 + 2*(280*a^5*x*cosh(x)^4 + 24*a^5*x - 16*a^5 + 80*a*b^4 + 35*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^4 + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 2*(224*a^5*x*cosh(x)^5 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 35*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 - 320*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^3 + 10*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^2 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x))*sinh(x)^3 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 2*(112*a^5*x*cosh(x)^6 - 16*a^5*x + 21*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 + 16*a^5 + 80*a^3*b^2 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^4 + 10*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 48*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^2 + 3*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^2 + 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x) - ((8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^8 + 8*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*sinh(x)^8 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^5 + 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + 6*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 2*(24*a^5 - 45*a^4*b + 30*a^2*b^3 - 9*b^5 + 35*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 - 30*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 - 10*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 - 15*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 9*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^7 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^8 + 8*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*sinh(x)^8 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^6 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 - 7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^5 + 8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + 6*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^

```
5)*cosh(x)^4 + 2*(24*a^5 + 45*a^4*b - 30*a^2*b^3 + 9*b^5 + 35*(8*a^5 + 15*a
^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 - 30*(8*a^5 + 15*a^4*b - 10*a^2*b^3 +
3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*
cosh(x)^5 - 10*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 + 3*(8*a^5
 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^3 - 4*(8*a^5 + 15*a^4*b
- 10*a^2*b^3 + 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b
^5)*cosh(x)^6 - 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 15*(8*a^5 + 15*a^4*
b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 + 9*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^
5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x
)^7 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^5 + 3*(8*a^5 + 15*a
^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 - (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b
^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(32*a^5*x*cosh(x)^7 +
7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 96*(a^5*x - a^5 - 5*a^3*b^2)*
cosh(x)^5 + 25*a^4*b + 10*a^2*b^3 - 3*b^5 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b
^5)*cosh(x)^4 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^3 + 3*(15*a^4*b + 7
0*a^2*b^3 + 11*b^5)*cosh(x)^2 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(
x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh
(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)
^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 -
1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cos
h(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18024, size = 316, normalized size = 2.55

$$\frac{1}{16}\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\log\left(e^{(-x)}+e^x+2\right)+\frac{1}{16}\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\log\left(e^{(-x)}+e^x-2\right)-\frac{3\,a^5\left(e^{(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(e^(-x) + e^x + 2) + 1/16*(
8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(e^(-x) + e^x - 2) - 1/4*(3*a^5*(
e^(-x) + e^x)^4 + 25*a^4*b*(e^(-x) + e^x)^3 + 10*a^2*b^3*(e^(-x) + e^x)^3 -
 3*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 80*a^3*b^2*(e^(-x) + e^x
)^2 - 60*a^4*b*(e^(-x) + e^x) + 40*a^2*b^3*(e^(-x) + e^x) + 20*b^5*(e^(-x)
+ e^x) - 160*a^3*b^2 + 80*a*b^4)/((e^(-x) + e^x)^2 - 4)^2
```

3.645 $\int (a\coth(x) + b\,\mathbf{csch}(x))^4\,dx$

Optimal. Leaf size=101

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x)-\frac{1}{3}\mathbf{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)+a^4x$$

```
[Out] a^4*x - ((b + a*Cosh[x])^2*(a*b + (3*a^2 - 2*b^2)*Cosh[x])*Csch[x])/3 - ((b
 + a*Cosh[x])^3*(a + b*Cosh[x])*Csch[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sinh[x]
)/3 + (a^2*(3*a^2 - 2*b^2)*Cosh[x]*Sinh[x])/3
```

Rubi [A] time = 0.238612, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4392, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x)-\frac{1}{3}\mathbf{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)+a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] a^4*x - ((b + a*Cosh[x])^2*(a*b + (3*a^2 - 2*b^2)*Cosh[x])*Csch[x])/3 - ((b
 + a*Cosh[x])^3*(a + b*Cosh[x])*Csch[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sinh[x]
)/3 + (a^2*(3*a^2 - 2*b^2)*Cosh[x]*Sinh[x])/3
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a\coth(x)+b\,\mathrm{csch}(x))^4\,dx=\int (ib+ia\cosh(x))^4\mathrm{csch}^4(x)\,dx$$
$$=-\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\mathrm{csch}^3(x)+\frac{1}{3}\int (ib+ia\cosh(x))^2\left(-3a^2+2b^2-ab\,\mathrm{co}\right.$$
$$=-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\mathrm{csch}(x)-\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x$$
$$=a^4x-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\mathrm{csch}(x)-\frac{1}{3}(b+a\cosh(x))^3(a+b\,\mathrm{co}$$

Mathematica [A] time = 0.265863, size = 95, normalized size = 0.94

$$-\frac{1}{12}\mathrm{csch}^3(x)\left(6a^2b^2\cosh(3x)+6b^2\left(3a^2+b^2\right)\cosh(x)+24a^3b\cosh(2x)-8a^3b+9a^4x\sinh(x)-3a^4x\sinh(3x)+4a^4\,\mathrm{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] -(Csch[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cosh[x] + 24*a^3*b*C
osh[2*x] + 4*a^4*Cosh[3*x] + 6*a^2*b^2*Cosh[3*x] - 2*b^4*Cosh[3*x] + 9*a^4*
x*Sinh[x] - 3*a^4*x*Sinh[3*x]))/12
```

Maple [A] time = 0.02, size = 123, normalized size = 1.2

$$a^4\left(x-\coth(x)-\frac{(\coth(x))^3}{3}\right)+4a^3b\left(-1/3\,\frac{(\cosh(x))^2}{(\sinh(x))^3}-2/3\,\frac{(\cosh(x))^2}{\sinh(x)}+2/3\,\sinh(x)\right)+6a^2b^2\left(-1/2\,\frac{\cosh(x)}{(\sinh(x))^3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^4,x)
```

```
[Out] a^4*(x-coth(x)-1/3*coth(x)^3)+4*a^3*b*(-1/3/sinh(x)^3*cosh(x)^2-2/3*cosh(x)
^2/sinh(x)+2/3*sinh(x))+6*a^2*b^2*(-1/2/sinh(x)^3*cosh(x)-1/2*(2/3-1/3*csch
(x)^2)*coth(x))+4*a*b^3*(-1/3/sinh(x)^3*cosh(x)^2+1/3*cosh(x)^2/sinh(x)-1/3
*sinh(x))+b^4*(2/3-1/3*csch(x)^2)*coth(x)
```

Maxima [B] time = 1.04159, size = 289, normalized size = 2.86

$$-2a^2b^2\coth(x)^3+\frac{1}{3}a^4\left(3x-\frac{4\left(3e^{(-2x)}-3e^{(-4x)}-2\right)}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}\right)+\frac{8}{3}a^3b\left(\frac{3e^{(-x)}}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}-\frac{2e^{(-3}}{3e^{(-2x)}-3e^{(-4x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*coth(x)^3 + 1/3*a^4*(3*x - 4*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^
(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 8/3*a^3*b*(3*e^(-x)/(3*e^(-2*x) - 3*
e^(-4*x) + e^(-6*x) - 1) - 2*e^(-3*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) -
 1) + 3*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 4/3*b^4*(3*e^(
-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 1/(3*e^(-2*x) - 3*e^(-4*x)
 + e^(-6*x) - 1)) + 32/3*a*b^3/(e^(-x) - e^x)^3
```

Fricas [B] time = 2.05329, size = 497, normalized size = 4.92

$$-\frac{24a^3b\cosh(x)^2-8a^3b+16ab^3+2\left(2a^4+3a^2b^2-b^4\right)\cosh(x)^3-\left(3a^4x+4a^4+6a^2b^2-2b^4\right)\sinh(x)^3+6\left(4a^3b\right.}{3\left(s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*a^3*b*cosh(x)^2 - 8*a^3*b + 16*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)
*cosh(x)^3 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4)*sinh(x)^3 + 6*(4*a^3*b +
 (2*a^4 + 3*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 6*(3*a^2*b^2 + b^4)*cosh(x)
 + 3*(3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(sinh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\coth(x) + b\,\mathrm{csch}(x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**4,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**4, x)
```

Giac [A] time = 1.22368, size = 151, normalized size = 1.5

$$a^4x - \frac{4\left(6\,a^3be^{(5\,x)} + 3\,a^4e^{(4\,x)} + 9\,a^2b^2e^{(4\,x)} - 4\,a^3be^{(3\,x)} + 8\,ab^3e^{(3\,x)} - 3\,a^4e^{(2\,x)} + 3\,b^4e^{(2\,x)} + 6\,a^3be^x + 2\,a^4 + 3\,a^2b^2 - b^4\right)}{3\left(e^{(2\,x)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] a^4*x - 4/3*(6*a^3*b*e^(5*x) + 3*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x) - 4*a^3*b*
e^(3*x) + 8*a*b^3*e^(3*x) - 3*a^4*e^(2*x) + 3*b^4*e^(2*x) + 6*a^3*b*e^x + 2
*a^4 + 3*a^2*b^2 - b^4)/(e^(2*x) - 1)^3
```

3.646 $\int (a\coth(x) + b\,\mathbf{csch}(x))^3\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)+a^3\log(\sinh(x))-\frac{1}{2}\mathbf{csch}^2(x)(a\cosh(x)+b)^2(a+b\cosh(x))$$

```
[Out] -(b*(3*a^2 - b^2)*ArcTanh[Cosh[x]])/2 + (a^2*b*Cosh[x])/2 - ((b + a*Cosh[x]
)^2*(a + b*Cosh[x])*Csch[x]^2)/2 + a^3*Log[Sinh[x]]
```

Rubi [A] time = 0.123469, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2668, 739, 774, 635, 204, 260}

$$-\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)+a^3\log(\sinh(x))-\frac{1}{2}\mathbf{csch}^2(x)(a\cosh(x)+b)^2(a+b\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^3,x]
```

```
[Out] -(b*(3*a^2 - b^2)*ArcTanh[Cosh[x]])/2 + (a^2*b*Cosh[x])/2 - ((b + a*Cosh[x]
)^2*(a + b*Cosh[x])*Csch[x]^2)/2 + a^3*Log[Sinh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (a\coth(x)+b\mathbf{csch}(x))^3\,dx = i\int (ib+ia\cosh(x))^3\mathbf{csch}^3(x)\,dx$$
$$= a^3\,\text{Subst}\left(\int \frac{(ib+x)^3}{\left(-a^2-x^2\right)^2}\,dx, x, ia\cosh(x)\right)$$
$$= -\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\mathbf{csch}^2(x)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{(ib+x)\left(-2a^2+b^2+ibx\right)}{-a^2-x^2}\right.$$
$$= \frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\mathbf{csch}^2(x)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{ia^2b-ib\left(-2\right.}{-a}\right.$$
$$= \frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\mathbf{csch}^2(x)-a^3\,\text{Subst}\left(\int \frac{x}{-a^2-x^2}\,dx,\right.$$
$$= -\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\mathbf{csc}$$

Mathematica [A] time = 0.240878, size = 99, normalized size = 1.68

$$-\frac{1}{4}\mathbf{csch}^2(x)\left(2b\left(3a^2+b^2\right)\cosh(x)+\cosh(2x)\left(b\left(b^2-3a^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-2a^3\log(\sinh(x))\right)+3a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^3,x]
```

```
[Out] -(Csch[x]^2*(2*a^3 + 6*a*b^2 + 2*b*(3*a^2 + b^2)*Cosh[x] + 2*a^3*Log[Sinh[x
]] + 3*a^2*b*Log[Tanh[x/2]] - b^3*Log[Tanh[x/2]] + Cosh[2*x]*(-2*a^3*Log[Si
nh[x]] + b*(-3*a^2 + b^2)*Log[Tanh[x/2]])))/4
```

Maple [A] time = 0.02, size = 79, normalized size = 1.3

$$a^3\ln(\sinh(x))-\frac{a^3(\coth(x))^2}{2}-3\frac{a^2b\cosh(x)}{(\sinh(x))^2}+\frac{3a^2b\operatorname{csch}(x)\coth(x)}{2}-3a^2b\operatorname{Artanh}(e^x)-\frac{3ab^2(\cosh(x))^2}{2(\sinh(x))^2}-\frac{b^3c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^3,x)
```

```
[Out] a^3*ln(sinh(x))-1/2*a^3*coth(x)^2-3*a^2*b/sinh(x)^2*cosh(x)+3/2*a^2*b*csch(
x)*coth(x)-3*a^2*b*arctanh(exp(x))-3/2*a*b^2*cosh(x)^2/sinh(x)^2-1/2*b^3*cs
```

```
ch(x)*coth(x)+b^3*arctanh(exp(x))
```

Maxima [B] time = 1.11341, size = 205, normalized size = 3.47

$$-\frac{3}{2}ab^2\coth(x)^2 + a^3\left(x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)\right) + \frac{1}{2}b^3\left(\frac{2\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(e^{(-x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*coth(x)^2 + a^3*(x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + lo
g(e^(-x) + 1) + log(e^(-x) - 1)) + 1/2*b^3*(2*(e^(-x) + e^(-3*x))/(2*e^(-2*
x) - e^(-4*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)) + 3/2*a^2*b*(2*(e^(
-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - log(e^(-x) + 1) + log(e^(-x)
- 1))
```

Fricas [B] time = 2.24344, size = 1712, normalized size = 29.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x*cosh(x)^4 + 2*a^3*x*sinh(x)^4 + 2*a^3*x + 2*(3*a^2*b + b^3)*c
osh(x)^3 + 2*(4*a^3*x*cosh(x) + 3*a^2*b + b^3)*sinh(x)^3 - 4*(a^3*x - a^3 -
 3*a*b^2)*cosh(x)^2 + 2*(6*a^3*x*cosh(x)^2 - 2*a^3*x + 2*a^3 + 6*a*b^2 + 3*
(3*a^2*b + b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^2*b + b^3)*cosh(x) - ((2*a^3 -
3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (2
*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3 - 3*a^2*
b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b + b^3)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^3 - 3*a^2
*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((2*a^3 + 3*a^2*b
- b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 - 2*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3 - (2*a^3 + 3*a^2*b - b^3
)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*a^3*x*cosh(x)^3 + 3*a
```

```
^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(x)^2 - 4*(a^3*x - a^3 - 3*a*b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\coth(x) + b\operatorname{csch}(x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**3,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**3, x)
```

Giac [B] time = 1.2006, size = 155, normalized size = 2.63

$$\frac{1}{4}\left(2a^3 - 3a^2b + b^3\right)\log\left(e^{(-x)} + e^x + 2\right) + \frac{1}{4}\left(2a^3 + 3a^2b - b^3\right)\log\left(e^{(-x)} + e^x - 2\right) - \frac{a^3\left(e^{(-x)} + e^x\right)^2 + 6a^2b\left(e^{(-x)} + e^x\right)}{2\left(\left(e^{(-x)} + e^x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3 - 3*a^2*b + b^3)*log(e^(-x) + e^x + 2) + 1/4*(2*a^3 + 3*a^2*b -
b^3)*log(e^(-x) + e^x - 2) - 1/2*(a^3*(e^(-x) + e^x)^2 + 6*a^2*b*(e^(-x) +
e^x) + 2*b^3*(e^(-x) + e^x) + 12*a*b^2)/((e^(-x) + e^x)^2 - 4)
```

3.647 $\int (a\coth(x) + b\mathbf{csch}(x))^2\,dx$

Optimal. Leaf size=27

$$a^2x + ab\sinh(x) - \mathbf{csch}(x)(a\cosh(x) + b)(a + b\cosh(x))$$

```
[Out] a^2*x - (b + a*Cosh[x])*(a + b*Cosh[x])*Csch[x] + a*b*Sinh[x]
```

Rubi [A] time = 0.0683047, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2691, 2637}

$$a^2x + ab\sinh(x) - \mathbf{csch}(x)(a\cosh(x) + b)(a + b\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^2,x]
```

```
[Out] a^2*x - (b + a*Cosh[x])*(a + b*Cosh[x])*Csch[x] + a*b*Sinh[x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\coth(x)+b\mathbf{csch}(x))^2\,dx &= -\int (ib+ia\cosh(x))^2\mathbf{csch}^2(x)\,dx\\ &= -(b+a\cosh(x))(a+b\cosh(x))\mathbf{csch}(x)-\int \left(-a^2-ab\cosh(x)\right)\,dx\\ &= a^2x-(b+a\cosh(x))(a+b\cosh(x))\mathbf{csch}(x)+(ab)\int \cosh(x)\,dx\\ &= a^2x-(b+a\cosh(x))(a+b\cosh(x))\mathbf{csch}(x)+ab\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.120875, size = 23, normalized size = 0.85

$$a(ax-2b\mathbf{csch}(x))-\left(a^2+b^2\right)\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^2,x]
```

```
[Out] -((a^2 + b^2)*Coth[x]) + a*(a*x - 2*b*Csch[x])
```

Maple [A] time = 0.01, size = 36, normalized size = 1.3

$$a^2(x-\coth(x))+2ab\left(-\frac{(\cosh(x))^2}{\sinh(x)}+\sinh(x)\right)-b^2\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^2,x)
```

```
[Out] a^2*(x-coth(x))+2*a*b*(-cosh(x)^2/sinh(x)+sinh(x))-b^2*coth(x)
```

Maxima [A] time = 1.0443, size = 61, normalized size = 2.26

$$a^2\left(x+\frac{2}{e^{(-2x)}-1}\right)+\frac{4ab}{e^{(-x)}-e^x}+\frac{2b^2}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + 2/(e^(-2*x) - 1)) + 4*a*b/(e^(-x) - e^x) + 2*b^2/(e^(-2*x) - 1)
```

Fricas [A] time = 1.9817, size = 95, normalized size = 3.52

$$-\frac{2\,ab + \left(a^2 + b^2\right)\cosh\left(x\right) - \left(a^2x + a^2 + b^2\right)\sinh\left(x\right)}{\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b + (a^2 + b^2)*cosh(x) - (a^2*x + a^2 + b^2)*sinh(x))/sinh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\coth\left(x\right) + b\,\mathrm{csch}\left(x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**2,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**2, x)
```

Giac [A] time = 1.13322, size = 39, normalized size = 1.44

$$a^2x - \frac{2\left(2\,abe^x + a^2 + b^2\right)}{e^{(2\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

```
[Out] a^2*x - 2*(2*a*b*e^x + a^2 + b^2)/(e^(2*x) - 1)
```

3.648 $\int (a\coth(x) + b\,\mathbf{csch}(x))\,dx$

Optimal. Leaf size=12

$$a\log(\sinh(x)) - b\tanh^{-1}(\cosh(x))$$

```
[Out] -(b*ArcTanh[Cosh[x]]) + a*Log[Sinh[x]]
```

Rubi [A] time = 0.0104265, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3475, 3770}

$$a\log(\sinh(x)) - b\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Coth[x] + b*Csch[x],x]
```

```
[Out] -(b*ArcTanh[Cosh[x]]) + a*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\coth(x) + b\,\mathbf{csch}(x))\,dx &= a\int \coth(x)\,dx + b\int \mathbf{csch}(x)\,dx\\ &= -b\tanh^{-1}(\cosh(x)) + a\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0037515, size = 15, normalized size = 1.25

$$a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Coth[x] + b*Csch[x],x]
```

```
[Out] a*Log[Sinh[x]] + b*Log[Tanh[x/2]]
```

Maple [A] time = 0.002, size = 14, normalized size = 1.2

$$a\ln\left(\sinh\left(x\right)\right)+b\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*coth(x)+b*csch(x),x)
```

```
[Out] a*ln(sinh(x))+b*ln(tanh(1/2*x))
```

Maxima [A] time = 1.05417, size = 18, normalized size = 1.5

$$a\log\left(\sinh\left(x\right)\right)+b\log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="maxima")
```

```
[Out] a*log(sinh(x)) + b*log(tanh(1/2*x))
```

Fricas [B] time = 2.10259, size = 108, normalized size = 9.

$$-ax+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="fricas")
```

```
[Out] -a*x + (a - b)*log(cosh(x) + sinh(x) + 1) + (a + b)*log(cosh(x) + sinh(x) -
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\coth(x) + b\operatorname{csch}(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*coth(x)+b*csch(x),x)
```

```
[Out] Integral(a*coth(x) + b*csch(x), x)
```

Giac [B] time = 1.14124, size = 45, normalized size = 3.75

$$-a\left(x - \log\left(\left|e^{(2x)} - 1\right|\right)\right) - b(\log(e^x + 1) - \log(|e^x - 1|))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="giac")
```

```
[Out] -a*(x - log(abs(e^(2*x) - 1))) - b*(log(e^x + 1) - log(abs(e^x - 1)))
```

3.649 $\int \frac{1}{a \coth(x)+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a\cosh(x)+b)}{a}$$

```
[Out] Log[b + a*Cosh[x]]/a
```

Rubi [A] time = 0.0462933, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3160, 2668, 31}

$$\frac{\log(a\cosh(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-1),x]
```

```
[Out] Log[b + a*Cosh[x]]/a
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = i\int \frac{\sinh(x)}{ib + ia\cosh(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{ib+x}\,dx, x, ia\cosh(x)\right)}{a}$$
$$= \frac{\log(b + a\cosh(x))}{a}$$

Mathematica [A] time = 0.0179501, size = 11, normalized size = 1.

$$\frac{\log(a\cosh(x) + b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-1),x]
```

```
[Out] Log[b + a*Cosh[x]]/a
```

Maple [B] time = 0.034, size = 51, normalized size = 4.6

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{a}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2 b+a+b\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)+1/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a*ln(t
anh(1/2*x)-1)
```

Maxima [B] time = 1.03671, size = 35, normalized size = 3.18

$$\frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Fricas [B] time = 2.06924, size = 72, normalized size = 6.55

$$-\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x)
```

```
[Out] Integral(1/(a*coth(x) + b*csch(x)), x)
```

Giac [A] time = 1.12907, size = 26, normalized size = 2.36

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

3.650 $\int \frac{1}{(a\coth(x)+b\mathbf{csch}(x))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{x}{a^2}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

```
[Out] x/a^2 - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]) - Sinh[x]/(a*(b + a*Cosh[x]))
```

Rubi [A] time = 0.131912, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4392, 2693, 2735, 2659, 205}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{x}{a^2}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-2),x]
```


```
[Out] x/a^2 - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]) - Sinh[x]/(a*(b + a*Cosh[x]))
```


Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx &= -\int \frac{\sinh^2(x)}{(ib+ia\cosh(x))^2}\,dx \\
&= -\frac{\sinh(x)}{a(b+a\cosh(x))}+\frac{i\int \frac{\cosh(x)}{ib+ia\cosh(x)}\,dx}{a} \\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(ib)\int \frac{1}{ib+ia\cosh(x)}\,dx}{a^2} \\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(2ib)\operatorname{Subst}\left(\int \frac{1}{ia+ib-(-ia+ib)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2} \\
&= \frac{x}{a^2}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{a(b+a\cosh(x))}
\end{aligned}$$

Mathematica [A] time = 0.333363, size = 61, normalized size = 0.91

$$\frac{\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{a\sinh(x)}{a\cosh(x)+b}+x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-2),x]
```

```
[Out] (x + (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
a*Sinh[x])/(b + a*Cosh[x]))/a^2
```

Maple [A] time = 0.05, size = 95, normalized size = 1.4

$$\frac{1}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{\tanh(x/2)}{a\left(a\left(\tanh(x/2)\right)^2-\left(\tanh(x/2)\right)^2b+a+b\right)}-2\frac{b}{a^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^2,x)
```

```
[Out] 1/a^2*ln(tanh(1/2*x)+1)-2/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+
b)-2/a^2*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-1/a^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19164, size = 1647, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 -
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b + 2*(a*b*cosh(x) + b^
2)*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cos
h(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*
cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*c
osh(x) + b)*sinh(x) + a)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3
)*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*
sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(
x)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*c
osh(x))*sinh(x)), ((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 +
2*a^3 - 2*a*b^2 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b +
2*(a*b*cosh(x) + b^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(
x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3)
*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*s
inh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(x
)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*co
sh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\coth\left(x\right)+b\operatorname{csch}\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**2,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-2), x)
```

Giac [A] time = 1.17056, size = 92, normalized size = 1.37

$$-\frac{2\,b\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}a^{2}}+\frac{x}{a^{2}}+\frac{2\,\left(be^{x}+a\right)}{\left(ae^{(2\,x)}+2\,be^{x}+a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) + x/a^2 + 2*
(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a^2)
```

3.651 $\int \frac{1}{(a\coth(x)+b\mathbf{csch}(x))^3}\,dx$

Optimal. Leaf size=50

$$\frac{a^2-b^2}{2a^3(a\cosh(x)+b)^2}+\frac{2b}{a^3(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^3}$$

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cosh[x])^2) + (2*b)/(a^3*(b + a*Cosh[x])) + Log[b
 + a*Cosh[x]]/a^3
```

Rubi [A] time = 0.107477, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{a^2-b^2}{2a^3(a\cosh(x)+b)^2}+\frac{2b}{a^3(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cosh[x])^2) + (2*b)/(a^3*(b + a*Cosh[x])) + Log[b
 + a*Cosh[x]]/a^3
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = -\left(i\int \frac{\sinh^3(x)}{(ib+ia\cosh(x))^3}\,dx\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{-a^2-x^2}{(ib+x)^3}\,dx, x, ia\cosh(x)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(ib+x)^3}+\frac{2ib}{(ib+x)^2}-\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^3}$$
$$= \frac{a^2-b^2}{2a^3(b+a\cosh(x))^2}+\frac{2b}{a^3(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^3}$$

Mathematica [A] time = 0.107567, size = 77, normalized size = 1.54

$$\frac{a^2\cosh(2x)\log(a\cosh(x)+b)+a^2\log(a\cosh(x)+b)+a^2+2b^2\log(a\cosh(x)+b)+4ab\cosh(x)(\log(a\cosh(x)+b)}{2a^3(a\cosh(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cosh[x]] + 2*b^2*Log[b + a*Cosh[x]] + a^2*Cosh
[2*x]*Log[b + a*Cosh[x]] + 4*a*b*Cosh[x]*(1 + Log[b + a*Cosh[x]]))/(2*a^3*(
b + a*Cosh[x])^2)
```

Maple [B] time = 0.055, size = 144, normalized size = 2.9

$$-\frac{1}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{1}{a^2\left(a(\tanh(x/2))^2-(\tanh(x/2))^2b+a+b\right)}+2\frac{1}{(a-b)\left(a(\tanh(x/2))^2-(\tanh(x/2))^2b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^3,x)
```

```
[Out] -1/a^3*ln(tanh(1/2*x)+1)-2/a^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+2/(a-b
)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^2+2/a/(a-b)/(a*tanh(1/2*x)^2-tanh(1
```

```
/2*x)^2*b+a+b)^2*b+1/a^3*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a^3*ln(t
anh(1/2*x)-1)
```

Maxima [B] time = 1.06751, size = 150, normalized size = 3.

$$\frac{2\left(2\,abe^{(-x)}+2\,abe^{(-3\,x)}+\left(a^2+3\,b^2\right)e^{(-2\,x)}\right)}{4\,a^4be^{(-x)}+4\,a^4be^{(-3\,x)}+a^5e^{(-4\,x)}+a^5+2\left(a^5+2\,a^3b^2\right)e^{(-2\,x)}}+\frac{x}{a^3}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*e^(-x) + 2*a*b*e^(-3*x) + (a^2 + 3*b^2)*e^(-2*x))/(4*a^4*b*e^(-x)
+ 4*a^4*b*e^(-3*x) + a^5*e^(-4*x) + a^5 + 2*(a^5 + 2*a^3*b^2)*e^(-2*x)) + x
/a^3 + log(2*b*e^(-x) + a*e^(-2*x) + a)/a^3
```

Fricas [B] time = 2.1715, size = 1362, normalized size = 27.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x*cosh(x)^4 + a^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(a^2*x*
cosh(x) + a*b*x - a*b)*sinh(x)^3 + a^2*x - 2*(a^2 + 3*b^2 - (a^2 + 2*b^2)*x
)*cosh(x)^2 + 2*(3*a^2*x*cosh(x)^2 - a^2 - 3*b^2 + (a^2 + 2*b^2)*x + 6*(a*b
*x - a*b)*cosh(x))*sinh(x)^2 + 4*(a*b*x - a*b)*cosh(x) - (a^2*cosh(x)^4 + a
^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(a^2*cosh(x) + a*b)*sinh(x)^3 + 4*a*b*co
sh(x) + 2*(a^2 + 2*b^2)*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + 6*a*b*cosh(x) + a^
2 + 2*b^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + 3*a*b*cosh(x)^2 + a*b + (a^
2 + 2*b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 4
*(a^2*x*cosh(x)^3 + a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 - a*b - (a^2 + 3*b^2
- (a^2 + 2*b^2)*x)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + a^5*sinh(x)^4 + 4*a^4
*b*cosh(x)^3 + 4*a^4*b*cosh(x) + a^5 + 4*(a^5*cosh(x) + a^4*b)*sinh(x)^3 +
2*(a^5 + 2*a^3*b^2)*cosh(x)^2 + 2*(3*a^5*cosh(x)^2 + 6*a^4*b*cosh(x) + a^5
+ 2*a^3*b^2)*sinh(x)^2 + 4*(a^5*cosh(x)^3 + 3*a^4*b*cosh(x)^2 + a^4*b + (a^
5 + 2*a^3*b^2)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**3,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-3), x)
```

Giac [A] time = 1.16278, size = 89, normalized size = 1.78

$$\frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^3} - \frac{3\,a\left(e^{(-x)} + e^{x}\right)^2 + 4\,b\left(e^{(-x)} + e^{x}\right) - 4\,a}{2\left(a\left(e^{(-x)} + e^{x}\right) + 2\,b\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^3 - 1/2*(3*a*(e^(-x) + e^x)^2 + 4*b*(e^(
-x) + e^x) - 4*a)/((a*(e^(-x) + e^x) + 2*b)^2*a^2)
```

3.652 $\int \frac{1}{(a\coth(x)+b\,\mathbf{csch}(x))^4}\,dx$

Optimal. Leaf size=159

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(a\cosh(x)+b)^2}-\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2a^3\left(a^2-b^2\right)(a\cosh(x)+b)}+\frac{x}{a^4}-\frac{\sin}{3a(a\cos}$$

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2
*a^3*(a^2 - b^2)*(b + a*Cosh[x])) - Sinh[x]^3/(3*a*(b + a*Cosh[x])^3) - (b*
Sinh[x]^3)/(2*a*(a^2 - b^2)*(b + a*Cosh[x])^2)
```

Rubi [A] time = 0.37267, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2693, 2864, 2863, 2735, 2659, 205}

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(a\cosh(x)+b)^2}-\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2a^3\left(a^2-b^2\right)(a\cosh(x)+b)}+\frac{x}{a^4}-\frac{\sin}{3a(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2
*a^3*(a^2 - b^2)*(b + a*Cosh[x])) - Sinh[x]^3/(3*a*(b + a*Cosh[x])^3) - (b*
Sinh[x]^3)/(2*a*(a^2 - b^2)*(b + a*Cosh[x])^2)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \int \frac{\sinh^4(x)}{(ib+ia\cosh(x))^4}\,dx$$

$$= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{i\int \frac{\cosh(x)\sinh^2(x)}{(ib+ia\cosh(x))^3}dx}{a}$$

$$= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2} + \frac{i\int \frac{(2ia+ib\cosh(x))\sinh^2(x)}{(ib+ia\cosh(x))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))}$$

$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh}$$

$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh}$$

$$= \frac{x}{a^4} - \frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sin}{3a(b+a}$$

Mathematica [A] time = 0.433528, size = 150, normalized size = 0.94

$$\frac{\sinh(x)\left(-\frac{a\left(8a^2-11b^2\right)(a\cosh(x)+b)^2}{(a-b)(a+b)} - \frac{6b\left(2b^2-3a^2\right)\text{csch}(x)(a\cosh(x)+b)^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 2a\left(a^2-b^2\right) + 7ab(a\cosh(x)+b) + 6x\text{c}\right.}{6a^4(a\cosh(x)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cosh[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cosh
[x])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cosh[x])^3*Csch[x] - (6*b*(-3*a^2 +
2*b^2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cosh[x])^3*Csch[
x])/(a^2 - b^2)^(3/2))*Sinh[x])/(6*a^4*(b + a*Cosh[x])^3)
```

Maple [B] time = 0.066, size = 507, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^4,x)
```

```
[Out] 1/a^4*ln(tanh(1/2*x)+1)-2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a+b)*tan
h(1/2*x)^5+1/a/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a+b)*tanh(1/2*x)^5*
b+3/a^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a+b)*tanh(1/2*x)^5*b^2-2/a
^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a+b)*tanh(1/2*x)^5*b^3-20/3/a/(
a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*tanh(1/2*x)^3+4/a^3/(a*tanh(1/2*x)^2
-tanh(1/2*x)^2*b+a+b)^3*tanh(1/2*x)^3*b^2-2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*
b+a+b)^3/(a-b)*tanh(1/2*x)-1/a/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a-b
)*tanh(1/2*x)*b+3/a^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a-b)*tanh(1/
2*x)*b^2+2/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a-b)*tanh(1/2*x)*b^
3-3/a^2*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-
b))^(1/2))+2/a^4*b^3/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)
/((a+b)*(a-b))^(1/2))-1/a^4*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.95758, size = 13226, normalized size = 83.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

[Out] [1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^6 + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*sinh(x)^6 + 16*a^7 - 38*a^5*b^2 + 22*a^3*b^4 + 6*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^5 + 6*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x) + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*sinh(x)^5 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^4 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x))*sinh(x)^4 + 4*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^3 + 4*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 30*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^3 + 15*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^2 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x))*sinh(x)^3 + 6*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 6*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^4 + 10*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^3 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x))*sinh(x)^2 - 3*((3*a^5*b - 2*a^3*b^3)*cosh(x)^6 + (3*a^5*b - 2*a^3*b^3)*sinh(x)^6 + 3*a^5*b - 2*a^3*b^3 + 6*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^5 + 6*(3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3)*cosh(x))*sinh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^4 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^2 + 10*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x))*sinh(x)^4 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x)^3 + 4*(9*a^4*b^2 - 4*b^6 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^3 + 15*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^4 + 20*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^3 + 6*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x))*sinh(x)^2 + 6*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x) + 6*(3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3)*cosh(x)^5 + 5*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^4 + 2*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^3 + 2*(9*a^4*b^2 - 4*b^6)*cosh(x)^2 + (3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x + 6*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x) + 6*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^5 + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^4 + 4*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^3 + 2*(24 *a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^2 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x + 2*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x))*sinh(x))/(a^11 - 2*a^9*b^2 + a^7*b^4 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^6 + (a^11 - 2*a^9*b^2 + a^7*b^4)*sinh(x)^6 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^5 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x))*sinh(x)^5 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^4 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^2 + 10*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x))*sinh(x)^4 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^3 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^3 + 15*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^2 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x)^3 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^4 + 20*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^3 + 6*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x))*sinh(x)^2 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x) + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^5 + 5*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^4 + 2*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^3 + 2*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^2 + (a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x)), 1/3*(3*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^6 + 3*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*sinh(x)^6 + 8*a^7 - 19*a^5*b^2 + 11*a^3*b^4 + 3*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^5 + 3*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x) + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*sinh(x)^5 + 3*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^4 + 3*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x))*sinh(x)^4 + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^3 + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 30*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^3 + 15*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^2 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 3*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^4 + 10*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^3 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x))*sinh(x)^2 + 3*((3*a^5*b - 2*a^3*b^3)*cosh(x)^6 + (3*a^5*b - 2*a^3*b^3)*sinh(x)^6 + 3*a^5*b - 2*a^3*b^3 + 6*(3*a^4*b

```
^2 - 2*a^2*b^4)*cosh(x)^5 + 6*(3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3
)*cosh(x))*sinh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^4 + 3*(3*
a^5*b + 10*a^3*b^3 - 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^2 + 10*(3*a^
4*b^2 - 2*a^2*b^4)*cosh(x))*sinh(x)^4 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x)^3 + 4
*(9*a^4*b^2 - 4*b^6 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^3 + 15*(3*a^4*b^2 - 2
*a^2*b^4)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x)^3
 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 -
 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^4 + 20*(3*a^4*b^2 - 2*a^2*b^4)*c
osh(x)^3 + 6*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 4*(9*a^4*b^2 - 4*
b^6)*cosh(x))*sinh(x)^2 + 6*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x) + 6*(3*a^4*b^2
- 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3)*cosh(x)^5 + 5*(3*a^4*b^2 - 2*a^2*b^4)*c
osh(x)^4 + 2*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^3 + 2*(9*a^4*b^2 - 4*
b^6)*cosh(x)^2 + (3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x))*sqrt(a^
2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 3*(a^7 - 2*
a^5*b^2 + a^3*b^4)*x + 3*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*x)*cosh(x) + 3*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*
(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^5 + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b
^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^4 + 4*(4*a^7 + 5*a^5*b^2 -
27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x
)^3 + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^
3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^2 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x + 2*(
4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4
+ 4*a*b^6)*x)*cosh(x))*sinh(x))/(a^11 - 2*a^9*b^2 + a^7*b^4 + (a^11 - 2*a^9
*b^2 + a^7*b^4)*cosh(x)^6 + (a^11 - 2*a^9*b^2 + a^7*b^4)*sinh(x)^6 + 6*(a^1
0*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^5 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a
^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x))*sinh(x)^5 + 3*(a^11 + 2*a^9*b^2 - 7*a^7
*b^4 + 4*a^5*b^6)*cosh(x)^4 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6 +
 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^2 + 10*(a^10*b - 2*a^8*b^3 + a^6*b^
5)*cosh(x))*sinh(x)^4 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh
(x)^3 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7 + 5*(a^11 - 2*a^9*b^2
 + a^7*b^4)*cosh(x)^3 + 15*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^2 + 3*(a^
11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x)^3 + 3*(a^11 + 2*a^
9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4
+ 4*a^5*b^6 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^4 + 20*(a^10*b - 2*a^8
*b^3 + a^6*b^5)*cosh(x)^3 + 6*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*co
sh(x)^2 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x))*sinh(x)^2
 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x) + 6*(a^10*b - 2*a^8*b^3 + a^6*b
^5 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^5 + 5*(a^10*b - 2*a^8*b^3 + a^6*b
^5)*cosh(x)^4 + 2*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^3 + 2*
(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^2 + (a^11 + 2*a^9*b^2
- 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**4,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-4), x)
```

Giac [A] time = 1.15381, size = 327, normalized size = 2.06

$$-\frac{\left(3\,a^2b - 2\,b^3\right)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^6 - a^4b^2\right)\sqrt{a^2-b^2}} + \frac{15\,a^4be^{(5\,x)} - 18\,a^2b^3e^{(5\,x)} + 12\,a^5e^{(4\,x)} + 27\,a^3b^2e^{(4\,x)} - 54\,ab^4e^{(4\,x)} + 48\,a^4be^{(3\,x)} - 34\,a}{3\left(a^6 - a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^6 - a^4*b^2)*sqr
t(a^2 - b^2)) + 1/3*(15*a^4*b*e^(5*x) - 18*a^2*b^3*e^(5*x) + 12*a^5*e^(4*x)
 + 27*a^3*b^2*e^(4*x) - 54*a*b^4*e^(4*x) + 48*a^4*b*e^(3*x) - 34*a^2*b^3*e^
(3*x) - 44*b^5*e^(3*x) + 12*a^5*e^(2*x) + 36*a^3*b^2*e^(2*x) - 78*a*b^4*e^(
2*x) + 33*a^4*b*e^x - 48*a^2*b^3*e^x + 8*a^5 - 11*a^3*b^2)/((a^6 - a^4*b^2)
*(a*e^(2*x) + 2*b*e^x + a)^3) + x/a^4
```

3.653 $\int \frac{1}{(a\coth(x)+b\mathbf{csch}(x))^5}\,dx$

Optimal. Leaf size=98

$$-\frac{\left(a^2-b^2\right)^2}{4a^5(a\cosh(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(a\cosh(x)+b)^3}+\frac{a^2-3b^2}{a^5(a\cosh(x)+b)^2}+\frac{4b}{a^5(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^5}$$

```
[Out] -(a^2 - b^2)^2/(4*a^5*(b + a*Cosh[x])^4) - (4*b*(a^2 - b^2))/(3*a^5*(b + a*
Cosh[x])^3) + (a^2 - 3*b^2)/(a^5*(b + a*Cosh[x])^2) + (4*b)/(a^5*(b + a*Cos
h[x])) + Log[b + a*Cosh[x]]/a^5
```

Rubi [A] time = 0.15647, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{4a^5(a\cosh(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(a\cosh(x)+b)^3}+\frac{a^2-3b^2}{a^5(a\cosh(x)+b)^2}+\frac{4b}{a^5(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] -(a^2 - b^2)^2/(4*a^5*(b + a*Cosh[x])^4) - (4*b*(a^2 - b^2))/(3*a^5*(b + a*
Cosh[x])^3) + (a^2 - 3*b^2)/(a^5*(b + a*Cosh[x])^2) + (4*b)/(a^5*(b + a*Cos
h[x])) + Log[b + a*Cosh[x]]/a^5
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = i\int \frac{\sinh^5(x)}{(ib+ia\cosh(x))^5}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{\left(-a^2-x^2\right)^2}{(ib+x)^5}\,dx, x, ia\cosh(x)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(ib+x)^5}+\frac{4ib\left(-a^2+b^2\right)}{(ib+x)^4}+\frac{2\left(a^2-3b^2\right)}{(ib+x)^3}-\frac{4ib}{(ib+x)^2}+\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^5}$$

$$= -\frac{\left(a^2-b^2\right)^2}{4a^5(b+a\cosh(x))^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(b+a\cosh(x))^3}+\frac{a^2-3b^2}{a^5(b+a\cosh(x))^2}+\frac{4b}{a^5(b+a\cosh(x))}$$

Mathematica [A] time = 0.305308, size = 138, normalized size = 1.41

$$\frac{12a^2\cosh^2(x)\left(a^2+6b^2\log(a\cosh(x)+b)+9b^2\right)+8ab\cosh(x)\left(a^2+6b^2\log(a\cosh(x)+b)+11b^2\right)+2a^2b^2+12a^4\text{co}}{12a^5(a\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] (-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cosh[x]] + 12*a^4*Cosh[x]^4
*Log[b + a*Cosh[x]] + 48*a^3*b*Cosh[x]^3*(1 + Log[b + a*Cosh[x]]) + 12*a^2*
Cosh[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cosh[x]]) + 8*a*b*Cosh[x]*(a^2 + 1
1*b^2 + 6*b^2*Log[b + a*Cosh[x]]))/(12*a^5*(b + a*Cosh[x])^4)
```

Maple [B] time = 0.07, size = 309, normalized size = 3.2

$$-\frac{1}{a^5}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{1}{a^4\left(a\left(\tanh(x/2)\right)^2-\left(\tanh(x/2)\right)^2b+a+b\right)}-4\frac{a}{(a-b)^2\left(a\left(\tanh(x/2)\right)^2-\left(\tanh(x/2)\right)^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^5,x)
```

```
[Out] -1/a^5*ln(tanh(1/2*x)+1)-2/a^4/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-4*a/(a
-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^4-8/(a-b)^2/(a*tanh(1/2*x)^2-ta
nh(1/2*x)^2*b+a+b)^4*b-4/a/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^4*
b^2-2/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^2+1/a^5*ln(a*tanh(1/2*x)^2-
tanh(1/2*x)^2*b+a+b)+8/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3+16/3
/a/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*b-8/3/a^2/(a-b)^2/(a*tan
h(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*b^2-1/a^5*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.24709, size = 385, normalized size = 3.93

$$\frac{4\left(6\,a^3be^{(-x)}+6\,a^3be^{(-7\,x)}+3\left(a^4+9\,a^2b^2\right)e^{(-2\,x)}+22\left(a^3b+2\,ab^3\right)e^{(-3\,x)}+\left(3\,a^4+56\,a^2b^2+25\,b^4\right)e^{(-4\,x)}+22}{3\left(8\,a^8be^{(-x)}+8\,a^8be^{(-7\,x)}+a^9e^{(-8\,x)}+a^9+4\left(a^9+6\,a^7b^2\right)e^{(-2\,x)}+8\left(3\,a^8b+4\,a^6b^3\right)e^{(-3\,x)}+2\left(3\,a^9+24\,a^7b^2+8\,a^5b^4\right)e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a^3*b*e^(-x) + 6*a^3*b*e^(-7*x) + 3*(a^4 + 9*a^2*b^2)*e^(-2*x) + 22*
(a^3*b + 2*a*b^3)*e^(-3*x) + (3*a^4 + 56*a^2*b^2 + 25*b^4)*e^(-4*x) + 22*(a
^3*b + 2*a*b^3)*e^(-5*x) + 3*(a^4 + 9*a^2*b^2)*e^(-6*x))/(8*a^8*b*e^(-x) +
8*a^8*b*e^(-7*x) + a^9*e^(-8*x) + a^9 + 4*(a^9 + 6*a^7*b^2)*e^(-2*x) + 8*(3
*a^8*b + 4*a^6*b^3)*e^(-3*x) + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*e^(-4*x)
+ 8*(3*a^8*b + 4*a^6*b^3)*e^(-5*x) + 4*(a^9 + 6*a^7*b^2)*e^(-6*x)) + x/a^5
+ log(2*b*e^(-x) + a*e^(-2*x) + a)/a^5
```

Fricas [B] time = 2.57025, size = 6251, normalized size = 63.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*x*cosh(x)^8 + 3*a^4*x*sinh(x)^8 + 24*(a^3*b*x - a^3*b)*cosh(x)^
7 + 24*(a^4*x*cosh(x) + a^3*b*x - a^3*b)*sinh(x)^7 - 12*(a^4 + 9*a^2*b^2 -
(a^4 + 6*a^2*b^2)*x)*cosh(x)^6 + 12*(7*a^4*x*cosh(x)^2 - a^4 - 9*a^2*b^2 +
(a^4 + 6*a^2*b^2)*x + 14*(a^3*b*x - a^3*b)*cosh(x))*sinh(x)^6 - 8*(11*a^3*b
```

+ 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^5 + 8*(21*a^4*x*cosh(x)^3 - 11*a^3*b - 22*a*b^3 + 63*(a^3*b*x - a^3*b)*cosh(x)^2 + 3*(3*a^3*b + 4*a*b^3)*x - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x)^5 + 3*a^4*x - 2*(6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x)^4 + 2*(105*a^4*x*cosh(x)^4 - 6*a^4 - 112*a^2*b^2 - 50*b^4 + 420*(a^3*b*x - a^3*b)*cosh(x)^3 - 90*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^2 + 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x - 20*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x))*sinh(x)^4 - 8*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^3 + 8*(21*a^4*x*cosh(x)^5 + 105*(a^3*b*x - a^3*b)*cosh(x)^4 - 11*a^3*b - 22*a*b^3 - 30*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^3 - 10*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 + 3*(3*a^3*b + 4*a*b^3)*x - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^2 + 4*(21*a^4*x*cosh(x)^6 + 126*(a^3*b*x - a^3*b)*cosh(x)^5 - 45*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^4 - 3*a^4 - 27*a^2*b^2 - 20*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^3 - 3*(6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x)^2 + 3*(a^4 + 6*a^2*b^2)*x - 6*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x))*sinh(x)^2 + 24*(a^3*b*x - a^3*b)*cosh(x) - 3*(a^4*cosh(x)^8 + a^4*sinh(x)^8 + 8*a^3*b*cosh(x)^7 + 8*(a^4*cosh(x) + a^3*b)*sinh(x)^7 + 4*(a^4 + 6*a^2*b^2)*cosh(x)^6 + 4*(7*a^4*cosh(x)^2 + 14*a^3*b*cosh(x) + a^4 + 6*a^2*b^2)*sinh(x)^6 + 8*(3*a^3*b + 4*a*b^3)*cosh(x)^5 + 8*(7*a^4*cosh(x)^3 + 21*a^3*b*cosh(x)^2 + 3*a^3*b + 4*a*b^3 + 3*(a^4 + 6*a^2*b^2)*cosh(x))*sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*a^4*cosh(x)^4 + 140*a^3*b*cosh(x)^3 + 3*a^4 + 24*a^2*b^2 + 8*b^4 + 30*(a^4 + 6*a^2*b^2)*cosh(x)^2 + 20*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + a^4 + 8*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 8*(7*a^4*cosh(x)^5 + 35*a^3*b*cosh(x)^4 + 3*a^3*b + 4*a*b^3 + 10*(a^4 + 6*a^2*b^2)*cosh(x)^3 + 10*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 6*a^2*b^2)*cosh(x)^2 + 4*(7*a^4*cosh(x)^6 + 42*a^3*b*cosh(x)^5 + 15*(a^4 + 6*a^2*b^2)*cosh(x)^4 + a^4 + 6*a^2*b^2 + 20*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^2 + 6*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 8*(a^4*cosh(x)^7 + 7*a^3*b*cosh(x)^6 + 3*(a^4 + 6*a^2*b^2)*cosh(x)^5 + 5*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + a^3*b + (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^3 + 3*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (a^4 + 6*a^2*b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 8*(3*a^4*x*cosh(x)^7 + 21*(a^3*b*x - a^3*b)*cosh(x)^6 - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^5 + 3*a^3*b*x - 5*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^4 - 3*a^3*b - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x)^3 - 3*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 - 3*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x))/(a^9*cosh(x)^8 + a^9*sinh(x)^8 + 8*a^8*b*cosh(x)^7 + 8*a^8*b*cosh(x) + a^9 + 8*(a^9*cosh(x) + a^8*b)*sinh(x)^7 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^6 + 4*(7*a^9*cosh(x)^2 + 14*a^8*b*cosh(x) + a^9 + 6*a^7*b^2)*sinh(x)^6 + 8*(3*a^8*b + 4*a^6*b^3)*cosh(x)^5 + 8*(7*a^9*cosh(x)^3 + 21*a^8*b*cosh(x)^2 + 3*a^8*b + 4*a^6*b^3 +

```
3*(a^9 + 6*a^7*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)
*cosh(x)^4 + 2*(35*a^9*cosh(x)^4 + 140*a^8*b*cosh(x)^3 + 3*a^9 + 24*a^7*b^2
 + 8*a^5*b^4 + 30*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 20*(3*a^8*b + 4*a^6*b^3)*co
sh(x))*sinh(x)^4 + 8*(3*a^8*b + 4*a^6*b^3)*cosh(x)^3 + 8*(7*a^9*cosh(x)^5 +
 35*a^8*b*cosh(x)^4 + 3*a^8*b + 4*a^6*b^3 + 10*(a^9 + 6*a^7*b^2)*cosh(x)^3
+ 10*(3*a^8*b + 4*a^6*b^3)*cosh(x)^2 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cos
h(x))*sinh(x)^3 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 4*(7*a^9*cosh(x)^6 + 42*a
^8*b*cosh(x)^5 + a^9 + 6*a^7*b^2 + 15*(a^9 + 6*a^7*b^2)*cosh(x)^4 + 20*(3*a
^8*b + 4*a^6*b^3)*cosh(x)^3 + 3*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^2
+ 6*(3*a^8*b + 4*a^6*b^3)*cosh(x))*sinh(x)^2 + 8*(a^9*cosh(x)^7 + 7*a^8*b*c
osh(x)^6 + a^8*b + 3*(a^9 + 6*a^7*b^2)*cosh(x)^5 + 5*(3*a^8*b + 4*a^6*b^3)*
cosh(x)^4 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^3 + 3*(3*a^8*b + 4*a^6
*b^3)*cosh(x)^2 + (a^9 + 6*a^7*b^2)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\coth\left(x\right) + b\operatorname{csch}\left(x\right)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**5,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-5), x)
```

Giac [A] time = 1.20338, size = 182, normalized size = 1.86

$$\frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^5} - \frac{25\,a^3\left(e^{(-x)} + e^{x}\right)^4 + 104\,a^2b\left(e^{(-x)} + e^{x}\right)^3 - 48\,a^3\left(e^{(-x)} + e^{x}\right)^2 + 168\,ab^2\left(e^{(-x)} + e^{x}\right)^2 - 64\,a^2b\left(e^{(-x)} + e^{x}\right) + 96\,b^3\left(e^{(-x)} + e^{x}\right) + 48\,a^3 - 32\,ab^2}{12\left(a\left(e^{(-x)} + e^{x}\right) + 2\,b\right)^4 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^5 - 1/12*(25*a^3*(e^(-x) + e^x)^4 + 104*
a^2*b*(e^(-x) + e^x)^3 - 48*a^3*(e^(-x) + e^x)^2 + 168*a*b^2*(e^(-x) + e^x)
^2 - 64*a^2*b*(e^(-x) + e^x) + 96*b^3*(e^(-x) + e^x) + 48*a^3 - 32*a*b^2)/(
(a*(e^(-x) + e^x) + 2*b)^4*a^4)
```

3.654 $\int (\coth(x) + \operatorname{csch}(x))^5 \, dx$

Optimal. Leaf size=28

$$\frac{4}{1-\cosh(x)} - \frac{2}{(1-\cosh(x))^2} + \log(1-\cosh(x))$$

```
[Out] -2/(1 - Cosh[x])^2 + 4/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0670859, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{1-\cosh(x)} - \frac{2}{(1-\cosh(x))^2} + \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^5,x]
```

```
[Out] -2/(1 - Cosh[x])^2 + 4/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^5\,dx &= -\left(i\int (i+i\cosh(x))^5\operatorname{csch}^5(x)\,dx\right)\\ &= -\operatorname{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx,x,i\cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)dx,x,i\cosh(x)\right)\\ &= \frac{2}{(i-i\cosh(x))^2}+\frac{4i}{i-i\cosh(x)}+\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0870054, size = 53, normalized size = 1.89

$$-\frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)-2\operatorname{csch}^2\left(\frac{x}{2}\right)+6\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\sinh(x))-5\log\left(\tanh\left(\frac{x}{2}\right)\right)-6\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^5,x]
```

```
[Out] -2*Csch[x/2]^2 - Csch[x/2]^4/2 - 6*Log[Cosh[x/2]] + 6*Log[Sinh[x/2]] + Log[
Sinh[x]] - 5*Log[Tanh[x/2]]
```

Maple [B] time = 0.021, size = 75, normalized size = 2.7

$$\ln(\sinh(x))-\frac{(\coth(x))^2}{2}-\frac{(\coth(x))^4}{4}-5\,\frac{(\cosh(x))^3}{(\sinh(x))^4}+\frac{5\cosh(x)}{3\,(\sinh(x))^4}+\frac{8\coth(x)}{3}\left(-\frac{(\operatorname{csch}(x))^3}{4}+\frac{3\operatorname{csch}(x)}{8}\right)-2\,\mathrm{Ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^5,x)
```

```
[Out] ln(sinh(x))-1/2*coth(x)^2-1/4*coth(x)^4-5/sinh(x)^4*cosh(x)^3+5/3/sinh(x)^4
*cosh(x)+8/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)-2*arctanh(exp(x))-15/4/si
nh(x)^4*cosh(x)^2-5/4*cosh(x)^2/sinh(x)^2
```

Maxima [B] time = 1.27624, size = 319, normalized size = 11.39

$$-\frac{5}{2}\coth(x)^4 + x + \frac{5\left(5e^{(-x)} + 3e^{(-3x)} + 3e^{(-5x)} + 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} - \frac{3e^{(-x)} - 11e^{(-3x)} - 11e^{(-5x)} + 3e^{(-7x)}}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} + \frac{5\left(e^{(}\right.}{2\left(4e^{(-2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*coth(x)^4 + x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*
e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x)
- e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e
^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 20/(e^(-x)
 - e^x)^4 + 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.08206, size = 925, normalized size = 33.04

$$-\frac{x\cosh(x)^4 + x\sinh(x)^4 - 4(x-2)\cosh(x)^3 + 4(x\cosh(x) - x + 2)\sinh(x)^3 + 2(3x-4)\cosh(x)^2 + 2\left(3x\cosh(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12696, size = 45, normalized size = 1.61

$$-x - \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{\left(e^{x} - 1\right)^{4}} + 2\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] -x - 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 + 2*log(abs(e^x - 1))
```

3.655 $\int (\coth(x) + \mathbf{csch}(x))^4 \, dx$

Optimal. Leaf size=30

$$x + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rubi [A] time = 0.113769, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^4,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^4\,dx &= \int (i+i\cosh(x))^4\operatorname{csch}^4(x)\,dx\\ &= \int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}\,dx\\ &= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}-\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx\\ &= \frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}\end{aligned}$$

Mathematica [A] time = 0.0479179, size = 30, normalized size = 1.

$$x-\frac{8}{3}\coth\left(\frac{x}{2}\right)-\frac{2}{3}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^4,x]
```

```
[Out] x - (8*Coth[x/2])/3 - (2*Coth[x/2]*Csch[x/2]^2)/3
```

Maple [A] time = 0.033, size = 57, normalized size = 1.9

$$x-\coth(x)-\frac{(\coth(x))^3}{3}-\frac{8\,(\cosh(x))^2}{3\,(\sinh(x))^3}-\frac{4\,(\cosh(x))^2}{3\,\sinh(x)}+\frac{4\,\sinh(x)}{3}-3\,\frac{\cosh(x)}{(\sinh(x))^3}-2\left(2/3-1/3\,(\operatorname{csch}(x))^2\right)\coth($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^4,x)
```

```
[Out] x-coth(x)-1/3*coth(x)^3-8/3/sinh(x)^3*cosh(x)^2-4/3*cosh(x)^2/sinh(x)+4/3*s
inh(x)-3/sinh(x)^3*cosh(x)-2*(2/3-1/3*csch(x)^2)*coth(x)
```

Maxima [B] time = 1.18459, size = 247, normalized size = 8.23

$$-2\coth(x)^3 + x - \frac{4\left(3e^{(-2x)} - 3e^{(-4x)} - 2\right)}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} + \frac{8e^{(-x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} + \frac{4e^{(-2x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) + 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) + 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 32/3/(e^(-x) -
e^x)^3
```

Fricas [B] time = 1.99215, size = 234, normalized size = 7.8

$$\frac{3x\cosh(x)^2 + 3x\sinh(x)^2 - 4(3x+10)\cosh(x) + 2(3x\cosh(x) - 3x - 4)\sinh(x) + 9x + 24}{3\left(\cosh(x)^2 + 2(\cosh(x) - 1)\sinh(x) + \sinh(x)^2 - 4\cosh(x) + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
- 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 4*cosh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**4,x)
```

```
[Out] Integral((coth(x) + csch(x))**4, x)
```

Giac [A] time = 1.11199, size = 30, normalized size = 1.

$$x - \frac{8\left(3\,e^{(2\,x)} - 3\,e^{x} + 2\right)}{3\,(e^{x} - 1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

3.656 $\int (\coth(x) + \mathbf{csch}(x))^3 \, dx$

Optimal. Leaf size=18

$$\frac{2}{1-\cosh(x)} + \log(1-\cosh(x))$$

```
[Out] 2/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rubi [A] time = 0.056183, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{1-\cosh(x)} + \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^3,x]
```

```
[Out] 2/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = i\int (i+i\cosh(x))^3\operatorname{csch}^3(x)\,dx$$
$$= \text{Subst}\left(\int \frac{i+x}{(i-x)^2}\,dx, x, i\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)dx, x, i\cosh(x)\right)$$
$$= \frac{2i}{i-i\cosh(x)}+\log(1-\cosh(x))$$

Mathematica [B] time = 0.0506899, size = 41, normalized size = 2.28

$$-\operatorname{csch}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\sinh(x))+3\log\left(\tanh\left(\frac{x}{2}\right)\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^3,x]
```

```
[Out] -Csch[x/2]^2 + 2*Log[Cosh[x/2]] - 2*Log[Sinh[x/2]] + Log[Sinh[x]] + 3*Log[T
anh[x/2]]
```

Maple [B] time = 0.018, size = 39, normalized size = 2.2

$$\ln(\sinh(x))-\frac{(\coth(x))^2}{2}-3\,\frac{\cosh(x)}{(\sinh(x))^2}+\operatorname{csch}(x)\coth(x)-2\,\text{Artanh}(e^x)-\frac{3\,(\cosh(x))^2}{2\,(\sinh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^3,x)
```

```
[Out] ln(sinh(x))-1/2*coth(x)^2-3/sinh(x)^2*cosh(x)+csch(x)*coth(x)-2*arctanh(exp
(x))-3/2*cosh(x)^2/sinh(x)^2
```

Maxima [B] time = 1.05576, size = 89, normalized size = 4.94

$$-\frac{3}{2}\coth(x)^2+x+\frac{4\left(e^{(-x)}+e^{(-3x)}\right)}{2\,e^{(-2x)}-e^{(-4x)}-1}+\frac{2\,e^{(-2x)}}{2\,e^{(-2x)}-e^{(-4x)}-1}+2\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*coth(x)^2 + x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*
e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.02317, size = 336, normalized size = 18.67

$$-\frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x) + 1\right)\log}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(
x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh
(x)^2 - 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + \operatorname{csch}(x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**3,x)
```

```
[Out] Integral((coth(x) + csch(x))**3, x)
```

Giac [A] time = 1.1337, size = 30, normalized size = 1.67

$$-x - \frac{4e^x}{(e^x - 1)^2} + 2\log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] -x - 4*e^x/(e^x - 1)^2 + 2*log(abs(e^x - 1))
```

3.657 $\int (\coth(x) + \operatorname{csch}(x))^2 \, dx$

Optimal. Leaf size=14

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rubi [A] time = 0.0822005, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^2,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\coth(x)+\mathrm{csch}(x))^2\,dx &= -\int (i+i\cosh(x))^2\mathrm{csch}^2(x)\,dx\\ &= -\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx\\ &= \frac{2\sinh(x)}{1-\cosh(x)}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0258454, size = 10, normalized size = 0.71

$$x-2\coth\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^2,x]
```

```
[Out] x - 2*Coth[x/2]
```

Maple [A] time = 0.009, size = 21, normalized size = 1.5

$$x-2\coth(x)-2\frac{(\cosh(x))^2}{\sinh(x)}+2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^2,x)
```

```
[Out] x-2*coth(x)-2*cosh(x)^2/sinh(x)+2*sinh(x)
```

Maxima [B] time = 1.04969, size = 34, normalized size = 2.43

$$x + \frac{4}{e^{(-x)} - e^{x}} + \frac{4}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

Fricas [A] time = 1.96929, size = 77, normalized size = 5.5

$$\frac{x\cosh(x) + x\sinh(x) - x - 4}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(\coth(x) + \operatorname{csch}(x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**2,x)
```

```
[Out] Integral((coth(x) + csch(x))**2, x)
```

Giac [A] time = 1.10915, size = 14, normalized size = 1.

$$x - \frac{4}{e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x - 4/(e^x - 1)
```

3.658 $\int (\coth(x) + \operatorname{csch}(x))\, dx$

Optimal. Leaf size=9

$$\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]] + Log[Sinh[x]]
```

Rubi [A] time = 0.0079258, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3475, 3770}

$$\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x] + Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] + Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\coth(x) + \operatorname{csch}(x))\, dx &= \int \coth(x)\, dx + \int \operatorname{csch}(x)\, dx \\ &= -\tanh^{-1}(\cosh(x)) + \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.003432, size = 11, normalized size = 1.22

$$\log(\sinh(x)) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x] + Csch[x],x]
```

```
[Out] Log[Sinh[x]] + Log[Tanh[x/2]]
```

Maple [A] time = 0.001, size = 10, normalized size = 1.1

$$\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)+csch(x),x)
```

```
[Out] ln(sinh(x))+ln(tanh(1/2*x))
```

Maxima [A] time = 1.04251, size = 12, normalized size = 1.33

$$\log\left(\sinh\left(x\right)\right)+\log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="maxima")
```

```
[Out] log(sinh(x)) + log(tanh(1/2*x))
```

Fricas [A] time = 2.05436, size = 47, normalized size = 5.22

$$-x+2\,\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="fricas")
```

```
[Out] -x + 2*log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth (x) + \operatorname{csch}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x)
```

```
[Out] Integral(coth(x) + csch(x), x)
```

Giac [B] time = 1.13858, size = 34, normalized size = 3.78

$$-x - \log (e^x + 1) + \log \left(\left|e^{(2\,x)} - 1\right|\right) + \log (|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="giac")
```

```
[Out] -x - log(e^x + 1) + log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

3.659 $\int \frac{1}{\coth(x)+\mathbf{csch}(x)} \, dx$

Optimal. Leaf size=5

$$\log(\cosh(x)+1)$$

```
[Out] Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0318504, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3160, 2667, 31}

$$\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-1),x]
```

```
[Out] Log[1 + Cosh[x]]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\coth(x)+\mathbf{csch}(x)}\,dx = i\int \frac{\sinh(x)}{i+i\cosh(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, i\cosh(x)\right)$$
$$= \log(1+\cosh(x))$$

Mathematica [A] time = 0.0188026, size = 9, normalized size = 1.8

$$2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-1),x]
```

```
[Out] 2*Log[Cosh[x/2]]
```

Maple [B] time = 0.027, size = 20, normalized size = 4.

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x)),x)
```

```
[Out] -ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

Maxima [B] time = 1.05773, size = 15, normalized size = 3.

$$x+2\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="maxima")
```

```
[Out] x + 2*log(e^(-x) + 1)
```

Fricas [B] time = 2.03081, size = 47, normalized size = 9.4

$$-x + 2 \log(\cosh(x) + \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="fricas")
```

```
[Out] -x + 2*log(cosh(x) + sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\coth(x) + \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x)
```

```
[Out] Integral(1/(coth(x) + csch(x)), x)
```

Giac [B] time = 1.12061, size = 15, normalized size = 3.

$$-x + 2 \log(e^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="giac")
```

```
[Out] -x + 2*log(e^x + 1)
```

3.660 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^2} dx$

Optimal. Leaf size=12

$$x - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rubi [A] time = 0.0494883, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x - \frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-2),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(\coth(x) + \mathbf{csch}(x))^2}\, dx = -\int \frac{\sinh^2(x)}{(i + i\cosh(x))^2}\, dx$$
$$= -\frac{2\sinh(x)}{1+\cosh(x)} + \int 1\, dx$$
$$= x - \frac{2\sinh(x)}{1+\cosh(x)}$$

Mathematica [A] time = 0.0202439, size = 10, normalized size = 0.83

$$x - 2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-2),x]
```

```
[Out] x - 2*Tanh[x/2]
```

Maple [A] time = 0.033, size = 24, normalized size = 2.

$$-2\tanh(x/2) - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^2,x)
```

```
[Out] -2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.11769, size = 16, normalized size = 1.33

$$x - \frac{4}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x - 4/(e^(-x) + 1)
```

Fricas [A] time = 2.01167, size = 77, normalized size = 6.42

$$\frac{x\cosh(x) + x\sinh(x) + x + 4}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**2,x)
```

```
[Out] Integral((coth(x) + csch(x))**(-2), x)
```

Giac [A] time = 1.12536, size = 14, normalized size = 1.17

$$x + \frac{4}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x + 4/(e^x + 1)
```

3.661 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^3} dx$

Optimal. Leaf size=14

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0574001, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-3),x]
```

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx &= -\left(i\int \frac{\sinh^3(x)}{(i+i\cosh(x))^3}\,dx\right)\\ &= -\operatorname{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx,x,i\cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)dx,x,i\cosh(x)\right)\\ &= \frac{2i}{i+i\cosh(x)}+\log(1+\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0200618, size = 18, normalized size = 1.29

$$\operatorname{sech}^2\left(\frac{x}{2}\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-3),x]
```

```
[Out] 2*Log[Cosh[x/2]] + Sech[x/2]^2
```

Maple [A] time = 0.034, size = 28, normalized size = 2.

$$-\left(\tanh\left(\frac{x}{2}\right)\right)^2-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^3,x)
```

```
[Out] -tanh(1/2*x)^2-ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

Maxima [B] time = 1.17757, size = 42, normalized size = 3.

$$x+\frac{4\,e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1}+2\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

Fricas [B] time = 2.10252, size = 336, normalized size = 24.

$$-\frac{x\cosh\left(x\right)^{2}+x\sinh\left(x\right)^{2}+2\,(x-2)\cosh\left(x\right)-2\left(\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)+1)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\log}{\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)+1)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(
x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh
(x)^2 + 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**3,x)
```

```
[Out] Integral((coth(x) + csch(x))**(-3), x)
```

Giac [A] time = 1.12657, size = 28, normalized size = 2.

$$-x + \frac{4\, e^{x}}{(e^{x} + 1)^{2}} + 2\,\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

3.662 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^4} \, dx$

Optimal. Leaf size=26

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rubi [A] time = 0.0848153, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx &= \int \frac{\sinh^4(x)}{(i+i\cosh(x))^4}\,dx\\ &= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3} + \int 1\,dx\\ &= x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}\end{aligned}$$

Mathematica [A] time = 0.0211706, size = 30, normalized size = 1.15

$$x-\frac{8}{3}\tanh\left(\frac{x}{2}\right)+\frac{2}{3}\tanh\left(\frac{x}{2}\right)\operatorname{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x - (8*Tanh[x/2])/3 + (2*Sech[x/2]^2*Tanh[x/2])/3
```

Maple [A] time = 0.06, size = 32, normalized size = 1.2

$$-\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3-2\,\tanh(x/2)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^4,x)
```

```
[Out] -2/3*tanh(1/2*x)^3-2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.28264, size = 51, normalized size = 1.96

$$x-\frac{8\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+2\right)}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] x - 8/3*(3*e^(-x) + 3*e^(-2*x) + 2)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1)
```

Fricas [B] time = 1.97454, size = 234, normalized size = 9.

$$\frac{3\,x\cosh\left(x\right)^{2}+3\,x\sinh\left(x\right)^{2}+4\left(3\,x+10\right)\cosh\left(x\right)+2\left(3\,x\cosh\left(x\right)+3\,x+4\right)\sinh\left(x\right)+9\,x+24}{3\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**4,x)
```

```
[Out] Integral((coth(x) + csch(x))**(-4), x)
```

Giac [A] time = 1.11941, size = 30, normalized size = 1.15

$$x+\frac{8\left(3\,e^{\left(2\,x\right)}+3\,e^{x}+2\right)}{3\left(e^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

3.663 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^5}\,dx$

Optimal. Leaf size=22

$$\frac{4}{\cosh(x)+1}-\frac{2}{(\cosh(x)+1)^2}+\log(\cosh(x)+1)$$

```
[Out] -2/(1 + Cosh[x])^2 + 4/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0631212, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{\cosh(x)+1}-\frac{2}{(\cosh(x)+1)^2}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-5),x]
```

```
[Out] -2/(1 + Cosh[x])^2 + 4/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\coth(x)+\mathrm{csch}(x))^5}\,dx &= i\int \frac{\sinh^5(x)}{(i+i\cosh(x))^5}\,dx\\ &= \mathrm{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, i\cosh(x)\right)\\ &= \mathrm{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)dx, x, i\cosh(x)\right)\\ &= \frac{2}{(i+i\cosh(x))^2}+\frac{4i}{i+i\cosh(x)}+\log(1+\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0216541, size = 32, normalized size = 1.45

$$-\frac{1}{2}\mathrm{sech}^4\left(\frac{x}{2}\right)+2\mathrm{sech}^2\left(\frac{x}{2}\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2*Log[Cosh[x/2]] + 2*Sech[x/2]^2 - Sech[x/2]^4/2
```

Maple [A] time = 0.041, size = 36, normalized size = 1.6

$$-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\left(\tanh\left(\frac{x}{2}\right)\right)^2-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^5,x)
```

```
[Out] -1/2*tanh(1/2*x)^4-tanh(1/2*x)^2-ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

Maxima [B] time = 1.10317, size = 70, normalized size = 3.18

$$x + \frac{8\left(e^{(-x)} + e^{(-2x)} + e^{(-3x)}\right)}{4e^{(-x)} + 6e^{(-2x)} + 4e^{(-3x)} + e^{(-4x)} + 1} + 2\log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] x + 8*(e^(-x) + e^(-2*x) + e^(-3*x))/(4*e^(-x) + 6*e^(-2*x) + 4*e^(-3*x) +
e^(-4*x) + 1) + 2*log(e^(-x) + 1)
```

Fricas [B] time = 2.00276, size = 925, normalized size = 42.05

$$-\frac{x\cosh(x)^4 + x\sinh(x)^4 + 4(x-2)\cosh(x)^3 + 4(x\cosh(x) + x - 2)\sinh(x)^3 + 2(3x-4)\cosh(x)^2 + 2\left(3x\cosh(x)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**5,x)
```

```
[Out] Integral((coth(x) + csch(x))**(-5), x)
```

Giac [A] time = 1.11155, size = 41, normalized size = 1.86

$$-x+\frac{8\left(e^{(3x)}+e^{(2x)}+e^{x}\right)}{(e^{x}+1)^{4}}+2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] -x + 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 + 2*log(e^x + 1)
```

3.664 $\int (-\coth(x) + \mathbf{csch}(x))^5 \, dx$

Optimal. Leaf size=24

$$-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

```
[Out] 2/(1 + Cosh[x])^2 - 4/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0626796, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^5,x]
```

```
[Out] 2/(1 + Cosh[x])^2 - 4/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = -\left(i\int (i-i\cosh(x))^5\operatorname{csch}^5(x)\,dx\right)$$
$$= \text{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx, x, -i\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)dx, x, -i\cosh(x)\right)$$
$$= -\frac{2}{(i+i\cosh(x))^2}-\frac{4i}{i+i\cosh(x)}-\log(1+\cosh(x))$$

Mathematica [B] time = 0.0846704, size = 55, normalized size = 2.29

$$\frac{1}{2}\text{sech}^4\left(\frac{x}{2}\right)-2\text{sech}^2\left(\frac{x}{2}\right)+6\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log(\sinh(x))-5\log\left(\tanh\left(\frac{x}{2}\right)\right)-6\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^5,x]
```

```
[Out] -6*Log[Cosh[x/2]] + 6*Log[Sinh[x/2]] - Log[Sinh[x]] - 5*Log[Tanh[x/2]] - 2*
Sech[x/2]^2 + Sech[x/2]^4/2
```

Maple [B] time = 0.022, size = 77, normalized size = 3.2

$$-\ln(\sinh(x))+\frac{(\coth(x))^2}{2}+\frac{(\coth(x))^4}{4}-5\frac{(\cosh(x))^3}{(\sinh(x))^4}+\frac{5\cosh(x)}{3(\sinh(x))^4}+\frac{8\coth(x)}{3}\left(-\frac{(\operatorname{csch}(x))^3}{4}+\frac{3\operatorname{csch}(x)}{8}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^5,x)
```

```
[Out] -ln(sinh(x))+1/2*coth(x)^2+1/4*coth(x)^4-5/sinh(x)^4*cosh(x)^3+5/3/sinh(x)^
4*cosh(x)+8/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)-2*arctanh(exp(x))+15/4/s
inh(x)^4*cosh(x)^2+5/4*cosh(x)^2/sinh(x)^2
```

Maxima [B] time = 1.23135, size = 321, normalized size = 13.38

$$\frac{5}{2}\coth(x)^4 - x + \frac{5\left(5e^{(-x)} + 3e^{(-3x)} + 3e^{(-5x)} + 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} - \frac{3e^{(-x)} - 11e^{(-3x)} - 11e^{(-5x)} + 3e^{(-7x)}}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} + \frac{5\left(e^{(-x)}\right.}{2\left(4e^{(-2x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*coth(x)^4 - x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/(
4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*e
^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) -
 e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-2
*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 4*(e^(-2*x) - e^(-4*x) + e^
(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 20/(e^(-x)
- e^x)^4 - 2*log(e^(-x) + 1)
```

Fricas [B] time = 2.04957, size = 923, normalized size = 38.46

$$\frac{x\cosh(x)^4 + x\sinh(x)^4 + 4(x-2)\cosh(x)^3 + 4(x\cosh(x) + x - 2)\sinh(x)^3 + 2(3x-4)\cosh(x)^2 + 2\left(3x\cosh(x)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 5\coth(x)\operatorname{csch}^4(x)\,dx - \int -10\coth^2(x)\operatorname{csch}^3(x)\,dx - \int 10\coth^3(x)\operatorname{csch}^2(x)\,dx - \int -5\coth^4(x)\operatorname{csch}(x)\,dx -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**5,x)
```

```
[Out] -Integral(5*coth(x)*csch(x)**4, x) - Integral(-10*coth(x)**2*csch(x)**3, x)
 - Integral(10*coth(x)**3*csch(x)**2, x) - Integral(-5*coth(x)**4*csch(x),
x) - Integral(coth(x)**5, x) - Integral(-csch(x)**5, x)
```

Giac [A] time = 1.12839, size = 38, normalized size = 1.58

$$x - \frac{8\left(e^{(3x)} + e^{(2x)} + e^{x}\right)}{\left(e^{x} + 1\right)^{4}} - 2\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] x - 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 - 2*log(e^x + 1)
```

3.665 $\int (-\coth(x) + \mathbf{csch}(x))^4 \, dx$

Optimal. Leaf size=26

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rubi [A] time = 0.117281, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^4,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (-\coth(x)+\operatorname{csch}(x))^4\,dx &= \int (i-i\cosh(x))^4\operatorname{csch}^4(x)\,dx\\ &= \int \frac{\sinh^4(x)}{(i+i\cosh(x))^4}\,dx\\ &= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}-\int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)}-\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}+\int 1\,dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}-\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}\end{aligned}$$

Mathematica [A] time = 0.0072997, size = 30, normalized size = 1.15

$$-\frac{2}{3}\tanh^3\left(\frac{x}{2}\right)+2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^4,x]
```

```
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2] - (2*Tanh[x/2]^3)/3
```

Maple [B] time = 0.031, size = 57, normalized size = 2.2

$$x-\coth(x)-\frac{(\coth(x))^3}{3}+\frac{8\,(\cosh(x))^2}{3\,(\sinh(x))^3}+\frac{4\,(\cosh(x))^2}{3\,\sinh(x)}-\frac{4\,\sinh(x)}{3}-3\,\frac{\cosh(x)}{(\sinh(x))^3}-2\,\left(2/3-1/3\,(\operatorname{csch}(x))^2\right)\operatorname{cotl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^4,x)
```

```
[Out] x-coth(x)-1/3*coth(x)^3+8/3/sinh(x)^3*cosh(x)^2+4/3*cosh(x)^2/sinh(x)-4/3*s
inh(x)-3/sinh(x)^3*cosh(x)-2*(2/3-1/3*csch(x)^2)*coth(x)
```

Maxima [B] time = 1.18983, size = 247, normalized size = 9.5

$$-2\,\coth\left(x\right)^{3} + x - \frac{4\,\left(3\,e^{\left(-2\,x\right)} - 3\,e^{\left(-4\,x\right)} - 2\right)}{3\,\left(3\,e^{\left(-2\,x\right)} - 3\,e^{\left(-4\,x\right)} + e^{\left(-6\,x\right)} - 1\right)} - \frac{8\,e^{\left(-x\right)}}{3\,e^{\left(-2\,x\right)} - 3\,e^{\left(-4\,x\right)} + e^{\left(-6\,x\right)} - 1} + \frac{4\,e^{\left(-2\,x\right)}}{3\,e^{\left(-2\,x\right)} - 3\,e^{\left(-4\,x\right)} + e^{\left(-6\,x\right)} - 1} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) - 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) - 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 32/3/(e^(-x) -
e^x)^3
```

Fricas [B] time = 1.89626, size = 234, normalized size = 9.

$$\frac{3\,x\cosh\left(x\right)^{2} + 3\,x\sinh\left(x\right)^{2} + 4\,\left(3\,x + 10\right)\cosh\left(x\right) + 2\,\left(3\,x\cosh\left(x\right) + 3\,x + 4\right)\sinh\left(x\right) + 9\,x + 24}{3\,\left(\cosh\left(x\right)^{2} + 2\,\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 4\,\cosh\left(x\right) + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\coth\left(x\right) + \operatorname{csch}\left(x\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**4,x)
```

```
[Out] Integral((-coth(x) + csch(x))**4, x)
```

Giac [A] time = 1.159, size = 30, normalized size = 1.15

$$x+\frac{8\left(3e^{(2x)}+3e^{x}+2\right)}{3\left(e^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

3.666 $\int (-\coth(x)+\text{csch}(x))^3 \, dx$

Optimal. Leaf size=16

$$-\frac{2}{\cosh(x)+1}-\log(\cosh(x)+1)$$

[Out] -2/(1 + Cosh[x]) - Log[1 + Cosh[x]]

Rubi [A] time = 0.0581354, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{2}{\cosh(x)+1}-\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(-Coth[x] + Csch[x])^3,x]

[Out] -2/(1 + Cosh[x]) - Log[1 + Cosh[x]]

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int(-\coth(x)+\mathbf{csch}(x))^3\,dx = i\int(i-i\cosh(x))^3\mathbf{csch}^3(x)\,dx$$
$$= -\operatorname{Subst}\left(\int\frac{i+x}{(i-x)^2}\,dx,x,-i\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int\left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)dx,x,-i\cosh(x)\right)$$
$$= -\frac{2i}{i+i\cosh(x)}-\log(1+\cosh(x))$$

Mathematica [B] time = 0.0519006, size = 43, normalized size = 2.69

$$-\mathbf{sech}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log(\sinh(x))+3\log\left(\tanh\left(\frac{x}{2}\right)\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^3,x]
```

```
[Out] 2*Log[Cosh[x/2]] - 2*Log[Sinh[x/2]] - Log[Sinh[x]] + 3*Log[Tanh[x/2]] - Sec
h[x/2]^2
```

Maple [B] time = 0.02, size = 41, normalized size = 2.6

$$-\ln(\sinh(x))+\frac{(\coth(x))^2}{2}-3\frac{\cosh(x)}{(\sinh(x))^2}+\operatorname{csch}(x)\coth(x)-2\operatorname{Artanh}(e^x)+\frac{3(\cosh(x))^2}{2(\sinh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^3,x)
```

```
[Out] -ln(sinh(x))+1/2*coth(x)^2-3/sinh(x)^2*cosh(x)+csch(x)*coth(x)-2*arctanh(ex
p(x))+3/2*cosh(x)^2/sinh(x)^2
```

Maxima [B] time = 1.19223, size = 92, normalized size = 5.75

$$\frac{3}{2}\coth(x)^2-x+\frac{4\left(e^{(-x)}+e^{(-3x)}\right)}{2e^{(-2x)}-e^{(-4x)}-1}-\frac{2e^{(-2x)}}{2e^{(-2x)}-e^{(-4x)}-1}-2\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*coth(x)^2 - x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 2*e
^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*log(e^(-x) + 1)
```

Fricas [B] time = 2.07745, size = 335, normalized size = 20.94

$$\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cosh(x) + 1\right)\log(\cdots}{\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cosh(x) + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
+ 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(x
*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 3\coth(x)\operatorname{csch}^2(x)\,dx - \int -3\coth^2(x)\operatorname{csch}(x)\,dx - \int \coth^3(x)\,dx - \int -\operatorname{csch}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**3,x)
```

```
[Out] -Integral(3*coth(x)*csch(x)**2, x) - Integral(-3*coth(x)**2*csch(x), x) - I
ntegral(coth(x)**3, x) - Integral(-csch(x)**3, x)
```

Giac [A] time = 1.11969, size = 26, normalized size = 1.62

$$x - \frac{4e^x}{(e^x+1)^2} - 2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] x - 4*e^x/(e^x + 1)^2 - 2*log(e^x + 1)
```

3.667 $\int (-\coth(x) + \mathbf{csch}(x))^2 \, dx$

Optimal. Leaf size=12

$$x - \frac{2\sinh(x)}{\cosh(x) + 1}$$

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rubi [A] time = 0.0836184, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2\sinh(x)}{\cosh(x) + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^2,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int(-\coth(x)+\operatorname{csch}(x))^2\,dx &= -\int(i-i\cosh(x))^2\operatorname{csch}^2(x)\,dx\\ &= -\int\frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)}+\int 1\,dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0050803, size = 18, normalized size = 1.5

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^2,x]
```

```
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

Maple [A] time = 0.01, size = 21, normalized size = 1.8

$$x-2\coth(x)+2\frac{(\cosh(x))^2}{\sinh(x)}-2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^2,x)
```

```
[Out] x-2*coth(x)+2*cosh(x)^2/sinh(x)-2*sinh(x)
```

Maxima [B] time = 1.22073, size = 34, normalized size = 2.83

$$x - \frac{4}{e^{(-x)} - e^{x}} + \frac{4}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x - 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

Fricas [A] time = 1.93748, size = 77, normalized size = 6.42

$$\frac{x\cosh(x) + x\sinh(x) + x + 4}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**2,x)
```

```
[Out] Integral((-coth(x) + csch(x))**2, x)
```

Giac [A] time = 1.12157, size = 14, normalized size = 1.17

$$x + \frac{4}{e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x + 4/(e^x + 1)
```

3.668 $\int (-\coth(x) + \mathbf{csch}(x))\,dx$

Optimal. Leaf size=11

$$-\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]] - Log[Sinh[x]]
```

Rubi [A] time = 0.0083718, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3475, 3770}

$$-\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[-Coth[x] + Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] - Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (-\coth(x) + \mathbf{csch}(x))\,dx &= -\int \coth(x)\,dx + \int \mathbf{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x)) - \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0045583, size = 13, normalized size = 1.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right) - \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[-Coth[x] + Csch[x],x]
```

```
[Out] -Log[Sinh[x]] + Log[Tanh[x/2]]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$-\ln(\sinh(x)) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(x)+csch(x),x)
```

```
[Out] -ln(sinh(x))+ln(tanh(1/2*x))
```

Maxima [A] time = 1.15103, size = 15, normalized size = 1.36

$$-\log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="maxima")
```

```
[Out] -log(sinh(x)) + log(tanh(1/2*x))
```

Fricas [A] time = 2.13488, size = 46, normalized size = 4.18

$$x - 2\log(\cosh(x) + \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="fricas")
```

```
[Out] x - 2*log(cosh(x) + sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x)
```

```
[Out] Integral(-coth(x) + csch(x), x)
```

Giac [B] time = 1.12717, size = 34, normalized size = 3.09

$$x - \log(e^x + 1) - \log\left(\left|e^{(2x)} - 1\right|\right) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="giac")
```

```
[Out] x - log(e^x + 1) - log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

3.669 $\int \frac{1}{-\coth(x)+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=9

$$-\log(1-\cosh(x))$$

```
[Out] -Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0342147, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3160, 2667, 31}

$$-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-1),x]
```

```
[Out] -Log[1 - Cosh[x]]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx &= i\int \frac{\sinh(x)}{i-i\cosh(x)}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{i+x}\,dx, x, -i\cosh(x)\right)\\ &= -\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0212703, size = 9, normalized size = 1.

$$-2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-1),x]
```

```
[Out] -2*Log[Sinh[x/2]]
```

Maple [B] time = 0.03, size = 23, normalized size = 2.6

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,\ln\left(\tanh\left(x/2\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x)),x)
```

```
[Out] ln(tanh(1/2*x)+1)-2*ln(tanh(1/2*x))+ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.22438, size = 18, normalized size = 2.

$$-x-2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="maxima")
```

```
[Out] -x - 2*log(e^(-x) - 1)
```

Fricas [A] time = 2.12802, size = 46, normalized size = 5.11

$$x - 2\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="fricas")
```

```
[Out] x - 2*log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\coth\left(x\right) - \operatorname{csch}\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x)
```

```
[Out] -Integral(1/(coth(x) - csch(x)), x)
```

Giac [A] time = 1.1381, size = 14, normalized size = 1.56

$$x - 2\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="giac")
```

```
[Out] x - 2*log(abs(e^x - 1))
```

3.670 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^2} dx$

Optimal. Leaf size=14

$$x + \frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rubi [A] time = 0.0513362, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2680, 8}

$$x + \frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-2),x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^2}\,dx = -\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx$$
$$= \frac{2\sinh(x)}{1-\cosh(x)} + \int 1\,dx$$
$$= x + \frac{2\sinh(x)}{1-\cosh(x)}$$

Mathematica [C] time = 0.0086892, size = 24, normalized size = 1.71

$$-2\coth\left(\frac{x}{2}\right) {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-2),x]
```

```
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

Maple [A] time = 0.036, size = 26, normalized size = 1.9

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\,(\tanh(x/2))^{-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-2/tanh(1/2*x)-ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.23996, size = 16, normalized size = 1.14

$$x + \frac{4}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - 1)
```

Fricas [A] time = 2.21153, size = 77, normalized size = 5.5

$$\frac{x\cosh(x) + x\sinh(x) - x - 4}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\coth(x) + \operatorname{csch}(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**2,x)
```

```
[Out] Integral((-coth(x) + csch(x))**(-2), x)
```

Giac [A] time = 1.13126, size = 14, normalized size = 1.

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x - 4/(e^x - 1)
```

3.671 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^3} dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0586499, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-3),x]
```

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx &= -\left(i\int \frac{\sinh^3(x)}{(i-i\cosh(x))^3}\,dx\right)\\ &= \operatorname{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx, x, -i\cosh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)dx, x, -i\cosh(x)\right)\\ &= -\frac{2i}{i-i\cosh(x)}-\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0227794, size = 18, normalized size = 0.9

$$\operatorname{csch}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-3),x]
```

```
[Out] Csch[x/2]^2 - 2*Log[Sinh[x/2]]
```

Maple [A] time = 0.04, size = 29, normalized size = 1.5

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-2\,\ln\left(\tanh\left(x/2\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^3,x)
```

```
[Out] ln(tanh(1/2*x)+1)+1/tanh(1/2*x)^2-2*ln(tanh(1/2*x))+ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.15291, size = 47, normalized size = 2.35

$$-x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.24225, size = 335, normalized size = 16.75

$$\frac{x\cosh\left(x\right)^{2} + x\sinh\left(x\right)^{2} - 2\,(x-2)\cosh\left(x\right) - 2\left(\cosh\left(x\right)^{2} + 2\,(\cosh\left(x\right)-1)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 2\,\cosh\left(x\right) + 1\right)\log}{\cosh\left(x\right)^{2} + 2\,(\cosh\left(x\right)-1)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 2\,\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\coth^{3}(x) - 3\coth^{2}(x)\operatorname{csch}(x) + 3\coth(x)\operatorname{csch}^{2}(x) - \operatorname{csch}^{3}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**3,x)
```

```
[Out] -Integral(1/(coth(x)**3 - 3*coth(x)**2*csch(x) + 3*coth(x)*csch(x)**2 - csc
h(x)**3), x)
```

Giac [A] time = 1.11473, size = 27, normalized size = 1.35

$$x + \frac{4 e^{x}}{(e^{x} - 1)^{2}} - 2 \log(|e^{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

3.672 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^4}\,dx$

Optimal. Leaf size=30

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rubi [A] time = 0.0830563, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2680, 8}

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx &= \int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}\,dx \\
&= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx \\
&= \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \int 1\,dx \\
&= x + \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}
\end{aligned}
$$

Mathematica [C] time = 0.0076706, size = 28, normalized size = 0.93

$$-\frac{2}{3}\coth^3\left(\frac{x}{2}\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-4),x]
```

```
[Out] (-2*Coth[x/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x/2]^2])/3
```

Maple [A] time = 0.068, size = 34, normalized size = 1.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-2\,(\tanh(x/2))^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^4,x)
```

```
[Out] ln(tanh(1/2*x)+1)-2/3/tanh(1/2*x)^3-2/tanh(1/2*x)-ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.30636, size = 51, normalized size = 1.7

$$x-\frac{8\left(3\,e^{(-x)}-3\,e^{(-2\,x)}-2\right)}{3\left(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] x - 8/3*(3*e^(-x) - 3*e^(-2*x) - 2)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
```

Fricas [B] time = 2.38497, size = 234, normalized size = 7.8

$$\frac{3\,x\cosh\left(x\right)^{2}+3\,x\sinh\left(x\right)^{2}-4\,(3\,x+10)\cosh\left(x\right)+2\,(3\,x\cosh\left(x\right)-3\,x-4)\sinh\left(x\right)+9\,x+24}{3\left(\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)-1)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
- 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 4*cosh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**4,x)
```

```
[Out] Integral((-coth(x) + csch(x))**(-4), x)
```

Giac [A] time = 1.16775, size = 30, normalized size = 1.

$$x-\frac{8\left(3\,e^{(2\,x)}-3\,e^{x}+2\right)}{3\,(e^{x}-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="giac")

[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

3.673 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^5} dx$

Optimal. Leaf size=30

$$-\frac{4}{1-\cosh(x)}+\frac{2}{(1-\cosh(x))^2}-\log(1-\cosh(x))$$

```
[Out] 2/(1 - Cosh[x])^2 - 4/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0624783, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{4}{1-\cosh(x)}+\frac{2}{(1-\cosh(x))^2}-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2/(1 - Cosh[x])^2 - 4/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx &= i\int \frac{\sinh^5(x)}{(i-i\cosh(x))^5}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, -i\cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)dx, x, -i\cosh(x)\right)\\ &= -\frac{2}{(i-i\cosh(x))^2}-\frac{4i}{i-i\cosh(x)}-\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0238201, size = 32, normalized size = 1.07

$$\frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)+2\operatorname{csch}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2*Csch[x/2]^2 + Csch[x/2]^4/2 - 2*Log[Sinh[x/2]]
```

Maple [A] time = 0.046, size = 37, normalized size = 1.2

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-4}+\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-2\ln\left(\tanh\left(x/2\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^5,x)
```

```
[Out] ln(tanh(1/2*x)+1)+1/2/tanh(1/2*x)^4+1/tanh(1/2*x)^2-2*ln(tanh(1/2*x))+ln(ta
nh(1/2*x)-1)
```

Maxima [B] time = 1.13417, size = 78, normalized size = 2.6

$$-x-\frac{8\left(e^{(-x)}-e^{(-2x)}+e^{(-3x)}\right)}{4e^{(-x)}-6e^{(-2x)}+4e^{(-3x)}-e^{(-4x)}-1}-2\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] -x - 8*(e^(-x) - e^(-2*x) + e^(-3*x))/(4*e^(-x) - 6*e^(-2*x) + 4*e^(-3*x) -
 e^(-4*x) - 1) - 2*log(e^(-x) - 1)
```

Fricas [B] time = 1.93872, size = 923, normalized size = 30.77

$$\frac{x\cosh(x)^4+x\sinh(x)^4-4(x-2)\cosh(x)^3+4(x\cosh(x)-x+2)\sinh(x)^3+2(3x-4)\cosh(x)^2+2\left(3x\cosh(x)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\coth^5(x)-5\coth^4(x)\operatorname{csch}(x)+10\coth^3(x)\operatorname{csch}^2(x)-10\coth^2(x)\operatorname{csch}^3(x)+5\coth(x)\operatorname{csch}^4(x)-\operatorname{csch}^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**5,x)
```

```
[Out] -Integral(1/(coth(x)**5 - 5*coth(x)**4*csch(x) + 10*coth(x)**3*csch(x)**2 -
 10*coth(x)**2*csch(x)**3 + 5*coth(x)*csch(x)**4 - csch(x)**5), x)
```

Giac [A] time = 1.14177, size = 42, normalized size = 1.4

$$x + \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{(e^{x} - 1)^{4}} - 2\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] x + 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 - 2*log(abs(e^x - 1))
```

3.674 $\int (\operatorname{csch}(x) + \sinh(x))\, dx$

Optimal. Leaf size=8

$$\cosh(x) - \tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x]
```

Rubi [A] time = 0.0075202, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3770, 2638}

$$\cosh(x) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x] + Sinh[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{csch}(x) + \sinh(x))\, dx &= \int \operatorname{csch}(x)\, dx + \int \sinh(x)\, dx \\ &= -\tanh^{-1}(\cosh(x)) + \cosh(x)\end{aligned}$$

Mathematica [A] time = 0.0035631, size = 10, normalized size = 1.25

$$\cosh(x) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x] + Sinh[x],x]
```

```
[Out] Cosh[x] + Log[Tanh[x/2]]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)+sinh(x),x)
```

```
[Out] ln(tanh(1/2*x))+cosh(x)
```

Maxima [A] time = 1.18091, size = 11, normalized size = 1.38

$$\cosh\left(x\right) + \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x) + log(tanh(1/2*x))
```

Fricas [B] time = 1.6715, size = 236, normalized size = 29.5

$$\frac{\cosh\left(x\right)^{2} - 2\left(\cosh\left(x\right) + \sinh\left(x\right)\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + 2\left(\cosh\left(x\right) + \sinh\left(x\right)\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right) + 2}{2\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh
(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\sinh(x) + \operatorname{csch}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x)
```

```
[Out] Integral(sinh(x) + csch(x), x)
```

Giac [B] time = 1.13579, size = 32, normalized size = 4.

$$\frac{1}{2}\, e^{(-x)} + \frac{1}{2}\, e^{x} - \log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - log(e^x + 1) + log(abs(e^x - 1))
```

3.675 $\int (\operatorname{csch}(x)+\sinh(x))^2\,dx$

Optimal. Leaf size=22

$$\frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)$$

```
[Out] (3*x)/2 - (3*Coth[x])/2 + (Cosh[x]^2*Coth[x])/2
```

Rubi [A] time = 0.0249942, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {290, 325, 206}

$$\frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^2,x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 + (Cosh[x]^2*Coth[x])/2
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (\operatorname{csch}(x)+\sinh(x))^2\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)^2}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\cosh^2(x)\coth(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\ &= -\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)\end{aligned}$$

Mathematica [A] time = 0.0043097, size = 18, normalized size = 0.82

$$\frac{3x}{2}+\frac{1}{4}\sinh(2x)-\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^2,x]
```

```
[Out] (3*x)/2 - Coth[x] + Sinh[2*x]/4
```

Maple [A] time = 0.011, size = 15, normalized size = 0.7

$$-\coth(x)+\frac{3\,x}{2}+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^2,x)
```

```
[Out] -coth(x)+3/2*x+1/2*cosh(x)*sinh(x)
```

Maxima [A] time = 1.1756, size = 35, normalized size = 1.59

$$\frac{3}{2}\,x + \frac{2}{e^{(-2\,x)} - 1} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 3/2*x + 2/(e^(-2*x) - 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 1.76542, size = 109, normalized size = 4.95

$$\frac{\cosh\left(x\right)^{3} + 3\,\cosh\left(x\right)\sinh\left(x\right)^{2} + 4\,(3\,x + 2)\sinh\left(x\right) - 9\,\cosh\left(x\right)}{8\,\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 4*(3*x + 2)*sinh(x) - 9*cosh(x))/sin
h(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sinh\left(x\right) + \operatorname{csch}\left(x\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**2,x)
```

```
[Out] Integral((sinh(x) + csch(x))**2, x)
```

Giac [B] time = 1.14688, size = 53, normalized size = 2.41

$$\frac{3}{2}\,x - \frac{3\,e^{(4\,x)} + 14\,e^{(2\,x)} - 1}{8\,\left(e^{(4\,x)} - e^{(2\,x)}\right)} + \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*x - 1/8*(3*e^(4*x) + 14*e^(2*x) - 1)/(e^(4*x) - e^(2*x)) + 1/8*e^(2*x)
```

3.676 $\int (\operatorname{csch}(x) + \sinh(x))^3\,dx$

Optimal. Leaf size=34

$$\frac{5\cosh^3(x)}{6} + \frac{5\cosh(x)}{2} - \frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] (-5*ArcTanh[Cosh[x]])/2 + (5*Cosh[x])/2 + (5*Cosh[x]^3)/6 - (Cosh[x]^3*Coth
[x]^2)/2
```

Rubi [A] time = 0.0517341, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4397, 2592, 288, 302, 206}

$$\frac{5\cosh^3(x)}{6} + \frac{5\cosh(x)}{2} - \frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^3,x]
```

```
[Out] (-5*ArcTanh[Cosh[x]])/2 + (5*Cosh[x])/2 + (5*Cosh[x]^3)/6 - (Cosh[x]^3*Coth
[x]^2)/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^3\,dx &= \int \cosh^3(x)\coth^3(x)\,dx\\
&= \operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(x)\right)\\
&= \frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{5}{2}\tanh^{-1}(\cosh(x))+\frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.0682244, size = 45, normalized size = 1.32

$$
\frac{1}{48}\operatorname{csch}^2(x)\left(-50\cosh(x)+25\cosh(3x)+\cosh(5x)-60\log\left(\tanh\left(\frac{x}{2}\right)\right)+60\cosh(2x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^3,x]
```

```
[Out] (Csch[x]^2*(-50*Cosh[x] + 25*Cosh[3*x] + Cosh[5*x] - 60*Log[Tanh[x/2]] + 60
*Cosh[2*x]*Log[Tanh[x/2]]))/48
```

Maple [A] time = 0.017, size = 28, normalized size = 0.8

$$-\frac{\operatorname{csch}(x)\coth(x)}{2}-5\operatorname{Artanh}(\mathrm{e}^{x})+3\cosh(x)+\left(-\frac{2}{3}+\frac{(\sinh(x))^{2}}{3}\right)\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^3,x)
```

```
[Out] -1/2*csch(x)*coth(x)-5*arctanh(exp(x))+3*cosh(x)+(-2/3+1/3*sinh(x)^2)*cosh(
x)
```

Maxima [B] time = 1.13593, size = 90, normalized size = 2.65

$$\frac{e^{(-x)}+e^{(-3x)}}{2e^{(-2x)}-e^{(-4x)}-1}+\frac{1}{24}e^{(3x)}+\frac{9}{8}e^{(-x)}+\frac{1}{24}e^{(-3x)}+\frac{9}{8}e^{x}-\frac{5}{2}\log\left(e^{(-x)}+1\right)+\frac{5}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/24*e^(3*x) + 9/8*e^(-x)
 + 1/24*e^(-3*x) + 9/8*e^x - 5/2*log(e^(-x) + 1) + 5/2*log(e^(-x) - 1)
```

Fricas [B] time = 1.89613, size = 2090, normalized size = 61.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 5)*
sinh(x)^8 + 25*cosh(x)^8 + 40*(3*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 10*(21*
cosh(x)^4 + 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 +
350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 175*cosh(x)^4 -
75*cosh(x)^2 - 5)*sinh(x)^4 - 50*cosh(x)^4 + 40*(3*cosh(x)^7 + 35*cosh(x)^5
```

```
 - 25*cosh(x)^3 - 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 140*cosh(x)^6 - 1
50*cosh(x)^4 - 60*cosh(x)^2 + 5)*sinh(x)^2 + 25*cosh(x)^2 - 60*(cosh(x)^7 +
 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)
^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 +
 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(
x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*log(cosh(x) + si
nh(x) + 1) + 60*(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^
2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (3
5*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*
cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^
2)*sinh(x))*log(cosh(x) + sinh(x) - 1) + 10*(cosh(x)^9 + 20*cosh(x)^7 - 30*
cosh(x)^5 - 20*cosh(x)^3 + 5*cosh(x))*sinh(x) + 1)/(cosh(x)^7 + 7*cosh(x)*s
inh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*co
sh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^
3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*co
sh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12487, size = 84, normalized size = 2.47

$$\frac{1}{24}\left(e^{(-x)}+e^{x}\right)^{3}-\frac{e^{(-x)}+e^{x}}{\left(e^{(-x)}+e^{x}\right)^{2}-4}+e^{(-x)}+e^{x}-\frac{5}{4}\log\left(e^{(-x)}+e^{x}+2\right)+\frac{5}{4}\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*(e^(-x) + e^x)^3 - (e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + e^(-x) + e^
x - 5/4*log(e^(-x) + e^x + 2) + 5/4*log(e^(-x) + e^x - 2)
```

3.677 $\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx$

Optimal. Leaf size=13

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rubi [A] time = 0.0700452, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4397, 4398, 4400, 2589}

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[x] + Sinh[x]],x]
```

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx &= \int \sqrt{\cosh(x)\coth(x)}\,dx\\ &= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\ &= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\ &= 2\sqrt{\cosh(x)\coth(x)}\tanh(x)\end{aligned}$$

Mathematica [B] time = 0.0693421, size = 35, normalized size = 2.69

$$\frac{2\left(\sqrt[4]{-\sinh^2(x)}-1\right)\tanh(x)\sqrt{\cosh(x)\coth(x)}}{\sqrt[4]{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[x] + Sinh[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

Maple [B] time = 0.121, size = 42, normalized size = 3.2

$$\frac{\sqrt{2}\left(e^{2x}-1\right)}{e^{2x}+1}\sqrt{\frac{\left(e^{2x}+1\right)^2 e^{-x}}{e^{2x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((exp(2*x)+1)^2*exp(-x)/(exp(2*x)-1))^(1/2)/(exp(2*x)+1)*(exp(2*x)-
1)
```

Maxima [B] time = 1.81146, size = 73, normalized size = 5.62

$$\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

Fricas [B] time = 1.71093, size = 201, normalized size = 15.46

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)}{\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh(x) + \operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x) + csch(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(x) + sinh(x)), x)
```

3.678 $\int (\mathbf{csch}(x) + \sinh(x))^{3/2}\, dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\mathbf{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rubi [A] time = 0.122285, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 4398, 4400, 2598, 2589}

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\mathbf{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^(3/2),x]
```

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx &= \int (\cosh(x)\coth(x))^{3/2}\,dx\\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int(-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}+\frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)
\end{aligned}$$

Mathematica [A] time = 0.0497481, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cosh^2(x)-4\right)\operatorname{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(x)+\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(3/2),x)
```

```
[Out] int((csch(x)+sinh(x))^(3/2),x)
```

Maxima [B] time = 1.84759, size = 147, normalized size = 4.74

$$\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

Fricas [B] time = 1.9067, size = 348, normalized size = 11.23

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-7\right)\sinh(x)^2-14\cosh(x)^2+4\left(\cosh(x)^3-7\cosh(x)\right)\sinh(x)+1\right)}{3\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csch(x) + sinh(x))^(3/2), x)
```

3.679 $\int (\operatorname{csch}(x) + \sinh(x))^{5/2} \, dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rubi [A] time = 0.156851, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4397, 4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int (\mathrm{csch}(x)+\sinh(x))^{5/2}\,dx &= \int (\cosh(x)\coth(x))^{5/2}\,dx \\
&= -\frac{\sqrt{\cosh(x)\coth(x)}\int (-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= -\frac{\sqrt{\cosh(x)\coth(x)}\int \cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int \sqrt{\cosh(x)}(-i\coth(x))^{5/2}}{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(32\sqrt{\cosh(x)\,c\right.}{15} \\
&= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\,c}
\end{aligned}
$$

Mathematica [A] time = 0.313063, size = 44, normalized size = 0.88

$$
\frac{1}{15}\sqrt{\cosh(x)\coth(x)}\left(64\tanh(x)-10\coth(x)+6\sinh(x)\cosh(x)+57\left(-\sinh^2(x)\right)^{3/4}\mathrm{csch}(x)\mathrm{sech}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(5/2),x)
```

```
[Out] int((csch(x)+sinh(x))^(5/2),x)
```

Maxima [B] time = 1.71256, size = 220, normalized size = 4.4

$$\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} + \frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} - \frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} + \frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

Fricas [B] time = 1.87019, size = 883, normalized size = 17.66

$$\frac{\sqrt{\frac{1}{2}}\left(3\,\cosh\left(x\right)^{8}+24\,\cosh\left(x\right)\sinh\left(x\right)^{7}+3\,\sinh\left(x\right)^{8}+12\left(7\,\cosh\left(x\right)^{2}+9\right)\sinh\left(x\right)^{6}+108\,\cosh\left(x\right)^{6}+24\left(7\,\cosh\left(x\right.\right.}{30\left(\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\operatorname{csch}(x)+\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csch(x) + sinh(x))^(5/2), x)
```

3.680 $\int (-\cosh(x) + \mathbf{sech}(x))\, dx$

Optimal. Leaf size=8

$$\tan^{-1}(\sinh(x)) - \sinh(x)$$

```
[Out] ArcTan[Sinh[x]] - Sinh[x]
```

Rubi [A] time = 0.0064484, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2637, 3770}

$$\tan^{-1}(\sinh(x)) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[-Cosh[x] + Sech[x],x]
```

```
[Out] ArcTan[Sinh[x]] - Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (-\cosh(x) + \mathbf{sech}(x))\, dx &= -\int \cosh(x)\, dx + \int \mathbf{sech}(x)\, dx \\ &= \tan^{-1}(\sinh(x)) - \sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0042033, size = 14, normalized size = 1.75

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cosh[x] + Sech[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - Sinh[x]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.1

$$\arctan(\sinh(x)) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)+sech(x),x)
```

```
[Out] arctan(sinh(x))-sinh(x)
```

Maxima [A] time = 1.18032, size = 11, normalized size = 1.38

$$\arctan(\sinh(x)) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x)) - sinh(x)
```

Fricas [B] time = 1.78094, size = 166, normalized size = 20.75

$$\frac{4\,(\cosh(x) + \sinh(x))\arctan(\cosh(x) + \sinh(x)) - \cosh(x)^2 - 2\,\cosh(x)\sinh(x) - \sinh(x)^2 + 1}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(x
)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cosh(x) + \operatorname{sech}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x)
```

```
[Out] Integral(-cosh(x) + sech(x), x)
```

Giac [A] time = 1.16438, size = 22, normalized size = 2.75

$$2 \arctan(e^x) + \frac{1}{2} e^{(-x)} - \frac{1}{2} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x) + 1/2*e^(-x) - 1/2*e^x
```

3.681 $\int (-\cosh(x) + \operatorname{sech}(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

```
[Out] (-3*x)/2 + (3*Tanh[x])/2 + (Sinh[x]^2*Tanh[x])/2
```

Rubi [A] time = 0.0249294, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 321, 206}

$$-\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[x])/2 + (Sinh[x]^2*Tanh[x])/2
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (-\cosh(x)+\mathbf{sech}(x))^2\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

Mathematica [A] time = 0.0274479, size = 16, normalized size = 0.73

$$-\frac{3x}{2} + \frac{1}{4}\sinh(2x) + \tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^2,x]
```

```
[Out] (-3*x)/2 + Sinh[2*x]/4 + Tanh[x]
```

Maple [A] time = 0.015, size = 13, normalized size = 0.6

$$\frac{\cosh(x)\sinh(x)}{2} - \frac{3x}{2} + \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^2,x)
```

```
[Out] 1/2*cosh(x)*sinh(x)-3/2*x+tanh(x)
```

Maxima [A] time = 1.1983, size = 35, normalized size = 1.59

$$-\frac{3}{2}\,x + \frac{2}{e^{(-2\,x)} + 1} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x + 2/(e^(-2*x) + 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 1.9106, size = 101, normalized size = 4.59

$$\frac{\sinh(x)^3 - 4\,(3\,x + 2)\cosh(x) + 3\left(\cosh(x)^2 + 3\right)\sinh(x)}{8\,\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sinh(x)^3 - 4*(3*x + 2)*cosh(x) + 3*(cosh(x)^2 + 3)*sinh(x))/cosh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cosh(x) + \operatorname{sech}(x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**2,x)
```

```
[Out] Integral((-cosh(x) + sech(x))**2, x)
```

Giac [B] time = 1.14546, size = 50, normalized size = 2.27

$$-\frac{3}{2}\,x + \frac{3\,e^{(4\,x)} - 14\,e^{(2\,x)} - 1}{8\left(e^{(4\,x)} + e^{(2\,x)}\right)} + \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="giac")
```

```
[Out] -3/2*x + 1/8*(3*e^(4*x) - 14*e^(2*x) - 1)/(e^(4*x) + e^(2*x)) + 1/8*e^(2*x)
```

3.682 $\int (-\cosh(x) + \textbf{sech}(x))^3 \, dx$

Optimal. Leaf size=34

$$-\frac{5\sinh^3(x)}{6} + \frac{5\sinh(x)}{2} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\tan^{-1}(\sinh(x))$$

```
[Out] (-5*ArcTan[Sinh[x]])/2 + (5*Sinh[x])/2 - (5*Sinh[x]^3)/6 + (Sinh[x]^3*Tanh[
x]^2)/2
```

Rubi [A] time = 0.0475908, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 203}

$$-\frac{5\sinh^3(x)}{6} + \frac{5\sinh(x)}{2} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^3,x]
```

```
[Out] (-5*ArcTan[Sinh[x]])/2 + (5*Sinh[x])/2 - (5*Sinh[x]^3)/6 + (Sinh[x]^3*Tanh[
x]^2)/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx &= -\int \sinh^3(x)\tanh^3(x)\,dx\\
&= -\operatorname{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\operatorname{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\operatorname{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \sinh(x)\right)\\
&= \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{5}{2}\tan^{-1}(\sinh(x)) + \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.0281505, size = 37, normalized size = 1.09

$$
-\frac{1}{48}\operatorname{sech}^2(x)\left(-50\sinh(x)-25\sinh(3x)+\sinh(5x)+60\tan^{-1}(\sinh(x))+60\cosh(2x)\tan^{-1}(\sinh(x))\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^3,x]
```

```
[Out] -(Sech[x]^2*(60*ArcTan[Sinh[x]] + 60*ArcTan[Sinh[x]]*Cosh[2*x] - 50*Sinh[x]
 - 25*Sinh[3*x] + Sinh[5*x]))/48
```

Maple [A] time = 0.018, size = 29, normalized size = 0.9

$$-\left(\frac{2}{3}+\frac{\left(\cosh\left(x\right)\right)^{2}}{3}\right)\sinh\left(x\right)+3\,\sinh\left(x\right)-5\,\arctan\left(\mathrm{e}^{x}\right)+\frac{\mathrm{sech}\left(x\right)\tanh\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^3,x)
```

```
[Out] -(2/3+1/3*cosh(x)^2)*sinh(x)+3*sinh(x)-5*arctan(exp(x))+1/2*sech(x)*tanh(x)
```

Maxima [B] time = 1.75132, size = 76, normalized size = 2.24

$$\frac{e^{(-x)}-e^{(-3\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+5\,\arctan\left(e^{(-x)}\right)-\frac{1}{24}\,e^{(3\,x)}-\frac{9}{8}\,e^{(-x)}+\frac{1}{24}\,e^{(-3\,x)}+\frac{9}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + 5*arctan(e^(-x)) - 1/24*e
^(3*x) - 9/8*e^(-x) + 1/24*e^(-3*x) + 9/8*e^x
```

Fricas [B] time = 2.07347, size = 1648, normalized size = 48.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 - 5)
*sinh(x)^8 - 25*cosh(x)^8 + 40*(3*cosh(x)^3 - 5*cosh(x))*sinh(x)^7 + 10*(21
*cosh(x)^4 - 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 -
 350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 - 175*cosh(x)^4 -
 75*cosh(x)^2 + 5)*sinh(x)^4 + 50*cosh(x)^4 + 40*(3*cosh(x)^7 - 35*cosh(x)^
5 - 25*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 - 140*cosh(x)^6 -
```

```
150*cosh(x)^4 + 60*cosh(x)^2 + 5)*sinh(x)^2 + 120*(cosh(x)^7 + 7*cosh(x)*si
nh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cos
h(x)^3 + 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3
 + cosh(x)^3 + (21*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cos
h(x)^6 + 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2
5*cosh(x)^2 + 10*(cosh(x)^9 - 20*cosh(x)^7 - 30*cosh(x)^5 + 20*cosh(x)^3 +
5*cosh(x))*sinh(x) - 1)/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*
cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cosh(x)^3 + 2*cosh(x))*sinh(x
)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)
^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 10*cosh(x)^4 + 3*
cosh(x)^2)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 3\cosh(x)\operatorname{sech}^2(x)\,dx - \int -3\cosh^2(x)\operatorname{sech}(x)\,dx - \int \cosh^3(x)\,dx - \int -\operatorname{sech}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**3,x)
```

```
[Out] -Integral(3*cosh(x)*sech(x)**2, x) - Integral(-3*cosh(x)**2*sech(x), x) - I
ntegral(cosh(x)**3, x) - Integral(-sech(x)**3, x)
```

Giac [B] time = 1.15187, size = 89, normalized size = 2.62

$$-\frac{5}{4}\pi + \frac{1}{24}\left(e^{(-x)} - e^x\right)^3 - \frac{e^{(-x)} - e^x}{\left(e^{(-x)} - e^x\right)^2 + 4} - \frac{5}{2}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - e^{(-x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="giac")
```

```
[Out] -5/4*pi + 1/24*(e^(-x) - e^x)^3 - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) - 5
/2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - e^(-x) + e^x
```

3.683 $\int \sqrt{-\cosh(x) + \mathbf{sech}(x)}\, dx$

Optimal. Leaf size=14

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

Rubi [A] time = 0.0568221, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cosh[x] + Sech[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx = \int \sqrt{-\sinh(x)\tanh(x)}\,dx$$
$$= \frac{\sqrt{-\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Mathematica [A] time = 0.0521067, size = 14, normalized size = 1.

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cosh[x] + Sech[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

Maple [B] time = 0.185, size = 43, normalized size = 3.1

$$\frac{\sqrt{2}\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{-\frac{\left(e^{2x}-1\right)^{2}e^{-x}}{e^{2x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(1/2),x)
```

```
[Out] 2^(1/2)*(-(exp(2*x)-1)^2*exp(-x)/(exp(2*x)+1))^(1/2)/(exp(2*x)-1)*(exp(2*x)
+1)
```

Maxima [B] time = 1.78975, size = 53, normalized size = 3.79

$$-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{-e^{(-2x)}-1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{-e^{(-2x)}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*e^(1/2*x)/sqrt(-e^(-2*x) - 1) - sqrt(2)*e^(-3/2*x)/sqrt(-e^(-2*x)
- 1)
```

Fricas [B] time = 1.82606, size = 208, normalized size = 14.86

$$2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\sqrt{-\frac{1}{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-1/(cosh(x
)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\cosh\left(x\right)+\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cosh(x) + sech(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\cosh\left(x\right)+\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cosh(x) + sech(x)), x)
```

3.684 $\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx$

Optimal. Leaf size=33

$$-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] (-8*Csch[x]*Sqrt[-(Sinh[x]*Tanh[x])])/3 - (2*Sinh[x]*Sqrt[-(Sinh[x]*Tanh[x]
)])/3
```

Rubi [A] time = 0.0967737, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^(3/2),x]
```

```
[Out] (-8*Csch[x]*Sqrt[-(Sinh[x]*Tanh[x])])/3 - (2*Sinh[x]*Sqrt[-(Sinh[x]*Tanh[x]
)])/3
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx &= \int (-\sinh(x)\tanh(x))^{3/2}\,dx\\ &= \frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= -\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{\left(4\sqrt{-\sinh(x)\tanh(x)}\right)\int \frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= -\frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.0831163, size = 24, normalized size = 0.73

$$\frac{2}{3}\coth(x)\left(4\operatorname{csch}^2(x)+1\right)(-\sinh(x)\tanh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^(3/2),x]
```

```
[Out] (2*Coth[x]*(1 + 4*Csch[x]^2)*(-(Sinh[x]*Tanh[x]))^(3/2))/3
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int (-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(3/2),x)
```

```
[Out] int((-cosh(x)+sech(x))^(3/2),x)
```

Maxima [B] time = 1.95826, size = 104, normalized size = 3.15

$$-\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*e^(3/2*x)/(-e^(-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(-e^(
-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(-e^(-2*x) - 1)^(3/2) - 1/6*sqrt(
2)*e^(-9/2*x)/(-e^(-2*x) - 1)^(3/2)
```

Fricas [B] time = 1.84747, size = 354, normalized size = 10.73

$$-\frac{\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\cosh\left(x\right)^{2}+7\right)\sinh\left(x\right)^{2}+14\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+7\cos\right.\right.}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)*sq
rt(-1/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh
(x) + cosh(x)))/(cosh(x) + sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cosh(x) + \mathrm{sech}(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cosh(x) + sech(x))^(3/2), x)
```

3.685 $\int (-\cosh(x) + \mathbf{sech}(x))^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] (-64*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])])/15 + (16*Tanh[x]*Sqrt[-(Sinh[x]*Tanh
[x])])/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[-(Sinh[x]*Tanh[x])])/5
```

Rubi [A] time = 0.125743, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^(5/2),x]
```


```
[Out] (-64*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])])/15 + (16*Tanh[x]*Sqrt[-(Sinh[x]*Tanh
[x])])/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[-(Sinh[x]*Tanh[x])])/5
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (-\cosh(x)+\operatorname{sech}(x))^{5/2}\,dx = \int (-\sinh(x)\tanh(x))^{5/2}\,dx$$
$$= \frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{\left(8\sqrt{-\sinh(x)\tanh(x)}\right)\int \sqrt{i\sinh(x)}(i\tanh(x}{5\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{\left(32\sqrt{-\sinh}\right.}{}$$
$$= -\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x$$

Mathematica [A] time = 0.129106, size = 30, normalized size = 0.57

$$\frac{2}{15}\operatorname{csch}(x)(-\sinh(x)\tanh(x))^{3/2}\left(-3\cosh^2(x)+32\coth^2(x)-5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^(5/2),x]
```

```
[Out] (2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Csch[x]*(-(Sinh[x]*Tanh[x]))^(3/2))/15
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(5/2),x)
```

```
[Out] int((-cosh(x)+sech(x))^(5/2),x)
```

Maxima [B] time = 1.76294, size = 155, normalized size = 2.92

$$-\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(-\frac{11}{2}x\right)}}{4\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{15}{2}x\right)}}{20\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(2)*e^(5/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(-e^(
-2*x) - 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(-e^(-2*x) - 1)^(5/2) + 41/6*sqr
t(2)*e^(-7/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(-e^(-2*x)
- 1)^(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(-e^(-2*x) - 1)^(5/2)
```

Fricas [B] time = 1.87972, size = 887, normalized size = 16.74

$$\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 - 9\right)\sinh(x)^6 - 108\cosh(x)^6 + 24\left(7\cosh(x)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
```

```
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)*sqrt(-1/(co
sh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cos
h(x)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sin
h(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-cosh(x) + sech(x))^(5/2), x)
```

3.686 $\int \frac{1}{\sinh(x)+\tanh(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2(\cosh(x)+1)}-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]/2 - 1/(2*(1 + Cosh[x]))
```

Rubi [A] time = 0.0754002, antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$\frac{\text{csch}^2(x)}{2}-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x] + Tanh[x])^(-1),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2 + Csch[x]^2/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh(x)+\tanh(x)}\,dx &= -\left(i\int \frac{\coth(x)}{-i-i\cosh(x)}\,dx\right)\\ &= \int \coth^2(x)\text{csch}(x)\,dx - \int \coth(x)\text{csch}^2(x)\,dx\\ &= -\frac{1}{2}\coth(x)\text{csch}(x) + \frac{1}{2}\int \text{csch}(x)\,dx - \text{Subst}\left(\int x\,dx, x, -i\text{csch}(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x)) - \frac{1}{2}\coth(x)\text{csch}(x) + \frac{\text{csch}^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.026307, size = 35, normalized size = 1.94

$$-\frac{1}{4}\text{sech}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x] + Tanh[x])^(-1),x]
```

```
[Out] -Log[Cosh[x/2]]/2 + Log[Sinh[x/2]]/2 - Sech[x/2]^2/4
```

Maple [A] time = 0.03, size = 17, normalized size = 0.9

$$\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)+tanh(x)),x)
```

```
[Out] 1/4*tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

Maxima [B] time = 1.13559, size = 53, normalized size = 2.94

$$-\frac{e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1}-\frac{1}{2}\,\log\left(e^{(-x)}+1\right)+\frac{1}{2}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*e^(-x) + e^(-2*x) + 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) -
1)
```

Fricas [B] time = 1.81944, size = 386, normalized size = 21.44

$$-\frac{\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\,(\cos\right.}{2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x) + \tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x)
```

```
[Out] Integral(1/(sinh(x) + tanh(x)), x)
```

Giac [B] time = 1.15286, size = 58, normalized size = 3.22

$$\frac{e^{(-x)} + e^{x} - 2}{4\left(e^{(-x)} + e^{x} + 2\right)} - \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) + e^x - 2)/(e^(-x) + e^x + 2) - 1/4*log(e^(-x) + e^x + 2) + 1/4
*log(e^(-x) + e^x - 2)
```

3.687 $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{1}{2(1-\cosh(x))}-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]/2 + 1/(2*(1 - Cosh[x]))
```

Rubi [A] time = 0.0732545, antiderivative size = 24, normalized size of antiderivative = 1.2, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\text{csch}^2(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x] - Tanh[x])^(-1),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2 - Csch[x]^2/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh(x)-\tanh(x)}\,dx &= -\left(i\int \frac{\coth(x)}{i-i\cosh(x)}\,dx\right)\\ &= \int \coth^2(x)\operatorname{csch}(x)\,dx+\int \coth(x)\operatorname{csch}^2(x)\,dx\\ &= -\frac{1}{2}\coth(x)\operatorname{csch}(x)+\frac{1}{2}\int \operatorname{csch}(x)\,dx+\operatorname{Subst}\left(\int x\,dx,x,-i\operatorname{csch}(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)-\frac{\operatorname{csch}^2(x)}{2}\end{aligned}$$

Mathematica [B] time = 0.0431123, size = 50, normalized size = 2.5

$$-\frac{1}{4}\operatorname{csch}^2\left(\frac{x}{2}\right)\left(\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\cosh(x)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x] - Tanh[x])^(-1),x]
```

```
[Out] -(Csch[x/2]^2*(1 - Log[Cosh[x/2]] + Cosh[x]*(Log[Cosh[x/2]] - Log[Sinh[x/2]
]) + Log[Sinh[x/2]]))/4
```

Maple [A] time = 0.032, size = 17, normalized size = 0.9

$$-\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)-tanh(x)),x)
```

```
[Out] -1/4/tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

Maxima [B] time = 1.22083, size = 54, normalized size = 2.7

$$\frac{e^{(-x)}}{2\,e^{(-x)}-e^{(-2\,x)}-1}-\frac{1}{2}\,\log\left(e^{(-x)}+1\right)+\frac{1}{2}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1
)
```

Fricas [B] time = 1.69748, size = 386, normalized size = 19.3

$$-\frac{\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\left(\cosh\right.\right.}{2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x)
```

```
[Out] Integral(1/(sinh(x) - tanh(x)), x)
```

Giac [B] time = 1.11335, size = 58, normalized size = 2.9

$$-\frac{e^{(-x)} + e^x + 2}{4\left(e^{(-x)} + e^x - 2\right)} - \frac{1}{4}\,\log\left(e^{(-x)} + e^x + 2\right) + \frac{1}{4}\,\log\left(e^{(-x)} + e^x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) + e^x + 2)/(e^(-x) + e^x - 2) - 1/4*log(e^(-x) + e^x + 2) + 1/
4*log(e^(-x) + e^x - 2)
```

3.688 $\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0667874, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3097, 3133}

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{bx}{a^2-b^2} + \frac{(ia)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$
$$= -\frac{bx}{a^2-b^2} + \frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.0609465, size = 29, normalized size = 0.74

$$\frac{a\log(a\cosh(x)+b\sinh(x))-bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-(b*x) + a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.037, size = 70, normalized size = 1.8

$$-4\,\frac{\ln(\tanh(x/2)+1)}{4\,a-4\,b}+\frac{a}{(a+b)\,(a-b)}\ln\left(a+2\,\tanh(x/2)\,b+a\left(\tanh\left(\frac{x}{2}\right)\right)^2\right)-4\,\frac{\ln(\tanh(x/2)-1)}{4\,a+4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/(4*a-4*b)*ln(tanh(1/2*x)+1)+a/(a+b)/(a-b)*ln(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)-4/(4*a+4*b)*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.18268, size = 54, normalized size = 1.38

$$\frac{a\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 1.77595, size = 109, normalized size = 2.79

$$-\frac{(a+b)x - a\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [A] time = 0.723252, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}x & \text{for}\ a=0 \wedge b=0 \\ \frac{\log(\cosh(x))}{a} & \text{for}\ b=0 \\ -\frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for}\ a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for}\ a=b \\ \frac{a\log\left(\frac{a\cosh(x)}{b}+\sinh(x)\right)}{a^2-b^2} - \frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (log(cosh(x))/a, Eq(b, 0)), (-x*sin
h(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x))
- cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2*b*sinh(x)
 + 2*b*cosh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + cosh(x)/(2*b*sinh
(x) + 2*b*cosh(x)), Eq(a, b)), (a*log(a*cosh(x)/b + sinh(x))/(a**2 - b**2)
- b*x/(a**2 - b**2), True))
```

Giac [A] time = 1.14103, size = 58, normalized size = 1.49

$$\frac{a\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2-b^2} - \frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

3.689 $\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{a^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -((a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0826827, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3099, 3074, 206, 2638}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{a^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\sinh(x)}{a^2-b^2}-\frac{a^2\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int\sinh(x)\,dx}{a^2-b^2}\\
&= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ia^2)\,\mathrm{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= -\frac{a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.193209, size = 89, normalized size = 1.2

$$
\frac{a\left(\sqrt{a-b}(a+b)\sinh(x)-2a\sqrt{a+b}\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)\right)-b\sqrt{a-b}(a+b)\cosh(x)}{(a-b)^{3/2}(a+b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-(Sqrt[a - b]*b*(a + b)*Cosh[x]) + a*(-2*a*Sqrt[a + b]*ArcTan[(b + a*Tanh[
x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*(a + b)*Sinh[x]))/((a - b)^(
3/2)*(a + b)^2)
```

Maple [A] time = 0.041, size = 93, normalized size = 1.3

$$
-2\,\frac{a^2}{(a+b)\,(a-b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)-8\,\frac{1}{(8\,a+8\,b)\,(\tanh(x/2)-1)}-8\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -2*a^2/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^
2)^(1/2))-8/(8*a+8*b)/(tanh(1/2*x)-1)-8/(8*a-8*b)/(tanh(1/2*x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88034, size = 1106, normalized size = 14.95

$$\left[-\frac{a^3 + a^2b - ab^2 - b^3 - \left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^2 - 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right) - \left(a^3 - a^2b - ab^2 + b\right.}{2\left(\left(a^4 - 2\,a^2b^2 + b^4\right)\mathrm{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a^2*cosh(x) + a^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14338, size = 82, normalized size = 1.11

$$-\frac{2\,a^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

3.690 $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=101

$$\frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 + (b*x)/(2*(a^2 - b^2)) - (a^3*Log[a*Cosh[x] + b*Si
nh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^2)
/(2*(a^2 - b^2))
```

Rubi [A] time = 0.132672, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3099, 3097, 3133, 2635, 8}

$$\frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 + (b*x)/(2*(a^2 - b^2)) - (a^3*Log[a*Cosh[x] + b*Si
nh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^2)
/(2*(a^2 - b^2))
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
```

```
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\sinh^2(x)}{2\left(a^2-b^2\right)}-\frac{a^2\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \sinh^2(x)\,dx}{a^2-b^2}\\ &= \frac{a^2bx}{\left(a^2-b^2\right)^2}-\frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh^2(x)}{2\left(a^2-b^2\right)}-\frac{\left(ia^3\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{b\int 1\,dx}{2\left(a^2-b^2\right)}\\ &= \frac{a^2bx}{\left(a^2-b^2\right)^2}+\frac{bx}{2\left(a^2-b^2\right)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}-\frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh^2(x)}{2\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.144926, size = 75, normalized size = 0.74

$$\frac{\left(b^3-a^2b\right)\sinh(2x)+a\left(a^2-b^2\right)\cosh(2x)+6a^2bx-4a^3\log(a\cosh(x)+b\sinh(x))-2b^3x}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (6*a^2*b*x - 2*b^3*x + a*(a^2 - b^2)*Cosh[2*x] - 4*a^3*Log[a*Cosh[x] + b*Si
nh[x]] + (-(a^2*b) + b^3)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.05, size = 175, normalized size = 1.7

$$-16\frac{1}{(32a-32b)(\tanh(x/2)+1)}+8\frac{1}{(16a-16b)(\tanh(x/2)+1)^2}+\frac{a}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{b}{2(a-b)^2}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -16/(32*a-32*b)/(tanh(1/2*x)+1)+8/(16*a-16*b)/(tanh(1/2*x)+1)^2+1/(a-b)^2*l
n(tanh(1/2*x)+1)*a-1/2/(a-b)^2*ln(tanh(1/2*x)+1)*b-a^3/(a-b)^2/(a+b)^2*ln(a
+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+8/(16*a+16*b)/(tanh(1/2*x)-1)^2+16/(32*a+
32*b)/(tanh(1/2*x)-1)+1/(a+b)^2*ln(tanh(1/2*x)-1)*a+1/2/(a+b)^2*ln(tanh(1/2
*x)-1)*b
```

Maxima [A] time = 1.14197, size = 117, normalized size = 1.16

$$-\frac{a^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4}-\frac{(2a+b)x}{2\left(a^2+2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}+\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(2*a + b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 1.88408, size = 818, normalized size = 8.1

$$\frac{\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4+4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3+\left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4+4\left(2a^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(2*a^3 + 3*a^
2*b - b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(2*a^3 + 3*a^2*b - b^3)*x)*sinh(x)^2 - 8*(a^3*cosh
(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(2*a^
3 + 3*a^2*b - b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15356, size = 154, normalized size = 1.52

$$-\frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{(2a-b)x}{2\left(a^2-2ab+b^2\right)}-\frac{\left(4ae^{(2x)}-2be^{(2x)}-a+b\right)e^{(-2x)}}{8\left(a^2-2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*
(2*a - b)*x/(a^2 - 2*a*b + b^2) - 1/8*(4*a*e^(2*x) - 2*b*e^(2*x) - a + b)*e
^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.691 $\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0617367, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3098, 3133}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$
$$= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.0416025, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.038, size = 71, normalized size = 1.8

$$2\,\frac{\ln(\tanh(x/2)+1)}{2\,a-2\,b} - \frac{b}{(a+b)\,(a-b)}\ln\left(a+2\,\tanh(x/2)\,b+a\left(\tanh\left(\frac{x}{2}\right)\right)^2\right) - 2\,\frac{\ln(\tanh(x/2)-1)}{2\,b+2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2/(2*a-2*b)*ln(tanh(1/2*x)+1)-b/(a-b)/(a+b)*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-2/(2*b+2*a)*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.15067, size = 55, normalized size = 1.41

$$-\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2} + \frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 1.83174, size = 108, normalized size = 2.77

$$\frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [A] time = 0.716007, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\log(\sinh(x)) & \text{for } a=0\wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{b\log\left(\frac{a\cosh(x)}{b}+\sinh(x)\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (x/a, Eq(b, 0)), (x*sinh
(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) -
 cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2*b*sinh(x)
+ 2*b*cosh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)/(2*b*sinh(
x) + 2*b*cosh(x)), Eq(a, b)), (a*x/(a**2 - b**2) - b*log(a*cosh(x)/b + sinh
(x))/(a**2 - b**2), True))
```

Giac [A] time = 1.14327, size = 58, normalized size = 1.49

$$-\frac{b\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.692 $\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -((b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0785008, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3100, 2637, 3074, 206}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{\left(ib^2\right)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\ &= -\frac{b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.156264, size = 80, normalized size = 1.08

$$\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{b^2-a^2}-\frac{2b^2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] time = 0.043, size = 93, normalized size = 1.3

$$-2\,\frac{b^2}{(a+b)\,(a-b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)-2\,\frac{1}{(2\,b+2\,a)\,(\tanh(x/2)-1)}-2\,\frac{1}{(2\,a-2\,b)\,(\tanh(x/2)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^
2)^(1/2))-2/(2*b+2*a)/(tanh(1/2*x)-1)-2/(2*a-2*b)/(tanh(1/2*x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83325, size = 1106, normalized size = 14.95

$$\left[-\frac{a^3 + a^2b - ab^2 - b^3 - \left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^2 - 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right) - \left(a^3 - a^2b - ab^2 + b\right.}{2\left(\left(a^4 - 2a^2b^2 + b^4\right)\text{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16549, size = 82, normalized size = 1.11

$$-\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

3.693 $\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (a*x)/(2*(a^2 - b^2)) - (b*Cosh[x]^2)/(2*(a^2
- b^2)) + (b^3*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[
x])/(2*(a^2 - b^2))
```

Rubi [A] time = 0.116145, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3100, 2635, 8, 3098, 3133}

$$-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (a*x)/(2*(a^2 - b^2)) - (b*Cosh[x]^2)/(2*(a^2
- b^2)) + (b^3*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[
x])/(2*(a^2 - b^2))
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx &= -\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\int\cosh^2(x)\,dx}{a^2-b^2}-\frac{b^2\int\frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{\left(ib^3\right)\int\frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{a\int 1\,dx}{2\left(a^2-b^2\right)}\\ &= -\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.10287, size = 75, normalized size = 0.74

$$\frac{a\left(a^2-b^2\right)\sinh(2x)+\left(b^3-a^2b\right)\cosh(2x)+2a^3x-6ab^2x+4b^3\log(a\cosh(x)+b\sinh(x))}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] + b
*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.049, size = 175, normalized size = 1.7

$$-\frac{1}{2a-2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+2\frac{1}{(4a-4b)(\tanh(x/2)+1)}+\frac{a}{2(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{b}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -1/(2*a-2*b)/(tanh(1/2*x)+1)^2+2/(4*a-4*b)/(tanh(1/2*x)+1)+1/2/(a-b)^2*ln(t
anh(1/2*x)+1)*a-1/(a-b)^2*ln(tanh(1/2*x)+1)*b+b^3/(a-b)^2/(a+b)^2*ln(a+2*ta
nh(1/2*x)*b+a*tanh(1/2*x)^2)+1/(2*b+2*a)/(tanh(1/2*x)-1)^2+2/(4*a+4*b)/(tan
h(1/2*x)-1)-1/2/(a+b)^2*ln(tanh(1/2*x)-1)*a-1/(a+b)^2*ln(tanh(1/2*x)-1)*b
```

Maxima [A] time = 1.20217, size = 116, normalized size = 1.15

$$\frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4}+\frac{(a+2b)x}{2\left(a^2+2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)*
x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 1.88098, size = 818, normalized size = 8.1

$$\frac{\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4+4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3+\left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4+4\left(a^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12158, size = 150, normalized size = 1.49

$$\frac{b^3 \log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{(a - 2b)x}{2\left(a^2 - 2ab + b^2\right)} - \frac{\left(2ae^{(2x)} - 4be^{(2x)} + a - b\right)e^{(-2x)}}{8\left(a^2 - 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(
a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*e^
(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.694 $\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0964272, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 204}

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx &= -\left(i\int\left(\frac{i\text{sech}(x)}{a}-\frac{ib}{a(b\cosh(x)+a\sinh(x))}\right)dx\right)\\
&= \frac{\int \text{sech}(x)\,dx}{a}-\frac{b\int \frac{1}{b\cosh(x)+a\sinh(x)}\,dx}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{(ib)\,\text{Subst}\left(\int \frac{1}{-a^2+b^2-x^2}\,dx,x,-ia\cosh(x)-ib\sinh(x)\right)}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}
\end{aligned}
$$

Mathematica [A] time = 0.122382, size = 60, normalized size = 1.2

$$
\frac{2\left(\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Maple [A] time = 0.049, size = 54, normalized size = 1.1

$$
2\,\frac{\arctan(\tanh(x/2))}{a}-2\,\frac{b}{a\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b+2\,a}{\sqrt{-a^2+b^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] 2/a*arctan(tanh(1/2*x))-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+
2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02501, size = 564, normalized size = 11.28

$$\left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)+2\left(a^2-b^2\right)\arctan\left(\cosh\left(x\right)+\right.}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{a\sinh(x)+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a*sinh(x) + b*cosh(x)), x)
```

Giac [A] time = 1.15066, size = 65, normalized size = 1.3

$$-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a}+\frac{2\,\arctan\left(e^x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

3.695 $\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal. Leaf size=51

$$\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] -(ArcTanh[Cosh[x]]/b) + (a*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
)/(b*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.100152, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (a*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
)/(b*Sqrt[a^2 - b^2])
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx &= i\int\left(-\frac{i\operatorname{csch}(x)}{b}-\frac{a}{b(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{csch}(x)\,dx}{b}-\frac{(ia)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{b}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] time = 0.0799109, size = 59, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*
Sqrt[a + b]) + Log[Tanh[x/2]])/b
```

Maple [A] time = 0.05, size = 53, normalized size = 1.

$$\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\,\frac{a}{b\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b+2\,a}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] 1/b*ln(tanh(1/2*x))-2*a/b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)
/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04369, size = 675, normalized size = 13.24

$$\left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)-\left(a^2-b^2\right)\log\left(\cosh(x)+\sinh\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - (a^2 -
 b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b^2)*log(cosh(x) + sinh(x) - 1))/
(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*a*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh
(x) + (a + b)*sinh(x))) + (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a^2 - b
^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b - b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{a\sinh(x) + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a*sinh(x) + b*cosh(x)), x)
```

Giac [A] time = 1.12607, size = 81, normalized size = 1.59

$$-\frac{2\,a\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b} - \frac{\log(e^x+1)}{b} + \frac{\log(|e^x-1|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) - log(e^
x + 1)/b + log(abs(e^x - 1))/b
```

3.696 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{a}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) -
a/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.0626764, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3154, 3074, 206}

$$-\frac{a}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) -
a/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} - \frac{b\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} - \frac{(ib)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(\right.}{a^2-b^2}$$

$$= -\frac{b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.189291, size = 125, normalized size = 1.89

$$-\frac{2b^2\sqrt{a+b}\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+2ab\sqrt{a+b}\cosh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+a\sqrt{a-b}(a+b)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt
[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/
(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b
*Sinh[x])))
```

Maple [A] time = 0.052, size = 99, normalized size = 1.5

$$4\,\frac{-2\,\tanh(x/2)\,b-2\,a}{\left(4\,a^2-4\,b^2\right)\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)}-8\,\frac{b}{\left(4\,a^2-4\,b^2\right)\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 4*(-2*tanh(1/2*x)*b-2*a)/(4*a^2-4*b^2)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-
8*b/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9003, size = 1473, normalized size = 22.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh
(x)^2 + a*b - b^2)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2)*sinh(x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^
5)*sinh(x)^2), 2*(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) +
(a*b + b^2)*sinh(x)^2 + a*b - b^2)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/(
(a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a*b^2)*cosh(x) - (a^3 - a*b^2)
*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b
 - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*
b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 -
 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1297, size = 97, normalized size = 1.47

$$-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{2\,ae^x}{\left(a^2-b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 2*a*e^x/((
a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.697 $\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=68

$$\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}-\frac{a}{\left(a^2-b^2\right)\left(a\coth(x)+b\right)}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - a/((a^2 - b^2)*(b + a*Coth[x])) - (2*a*b*Lo
g[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2
```

Rubi [A] time = 0.138031, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3085, 3483, 3531, 3530}

$$\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}-\frac{a}{\left(a^2-b^2\right)\left(a\coth(x)+b\right)}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - a/((a^2 - b^2)*(b + a*Coth[x])) - (2*a*b*Lo
g[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\int \frac{1}{(-ib-ia\coth(x))^2}\,dx \\
&= -\frac{a}{\left(a^2-b^2\right)(b+a\coth(x))}-\frac{\int \frac{-ib+ia\coth(x)}{-ib-ia\coth(x)}\,dx}{a^2-b^2} \\
&= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}-\frac{a}{\left(a^2-b^2\right)(b+a\coth(x))}-\frac{(2iab)\int \frac{-a-b\coth(x)}{-ib-ia\coth(x)}\,dx}{\left(a^2-b^2\right)^2} \\
&= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}-\frac{a}{\left(a^2-b^2\right)(b+a\coth(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.229815, size = 61, normalized size = 0.9

$$\frac{x\left(a^2+b^2\right)-\frac{a(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}-2ab\log(a\cosh(x)+b\sinh(x))}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] - (a*(a - b)*(a + b)*Sinh
[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

Maple [B] time = 0.074, size = 146, normalized size = 2.2

$$\frac{1}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{a^3\tanh(x/2)}{(a-b)^2(a+b)^2\left(a+2\tanh(x/2)b+a(\tanh(x/2))^2\right)}+2\frac{a\tanh(x}{(a-b)^2(a+b)^2\left(a+2\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/(a-b)^2*ln(tanh(1/2*x)+1)-2*a^3/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)+2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)*b^2-2*a/(a-b)^2/(a+b)^2*b*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x
)^2)-1/(a+b)^2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.25772, size = 140, normalized size = 2.06

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}-\frac{2\,a^2}{a^4-2\,a^2b^2+b^4+\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}}+\frac{x}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*a^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

Fricas [B] time = 1.82946, size = 826, normalized size = 12.15

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\sinh(x}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+\left(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
```

```
^3 - 2*a^2*b + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

Sympy [A] time = 143.99, size = 2574, normalized size = 37.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((x - sinh(x)/cosh(x))/a**2, Eq(b,
0)), (x*sinh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x) + 4*b**2*cos
h(x)**2) - 2*x*sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)
+ 4*b**2*cosh(x)**2) + x*cosh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cos
h(x) + 4*b**2*cosh(x)**2) + 3*sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*s
inh(x)*cosh(x) + 4*b**2*cosh(x)**2) - 2*cosh(x)**2/(4*b**2*sinh(x)**2 - 8*b
**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -b)), (x*sinh(x)**2/(4*b**2
*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*sinh(x)*cos
h(x)/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*c
osh(x)**2/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2)
+ 3*sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*co
sh(x)**2) + 2*cosh(x)**2/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b*
*2*cosh(x)**2), Eq(a, b)), (x*exp(4*x)*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)*
*2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*
exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b*
*2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - 2*x*exp(4*x)*sinh(x)*cosh(x)/(4*b
**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)
*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b
**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*exp(4*x)*c
osh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4
*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*co
sh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2)
+ 4*x*exp(2*x)*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sin
h(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*
b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b
**2*cosh(x)**2) - 4*x*exp(2*x)*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b
```

```
**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)
*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(
x)*cosh(x) + 4*b**2*cosh(x)**2) + x*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2
- 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp
(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*
sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*sinh(x)*cosh(x)/(4*b**2*exp(4*x)
*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2
+ 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)*
*2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*cosh(x)**2/(4*b**2*exp
(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x
)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sin
h(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 3*exp(4*x)*sinh(x)*
cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b
**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh
(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) -
2*exp(4*x)*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)
*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2
*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*
cosh(x)**2) + 4*exp(2*x)*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b*
*2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*
sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x
)*cosh(x) + 4*b**2*cosh(x)**2) + 3*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)
**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2
*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b
**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*cosh(x)**2/(4*b**2*exp(4*x)*si
nh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8
*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2
+ 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -(b*exp(2*x) - b)/(exp
(2*x) + 1))), (a**4*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b*
*3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + a**3*b*
x*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**
2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + a**2*b**2*x*sinh(x)/(a**5
*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x)
+ a*b**5*cosh(x) + b**6*sinh(x)) - 2*a**2*b**2*log(a*cosh(x)/b + sinh(x))*c
osh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b
**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) - a**2*b**2*cosh(x)/(a**5*b*co
sh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b
**5*cosh(x) + b**6*sinh(x)) + a*b**3*x*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*
sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6
*sinh(x)) - 2*a*b**3*log(a*cosh(x)/b + sinh(x))*sinh(x)/(a**5*b*cosh(x) + a
**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(
x) + b**6*sinh(x)) + b**4*x*sinh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2
*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)),
True))
```

Giac [A] time = 1.13107, size = 153, normalized size = 2.25

$$-\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2-2\,ab+b^2}+\frac{2\left(abe^{(2\,x)}+a^2-ab\right)}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a^2 - a*b)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.698 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=195

$$\frac{a\left(a^2+2b^2\right)\sinh(x)}{b^3\left(a^2-b^2\right)}+\frac{\left(2a^2+b^2\right)\cosh(x)}{b^4-a^2b^2}+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+$$

```
[Out] (3*a^2*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + ((2*a^2 + b^2)*Cosh[x])/(-(a^2*b^2) + b^4) + (a*(a^2 + 2*b^2)*Sinh[x])/(
b^3*(a^2 - b^2)) - a^3/(b^3*(a + b)^2*(1 - Tanh[x/2])) + a^3/((a - b)^2*b^3
*(1 + Tanh[x/2])) + (2*a^2*(a + b*Tanh[x/2]))/((a^2 - b^2)^2*(a + 2*b*Tanh[
x/2] + a*Tanh[x/2]^2))
```

Rubi [A] time = 1.22499, antiderivative size = 301, normalized size of antiderivative = 1.54, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4401, 2637, 2638, 6742, 638, 618, 204, 3100, 3074, 206}

$$\frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{b^3(a-b)^2\left(\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/
2)) + (2*a^2*b*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (2*a^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2
- b^2)^(5/2)) + Cosh[x]/b^2 - (3*a^2*Cosh[x])/(b^2*(a^2 - b^2)) - (2*a*Sinh
[x])/b^3 + (3*a^3*Sinh[x])/(b^3*(a^2 - b^2)) - a^3/(b^3*(a + b)^2*(1 - Tanh
[x/2])) + a^3/((a - b)^2*b^3*(1 + Tanh[x/2])) + (2*a^2*(a + b*Tanh[x/2]))/(
(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/2]^2))
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = i\int\left(\frac{2ia\cosh(x)}{b^3}-\frac{i\sinh(x)}{b^2}-\frac{ia^3\cosh^3(x)}{b^3(ia\cosh(x)+ib\sinh(x))^2}-\frac{3ia^2\cosh^2(x)}{b^3(a\cosh(x)+b\sinh(x}\right.$$

$$=-\frac{(2a)\int\cosh(x)\,dx}{b^3}+\frac{\left(3a^2\right)\int\frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{b^3}+\frac{a^3\int\frac{\cosh^3(x)}{(ia\cosh(x)+ib\sinh(x))^2}\,dx}{b^3}+\frac{\int \mathrm{si}}{}$$

$$=\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}-\frac{2a\sinh(x)}{b^3}+\frac{\left(2a^3\right)\mathrm{Subst}\left(\int\frac{\left(-1-x^2\right)^3}{\left(1-x^2\right)^2\left(a+2bx+ax^2\right)^2}\,dx,x,\tanh\left(\right.\right.}{b^3}$$

$$=\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)}+\frac{\left(2a^3\right)\mathrm{Subst}\left(\int\left(-\frac{1}{2(a+b)^2(-1+x)^2}\right.\right.}{}$$

$$=-\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)}$$

$$=-\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)}$$

$$=-\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{2a^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}}+\frac{\cosh(x)}{b^2}-\frac{3a^2\,\mathrm{c}}{b^2\left(a\right.}$$

$$=-\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{2a^2b\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{2a^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a}{\sqrt{}}\right.}{b\left(a^2-b^2\right)^{5/2}}$$

Mathematica [A] time = 0.4389, size = 205, normalized size = 1.05

$$\frac{a\sqrt{a-b}\left(a^2b+a^3+ab^2+b^3\right)\cosh^2(x)+a\left(a^2\sqrt{a-b}(a+b)-2b^2\sqrt{a-b}(a+b)\sinh^2(x)+6ab^2\sqrt{a+b}\sinh(x)\tan^{-1}\left(\frac{a}{\ldots}\right.\right.}{(a-b)^{5/2}(a+b)^3(a\cosh(x)+b\sinh(x)\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Sqrt[a - b]*(a^3 + a^2*b + a*b^2 + b^3)*Cosh[x]^2 - b*Cosh[x]*(-6*a^3*Sq
rt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^(3/
2)*(a + b)^2*Sinh[x]) + a*(a^2*Sqrt[a - b]*(a + b) + 6*a*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] - 2*Sqrt[a - b]*b
^2*(a + b)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] time = 0.072, size = 164, normalized size = 0.8

$$\frac{1}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,\frac{a^2\tanh(x/2)\,b}{(a-b)^2\,(a+b)^2\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)}+2\,\frac{a^3}{(a-b)^2\,(a+b)^2\left(a+2\,\tanh(x\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/(a-b)^2/(tanh(1/2*x)+1)+2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1
/2*x)^2)*tanh(1/2*x)*b+2*a^3/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2*
x)^2)+6*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2
*b)/(a^2-b^2)^(1/2))-1/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 2.0956, size = 3664, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")

```
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b
^4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)
^2 - 6*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b
^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b^
4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^
2 - 12*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^
2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/(
(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)
*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
```

```
3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)
*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15241, size = 235, normalized size = 1.21

$$\frac{6\,a^2b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}+\frac{5\,a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 6*a^2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sq
rt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(5*a^3*e^(2*x) + 3*a^2*b
*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^4
 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.699 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=64

$$\frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{a\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] (a*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + b/(
(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.0550593, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3155, 3074, 206}

$$\frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{a\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + b/(
(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{a\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{(ia)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x\right.}{a^2-b^2}$$

$$= \frac{a\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.135574, size = 124, normalized size = 1.94

$$\frac{2a^2\sqrt{a+b}\cosh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+2ab\sqrt{a+b}\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+b\sqrt{a-b}(a+b)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a
 - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b*S
inh[x]))
```

Maple [A] time = 0.056, size = 98, normalized size = 1.5

$$2\,\frac{1}{a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2}\left(\frac{b^2\tanh(x/2)}{a\left(a^2-b^2\right)}+\frac{b}{a^2-b^2}\right)+2\,\frac{a}{\left(a^2-b^2\right)^{3/2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tanh(1/2*x)+b/(a^2-b^2))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x
)^2)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92698, size = 1474, normalized size = 23.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh
(x)^2 + a^2 - a*b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) + 2*(a^2*b - b^3)*cosh(x) + 2*(a^2*b - b^3)*sinh(x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^
5)*sinh(x)^2), -2*(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) +
 (a^2 + a*b)*sinh(x)^2 + a^2 - a*b)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/
((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3
)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3
*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2
- 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15309, size = 97, normalized size = 1.52

$$\frac{2\,a\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{2\,be^x}{\left(a^2-b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*b*e^x/((a
^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.700 $\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b
^2)^2 + b/((a^2 - b^2)*(a + b*Tanh[x]))
```

Rubi [A] time = 0.126474, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3086, 3483, 3531, 3530}

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b
^2)^2 + b/((a^2 - b^2)*(a + b*Tanh[x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \int \frac{1}{(a+b\tanh(x))^2}\,dx\\ &= \frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}+\frac{\int \frac{a-b\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}-\frac{(2iab)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}\\ &= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.23265, size = 66, normalized size = 0.99

$$\frac{x\left(a^2+b^2\right)+\frac{b^2\left(b^2-a^2\right)\sinh(x)}{a(a\cosh(x)+b\sinh(x))}-2ab\log(a\cosh(x)+b\sinh(x))}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] + (b^2*(-a^2 + b^2)*Sinh[
x])/(a*(a*Cosh[x] + b*Sinh[x])))/((a - b)^2*(a + b)^2)
```

Maple [B] time = 0.067, size = 149, normalized size = 2.2

$$\frac{1}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,\frac{a\tanh(x/2)\,b^2}{(a-b)^2\,(a+b)^2\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)}+2\,\frac{b^4\tanh}{(a-b)^2\,(a+b)^2\,a\left(a+2\,\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/(a-b)^2*ln(tanh(1/2*x)+1)-2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)*b^2+2*b^4/(a-b)^2/(a+b)^2/a*tanh(1/2*x)/(a+2*tanh(1/2*
x)*b+a*tanh(1/2*x)^2)-2*a/(a-b)^2/(a+b)^2*b*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-1/(a+b)^2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.25549, size = 140, normalized size = 2.09

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}-\frac{2\,b^2}{a^4-2\,a^2b^2+b^4+\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}}+\frac{x}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*b^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

Fricas [B] time = 1.87638, size = 826, normalized size = 12.33

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\sinh(x}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+\left(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
```

```
*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

Sympy [A] time = 141.206, size = 2533, normalized size = 37.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), (x/a**2, Eq(b,
0)), (x*sinh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x) + 4*b**2*cos
h(x)**2) - 2*x*sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)
+ 4*b**2*cosh(x)**2) + x*cosh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cos
h(x) + 4*b**2*cosh(x)**2) - sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*sin
h(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*cosh(x)**2/(4*b**2*sinh(x)**2 - 8*b**
2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -b)), (x*sinh(x)**2/(4*b**2*s
inh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*sinh(x)*cosh(
x)/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*cos
h(x)**2/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) -
sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x
)**2) - 2*cosh(x)**2/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*c
osh(x)**2), Eq(a, b)), (x*exp(4*x)*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 -
 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(
2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*s
inh(x)*cosh(x) + 4*b**2*cosh(x)**2) - 2*x*exp(4*x)*sinh(x)*cosh(x)/(4*b**2*
exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cos
h(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*
sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*exp(4*x)*cosh(
x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**
2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x
)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - 4*
x*exp(2*x)*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)
*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2
*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*
cosh(x)**2) + 4*x*exp(2*x)*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*
```

```
exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sin
h(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*c
osh(x) + 4*b**2*cosh(x)**2) + x*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*
b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x
)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh
(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sin
h(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*
b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 +
 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*cosh(x)**2/(4*b**2*exp(4*x
)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2
 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)
**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - exp(4*x)*sinh(x)*cosh(x
)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*ex
p(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2
 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*exp(
4*x)*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(
x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2
*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x
)**2) + 4*exp(2*x)*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp
(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x
)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh
(x) + 4*b**2*cosh(x)**2) - sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*
b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x
)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh
(x)*cosh(x) + 4*b**2*cosh(x)**2) - 2*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2
 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*ex
p(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2
*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -(b*exp(2*x) - b)/(exp(2*x) +
1))), (a**3*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x)
- 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*x*sinh(x)/(
a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) +
 a*b**4*cosh(x) + b**5*sinh(x)) - 2*a**2*b*log(a*cosh(x)/b + sinh(x))*cosh(
x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(
x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*cosh(x)/(a**5*cosh(x) + a**4*b
*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**
5*sinh(x)) + a*b**2*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*
cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*l
og(a*cosh(x)/b + sinh(x))*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b
**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + b**3*x
*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3
*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - b**3*cosh(x)/(a**5*cosh(x) + a*
*4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) +
 b**5*sinh(x)), True))
```

Giac [A] time = 1.15571, size = 154, normalized size = 2.3

$$-\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2-2\,ab+b^2}+\frac{2\left(abe^{(2\,x)}+ab-b^2\right)}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a*b - b^2)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.701 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{3ab^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

```
[Out] (-3*a*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) + 1/((a + b)^2*(1 - Tanh[x/2])) - 1/((a - b)^2*(1 + Tanh[x/2])) - (2*b^3*
(a + b*Tanh[x/2]))/(a*(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/2]^2))
```

Rubi [A] time = 0.781512, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6742, 638, 618, 204}

$$-\frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{2b^4\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) -
(2*b^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b
^2)^(5/2)) + 1/((a + b)^2*(1 - Tanh[x/2])) - 1/((a - b)^2*(1 + Tanh[x/2]))
- (2*b^3*(a + b*Tanh[x/2]))/(a*(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/
2]^2))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
```

```
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(1-x^2\right)^2\left(a+2bx+ax^2\right)^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{1}{2(a+b)^2(-1+x)^2}+\frac{1}{2(a-b)^2(1+x)^2}-\frac{2b^3x}{a\left(-a^2+b^2\right)\left(a+2bx+ax^2\right)^2}+\right.\right.$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{\left(4b^3\right)\text{Subst}\left(\int \frac{x}{\left(a+2bx+ax^2\right)^2}\,dx, x, \text{t}\right.}{a\left(a^2-b^2\right)}$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\,\text{ta}\right.}$$

$$= -\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)}$$

$$= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}$$

Mathematica [A] time = 0.349257, size = 204, normalized size = 1.53

$$\frac{b\sqrt{a-b}\left(a^2b+a^3+ab^2+b^3\right)\sinh^2(x)-2a^2b\sqrt{a-b}(a+b)\cosh^2(x)-6ab^3\sqrt{a+b}\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+a\cosh}{(a-b)^{5/2}(a+b)^3(a\cosh(x)+b\sinh}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-(Sqrt[a - b]*b^3*(a + b)) - 2*a^2*Sqrt[a - b]*b*(a + b)*Cosh[x]^2 - 6*a*b
^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]
+ Sqrt[a - b]*b*(a^3 + a^2*b + a*b^2 + b^3)*Sinh[x]^2 + a*Cosh[x]*(-6*a*b^2
*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^
(3/2)*(a + b)^2*Sinh[x]))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] time = 0.072, size = 167, normalized size = 1.3

$$-\frac{1}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\frac{b^4\tanh(x/2)}{(a-b)^2(a+b)^2a\left(a+2\tanh(x/2)b+a(\tanh(x/2))^2\right)}-2\frac{b}{(a-b)^2(a+b)^2\left(a+2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a-b)^2/(tanh(1/2*x)+1)-2*b^4/(a-b)^2/(a+b)^2/a*tanh(1/2*x)/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)-2*b^3/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-6*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))-1/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0218, size = 3667, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 + 6*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a*
b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 +
b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 - 12*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)
^2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a
*b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(
a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))
/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3
```

```
+ 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b
- 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
2)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14985, size = 235, normalized size = 1.77

$$-\frac{6\,ab^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}-\frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+5\,b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -6*a*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x) + 3*a^2*b*
e^(2*x) + 3*a*b^2*e^(2*x) + 5*b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^
4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.702 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=19

$$\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

```
[Out] Tanh[x]^2/(2*a*(a + b*Tanh[x])^2)
```

Rubi [A] time = 0.0321619, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 37}

$$\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] Tanh[x]^2/(2*a*(a + b*Tanh[x])^2)
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\text{Subst}\left(\int \frac{x}{(a-ibx)^3}\,dx, x, i\tanh(x)\right)$$
$$= \frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

Mathematica [B] time = 0.114617, size = 54, normalized size = 2.84

$$-\frac{a^2+ab\sinh(2x)+b^2\cosh(2x)-b^2}{2a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -(a^2 - b^2 + b^2*Cosh[2*x] + a*b*Sinh[2*x])/(2*a*(a - b)*(a + b)*(a*Cosh[x
] + b*Sinh[x])^2)
```

Maple [A] time = 0.068, size = 31, normalized size = 1.6

$$2\,\frac{(\tanh(x/2))^2}{a\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] 2/a*tanh(1/2*x)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^2
```

Maxima [B] time = 1.26658, size = 225, normalized size = 11.84

$$-\frac{2\,(a-b)e^{(-2\,x)}}{a^4-2\,a^2b^2+b^4+2\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}+\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)e^{(-4\,x)}} - \frac{}{a^4-2\,a^2b^2+b^4+2\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b
^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) - 2*b/
(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

Fricas [B] time = 1.78601, size = 518, normalized size = 27.26

$$-\frac{}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh\left(x\right)^3+3\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh\left(x\right)\sinh\left(x\right)^2+\left(a^4+4\,a^3b+6\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + (a + 2*b)*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16817, size = 68, normalized size = 3.58

$$-\frac{2\left(ae^{(2\,x)}+be^{(2\,x)}-b\right)}{\left(a^2+2\,ab+b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*x) + b*e^(2*x) - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

3.703 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=104

$$-\frac{bx\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{a}{2\left(a^2-b^2\right)(a\coth(x)+b)^2}+\frac{a\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

```
[Out] -((b*(3*a^2 + b^2)*x)/(a^2 - b^2)^3) - a/(2*(a^2 - b^2)*(b + a*Coth[x])^2)
+ (2*a*b)/((a^2 - b^2)^2*(b + a*Coth[x])) + (a*(a^2 + 3*b^2)*Log[a*Cosh[x]
+ b*Sinh[x]])/(a^2 - b^2)^3
```

Rubi [A] time = 0.237618, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3085, 3483, 3529, 3531, 3530}

$$-\frac{bx\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{a}{2\left(a^2-b^2\right)(a\coth(x)+b)^2}+\frac{a\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -((b*(3*a^2 + b^2)*x)/(a^2 - b^2)^3) - a/(2*(a^2 - b^2)*(b + a*Coth[x])^2)
+ (2*a*b)/((a^2 - b^2)^2*(b + a*Coth[x])) + (a*(a^2 + 3*b^2)*Log[a*Cosh[x]
+ b*Sinh[x]])/(a^2 - b^2)^3
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = i\int \frac{1}{(-ib-ia\coth(x))^3}\,dx$$

$$= -\frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{i\int \frac{-ib+ia\coth(x)}{(-ib-ia\coth(x))^2}\,dx}{a^2-b^2}$$

$$= -\frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{i\int \frac{-a^2-b^2+2ab\coth(x)}{-ib-ia\coth(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= -\frac{b\left(3a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{\left(ia\left(a^2+\right.\right.}{}$$

$$= -\frac{b\left(3a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{a\left(a^2+3\right.}{}$$

Mathematica [A] time = 0.6463, size = 117, normalized size = 1.12

$$-\frac{bx\left(3a^2+b^2\right)}{(a-b)^3(a+b)^3}+\frac{\left(a^3+3ab^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}+\frac{a^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2}+\frac{3}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -((b*(3*a^2 + b^2)*x)/((a - b)^3*(a + b)^3)) + ((a^3 + 3*a*b^2)*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 + a^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*
Sinh[x])^2) + (3*a*b*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

Maple [B] time = 0.092, size = 404, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] -1/(a-b)^3*ln(tanh(1/2*x)+1)+4*a^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)^2*tanh(1/2*x)^3*b-4*a^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*ta
nh(1/2*x)^2)^2*tanh(1/2*x)^3*b^3-2*a^5/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)^2*tanh(1/2*x)^2+12*a^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)^2*tanh(1/2*x)^2*b^2-10*a/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b
+a*tanh(1/2*x)^2)^2*tanh(1/2*x)^2*b^4+4*a^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)*b-4*a^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)
*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)*b^3+a^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)+3*a/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)
^2)*b^2-1/(a+b)^3*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.33858, size = 390, normalized size = 3.75

$$\frac{\left(a^3+3\,ab^2\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{2\left(3\,a^3b-\right.}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\,a^5b^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] (a^3 + 3*a*b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 2*(3*a^3*b + 3*a^2*b^2 + (a^4 + 2*a^3*b - 3*a^2*b^2)*e^(-2*x))/(a
^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^
7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Fricas [B] time = 2.03017, size = 2854, normalized size = 27.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
 + 6*a^4*b - 12*a^3*b^2 + 6*a^2*b^3 - 2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^
3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 -
2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^3 - 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
a*b^4 + b^5)*x - (a^5 - 2*a^4*b + 4*a^3*b^2 - 6*a^2*b^3 + 3*a*b^4 + (a^5 +
2*a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + 4
*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + 4*a^3*
b^2 + 6*a^2*b^3 + 3*a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x
)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 + 3*(a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*b
^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*
b^3 + 3*a*b^4)*cosh(x)^3 + (a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x))*lo
g(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 - (a^5 - 3*a^4*b - a^3*b^2
 + 3*a^2*b^3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*c
osh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2
*b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^
5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2
*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sin
h(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a
^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
```

```
 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2
+ 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b
^7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(
x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14466, size = 339, normalized size = 3.26

$$\frac{\left(a^3 + 3\,ab^2\right)\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6} - \frac{x}{a^3 - 3\,a^2b + 3\,ab^2 - b^3} - \frac{3\,a^4e^{(4\,x)} + 3\,a^3be^{(4\,x)} + 9\,a^2b^2e^{(4\,x)} + 9\,ab^3e^{(4\,x)} + 2\,a}{2\left(a^5 - a^4b - 2\,a^3b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a*b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6) - x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/2*(3*a^4*e^(4*x) +
 3*a^3*b*e^(4*x) + 9*a^2*b^2*e^(4*x) + 9*a*b^3*e^(4*x) + 2*a^4*e^(2*x) + 10
*a^3*b*e^(2*x) + 6*a^2*b^2*e^(2*x) - 18*a*b^3*e^(2*x) + 3*a^4 + 3*a^3*b - 1
5*a^2*b^2 + 9*a*b^3)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*(
a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.704 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=19

$$-\frac{\coth^2(x)}{2b(a\coth(x)+b)^2}$$

```
[Out] -Coth[x]^2/(2*b*(b + a*Coth[x])^2)
```

Rubi [A] time = 0.030686, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3088, 37}

$$-\frac{\coth^2(x)}{2b(a\coth(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -Coth[x]^2/(2*b*(b + a*Coth[x])^2)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = i\,\mathrm{Subst}\left(\int \frac{x}{(-ib+ax)^3}\,dx, x, -i\coth(x)\right)$$
$$= -\frac{\coth^2(x)}{2b(b+a\coth(x))^2}$$

Mathematica [B] time = 0.0576048, size = 40, normalized size = 2.11

$$\frac{a\sinh(2x)+b\cosh(2x)}{2(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (b*Cosh[2*x] + a*Sinh[2*x])/(2*(a - b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^2)
```

Maple [B] time = 0.065, size = 55, normalized size = 2.9

$$-2\,\frac{1}{\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)^2}\left(-\frac{(\tanh(x/2))^3}{a}-\frac{(\tanh(x/2))^2\,b}{a^2}-\frac{\tanh(x/2)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] -2*(-1/a*tanh(1/2*x)^3-1/a^2*b*tanh(1/2*x)^2-1/a*tanh(1/2*x))/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)^2
```

Maxima [B] time = 1.26434, size = 225, normalized size = 11.84

$$\frac{2\,(a-b)e^{(-2\,x)}}{a^4-2\,a^2b^2+b^4+2\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}+\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)e^{(-4\,x)}}+\frac{}{a^4-2\,a^2b^2+b^4+2\left(a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^
4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) + 2*a/(
a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

Fricas [B] time = 1.72395, size = 518, normalized size = 27.26

$$-\frac{}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh\left(x\right)^3+3\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh\left(x\right)\sinh\left(x\right)^2+\left(a^4+4\,a^3b+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*((2*a + b)*cosh(x) + b*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14773, size = 65, normalized size = 3.42

$$-\frac{2\left(ae^{(2\,x)}+be^{(2\,x)}+a\right)}{\left(a^2+2\,ab+b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*x) + b*e^(2*x) + a)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

3.705 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=104

$$\frac{ax\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh
[x]])/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*(a + b*Tanh[x])^2) + (2*a*b)/((a^2 -
 b^2)^2*(a + b*Tanh[x]))
```

Rubi [A] time = 0.200515, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3086, 3483, 3529, 3531, 3530}

$$\frac{ax\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh
[x]])/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*(a + b*Tanh[x])^2) + (2*a*b)/((a^2 -
 b^2)^2*(a + b*Tanh[x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx &= \int \frac{1}{(a+b\tanh(x))^3}\,dx\\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{\int \frac{a-b\tanh(x)}{(a+b\tanh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{\int \frac{a^2+b^2-2ab\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}\\
&= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{\left(ib\left(3a^2+\right.\right.}{}\\
&= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}
\end{aligned}$$

Mathematica [A] time = 0.85633, size = 119, normalized size = 1.14

$$\frac{ax\left(a^2+3b^2\right)}{(a-b)^3(a+b)^3}+\frac{\left(-3a^2b-b^3\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}-\frac{b^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2}-\frac{}{(a-b)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/((a - b)^3*(a + b)^3) + ((-3*a^2*b - b^3)*Log[a*Cosh[x]
 + b*Sinh[x]])/(a^2 - b^2)^3 - b^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*Si
nh[x])^2) - (3*b^2*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

Maple [B] time = 0.085, size = 494, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] 1/(a-b)^3*ln(tanh(1/2*x)+1)-6*b^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tanh
(1/2*x)^2)^2*a^3*tanh(1/2*x)^3+8*b^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*t
anh(1/2*x)^2)^2*a*tanh(1/2*x)^3-2*b^6/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*
tanh(1/2*x)^2)^2/a*tanh(1/2*x)^3-10*b^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+
a*tanh(1/2*x)^2)^2*a^2*tanh(1/2*x)^2+12*b^5/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)^2-2*b^7/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)
*b+a*tanh(1/2*x)^2)^2/a^2*tanh(1/2*x)^2-6*b^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)^2*a^3*tanh(1/2*x)+8*b^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)^2*a*tanh(1/2*x)-2*b^6/(a-b)^3/(a+b)^3/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)^2/a*tanh(1/2*x)-3*b/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*
x)*b+a*tanh(1/2*x)^2)*a^2-b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1
/2*x)^2)-1/(a+b)^3*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.34003, size = 394, normalized size = 3.79

$$-\frac{\left(3\,a^2b+b^3\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{2\left(3\,a^2\right.}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\,a^5b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)*e^(-2*x))/(
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b
^7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
 3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Fricas [B] time = 2.00484, size = 2853, normalized size = 27.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x)
^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
+ 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5
 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 + 2
*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a
*b^4 + b^5)*x - (3*a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (3*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^4 + 2*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)
^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2*
a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2
*a*b^4 + b^5)*cosh(x)^3 + (3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh(x))*log
(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 + (3*a^3*b^2 - a^2*b^3 - 3*
a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*co
sh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5
*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2*
a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sinh
```

```
(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a^
8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
- 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2 +
 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^
7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x
))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)

[Out] Timed out
```

Giac [B] time = 1.17902, size = 339, normalized size = 3.26

$$-\frac{\left(3\,a^{2}b+b^{3}\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{x}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{9\,a^{3}be^{(4\,x)}+9\,a^{2}b^{2}e^{(4\,x)}+3\,ab^{3}e^{(4\,x)}+3\,b^{4}e^{(4\,x)}+\cdots}{2\left(a^{5}-a^{4}b-2\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")

[Out] -(3*a^2*b + b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6) + x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/2*(9*a^3*b*e^(4*x
) + 9*a^2*b^2*e^(4*x) + 3*a*b^3*e^(4*x) + 3*b^4*e^(4*x) + 18*a^3*b*e^(2*x)
- 6*a^2*b^2*e^(2*x) - 10*a*b^3*e^(2*x) - 2*b^4*e^(2*x) + 9*a^3*b - 15*a^2*b
^2 + 3*a*b^3 + 3*b^4)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.706 $\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=72

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0891185, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3109, 2637, 2638, 3074, 206}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= \frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{(iab)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\ &= \frac{ab\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.202041, size = 79, normalized size = 1.1

$$\frac{b\sinh(x)}{b^2-a^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*
(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

Maple [A] time = 0.033, size = 92, normalized size = 1.3

$$2\,{\frac {ab}{ \left(a+b \right) \left(a-b \right) \sqrt {{a}^{2}-{b}^{2}}}\arctan \left(1/2\,{\frac {2\,a\tanh \left(x/2 \right) +2\,b}{\sqrt {{a}^{2}-{b}^{2}}}} \right) }-4\,{\frac {1}{ \left(4\,a+4\,b \right) \left(\tanh \left(x/2 \right) -1 \right) }}+4\,{\frac {1}{ \left(4\,a-4\,b \right) \left(\tanh \left(x/2 \right) +1 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))-4/(4*a+4*b)/(tanh(1/2*x)-1)+4/(4*a-4*b)/(tanh(1/2*x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94209, size = 1103, normalized size = 15.32

$$\left[\frac{a^{3} + a^{2} b - a b^{2} - b^{3} + \left(a^{3} - a^{2} b - a b^{2} + b^{3}\right) \cosh\left(x\right)^{2} + 2 \, \left(a^{3} - a^{2} b - a b^{2} + b^{3}\right) \cosh\left(x\right) \sinh\left(x\right) + \left(a^{3} - a^{2} b - a b^{2} + b^{3}\right)}{2 \, \left(\left(a^{4} - 2 \, a^{2} b^{2} + b^{4}\right) \cosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)
```

```
*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x
) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^
2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14099, size = 81, normalized size = 1.12

$$\frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-x
)/(a - b) + 1/2*e^x/(a + b)
```

3.707 $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{ax}{2\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\sinh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) - (a*x)/(2*(a^2 - b^2)) + (a^2*b*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) - (b*Sinh[
x]^2)/(2*(a^2 - b^2))
```

Rubi [A] time = 0.16024, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2564, 30, 2635, 8, 3097, 3133}

$$-\frac{ax}{2\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\sinh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) - (a*x)/(2*(a^2 - b^2)) + (a^2*b*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) - (b*Sinh[
x]^2)/(2*(a^2 - b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} + \frac{\left(ia^2b\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a\int 1\,dx}{2\left(a^2-b^2\right)} + \frac{b\,\text{Subst}}{}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} - \frac{ax}{2\left(a^2-b^2\right)} + \frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2} + \frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} - \frac{b\sinh^2}{2\left(a^2-\right.}$$

Mathematica [A] time = 0.229039, size = 73, normalized size = 0.72

$$\frac{\left(b^3-a^2b\right)\cosh(2x)+a\left(-2x\left(a^2+b^2\right)+\left(a^2-b^2\right)\sinh(2x)+4ab\log(a\cosh(x)+b\sinh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] + b
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.044, size = 145, normalized size = 1.4

$$-4\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)^2}+8\,\frac{1}{(16\,a-16\,b)\,(\tanh(x/2)+1)}-\frac{a}{2\,(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{a^2b}{(a-b)^2\,(a+b)^2}\ln\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/(8*a-8*b)/(tanh(1/2*x)+1)^2+8/(16*a-16*b)/(tanh(1/2*x)+1)-1/2/(a-b)^2*ln
(tanh(1/2*x)+1)*a+a^2*b/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^
2)+4/(8*a+8*b)/(tanh(1/2*x)-1)^2+8/(16*a+16*b)/(tanh(1/2*x)-1)+1/2/(a+b)^2*
ln(tanh(1/2*x)-1)*a
```

Maxima [A] time = 1.2138, size = 112, normalized size = 1.1

$$\frac{a^2 b \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^4 - 2a^2b^2 + b^4} - \frac{ax}{2\left(a^2 + 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^2
 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 1.84974, size = 826, normalized size = 8.1

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)\sinh(x)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh(x)^4 - 4\left(a^3 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12046, size = 136, normalized size = 1.33

$$\frac{a^2b\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}-\frac{ax}{2\left(a^2-2ab+b^2\right)}+\frac{\left(2ae^{(2x)}-a+b\right)e^{(-2x)}}{8\left(a^2-2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
+ b^2) + 1/8*e^(2*x)/(a + b)
```

3.708 $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rubi [A] time = 0.202991, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```


```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{\left(a^3b\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx\right.}{a^2-b^2}$$

$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ia^3b\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^3b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} -$$

Mathematica [A] time = 1.10541, size = 180, normalized size = 1.31

$$\frac{-3a\sqrt{a-b}\sqrt{a+b}\left(3a^2+b^2\right)\cosh(x)+a\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)\cosh(3x)+b\left(3\sqrt{a-b}\sqrt{a+b}\left(5a^2-b^2\right)\sinh(x)-\sqrt{a}\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a
+ b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

Maple [A] time = 0.049, size = 166, normalized size = 1.2

$$-8\,\frac{1}{(16\,a-16\,b)\,(\tanh(x/2)+1)^2} + \frac{16}{48\,a-48\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \frac{a}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - 2\,\frac{a^3b}{(a-b)^2\,(a+b)^2\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/(tanh(1/2*x)+1)^3/(16*a-16*b)-1/2*a/(
a-b)^2/(tanh(1/2*x)+1)-2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)-8/
(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2*a/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13573, size = 4208, normalized size = 30.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*
```

```
a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5
*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(
x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 - 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - 5*
a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5
+ 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cos
h(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15681, size = 220, normalized size = 1.61

$$-\frac{2\,a^3b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-12\,abe^x-3\,b^2e}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^x
- 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.709 $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=102

$$\frac{a^2bx}{(a^2-b^2)^2}-\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 - (b*x)/(2*(a^2 - b^2)) - (a*b^2*Log[a*Cosh[x] + b*
Sinh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^
2)/(2*(a^2 - b^2))
```

Rubi [A] time = 0.162841, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2635, 8, 2564, 30, 3098, 3133}

$$\frac{a^2bx}{(a^2-b^2)^2}-\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 - (b*x)/(2*(a^2 - b^2)) - (a*b^2*Log[a*Cosh[x] + b*
Sinh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^
2)/(2*(a^2 - b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int\cosh(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b\int\cosh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int\frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$
$$= \frac{a^2bx}{\left(a^2-b^2\right)^2} - \frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} - \frac{\left(iab^2\right)\int\frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}(\int x\,dx, x, i\sin$$
$$= \frac{a^2bx}{\left(a^2-b^2\right)^2} - \frac{bx}{2\left(a^2-b^2\right)} - \frac{ab^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2} - \frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} + \frac{a\sinh^2}{2\left(a^2-\right.}$$

Mathematica [A] time = 0.223209, size = 73, normalized size = 0.72

$$\frac{a\left(a^2-b^2\right)\cosh(2x)+b\left(2x\left(a^2+b^2\right)+\left(b^2-a^2\right)\sinh(2x)-4ab\log(a\cosh(x)+b\sinh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 - b^2)*Cosh[2*x] + b*(2*(a^2 + b^2)*x - 4*a*b*Log[a*Cosh[x] + b*Sin
h[x]] + (-a^2 + b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.04, size = 146, normalized size = 1.4

$$-4\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)} + 2\,\frac{1}{(4\,a-4\,b)\,(\tanh(x/2)+1)^2} + \frac{b}{2\,(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{ab^2}{(a-b)^2\,(a+b)^2}\ln\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/(8*a-8*b)/(tanh(1/2*x)+1)+2/(4*a-4*b)/(tanh(1/2*x)+1)^2+1/2/(a-b)^2*ln(t
anh(1/2*x)+1)*b-a*b^2/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)
+2/(4*a+4*b)/(tanh(1/2*x)-1)^2+4/(8*a+8*b)/(tanh(1/2*x)-1)-1/2/(a+b)^2*ln(t
anh(1/2*x)-1)*b
```

Maxima [A] time = 1.17835, size = 113, normalized size = 1.11

$$-\frac{ab^2 \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^4 - 2a^2b^2 + b^4} + \frac{bx}{2\left(a^2 + 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)} + \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*b*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 1.83732, size = 826, normalized size = 8.1

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)\sinh(x)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh(x)^4 + 4\left(a^2b + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^2*b + 2*a*
b^2 + b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^2*b + 2*a*b^2 + b^3)*x)*sinh(x)^2 - 8*(a*b^2*co
sh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^2*b + 2*a*b^2 + b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

[Out] Timed out

Giac [A] time = 1.14789, size = 138, normalized size = 1.35

$$-\frac{ab^2 \log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{bx}{2\left(a^2 - 2ab + b^2\right)} - \frac{\left(2be^{(2x)} - a + b\right)e^{(-2x)}}{8\left(a^2 - 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/
2*b*x/(a^2 - 2*a*b + b^2) - 1/8*(2*b*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
 + b^2) + 1/8*e^(2*x)/(a + b)
```

3.710 $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=122

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^2b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] (a^2*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*S
inh[x])/(a^2 - b^2)^2 + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rubi [A] time = 0.228581, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3109, 2565, 30, 2564, 2637, 2638, 3074, 206}

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^2b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```


```
[Out] (a^2*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*S
inh[x])/(a^2 - b^2)^2 + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{(a^2b)\int \sinh(x)\,dx}{(a^2-b^2)^2} - \frac{(ab^2)\int \cosh(x)\,dx}{(a^2-b^2)^2} + \frac{(a^2b^2)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{(ia)\,\text{Subst}\left(\int x^{\cdots}\right.}{a^2}$$

$$= \frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)} + \frac{(ia^2b^2)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -\cdots\right.}{(a^2-b^2)^{\cdots}}$$

$$= \frac{a^2b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}$$

Mathematica [A] time = 0.979168, size = 179, normalized size = 1.47

$$\frac{3b\sqrt{a-b}\sqrt{a+b}\left(3a^2+b^2\right)\cosh(x) - b\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)\cosh(3x) + a\left(-3\sqrt{a-b}\sqrt{a+b}\left(a^2+3b^2\right)\sinh(x) + \sqrt{a-\cdots}\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (3*Sqrt[a - b]*b*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] - Sqrt[a - b]*b*Sqrt[a +
 b]*(a^2 - b^2)*Cosh[3*x] + a*(24*a*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] - 3*Sqrt[a - b]*Sqrt[a + b]*(a^2 + 3*b^2)*Sinh[x] + Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

Maple [A] time = 0.046, size = 168, normalized size = 1.4

$$-\frac{8}{24a-24b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + 4\frac{1}{(8a-8b)(\tanh(x/2)+1)^2} + \frac{b}{2(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + 2\frac{a^2b^2}{(a-b)^2(a+b)^2\sqrt{a^2-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -8/3/(tanh(1/2*x)+1)^3/(8*a-8*b)+4/(8*a-8*b)/(tanh(1/2*x)+1)^2+1/2*b/(a-b)^
2/(tanh(1/2*x)+1)+2*a^2*b^2/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
```

```
*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-8/3/(tanh(1/2*x)-1)^3/(8*a+8*b)-4/(8*a+8
*b)/(tanh(1/2*x)-1)^2-1/2*b/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15093, size = 4230, normalized size = 34.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3
*a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x
))*sinh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*co
sh(x)^2 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*
b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^2*b
^2*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 +
a^2*b^2*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a
^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a
```

```
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*
b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*
b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^
5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 +
 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^
5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x))*sinh(x)^3 + 3
*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 3*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 +
2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^2*b^2*cosh(x)^3 + 3
*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 + a^2*b^2*sinh(x)^
3)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x
))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15182, size = 215, normalized size = 1.76

$$\frac{2\,a^2b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-3\,a^2e^x+3\,b^2e^x}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 3*a^2*e^x
 + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.711 $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=194

$$\frac{bx}{8\left(a^2-b^2\right)}-\frac{a^2bx}{2\left(a^2-b^2\right)^2}-\frac{a^2b^3x}{\left(a^2-b^2\right)^3}+\frac{a\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ab^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}+\frac{b\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}+\frac{a^2b\sin}{2(}$$

```
[Out] -((a^2*b^3*x)/(a^2 - b^2)^3) - (a^2*b*x)/(2*(a^2 - b^2)^2) + (b*x)/(8*(a^2
- b^2)) + (a^3*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*b*Cosh[
x]*Sinh[x])/(2*(a^2 - b^2)^2) + (b*Cosh[x]*Sinh[x])/(8*(a^2 - b^2)) - (b*Co
sh[x]^3*Sinh[x])/(4*(a^2 - b^2)) - (a*b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (a
*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rubi [A] time = 0.34237, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3109, 2568, 2635, 8, 2564, 30, 3097, 3133}

$$\frac{bx}{8\left(a^2-b^2\right)}-\frac{a^2bx}{2\left(a^2-b^2\right)^2}-\frac{a^2b^3x}{\left(a^2-b^2\right)^3}+\frac{a\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ab^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}+\frac{b\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}+\frac{a^2b\sin}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```



```
[Out] -((a^2*b^3*x)/(a^2 - b^2)^3) - (a^2*b*x)/(2*(a^2 - b^2)^2) + (b*x)/(8*(a^2
- b^2)) + (a^3*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*b*Cosh[
x]*Sinh[x])/(2*(a^2 - b^2)^2) + (b*Cosh[x]*Sinh[x])/(8*(a^2 - b^2)) - (b*Co
sh[x]^3*Sinh[x])/(4*(a^2 - b^2)) - (a*b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (a
*Sinh[x]^4)/(4*(a^2 - b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
```

```
), 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh(x)\sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{b\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{\left(a^2b\right)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{}{a\cosh}}{\left(a^2-\right.}$$

$$= -\frac{a^2b^3x}{\left(a^2-b^2\right)^3} + \frac{a^2b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{b\cosh(x)\sinh(x)}{8\left(a^2-b^2\right)} - \frac{b\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{a\sinh^4(x}{4\left(a^2-b^2\right.}$$

$$= -\frac{a^2b^3x}{\left(a^2-b^2\right)^3} - \frac{a^2bx}{2\left(a^2-b^2\right)^2} + \frac{bx}{8\left(a^2-b^2\right)} + \frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3} + \frac{a^2b\cosh(x)}{2\left(a^2-\right.}$$

Mathematica [A] time = 0.602485, size = 128, normalized size = 0.66

$$\frac{a\left(a^2-b^2\right)^2\cosh(4x) - 4a\left(a^4-b^4\right)\cosh(2x) - b\left(-8a^2\left(a^2-b^2\right)\sinh(2x) + \left(a^2-b^2\right)^2\sinh(4x) + 4\left(6a^2b^2x - 8a^3b\log(a\right.\right.}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-4*a*(a^4 - b^4)*Cosh[2*x] + a*(a^2 - b^2)^2*Cosh[4*x] - b*(4*(3*a^4*x + 6
*a^2*b^2*x - b^4*x - 8*a^3*b*Log[a*Cosh[x] + b*Sinh[x]]) - 8*a^2*(a^2 - b^2
)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

Maple [A] time = 0.058, size = 321, normalized size = 1.7

$$4\,\frac{1}{(16\,a-16\,b)\left(\tanh(x/2)+1\right)^4} - 16\,\frac{1}{(32\,a-32\,b)\left(\tanh(x/2)+1\right)^3} + \frac{a}{8\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{b}{8\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 4/(16*a-16*b)/(tanh(1/2*x)+1)^4-16/(32*a-32*b)/(tanh(1/2*x)+1)^3+1/8*a/(a-b
)^2/(tanh(1/2*x)+1)+1/8*b/(a-b)^2/(tanh(1/2*x)+1)+1/8/(a-b)^2/(tanh(1/2*x)+
1)^2*a-3/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-3/8*b/(a-b)^3*ln(tanh(1/2*x)+1)*a+1/
8*b^2/(a-b)^3*ln(tanh(1/2*x)+1)+a^3*b^2/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*
b+a*tanh(1/2*x)^2)+4/(16*a+16*b)/(tanh(1/2*x)-1)^4+16/(32*a+32*b)/(tanh(1/2
*x)-1)^3+1/8/(a+b)^2/(tanh(1/2*x)-1)^2*a+3/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-1/
8*a/(a+b)^2/(tanh(1/2*x)-1)+1/8*b/(a+b)^2/(tanh(1/2*x)-1)+3/8*b/(a+b)^3*ln(
tanh(1/2*x)-1)*a+1/8*b^2/(a+b)^3*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.27258, size = 207, normalized size = 1.07

$$\frac{a^3b^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\left(3ab+b^2\right)x}{8\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(4ae^{(-2x)}-a-b\right)e^{(4x)}}{64\left(a^2+2ab+b^2\right)}-\frac{4ae^{(-2x)}-(a-b)e^{(-4x)}}{64\left(a^2-2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^3*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- 1/8*(3*a*b + b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*a*e^(-2*x)
- a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*a*e^(-2*x) - (a - b)*e^(-4*x
))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 1.94103, size = 2612, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(a^5 - 2*a^4*b
+ 2*a^2*b^3 - a*b^4)*cosh(x)^6 - 4*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4 - 7*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^5 - 2*a^4*b + 2*a^2*b
^3 - a*b^4)*cosh(x))*sinh(x)^5 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^
```

```
4 + b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 - 30*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^2 - 4*(3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh
(x)^3 - 4*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x))*sinh(x)^3 - 4*
(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 - a^5 - 2*a^4*b + 2*a^2*b^3 + a*b^4
 - 15*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^4 - 12*(3*a^4*b + 8*a^3*b
^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^2)*sinh(x)^2 + 64*(a^3*b^2*cosh(x)^4 + 4*a^
3*b^2*cosh(x)^3*sinh(x) + 6*a^3*b^2*cosh(x)^2*sinh(x)^2 + 4*a^3*b^2*cosh(x)
*sinh(x)^3 + a^3*b^2*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
- 3*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^5 - 4*(3*a^4*b + 8*a^3*b^2
+ 6*a^2*b^3 - b^5)*x*cosh(x)^3 - (a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14632, size = 269, normalized size = 1.39

$$\frac{a^3b^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,ab-b^2\right)x}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(18\,abe^{(4x)}-6\,b^2e^{(4x)}-4\,a^2e^{(2x)}+4\,abe^{(2x)}+a^2-2\,ab\right.}{64\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^3*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 1/8*(3*a*b - b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(18*a
```

```
*b*e^(4*x) - 6*b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*a*b*e^(2*x) + a^2 - 2*a*b +
b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x)
 - 4*a*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.712 $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{ab^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -((a*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 + (a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a^2*b*
Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x])/(a^2 - b^2) - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rubi [A] time = 0.195335, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{ab^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 + (a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a^2*b*
Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x])/(a^2 - b^2) - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^3\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\mathrm{Subst}\left(\int x^2\,dx, x, \mathrm{c}\right.}{a^2-b^2}$$

$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(iab^3\right)\mathrm{Subst}\left(\int \frac{}{a^2}\right.}{}$$

$$= -\frac{ab^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b}{3}$$

Mathematica [A] time = 1.13356, size = 167, normalized size = 1.22

$$\frac{1}{12}\left(\frac{3b\left(a^2+3b^2\right)\sinh(x)}{(a-b)^2(a+b)^2} + \frac{3a\left(a^2-5b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{a^2b\sinh(3x)}{(a-b)^2(a+b)^2} + \frac{b^3\sinh(3x)}{(a-b)^2(a+b)^2} - \frac{24ab^3\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{}{(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((-24*a*b^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(
5/2)*(a + b)^(5/2)) + (3*a*(a^2 - 5*b^2)*Cosh[x])/((a - b)^2*(a + b)^2) + (
a*Cosh[3*x])/((a - b)*(a + b)) + (3*b*(a^2 + 3*b^2)*Sinh[x])/((a - b)^2*(a
+ b)^2) - (a^2*b*Sinh[3*x])/((a - b)^2*(a + b)^2) + (b^3*Sinh[3*x])/((a - b
)^2*(a + b)^2))/12
```

Maple [A] time = 0.045, size = 200, normalized size = 1.5

$$-2\,\frac{1}{(4\,a-4\,b)\,(\tanh(x/2)+1)^2} + \frac{4}{12\,a-12\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{a}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{b}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -2/(4*a-4*b)/(tanh(1/2*x)+1)^2+4/3/(tanh(1/2*x)+1)^3/(4*a-4*b)+1/2*a/(a-b)^
2/(tanh(1/2*x)+1)-b/(a-b)^2/(tanh(1/2*x)+1)-2*a*b^3/(a-b)^2/(a+b)^2/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-4/3/(tanh(1/2*x)
-1)^3/(4*a+4*b)-2/(4*a+4*b)/(tanh(1/2*x)-1)^2-1/2*a/(a+b)^2/(tanh(1/2*x)-1)
-b/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14635, size = 4208, normalized size = 30.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5)*cosh(x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a
*b^4 - 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x
))*sinh(x)^3 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*co
sh(x)^2 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b
- 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a*b^3
*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*
```

```
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^
5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4
*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(
x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5 + 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 + a^4*
b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(a^5 -
a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x)^2 + 3*(a^5 - a^4*b
 - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a*b^3*cosh(x)^3 + 3*a*b^3*cos
h(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^5 + a^4*b - 6
*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4*b - 6*a^3*b^
2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15688, size = 220, normalized size = 1.61

$$-\frac{2\,ab^{3}\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{\left(3\,ae^{(2\,x)}-9\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,\left(a^{2}-2\,ab+b^{2}\right)}+\frac{a^{2}e^{(3\,x)}+2\,abe^{(3\,x)}+b^{2}e^{(3\,x)}+3\,a^{2}e^{x}+12\,abe^{x}+9\,b^{2}}{24\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) - 9*b*e^(2*x) + a - b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 3*a^2*e^x
+ 12*a*b*e^x + 9*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.713 $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=194

$$-\frac{ax}{8\left(a^2-b^2\right)}-\frac{ab^2x}{2\left(a^2-b^2\right)^2}+\frac{a^3b^2x}{\left(a^2-b^2\right)^3}+\frac{a^2b\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\cosh^4(x)}{4\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}-\frac{a\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}-\frac{ab^2\,\mathrm{si}}{2}$$

```
[Out] (a^3*b^2*x)/(a^2 - b^2)^3 - (a*b^2*x)/(2*(a^2 - b^2)^2) - (a*x)/(8*(a^2 - b
^2)) - (b*Cosh[x]^4)/(4*(a^2 - b^2)) - (a^2*b^3*Log[a*Cosh[x] + b*Sinh[x]])
/(a^2 - b^2)^3 - (a*b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*Cosh[x]*Sin
h[x])/(8*(a^2 - b^2)) + (a*Cosh[x]^3*Sinh[x])/(4*(a^2 - b^2)) + (a^2*b*Sinh
[x]^2)/(2*(a^2 - b^2)^2)
```

Rubi [A] time = 0.33815, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133}

$$-\frac{ax}{8\left(a^2-b^2\right)}-\frac{ab^2x}{2\left(a^2-b^2\right)^2}+\frac{a^3b^2x}{\left(a^2-b^2\right)^3}+\frac{a^2b\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\cosh^4(x)}{4\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}-\frac{a\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}-\frac{ab^2\,\mathrm{si}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^3*b^2*x)/(a^2 - b^2)^3 - (a*b^2*x)/(2*(a^2 - b^2)^2) - (a*x)/(8*(a^2 - b
^2)) - (b*Cosh[x]^4)/(4*(a^2 - b^2)) - (a^2*b^3*Log[a*Cosh[x] + b*Sinh[x]])
/(a^2 - b^2)^3 - (a*b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*Cosh[x]*Sin
h[x])/(8*(a^2 - b^2)) + (a*Cosh[x]^3*Sinh[x])/(4*(a^2 - b^2)) + (a^2*b*Sinh
[x]^2)/(2*(a^2 - b^2)^2)
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{a\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{\left(a^2b\right)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{\text{co}}{a\cosh(}}{\left(a^2-\text{b}\right.}$$

$$= \frac{a^3b^2x}{\left(a^2-b^2\right)^3} - \frac{b\cosh^4(x)}{4\left(a^2-b^2\right)} - \frac{ab^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} - \frac{a\cosh(x)\sinh(x)}{8\left(a^2-b^2\right)} + \frac{a\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)}$$

$$= \frac{a^3b^2x}{\left(a^2-b^2\right)^3} - \frac{ab^2x}{2\left(a^2-b^2\right)^2} - \frac{ax}{8\left(a^2-b^2\right)} - \frac{b\cosh^4(x)}{4\left(a^2-b^2\right)} - \frac{a^2b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3} -$$

Mathematica [A] time = 0.569968, size = 126, normalized size = 0.65

$$\frac{-b\left(a^2-b^2\right)^2\cosh(4x)+4b\left(a^4-b^4\right)\cosh(2x)+a\left(8b^2\left(b^2-a^2\right)\sinh(2x)+\left(a^2-b^2\right)^2\sinh(4x)-4\left(x\left(-6a^2b^2+a^4-3b\right.\right.\right.}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (4*b*(a^4 - b^4)*Cosh[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] + a*(-4*((a^4 - 6*a^
2*b^2 - 3*b^4)*x + 8*a*b^3*Log[a*Cosh[x] + b*Sinh[x]]) + 8*b^2*(-a^2 + b^2)
*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

Maple [A] time = 0.051, size = 322, normalized size = 1.7

$$-2\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)^4} + 8\,\frac{1}{(\tanh(x/2)+1)^3\,(16\,a-16\,b)} + \frac{a}{8\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{3\,b}{8\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -2/(8*a-8*b)/(tanh(1/2*x)+1)^4+8/(tanh(1/2*x)+1)^3/(16*a-16*b)+1/8*a/(a-b)^
2/(tanh(1/2*x)+1)-3/8*b/(a-b)^2/(tanh(1/2*x)+1)-3/8/(a-b)^2/(tanh(1/2*x)+1)
^2*a+5/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-1/8*a^2/(a-b)^3*ln(tanh(1/2*x)+1)+3/8*
b/(a-b)^3*ln(tanh(1/2*x)+1)*a-a^2*b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+
a*tanh(1/2*x)^2)+2/(8*a+8*b)/(tanh(1/2*x)-1)^4+8/(tanh(1/2*x)-1)^3/(16*a+16
*b)+3/8/(a+b)^2/(tanh(1/2*x)-1)^2*a+5/8/(a+b)^2/(tanh(1/2*x)-1)^2*b+1/8*a/(
a+b)^2/(tanh(1/2*x)-1)+3/8*b/(a+b)^2/(tanh(1/2*x)-1)+1/8*a^2/(a+b)^3*ln(tan
h(1/2*x)-1)+3/8*b/(a+b)^3*ln(tanh(1/2*x)-1)*a
```

Maxima [A] time = 1.3266, size = 203, normalized size = 1.05

$$-\frac{a^2 b^3 \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} - \frac{\left(a^2 + 3ab\right)x}{8\left(a^3 + 3a^2b + 3ab^2 + b^3\right)} + \frac{\left(4be^{(-2x)} + a + b\right)e^{(4x)}}{64\left(a^2 + 2ab + b^2\right)} + \frac{4be^{(-2x)} - (a-b)e^{(-4x)}}{64\left(a^2 - 2ab + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - 1/8*(a^2 + 3*a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*b*e^(-2*x)
 + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*b*e^(-2*x) - (a - b)*e^(-4*
x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.2313, size = 2612, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(a^4*b - 2*a^3*
b^2 + 2*a*b^4 - b^5)*cosh(x)^6 + 4*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5 + 7*(
```

```
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^4*b - 2*a^3*b^2 + 2*a
*b^4 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 + 30*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 - 6*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh
(x)^3 - 4*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x))*sinh(x)^3 + 4*
(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5
 + 15*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^4 - 12*(a^5 - 6*a^3*b^2 -
 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^2*b^3*cosh(x)^4 + 4*a^
2*b^3*cosh(x)^3*sinh(x) + 6*a^2*b^3*cosh(x)^2*sinh(x)^2 + 4*a^2*b^3*cosh(x)
*sinh(x)^3 + a^2*b^3*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
+ 3*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^5 - 4*(a^5 - 6*a^3*b^2 - 8*
a^2*b^3 - 3*a*b^4)*x*cosh(x)^3 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15873, size = 273, normalized size = 1.41

$$-\frac{a^{2}b^{3}\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}-\frac{\left(a^{2}-3ab\right)x}{8\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)}+\frac{\left(6a^{2}e^{(4x)}-18abe^{(4x)}+4abe^{(2x)}-4b^{2}e^{(2x)}-a^{2}+2\right.}{64\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^2*b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - 1/8*(a^2 - 3*a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(6*a
^2*e^(4*x) - 18*a*b*e^(4*x) + 4*a*b*e^(2*x) - 4*b^2*e^(2*x) - a^2 + 2*a*b -
 b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x
) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.714 $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=212

$$-\frac{b\sinh^5(x)}{5\left(a^2-b^2\right)}-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2b^3\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{a\cosh^5(x)}{5\left(a^2-b^2\right)}-\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^3b^2\cosh(x)}{\left(a^2-b^2\right)^3}+\cdots$$

```
[Out] (a^3*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2)
 + (a^3*b^2*Cosh[x])/(a^2 - b^2)^3 - (a*b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) -
(a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*Cosh[x]^5)/(5*(a^2 - b^2)) - (a^2*b^3*Si
nh[x])/(a^2 - b^2)^3 + (a^2*b*Sinh[x]^3)/(3*(a^2 - b^2)^2) - (b*Sinh[x]^3)/
(3*(a^2 - b^2)) - (b*Sinh[x]^5)/(5*(a^2 - b^2))
```

Rubi [A] time = 0.432444, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3109, 2564, 14, 2565, 30, 2637, 2638, 3074, 206}

$$-\frac{b\sinh^5(x)}{5\left(a^2-b^2\right)}-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2b^3\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{a\cosh^5(x)}{5\left(a^2-b^2\right)}-\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^3b^2\cosh(x)}{\left(a^2-b^2\right)^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```


```
[Out] (a^3*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2)
 + (a^3*b^2*Cosh[x])/(a^2 - b^2)^3 - (a*b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) -
(a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*Cosh[x]^5)/(5*(a^2 - b^2)) - (a^2*b^3*Si
nh[x])/(a^2 - b^2)^3 + (a^2*b*Sinh[x]^3)/(3*(a^2 - b^2)^2) - (b*Sinh[x]^3)/
(3*(a^2 - b^2)) - (b*Sinh[x]^5)/(5*(a^2 - b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
```

```
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{\left(a^2b\right)\int \cosh(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh^2(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(a^3b^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^3} - \frac{\left(a^2b^3\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^3} + \frac{\left(a^3b^3\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} + \frac{\left(ia^2b\right)\text{Subst}}{(}$$

$$= \frac{a^3b^2\cosh(x)}{\left(a^2-b^2\right)^3} - \frac{ab^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a\cosh^5(x)}{5\left(a^2-b^2\right)} - \frac{a^2b^3\sinh(x)}{\left(a^2-b^2\right)^3} + \frac{a^2b\sinh^3(x)}{3\left(a^2-b^2\right)^2} -$$

$$= \frac{a^3b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{a^3b^2\cosh(x)}{\left(a^2-b^2\right)^3} - \frac{ab^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a\cosh^5(x)}{5\left(a^2-b^2\right)}$$

Mathematica [A] time = 2.2181, size = 325, normalized size = 1.53

$$\frac{1}{32}\left(\frac{2b\left(10a^2b^2+5a^4+b^4\right)\sinh(x)}{(b-a)^3(a+b)^3} + \frac{2b\left(3a^2+b^2\right)\sinh(3x)}{3(a-b)^2(a+b)^2} + \frac{2a\left(10a^2b^2+a^4+5b^4\right)\cosh(x)}{(a-b)^3(a+b)^3} - \frac{2a\left(a^2+3b^2\right)\cosh(3x)}{3(a-b)^2(a+b)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((4*a*b*(3*a^4 + 10*a^2*b^2 + 3*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*
Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) + (2*a*(a^4 + 10*a^2*b^2 + 5*b
^4)*Cosh[x])/((a - b)^3*(a + b)^3) - (2*a*(a^2 + 3*b^2)*Cosh[3*x])/(3*(a -
b)^2*(a + b)^2) + (2*a*Cosh[5*x])/(5*(a - b)*(a + b)) + (2*b*(5*a^4 + 10*a^
2*b^2 + b^4)*Sinh[x])/((-a + b)^3*(a + b)^3) - 3*((4*a*b*ArcTan[(b + a*Tanh
[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a + b)^(3/2)) + (2*a*Cos
h[x])/(a^2 - b^2) + (2*b*Sinh[x])/(-a^2 + b^2)) + (2*b*(3*a^2 + b^2)*Sinh[3
```

```
*x])/(3*(a - b)^2*(a + b)^2) - (2*b*Sinh[5*x])/(5*(a - b)*(a + b)))/32
```

Maple [A] time = 0.056, size = 344, normalized size = 1.6

$$-4\frac{1}{(8a-8b)(\tanh(x/2)+1)^4}+\frac{16}{80a-80b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}+\frac{5a}{12(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{7b}{12(a-b)^2}\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/(8*a-8*b)/(tanh(1/2*x)+1)^4+16/5/(16*a-16*b)/(tanh(1/2*x)+1)^5+5/12/(a-b
)^2/(tanh(1/2*x)+1)^3*a-7/12/(a-b)^2/(tanh(1/2*x)+1)^3*b-1/8/(a-b)^2/(tanh(
1/2*x)+1)^2*a+3/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-1/8*a^2/(a-b)^3/(tanh(1/2*x)+
1)+3/8*a/(a-b)^3/(tanh(1/2*x)+1)*b+2*a^3*b^3/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-16/5/(16*a+16*b)/(tanh(
1/2*x)-1)^5-4/(8*a+8*b)/(tanh(1/2*x)-1)^4-1/8/(a+b)^2/(tanh(1/2*x)-1)^2*a-3
/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-5/12/(a+b)^2/(tanh(1/2*x)-1)^3*a-7/12/(a+b)^
2/(tanh(1/2*x)-1)^3*b+1/8*a^2/(a+b)^3/(tanh(1/2*x)-1)+3/8*a/(a+b)^3/(tanh(1
/2*x)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.24966, size = 10982, normalized size = 51.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")

[Out] [1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a^3*b^4 + 27*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^2)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x))*sinh(x)^3 - 5*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 90*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 480*(a^3*b^3*cosh(x)^5 + 5*a^3*b^3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-

a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh (x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 10*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 4*(a^7 - 3*a ^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9* a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5 *a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^8 - 4*a^6*b^2 + 6 *a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4* a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2* b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^ 6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*s inh(x)^5), 1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^ 2*b^5 - a*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5 *(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a ^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^ 6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*s inh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9 *a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a ^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^ 5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^ 7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*co sh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^ 4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^ 4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a* b^6 - b^7)*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a ^3*b^4 + 27*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4 *b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45* (a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7) *cosh(x)^2)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3* b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a ^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3 *(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x))*sinh(x)^3 - 5*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 +

```
 a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^
5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b
 + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 9
0*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*
b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 - 960*(a^3*b^3*cosh(x)^5 + 5*a^3*b^
3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2
*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 10*(3*(a^7
 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cos
h(x)^9 - 4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a
*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4
*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^
5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6
*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 +
 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sinh(x)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18073, size = 439, normalized size = 2.07

$$\frac{2\,a^3b^3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{\left(30\,a^2e^{(4\,x)}-120\,abe^{(4\,x)}+30\,b^2e^{(4\,x)}+5\,a^2e^{(2\,x)}-5\,b^2e^{(2\,x)}-3\,a^2+6\,ab-3\,b^2\right)e^{(-5}}{480\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*sqrt(a^2 - b^2)) - 1/480*(30*a^2*e^(4*x) - 120*a*b*e^(4*x) + 30
*b^2*e^(4*x) + 5*a^2*e^(2*x) - 5*b^2*e^(2*x) - 3*a^2 + 6*a*b - 3*b^2)*e^(-5
*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/480*(3*a^4*e^(5*x) + 12*a^3*b*e^(5*
x) + 18*a^2*b^2*e^(5*x) + 12*a*b^3*e^(5*x) + 3*b^4*e^(5*x) - 5*a^4*e^(3*x)
- 10*a^3*b*e^(3*x) + 10*a*b^3*e^(3*x) + 5*b^4*e^(3*x) - 30*a^4*e^x - 180*a^
3*b*e^x - 300*a^2*b^2*e^x - 180*a*b^3*e^x - 30*b^4*e^x)/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

3.715 $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2abx}{(a^2-b^2)^2}+\frac{b\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (-2*a*b*x)/(a^2 - b^2)^2 + (a^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 +
 (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (b*Sinh[x])/((a^2 - b^2)*
(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.204338, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3111, 3098, 3133, 3097, 3075}

$$-\frac{2abx}{(a^2-b^2)^2}+\frac{b\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*x)/(a^2 - b^2)^2 + (a^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 +
 (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (b*Sinh[x])/((a^2 - b^2)*
(a*Cosh[x] + b*Sinh[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
```

```
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{a\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2} \\
&= -\frac{2abx}{\left(a^2-b^2\right)^2} + \frac{b\sinh(x)}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} + \frac{\left(ia^2\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(ib^2 \right.}{} \\
&= -\frac{2abx}{\left(a^2-b^2\right)^2} + \frac{a^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2} + \frac{b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2} + \frac{}{\left(a^2-\right.}
\end{aligned}
$$

Mathematica [A] time = 0.183565, size = 60, normalized size = 0.65

$$
\frac{\left(a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))-2abx+\frac{b(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*x + (a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]] + ((a - b)*b*(a + b)*Sin
h[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

Maple [A] time = 0.063, size = 181, normalized size = 2.

$$-\frac{1}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2\frac{a^2\tanh(x/2)\,b}{(a-b)^2(a+b)^2\left(a+2\tanh(x/2)\,b+a(\tanh(x/2))^2\right)}-2\frac{b^3\tanh(}{(a-b)^2(a+b)^2\left(a+2\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a-b)^2*ln(tanh(1/2*x)+1)+2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)*tanh(1/2*x)*b-2/(a-b)^2/(a+b)^2*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)+1/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2
)*a^2+1/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2-1/(a+b)^2
*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.23402, size = 144, normalized size = 1.55

$$\frac{2\,ab}{a^4-2\,a^2b^2+b^4+\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}}+\frac{\left(a^2+b^2\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b/(a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) +
(a^2 + b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + x/(a^2
 + 2*a*b + b^2)
```

Fricas [B] time = 2.41042, size = 892, normalized size = 9.59

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh\left(x\right)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh\left(x\right)\sinh\left(x\right)+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\sinh}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+\left(a^5+a^4b-2\,a^3b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*a*b^2 + (a^3 + a^2*b - a*b^2 - b^3)*x - (a^3 - a^2*b + a*b^2 - b^
3 + (a^3 + a^2*b + a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 + a^2*b + a*b^2 + b^3)*c
osh(x)*sinh(x) + (a^3 + a^2*b + a*b^2 + b^3)*sinh(x)^2)*log(2*(a*cosh(x) +
b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 +
2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^
5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)
```

Sympy [A] time = 146.537, size = 1986, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(cosh(x))/a**2, Eq(b
, 0)), (-x*sinh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x) + 4*b**2*
cosh(x)**2) + 2*x*sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(
x) + 4*b**2*cosh(x)**2) - x*cosh(x)**2/(4*b**2*sinh(x)**2 - 8*b**2*sinh(x)*
cosh(x) + 4*b**2*cosh(x)**2) + sinh(x)*cosh(x)/(4*b**2*sinh(x)**2 - 8*b**2*
sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -b)), (x*sinh(x)**2/(4*b**2*sin
h(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*sinh(x)*cosh(x)
/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*cosh(
x)**2/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - si
nh(x)*cosh(x)/(4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)*
*2), Eq(a, b)), (-x*exp(4*x)*sinh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**
2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*s
inh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)
*cosh(x) + 4*b**2*cosh(x)**2) + 2*x*exp(4*x)*sinh(x)*cosh(x)/(4*b**2*exp(4*
```

```
x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**
2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x
)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - x*exp(4*x)*cosh(x)**2/
(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(
4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 +
 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + x*sinh(x
)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2
*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)
**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) + 2*x
*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh
(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(
2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(
x)**2) + x*cosh(x)**2/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)
*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2
*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*
cosh(x)**2) + exp(4*x)*sinh(x)*cosh(x)/(4*b**2*exp(4*x)*sinh(x)**2 - 8*b**2
*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b**2*exp(2*x)*si
nh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8*b**2*sinh(x)*
cosh(x) + 4*b**2*cosh(x)**2) + 4*exp(2*x)*cosh(x)**2/(4*b**2*exp(4*x)*sinh(
x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**2 + 8*b*
*2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x)**2 + 8
*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2) - sinh(x)*cosh(x)/(4*b**2*exp(4*
x)*sinh(x)**2 - 8*b**2*exp(4*x)*sinh(x)*cosh(x) + 4*b**2*exp(4*x)*cosh(x)**
2 + 8*b**2*exp(2*x)*sinh(x)**2 - 8*b**2*exp(2*x)*cosh(x)**2 + 4*b**2*sinh(x
)**2 + 8*b**2*sinh(x)*cosh(x) + 4*b**2*cosh(x)**2), Eq(a, -(b*exp(2*x) - b)
/(exp(2*x) + 1))), (a**3*log(a*cosh(x)/b + sinh(x))*cosh(x)/(a**5*cosh(x) +
 a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x
) + b**5*sinh(x)) - a**3*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b*
*2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - 2*a**2*
b*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b
**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*log(a*cosh(x)/b + sin
h(x))*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2
*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*x*sinh(x)/(a**5*c
osh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**
4*cosh(x) + b**5*sinh(x)) + a*b**2*log(a*cosh(x)/b + sinh(x))*cosh(x)/(a**5
*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b
**4*cosh(x) + b**5*sinh(x)) + a*b**2*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
 - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x
)) + b**3*log(a*cosh(x)/b + sinh(x))*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
 - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x
)), True))
```

Giac [A] time = 1.14981, size = 173, normalized size = 1.86

$$\frac{\left(a^2+b^2\right)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}-\frac{x}{a^2-2ab+b^2}-\frac{a^2e^{(2x)}+b^2e^{(2x)}+a^2-b^2}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2x)}+be^{(2x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4)
 - x/(a^2 - 2*a*b + b^2) - (a^2*e^(2*x) + b^2*e^(2*x) + a^2 - b^2)/((a^3 -
a^2*b - a*b^2 + b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.716 $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=165

$$\frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}-\frac{a^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{2ab^2\tan^{-1}\left(\right.}{\left(a^2\right.}$$

```
[Out] -((a^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2))
- (2*a*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) - (2*a*b*Cosh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x])/(a^2 - b^2)^2 + (b^2*Sin
h[x])/(a^2 - b^2)^2 - (a^2*b)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.307308, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3109, 2637, 2638, 3074, 206, 3099, 3154}

$$\frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}-\frac{a^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{2ab^2\tan^{-1}\left(\right.}{\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((a^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2))
- (2*a*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) - (2*a*b*Cosh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x])/(a^2 - b^2)^2 + (b^2*Sin
h[x])/(a^2 - b^2)^2 - (a^2*b)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
```

```
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{a^2 b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} - \frac{a^3\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \sin}{\left(a^2-\right.}$$

$$= -\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{a^2 b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} - \frac{\left(ia^3\right)\mathrm{S}}{}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{2ab^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh(}{\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.03944, size = 222, normalized size = 1.35

$$\frac{b\left(\sqrt{a-b}\sqrt{a+b}\left(a^2+b^2\right)\sinh^2(x) - 2a\left(a^2+2b^2\right)\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right) + a^2\left(-\sqrt{a-b}\right)\sqrt{a+b}\right) + a\cosh(x)\left(\sqrt{a-\right.}{(a-b)^{5/2}(a+b)^{5/2}(a\cosh(x)+b\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^2*Sqrt[a - b]*b*Sqrt[a + b]*Cosh[x]^2 + a*Cosh[x]*(-2*a*(a^2 + 2*b^2)
*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)*Sinh[x]) + b*(-(a^2*Sqrt[a - b]*Sqrt[a + b]) - 2*a*(a^2 + 2
*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] + Sqrt[a
- b]*Sqrt[a + b]*(a^2 + b^2)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^(5/2)*(a*Co
sh[x] + b*Sinh[x]))
```

Maple [A] time = 0.07, size = 219, normalized size = 1.3

$$-\frac{1}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\,\frac{a\tanh(x/2)\,b^2}{(a-b)^2\,(a+b)^2\left(a+2\,\tanh(x/2)\,b+a\,(\tanh(x/2))^2\right)}-2\,\frac{a^2b}{(a-b)^2\,(a+b)^2\left(a+2\,\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a-b)^2/(tanh(1/2*x)+1)-2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*x)
*b+a*tanh(1/2*x)^2)*b^2-2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)*b-2*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))-4*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61199, size = 4070, normalized size = 24.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
```

```
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((
a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/
((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^
5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2
)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

[Out] Timed out

Giac [A] time = 1.16894, size = 242, normalized size = 1.47

$$-\frac{2\left(a^{3}+2\,ab^{2}\right)\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{e^{x}}{2\left(a^{2}+2\,ab+b^{2}\right)}-\frac{a^{3}e^{(2\,x)}+7\,a^{2}be^{(2\,x)}+3\,ab^{2}e^{(2\,x)}+b^{3}e^{(2\,x)}+a^{3}+a^{2}b-ab^{2}-b^{3}}{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^{x}-be^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 + 2*a*b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x)
+ 7*a^2*b*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b
^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.717 $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=215

$$\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{abx}{\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{ab\sin}{(a}$$

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 + (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 - (a^2*b)/((a^2 - b^2)^2*(b + a*Coth[x])
) - (a^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (3*a^2*b^2*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2
*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2)
```

Rubi [A] time = 0.536012, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3109, 2564, 30, 2635, 8, 3097, 3133, 3099, 3085, 3483, 3531, 3530}

$$\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{abx}{\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{ab\sin}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 + (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 - (a^2*b)/((a^2 - b^2)^2*(b + a*Coth[x])
) - (a^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (3*a^2*b^2*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2
*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2)
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{a^3\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \cosh(x)\sinh(x)\,d}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{\left(ia^4\right)\int \frac{-ib\cosh(x}{a\cosh(x)}}{\left(a^2-b\right.}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} - \frac{a^4\log(a\cosh}{\left(a^2\right.}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} - \frac{a^4\log(a\cosh}{\left(a^2\right.}$$

Mathematica [A] time = 0.950532, size = 176, normalized size = 0.82

$$\frac{a\left(a^2-b^2\right)^2\cosh(3x) + a\cosh(x)\left(-8a^2\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x)) + 2a^2b^2 + 24a^3bx + a^4 + 8ab^3x - 3b^4\right) -}{8(a-b)^3(a+b)^3(a\cosh(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*(a^2 - b^2)^2*Cosh[3*x] + a*Cosh[x]*(a^4 + 2*a^2*b^2 - 3*b^4 + 24*a^3*b*
x + 8*a*b^3*x - 8*a^2*(a^2 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*b*((a^2
 - b^2)^2*Cosh[2*x] + 2*a*(3*a^3 - 3*a*b^2 - 6*a^2*b*x - 2*b^3*x + 2*a*(a^2
 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]))*Sinh[x])/(8*(a - b)^3*(a + b)^3*(a*C
osh[x] + b*Sinh[x]))
```

Maple [A] time = 0.081, size = 253, normalized size = 1.2

$$\frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{a}{(a-b)^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\,\frac{a^4b\,\mathrm{ta}}{(a-b)^3\,(a+b)^3\left(a+2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+a/(a-b)^3*ln(tanh
(1/2*x)+1)-2*a^4/(a-b)^3/(a+b)^3*b*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)+2*a^2/(a-b)^3/(a+b)^3*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-a^4/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-3*a^2/(a-b
)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2+1/2/(a+b)^2/(tanh(1/2
*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)+a/(a+b)^3*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.27537, size = 325, normalized size = 1.51

$$-\frac{ax}{a^3+3a^2b+3ab^2+b^3}-\frac{\left(a^4+3a^2b^2\right)\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a^4-2a^3b+2ab^3-b^4+\left(a^4-20a^3b\right.}{8\left(\left(a^6-3a^4b^2+3a^2b^4-b^6\right)e^{(-2x)}+\left(a^6-2a^5b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -a*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a^4 + 3*a^2*b^2)*log(-(a - b)*e^(-2
*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a
*b^3 - b^4 + (a^4 - 20*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b
^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.68962, size = 3650, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^5 + 4*a^4*b + 6
```

```
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 19*a^4*b - 14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*cosh(x)^2 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(
x)^2 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*sinh(x)^2 - 8*((a^5 + a^4*b
 + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^4 + 4*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^
3)*cosh(x)*sinh(x)^3 + (a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*sinh(x)^4 + (a
^5 - a^4*b + 3*a^3*b^2 - 3*a^2*b^3)*cosh(x)^2 + (a^5 - a^4*b + 3*a^3*b^2 -
3*a^2*b^3 + 6*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^2)*sinh(x)^2 +
2*(2*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b + 3*a^3
*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x)^3 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b
^4)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17677, size = 321, normalized size = 1.49

$$\frac{ax}{a^3-3a^2b+3ab^2-b^3}-\frac{\left(a^4+3a^2b^2\right)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{e^{(2x)}}{8\left(a^2+2ab+b^2\right)}+\frac{2a^3e^{(4x)}-4a^2be^{(4x)}+2ab^2e}{8\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] a*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a^4 + 3*a^2*b^2)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2
 + 2*a*b + b^2) + 1/8*(2*a^3*e^(4*x) - 4*a^2*b*e^(4*x) + 2*a*b^2*e^(4*x) +
3*a^3*e^(2*x) + 11*a^2*b*e^(2*x) + a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*
b - a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x
) - b*e^(2*x)))
```

3.718 $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\cosh(x)}{(a^2-b^2)^2}+\frac{a^2\cosh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{b^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{2a^2b\tan$$

```
[Out] (2*a^2*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cosh[x])/(a^2 - b^2)^2 + (b^2*Cosh[x])/(a^2 - b^2)^2 - (2*a*b*Sinh[x
])/(a^2 - b^2)^2 + (a*b^2)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.315876, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3100, 2637, 3074, 206, 3109, 2638, 3155}

$$-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\cosh(x)}{(a^2-b^2)^2}+\frac{a^2\cosh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{b^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{2a^2b\tan$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (2*a^2*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cosh[x])/(a^2 - b^2)^2 + (b^2*Cosh[x])/(a^2 - b^2)^2 - (2*a*b*Sinh[x
])/(a^2 - b^2)^2 + (a*b^2)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
```

```
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{b^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} + \frac{a^2\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh(x)\,d}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{2ab\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} + 2\frac{\left(ia^2b\right.}{}$$

$$= \frac{2a^2b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2\cosh(x}{\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.871653, size = 264, normalized size = 1.62

$$\frac{1}{4}\left(\frac{4\left(a^2+b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{6b\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{a\left(a^2+3b^2\right)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\sinh(x)}{(a-b)^2(a+b)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[
a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(
Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/(4*(a - b)^(3/2)*(a + b)^2*(a*Cosh[x] +
b*Sinh[x])) + ((6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqr
t[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) + (4*(a^2 + b^2)*Cosh[x])/((a - b
)^2*(a + b)^2) - (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (a*(a^2 + 3*b^2))/
((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/4
```

Maple [A] time = 0.067, size = 217, normalized size = 1.3

$$\frac{1}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,\frac{b^3\tanh(x/2)}{(a-b)^2(a+b)^2\left(a+2\,\tanh(x/2)\,b+a(\tanh(x/2))^2\right)}+2\,\frac{ab^2}{(a-b)^2(a+b)^2\left(a+2\,\tanh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/(a-b)^2/(tanh(1/2*x)+1)+2/(a-b)^2/(a+b)^2*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)+2*b^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)
^2)*a+4*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2
*b)/(a^2-b^2)^(1/2))+2*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/(a+b)^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69667, size = 4068, normalized size = 24.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2
+ 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 2*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*c
```

```
osh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3
*b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^
4)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 +
a*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x)
 + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2 +
 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*
b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*co
sh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3*
b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4
)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 + a
*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a
+ b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3
+ a*b^4 - b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*
cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
- a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3
*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b
 - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*
sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15143, size = 242, normalized size = 1.48

$$\frac{2\left(2\,a^2b+b^3\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}+\frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+7\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^2*b + b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2
 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(a^3*e^(2*x) +
 3*a^2*b*e^(2*x) + 7*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^
3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.719 $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=205

$$-\frac{a^2x}{2\left(a^2-b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2-b^2\right)^3}+\frac{b^2x}{2\left(a^2-b^2\right)^2}-\frac{ab\sinh^2(x)}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}+\ldots$$

```
[Out] (-4*a^2*b^2*x)/(a^2 - b^2)^3 - (a^2*x)/(2*(a^2 - b^2)^2) + (b^2*x)/(2*(a^2
- b^2)^2) + (2*a^3*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (2*a*b^3*L
og[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*Cosh[x]*Sinh[x])/(2*(a^2 -
b^2)^2) + (b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*b*Sinh[x]^2)/(a^2 -
b^2)^2 + (a*b^2*Sinh[x])/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.659398, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3111, 3109, 2635, 8, 2564, 30, 3098, 3133, 3097, 3075}

$$-\frac{a^2x}{2\left(a^2-b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2-b^2\right)^3}+\frac{b^2x}{2\left(a^2-b^2\right)^2}-\frac{ab\sinh^2(x)}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (-4*a^2*b^2*x)/(a^2 - b^2)^3 - (a^2*x)/(2*(a^2 - b^2)^2) + (b^2*x)/(2*(a^2
- b^2)^2) + (2*a^3*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (2*a*b^3*L
og[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*Cosh[x]*Sinh[x])/(2*(a^2 -
b^2)^2) + (b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*b*Sinh[x]^2)/(a^2 -
b^2)^2 + (a*b^2*Sinh[x])/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \mathrm{c}}{(a}$$

$$= \frac{a^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{b^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} + 2\Bigg(-$$

$$= -\frac{a^2x}{2\left(a^2-b^2\right)^2} + \frac{b^2x}{2\left(a^2-b^2\right)^2} + 2\left(-\frac{a^2b^2x}{\left(a^2-b^2\right)^3} + \frac{a^3b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}\right) - 2\Bigg($$

Mathematica [A] time = 1.86382, size = 174, normalized size = 0.85

$$\frac{1}{8}\left(-\frac{4x\left(6a^2b^2+a^4+b^4\right)}{(a-b)^3(a+b)^3} + \frac{2\left(a^2+b^2\right)\sinh(2x)}{(a-b)^2(a+b)^2} + \frac{\left(6a^2b^2+a^4+b^4\right)\sinh(x)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} + \frac{16ab\left(a^2+b^2\right)\log(a\cos}{\left(a^2-b^2\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-4*(a^4 + 6*a^2*b^2 + b^4)*x)/((a - b)^3*(a + b)^3) - (4*a*b*Cosh[2*x])/(
(a - b)^2*(a + b)^2) + (16*a*b*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]])/(a^2
 - b^2)^3 + ((a^4 + 6*a^2*b^2 + b^4)*Sinh[x])/(a*(a - b)^2*(a + b)^2*(a*Cos
h[x] + b*Sinh[x])) - Sinh[x]/(a^2*Cosh[x] + a*b*Sinh[x]) + (2*(a^2 + b^2)*S
inh[2*x])/((a - b)^2*(a + b)^2))/8
```

Maple [A] time = 0.078, size = 286, normalized size = 1.4

$$-\frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a}{2\,(a-b)^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{b}{2\,(a-b)^3}\ln\left(\tanh\left(\frac{x}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/2/(a-b)^2/(tanh(1/2*x)+1)-1/2*a/(a-b)^3*ln
(tanh(1/2*x)+1)-1/2/(a-b)^3*ln(tanh(1/2*x)+1)*b+2*a^3*b^2/(a-b)^3/(a+b)^3*t
anh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a*b^4/(a-b)^3/(a+b)^3*tanh
(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+2*a^3*b/(a-b)^3/(a+b)^3*ln(a+2*
tanh(1/2*x)*b+a*tanh(1/2*x)^2)+2*a*b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b
+a*tanh(1/2*x)^2)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)
+1/2*a/(a+b)^3*ln(tanh(1/2*x)-1)-1/2/(a+b)^3*ln(tanh(1/2*x)-1)*b
```

Maxima [A] time = 1.21942, size = 329, normalized size = 1.6

$$-\frac{(a-b)x}{2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{2\left(a^3b+ab^3\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{a^4-2\,a^3b+2\,ab^3-b^4+\left(a^4-4\right.}{8\left(\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)e^{(-2\,x)}+\left(a^6-2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*(a - b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*b + a*b^3)*log(-(a
- b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a
^3*b + 2*a*b^3 - b^4 + (a^4 - 4*a^3*b + 22*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x
```

```
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2
 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) - 1/8*e^(-2*x)/(a^2 - 2*a
*b + b^2)
```

Fricas [B] time = 2.69527, size = 3779, normalized size = 18.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^
3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*x)*cosh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 + 5
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b +
 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x))*sinh(x)^3 - (a^5 + 3*a^4*b + 18*a^3*b
^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*x)*cosh(x)^2 - (a^5 + 3*a^4*b + 18*a^3*b^2 - 18*a^2*b^3 -
3*a*b^4 - b^5 - 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 +
 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^2 + 4*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + 16*((a^4*b
 + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^4 + 4*(a^4*b + a^3*b^2 + a^2*b^3 + a*
b^4)*cosh(x)*sinh(x)^3 + (a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*sinh(x)^4 + (a
^4*b - a^3*b^2 + a^2*b^3 - a*b^4)*cosh(x)^2 + (a^4*b - a^3*b^2 + a^2*b^3 -
a*b^4 + 6*(a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^2 + 2*(2*(
a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^3 + (a^4*b - a^3*b^2 + a^2*b^3 -
 a*b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))
) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^3 - (a^5 + 3*a^4*b + 1
8*a^3*b^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b -
```

```
 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(
x)^3 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6
 - b^7)*sinh(x)^4 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^
2*b^5 - a*b^6 + b^7)*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a
^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
b^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17331, size = 313, normalized size = 1.53

$$-\frac{(a+b)x}{2\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{2\left(a^3b+ab^3\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{e^{(2\,x)}}{8\left(a^2+2\,ab+b^2\right)}+\frac{a^3e^{(4\,x)}-3\,a^2be^{(4\,x)}+3\,\cdots}{8\left(a^4-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(a^3*b + a*b^3)*log(abs(
a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e
^(2*x)/(a^2 + 2*a*b + b^2) + 1/8*(a^3*e^(4*x) - 3*a^2*b*e^(4*x) + 3*a*b^2*e
^(4*x) - b^3*e^(4*x) - 8*a^2*b*e^(2*x) - 8*a*b^2*e^(2*x) - a^3 - a^2*b + a*
b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x) - b*
e^(2*x)))
```

3.720 $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2a^3b\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{2ab^3\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\cosh(x)}{\left(a^2-b^2\right)^3}-\frac{}{\left(a^2-b^2\right)^3}$$

```
[Out] (-2*a^4*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2
) - (3*a^2*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - (4*a^2*b^2*Cosh[x])/(a^2 - b^2)^3 - (a^2*Cosh[x])/(a^2 - b^2)^2 +
(a^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (2*
a^3*b*Sinh[x])/(a^2 - b^2)^3 + (2*a*b^3*Sinh[x])/(a^2 - b^2)^3 - (2*a*b*Sin
h[x]^3)/(3*(a^2 - b^2)^2) - (a^3*b^2)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x]
))
```

Rubi [A] time = 1.03437, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2565, 30, 2564, 2637, 2638, 3074, 206, 2633, 3099, 3154}

$$-\frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2a^3b\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{2ab^3\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\cosh(x)}{\left(a^2-b^2\right)^3}-\frac{}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (-2*a^4*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2
) - (3*a^2*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - (4*a^2*b^2*Cosh[x])/(a^2 - b^2)^3 - (a^2*Cosh[x])/(a^2 - b^2)^2 +
(a^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (2*
a^3*b*Sinh[x])/(a^2 - b^2)^3 + (2*a*b^3*Sinh[x])/(a^2 - b^2)^3 - (2*a*b*Sin
h[x]^3)/(3*(a^2 - b^2)^2) - (a^3*b^2)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x]
))
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
```

```
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \sinh^3(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int c}{}$$

$$= -\frac{a^3b^2}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))} + 2\left(\frac{a^3b\sinh(x)}{\left(a^2-b^2\right)^3} - \frac{\left(a^4b\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} - \frac{(a}{}\right.$$

$$= -\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{a^3b^2}{\left(a^2-b^2\right)^3(a\cosh(x)+b\,\mathrm{si}}$$

$$= -\frac{a^2b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{2ab\sinh^3(}{3\left(a^2-b^2\right.}$$

Mathematica [A] time = 3.77175, size = 474, normalized size = 1.82

$$\frac{1}{16}\left(\frac{32ab\left(a^2+b^2\right)\sinh(x)}{(a-b)^3(a+b)^3} - \frac{4\left(a^2+b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{8\left(6a^2b^2+a^4+b^4\right)\cosh(x)}{(a-b)^3(a+b)^3} + \frac{4\left(a^2+b^2\right)\cosh(3x)}{3(a-b)^2(a+b)^2} - \frac{6b\left(3a^2+b^2\right)t}{(a-b)^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(
(a - b)^(5/2)*(a + b)^(5/2)) - (10*b*(5*a^4 + 10*a^2*b^2 + b^4)*ArcTan[(b +
 a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) - (
4*(a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*C
osh[x])/((a - b)^3*(a + b)^3) + (4*(a^2 + b^2)*Cosh[3*x])/(3*(a - b)^2*(a +
 b)^2) + (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (32*a*b*(a^2 + b^2)*Sinh[x
])/((a - b)^3*(a + b)^3) - (a*(a^2 + 3*b^2))/((a - b)^2*(a + b)^2*(a*Cosh[x
] + b*Sinh[x])) - (a*(a^4 + 10*a^2*b^2 + 5*b^4))/((a - b)^3*(a + b)^3*(a*Co
sh[x] + b*Sinh[x])) + (2*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[
(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]))/((a - b)^(3/2)*
(a + b)^2*(a*Cosh[x] + b*Sinh[x])) - (8*a*b*Sinh[3*x])/(3*(a - b)^2*(a + b)
```

```
^2))/16
```

Maple [A] time = 0.081, size = 326, normalized size = 1.3

$$\frac{1}{3\left(a-b\right)^{2}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{1}{2\left(a-b\right)^{2}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{a}{2\left(a-b\right)^{3}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{2\left(a-b\right)^{3}}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/3/(a-b)^2/(tanh(1/2*x)+1)^3-1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^3/(ta
nh(1/2*x)+1)*a-1/2/(a-b)^3/(tanh(1/2*x)+1)*b-2*a^2/(a-b)^3/(a+b)^3*b^3*tanh
(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a^3*b^2/(a-b)^3/(a+b)^3/(a+2*
tanh(1/2*x)*b+a*tanh(1/2*x)^2)-4*a^4*b/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-6*a^2*b^3/(a-b)^3/(a+b)^3/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/3/(a+b)^2/
(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^3/(tanh(1/2*x)-1)
*a-1/2/(a+b)^3/(tanh(1/2*x)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.16211, size = 11158, normalized size = 42.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 + a^7 + a^6*b - 3*a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 2*(4*a^7 - 9*a^6*b
 - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^
6 - 2*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6
*a*b^6 - b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(4*a^7 - 9*a
^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh
(x))*sinh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x)^4 -
2*(9*a^7 + 81*a^5*b^2 - 69*a^3*b^4 - 21*a*b^6 - 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 15*(4*a^7 -
 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 1
7*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^3 - 3*(3*a^7 + 2
7*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^3 - 2*(4*a^7 + 9*a^6*b -
 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x)^2
- 2*(4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a
*b^6 + b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 + 15*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 -
 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^4 + 18*(3*a^7 + 27*a^5*b^2
- 23*a^3*b^4 - 7*a*b^6)*cosh(x)^2)*sinh(x)^2 + 24*((2*a^5*b + 2*a^4*b^2 + 3
*a^3*b^3 + 3*a^2*b^4)*cosh(x)^5 + 5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^
2*b^4)*cosh(x)*sinh(x)^4 + (2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*si
nh(x)^5 + (2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^3 + (2*a^5*
b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4 + 10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^3 + (10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 +
 3*a^2*b^4)*cosh(x)^3 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cos
h(x))*sinh(x)^2 + (5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^
4 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^2)*sinh(x))*sqr
t(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*
sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 4*(2*(a^7 -
 a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(
x)^7 - 3*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5
+ 6*a*b^6 - b^7)*cosh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*
cosh(x)^3 - (4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b
^5 + 6*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
```

+ 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 + 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sinh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)), 1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 + a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 2*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^6 - 2*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x))*sinh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x)^4 - 2*(9*a^7 + 81*a^5*b^2 - 69*a^3*b^4 - 21*a*b^6 - 35*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 15*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^3 - 3*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^3 - 2*(4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x)^2 - 2*(4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a*b^6 + b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 15*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^4 + 18*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x)^2)*sinh(x)^2 + 48*((2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^5 + 5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)*sinh(x)^4 + (2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*sinh(x)^5 + (2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^3 + (2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4 + 10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^3 + (10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 +

```
3*a^2*b^4)*cosh(x)^3 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cos
h(x))*sinh(x)^2 + (5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^
4 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^2)*sinh(x))*sqr
t(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) +
4*(2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^7 - 3*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 -
 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4
- 7*a*b^6)*cosh(x)^3 - (4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^
4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*
cosh(x)^5 + 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5
- 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4
*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^
8 + b^9)*sinh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a
^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*
a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8
 - b^9 + 10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 -
4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*
b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 +
 a*b^8 + b^9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (
5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2
)*sinh(x))]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]   time = 1.19204, size = 419, normalized size = 1.61

$$-\frac{2\,a^3b^2e^x}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}-\frac{\left(9\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}-\frac{2\left(2\,a^4b+3\,a^2b^3\right)\arctan\left(\frac{ae^x\cdots}{\sqrt{a^2\cdots}}\right.}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a^3*b^2*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x)
+ a - b)) - 1/24*(9*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - 2*(2*a^4*b + 3*a^2*b^3)*arctan((a*e^x + b*e^x)/sqrt(a^
2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4
*e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3
*x) - 9*a^4*e^x - 24*a^3*b*e^x - 18*a^2*b^2*e^x + 3*b^4*e^x)/(a^6 + 6*a^5*b
 + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.721 $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=215

$$\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{abx}{\left(a^2-b^2\right)^2}+\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{ab\sin}{(}$$

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 - (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 + (b^2*Cosh[x]^2)/(2*(a^2 - b^2)^2) - (3
*a^2*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (b^4*Log[a*Cosh[x] + b
*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[
x]^2)/(2*(a^2 - b^2)^2) + (a*b^2)/((a^2 - b^2)^2*(a + b*Tanh[x]))
```

Rubi [A] time = 0.558087, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3100, 2635, 8, 3098, 3133, 3109, 2564, 30, 3086, 3483, 3531, 3530}

$$\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{abx}{\left(a^2-b^2\right)^2}+\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{ab\sin}{(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 - (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 + (b^2*Cosh[x]^2)/(2*(a^2 - b^2)^2) - (3
*a^2*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (b^4*Log[a*Cosh[x] + b
*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[
x]^2)/(2*(a^2 - b^2)^2) + (a*b^2)/((a^2 - b^2)^2*(a + b*Tanh[x]))
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
```

```
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} + \frac{a^2\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh}}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))} - \frac{\left(ia^2b^2\right)\int \frac{-ib\cos}{a\cosh}}{(a^2-}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{a^2b^2\log(a\cosh(x)+b\sinh}{\left(a^2-b^2\right)^3}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{3a^2b^2\log(a\cosh(x)+b\sin}{\left(a^2-b^2\right)^3}$$

Mathematica [A] time = 1.19962, size = 183, normalized size = 0.85

$$\frac{a\cosh(x)\left(\left(a^4-b^4\right)\cosh(2x)-4b\left(-ax\left(a^2+3b^2\right)+a\left(a^2-b^2\right)\sinh(x)\cosh(x)+b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(\right.\right.\right.}{4(a-b)^3(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Cosh[x]*((a^4 - b^4)*Cosh[2*x] - 4*b*(-(a*(a^2 + 3*b^2)*x) + b*(3*a^2 +
b^2)*Log[a*Cosh[x] + b*Sinh[x]] + a*(a^2 - b^2)*Cosh[x]*Sinh[x])) + b*Sinh[
x]*((a^4 - b^4)*Cosh[2*x] + 4*b*(-(a^2*b) + b^3 + a^3*x + 3*a*b^2*x - b*(3*
a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*a*b*(a^2 - b^2)*Sinh[2*x]))/(4*(
a - b)^3*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] time = 0.079, size = 253, normalized size = 1.2

$$\frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{1}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{b}{(a-b)^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\,\frac{a^2b^3\,\mathrm{ta}}{(a-b)^3\,(a+b)^3\,\left(a+2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+1/(a-b)^3*ln(tanh
(1/2*x)+1)*b-2*a^2/(a-b)^3/(a+b)^3*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)+2*b^5/(a-b)^3/(a+b)^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-3*a^2/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2-b^4/
(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+1/2/(a+b)^2/(tanh(1/2
*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)-1/(a+b)^3*ln(tanh(1/2*x)-1)*b
```

Maxima [A] time = 1.222, size = 324, normalized size = 1.51

$$\frac{bx}{a^3+3a^2b+3ab^2+b^3}-\frac{\left(3a^2b^2+b^4\right)\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a^4-2a^3b+2ab^3-b^4+\left(a^4-4a^3b+\right.}{8\left(\left(a^6-3a^4b^2+3a^2b^4-b^6\right)e^{(-2x)}+\left(a^6-2a^5b-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*
x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a*
b^3 - b^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 20*a*b^3 + b^4)*e^(-2*x))/((a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^
3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.63856, size = 3650, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^4*b + 4*a^3*b^2
```

```
 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^
5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*cosh(x)^2 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(
x)^2 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*sinh(x)^2 - 8*((3*a^3*b^2 +
 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^
5)*cosh(x)*sinh(x)^3 + (3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*sinh(x)^4 + (3
*a^3*b^2 - 3*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + (3*a^3*b^2 - 3*a^2*b^3 + a*
b^4 - b^5 + 6*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 +
2*(2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 - 3*a^2*b
^3 + a*b^4 - b^5)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^4*b
 + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x)^3 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b
^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15, size = 324, normalized size = 1.51

$$\frac{bx}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(3\,a^2b^2+b^4\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{e^{(2\,x)}}{8\left(a^2+2\,ab+b^2\right)}-\frac{2\,a^2be^{(4\,x)}-4\,ab^2e^{(4\,x)}+2\,b^3e}{8\left(a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2*b^2 + b^4)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2
 + 2*a*b + b^2) - 1/8*(2*a^2*b*e^(4*x) - 4*a*b^2*e^(4*x) + 2*b^3*e^(4*x) -
a^3*e^(2*x) - a^2*b*e^(2*x) - 11*a*b^2*e^(2*x) - 3*b^3*e^(2*x) - a^3 - a^2*
b + a*b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x
) - b*e^(2*x)))
```

3.722 $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=259

$$\frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\sinh(x)}{\left(a^2-b^2\right)^3}-\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2ab^3\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{2a^3b\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{}{\left(a^2-b^2\right)^3}$$

```
[Out] (3*a^3*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/
2) + (2*a*b^4*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(7/2) + (2*a^3*b*Cosh[x])/(a^2 - b^2)^3 + (2*a*b^3*Cosh[x])/(a^2 - b^2)^3 -
 (2*a*b*Cosh[x]^3)/(3*(a^2 - b^2)^2) - (4*a^2*b^2*Sinh[x])/(a^2 - b^2)^3 +
(b^2*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Sinh
[x]^3)/(3*(a^2 - b^2)^2) + (a^2*b^3)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x])
)
```

Rubi [A] time = 0.907976, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206, 2564, 2638, 3155}

$$\frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\sinh(x)}{\left(a^2-b^2\right)^3}-\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2ab^3\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{2a^3b\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (3*a^3*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/
2) + (2*a*b^4*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(7/2) + (2*a^3*b*Cosh[x])/(a^2 - b^2)^3 + (2*a*b^3*Cosh[x])/(a^2 - b^2)^3 -
 (2*a*b*Cosh[x]^3)/(3*(a^2 - b^2)^2) - (4*a^2*b^2*Sinh[x])/(a^2 - b^2)^3 +
(b^2*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Sinh
[x]^3)/(3*(a^2 - b^2)^2) + (a^2*b^3)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x])
)
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
```

```
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \cosh(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^2b^3}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))} + \frac{\left(a^3b^2\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} + 2\left(\frac{\left(a^3b\right)\int \sinh(x)\,d}{\left(a^2-b^2\right)^3}\right.$$

$$= -\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{a^2b^3}{\left(a^2-b^2\right)^3(a\cosh(x)+b\,\mathrm{s}}$$

$$= \frac{a^3b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh^3(x}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 2.0564, size = 481, normalized size = 1.86

$$\frac{1}{16}\left(\frac{4\left(a^2+b^2\right)\sinh(x)}{(a-b)^2(a+b)^2} - \frac{6a\left(a^2+3b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{b\left(3a^2+b^2\right)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\cosh(x)}{(a-b)^2(a+b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -(Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[
a - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(
Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/(8*(a - b)^(3/2)*(a + b)^2*(a*Cosh[x] +
b*Sinh[x])) + ((-6*a*(a^2 + 3*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sq
rt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) - (8*a*b*Cosh[x])/((a - b)^2*(a
+ b)^2) + (4*(a^2 + b^2)*Sinh[x])/((a - b)^2*(a + b)^2) - (b*(3*a^2 + b^2))
/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/16 + ((10*a*(a^4 + 10*a^2*b
^2 + 5*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(
7/2)*(a + b)^(7/2)) + (32*a*b*(a^2 + b^2)*Cosh[x])/((a - b)^3*(a + b)^3) -
(8*a*b*Cosh[3*x])/(3*(a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*Sinh
[x])/((a - b)^3*(a + b)^3) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/((a - b)^3*(a +
 b)^3*(a*Cosh[x] + b*Sinh[x])) + (4*(a^2 + b^2)*Sinh[3*x])/(3*(a - b)^2*(a
```

```
+ b)^2))/16
```

Maple [A] time = 0.08, size = 289, normalized size = 1.1

$$-\frac{1}{3(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{2(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{b}{(a-b)^3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,\frac{b^4a\,\mathrm{t}}{(a-b)^3(a+b)^3\left(a+2\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/3/(a-b)^2/(tanh(1/2*x)+1)^3+1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/(a-b)^3/(tan
h(1/2*x)+1)*b+2*a*b^4/(a-b)^3/(a+b)^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh
(1/2*x)^2)+2*a^2*b^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+6*
a^3*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a
^2-b^2)^(1/2))+4*a*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh
(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(ta
nh(1/2*x)-1)^2-1/(a+b)^3/(tanh(1/2*x)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.4068, size = 11158, normalized size = 43.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 - a^7 - a^6*b + 3*a^5
*b^2 + 3*a^4*b^3 - 3*a^3*b^4 - 3*a^2*b^5 + a*b^6 + b^7 - 2*(a^7 - 6*a^6*b +
 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^
6 - 2*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a
*b^6 - 4*b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - 6*a^6
*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh
(x))*sinh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^4 +
2*(21*a^6*b + 69*a^4*b^3 - 81*a^2*b^5 - 9*b^7 + 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 - 15*(a^7 - 6
*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^3 + 3*(7*a^6*b +
 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 + 2*(a^7 + 6*a^6*b + 7
*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x)^2
+ 2*(a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b
^6 + 4*b^7 + 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 - 15*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17
*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^4 + 18*(7*a^6*b + 23*a^4*b^
3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^2)*sinh(x)^2 + 24*((3*a^4*b^2 + 3*a^3*b^3 +
 2*a^2*b^4 + 2*a*b^5)*cosh(x)^5 + 5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*
a*b^5)*cosh(x)*sinh(x)^4 + (3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*si
nh(x)^5 + (3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^3 + (3*a^4*
b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 + 10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b
^4 + 2*a*b^5)*cosh(x)^2)*sinh(x)^3 + (10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4
 + 2*a*b^5)*cosh(x)^3 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cos
h(x))*sinh(x)^2 + (5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*cosh(x)^
4 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^2)*sinh(x))*sqr
t(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*
sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 4*(2*(a^7 -
 a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(
x)^7 - 3*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 +
9*a*b^6 - 4*b^7)*cosh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*
cosh(x)^3 + (a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5
 + 9*a*b^6 + 4*b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 +
 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4*a^7*b^2 -
4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*si
```

```
nh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*
a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4
*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 10
*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*b - 4*a^7*b
^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^
9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (5*(a^9 + a^
8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7
 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)),
 1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 - a^7 - a^6*b + 3*a^5
*b^2 + 3*a^4*b^3 - 3*a^3*b^4 - 3*a^2*b^5 + a*b^6 + b^7 - 2*(a^7 - 6*a^6*b +
 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^
6 - 2*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a
*b^6 - 4*b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - 6*a^6
*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh
(x))*sinh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^4 +
2*(21*a^6*b + 69*a^4*b^3 - 81*a^2*b^5 - 9*b^7 + 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 - 15*(a^7 - 6
*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^3 + 3*(7*a^6*b +
 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 + 2*(a^7 + 6*a^6*b + 7
*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x)^2
+ 2*(a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b
^6 + 4*b^7 + 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 - 15*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17
*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^4 + 18*(7*a^6*b + 23*a^4*b^
3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^2)*sinh(x)^2 - 48*((3*a^4*b^2 + 3*a^3*b^3 +
 2*a^2*b^4 + 2*a*b^5)*cosh(x)^5 + 5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*
a*b^5)*cosh(x)*sinh(x)^4 + (3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*si
nh(x)^5 + (3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^3 + (3*a^4*
b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 + 10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b
^4 + 2*a*b^5)*cosh(x)^2)*sinh(x)^3 + (10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4
 + 2*a*b^5)*cosh(x)^3 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cos
h(x))*sinh(x)^2 + (5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*cosh(x)^
4 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^2)*sinh(x))*sqr
t(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) +
```

```
4*(2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^7 - 3*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2
*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^
5 - 3*b^7)*cosh(x)^3 + (a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4
- 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*
cosh(x)^5 + 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5
- 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4
*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^
8 + b^9)*sinh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a
^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*
a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8
 - b^9 + 10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 -
4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*
b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 +
 a*b^8 + b^9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (
5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2
)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15524, size = 419, normalized size = 1.62

$$\frac{2\,a^2b^3e^x}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}+\frac{\left(3\,ae^{(2\,x)}+9\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{2\left(3\,a^3b^2+2\,ab^4\right)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-}}\right.}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a^2*b^3*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x) +
 a - b)) + 1/24*(3*a*e^(2*x) + 9*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + 2*(3*a^3*b^2 + 2*a*b^4)*arctan((a*e^x + b*e^x)/sqrt(a^2
 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4*
e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3*
x) - 3*a^4*e^x + 18*a^2*b^2*e^x + 24*a*b^3*e^x + 9*b^4*e^x)/(a^6 + 6*a^5*b
+ 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.723 $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=314

$$-\frac{a^3bx}{\left(a^2-b^2\right)^3}-\frac{6a^3b^3x}{\left(a^2-b^2\right)^4}+\frac{abx}{4\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{2a^2b^2\sinh^2(x)}{\left(a^2-b^2\right)^3}+\frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{ab\sinh(x)\cosh^3}{2\left(a^2-b^2\right)^2}$$

```
[Out] (-6*a^3*b^3*x)/(a^2 - b^2)^4 - (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b
^2)^3 + (a*b*x)/(4*(a^2 - b^2)^2) + (b^2*Cosh[x]^4)/(4*(a^2 - b^2)^2) + (3*
a^4*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^4 + (3*a^2*b^4*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)^4 + (a^3*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*
b^3*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*b*Cosh[x]*Sinh[x])/(4*(a^2 - b^2)^2
) - (a*b*Cosh[x]^3*Sinh[x])/(2*(a^2 - b^2)^2) - (2*a^2*b^2*Sinh[x]^2)/(a^2
- b^2)^3 + (a^2*Sinh[x]^4)/(4*(a^2 - b^2)^2) + (a^2*b^3*Sinh[x])/((a^2 - b^
2)^3*(a*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 1.73207, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133, 3097, 3075}

$$-\frac{a^3bx}{\left(a^2-b^2\right)^3}-\frac{6a^3b^3x}{\left(a^2-b^2\right)^4}+\frac{abx}{4\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{2a^2b^2\sinh^2(x)}{\left(a^2-b^2\right)^3}+\frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{ab\sinh(x)\cosh^3}{2\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```


```
[Out] (-6*a^3*b^3*x)/(a^2 - b^2)^4 - (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b
^2)^3 + (a*b*x)/(4*(a^2 - b^2)^2) + (b^2*Cosh[x]^4)/(4*(a^2 - b^2)^2) + (3*
a^4*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^4 + (3*a^2*b^4*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)^4 + (a^3*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*
b^3*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*b*Cosh[x]*Sinh[x])/(4*(a^2 - b^2)^2
) - (a*b*Cosh[x]^3*Sinh[x])/(2*(a^2 - b^2)^2) - (2*a^2*b^2*Sinh[x]^2)/(a^2
- b^2)^3 + (a^2*Sinh[x]^4)/(4*(a^2 - b^2)^2) + (a^2*b^3*Sinh[x])/((a^2 - b^
2)^3*(a*Cosh[x] + b*Sinh[x]))
```


Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
```

```
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \cosh(x)\sinh^3(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,c}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(a^3b^2\right)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} + 2\left(\frac{\left(a^3b\right)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^3} - \frac{\left(a^2b^2\right)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^3} + \right.$$

$$= -\frac{2a^3b^3x}{\left(a^2-b^2\right)^4} + \frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^2b^3\sinh(x)}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))} + \frac{(ia^4}{}$$

$$= -\frac{2a^3b^3x}{\left(a^2-b^2\right)^4} + \frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^4b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^4} + \frac{a^2b^4\log(a\cosh(x)+}{\left(a^2-b^2\right)^4}$$

Mathematica [A] time = 1.37939, size = 366, normalized size = 1.17

$$\frac{-48a^5b^2x\sinh(x) + 84a^4b^3\sinh(x) - 15a^4b^3\sinh(3x) + 3a^4b^3\sinh(5x) - 288a^3b^4x\sinh(x) - 100a^2b^5\sinh(x) + 3a^2b^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a*(a^2 - b^2)^2*(a^2 + 3*b^2)*Cosh[3*x] + a^7*Cosh[5*x] - 3*a^5*b^2*Cos
h[5*x] + 3*a^3*b^4*Cosh[5*x] - a*b^6*Cosh[5*x] - 4*a*Cosh[x]*(a^6 + 9*a^4*b
^2 - 5*a^2*b^4 - 5*b^6 + 12*a^5*b*x + 72*a^3*b^3*x + 12*a*b^5*x - 48*a^2*b^
2*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) + 20*a^6*b*Sinh[x] + 84*a^4*b^3*S
inh[x] - 100*a^2*b^5*Sinh[x] - 4*b^7*Sinh[x] - 48*a^5*b^2*x*Sinh[x] - 288*a
^3*b^4*x*Sinh[x] - 48*a*b^6*x*Sinh[x] + 192*a^4*b^3*Log[a*Cosh[x] + b*Sinh[
x]]*Sinh[x] + 192*a^2*b^5*Log[a*Cosh[x] + b*Sinh[x]]*Sinh[x] + 9*a^6*b*Sinh
[3*x] - 15*a^4*b^3*Sinh[3*x] + 3*a^2*b^5*Sinh[3*x] + 3*b^7*Sinh[3*x] - a^6*
b*Sinh[5*x] + 3*a^4*b^3*Sinh[5*x] - 3*a^2*b^5*Sinh[5*x] + b^7*Sinh[5*x])/(6
4*(a - b)^4*(a + b)^4*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] time = 0.089, size = 398, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/4/(a-b)^2/(tanh(1/2*x)+1)^4-1/2/(a-b)^2/(tanh(1/2*x)+1)^3+1/8/(a-b)^3/(ta
nh(1/2*x)+1)*a+3/8/(a-b)^3/(tanh(1/2*x)+1)*b+1/8/(a-b)^3/(tanh(1/2*x)+1)^2*
a-5/8/(a-b)^3/(tanh(1/2*x)+1)^2*b-3/4*a*b/(a-b)^4*ln(tanh(1/2*x)+1)+2*a^4*b
^3/(a-b)^4/(a+b)^4*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a^2*b^
5/(a-b)^4/(a+b)^4*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+3*a^4*b^2
/(a-b)^4/(a+b)^4*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+3*a^2*b^4/(a-b)^4/(a
+b)^4*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+1/4/(a+b)^2/(tanh(1/2*x)-1)^4+1
/2/(a+b)^2/(tanh(1/2*x)-1)^3+1/8/(a+b)^3/(tanh(1/2*x)-1)^2*a+5/8/(a+b)^3/(t
anh(1/2*x)-1)^2*b-1/8/(a+b)^3/(tanh(1/2*x)-1)*a+3/8/(a+b)^3/(tanh(1/2*x)-1)
*b+3/4*a*b/(a+b)^4*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.15133, size = 518, normalized size = 1.65

$$-\frac{3\,abx}{4\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)}+\frac{3\left(a^4b^2+a^2b^4\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(a+b)e^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{a^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -3/4*a*b*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b
^4)*log(-(a - b)*e^(-2*x) - a - b)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8) - 1/64*(4*(a + b)*e^(-2*x) - (a - b)*e^(-4*x))/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + 1/64*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5
 + b^6 - 3*(a^6 - 4*a^5*b + 5*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*e^(-2*x)
 - 4*(a^6 - 6*a^5*b + 15*a^4*b^2 - 52*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)
*e^(-4*x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*e^(-4*x) + (a^8
 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)
*e^(-6*x))
```

Fricas [B] time = 3.16378, size = 8631, normalized size = 27.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
+ b^7)*cosh(x)^10 + 10*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3
*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 3*(a^7 - 3*a^6*b
+ a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 3*
(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7
- 15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^2)*sinh(x)^8 + 24*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 +
5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + a^7 +
 a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 4*(a
^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^
7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)
*cosh(x)^6 - 2*(2*a^7 - 10*a^6*b + 18*a^5*b^2 - 10*a^4*b^3 - 10*a^3*b^4 + 1
8*a^2*b^5 - 10*a*b^6 + 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 42*(a^7 - 3*a^6*b + a^5*b^2
 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2 + 24*(a^6*b +
 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*sinh(x)^6 + 12
*(21*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^5 - 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2
*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 2*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3
- 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^
3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x))*sinh(x)^5 - 4*(a^7 + 5*a^6*
b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a
^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x)^4
- 2*(2*a^7 + 10*a^6*b + 18*a^5*b^2 + 74*a^4*b^3 - 74*a^3*b^4 - 18*a^2*b^5 -
 10*a*b^6 - 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 105*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b
^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 30*(a^7 - 5*a^6*b + 9
*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b +
5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^2 + 24*
(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*sinh(x)^
4 + 8*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*
b^6 + b^7)*cosh(x)^7 - 21*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4
- a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 10*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*
a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^3 - 2*(a^7 + 5*a^6*b +
 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a^6*
b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x))*sinh
(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*
b^6 + b^7)*cosh(x)^2 + 3*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b
```

^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b ^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5* a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 20*(a^7 - 5*a^6* b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6 *b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^4 - 8*(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a* b^6 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b ^6)*x)*cosh(x)^2)*sinh(x)^2 + 192*((a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)* cosh(x)^6 + 6*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)*sinh(x)^5 + (a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*sinh(x)^6 + (a^5*b^2 - a^4*b^3 + a^3 *b^4 - a^2*b^5)*cosh(x)^4 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5 + 15*(a^ 5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^3 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^ 2*b^5)*cosh(x))*sinh(x)^3 + 3*(5*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*co sh(x)^4 + 2*(a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^5 + 2*(a^5*b^2 - a^4*b ^3 + a^3*b^4 - a^2*b^5)*cosh(x)^3)*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 2*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 12*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4 *b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 12*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^5 - 8 *(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6) *x)*cosh(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^ 5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^6 + 6*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^5 + (a^9 + a^8*b - 4*a^7*b^2 - 4 *a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin h(x)^6 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a ^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^4 + (a^9 - a^8*b - 4*a^7*b^2 + 4* a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 15* (a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^9 + a^8*b - 4*a^7*b ^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^ 9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^3 + 3*(5*(a^9 + a^ 8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5* b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^ 3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4 *a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cos h(x)^3)*sinh(x))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18333, size = 518, normalized size = 1.65

$$-\frac{3\,abx}{4\,\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)}+\frac{\left(36\,abe^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-4\,x)}}{64\,\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)}+\frac{3\,\left(a^4b^2+a^2b^4\right)\log\left(\right.}{a^8-4\,a^6b^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*a*b*x/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 1/64*(36*a*b*e^(4*
x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^4 - 4*a
^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b^4)*log(abs(a*e^(2*x)
 + b*e^(2*x) + a - b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 1/
64*(a^2*e^(4*x) + 2*a*b*e^(4*x) + b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*
x))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (3*a^5*b^2*e^(2*x) + 3*a^
4*b^3*e^(2*x) + 3*a^3*b^4*e^(2*x) + 3*a^2*b^5*e^(2*x) + 3*a^5*b^2 - a^4*b^3
 + a^3*b^4 - 3*a^2*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a
*e^(2*x) + b*e^(2*x) + a - b))
```

3.724 $\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] -((c*C*x)/(b^2 - c^2)) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]]
)/Sqrt[b^2 - c^2] + (b*C*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.0816141, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3137, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]]
)/Sqrt[b^2 - c^2] + (b*C*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx\\ &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\mathrm{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh(x\right.\\ &= -\frac{cCx}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.224408, size = 78, normalized size = 0.98

$$\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{\sqrt{b-c}\sqrt{b+c}}+\frac{C(b\log(b\cosh(x)+c\sinh(x))-cx)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/(Sqrt[b - c]*Sqrt
[b + c]) + (C*(-(c*x) + b*Log[b*Cosh[x] + c*Sinh[x]]))/(b^2 - c^2)
```

Maple [B] time = 0.057, size = 181, normalized size = 2.3

$$-2\,\frac{C\ln(\tanh(x/2)+1)}{2\,b-2\,c}+\frac{bC}{(b-c)\,(b+c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b+2\,c\tanh(x/2)+b\right)+2\,\frac{Ab^2}{(b-c)\,(b+c)\,\sqrt{b^2-c^2}}\arctan\left(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)+1/(b-c)/(b+c)*b*C*ln(tanh(1/2*x)^2*b+2*c*t
anh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c
)/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(1
/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49732, size = 644, normalized size = 8.05

$$\left[\frac{Cb\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right)-}{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - sqrt(-b^2 + c^2)
*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (C*b + C*c)*x)/(b^2 -
c^2), (C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - 2*sqrt(b^2
- c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (C*b
 + C*c)*x)/(b^2 - c^2)]
```

Sympy [A] time = 47.2688, size = 367, normalized size = 4.59

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}-\frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{C\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{C\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{Cbx}{b^2-c^2}-\frac{2Cb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2}+\frac{Cb\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+}}{b}\right.}{b^2-c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh
(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh(x
)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) -
 C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x) + 2
*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*si
nh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), (-
A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c*
*2) + A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**
2 - c**2) + C*b*x/(b**2 - c**2) - 2*C*b*log(tanh(x/2) + 1)/(b**2 - c**2) +
C*b*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*log(tan
h(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - C*c*x/(b**2 - c**2), T
rue))
```

Giac [A] time = 1.15959, size = 108, normalized size = 1.35

$$\frac{Cb\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Cx}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c*
e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) - C*x/(b - c)
```

3.725 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=82

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}-\frac{cC\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

```
[Out] -((c*C*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2))
- (b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.0790252, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3154, 3074, 206}

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}-\frac{cC\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((c*C*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2))
- (b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{(cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{(icC)\,\mathrm{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x)-ib\sin$$

$$= -\frac{cC\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}-\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.439, size = 155, normalized size = 1.89

$$-\frac{\sinh(x)\left(2bc^2C\sqrt{b+c}\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)-A(b-c)^{3/2}(b+c)^2\right)+2b^2cC\sqrt{b+c}\cosh(x)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)+b^2C\sqrt{b}}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((b^2*Sqrt[b - c]*(b + c)*C + 2*b^2*c*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2
])/(Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (-(A*(b - c)^(3/2)*(b + c)^2) + 2*b
*c^2*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sin
h[x])/(b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x])))
```

Maple [A] time = 0.067, size = 115, normalized size = 1.4

$$-2\,\frac{1}{(\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b}\left(-\frac{(Ab^2-Ac^2-Ccb)\tanh(x/2)}{(b^2-c^2)\,b}+\frac{bC}{b^2-c^2}\right)-2\,\frac{Cc}{(b^2-c^2)^{3/2}}\arctan\left(1/2\,\frac{2\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2-C*b*c)/(b^2-c^2)/b*tanh(1/2*x)+b*C/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+
2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42513, size = 1648, normalized size = 20.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (C*b*c - C*c^2 + (C*b*c + C*
c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh(x
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(
C*b^3 - C*b*c^2)*cosh(x) + 2*(C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - (C*b*c - C*c^2 + (C*b*c +
C*c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) + (C*b^3 - C*b*c^2)*cosh(x) + (C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
```

```
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14731, size = 112, normalized size = 1.37

$$-\frac{2\,Cc\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\,(Cbe^x+Ab-Ac)}{\left(b^2-c^2\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*C*c*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2*(C*b*e
^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.726 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=123

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bC\sinh(x)+c^2C\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) -
 (b*C - A*c*Cosh[x] - A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) - (c^2*C*Cosh[x] + b*c*C*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.126653, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3157, 3153, 3074, 206}

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bC\sinh(x)+c^2C\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) -
 (b*C - A*c*Cosh[x] - A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) - (c^2*C*Cosh[x] + b*c*C*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int\frac{-2cC+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+bcC\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{A\int\frac{}{b\cosh}}{2(b}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+bcC\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{(iA)\,\mathrm{Subs}}{}$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+}{\left(b^2-c^2\right)^2(b\cosh(}$$

Mathematica [A] time = 1.12868, size = 134, normalized size = 1.09

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)-b^2C}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{c(A-2C\sinh(x))}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (-(b^2*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Co
sh[x] + c*Sinh[x])^2) + (c*(A - 2*C*Sinh[x]))/(b*(b - c)*(b + c)*(b*Cosh[x]
 + c*Sinh[x])))/2
```

Maple [A] time = 0.082, size = 187, normalized size = 1.5

$$2\,\frac{1}{\left((\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b\right)^2}\left(-1/2\,\frac{A\left(b^2-2\,c^2\right)(\tanh(x/2))^3}{\left(b^2-c^2\right)b}+1/2\,\frac{\left(Ab^2c+2\,Ac^3+2\,Cb^3-2\,Cbc^2\right)(\tanh(\right.}{\left(b^2-c^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*A*(b^2-2*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^2*c+2*A*c^3+2*C*b^
3-2*C*b*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^2+1/2*A*(b^2+2*c^2)/(b^2-c^2)/b*tanh
(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2
)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7135, size = 4243, normalized size = 34.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(4*C*b^2*c - 8*C*b*c^2 + 4*C*c^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^
3)*cosh(x)^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 4*(C*b^3 -
 C*b^2*c - C*b*c^2 + C*c^3)*cosh(x)^2 - 2*(2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2
+ 2*C*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) - 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
 4*(C*b^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b
^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4
*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
- 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c -
 b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*
c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3
*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)
*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), (2*
C*b^2*c - 4*C*b*c^2 + 2*C*c^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)
^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 2*(C*b^3 - C*b^2*c - C
*b*c^2 + C*c^3)*cosh(x)^2 - (2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2 + 2*C*c^3 - 3*
(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*b^2 + 2*A*b*c
+ A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b^2
 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^2)
*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*sinh
(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))*s
inh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*s
inh(x))) - (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - (A*b^3 - A*b^2*c -
 A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(C*b
^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2
+ 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3
*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^
2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3
*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 +
 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x
```

```
)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*co
sh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14862, size = 205, normalized size = 1.67

$$\frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Cb^2e^{(2x)}+2\,Cc^2e^{(2x)}-Ab^2e^x+Ac^2e^x+2\,Cbc-2\,Cc^2}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*C*b^2*e^(2*x) + 2*C*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

3.727 $\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (b*B*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] - (B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.0614946, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3138, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] - (B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx\\ &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\mathrm{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh(x)-\right.\\ &= \frac{bBx}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.170774, size = 78, normalized size = 0.98

$$\frac{2A\sqrt{b-c}\sqrt{b+c}\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)-Bc\log(b\cosh(x)+c\sinh(x))+bBx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*
Sqrt[b + c])] - B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Maple [B] time = 0.055, size = 182, normalized size = 2.3

$$2\,\frac{B\ln(\tanh(x/2)+1)}{2\,b-2\,c}-\frac{Bc}{(b-c)\,(b+c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b+2\,c\tanh(x/2)+b\right)+2\,\frac{Ab^2}{(b-c)\,(b+c)\,\sqrt{b^2-c^2}}\arctan\left(1/2\,\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b+c)*B*c*ln(tanh(1/2*x)^2*b+2*c*ta
nh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)
```

```
/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(1/
2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2-2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46231, size = 647, normalized size = 8.09

$$\left[-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right.}{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + sqrt(-b^2 + c^2
)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2
- 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
 + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (B*b + B*c)*x)/(b^2 -
 c^2), -(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + 2*sqrt(b^
2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (B
*b + B*c)*x)/(b^2 - c^2)]
```

Sympy [A] time = 46.9949, size = 408, normalized size = 5.1

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{Bbx}{b^2-c^2}-\frac{Bcx}{b^2-c^2}+\frac{2Bc\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2}-\frac{Bc\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}\cdots\right.}{b^2-c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x))
 + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x) + 2
*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*
c*sinh(x) + 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*co
sh(x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)),
Eq(b, c)), (-A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/
b)/(b**2 - c**2) + A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 +
c**2)/b)/(b**2 - c**2) + B*b*x/(b**2 - c**2) - B*c*x/(b**2 - c**2) + 2*B*c*
log(tanh(x/2) + 1)/(b**2 - c**2) - B*c*log(tanh(x/2) + c/b - sqrt(-b**2 + c
**2)/b)/(b**2 - c**2) - B*c*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b*
*2 - c**2), True))
```

Giac [A] time = 1.13014, size = 108, normalized size = 1.35

$$-\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{Bx}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c
*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + B*x/(b - c)
```

3.728 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=78

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)}+\frac{bB\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

```
[Out] (b*B*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
B*c + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.0613427, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3155, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)}+\frac{bB\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
B*c + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx &= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB)\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}\\
&= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(ibB)\,\text{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh(x)-ib\sinh(x
\end{aligned}
$$

$$
= \frac{bB\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}
$$

Mathematica [A] time = 0.281115, size = 151, normalized size = 1.94

$$
\frac{\sinh(x)\left(A(b-c)^{3/2}(b+c)^2+2b^2Bc\sqrt{b+c}\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)\right)+2b^3B\sqrt{b+c}\cosh(x)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)+bBc\sqrt{b-c}(b}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*Sqrt[b - c]*c*(b + c) + 2*b^3*B*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(
Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (A*(b - c)^(3/2)*(b + c)^2 + 2*b^2*B*c*
Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sinh[x])/(
b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x]))
```

Maple [A] time = 0.062, size = 116, normalized size = 1.5

$$
-2\,\frac{1}{(\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b}\left(-\frac{\left(Ab^2-Ac^2+Bc^2\right)\tanh(x/2)}{b\left(b^2-c^2\right)}-\frac{Bc}{b^2-c^2}\right)+2\,\frac{Bb}{\left(b^2-c^2\right)^{3/2}}\arctan\left(1/2\,\frac{2\tanh(}{\sqrt{b}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2)/b/(b^2-c^2)*tanh(1/2*x)-B*c/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+
2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.416, size = 1648, normalized size = 21.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (B*b^2 - B*b*c + (B*b^2 + B*
b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh(x
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - 2*(
B*b^2*c - B*c^3)*cosh(x) - 2*(B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 - B*b*c + (B*b^2 +
B*b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) - (B*b^2*c - B*c^3)*cosh(x) - (B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
```

```
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14298, size = 112, normalized size = 1.44

$$\frac{2\,Bb\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\,(Bce^x-Ab+Ac)}{\left(b^2-c^2\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*B*b*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + 2*(B*c*e^
x - A*b + A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.729 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=120

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b^2B\sinh(x)+bBc\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) + (b*B*c*Cosh[x] + b^2*B*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.118954, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3158, 3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b^2B\sinh(x)+bBc\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) + (b*B*c*Cosh[x] + b^2*B*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2bB+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cosh(x)+b^2B\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{A\int \frac{1}{b\cosh(x)+\cdots}}{2\left(b^2-\cdots\right.}$$

$$= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cosh(x)+b^2B\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{(iA)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cosh(x)+b^2\cdots}{\left(b^2-c^2\right)^2(b\cosh(x\cdots}$$

Mathematica [A] time = 1.03738, size = 134, normalized size = 1.12

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)+bBc}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{Ac+2bB\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (A*c + 2*b*B*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*Sin
h[x])) + (b*B*c + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*
Sinh[x])^2))/2
```

Maple [A] time = 0.094, size = 214, normalized size = 1.8

$$2\,\frac{1}{\left((\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b\right)^2}\left(-1/2\,\frac{\left(Ab^2-2\,Ac^2-2\,Bb^2+2\,Bc^2\right)(\tanh(x/2))^3}{\left(b^2-c^2\right)b}+1/2\,\frac{c\left(Ab^2+2\,Ac^2+2\,Bb\right.}{\left(b^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*c*(A*
b^2+2*A*c^2+2*B*b^2-2*B*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^2+1/2*(A*b^2+2*A*c^2
+2*B*b^2-2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2
*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/
(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7187, size = 4246, normalized size = 35.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*B*b^3 - 8*B*b^2*c + 4*B*b*c^2 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c
^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 4*(B*b^3
- B*b^2*c - B*b*c^2 + B*c^3)*cosh(x)^2 + 2*(2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2
 + 2*B*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A
*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sin
h(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*
(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*
cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*
c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*
cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)
) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)
^2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A
*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2
+ 4*(B*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c -
b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 -
4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c
- b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4
*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 +
3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2
)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
 c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), -(
2*B*b^3 - 4*B*b^2*c + 2*B*b*c^2 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(
x)^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 2*(B*b^3 - B*b^2*c -
 B*b*c^2 + B*c^3)*cosh(x)^2 + (2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2 + 2*B*c^3 -
3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*b^2 + 2*A*b*
c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b
^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^
2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*si
nh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))
*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)
*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + (A*b^3 - A*b^2*c
 - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(B
*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^
2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b
^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*
c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 +
 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh
```

```
(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*
cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16551, size = 205, normalized size = 1.71

$$\frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}+2\,Bc^2e^{(2x)}-Ab^2e^x+Ac^2e^x-2\,Bb^2+2\,Bbc}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) + 2*B*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*c)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

3.730 $\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}(\sinh(x)+\cosh(x))^2$$

```
[Out] (Cosh[x] + Sinh[x])^2/2
```

Rubi [A] time = 0.0389334, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4385}

$$\frac{1}{2}(\sinh(x)+\cosh(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] (Cosh[x] + Sinh[x])^2/2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{1}{2}(\cosh(x)+\sinh(x))^2$$

Mathematica [A] time = 0.0034965, size = 17, normalized size = 1.55

$$\frac{1}{2}\sinh(2x)+\frac{1}{2}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] Cosh[2*x]/2 + Sinh[2*x]/2
```

Maple [A] time = 0., size = 17, normalized size = 1.6

$$\frac{\cosh(x) + \sinh(x)}{2\cosh(x) - 2\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] 1/2*(cosh(x)+sinh(x))/(cosh(x)-sinh(x))
```

Maxima [A] time = 1.02184, size = 8, normalized size = 0.73

$$\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*x)
```

Fricas [A] time = 2.2091, size = 61, normalized size = 5.55

$$\frac{\cosh(x) + \sinh(x)}{2\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.376119, size = 8, normalized size = 0.73

$$\frac{\cosh(x)}{-\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)

[Out] cosh(x)/(-sinh(x) + cosh(x))
```

Giac [A] time = 1.14802, size = 8, normalized size = 0.73

$$\frac{1}{2}\, e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")

[Out] 1/2*e^(2*x)
```

3.731 $\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

```
[Out] -1/(2*(Cosh[x] + Sinh[x])^2)
```

Rubi [A] time = 0.0344315, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4385}

$$-\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -1/(2*(Cosh[x] + Sinh[x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2(\cosh(x)+\sinh(x))^2}$$

Mathematica [A] time = 0.0049873, size = 17, normalized size = 1.55

$$\frac{1}{2}\sinh(2x)-\frac{1}{2}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -Cosh[2*x]/2 + Sinh[2*x]/2
```

Maple [A] time = 0.002, size = 17, normalized size = 1.6

$$-\frac{\cosh(x) - \sinh(x)}{2\cosh(x) + 2\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
```

```
[Out] -1/2*(cosh(x)-sinh(x))/(cosh(x)+sinh(x))
```

Maxima [A] time = 1.08188, size = 8, normalized size = 0.73

$$-\frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*x)
```

Fricas [B] time = 2.28627, size = 68, normalized size = 6.18

$$-\frac{1}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [A] time = 0.331629, size = 10, normalized size = 0.91

$$-\frac{\cosh(x)}{\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
```

```
[Out] -cosh(x)/(sinh(x) + cosh(x))
```

Giac [A] time = 1.12036, size = 8, normalized size = 0.73

$$-\frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*x)
```

3.732 $\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx$

Optimal. Leaf size=14

$$-i\log(\cosh(x)+i\sinh(x))$$

```
[Out] (-I)*Log[Cosh[x] + I*Sinh[x]]
```

Rubi [A] time = 0.0266594, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$-i\log(\cosh(x)+i\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

```
[Out] (-I)*Log[Cosh[x] + I*Sinh[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx = -i\log(\cosh(x)+i\sinh(x))$$

Mathematica [A] time = 0.0337679, size = 15, normalized size = 1.07

$$\tan^{-1}(\tanh(x))-\frac{1}{2}i\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

```
[Out] ArcTan[Tanh[x]] - (I/2)*Log[Cosh[2*x]]
```

Maple [A] time = 0.027, size = 13, normalized size = 0.9

$$-i\ln\left(\cosh\left(x\right)+i\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
```

```
[Out] -I*ln(cosh(x)+I*sinh(x))
```

Maxima [A] time = 1.0486, size = 14, normalized size = 1.

$$-i\log\left(\cosh\left(x\right)+i\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*log(cosh(x) + I*sinh(x))
```

Fricas [A] time = 2.32311, size = 35, normalized size = 2.5

$$ix - i\log\left(e^{(2x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="fricas")
```

```
[Out] I*x - I*log(e^(2*x) - I)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [A] time = 1.11476, size = 18, normalized size = 1.29

$$ix - i \log\left(e^{(2x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="giac")
```

```
[Out] I*x - I*log(e^(2*x) - I)
```

3.733 $\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2
 - c^2)
```

Rubi [A] time = 0.0464645, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2
 - c^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.114514, size = 43, normalized size = 0.81

$$\frac{x(bB-cC)+(bC-Bc)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c
))
```

Maple [B] time = 0.057, size = 145, normalized size = 2.7

$$2\,\frac{B\ln(\tanh(x/2)+1)}{2\,b-2\,c}-2\,\frac{C\ln(\tanh(x/2)+1)}{2\,b-2\,c}-\frac{Bc}{(b-c)\,(b+c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}b+2\,c\tanh(x/2)+b\right)+\frac{bC}{(b-c)\,(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b+
c)*B*c*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+1/(b-c)/(b+c)*b*C*ln(tanh(1/2*
x)^2*b+2*c*tanh(1/2*x)+b)-2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(
tanh(1/2*x)-1)
```

Maxima [A] time = 1.11514, size = 117, normalized size = 2.21

$$C\left(\frac{b\log\left(-(b-c)e^{(-2\,x)}-b-c\right)}{b^{2}-c^{2}}+\frac{x}{b+c}\right)-B\left(\frac{c\log\left(-(b-c)e^{(-2\,x)}-b-c\right)}{b^{2}-c^{2}}-\frac{x}{b+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima"
)
```

```
[Out] C*(b*log(-(b - c)*e^(-2*x) - b - c)/(b^2 - c^2) + x/(b + c)) - B*(c*log(-(b
 - c)*e^(-2*x) - b - c)/(b^2 - c^2) - x/(b + c))
```

Fricas [A] time = 2.24471, size = 143, normalized size = 2.7

$$\frac{((B-C)b+(B-C)c)x+(Cb-Bc)\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas"
)
```

```
[Out] (((B - C)*b + (B - C)*c)*x + (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cos
h(x) - sinh(x))))/(b^2 - c^2)
```

Sympy [A] time = 0.883475, size = 326, normalized size = 6.15

$$\begin{cases} \tilde{\infty}\,(B\log(\sinh(x))+Cx) \\ \frac{Bx+C\log(\cosh(x))}{b} \\ \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{C\cos}{-2c\sinh(x)} \\ \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{C\cosh(x)}{2c\sinh(x)+2c\cosh} \\ \frac{Bbx}{b^2-c^2}-\frac{Bc\log\left(\frac{b\cosh(x)}{c}+\sinh(x)\right)}{b^2-c^2}+\frac{Cb\log\left(\frac{b\cosh(x)}{c}+\sinh(x)\right)}{b^2-c^2}-\frac{Ccx}{b^2-c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sinh(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*x + C*log(
cosh(x)))/b, Eq(c, 0)), (B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cos
h(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x))
- C*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*
c*cosh(x)) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (B*x*sinh(
x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) -
B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh
(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2
*c*cosh(x)), Eq(b, c)), (B*b*x/(b**2 - c**2) - B*c*log(b*cosh(x)/c + sinh(x
))/(b**2 - c**2) + C*b*log(b*cosh(x)/c + sinh(x))/(b**2 - c**2) - C*c*x/(b*
*2 - c**2), True))
```

Giac [A] time = 1.13779, size = 73, normalized size = 1.38

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(\left|be^{(2x)}+ce^{(2x)}+b-c\right|\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(abs(b*e^(2*x) + c*e^(2*x) + b - c))/(b^
2 - c^2)
```

3.734 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=78

$$\frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C)/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.0718705, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3153, 3074, 206}

$$\frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C)/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$
$$= \frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(i(bB-cC))\,\mathrm{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x)-ib$$
$$= \frac{(bB-cC)\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.228144, size = 87, normalized size = 1.12

$$\frac{2(bB-cC)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}+\frac{Bc-bC}{(b-c)(b+c)(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (B*c - b*C)/((b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x
]))
```

Maple [B] time = 0.073, size = 152, normalized size = 2.

$$2\,\frac{1}{(\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b}\left(\frac{c\,(Bc-bC)\tanh(x/2)}{\left(b^2-c^2\right)b}+\frac{Bc-bC}{b^2-c^2}\right)+2\,\frac{Bb}{\left(b^2-c^2\right)^{3/2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b+2}{\sqrt{b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(c*(B*c-C*b)/(b^2-c^2)/b*tanh(1/2*x)+(B*c-C*b)/(b^2-c^2))/(tanh(1/2*x)^2*
b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)
/(b^2-c^2)^(1/2))-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b
^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21843, size = 1789, normalized size = 22.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="frica
s")
```

```
[Out] [-((B*b^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 +
 2*(B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C
*c^2)*sinh(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x
)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b
+ c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b
 - c)) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c
 - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 -
 c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b
^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2), -2*((B*b^2 - (B +
```

```
C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B -
C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*
sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x)))
 + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*b*c^2
 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^
5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*
b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13586, size = 119, normalized size = 1.53

$$\frac{2\,(Bb-Cc)\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\,(Cbe^x-Bce^x)}{\left(b^2-c^2\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac"
)
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.735 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=71

$$\frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\sinh(x)(bB-cC)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

```
[Out] (B*c - b*C)/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2) + ((b*B - c*C)*Sinh[x
])/(b*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.0687751, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3156, 12, 3075}

$$\frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\sinh(x)(bB-cC)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (B*c - b*C)/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2) + ((b*B - c*C)*Sinh[x
])/(b*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx &= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}\\ &= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\int \frac{1}{(b\cosh(x)+c\sinh(x))^2}\,dx}{b^2-c^2}\\ &= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\sinh(x)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.158388, size = 70, normalized size = 0.99

$$\frac{C\left(c^2-b^2\right)+b\sinh(2x)(bB-cC)+c\cosh(2x)(bB-cC)}{2b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((-b^2 + c^2)*C + c*(b*B - c*C)*Cosh[2*x] + b*(b*B - c*C)*Sinh[2*x])/(2*b*(
b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x])^2)
```

Maple [A] time = 0.083, size = 63, normalized size = 0.9

$$-2\,\frac{1}{\left((\tanh(x/2))^2 b+2\,c\tanh(x/2)+b\right)^2}\left(-\frac{B\,(\tanh(x/2))^3}{b}-\frac{(Bc+bC)\,(\tanh(x/2))^2}{b^2}-\frac{B\tanh(x/2)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-B/b*tanh(1/2*x)^3-(B*c+C*b)/b^2*tanh(1/2*x)^2-B/b*tanh(1/2*x))/(tanh(1
/2*x)^2*b+2*c*tanh(1/2*x)+b)^2
```

Maxima [B] time = 1.11157, size = 455, normalized size = 6.41

$$2\,B\left(\frac{(b-c)e^{(-2\,x)}}{b^4-2\,b^2c^2+c^4+2\left(b^4-2\,b^3c+2\,bc^3-c^4\right)e^{(-2\,x)}+\left(b^4-4\,b^3c+6\,b^2c^2-4\,bc^3+c^4\right)e^{(-4\,x)}}+\frac{}{b^4-2\,b^2c^2+c^4+2\,(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxim
a")
```

```
[Out] 2*B*((b - c)*e^(-2*x)/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 -
 c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + b/
(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4
- 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x))) - 2*C*((b - c)*e^(-2*x)/(
b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 -
 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + c/(b^4 - 2*b^2*c^2 + c^4
+ 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 -
 4*b*c^3 + c^4)*e^(-4*x)))
```

Fricas [B] time = 1.98492, size = 556, normalized size = 7.83

$$-\frac{2\,(((2\,}{\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh\left(x\right)^3+3\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh\left(x\right)\sinh\left(x\right)^2+\left(b^4+4\,b^3c+6\,b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="frica
s")
```

```
[Out] -2*(((2*B + C)*b + B*c)*cosh(x) + (C*b + (B + 2*C)*c)*sinh(x))/((b^4 + 4*b^
3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)*sinh(x)^2 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 +
c^4)*sinh(x)^3 + (3*b^4 + 4*b^3*c - 2*b^2*c^2 - 4*b*c^3 - c^4)*cosh(x) + (b
^4 + 4*b^3*c + 2*b^2*c^2 - 4*b*c^3 - 3*c^4 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20253, size = 95, normalized size = 1.34

$$-\frac{2\left(Bbe^{(2x)}+Cbe^{(2x)}+Bce^{(2x)}+Cce^{(2x)}+Bb-Cc\right)}{\left(b^2+2bc+c^2\right)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac"
)
```

```
[Out] -2*(B*b*e^(2*x) + C*b*e^(2*x) + B*c*e^(2*x) + C*c*e^(2*x) + B*b - C*c)/((b^
2 + 2*b*c + c^2)*(b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

3.736 $\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=92

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 -
c^2]])/Sqrt[b^2 - c^2] - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^
2)
```

Rubi [A] time = 0.0696699, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3136, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 -
c^2]])/Sqrt[b^2 - c^2] - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^
2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx &= \frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx\\ &= \frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx\right.\\ &= \frac{(bB-cC)x}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.255864, size = 90, normalized size = 0.98

$$\frac{2A\sqrt{b-c}\sqrt{b+c}\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)+x(bB-cC)+(bC-Bc)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt
[b - c]*Sqrt[b + c])] + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)
*(b + c))
```

Maple [B] time = 0.058, size = 253, normalized size = 2.8

$$2\,\frac{B\ln(\tanh(x/2)+1)}{2\,b-2\,c}-2\,\frac{C\ln(\tanh(x/2)+1)}{2\,b-2\,c}-\frac{Bc}{(b-c)(b+c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b+2\,c\tanh(x/2)+b\right)+\frac{bC}{(b-c)(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b+
c)*B*c*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+1/(b-c)/(b+c)*b*C*ln(tanh(1/2*
x)^2*b+2*c*tanh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(
1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1
/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2-2*B/(2*b+2*c)*ln(tanh(1/2*x
)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22185, size = 701, normalized size = 7.62

$$\left[-\frac{\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right)-((B-C)b+(B-C)c)x-}{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="frica
s")
```

```
[Out] [-(sqrt(-b^2 + c^2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - ((B
 - C)*b + (B - C)*c)*x - (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x)
 - sinh(x))))/(b^2 - c^2), -(2*sqrt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b
 + c)*cosh(x) + (b + c)*sinh(x))) - ((B - C)*b + (B - C)*c)*x - (C*b - B*c)
*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))))/(b^2 - c^2)]
```

Sympy [A] time = 80.0006, size = 643, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(ta
nh(x/2) + 1) + B*log(tanh(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) +
2*c*cosh(x)) + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c
*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh
(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x))
 - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x) +
 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*
sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x
)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C
*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), (-A*sqrt(-b**2 + c**2)*log
(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + A*sqrt(-b**2 + c**
2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + B*b*x/(b**2
- c**2) - B*c*x/(b**2 - c**2) + 2*B*c*log(tanh(x/2) + 1)/(b**2 - c**2) - B*
c*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - B*c*log(tanh(
x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*x/(b**2 - c**2) - 2*
C*b*log(tanh(x/2) + 1)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b - sqrt(-b**2
 + c**2)/b)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)
/(b**2 - c**2) - C*c*x/(b**2 - c**2), True))
```

Giac [A] time = 1.13483, size = 120, normalized size = 1.3

$$\frac{2\,A\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\sqrt{b^{2}-c^{2}}}+\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}{b^{2}-c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] 2*A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + (B - C)*x/(b
- c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2)
```

3.737 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=88

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*
Sinh[x]))
```

Rubi [A] time = 0.0686693, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```


```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*
Sinh[x]))
```


Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$= \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(i(bB-cC))\operatorname{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh$$

$$= \frac{(bB-cC)\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.261468, size = 106, normalized size = 1.2

$$\frac{A\left(b^2-c^2\right)\sinh(x)+b(Bc-bC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2(bB-cC)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)
*(b + c)*(b*Cosh[x] + c*Sinh[x]))
```

Maple [A] time = 0.073, size = 167, normalized size = 1.9

$$-2\,\frac{1}{(\tanh(x/2))^2\,b+2\,c\tanh(x/2)+b}\left(-\frac{\left(Ab^2-Ac^2+Bc^2-Ccb\right)\tanh(x/2)}{b\left(b^2-c^2\right)}-\frac{Bc-bC}{b^2-c^2}\right)+2\,\frac{Bb}{\left(b^2-c^2\right)^{3/2}}\arctan\left(1/2\,\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)-(B*c-C*b)/(b^2-c^2))
/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*ta
nh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(
1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20108, size = 1897, normalized size = 21.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 + (B*b^2 - (B + C)*b*c + C*c^2
 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B - C)*b*c - C*c^2
)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*sqrt(-b^2 + c^
2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(C*b^3 - B*b^2*c - C
*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2
 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*
c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 +
```

```
b*c^4 + c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 -
(B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 +
(B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x
)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh
(x))) + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*
b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5
+ (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20307, size = 128, normalized size = 1.45

$$\frac{2\,(Bb-Cc)\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\,(Cbe^{x}-Bce^{x}+Ab-Ac)}{\left(b^{2}-c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b -
c))
```

3.738 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=135

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh
[x])^2) + (c*(b*B - c*C)*Cosh[x] + b*(b*B - c*C)*Sinh[x])/((b^2 - c^2)^2*(b
*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.139993, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3156, 3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh
[x])^2) + (c*(b*B - c*C)*Cosh[x] + b*(b*B - c*C)*Sinh[x])/((b^2 - c^2)^2*(b
*Cosh[x] + c*Sinh[x]))
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\ldots$$

$$= \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\ldots$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-cC)\cosh \ldots}{\left(b^2-c^2\right)^2 \ldots}$$

Mathematica [A] time = 0.828333, size = 146, normalized size = 1.08

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)+b(Bc-bC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{Ac+2\sinh(x)(bB-cC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*
(b*Cosh[x] + c*Sinh[x])^2) + (A*c + 2*(b*B - c*C)*Sinh[x])/(b*(b - c)*(b +
c)*(b*Cosh[x] + c*Sinh[x])))/2
```

Maple [A] time = 0.087, size = 228, normalized size = 1.7

$$2\,\frac{1}{\left(\left(\tanh\left(x/2\right)\right)^2b+2\,c\tanh\left(x/2\right)+b\right)^2}\left(-1/2\,\frac{\left(Ab^2-2\,Ac^2-2\,Bb^2+2\,Bc^2\right)\left(\tanh\left(x/2\right)\right)^3}{\left(b^2-c^2\right)b}+1/2\,\frac{\left(Ab^2c+2\,Ac^3+2\,Bcb\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^
2*c+2*A*c^3+2*B*b^2*c-2*B*c^3+2*C*b^3-2*C*b*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^
2+1/2*(A*b^2+2*A*c^2+2*B*b^2-2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)+1/2*A*c/(b^2-
c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/2*(2
*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39741, size = 4510, normalized size = 33.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")
```

```
[Out] [-1/2*(4*B*b^3 - 4*(2*B + C)*b^2*c + 4*(B + 2*C)*b*c^2 - 4*C*c^3 - 2*(A*b^3
 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*
c^3)*sinh(x)^3 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c
^3)*cosh(x)^2 + 2*(2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*c^2 + 2*(B
 + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c^3)*cosh(x))*sin
h(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(
b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh
(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sin
h(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c
^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6
)*cosh(x))*sinh(x)), -(2*B*b^3 - 2*(2*B + C)*b^2*c + 2*(B + 2*C)*b*c^2 - 2*
C*c^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - (A*b^3 + A*b^2*c -
A*b*c^2 - A*c^3)*sinh(x)^3 + 2*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2
 + (B + C)*c^3)*cosh(x)^2 + (2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*
c^2 + 2*(B + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x
)^2 + ((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*co
sh(x)*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A
*c^2 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c
```

```
+ A*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A
*b^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b
+ c)*cosh(x) + (b + c)*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh
(x) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A
*c^3)*cosh(x)^2 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*
c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b
*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6
)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 +
 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b
^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2
+ 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16206, size = 247, normalized size = 1.83

$$\frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}-2\,Cb^2e^{(2x)}+2\,Bc^2e^{(2x)}+2\,Cc^2e^{(2x)}-Ab^2e^x+Ac^2}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) - 2*C*b^2*e^(2*x) + 2
*B*c^2*e^(2*x) + 2*C*c^2*e^(2*x) - A*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*
```

```
c + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e^(2*x) + c*e^(2*x)
+ b - c)^2)
```

3.739 $\int (a+b\cosh(x)+c\sinh(x))^3\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}ax\left(2a^2+3b^2-3c^2\right)+\frac{1}{6}b\sinh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{6}c\cosh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+$$

```
[Out] (a*(2*a^2 + 3*b^2 - 3*c^2)*x)/2 + (c*(11*a^2 + 4*b^2 - 4*c^2)*Cosh[x])/6 +
(b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a*b*Sinh[x])*(a
+ b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*S
inh[x])^2)/3
```

Rubi [A] time = 0.131999, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3120, 3146, 2637, 2638}

$$\frac{1}{2}ax\left(2a^2+3b^2-3c^2\right)+\frac{1}{6}b\sinh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{6}c\cosh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2 - 3*c^2)*x)/2 + (c*(11*a^2 + 4*b^2 - 4*c^2)*Cosh[x])/6 +
(b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a*b*Sinh[x])*(a
+ b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*S
inh[x])^2)/3
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2+\frac{1}{3}\int (a+b\cosh(x)+c\sinh(x$$

$$=\frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x))+\frac{1}{3}(c\cosh(x)+b\sinh(x))(a$$

$$=\frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x))+\frac{1}{3}($$

$$=\frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{1}{6}c\left(11a^2+4b^2-4c^2\right)\cosh(x)+\frac{1}{6}b\left(11a^2+4b^2-4c^2\right)\sinh$$

Mathematica [A] time = 0.186675, size = 116, normalized size = 0.97

$$\frac{1}{12}\left(6ax\left(2a^2+3b^2-3c^2\right)+9b\sinh(x)\left(4a^2+b^2-c^2\right)+9c\cosh(x)\left(4a^2+b^2-c^2\right)+9a\left(b^2+c^2\right)\sinh(2x)+18abc\cosh$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2 - 3*c^2)*x + 9*c*(4*a^2 + b^2 - c^2)*Cosh[x] + 18*a*b*c
*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 9*b*(4*a^2 + b^2 - c^2)*Sinh[x] +
9*a*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])/12
```

Maple [A] time = 0.028, size = 130, normalized size = 1.1

$$a^3x+3\,a^2b\sinh(x)+3\,a^2c\cosh(x)+3\,ab^2\left(1/2\,\cosh(x)\sinh(x)+x/2\right)+3\,abc\left(\cosh(x)\right)^2+3\,ac^2\left(1/2\,\cosh(x)\sinh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] a^3*x+3*a^2*b*sinh(x)+3*a^2*c*cosh(x)+3*a*b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+3
*a*b*c*cosh(x)^2+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+b^3*(2/3+1/3*cosh(x)^2
)*sinh(x)+3*c*b^2*(1/3*cosh(x)*sinh(x)^2+1/3*cosh(x))+3*b*c^2*(1/3*sinh(x)*
cosh(x)^2-1/3*sinh(x))+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)
```

Maxima [A] time = 1.02551, size = 185, normalized size = 1.55

$$b^2c\cosh(x)^3+bc^2\sinh(x)^3+a^3x+\frac{1}{24}\,c^3\left(e^{(3\,x)}-9\,e^{(-x)}+e^{(-3\,x)}-9\,e^x\right)+\frac{1}{24}\,b^3\left(e^{(3\,x)}-9\,e^{(-x)}-e^{(-3\,x)}+9\,e^x\right)+3\,(c\,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + b*sinh(x))*a^2 + 3/8*(8*b*c*cosh(x)^2 + b^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

Fricas [A] time = 1.98616, size = 421, normalized size = 3.54

$$\frac{3}{2}\,abc\cosh(x)^2+\frac{1}{12}\left(3\,b^2c+c^3\right)\cosh(x)^3+\frac{1}{12}\left(b^3+3\,bc^2\right)\sinh(x)^3+\frac{1}{4}\left(6\,abc+\left(3\,b^2c+c^3\right)\cosh(x)\right)\sinh(x)^2+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 3/2*a*b*c*cosh(x)^2 + 1/12*(3*b^2*c + c^3)*cosh(x)^3 + 1/12*(b^3 + 3*b*c^2)
*sinh(x)^3 + 1/4*(6*a*b*c + (3*b^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(2*a^3
```

```
 + 3*a*b^2 - 3*a*c^2)*x - 3/4*(c^3 - (4*a^2 + b^2)*c)*cosh(x) + 1/4*(12*a^2
*b + 3*b^3 - 3*b*c^2 + (b^3 + 3*b*c^2)*cosh(x)^2 + 6*(a*b^2 + a*c^2)*cosh(x
))*sinh(x)
```

Sympy [A] time = 0.722612, size = 196, normalized size = 1.65

$$a^3x + 3a^2b\sinh(x) + 3a^2c\cosh(x) - \frac{3ab^2x\sinh^2(x)}{2} + \frac{3ab^2x\cosh^2(x)}{2} + \frac{3ab^2\sinh(x)\cosh(x)}{2} + 3abc\sinh^2(x) + \frac{3ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] a**3*x + 3*a**2*b*sinh(x) + 3*a**2*c*cosh(x) - 3*a*b**2*x*sinh(x)**2/2 + 3*
a*b**2*x*cosh(x)**2/2 + 3*a*b**2*sinh(x)*cosh(x)/2 + 3*a*b*c*sinh(x)**2 + 3
*a*c**2*x*sinh(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + 3*a*c**2*sinh(x)*cosh(x)
/2 - 2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + b**2*c*cosh(x)**3 + b*
c**2*sinh(x)**3 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

Giac [B] time = 1.12245, size = 296, normalized size = 2.49

$$\frac{1}{24}b^3e^{(3x)} + \frac{1}{8}b^2ce^{(3x)} + \frac{1}{8}bc^2e^{(3x)} + \frac{1}{24}c^3e^{(3x)} + \frac{3}{8}ab^2e^{(2x)} + \frac{3}{4}abce^{(2x)} + \frac{3}{8}ac^2e^{(2x)} + \frac{3}{2}a^2be^x + \frac{3}{8}b^3e^x + \frac{3}{2}a^2ce^x + \frac{3}{8}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*x) + 1/8*b^2*c*e^(3*x) + 1/8*b*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a*b^2*e^(2*x) + 3/4*a*b*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 3/2*a^2*b*e^
x + 3/8*b^3*e^x + 3/2*a^2*c*e^x + 3/8*b^2*c*e^x - 3/8*b*c^2*e^x - 3/8*c^3*e
^x + 1/2*(2*a^3 + 3*a*b^2 - 3*a*c^2)*x - 1/24*(b^3 - 3*b^2*c + 3*b*c^2 - c^
3 + 9*(4*a^2*b + b^3 - 4*a^2*c - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(a*b^2 -
2*a*b*c + a*c^2)*e^x)*e^(-3*x)
```

3.740 $\int (a+b\cosh(x)+c\sinh(x))^2\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}x\left(2a^2+b^2-c^2\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))+\frac{3}{2}ab\sinh(x)+\frac{3}{2}ac\cosh(x)$$

```
[Out] ((2*a^2 + b^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a*b*Sinh[x])/2 + ((c*Cos
h[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2
```

Rubi [A] time = 0.0354636, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(2a^2+b^2-c^2\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))+\frac{3}{2}ab\sinh(x)+\frac{3}{2}ac\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((2*a^2 + b^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a*b*Sinh[x])/2 + ((c*Cos
h[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a + b\cosh(x) + c\sinh(x))^2\, dx = \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x)) + \frac{1}{2}\int \left(2a^2 + b^2 - c^2 + 3ab\cosh\right.$$
$$= \frac{1}{2}\left(2a^2 + b^2 - c^2\right)x + \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x)) + \frac{1}{2}(3ab)\int$$
$$= \frac{1}{2}\left(2a^2 + b^2 - c^2\right)x + \frac{3}{2}ac\cosh(x) + \frac{3}{2}ab\sinh(x) + \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\,\mathrm{co}$$

Mathematica [A] time = 0.0814906, size = 54, normalized size = 0.92

$$\frac{1}{4}\left(2x\left(2a^2 + b^2 - c^2\right) + 8ab\sinh(x) + 8ac\cosh(x) + \left(b^2 + c^2\right)\sinh(2x) + 2bc\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 - c^2)*x + 8*a*c*Cosh[x] + 2*b*c*Cosh[2*x] + 8*a*b*Sinh[x]
+ (b^2 + c^2)*Sinh[2*x])/4
```

Maple [A] time = 0.023, size = 54, normalized size = 0.9

$$a^2x + 2\,ab\sinh(x) + 2\,ac\cosh(x) + b^2\left(\frac{\cosh(x)\sinh(x)}{2} + \frac{x}{2}\right) + b\,(\cosh(x))^2\,c + c^2\left(\frac{\cosh(x)\sinh(x)}{2} - \frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] a^2*x+2*a*b*sinh(x)+2*a*c*cosh(x)+b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+b*cosh(x)
^2*c+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Maxima [A] time = 1.00798, size = 85, normalized size = 1.44

$$bc\cosh(x)^2 + \frac{1}{8}\,b^2\left(4\,x + e^{(2\,x)} - e^{(-2\,x)}\right) - \frac{1}{8}\,c^2\left(4\,x - e^{(2\,x)} + e^{(-2\,x)}\right) + a^2x + 2\,(c\cosh(x) + b\sinh(x))a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + b*sinh(x))*a
```

Fricas [A] time = 1.95671, size = 171, normalized size = 2.9

$$\frac{1}{2}\,bc\cosh\left(x\right)^{2}+\frac{1}{2}\,bc\sinh\left(x\right)^{2}+2\,ac\cosh\left(x\right)+\frac{1}{2}\left(2\,a^{2}+b^{2}-c^{2}\right)x+\frac{1}{2}\left(4\,ab+\left(b^{2}+c^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*c*cosh(x)^2 + 1/2*b*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(2*a^2 + b^2 -
c^2)*x + 1/2*(4*a*b + (b^2 + c^2)*cosh(x))*sinh(x)
```

Sympy [A] time = 0.352485, size = 100, normalized size = 1.69

$$a^{2}x+2ab\sinh\left(x\right)+2ac\cosh\left(x\right)-\frac{b^{2}x\sinh^{2}\left(x\right)}{2}+\frac{b^{2}x\cosh^{2}\left(x\right)}{2}+\frac{b^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}+bc\sinh^{2}\left(x\right)+\frac{c^{2}x\sinh^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] a**2*x + 2*a*b*sinh(x) + 2*a*c*cosh(x) - b**2*x*sinh(x)**2/2 + b**2*x*cosh(
x)**2/2 + b**2*sinh(x)*cosh(x)/2 + b*c*sinh(x)**2 + c**2*x*sinh(x)**2/2 - c
**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

Giac [A] time = 1.13587, size = 112, normalized size = 1.9

$$\frac{1}{8}\,b^{2}e^{\left(2\,x\right)}+\frac{1}{4}\,bce^{\left(2\,x\right)}+\frac{1}{8}\,c^{2}e^{\left(2\,x\right)}+abe^{x}+ace^{x}+\frac{1}{2}\left(2\,a^{2}+b^{2}-c^{2}\right)x-\frac{1}{8}\left(b^{2}-2\,bc+c^{2}+8\left(ab-ac\right)e^{x}\right)e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e^(2*x) + 1/4*b*c*e^(2*x) + 1/8*c^2*e^(2*x) + a*b*e^x + a*c*e^x + 1
/2*(2*a^2 + b^2 - c^2)*x - 1/8*(b^2 - 2*b*c + c^2 + 8*(a*b - a*c)*e^x)*e^(-
2*x)
```

3.741 $\int (a+b\cosh(x)+c\sinh(x))\,dx$

Optimal. Leaf size=12

$$ax+b\sinh(x)+c\cosh(x)$$

```
[Out] a*x + c*Cosh[x] + b*Sinh[x]
```

Rubi [A] time = 0.0087491, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2637, 2638}

$$ax+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + b*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh(x)+c\sinh(x))\,dx &= ax+b\int \cosh(x)\,dx+c\int \sinh(x)\,dx\\ &= ax+c\cosh(x)+b\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.001935, size = 12, normalized size = 1.

$$ax+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + b*Sinh[x]
```

Maple [A] time = 0.002, size = 13, normalized size = 1.1

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(x)+c*sinh(x),x)
```

```
[Out] a*x+c*cosh(x)+b*sinh(x)
```

Maxima [A] time = 1.04637, size = 16, normalized size = 1.33

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="maxima")
```

```
[Out] a*x + c*cosh(x) + b*sinh(x)
```

Fricas [A] time = 1.96078, size = 39, normalized size = 3.25

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="fricas")
```

```
[Out] a*x + c*cosh(x) + b*sinh(x)
```

Sympy [A] time = 0.164501, size = 12, normalized size = 1.

$$ax + b\sinh(x) + c\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x)
```

```
[Out] a*x + b*sinh(x) + c*cosh(x)
```

Giac [B] time = 1.15804, size = 35, normalized size = 2.92

$$ax + \frac{1}{2}c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}b\left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="giac")
```

```
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x)
```

3.742 $\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] (-2*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rubi [A] time = 0.0687801, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)\right)$$

$$= -\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Mathematica [A] time = 0.0729275, size = 54, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (2*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
 - c^2]
```

Maple [A] time = 0.041, size = 53, normalized size = 1.

$$-2\,\frac{1}{\sqrt{-a^2+b^2-c^2}}\arctan\left(1/2\,\frac{2\,(a-b)\tanh(x/2)-2\,c}{\sqrt{-a^2+b^2-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09105, size = 660, normalized size = 12.94

$$\left[\frac{\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+2\,bc+c^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{a^2-b^2+c^2}((b+c)\cosh(x)+(}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)+b-c}\right.}{\sqrt{a^2-b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2
 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*c
osh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x
) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*c
osh(x) + a)*sinh(x) + b - c))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^
2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a
^2 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1667, size = 62, normalized size = 1.22

$$\frac{2 \arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

3.743 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{b\sinh(x)+c\cosh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] (-2*a*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 +
c^2)^(3/2) - (c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*
Sinh[x]))
```

Rubi [A] time = 0.0875916, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 12, 3124, 618, 206}

$$-\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{b\sinh(x)+c\cosh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 +
c^2)^(3/2) - (c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*
Sinh[x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx &= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{\int \frac{a}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\mathrm{t}\right.}{a^2-b^2+c^2}\\
&= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx,x,2c\right.}{a^2-b^2+c^2}\\
&= -\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.267378, size = 105, normalized size = 1.17

$$\frac{(b^2-c^2)\sinh(x)-ac}{b(-a^2+b^2-c^2)(a+b\cosh(x)+c\sinh(x))}-\frac{2a\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2
 - c^2)^(3/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x]))
```

Maple [B] time = 0.061, size = 191, normalized size = 2.1

$$-2\frac{1}{a(\tanh(x/2))^2-(\tanh(x/2))^2b-2c\tanh(x/2)-a-b}\left(-\frac{(ab-b^2+c^2)\tanh(x/2)}{a^3-a^2b-ab^2+ac^2+b^3-bc^2}-\frac{ac}{a^3-a^2b-ab^2+ac^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(a*b-b^2+c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(1/2*x)-a*c/(a^3-a
^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*
x)-a-b)-2*a/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2
*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19876, size = 2912, normalized size = 32.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 + 2*b*c^2 - 2*c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^
2 + (a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c
 + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*
c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)
*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2
*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(a^2 - b^2)*c +
2*(a^3 - a*b^2 + a*c^2)*cosh(x) + 2*(a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b -
 2*a^2*b^3 + b^5 + b*c^4 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b
*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b
^4)*c)*sinh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 +
 a*c^4 + 2*(a^3 - a*b^2)*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4
+ 2*(a^3 - a*b^2)*c^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b
^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x)
), 2*(a^2*b - b^3 + b*c^2 - c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^2 +
(a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
+ c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2 - b^2)*c + (a^3 - a*b^2 + a*c^2
)*cosh(x) + (a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b - 2*a^2*b^3 + b^5 + b*c^4
 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4*b - 2*a^2*b^3 + b^5
 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2
 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^
2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*sinh(x)^2 - (a^4
- 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)
*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13329, size = 150, normalized size = 1.67

$$\frac{2\,a\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}+\frac{2\,(ae^{x}+b-c)}{\left(a^{2}-b^{2}+c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2 + c^2)*s
qrt(-a^2 + b^2 - c^2)) + 2*(a*e^x + b - c)/((a^2 - b^2 + c^2)*(b*e^(2*x) +
c*e^(2*x) + 2*a*e^x + b - c))
```

3.744 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=146

$$-\frac{(2a^2+b^2-c^2)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}-\frac{b\sinh(x)+c\cos\ldots}{2(a^2-b^2+c^2)(a+b\cosh(\ldots}$$

```
[Out] -(((2*a^2 + b^2 - c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2
]])/(a^2 - b^2 + c^2)^(5/2)) - (c*Cosh[x] + b*Sinh[x])/(2*(a^2 - b^2 + c^2)
*(a + b*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a*b*Sinh[x]))/(2*(a^2 -
 b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.165141, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 3153, 3124, 618, 206}

$$-\frac{(2a^2+b^2-c^2)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}-\frac{b\sinh(x)+c\cos\ldots}{2(a^2-b^2+c^2)(a+b\cosh(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] -(((2*a^2 + b^2 - c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2
]])/(a^2 - b^2 + c^2)^(5/2)) - (c*Cosh[x] + b*Sinh[x])/(2*(a^2 - b^2 + c^2)
*(a + b*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a*b*Sinh[x]))/(2*(a^2 -
 b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2a+b\cosh(x)+c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+}$$

$$= -\frac{\left(2a^2+b^2-c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh}$$

Mathematica [A] time = 0.506649, size = 183, normalized size = 1.25

$$\frac{1}{2}\left(\frac{2\left(2a^2+b^2-c^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{\left(b^2-c^2\right)\sinh(x)-ac}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{c\left(2a^2+b^2-c^2\right)-3a\,(}{b\left(a^2-b^2+c^2\right)^2(a+b\,\mathrm{co}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2 - c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c
^2]])/(-a^2 + b^2 - c^2)^(5/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 +
b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (c*(2*a^2 + b^2 - c^2) - 3*a*(b
^2 - c^2)*Sinh[x])/(b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])))/2
```

Maple [B] time = 0.083, size = 747, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*a^3*b-7*a^2*b^2+5*a^2*c^2+2*a*b^3-2*a*b*c^2+b^4-3*b^2*c^2+2*c^4
)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*c*(4*
a^4-12*a^3*b+13*a^2*b^2-7*a^2*c^2-6*a*b^3+6*a*b*c^2+b^4+b^2*c^2-2*c^4)/(a^4
-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(
4*a^4*b-5*a^3*b^2+11*a^3*c^2-3*a^2*b^3-3*a^2*b*c^2+5*a*b^4-7*a*b^2*c^2+2*a*
c^4-b^5-b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tanh(1/2*x)+1/2*c*(4*a^4-3*a^2*b^2+a^2*c^2-b^4+b^2*c^2)/(a^4-2*a^
2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1
/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c
^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2
)^(1/2))*a^2-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b^2+1/(a^4-
2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.18564, size = 15389, normalized size = 105.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + 6*a^3*c^2 - 12*a*b*c^3 + 6*a*c^4 + 2*(2*a^4*b -
 a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2
*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^3 + 2*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c
^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*s
```

inh(x)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*cosh(x
)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2 + (2*a^4*b -
 a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2
*a^4 - a^2*b^2 - b^4)*c)*cosh(x))*sinh(x)^2 - ((2*a^2*b^2 + b^4 + 2*a^2*c^2
 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^4 + (2*a^2*b^2 + b^4 + 2*a^
2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*sinh(x)^4 + 2*a^2*b^2 + b^4 +
2*a^2*c^2 + 2*b*c^3 - c^4 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 +
 a*b^2)*c)*cosh(x)^3 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^
2)*c + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*
cosh(x))*sinh(x)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2
)*cosh(x)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2 + 3*(2
*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^2
 + 6*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x))*sinh(
x)^2 - 2*(2*a^2*b + b^3)*c + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 - (2*a^3
+ a*b^2)*c)*cosh(x) + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 + (2*a^2*b^2 + b
^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^3 + 3*(2*a^3*
b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x)^2 - (2*a^3 + a*b^2
)*c + (4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2)*cosh(x))*sinh(x
))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c
+ c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c
 + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*
cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*
a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 12*(a^3*b - a*b^3)*
c + 2*(10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (
11*a^2*b - 2*b^3)*c^2 - (10*a^4 - 11*a^2*b^2 + b^4)*c)*cosh(x) + 2*(10*a^4*
b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (11*a^2*b - 2*b
^3)*c^2 + 3*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a
^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^2 - (10*a^4 - 11*a^2
*b^2 + b^4)*c + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*c
osh(x))*sinh(x))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 - 2*b*c^7 + c^8 + (
3*a^2 - 2*b^2)*c^6 - 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + (a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a
^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 +
 (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)
*cosh(x)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^
2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2
*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7)*c)*sinh(x)^4 - 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + 4*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 +
 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3
*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x)^3 + 4*
(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*
c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b -
2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c + (a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a

^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c) *cosh(x))*sinh(x)^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x)^2 + 2*(2* a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)* c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3* b^4 - a*b^6)*c)*cosh(x))*sinh(x)^2 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 - a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*c osh(x) + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 - a*c^7 - 3*(a^ 3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^ 6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^ 5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + 3*(a^7*b - 3*a^5*b ^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^ 5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x)^2 - (a^7 - 3*a^5* b^2 + 3*a^3*b^4 - a*b^6)*c + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4 *b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x))*sinh(x)), (3*a^3*b^2 - 3*a*b^4 + 3* a^3*c^2 - 6*a*b*c^3 + 3*a*c^4 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a ^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^ 3 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2 *b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*sinh(x)^3 + 3*(2*a^5 - a^3*b^2 - a*b ^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*cosh(x)^2 + 3*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x))*s inh(x)^2 + ((2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^4 + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*sinh(x)^4 + 2*a^2*b^2 + b^4 + 2*a^2*c^2 + 2*b*c^3 - c^4 + 4*(2*a^ 3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x)^3 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x))*sinh(x)^3 + 2*(4*a^4 + 4*a ^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2)*cosh(x)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2 + 3*(2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^ 3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^2 + 6*(2*a^3*b + a*b^3 - a*b*c^2 - a

```
*c^3 + (2*a^3 + a*b^2)*c)*cosh(x))*sinh(x)^2 - 2*(2*a^2*b + b^3)*c + 4*(2*a
^3*b + a*b^3 - a*b*c^2 + a*c^3 - (2*a^3 + a*b^2)*c)*cosh(x) + 4*(2*a^3*b +
a*b^3 - a*b*c^2 + a*c^3 + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*
(2*a^2*b + b^3)*c)*cosh(x)^3 + 3*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^
3 + a*b^2)*c)*cosh(x)^2 - (2*a^3 + a*b^2)*c + (4*a^4 + 4*a^2*b^2 + b^4 + c^
4 - 2*(2*a^2 + b^2)*c^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sq
rt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c
^2)) - 6*(a^3*b - a*b^3)*c + (10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (
11*a^2 - 2*b^2)*c^3 + (11*a^2*b - 2*b^3)*c^2 - (10*a^4 - 11*a^2*b^2 + b^4)*
c)*cosh(x) + (10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*
c^3 + (11*a^2*b - 2*b^3)*c^2 + 3*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (
a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)
^2 - (10*a^4 - 11*a^2*b^2 + b^4)*c + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (
a^3 + 2*a*b^2)*c^2)*cosh(x))*sinh(x))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8 - 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 - 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^
2*b^2)*c^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^
2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2
*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7)*c)*cosh(x)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 +
2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^
2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 +
2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*sinh(x)^4 - 6*(a^4*b - 2*a^2*b^3
 + b^5)*c^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 +
3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c
^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*c)*cosh(x)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c
^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b
^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*c + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^
2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2
*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7)*c)*cosh(x))*sinh(x)^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 +
2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6
+ 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c
^2)*cosh(x)^2 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a
^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^
2*b^4 + 4*b^6)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c
^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*
(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b -
3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 +
 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x))*sinh(x)^2 - 2*(a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*c + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c
^6 - a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b
```

```
^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 - (a^7 - 3*a^5*b^2 + 3*
a^3*b^4 - a*b^6)*c)*cosh(x) + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*
b*c^6 - a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^
3*b^2 + a*b^4)*c^3 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8
 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a
^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2
 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*
b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5
*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh
(x)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c + (2*a^8 - 5*a^6*b^2 + 3*a^
4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)
*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18149, size = 410, normalized size = 2.81

$$\frac{\left(2\,a^2+b^2-c^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,a^2be^{(3\,x)}+b^3e^{(3\,x)}+2\,a^2ce^{(3\,x)}+b^2ce^{(3\,x)}-bc^2e^{(3\,x)}-c^3e^{(3\,x)}+\cdots}{\left(a^4-2\,a^2b^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*a^2 + b^2 - c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2))
+ (2*a^2*b*e^(3*x) + b^3*e^(3*x) + 2*a^2*c*e^(3*x) + b^2*c*e^(3*x) - b*c^2*
e^(3*x) - c^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a*b^2*e^(2*x) - 3*a*c^2*e^(2*x) +
 10*a^2*b*e^x - b^3*e^x - 10*a^2*c*e^x + b^2*c*e^x + b*c^2*e^x - c^3*e^x +
```

```
3*a*b^2 - 6*a*b*c + 3*a*c^2)/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^
2 + c^4)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

3.745 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

Optimal. Leaf size=220

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{6\left(a^2-b^2+c^2\right)^3(a+b\cosh(x)+c\sinh(x))}-\frac{5(\ldots}{6\left(a^2-b^2+\ldots\right.}$$

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2
 + c^2]])/(a^2 - b^2 + c^2)^(7/2)) - (c*Cosh[x] + b*Sinh[x])/(3*(a^2 - b^2
+ c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a*b*Sinh[x]))/(6*
(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2) - (c*(11*a^2 + 4*b^2 - 4
*c^2)*Cosh[x] + b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/(6*(a^2 - b^2 + c^2)^3*
(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.301706, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3129, 3156, 3153, 3124, 618, 206}

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{6\left(a^2-b^2+c^2\right)^3(a+b\cosh(x)+c\sinh(x))}-\frac{5(\ldots}{6\left(a^2-b^2+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```


```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2
 + c^2]])/(a^2 - b^2 + c^2)^(7/2)) - (c*Cosh[x] + b*Sinh[x])/(3*(a^2 - b^2
+ c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a*b*Sinh[x]))/(6*
(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2) - (c*(11*a^2 + 4*b^2 - 4
*c^2)*Cosh[x] + b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/(6*(a^2 - b^2 + c^2)^3*
(a + b*Cosh[x] + c*Sinh[x]))
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
```

```
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{\int \frac{-3a+2b\cosh(x)+2c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx}{3\left(a^2-b^2+c^2\right)}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+ab\sinh(x)}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+ab\sinh(x)}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+ab\sinh(x)}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+ab\sinh(x)}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c}$$

$$= -\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}} - \frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\,\mathrm{si}}$$

Mathematica [B] time = 0.996838, size = 488, normalized size = 2.22

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{7/2}} - \frac{-72a^2b^2c^2\sinh(x)-30a^2bc\cosh(x)\left(2a^2+3b^2-3c^2\right)-6ac\cosh(2x)\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^
2 - c^2]])/(-a^2 + b^2 - c^2)^(7/2)) - (-44*a^5*c - 82*a^3*b^2*c - 24*a*b^4
*c + 82*a^3*c^3 + 48*a*b^2*c^3 - 24*a*c^5 - 30*a^2*b*c*(2*a^2 + 3*b^2 - 3*c
^2)*Cosh[x] - 6*a*c*(a^2*(-7*b^2 + 11*c^2) + 2*(b^4 + b^2*c^2 - 2*c^4))*Cos
h[2*x] + 22*a^2*b^3*c*Cosh[3*x] + 8*b^5*c*Cosh[3*x] - 22*a^2*b*c^3*Cosh[3*x
] - 16*b^3*c^3*Cosh[3*x] + 8*b*c^5*Cosh[3*x] + 72*a^4*b^2*Sinh[x] - 9*a^2*b
^4*Sinh[x] + 12*b^6*Sinh[x] - 132*a^4*c^2*Sinh[x] - 72*a^2*b^2*c^2*Sinh[x]
- 36*b^4*c^2*Sinh[x] + 81*a^2*c^4*Sinh[x] + 36*b^2*c^4*Sinh[x] - 12*c^6*Sin
```

```
h[x] + 54*a^3*b^3*Sinh[2*x] + 6*a*b^5*Sinh[2*x] - 78*a^3*b*c^2*Sinh[2*x] -
48*a*b^3*c^2*Sinh[2*x] + 42*a*b*c^4*Sinh[2*x] + 11*a^2*b^4*Sinh[3*x] + 4*b^
6*Sinh[3*x] - 4*b^4*c^2*Sinh[3*x] - 11*a^2*c^4*Sinh[3*x] - 4*b^2*c^4*Sinh[3
*x] + 4*c^6*Sinh[3*x])/(24*b*(a^2 - b^2 + c^2)^3*(a + b*Cosh[x] + c*Sinh[x]
)^3)
```

Maple [B] time = 0.118, size = 1842, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^4,x)
```

```
[Out] -2*(-1/2*(6*a^5*b-15*a^4*b^2+9*a^4*c^2+11*a^3*b^3-9*a^3*b*c^2-3*a^2*b^4-3*a
^2*b^2*c^2+6*a^2*c^4+3*a*b^5-3*a*b^3*c^2-2*b^6+6*b^4*c^2-6*b^2*c^4+2*c^6)/(
a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b
^2*c^4+c^6)/(a-b)*tanh(1/2*x)^5-1/2*c*(6*a^6-30*a^5*b+57*a^4*b^2-27*a^4*c^2
-55*a^3*b^3+45*a^3*b*c^2+33*a^2*b^4-21*a^2*b^2*c^2-12*a^2*c^4-15*a*b^5+15*a
*b^3*c^2+4*b^6-12*b^4*c^2+12*b^2*c^4-4*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*
b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^2-2*a*b+b^2)*ta
nh(1/2*x)^4+1/3*(18*a^7*b-54*a^6*b^2+54*a^6*c^2+38*a^5*b^3-120*a^5*b*c^2+30
*a^4*b^4+81*a^4*b^2*c^2-21*a^4*c^4-50*a^3*b^5-61*a^3*b^3*c^2+81*a^3*b*c^4+2
2*a^2*b^6+87*a^2*b^4*c^2-105*a^2*b^2*c^4-4*a^2*c^6-6*a*b^7-39*a*b^5*c^2+51*
a*b^3*c^4-6*a*b*c^6+2*b^8-2*b^6*c^2-6*b^4*c^4+10*b^2*c^6-4*c^8)/(a^6-3*a^4*
b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*x)^3+c*(6*a^7-18*a^6*b+18*a^5*b^2-20*a
^5*c^2-2*a^4*b^3+22*a^4*b*c^2-14*a^3*b^4+7*a^3*b^2*c^2-3*a^3*c^4+18*a^2*b^5
-6*a^2*b^3*c^2-12*a^2*b*c^4-10*a*b^6+3*a*b^4*c^2+9*a*b^2*c^4-2*a*c^6+2*b^7-
6*b^5*c^2+6*b^3*c^4-2*b*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c
^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^2-2*a*b+b^2)/(a-b)*tanh(1/2*x)
^2-1/2*(6*a^7*b-9*a^6*b^2+27*a^6*c^2-7*a^5*b^3-9*a^5*b*c^2+16*a^4*b^4-30*a^
4*b^2*c^2+4*a^4*c^4-4*a^3*b^5+14*a^3*b*c^4-5*a^2*b^6-3*a^2*b^4*c^2+6*a^2*b^
2*c^4+2*a^2*c^6+5*a*b^7+9*a*b^5*c^2-18*a*b^3*c^4+4*a*b*c^6-2*b^8+6*b^6*c^2-
6*b^4*c^4+2*b^2*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2
*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*x)-1/6
*a*c*(18*a^6-21*a^4*b^2+5*a^4*c^2-12*a^2*b^4+16*a^2*b^2*c^2+2*a^2*c^4+15*b^
6-21*b^4*c^2+6*b^2*c^4)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*
a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^3-3*a^2*b+3*a*b^2-b^3))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^3-2*a^3/(a^6-3*a^4*b^2+3*a^4*c^2
+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(-a^2+b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))-3*a/(a
```

```
^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^
2*c^4+c^6)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*b^2+3*a/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*
a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20635, size = 968, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

```
[Out] (2*a^3 + 3*a*b^2 - 3*a*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2 - 6*a^2*b^2*c^2 + 3*b^4
*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*sqrt(-a^2 + b^2 - c^2)) + 1/3*(6*a^3*b^
2*e^(5*x) + 9*a*b^4*e^(5*x) + 12*a^3*b*c*e^(5*x) + 18*a*b^3*c*e^(5*x) + 6*a
^3*c^2*e^(5*x) - 18*a*b*c^3*e^(5*x) - 9*a*c^4*e^(5*x) + 30*a^4*b*e^(4*x) +
45*a^2*b^3*e^(4*x) + 30*a^4*c*e^(4*x) + 45*a^2*b^2*c*e^(4*x) - 45*a^2*b*c^2
*e^(4*x) - 45*a^2*c^3*e^(4*x) + 44*a^5*e^(3*x) + 82*a^3*b^2*e^(3*x) + 24*a*
b^4*e^(3*x) - 82*a^3*c^2*e^(3*x) - 48*a*b^2*c^2*e^(3*x) + 24*a*c^4*e^(3*x)
+ 102*a^4*b*e^(2*x) + 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) - 102*a^4*c*e^(2*
x) - 36*a^2*b^2*c*e^(2*x) - 12*b^4*c*e^(2*x) - 36*a^2*b*c^2*e^(2*x) - 24*b^
3*c^2*e^(2*x) + 36*a^2*c^3*e^(2*x) + 24*b^2*c^3*e^(2*x) + 12*b*c^4*e^(2*x)
- 12*c^5*e^(2*x) + 60*a^3*b^2*e^x + 15*a*b^4*e^x - 120*a^3*b*c*e^x - 30*a*b
^3*c*e^x + 60*a^3*c^2*e^x + 30*a*b*c^3*e^x - 15*a*c^4*e^x + 11*a^2*b^3 + 4*
b^5 - 33*a^2*b^2*c - 12*b^4*c + 33*a^2*b*c^2 + 8*b^3*c^2 - 11*a^2*c^3 + 8*b
^2*c^3 - 12*b*c^4 + 4*c^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2
- 6*a^2*b^2*c^2 + 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*(b*e^(2*x) + c*e
^(2*x) + 2*a*e^x + b - c)^3)
```

3.746 $\int (a + a\cosh(x) + c\sinh(x))^3\,dx$

Optimal. Leaf size=105

$$\frac{1}{2}ax\left(5a^2-3c^2\right)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)+\frac{5}{6}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)+a+c\sin$$

```
[Out] (a*(5*a^2 - 3*c^2)*x)/2 + (c*(15*a^2 - 4*c^2)*Cosh[x])/6 + (a*(15*a^2 - 4*c
^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a^2*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])
)/6 + ((c*Cosh[x] + a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])^2)/3
```

Rubi [A] time = 0.116252, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3120, 3146, 2637, 2638}

$$\frac{1}{2}ax\left(5a^2-3c^2\right)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)+\frac{5}{6}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)+a+c\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*x)/2 + (c*(15*a^2 - 4*c^2)*Cosh[x])/6 + (a*(15*a^2 - 4*c
^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a^2*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])
)/6 + ((c*Cosh[x] + a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])^2)/3
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
```

```
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a + a\cosh(x) + c\sinh(x))^3\,dx = \frac{1}{3}(c\cosh(x) + a\sinh(x))(a + a\cosh(x) + c\sinh(x))^2 + \frac{1}{3}\int (a + a\cosh(x) + c\sinh$$

$$= \frac{5}{6}\left(ac\cosh(x) + a^2\sinh(x)\right)(a + a\cosh(x) + c\sinh(x)) + \frac{1}{3}(c\cosh(x) + a\sinh(x))($$

$$= \frac{1}{2}a\left(5a^2 - 3c^2\right)x + \frac{5}{6}\left(ac\cosh(x) + a^2\sinh(x)\right)(a + a\cosh(x) + c\sinh(x)) + \frac{1}{3}(c\,\mathrm{co}$$

$$= \frac{1}{2}a\left(5a^2 - 3c^2\right)x + \frac{1}{6}c\left(15a^2 - 4c^2\right)\cosh(x) + \frac{1}{6}a\left(15a^2 - 4c^2\right)\sinh(x) + \frac{5}{6}\left(ac\cosh$$

Mathematica [A] time = 0.163128, size = 112, normalized size = 1.07

$$\frac{1}{12}\left(-9c\left(c^2 - 5a^2\right)\cosh(x) + 18a^2c\cosh(2x) + 3a^2c\cosh(3x) + 30a^3x + 45a^3\sinh(x) + 9a^3\sinh(2x) + a^3\sinh(3x) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*a^3*x - 18*a*c^2*x - 9*c*(-5*a^2 + c^2)*Cosh[x] + 18*a^2*c*Cosh[2*x] +
3*a^2*c*Cosh[3*x] + c^3*Cosh[3*x] + 45*a^3*Sinh[x] - 9*a*c^2*Sinh[x] + 9*a^
3*Sinh[2*x] + 9*a*c^2*Sinh[2*x] + a^3*Sinh[3*x] + 3*a*c^2*Sinh[3*x])/12
```

Maple [A] time = 0.035, size = 129, normalized size = 1.2

$$a^3x + 3\,a^3\sinh(x) + 3\,a^2c\cosh(x) + 3\,a^3\,(1/2\,\cosh(x)\sinh(x) + x/2) + 3\,a^2c\,(\cosh(x))^2 + 3\,ac^2\,(1/2\,\cosh(x)\sinh(x) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] a^3*x+3*a^3*sinh(x)+3*a^2*c*cosh(x)+3*a^3*(1/2*cosh(x)*sinh(x)+1/2*x)+3*a^2
*c*cosh(x)^2+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+a^3*(2/3+1/3*cosh(x)^2)*si
nh(x)+3*a^2*c*(1/3*cosh(x)*sinh(x)^2+1/3*cosh(x))+3*a*c^2*(1/3*sinh(x)*cosh
(x)^2-1/3*sinh(x))+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)
```

Maxima [A] time = 1.0289, size = 185, normalized size = 1.76

$$a^2c\cosh(x)^3 + ac^2\sinh(x)^3 + a^3x + \frac{1}{24}\,c^3\left(e^{(3\,x)} - 9\,e^{(-x)} + e^{(-3\,x)} - 9\,e^x\right) + \frac{1}{24}\,a^3\left(e^{(3\,x)} - 9\,e^{(-x)} - e^{(-3\,x)} + 9\,e^x\right) + 3\,(c\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*c*cosh(x)^3 + a*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + a*sinh(x))*a^2 + 3/8*(8*a*c*cosh(x)^2 + a^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

Fricas [A] time = 2.41206, size = 381, normalized size = 3.63

$$\frac{3}{2}\,a^2c\cosh(x)^2 + \frac{1}{12}\left(3\,a^2c + c^3\right)\cosh(x)^3 + \frac{1}{12}\left(a^3 + 3\,ac^2\right)\sinh(x)^3 + \frac{1}{4}\left(6\,a^2c + \left(3\,a^2c + c^3\right)\cosh(x)\right)\sinh(x)^2 + \frac{1}{2}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 3/2*a^2*c*cosh(x)^2 + 1/12*(3*a^2*c + c^3)*cosh(x)^3 + 1/12*(a^3 + 3*a*c^2)
*sinh(x)^3 + 1/4*(6*a^2*c + (3*a^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(5*a^3
```

```
 - 3*a*c^2)*x + 3/4*(5*a^2*c - c^3)*cosh(x) + 1/4*(15*a^3 - 3*a*c^2 + (a^3
+ 3*a*c^2)*cosh(x)^2 + 6*(a^3 + a*c^2)*cosh(x))*sinh(x)
```

Sympy [A] time = 1.06227, size = 189, normalized size = 1.8

$$-\frac{3a^3x\sinh^2(x)}{2}+\frac{3a^3x\cosh^2(x)}{2}+a^3x-\frac{2a^3\sinh^3(x)}{3}+a^3\sinh(x)\cosh^2(x)+\frac{3a^3\sinh(x)\cosh(x)}{2}+3a^3\sinh(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] -3*a**3*x*sinh(x)**2/2 + 3*a**3*x*cosh(x)**2/2 + a**3*x - 2*a**3*sinh(x)**3
/3 + a**3*sinh(x)*cosh(x)**2 + 3*a**3*sinh(x)*cosh(x)/2 + 3*a**3*sinh(x) +
3*a**2*c*sinh(x)**2 + a**2*c*cosh(x)**3 + 3*a**2*c*cosh(x) + 3*a*c**2*x*sin
h(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + a*c**2*sinh(x)**3 + 3*a*c**2*sinh(x)*
cosh(x)/2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

Giac [A] time = 1.11794, size = 251, normalized size = 2.39

$$\frac{1}{24}a^3e^{(3x)}+\frac{1}{8}a^2ce^{(3x)}+\frac{1}{8}ac^2e^{(3x)}+\frac{1}{24}c^3e^{(3x)}+\frac{3}{8}a^3e^{(2x)}+\frac{3}{4}a^2ce^{(2x)}+\frac{3}{8}ac^2e^{(2x)}+\frac{15}{8}a^3e^x+\frac{15}{8}a^2ce^x-\frac{3}{8}ac^2e^x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*c*e^(3*x) + 1/8*a*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a^3*e^(2*x) + 3/4*a^2*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 15/8*a^3*e^x +
 15/8*a^2*c*e^x - 3/8*a*c^2*e^x - 3/8*c^3*e^x + 1/2*(5*a^3 - 3*a*c^2)*x - 1
/24*(a^3 - 3*a^2*c + 3*a*c^2 - c^3 + 9*(5*a^3 - 5*a^2*c - a*c^2 + c^3)*e^(2
*x) + 9*(a^3 - 2*a^2*c + a*c^2)*e^x)*e^(-3*x)
```

3.747 $\int (a + a\cosh(x) + c\sinh(x))^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}x\left(3a^2-c^2\right)+\frac{3}{2}a^2\sinh(x)+\frac{3}{2}ac\cosh(x)+\frac{1}{2}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))$$

```
[Out] ((3*a^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a^2*Sinh[x])/2 + ((c*Cosh[x] +
 a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x]))/2
```

Rubi [A] time = 0.0345935, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(3a^2-c^2\right)+\frac{3}{2}a^2\sinh(x)+\frac{3}{2}ac\cosh(x)+\frac{1}{2}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((3*a^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a^2*Sinh[x])/2 + ((c*Cosh[x] +
 a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x]))/2
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))+\frac{1}{2}\int \left(3a^2-c^2+3a^2\cosh(x)\right.$$
$$= \frac{1}{2}\left(3a^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))+\frac{1}{2}\left(3a^2\right)\int c$$
$$= \frac{1}{2}\left(3a^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}a^2\sinh(x)+\frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh($$

Mathematica [A] time = 0.0697247, size = 55, normalized size = 0.96

$$\frac{1}{2}x\left(3a^2-c^2\right)+\frac{1}{4}\left(a^2+c^2\right)\sinh(2x)+2a^2\sinh(x)+2ac\cosh(x)+\frac{1}{2}ac\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((3*a^2 - c^2)*x)/2 + 2*a*c*Cosh[x] + (a*c*Cosh[2*x])/2 + 2*a^2*Sinh[x] + (
(a^2 + c^2)*Sinh[2*x])/4
```

Maple [A] time = 0.02, size = 55, normalized size = 1.

$$a^2x+2\,a^2\sinh(x)+2\,ac\cosh(x)+a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ac\left(\cosh(x)\right)^2+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] a^2*x+2*a^2*sinh(x)+2*a*c*cosh(x)+a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*c*cosh(
x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Maxima [A] time = 0.98096, size = 85, normalized size = 1.49

$$ac\cosh(x)^2+\frac{1}{8}a^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-\frac{1}{8}c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)+a^2x+2\,(c\cosh(x)+a\sinh(x))a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] a*c*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + a*sinh(x))*a
```

Fricas [A] time = 2.32193, size = 163, normalized size = 2.86

$$\frac{1}{2}\,ac\cosh\left(x\right)^{2}+\frac{1}{2}\,ac\sinh\left(x\right)^{2}+2\,ac\cosh\left(x\right)+\frac{1}{2}\left(3\,a^{2}-c^{2}\right)x+\frac{1}{2}\left(4\,a^{2}+\left(a^{2}+c^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*c*cosh(x)^2 + 1/2*a*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(3*a^2 - c^2)*x
 + 1/2*(4*a^2 + (a^2 + c^2)*cosh(x))*sinh(x)
```

Sympy [A] time = 0.433168, size = 100, normalized size = 1.75

$$-\frac{a^{2}x\sinh^{2}\left(x\right)}{2}+\frac{a^{2}x\cosh^{2}\left(x\right)}{2}+a^{2}x+\frac{a^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}+2a^{2}\sinh\left(x\right)+ac\sinh^{2}\left(x\right)+2ac\cosh\left(x\right)+\frac{c^{2}x\sinh^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*x + a**2*sinh(x)*cosh(x)/
2 + 2*a**2*sinh(x) + a*c*sinh(x)**2 + 2*a*c*cosh(x) + c**2*x*sinh(x)**2/2 -
 c**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

Giac [A] time = 1.14582, size = 109, normalized size = 1.91

$$\frac{1}{8}\,a^{2}e^{\left(2\,x\right)}+\frac{1}{4}\,ace^{\left(2\,x\right)}+\frac{1}{8}\,c^{2}e^{\left(2\,x\right)}+a^{2}e^{x}+ace^{x}+\frac{1}{2}\left(3\,a^{2}-c^{2}\right)x-\frac{1}{8}\left(a^{2}-2\,ac+c^{2}+8\left(a^{2}-ac\right)e^{x}\right)e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*c*e^(2*x) + 1/8*c^2*e^(2*x) + a^2*e^x + a*c*e^x + 1
/2*(3*a^2 - c^2)*x - 1/8*(a^2 - 2*a*c + c^2 + 8*(a^2 - a*c)*e^x)*e^(-2*x)
```

3.748 $\int (a + a\cosh(x) + c\sinh(x))\,dx$

Optimal. Leaf size=12

$$ax + a\sinh(x) + c\cosh(x)$$

```
[Out] a*x + c*Cosh[x] + a*Sinh[x]
```

Rubi [A] time = 0.0091605, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2637, 2638}

$$ax + a\sinh(x) + c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + a*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\cosh(x) + c\sinh(x))\,dx &= ax + a\int \cosh(x)\,dx + c\int \sinh(x)\,dx\\ &= ax + c\cosh(x) + a\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0019364, size = 12, normalized size = 1.

$$ax + a\sinh(x) + c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + a*Sinh[x]
```

Maple [A] time = 0., size = 13, normalized size = 1.1

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*cosh(x)+c*sinh(x),x)
```

```
[Out] a*x+c*cosh(x)+a*sinh(x)
```

Maxima [A] time = 0.998106, size = 16, normalized size = 1.33

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="maxima")
```

```
[Out] a*x + c*cosh(x) + a*sinh(x)
```

Fricas [A] time = 2.32872, size = 39, normalized size = 3.25

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="fricas")
```

```
[Out] a*x + c*cosh(x) + a*sinh(x)
```

Sympy [A] time = 0.27556, size = 12, normalized size = 1.

$$ax + a\sinh(x) + c\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x)
```

```
[Out] a*x + a*sinh(x) + c*cosh(x)
```

Giac [B] time = 1.13995, size = 35, normalized size = 2.92

$$ax + \frac{1}{2}c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}a\left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="giac")
```

```
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

3.749 $\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

```
[Out] Log[a + c*Tanh[x/2]]/c
```

Rubi [A] time = 0.0187025, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3124, 31}

$$\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] Log[a + c*Tanh[x/2]]/c
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}\end{aligned}$$

Mathematica [B] time = 0.0366816, size = 35, normalized size = 2.33

$$\frac{\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}{c}-\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] -(Log[Cosh[x/2]]/c) + Log[a*Cosh[x/2] + c*Sinh[x/2]]/c
```

Maple [A] time = 0.036, size = 14, normalized size = 0.9

$$\frac{1}{c}\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x)),x)
```

```
[Out] ln(a+c*tanh(1/2*x))/c
```

Maxima [B] time = 1.02476, size = 49, normalized size = 3.27

$$\frac{\log\left(-(a-c)e^{(-x)}-a-c\right)}{c}-\frac{\log\left(e^{(-x)}+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] log(-(a - c)*e^(-x) - a - c)/c - log(e^(-x) + 1)/c
```

Fricas [B] time = 2.29608, size = 109, normalized size = 7.27

$$\frac{\log\left((a+c)\cosh\left(x\right)+(a+c)\sinh\left(x\right)+a-c\right)-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] (log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - log(cosh(x) + sinh(x) + 1
))/c
```

Sympy [A] time = 1.65989, size = 17, normalized size = 1.13

$$\begin{cases} \frac{\log\left(\frac{a}{c}+\tanh\left(\frac{x}{2}\right)\right)}{c} & \text{for } c \neq 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((log(a/c + tanh(x/2))/c, Ne(c, 0)), (tanh(x/2)/a, True))
```

Giac [B] time = 1.15186, size = 53, normalized size = 3.53

$$\frac{(a+c)\log\left(|ae^x+ce^x+a-c|\right)}{ac+c^2}-\frac{\log\left(e^x+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] (a + c)*log(abs(a*e^x + c*e^x + a - c))/(a*c + c^2) - log(e^x + 1)/c
```

3.750 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=43

$$\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

```
[Out] (a*Log[a + c*Tanh[x/2]])/c^3 - (c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x]
+ c*Sinh[x]))
```

Rubi [A] time = 0.0406585, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3129, 12, 3124, 31}

$$\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (a*Log[a + c*Tanh[x/2]])/c^3 - (c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x]
+ c*Sinh[x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx &= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{\int \frac{a}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a+2cx}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^2}\\
&= \frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [B] time = 0.291693, size = 87, normalized size = 2.02

$$
\frac{\frac{c(c^2-a^2)\sinh\left(\frac{x}{2}\right)}{a\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}+2a\left(\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)-c\tanh\left(\frac{x}{2}\right)}{2c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (2*a*(-Log[Cosh[x/2]] + Log[a*Cosh[x/2] + c*Sinh[x/2]]) + (c*(-a^2 + c^2)*S
inh[x/2])/(a*(a*Cosh[x/2] + c*Sinh[x/2])) - c*Tanh[x/2])/(2*c^3)
```

Maple [A] time = 0.063, size = 58, normalized size = 1.4

$$
-\frac{1}{2c^2}\tanh\left(\frac{x}{2}\right)+\frac{a^2}{2c^3}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{2c}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{a}{c^3}\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -1/2/c^2*tanh(1/2*x)+1/2/c^3/(a+c*tanh(1/2*x))*a^2-1/2/c/(a+c*tanh(1/2*x))+
a*ln(a+c*tanh(1/2*x))/c^3
```

Maxima [B] time = 1.04669, size = 116, normalized size = 2.7

$$-\frac{2\left(ae^{(-x)}+a+c\right)}{2\,ac^2e^{(-x)}+ac^2+c^3+\left(ac^2-c^3\right)e^{(-2\,x)}}+\frac{a\log\left(-(a-c)e^{(-x)}-a-c\right)}{c^3}-\frac{a\log\left(e^{(-x)}+1\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + a + c)/(2*a*c^2*e^(-x) + a*c^2 + c^3 + (a*c^2 - c^3)*e^(-2*x
)) + a*log(-(a - c)*e^(-x) - a - c)/c^3 - a*log(e^(-x) + 1)/c^3
```

Fricas [B] time = 2.41647, size = 657, normalized size = 15.28

$$\frac{2\,ac\cosh\left(x\right)+2\,ac\sinh\left(x\right)+2\,ac-2\,c^2+\left(2\,a^2\cosh\left(x\right)+\left(a^2+ac\right)\cosh\left(x\right)^2+\left(a^2+ac\right)\sinh\left(x\right)^2+a^2-ac+2\left(a^2+\right.\right.}{2\,ac^3\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*c*cosh(x) + 2*a*c*sinh(x) + 2*a*c - 2*c^2 + (2*a^2*cosh(x) + (a^2 + a*
c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 + (a^2 + a*c)*cos
h(x))*sinh(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - (2*a^2*cosh
(x) + (a^2 + a*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 +
(a^2 + a*c)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1))/(2*a*c^3*cosh(x)
+ a*c^3 - c^4 + (a*c^3 + c^4)*cosh(x)^2 + (a*c^3 + c^4)*sinh(x)^2 + 2*(a*c^
3 + (a*c^3 + c^4)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13695, size = 113, normalized size = 2.63

$$\frac{\left(a^{2}+ac\right)\log\left(|ae^{x}+ce^{x}+a-c|\right)}{ac^{3}+c^{4}}-\frac{a\log\left(e^{x}+1\right)}{c^{3}}+\frac{2\left(ae^{x}+a-c\right)}{\left(ae^{(2x)}+ce^{(2x)}+2\,ae^{x}+a-c\right)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + a*c)*log(abs(a*e^x + c*e^x + a - c))/(a*c^3 + c^4) - a*log(e^x + 1)/
c^3 + 2*(a*e^x + a - c)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)*c^2)
```

3.751 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=89

$$\frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{3\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^4(a\cosh(x)+a+c\sinh(x))}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

```
[Out] ((3*a^2 - c^2)*Log[a + c*Tanh[x/2]])/(2*c^5) - (c*Cosh[x] + a*Sinh[x])/(2*c
^2*(a + a*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a^2*Sinh[x]))/(2*c^4*
(a + a*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.0936418, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3129, 3153, 3124, 31}

$$\frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{3\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^4(a\cosh(x)+a+c\sinh(x))}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((3*a^2 - c^2)*Log[a + c*Tanh[x/2]])/(2*c^5) - (c*Cosh[x] + a*Sinh[x])/(2*c
^2*(a + a*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a^2*Sinh[x]))/(2*c^4*
(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx &= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{\int \frac{-2a+a\cosh(x)+c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^2}\,dx}{2c^2}\\ &= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{3\left(ac\cosh(x)+a^2\sinh(x)\right)}{2c^4(a+a\cosh(x)+c\sinh(x))}+\frac{\left(3a^2-c^2\right)\int\\ &= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{3\left(ac\cosh(x)+a^2\sinh(x)\right)}{2c^4(a+a\cosh(x)+c\sinh(x))}+\frac{\left(3a^2-c^2\right)\mathrm{Su}\\ &= \frac{\left(3a^2-c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{3\left(ac\cosh(x\right.}{2c^4(a+a\cos\end{aligned}$$

Mathematica [A] time = 0.494929, size = 148, normalized size = 1.66

$$\frac{4\left(c^2-3a^2\right)\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{6c\left(c^2-a^2\right)\sinh\left(\frac{x}{2}\right)}{a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)}+4\left(3a^2-c^2\right)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)+\frac{c^2(a-c)(a+c)}{\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)^2}-6ac\,t}{8c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (4*(-3*a^2 + c^2)*Log[Cosh[x/2]] + 4*(3*a^2 - c^2)*Log[a*Cosh[x/2] + c*Sinh
[x/2]] - c^2*Sech[x/2]^2 + ((a - c)*c^2*(a + c))/(a*Cosh[x/2] + c*Sinh[x/2]
)^2 + (6*c*(-a^2 + c^2)*Sinh[x/2])/(a*Cosh[x/2] + c*Sinh[x/2]) - 6*a*c*Tanh
[x/2])/(8*c^5)
```

Maple [A] time = 0.08, size = 138, normalized size = 1.6

$$\frac{1}{8c^3}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{3a}{4c^4}\tanh\left(\frac{x}{2}\right)-\frac{a^4}{8c^5}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a^2}{4c^3}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{1}{8c}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a^3}{c^5}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 1/8/c^3*tanh(1/2*x)^2-3/4/c^4*a*tanh(1/2*x)-1/8/c^5/(a+c*tanh(1/2*x))^2*a^4
+1/4/c^3/(a+c*tanh(1/2*x))^2*a^2-1/8/c/(a+c*tanh(1/2*x))^2+a^3/c^5/(a+c*tan
h(1/2*x))-a/c^3/(a+c*tanh(1/2*x))+3/2/c^5*ln(a+c*tanh(1/2*x))*a^2-1/2/c^3*l
n(a+c*tanh(1/2*x))
```

Maxima [B] time = 1.12245, size = 335, normalized size = 3.76

$$-\frac{3a^3+6a^2c+3ac^2+\left(9a^3+9a^2c+ac^2+c^3\right)e^{(-x)}+3\left(3a^3-ac^2\right)e^{(-2x)}+\left(3a^3-3a^2c-ac^2+c^3\right)e^{(-3x)}}{a^2c^4+2ac^5+c^6+4\left(a^2c^4+ac^5\right)e^{(-x)}+2\left(3a^2c^4-c^6\right)e^{(-2x)}+4\left(a^2c^4-ac^5\right)e^{(-3x)}+\left(a^2c^4-2ac^5+c^6\right)e^{(-4x)}}+\frac{\left(3a^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^3 + 6*a^2*c + 3*a*c^2 + (9*a^3 + 9*a^2*c + a*c^2 + c^3)*e^(-x) + 3*(3
*a^3 - a*c^2)*e^(-2*x) + (3*a^3 - 3*a^2*c - a*c^2 + c^3)*e^(-3*x))/(a^2*c^4
 + 2*a*c^5 + c^6 + 4*(a^2*c^4 + a*c^5)*e^(-x) + 2*(3*a^2*c^4 - c^6)*e^(-2*x
) + 4*(a^2*c^4 - a*c^5)*e^(-3*x) + (a^2*c^4 - 2*a*c^5 + c^6)*e^(-4*x)) + 1/
2*(3*a^2 - c^2)*log(-(a - c)*e^(-x) - a - c)/c^5 - 1/2*(3*a^2 - c^2)*log(e^
(-x) + 1)/c^5
```

Fricas [B] time = 2.82751, size = 3359, normalized size = 37.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*c - 12*a^2*c^2 + 6*a*c^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)
*cosh(x)^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*sinh(x)^3 + 6*(3*a^3*c -
 a*c^3)*cosh(x)^2 + 6*(3*a^3*c - a*c^3 + (3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4
)*cosh(x))*sinh(x)^2 + 2*(9*a^3*c - 9*a^2*c^2 + a*c^3 - c^4)*cosh(x) + ((3*
a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2
*a^2*c^2 - 2*a*c^3 - c^4)*sinh(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3
 - c^4 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3
*c - a^2*c^2 - a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x
))*sinh(x)^3 + 2*(9*a^4 - 6*a^2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2
 + c^4 + 3*(3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a
^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a
^2*c^2 + a*c^3)*cosh(x) + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6
*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^
2 - a*c^3)*cosh(x)^2 + (9*a^4 - 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log((a +
 c)*cosh(x) + (a + c)*sinh(x) + a - c) - ((3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*
a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*sinh
(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3 - c^4 + 4*(3*a^4 + 3*a^3*c -
a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3 + (3*a^4
+ 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x))*sinh(x)^3 + 2*(9*a^4 - 6*a^
2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2 + c^4 + 3*(3*a^4 + 6*a^3*c +
2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3
)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3)*cosh(x) + 4*(3
*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 -
 c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^2 + (9*a^4
- 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(9*a^3*
c - 9*a^2*c^2 + a*c^3 - c^4 + 3*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*cosh(x)
^2 + 6*(3*a^3*c - a*c^3)*cosh(x))*sinh(x))/(a^2*c^5 - 2*a*c^6 + c^7 + (a^2*
c^5 + 2*a*c^6 + c^7)*cosh(x)^4 + (a^2*c^5 + 2*a*c^6 + c^7)*sinh(x)^4 + 4*(a
^2*c^5 + a*c^6)*cosh(x)^3 + 4*(a^2*c^5 + a*c^6 + (a^2*c^5 + 2*a*c^6 + c^7)*
cosh(x))*sinh(x)^3 + 2*(3*a^2*c^5 - c^7)*cosh(x)^2 + 2*(3*a^2*c^5 - c^7 + 3
*(a^2*c^5 + 2*a*c^6 + c^7)*cosh(x)^2 + 6*(a^2*c^5 + a*c^6)*cosh(x))*sinh(x)
^2 + 4*(a^2*c^5 - a*c^6)*cosh(x) + 4*(a^2*c^5 - a*c^6 + (a^2*c^5 + 2*a*c^6
+ c^7)*cosh(x)^3 + 3*(a^2*c^5 + a*c^6)*cosh(x)^2 + (3*a^2*c^5 - c^7)*cosh(x
))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17161, size = 277, normalized size = 3.11

$$\frac{\left(3\,a^{3}+3\,a^{2}c-ac^{2}-c^{3}\right)\log\left(|ae^{x}+ce^{x}+a-c|\right)}{2\left(ac^{5}+c^{6}\right)}-\frac{\left(3\,a^{2}-c^{2}\right)\log\left(e^{x}+1\right)}{2\,c^{5}}+\frac{3\,a^{3}e^{(3\,x)}+3\,a^{2}ce^{(3\,x)}-ac^{2}e^{(3\,x)}-c^{3}e^{(3\,x)}+9\,a}{\left(ae\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^3 + 3*a^2*c - a*c^2 - c^3)*log(abs(a*e^x + c*e^x + a - c))/(a*c^5
+ c^6) - 1/2*(3*a^2 - c^2)*log(e^x + 1)/c^5 + (3*a^3*e^(3*x) + 3*a^2*c*e^(3
*x) - a*c^2*e^(3*x) - c^3*e^(3*x) + 9*a^3*e^(2*x) - 3*a*c^2*e^(2*x) + 9*a^3
*e^x - 9*a^2*c*e^x + a*c^2*e^x - c^3*e^x + 3*a^3 - 6*a^2*c + 3*a*c^2)/((a*e
^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^2*c^4)
```

3.752 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$

Optimal. Leaf size=140

$$\frac{a(5a^2-3c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7}-\frac{a(15a^2-4c^2)\sinh(x)+c(15a^2-4c^2)\cosh(x)}{6c^6(a\cosh(x)+a+c\sinh(x))}-\frac{5(a^2\sinh(x)+ac\cosh(x))}{6c^4(a\cosh(x)+a+c\sinh(x))^2}$$

```
[Out] (a*(5*a^2 - 3*c^2)*Log[a + c*Tanh[x/2]])/(2*c^7) - (c*Cosh[x] + a*Sinh[x])/
(3*c^2*(a + a*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a^2*Sinh[x]))/(6*
c^4*(a + a*Cosh[x] + c*Sinh[x])^2) - (c*(15*a^2 - 4*c^2)*Cosh[x] + a*(15*a^
2 - 4*c^2)*Sinh[x])/(6*c^6*(a + a*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.212702, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 3156, 3153, 3124, 31}

$$\frac{a(5a^2-3c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7}-\frac{a(15a^2-4c^2)\sinh(x)+c(15a^2-4c^2)\cosh(x)}{6c^6(a\cosh(x)+a+c\sinh(x))}-\frac{5(a^2\sinh(x)+ac\cosh(x))}{6c^4(a\cosh(x)+a+c\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*Log[a + c*Tanh[x/2]])/(2*c^7) - (c*Cosh[x] + a*Sinh[x])/
(3*c^2*(a + a*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a^2*Sinh[x]))/(6*
c^4*(a + a*Cosh[x] + c*Sinh[x])^2) - (c*(15*a^2 - 4*c^2)*Cosh[x] + a*(15*a^
2 - 4*c^2)*Sinh[x])/(6*c^6*(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
```

```
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{\int \frac{-3a+2a\cosh(x)+2c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^3}\,dx}{3c^2}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} + \frac{\int \frac{2\left(5a^2-2c^2\right)-}{(a+a\,c}}{}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c\left(15a^2-4c\right.}{6c^6}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c\left(15a^2-4c\right.}{6c^6}$$

$$= -\frac{a\left(3-\frac{5a^2}{c^2}\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh\right.}{6c^4(a+a\,c}$$

Mathematica [B] time = 0.574044, size = 300, normalized size = 2.14

$$\frac{192\left(3ac^2-5a^3\right)\log\left(\cosh\left(\frac{x}{2}\right)\right)+192a\left(5a^2-3c^2\right)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-\frac{c\,\mathrm{sech}^6\left(\frac{x}{2}\right)\left(-255a^4c^2\sinh(x)-72a^4c^2\sinh(2x)+\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (192*(-5*a^3 + 3*a*c^2)*Log[Cosh[x/2]] + 192*a*(5*a^2 - 3*c^2)*Log[a*Cosh[x
/2] + c*Sinh[x/2]] - (c*Sech[x/2]^6*(-150*a^5*c + 130*a^3*c^3 - 24*a*c^5 +
(-75*a^5*c + 75*a^3*c^3 + 12*a*c^5)*Cosh[x] + 6*a*c*(25*a^4 - 15*a^2*c^2 +
4*c^4)*Cosh[2*x] + 75*a^5*c*Cosh[3*x] - 35*a^3*c^3*Cosh[3*x] + 4*a*c^5*Cosh
[3*x] + 150*a^6*Sinh[x] - 255*a^4*c^2*Sinh[x] + 129*a^2*c^4*Sinh[x] - 12*c^
6*Sinh[x] + 120*a^6*Sinh[2*x] - 72*a^4*c^2*Sinh[2*x] + 36*a^2*c^4*Sinh[2*x]
 + 30*a^6*Sinh[3*x] + 37*a^4*c^2*Sinh[3*x] - 27*a^2*c^4*Sinh[3*x] + 4*c^6*S
inh[3*x]))/(a*(a + c*Tanh[x/2])^3))/(384*c^7)
```

Maple [A] time = 0.098, size = 250, normalized size = 1.8

$$-\frac{1}{24\,c^4}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{a}{4\,c^5}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{5\,a^2}{4\,c^6}\tanh\left(\frac{x}{2}\right)+\frac{3}{8\,c^4}\tanh\left(\frac{x}{2}\right)+\frac{a^6}{24\,c^7}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{a^4}{8\,c^5}\left(a+c\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^4,x)
```

```
[Out] -1/24/c^4*tanh(1/2*x)^3+1/4/c^5*a*tanh(1/2*x)^2-5/4/c^6*a^2*tanh(1/2*x)+3/8
/c^4*tanh(1/2*x)+1/24/c^7/(a+c*tanh(1/2*x))^3*a^6-1/8/c^5/(a+c*tanh(1/2*x))
^3*a^4+1/8/c^3/(a+c*tanh(1/2*x))^3*a^2-1/24/c/(a+c*tanh(1/2*x))^3-3/8*a^5/c
^7/(a+c*tanh(1/2*x))^2+3/4*a^3/c^5/(a+c*tanh(1/2*x))^2-3/8*a/c^3/(a+c*tanh(
1/2*x))^2+15/8/c^7/(a+c*tanh(1/2*x))*a^4-9/4/c^5/(a+c*tanh(1/2*x))*a^2+3/8/
c^3/(a+c*tanh(1/2*x))+5/2*a^3/c^7*ln(a+c*tanh(1/2*x))-3/2*a/c^5*ln(a+c*tanh
(1/2*x))
```

Maxima [B] time = 1.21488, size = 657, normalized size = 4.69

$$-\frac{15\,a^5+45\,a^4c+41\,a^3c^2+3\,a^2c^3-12\,ac^4-4\,c^5+15\left(5\,a^5+10\,a^4c+4\,a^3c^2-2\,a^2c^3-ac^4\right)e^{(-x)}+6\left(25\,a^5+25\,a^4c-10\right.}{3\left(a^3c^6+3\,a^2c^7+3\,ac^8+c^9+6\left(a^3c^6+2\,a^2c^7+ac^8\right)e^{(-x)}+3\left(5\,a^3c^6+5\,a^2c^7-ac^8-c^9\right)e^{(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(15*a^5 + 45*a^4*c + 41*a^3*c^2 + 3*a^2*c^3 - 12*a*c^4 - 4*c^5 + 15*(5
*a^5 + 10*a^4*c + 4*a^3*c^2 - 2*a^2*c^3 - a*c^4)*e^(-x) + 6*(25*a^5 + 25*a^
4*c - 10*a^3*c^2 - 10*a^2*c^3 + 2*a*c^4 + 2*c^5)*e^(-2*x) + 2*(75*a^5 - 65*
a^3*c^2 + 12*a*c^4)*e^(-3*x) + 15*(5*a^5 - 5*a^4*c - 3*a^3*c^2 + 3*a^2*c^3)
*e^(-4*x) + 3*(5*a^5 - 10*a^4*c + 2*a^3*c^2 + 6*a^2*c^3 - 3*a*c^4)*e^(-5*x)
)/(a^3*c^6 + 3*a^2*c^7 + 3*a*c^8 + c^9 + 6*(a^3*c^6 + 2*a^2*c^7 + a*c^8)*e^
(-x) + 3*(5*a^3*c^6 + 5*a^2*c^7 - a*c^8 - c^9)*e^(-2*x) + 4*(5*a^3*c^6 - 3*
a*c^8)*e^(-3*x) + 3*(5*a^3*c^6 - 5*a^2*c^7 - a*c^8 + c^9)*e^(-4*x) + 6*(a^3
*c^6 - 2*a^2*c^7 + a*c^8)*e^(-5*x) + (a^3*c^6 - 3*a^2*c^7 + 3*a*c^8 - c^9)*
e^(-6*x)) + 1/2*(5*a^3 - 3*a*c^2)*log(-(a - c)*e^(-x) - a - c)/c^7 - 1/2*(5
*a^3 - 3*a*c^2)*log(e^(-x) + 1)/c^7
```

Fricas [B] time = 3.32946, size = 9150, normalized size = 65.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

[Out] 1/6*(30*a^5*c - 90*a^4*c^2 + 82*a^3*c^3 - 6*a^2*c^4 - 24*a*c^5 + 8*c^6 + 6*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 6*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*sinh(x)^5 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4 + (5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x)^4 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^3 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5 + 15*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^3 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x)^2 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + (75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x))*sinh(x)^2 + 30*(5*a^5*c - 10*a^4*c^2 + 4*a^3*c^3 + 2*a^2*c^4 - a*c^5)*cosh(x) + 3*((5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^6 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x)^5 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^4 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 10*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^4 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x))*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 5*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 2*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^3 + 2*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - 3*((5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^6 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^

```
2*c^4)*cosh(x)^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4
+ (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)
)*sinh(x)^5 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 +
3*a*c^5)*cosh(x)^4 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2
*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 -
 3*a*c^5)*cosh(x)^2 + 10*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*
c^4)*cosh(x))*sinh(x)^4 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4
*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^
3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 -
 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*
a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c -
20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a
^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c
+ 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*
a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c
 - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 3
0*a^4*c^2 + 9*a^2*c^4)*cosh(x))*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2
 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3
*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 -
 3*a*c^5)*cosh(x)^5 + 5*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c
^4)*cosh(x)^4 + 2*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4
+ 3*a*c^5)*cosh(x)^3 + 2*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*
a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*si
nh(x))*log(cosh(x) + sinh(x) + 1) + 6*(25*a^5*c - 50*a^4*c^2 + 20*a^3*c^3 +
 10*a^2*c^4 - 5*a*c^5 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3
*a*c^5)*cosh(x)^4 + 20*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x
)^3 + 2*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^2 + 4*(25*a^5*c - 25*a^4
*c^2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x))*sinh(x))/(a^3*c^
7 - 3*a^2*c^8 + 3*a*c^9 - c^10 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cos
h(x)^6 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*sinh(x)^6 + 6*(a^3*c^7 + 2*
a^2*c^8 + a*c^9)*cosh(x)^5 + 6*(a^3*c^7 + 2*a^2*c^8 + a*c^9 + (a^3*c^7 + 3*
a^2*c^8 + 3*a*c^9 + c^10)*cosh(x))*sinh(x)^5 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a
*c^9 - c^10)*cosh(x)^4 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10 + 5*(a^3*c
^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^2 + 10*(a^3*c^7 + 2*a^2*c^8 + a*c^
9)*cosh(x))*sinh(x)^4 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^3 + 4*(5*a^3*c^7 -
3*a*c^9 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^3 + 15*(a^3*c^7
+ 2*a^2*c^8 + a*c^9)*cosh(x)^2 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*c
osh(x))*sinh(x)^3 + 3*(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10)*cosh(x)^2 + 3*
(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 +
c^10)*cosh(x)^4 + 20*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^3 + 6*(5*a^3*c^7
 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x)^2 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x))*s
inh(x)^2 + 6*(a^3*c^7 - 2*a^2*c^8 + a*c^9)*cosh(x) + 6*(a^3*c^7 - 2*a^2*c^8
 + a*c^9 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^5 + 5*(a^3*c^7 +
2*a^2*c^8 + a*c^9)*cosh(x)^4 + 2*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cos
h(x)^3 + 2*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^2 + (5*a^3*c^7 - 5*a^2*c^8 - a*c^9
```

```
+ c^10)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16525, size = 509, normalized size = 3.64

$$\frac{\left(5\,a^4+5\,a^3c-3\,a^2c^2-3\,ac^3\right)\log\left(|ae^x+ce^x+a-c|\right)}{2\left(ac^7+c^8\right)}-\frac{\left(5\,a^3-3\,ac^2\right)\log\left(e^x+1\right)}{2\,c^7}+\frac{15\,a^5e^{(5\,x)}+30\,a^4ce^{(5\,x)}+6\,a^3c^2e^{(5\,x)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^4 + 5*a^3*c - 3*a^2*c^2 - 3*a*c^3)*log(abs(a*e^x + c*e^x + a - c))
/(a*c^7 + c^8) - 1/2*(5*a^3 - 3*a*c^2)*log(e^x + 1)/c^7 + 1/3*(15*a^5*e^(5*
x) + 30*a^4*c*e^(5*x) + 6*a^3*c^2*e^(5*x) - 18*a^2*c^3*e^(5*x) - 9*a*c^4*e^
(5*x) + 75*a^5*e^(4*x) + 75*a^4*c*e^(4*x) - 45*a^3*c^2*e^(4*x) - 45*a^2*c^3
*e^(4*x) + 150*a^5*e^(3*x) - 130*a^3*c^2*e^(3*x) + 24*a*c^4*e^(3*x) + 150*a
^5*e^(2*x) - 150*a^4*c*e^(2*x) - 60*a^3*c^2*e^(2*x) + 60*a^2*c^3*e^(2*x) +
12*a*c^4*e^(2*x) - 12*c^5*e^(2*x) + 75*a^5*e^x - 150*a^4*c*e^x + 60*a^3*c^2
*e^x + 30*a^2*c^3*e^x - 15*a*c^4*e^x + 15*a^5 - 45*a^4*c + 41*a^3*c^2 - 3*a
^2*c^3 - 12*a*c^4 + 4*c^5)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^3*c^6
)
```

3.753 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx$

Optimal. Leaf size=188

$$\frac{35}{8}x\left(b^2-c^2\right)^2+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\right.$$

```
[Out] (35*(b^2 - c^2)^2*x)/8 + (35*c*(b^2 - c^2)^(3/2)*Cosh[x])/8 + (35*b*(b^2 -
c^2)^(3/2)*Sinh[x])/8 + (35*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x]))/24 + (7*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2)/12 + ((c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3)/4
```

Rubi [A] time = 0.147956, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{35}{8}x\left(b^2-c^2\right)^2+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```



```
[Out] (35*(b^2 - c^2)^2*x)/8 + (35*c*(b^2 - c^2)^(3/2)*Cosh[x])/8 + (35*b*(b^2 -
c^2)^(3/2)*Sinh[x])/8 + (35*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x]))/24 + (7*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2)/12 + ((c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3)/4
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx &= \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3+\frac{1}{4}\left(7\sqrt{b^2-\right.\\ &= \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\\ &= \frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\\ &= \frac{35}{8}\left(b^2-c^2\right)^2x+\frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(\right.\\ &= \frac{35}{8}\left(b^2-c^2\right)^2x+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)+\frac{35}{24}\end{aligned}$$

Mathematica [A] time = 0.493392, size = 208, normalized size = 1.11

$$7b(b-c)\sqrt{b^2-c^2}(b+c)\sinh(x)+\frac{7}{4}\left(b^4-c^4\right)\sinh(2x)+\frac{1}{3}b\sqrt{b^2-c^2}\left(b^2+3c^2\right)\sinh(3x)+\frac{1}{32}\left(6b^2c^2+b^4+c^4\right)\sinh($$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

```
[Out] (35*(b - c)^2*(b + c)^2*x)/8 + 7*(b - c)*c*(b + c)*Sqrt[b^2 - c^2]*Cosh[x]
+ (7*b*c*(b^2 - c^2)*Cosh[2*x])/2 + (c*Sqrt[b^2 - c^2]*(3*b^2 + c^2)*Cosh[3
*x])/3 + (b*c*(b^2 + c^2)*Cosh[4*x])/8 + 7*b*(b - c)*(b + c)*Sqrt[b^2 - c^2
]*Sinh[x] + (7*(b^4 - c^4)*Sinh[2*x])/4 + (b*Sqrt[b^2 - c^2]*(b^2 + 3*c^2)*
Sinh[3*x])/3 + ((b^4 + 6*b^2*c^2 + c^4)*Sinh[4*x])/32
```

Maple [B] time = 0.082, size = 409, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x)
```

```
[Out] -2*c^2*b^2*x+4*(b^2-c^2)^(1/2)*b^3*(2/3+1/3*cosh(x)^2)*sinh(x)-6*c^2*b^2*(1
/2*cosh(x)*sinh(x)+1/2*x)+4*(b^2-c^2)^(1/2)*b^3*sinh(x)+4*(b^2-c^2)^(1/2)*c
^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+6*c^2*b^2*(1/2*cosh(x)*sinh(x)-1/2*x)-4*(b^
2-c^2)^(1/2)*c^3*cosh(x)+b^4*x+4*b^3*c*(1/4*sinh(x)^2*cosh(x)^2+1/4*cosh(x)
^2)+6*c^2*b^2*(1/4*sinh(x)*cosh(x)^3-1/8*cosh(x)*sinh(x)-1/8*x)+4*b*c^3*(1/
4*sinh(x)^2*cosh(x)^2-1/4*cosh(x)^2)+c^4*x+6*b^3*cosh(x)^2*c-6*b*cosh(x)^2*
c^3-4*(b^2-c^2)^(1/2)*b*c^2*sinh(x)+4*(b^2-c^2)^(1/2)*b^2*c*cosh(x)+12*(b^2
-c^2)^(1/2)*b^2*c*(1/3*cosh(x)*sinh(x)^2+1/3*cosh(x))+12*(b^2-c^2)^(1/2)*b*
c^2*(1/3*sinh(x)*cosh(x)^2-1/3*sinh(x))+b^4*((1/4*cosh(x)^3+3/8*cosh(x))*si
nh(x)+3/8*x)+6*b^4*(1/2*cosh(x)*sinh(x)+1/2*x)+c^4*((1/4*sinh(x)^3-3/8*sinh
(x))*cosh(x)+3/8*x)-6*c^4*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Maxima [A] time = 1.02717, size = 374, normalized size = 1.99

$$b^3 c \cosh(x)^4 + bc^3 \sinh(x)^4 + \frac{1}{64} b^4 \Big(24x + e^{(4x)} + 8e^{(2x)} - 8e^{(-2x)} - e^{(-4x)}\Big) + \frac{1}{64} c^4 \Big(24x + e^{(4x)} - 8e^{(2x)} + 8e^{(-2x)} - e^{(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

```
[Out] b^3*c*cosh(x)^4 + b*c^3*sinh(x)^4 + 1/64*b^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*c^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*b^2*c^2*(8*x - e^(4*x) + e^(-4*x)) + (b^2 - c^2)^2*x + 4
*(b^2 - c^2)^(3/2)*(c*cosh(x) + b*sinh(x)) + 3/4*(8*b*c*cosh(x)^2 + b^2*(4*
x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*(b^2 - c^2) + 1/6
*(24*b^2*c*cosh(x)^3 + 24*b*c^2*sinh(x)^3 + c^3*(e^(3*x) - 9*e^(-x) + e^(-3
*x) - 9*e^x) + b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x))*sqrt(b^2 - c^2)
```

Fricas [B] time = 2.83112, size = 3148, normalized size = 16.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 24*(b^4 +
4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^7 + 3*(b^4 + 4*b^3*c +
 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^8 + 168*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)
```

```
*cosh(x)^6 + 84*(2*b^4 + 4*b^3*c - 4*b*c^3 - 2*c^4 + (b^4 + 4*b^3*c + 6*b^2
*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^6 + 840*(b^4 - 2*b^2*c^2 + c^4)*x*
cosh(x)^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 6*
(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^5 + 210*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + 12*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^2 + 4*(b^4 - 2*b^2*c^2 + c^4)*x)*sinh(x)^4 - 3*b^4 + 12*b^3*c - 18*b
^2*c^2 + 12*b*c^3 - 3*c^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4
)*cosh(x)^5 + 20*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 20*(b^4 - 2*b^
2*c^2 + c^4)*x*cosh(x))*sinh(x)^3 - 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cos
h(x)^2 + 84*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 30*(b^
4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4
+ 60*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^2)*sinh(x)^2 + 24*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 42*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^5 + 140*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^3 - 14*(b^4 - 2*b^3*c + 2*
b*c^3 - c^4)*cosh(x))*sinh(x) + 32*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)
^7 + 7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^6 + (b^3 + 3*b^2*c +
 3*b*c^2 + c^3)*sinh(x)^7 + 21*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^5 + 21*(
b^3 + b^2*c - b*c^2 - c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh
(x)^5 + 35*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 3*(b^3 + b^2*c - b*
c^2 - c^3)*cosh(x))*sinh(x)^4 - 21*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^3 +
7*(5*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c + 3*b*c^2
- 3*c^3 + 30*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^3 + 21*((b^3 +
3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 10*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)
^3 - 3*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x)^2 - (b^3 - 3*b^2*c + 3*
b*c^2 - c^3)*cosh(x) + (7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 105*(
b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 63*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)
```

Sympy [B] time = 2.63169, size = 626, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] 3*b**4*x*sinh(x)**4/8 - 3*b**4*x*sinh(x)**2*cosh(x)**2/4 - 3*b**4*x*sinh(x)
**2 + 3*b**4*x*cosh(x)**4/8 + 3*b**4*x*cosh(x)**2 + b**4*x - 3*b**4*sinh(x)
**3*cosh(x)/8 + 5*b**4*sinh(x)*cosh(x)**3/8 + 3*b**4*sinh(x)*cosh(x) + 6*b*
*3*c*sinh(x)**2 + b**3*c*cosh(x)**4 - 8*b**3*sqrt(b**2 - c**2)*sinh(x)**3/3
```

```
 + 4*b**3*sqrt(b**2 - c**2)*sinh(x)*cosh(x)**2 + 4*b**3*sqrt(b**2 - c**2)*s
inh(x) - 3*b**2*c**2*x*sinh(x)**4/4 + 3*b**2*c**2*x*sinh(x)**2*cosh(x)**2/2
 + 6*b**2*c**2*x*sinh(x)**2 - 3*b**2*c**2*x*cosh(x)**4/4 - 6*b**2*c**2*x*co
sh(x)**2 - 2*b**2*c**2*x + 3*b**2*c**2*sinh(x)**3*cosh(x)/4 + 3*b**2*c**2*s
inh(x)*cosh(x)**3/4 + 4*b**2*c*sqrt(b**2 - c**2)*cosh(x)**3 + 4*b**2*c*sqrt
(b**2 - c**2)*cosh(x) + b*c**3*sinh(x)**4 - 6*b*c**3*sinh(x)**2 + 4*b*c**2*
sqrt(b**2 - c**2)*sinh(x)**3 - 4*b*c**2*sqrt(b**2 - c**2)*sinh(x) + 3*c**4*
x*sinh(x)**4/8 - 3*c**4*x*sinh(x)**2*cosh(x)**2/4 - 3*c**4*x*sinh(x)**2 + 3
*c**4*x*cosh(x)**4/8 + 3*c**4*x*cosh(x)**2 + c**4*x + 5*c**4*sinh(x)**3*cos
h(x)/8 - 3*c**4*sinh(x)*cosh(x)**3/8 - 3*c**4*sinh(x)*cosh(x) + 4*c**3*sqrt
(b**2 - c**2)*sinh(x)**2*cosh(x) - 8*c**3*sqrt(b**2 - c**2)*cosh(x)**3/3 -
4*c**3*sqrt(b**2 - c**2)*cosh(x)
```

Giac [B] time = 1.17548, size = 527, normalized size = 2.8

$$\frac{7}{2}\left(b^3+b^2c-bc^2-c^3\right)\sqrt{b^2-c^2}e^x+\frac{35}{8}\left(b^4-2\,b^2c^2+c^4\right)x+\frac{1}{64}\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)e^{(4\,x)}+\frac{1}{6}\left(\sqrt{b^2-c^2}b^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] 7/2*(b^3 + b^2*c - b*c^2 - c^3)*sqrt(b^2 - c^2)*e^x + 35/8*(b^4 - 2*b^2*c^2
 + c^4)*x + 1/64*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*e^(4*x) + 1/6*
(sqrt(b^2 - c^2)*b^3 + 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 +
sqrt(b^2 - c^2)*c^3)*e^(3*x) + 7/8*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*e^(2*x)
- 1/192*(3*b^4 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 + 672*(sqrt(b^2 -
 c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c^2 + sqrt(b^2 - c^2)
*c^3)*e^(3*x) + 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(2*x) + 32*(sqrt(b^2
- c^2)*b^3 - 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 - sqrt(b^2 -
 c^2)*c^3)*e^x)*e^(-4*x)
```

3.754 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$

Optimal. Leaf size=136

$$\frac{5}{2}x\left(b^2-c^2\right)^{3/2}+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh\right.$$

```
[Out] (5*(b^2 - c^2)^(3/2)*x)/2 + (5*c*(b^2 - c^2)*Cosh[x])/2 + (5*b*(b^2 - c^2)*
Sinh[x])/2 + (5*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] +
b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x])^2)/3
```

Rubi [A] time = 0.0896551, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{5}{2}x\left(b^2-c^2\right)^{3/2}+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (5*(b^2 - c^2)^(3/2)*x)/2 + (5*c*(b^2 - c^2)*Cosh[x])/2 + (5*b*(b^2 - c^2)*
Sinh[x])/2 + (5*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] +
b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x])^2)/3
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx = \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\frac{1}{3}\left(5\sqrt{b^2-c\right.$$
$$=\frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\frac{1}{3}(c$$
$$=\frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+\right.$$
$$=\frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{6}\sqrt{b^2-c^2}($$

Mathematica [A] time = 0.257189, size = 134, normalized size = 0.99

$$\frac{1}{12}\left(30x(b-c)(b+c)\sqrt{b^2-c^2}+45b\left(b^2-c^2\right)\sinh(x)+9\sqrt{b^2-c^2}\left(b^2+c^2\right)\sinh(2x)+b\left(b^2+3c^2\right)\sinh(3x)+45c\left(b^2-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*(b - c)*(b + c)*Sqrt[b^2 - c^2]*x + 45*c*(b^2 - c^2)*Cosh[x] + 18*b*c*S
qrt[b^2 - c^2]*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 45*b*(b^2 - c^2)*Sin
h[x] + 9*Sqrt[b^2 - c^2]*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])
/12
```

Maple [A] time = 0.046, size = 202, normalized size = 1.5

$$b^3\left(\frac{2}{3}+\frac{(\cosh(x))^2}{3}\right)\sinh(x)+3cb^2\left(1/3\cosh(x)(\sinh(x))^2+1/3\cosh(x)\right)+3\sqrt{b^2-c^2}b^2(1/2\cosh(x)\sinh(x)+x/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x)
```

```
[Out] b^3*(2/3+1/3*cosh(x)^2)*sinh(x)+3*c*b^2*(1/3*cosh(x)*sinh(x)^2+1/3*cosh(x))
+3*(b^2-c^2)^(1/2)*b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+3*b*c^2*(1/3*sinh(x)*cos
h(x)^2-1/3*sinh(x))+3*(b^2-c^2)^(1/2)*b*c*cosh(x)^2+3*b^3*sinh(x)-3*b*c^2*s
inh(x)+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+3*(b^2-c^2)^(1/2)*c^2*(1/2*cosh(x)*
sinh(x)-1/2*x)+3*b^2*cosh(x)*c-3*c^3*cosh(x)+(b^2-c^2)^(1/2)*b^2*x-(b^2-c^2
)^(1/2)*c^2*x
```

Maxima [A] time = 1.03399, size = 217, normalized size = 1.6

$$b^2c\cosh(x)^3 + bc^2\sinh(x)^3 + \frac{1}{24}c^3\left(e^{(3x)} - 9e^{(-x)} + e^{(-3x)} - 9e^x\right) + \frac{1}{24}b^3\left(e^{(3x)} - 9e^{(-x)} - e^{(-3x)} + 9e^x\right) + \left(b^2 - c^2\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + 1/24*c^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + (b^2 - c^2)^
(3/2)*x + 3*(b^2 - c^2)*(c*cosh(x) + b*sinh(x)) + 3/8*(8*b*c*cosh(x)^2 + b^
2*(4*x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*sqrt(b^2 - c
^2)
```

Fricas [B] time = 2.64613, size = 1665, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*(b^3 + 3*b^2*c + 3*b*c^
2 + c^3)*cosh(x)*sinh(x)^5 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 45
*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 15*(3*b^3 + 3*b^2*c - 3*b*c^2 - 3*
c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*((b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 9*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*si
nh(x)^3 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 45*(b^3 - b^2*c - b*c^2 + c^3)*co
sh(x)^2 + 15*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c +
 3*b*c^2 - 3*c^3 + 18*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 6*
((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 30*(b^3 + b^2*c - b*c^2 - c^3)
*cosh(x)^3 - 15*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + 3*(3*(b^2 +
```

```
2*b*c + c^2)*cosh(x)^5 + 15*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + 3*(b^2
+ 2*b*c + c^2)*sinh(x)^5 + 20*(b^2 - c^2)*x*cosh(x)^3 + 10*(3*(b^2 + 2*b*c
+ c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^3 + 30*((b^2 + 2*b*c + c^2)*cos
h(x)^3 + 2*(b^2 - c^2)*x*cosh(x))*sinh(x)^2 - 3*(b^2 - 2*b*c + c^2)*cosh(x)
 + 3*(5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 20*(b^2 - c^2)*x*cosh(x)^2 - b^2 +
2*b*c - c^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3
*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] time = 2.25663, size = 298, normalized size = 2.19

$$-\frac{2b^3\sinh^3(x)}{3}+b^3\sinh(x)\cosh^2(x)+3b^3\sinh(x)+b^2c\cosh^3(x)+3b^2c\cosh(x)-\frac{3b^2x\sqrt{b^2-c^2}\sinh^2(x)}{2}+\frac{3b^2x\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] -2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + 3*b**3*sinh(x) + b**2*c*co
sh(x)**3 + 3*b**2*c*cosh(x) - 3*b**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 + 3*b
**2*x*sqrt(b**2 - c**2)*cosh(x)**2/2 + b**2*x*sqrt(b**2 - c**2) + 3*b**2*sq
rt(b**2 - c**2)*sinh(x)*cosh(x)/2 + b*c**2*sinh(x)**3 - 3*b*c**2*sinh(x) +
3*b*c*sqrt(b**2 - c**2)*sinh(x)**2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(
x)**3/3 - 3*c**3*cosh(x) + 3*c**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 - 3*c**2
*x*sqrt(b**2 - c**2)*cosh(x)**2/2 - c**2*x*sqrt(b**2 - c**2) + 3*c**2*sqrt(
b**2 - c**2)*sinh(x)*cosh(x)/2
```

Giac [A] time = 1.15932, size = 262, normalized size = 1.93

$$\frac{5}{2}\left(b^2-c^2\right)^{\frac{3}{2}}x+\frac{3}{8}\left(b^2+2\,bc+c^2\right)\sqrt{b^2-c^2}e^{(2\,x)}+\frac{1}{24}\left(b^3+3\,b^2c+3\,bc^2+c^3\right)e^{(3\,x)}-\frac{1}{24}\left(b^3-3\,b^2c+3\,bc^2-c^3+45\left(b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 5/2*(b^2 - c^2)^(3/2)*x + 3/8*(b^2 + 2*b*c + c^2)*sqrt(b^2 - c^2)*e^(2*x) +
 1/24*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*e^(3*x) - 1/24*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3 + 45*(b^3 - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(sqrt(b^2 - c^2)*b^2
- 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^2)*e^x)*e^(-3*x) + 15/8*(b^3 +
b^2*c - b*c^2 - c^3)*e^x
```

3.755 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{3}{2}x\left(b^2-c^2\right)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

```
[Out] (3*(b^2 - c^2)*x)/2 + (3*c*Sqrt[b^2 - c^2]*Cosh[x])/2 + (3*b*Sqrt[b^2 - c^2
]*Sinh[x])/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Si
nh[x]))/2
```

Rubi [A] time = 0.048117, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{3}{2}x\left(b^2-c^2\right)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (3*(b^2 - c^2)*x)/2 + (3*c*Sqrt[b^2 - c^2]*Cosh[x])/2 + (3*b*Sqrt[b^2 - c^2
]*Sinh[x])/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Si
nh[x]))/2
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = \frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\frac{1}{2}\left(3\sqrt{b^2-c^2}\right.$$
$$=\frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$
$$=\frac{3}{2}\left(b^2-c^2\right)x+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{1}{2}(c\cosh(x)+b$$

Mathematica [A] time = 0.113133, size = 72, normalized size = 0.8

$$\frac{1}{4}\left(8b\sqrt{b^2-c^2}\sinh(x)+\left(b^2+c^2\right)\sinh(2x)+8c\sqrt{b^2-c^2}\cosh(x)+6x(b-c)(b+c)+2bc\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (6*(b - c)*(b + c)*x + 8*c*Sqrt[b^2 - c^2]*Cosh[x] + 2*b*c*Cosh[2*x] + 8*b*
Sqrt[b^2 - c^2]*Sinh[x] + (b^2 + c^2)*Sinh[2*x])/4
```

Maple [A] time = 0.033, size = 80, normalized size = 0.9

$$b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+b(\cosh(x))^2c+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+2b\sinh(x)\sqrt{b^2-c^2}+2c\cosh(x)\sqrt{b^2-c^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+b*cosh(x)^2*c+c^2*(1/2*cosh(x)*sinh(x)-1/2*
x)+2*b*sinh(x)*(b^2-c^2)^(1/2)+2*c*cosh(x)*(b^2-c^2)^(1/2)+b^2*x-c^2*x
```

Maxima [A] time = 1.17802, size = 107, normalized size = 1.19

$$bc\cosh(x)^2 + \frac{1}{8}b^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - \frac{1}{8}c^2\left(4x - e^{(2x)} + e^{(-2x)}\right) + b^2x - c^2x + 2\sqrt{b^2 - c^2}(c\cosh(x) + b\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + b^2*x - c^2*x + 2*sqrt(b^2 - c^2)*(c*cosh(x) + b*sinh(x))
```

Fricas [B] time = 2.59504, size = 656, normalized size = 7.29

$$\frac{\left(b^2 + 2bc + c^2\right)\cosh(x)^4 + 4\left(b^2 + 2bc + c^2\right)\cosh(x)\sinh(x)^3 + \left(b^2 + 2bc + c^2\right)\sinh(x)^4 + 12\left(b^2 - c^2\right)x\cosh(x)^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^
3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 + 12*(b^2 - c^2)*x*cosh(x)^2 + 6*((b^2 +
2*b*c + c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 4
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + 6*(b^2 - c^2)*x*cosh(x))*sinh(x) + 8*((b
+ c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*
cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [A] time = 0.497405, size = 122, normalized size = 1.36

$$-\frac{b^2x\sinh^2(x)}{2} + \frac{b^2x\cosh^2(x)}{2} + b^2x + \frac{b^2\sinh(x)\cosh(x)}{2} + bc\sinh^2(x) + 2b\sqrt{b^2 - c^2}\sinh(x) + \frac{c^2x\sinh^2(x)}{2} - \frac{c^2x}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] -b**2*x*sinh(x)**2/2 + b**2*x*cosh(x)**2/2 + b**2*x + b**2*sinh(x)*cosh(x)/
2 + b*c*sinh(x)**2 + 2*b*sqrt(b**2 - c**2)*sinh(x) + c**2*x*sinh(x)**2/2 -
```

```
c**2*x*cosh(x)**2/2 - c**2*x + c**2*sinh(x)*cosh(x)/2 + 2*c*sqrt(b**2 - c**
2)*cosh(x)
```

Giac [A] time = 1.16837, size = 130, normalized size = 1.44

$$\sqrt{b^2 - c^2}(b + c)e^x + \frac{3}{2}\left(b^2 - c^2\right)x + \frac{1}{8}\left(b^2 + 2\,bc + c^2\right)e^{(2\,x)} - \frac{1}{8}\left(b^2 - 2\,bc + c^2 + 8\left(\sqrt{b^2 - c^2}b - \sqrt{b^2 - c^2}c\right)e^x\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(b^2 - c^2)*(b + c)*e^x + 3/2*(b^2 - c^2)*x + 1/8*(b^2 + 2*b*c + c^2)*e
^(2*x) - 1/8*(b^2 - 2*b*c + c^2 + 8*(sqrt(b^2 - c^2)*b - sqrt(b^2 - c^2)*c)
*e^x)*e^(-2*x)
```

3.756 $\int \left(\sqrt{b^2-c^2}+b \cosh(x)+c \sinh(x)\right) \, dx$

Optimal. Leaf size=24

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Rubi [A] time = 0.0108072, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)dx &= \sqrt{b^2-c^2}x+b\int \cosh(x)\,dx+c\int \sinh(x)\,dx\\ &= \sqrt{b^2-c^2}x+c\cosh(x)+b\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0080919, size = 24, normalized size = 1.

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$c\cosh(x) + b\sinh(x) + x\sqrt{b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x)
```

```
[Out] c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

Maxima [A] time = 1.00005, size = 30, normalized size = 1.25

$$c\cosh(x) + b\sinh(x) + \sqrt{b^2 - c^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="maxima")
```

```
[Out] c*cosh(x) + b*sinh(x) + sqrt(b^2 - c^2)*x
```

Fricas [B] time = 2.72436, size = 196, normalized size = 8.17

$$\frac{(b+c)\cosh(x)^2 + 2(b+c)\cosh(x)\sinh(x) + (b+c)\sinh(x)^2 + 2\sqrt{b^2-c^2}(x\cosh(x) + x\sinh(x)) - b + c}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*
sqrt(b^2 - c^2)*(x*cosh(x) + x*sinh(x)) - b + c)/(cosh(x) + sinh(x))
```

Sympy [A] time = 0.268791, size = 20, normalized size = 0.83

$$b\sinh(x) + c\cosh(x) + x\sqrt{b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2),x)
```

```
[Out] b*sinh(x) + c*cosh(x) + x*sqrt(b**2 - c**2)
```

Giac [A] time = 1.14882, size = 49, normalized size = 2.04

$$\frac{1}{2}\,c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}\,b\left(e^{(-x)} - e^{x}\right) + \sqrt{b^2 - c^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x) + sqrt(b^2 - c^2)*x
```

3.757 $\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

```
[Out] -((c + Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x])))
```

Rubi [A] time = 0.0367554, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3114}

$$-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] -((c + Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x])))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=-\frac{c+\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.0734254, size = 36, normalized size = 1.06

$$\frac{-\sqrt{b^2-c^2}\sinh(x)-c}{c(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (-c - Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x]))
```

Maple [C] time = 0.254, size = 596, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x)
```

```
[Out] 1/2*sum((2*b*_R^2+4*c*_R+2*b+abs(b^2-c^2)^(1/2)*(1-I+_R^2*(-1+I-I*signum(b^
2-c^2)-signum(b^2-c^2))+I*signum(b^2-c^2)+signum(b^2-c^2)))/(abs((b-c)*(b+c
))*_R^3*signum((b-c)*(b+c))^2-abs((b-c)*(b+c))*_R*signum((b-c)*(b+c))^2+abs
((b-c)*(b+c))*_R^3+2*_R^3*b^2+6*_R^2*b*c-abs((b-c)*(b+c))*_R+2*b^2*_R+4*c^2
*_R+2*c*b+abs((b-c)*(b+c))^(1/2)*(-2*signum((b-c)*(b+c))*b*_R^3-3*signum((b
-c)*(b+c))*c*_R^2-2*b*_R^3-3*c*_R^2+signum((b-c)*(b+c))*c+c))*ln(tanh(1/2*x
)-_R),_R=RootOf((abs((b-c)*(b+c))*signum((b-c)*(b+c))^2-2*abs((b-c)*(b+c))^
(1/2)*signum((b-c)*(b+c))*b+abs((b-c)*(b+c))-2*abs((b-c)*(b+c))^(1/2)*b+2*b
^2)*_Z^4+(-4*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*c-4*abs((b-c)*(b+c)
)^(1/2)*c+8*c*b)*_Z^3+(-2*abs((b-c)*(b+c))*signum((b-c)*(b+c))^2-2*abs((b-c
)*(b+c))+4*b^2+8*c^2)*_Z^2+(4*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*c+
4*abs((b-c)*(b+c))^(1/2)*c+8*c*b)*_Z+abs((b-c)*(b+c))*signum((b-c)*(b+c))^2
+2*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*b+abs((b-c)*(b+c))+2*abs((b-c
)*(b+c))^(1/2)*b+2*b^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.98676, size = 235, normalized size = 6.91

$$-\frac{2\left((b+c)\cosh(x)+(b+c)\sinh(x)-\sqrt{b^2-c^2}\right)}{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+2\left(b^2+2\,bc+c^2\right)\cosh(x)\sinh(x)+\left(b^2+2\,bc+c^2\right)\sinh(x)^2-b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b + c)*cosh(x) + (b + c)*sinh(x) - sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^
2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b^2 + 2*b*c + c^2)*
sinh(x)^2 - b^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.758 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}$$

```
[Out] (c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh[x
] + b*Sinh[x]))
```

Rubi [A] time = 0.0806866, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh[x
] + b*Sinh[x]))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
```

```
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} + \frac{\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,d}{3\sqrt{b^2-c^2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c+\sqrt{b^2-c^2}\sinh(x}{3c\sqrt{b^2-c^2}(c\cosh(x)+b\,s}$$

Mathematica [A] time = 0.143531, size = 68, normalized size = 0.68

$$-\frac{-2c\sqrt{b^2-c^2}+b^2\sinh^3(x)+2bc\cosh^3(x)+2c^2\sinh(x)+c^2\sinh(x)\cosh^2(x)}{3c(b\sinh(x)+c\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] -(-2*c*Sqrt[b^2 - c^2] + 2*b*c*Cosh[x]^3 + 2*c^2*Sinh[x] + c^2*Cosh[x]^2*Si
nh[x] + b^2*Sinh[x]^3)/(3*c*(c*Cosh[x] + b*Sinh[x])^3)
```

Maple [B] time = 0.086, size = 217, normalized size = 2.2

$$2\,\frac{\sqrt{b^2-c^2}+b}{c^2}\left(\frac{\left(\sqrt{b^2-c^2}+b\right)(\tanh(x/2))^2}{c^2}+\frac{\left(2\,b^2-c^2+2\,\sqrt{b^2-c^2}b\right)\tanh(x/2)}{c^3}+2/3\,\frac{2\,\sqrt{b^2-c^2}b^2-\sqrt{b^2-c^2}c^2+2}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x)
```

```
[Out] 2*((b^2-c^2)^(1/2)+b)/c^2*(((b^2-c^2)^(1/2)+b)/c^2*tanh(1/2*x)^2+(2*b^2-c^2
+2*(b^2-c^2)^(1/2)*b)/c^3*tanh(1/2*x)+2/3*(2*(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^
(1/2)*c^2+2*b^3-2*b*c^2)/c^4)/(tanh(1/2*x)^2+2/c*((b-c)*(b+c))^(1/2)*tanh(1
/2*x)+2*b/c*tanh(1/2*x)+2/c^2*((b-c)*(b+c))^(1/2)*b+2/c^2*b^2-1)/(tanh(1/2*
```

```
x)+1/c*((b-c)*(b+c))^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 3.05014, size = 1639, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 12*(b^2 + 2*b*c + c^2)*cosh(x)*sinh
(x)^3 + 3*(b^2 + 2*b*c + c^2)*sinh(x)^4 + 6*(b^2 - c^2)*cosh(x)^2 + 6*(3*(b
^2 + 2*b*c + c^2)*cosh(x)^2 + b^2 - c^2)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 12
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + (b^2 - c^2)*cosh(x))*sinh(x) - 8*((b + c)
*cosh(x)^3 + 3*(b + c)*cosh(x)^2*sinh(x) + 3*(b + c)*cosh(x)*sinh(x)^2 + (b
 + c)*sinh(x)^3)*sqrt(b^2 - c^2))/((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c
^4)*cosh(x)^6 + 6*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(
x)^5 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^6 - 3*(b^4 + 2*b
^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4 - 5*(b^4
 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^4 - b^4 + 2*b^3*
c - 2*b*c^3 + c^4 + 4*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x
)^3 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^3 + 3*(b^4 - 2*b^2
*c^2 + c^4)*cosh(x)^2 + 3*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*co
sh(x)^4 + b^4 - 2*b^2*c^2 + c^4 - 6*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)
^2)*sinh(x)^2 + 6*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 -
2*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + (b^4 - 2*b^2*c^2 + c^4)*cosh(
x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.759 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$

Optimal. Leaf size=146

$$\frac{2(b\sinh(x)+c\cosh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}-\frac{2\left(\sqrt{b^2-c^2}\,\text{s}\right.}{15c\left(b^2-c^2\right)(b\sin}$$

```
[Out] (c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(15*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c*(b^2
 - c^2)*(c*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.119504, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{2(b\sinh(x)+c\cosh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}-\frac{2\left(\sqrt{b^2-c^2}\,\text{s}\right.}{15c\left(b^2-c^2\right)(b\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(15*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c*(b^2
 - c^2)*(c*Cosh[x] + b*Sinh[x]))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x\right.}}{5\sqrt{b^2-c^2}}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2(c\cosh(x)+}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\right.}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2(c\cosh(x)+}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\right.}$$

Mathematica [A] time = 0.37921, size = 184, normalized size = 1.26

$$\frac{-12\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}\sinh(x)+c\right)-\frac{2\sqrt{b^2-c^2}\sinh(x)(b\sinh(x)+c\cosh(x))^4}{(b-c)(b+c)}-\frac{b\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))^3}{(b-c)(b+c)}+\left(\sqrt{b^2-c^2}\sinh(x)-5c\right.}{15c(b\sinh(x)+c\cosh(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (12*b*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]) - (b*Sqrt[b^2 - c^2]*(c*Cosh[
x] + b*Sinh[x])^3)/((b - c)*(b + c)) - (2*Sqrt[b^2 - c^2]*Sinh[x]*(c*Cosh[x
] + b*Sinh[x])^4)/((b - c)*(b + c)) + (c*Cosh[x] + b*Sinh[x])^2*(-5*c + Sqr
t[b^2 - c^2]*Sinh[x]) - 12*(b^2 - c^2)*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c
*(c*Cosh[x] + b*Sinh[x])^5)
```

Maple [B] time = 0.132, size = 488, normalized size = 3.3

$$2\frac{1}{c^4}\left(-\frac{\left(4\sqrt{b^2-c^2}b^2-\sqrt{b^2-c^2}c^2+4b^3-3bc^2\right)(\tanh(x/2))^4}{c^2}-2\frac{\left(8b^4-8c^2b^2+c^4+8\sqrt{b^2-c^2}b^3-4\sqrt{b^2-c^2}c^2b\right)(\mathrm{t}}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x)
```

```
[Out] 2/c^4*(-(4*(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^(1/2)*c^2+4*b^3-3*b*c^2)/c^2*tanh(
1/2*x)^4-2*(8*b^4-8*c^2*b^2+c^4+8*(b^2-c^2)^(1/2)*b^3-4*(b^2-c^2)^(1/2)*c^2
*b)/c^3*tanh(1/2*x)^3-4/3*(24*(b^2-c^2)^(1/2)*b^4-20*(b^2-c^2)^(1/2)*b^2*c^
2+2*(b^2-c^2)^(1/2)*c^4+24*b^5-32*b^3*c^2+9*c^4*b)/c^4*tanh(1/2*x)^2-2/3*(4
8*b^6-76*b^4*c^2+31*b^2*c^4-2*c^6+48*(b^2-c^2)^(1/2)*b^5-52*(b^2-c^2)^(1/2)
*b^3*c^2+11*(b^2-c^2)^(1/2)*b*c^4)/c^5*tanh(1/2*x)-1/15/c^6*(192*(b^2-c^2)^
(1/2)*b^6-256*(b^2-c^2)^(1/2)*b^4*c^2+96*(b^2-c^2)^(1/2)*b^2*c^4-7*(b^2-c^2
)^(1/2)*c^6+192*b^7-352*b^5*c^2+200*b^3*c^4-35*b*c^6))/(tanh(1/2*x)^2+2/c*(
b^2-c^2)^(1/2)*tanh(1/2*x)+2*b/c*tanh(1/2*x)+2*b/c^2*(b^2-c^2)^(1/2)+2/c^2*
b^2-1)^2/(tanh(1/2*x)+1/c*(b^2-c^2)^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 3.12206, size = 6947, normalized size = 47.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(10*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 70*(b^4 +
 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^6 + 10*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^7 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4
)*cosh(x)^5 + 2*(38*b^4 + 76*b^3*c - 76*b*c^3 - 38*c^4 + 105*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^5 + 10*(35*(b^4 + 4*b^3*c
+ 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 38*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)
```

*cosh(x))*sinh(x)^4 + 10*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 + 10*(35*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 + 76 *(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^3 + 10*(21*(b^4 + 4*b^3 *c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c ^4)*cosh(x)^3 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^2 + 10*(7*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 38*(b^4 + 2*b^3*c - 2*b*c ^3 - c^4)*cosh(x)^4 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x) - (45*(b ^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 270*(b^3 + 3*b^2*c + 3*b*c^2 + c^ 3)*cosh(x)*sinh(x)^5 + 45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 55*(b ^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(11*b^3 + 11*b^2*c - 11*b*c^2 - 11* c^3 + 135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*(45*(b^ 3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 11*(b^3 + b^2*c - b*c^2 - c^3)*cos h(x))*sinh(x)^3 + b^3 - 3*b^2*c + 3*b*c^2 - c^3 - 5*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - b^3 + b ^2*c + b*c^2 - c^3 + 66*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 10*(27*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 22*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 - (b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x))*sqrt(b^2 - c^2))/((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^10 + 10*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)*sinh(x)^9 + (b^7 + 7*b^6* c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*sinh (x)^10 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^8 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^ 3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7 - 9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4* c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^8 + 40*(3 *(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c ^6 + c^7)*cosh(x)^3 - (b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x))*sinh(x)^7 - b^7 + 3*b^6*c - b^5*c^2 - 5 *b^4*c^3 + 5*b^3*c^4 + b^2*c^5 - 3*b*c^6 + c^7 + 10*(b^7 + 3*b^6*c + b^5*c^ 2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^6 + 10*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7 + 21*(b ^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^4 - 14*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^7 + 7*b^6*c + 21 *b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^5 - 70*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c ^6 - c^7)*cosh(x)^3 + 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x))*sinh(x)^5 - 10*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^4 - 10*(b^7 + b^ 6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 - 21*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^6 + 35*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9* b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^4 - 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^ 3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^4 + 40*(3*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c

```
^7)*cosh(x)^7 - 7*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^
2*c^5 - 5*b*c^6 - c^7)*cosh(x)^5 + 5*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 -
 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^3 - (b^7 + b^6*c - 3*b^5*c^2
- 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x))*sinh(x)^3 + 5*(
b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*
cosh(x)^2 + 5*(9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21
*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^8 + b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 +
 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7 - 28*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^
4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^6 + 30*(b^7 + 3*b^6*
c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^4 -
12*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c
^7)*cosh(x)^2)*sinh(x)^2 + 10*((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 3
5*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^9 - 4*(b^7 + 5*b^6*c + 9*b^
5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^7 + 6*(b
^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*c
osh(x)^5 - 4*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 -
 b*c^6 - c^7)*cosh(x)^3 + (b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4
- 3*b^2*c^5 - b*c^6 + c^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.760 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$

Optimal. Leaf size=198

$$\frac{2(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{3(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{b\,\mathrm{si}}{7\sqrt{b^2-c^2}\left(\sqrt{b^2\right.}}$$

```
[Out] (c*Cosh[x] + b*Sinh[x])/(7*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^4) + (3*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)^
(3/2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2
]*Sinh[x]))/(35*c*(b^2 - c^2)^(3/2)*(c*Cosh[x] + b*Sinh[x]))
```

Rubi [A] time = 0.177855, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{2(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{3(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{b\,\mathrm{si}}{7\sqrt{b^2-c^2}\left(\sqrt{b^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```


```
[Out] (c*Cosh[x] + b*Sinh[x])/(7*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^4) + (3*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)^
(3/2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2
]*Sinh[x]))/(35*c*(b^2 - c^2)^(3/2)*(c*Cosh[x] + b*Sinh[x]))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
```

```
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}dx = \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh\right.}}{7\sqrt{b^2-c^2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3(c\cosh(x)}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3(c\cosh(x)}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3(c\cosh(x)}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+\right.}$$

Mathematica [B] time = 0.79379, size = 425, normalized size = 2.15

$$-\frac{1295b^4c^2\sinh(x)+189b^4c^2\sinh(3x)-35b^4c^2\sinh(5x)+15b^4c^2\sinh(7x)-896b^3c^2\sqrt{b^2-c^2}\sinh(2x)-2485b^2c^4\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -(-832*b^4*c*Sqrt[b^2 - c^2] + 1664*b^2*c^3*Sqrt[b^2 - c^2] - 832*c^5*Sqrt[
b^2 - c^2] + 1190*b*c*(b^2 - c^2)^2*Cosh[x] + 448*c*Sqrt[b^2 - c^2]*(-b^4 +
 c^4)*Cosh[2*x] + 112*b^5*c*Cosh[3*x] + 56*b^3*c^3*Cosh[3*x] - 168*b*c^5*Co
sh[3*x] - 28*b^5*c*Cosh[5*x] + 28*b*c^5*Cosh[5*x] + 6*b^5*c*Cosh[7*x] + 20*
b^3*c^3*Cosh[7*x] + 6*b*c^5*Cosh[7*x] - 35*b^6*Sinh[x] + 1295*b^4*c^2*Sinh[
```

```
x] - 2485*b^2*c^4*Sinh[x] + 1225*c^6*Sinh[x] - 896*b^3*c^2*Sqrt[b^2 - c^2]*
Sinh[2*x] + 896*b*c^4*Sqrt[b^2 - c^2]*Sinh[2*x] + 21*b^6*Sinh[3*x] + 189*b^
4*c^2*Sinh[3*x] - 161*b^2*c^4*Sinh[3*x] - 49*c^6*Sinh[3*x] - 7*b^6*Sinh[5*x
] - 35*b^4*c^2*Sinh[5*x] + 35*b^2*c^4*Sinh[5*x] + 7*c^6*Sinh[5*x] + b^6*Sin
h[7*x] + 15*b^4*c^2*Sinh[7*x] + 15*b^2*c^4*Sinh[7*x] + c^6*Sinh[7*x])/(1120
*(b - c)*c*(b + c)*(c*Cosh[x] + b*Sinh[x])^7)
```

Maple [B] time = 0.229, size = 828, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x)
```

```
[Out] 2/c^6*((8*b^4-8*c^2*b^2+c^4+8*(b^2-c^2)^(1/2)*b^3-4*(b^2-c^2)^(1/2)*c^2*b)/
c^2*tanh(1/2*x)^6+3*(16*(b^2-c^2)^(1/2)*b^4-12*(b^2-c^2)^(1/2)*b^2*c^2+(b^2
-c^2)^(1/2)*c^4+16*b^5-20*b^3*c^2+5*c^4*b)/c^3*tanh(1/2*x)^5+2*(80*(b^2-c^2
)^(1/2)*b^5-84*(b^2-c^2)^(1/2)*b^3*c^2+17*(b^2-c^2)^(1/2)*b*c^4+80*b^6-124*
b^4*c^2+49*b^2*c^4-3*c^6)/c^4*tanh(1/2*x)^4+2*(160*b^7-288*b^5*c^2+150*b^3*
c^4-20*b*c^6+160*(b^2-c^2)^(1/2)*b^6-208*(b^2-c^2)^(1/2)*b^4*c^2+66*(b^2-c^
2)^(1/2)*b^2*c^4-3*(b^2-c^2)^(1/2)*c^6)/c^5*tanh(1/2*x)^3+3/5*(640*b^7*(b^2
-c^2)^(1/2)-992*(b^2-c^2)^(1/2)*b^5*c^2+440*(b^2-c^2)^(1/2)*b^3*c^4-50*(b^2
-c^2)^(1/2)*b*c^6+640*b^8-1312*b^6*c^2+856*c^4*b^4-186*b^2*c^6+7*c^8)/c^6*t
anh(1/2*x)^2+1/5*(1280*b^9-2944*b^7*c^2+2288*b^5*c^4-676*b^3*c^6+57*b*c^8+1
280*(b^2-c^2)^(1/2)*b^8-2304*(b^2-c^2)^(1/2)*b^6*c^2+1296*(b^2-c^2)^(1/2)*b
^4*c^4-236*(b^2-c^2)^(1/2)*b^2*c^6+7*(b^2-c^2)^(1/2)*c^8)/c^7*tanh(1/2*x)+4
/35*(640*(b^2-c^2)^(1/2)*b^9-1312*(b^2-c^2)^(1/2)*b^7*c^2+896*(b^2-c^2)^(1/
2)*b^5*c^4-238*(b^2-c^2)^(1/2)*b^3*c^6+21*(b^2-c^2)^(1/2)*b*c^8+640*b^10-16
32*b^8*c^2+1472*b^6*c^4-562*b^4*c^6+85*b^2*c^8-3*c^10)/c^8)/(tanh(1/2*x)^2+
2/c*(b^2-c^2)^(1/2)*tanh(1/2*x)+2*b/c*tanh(1/2*x)+2*b/c^2*(b^2-c^2)^(1/2)+2
/c^2*b^2-1)^3/(tanh(1/2*x)+1/c*(b^2-c^2)^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [B] time = 4.05519, size = 15194, normalized size = 76.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")

```
[Out] -4/35*(35*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)
^10 + 350*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)
*sinh(x)^9 + 35*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*s
inh(x)^10 + 595*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cos
h(x)^8 + 35*(17*b^5 + 51*b^4*c + 34*b^3*c^2 - 34*b^2*c^3 - 51*b*c^4 - 17*c^
5 + 45*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^2)
*sinh(x)^8 + 280*(15*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c
^5)*cosh(x)^3 + 17*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*
cosh(x))*sinh(x)^7 + 630*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)^6 + 70*(9*b^5 + 9*b^4*c - 18*b^3*c^2 - 18*b^2*c^3 + 9*b*c^4 + 9*c
^5 + 105*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^
4 + 238*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*
sinh(x)^6 + 140*(63*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^
5)*cosh(x)^5 + 238*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*
cosh(x)^3 + 27*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))
*sinh(x)^5 - b^5 + 5*b^4*c - 10*b^3*c^2 + 10*b^2*c^3 - 5*b*c^4 + c^5 + 14*(
b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^4 + 14*(525*(b^5
 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^6 + b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + 2975*(b^5 + 3*b^4*c + 2*b^3*c^2
 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^4 + 675*(b^5 + b^4*c - 2*b^3*c^2 - 2*b
^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*sinh(x)^4 + 56*(75*(b^5 + 5*b^4*c + 10*b^3
*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^7 + 595*(b^5 + 3*b^4*c + 2*b^3*c
^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^5 + 225*(b^5 + b^4*c - 2*b^3*c^2 -
2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 +
 b*c^4 - c^5)*cosh(x))*sinh(x)^3 + 7*(b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3
 - 3*b*c^4 + c^5)*cosh(x)^2 + 7*(225*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c
^3 + 5*b*c^4 + c^5)*cosh(x)^8 + 2380*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3
 - 3*b*c^4 - c^5)*cosh(x)^6 + b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c
^4 + c^5 + 1350*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)
^4 + 12*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^2)*sinh
(x)^2 + 14*(25*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*co
sh(x)^9 + 340*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(
```

```
x)^7 + 270*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^5 +
4*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^3 + (b^5 - 3*
b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh(x))*sinh(x) - 32*(7*(b^
4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^9 + 63*(b^4 + 4*b^3*c + 6*
b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^8 + 7*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*sinh(x)^9 + 26*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^7 +
2*(13*b^4 + 26*b^3*c - 26*b*c^3 - 13*c^4 + 126*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^7 + 14*(42*(b^4 + 4*b^3*c + 6*b^2*c^2 +
4*b*c^3 + c^4)*cosh(x)^3 + 13*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh
(x)^6 + 7*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^5 + 7*(126*(b^4 + 4*b^3*c + 6*b^2
*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 + 78*(b^4 + 2*b^3*c
 - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^5 + 7*(126*(b^4 + 4*b^3*c + 6*b^2*c^2
+ 4*b*c^3 + c^4)*cosh(x)^5 + 130*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3
+ 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^4 + 14*(42*(b^4 + 4*b^3*c + 6*
b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 65*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cos
h(x)^4 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x)^3 + 14*(18*(b^4 + 4*b
^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 39*(b^4 + 2*b^3*c - 2*b*c^3 -
 c^4)*cosh(x)^5 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3)*sinh(x)^2 + 7*(9*(b^
4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 26*(b^4 + 2*b^3*c - 2*
b*c^3 - c^4)*cosh(x)^6 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^4)*sinh(x))*sqrt
(b^2 - c^2))/((b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*
b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^14 + 14*(b^9 + 9
*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 +
 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)*sinh(x)^13 + (b^9 + 9*b^8*c + 36*b^7*c
^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b
*c^8 + c^9)*sinh(x)^14 - 7*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^
5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^12 -
7*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b
^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9 - 13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b
^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^
9)*cosh(x)^2)*sinh(x)^12 + 28*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3
+ 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh
(x)^3 - 3*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^
5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x))*sinh(x)^11 + 21*(b^9
+ 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9
)*cosh(x)^10 + 7*(3*b^9 + 15*b^8*c + 24*b^7*c^2 - 42*b^5*c^4 - 42*b^4*c^5 +
 24*b^2*c^7 + 15*b*c^8 + 3*c^9 + 143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c
^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*c
osh(x)^4 - 66*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^
4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^2)*sinh(x)^10 + 14
*(143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5
+ 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^5 - 110*(b^9 + 7*b^8*c +
 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^
7 - 7*b*c^8 - c^9)*cosh(x)^3 + 15*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 -
 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x))*sinh(x)^9 - b^9 + 5*b^8*c
```

- 8*b^7*c^2 + 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 - 5*b*c^8 + c^9 - 35*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^8 - 7*(5*b^9 + 15*b^8*c - 40*b^6*c^3 - 30*b^5*c^4 + 30*b^4*c^5 + 40*b^3*c^6 - 15*b*c^8 - 5*c^9 - 429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^6 + 495*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^4 - 135*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^2)*sinh(x)^8 + 8*(429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^7 - 693*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^5 + 315*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^3 - 35*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x))*sinh(x)^7 + 35*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^6 + 7*(5*b^9 + 5*b^8*c - 20*b^7*c^2 - 20*b^6*c^3 + 30*b^5*c^4 + 30*b^4*c^5 - 20*b^3*c^6 - 20*b^2*c^7 + 5*b*c^8 + 5*c^9 + 429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^8 - 924*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^6 + 630*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^4 - 140*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^2)*sinh(x)^6 + 14*(143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^9 - 396*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^7 + 378*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^5 - 140*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^3 + 15*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x))*sinh(x)^5 - 21*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 - 6*b^4*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^4 + 7*(143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^10 - 3*b^9 + 3*b^8*c + 12*b^7*c^2 - 12*b^6*c^3 - 18*b^5*c^4 + 18*b^4*c^5 + 12*b^3*c^6 - 12*b^2*c^7 - 3*b*c^8 + 3*c^9 - 495*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^8 + 630*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^6 - 350*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^4 + 75*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^2)*sinh(x)^4 + 28*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^11 - 55*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^

```
2*c^7 - 7*b*c^8 - c^9)*cosh(x)^9 + 90*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c
^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^7 - 70*(b^9 + 3*b^8*c
- 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^5
+ 25*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c
^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^3 - 3*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^
6*c^3 + 6*b^5*c^4 - 6*b^4*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x
))*sinh(x)^3 + 7*(b^9 - 3*b^8*c + 8*b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3
*c^6 - 3*b*c^8 + c^9)*cosh(x)^2 + 7*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^
6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9
)*cosh(x)^12 - 66*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 1
4*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^10 + b^9 - 3*b
^8*c + 8*b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 + c^9 + 135*
(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8
+ c^9)*cosh(x)^8 - 140*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 +
 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^6 + 75*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6
*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)
^4 - 18*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 - 6*b^4*c^5 - 4*b^
3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^2)*sinh(x)^2 + 14*((b^9 + 9*b^8*c
+ 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2
*c^7 + 9*b*c^8 + c^9)*cosh(x)^13 - 6*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c
^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cos
h(x)^11 + 15*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c
^7 + 5*b*c^8 + c^9)*cosh(x)^9 - 20*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 +
 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^7 + 15*(b^9 + b^8*c - 4*b^7
*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 +
c^9)*cosh(x)^5 - 6*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 - 6*b^4
*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^3 + (b^9 - 3*b^8*c + 8*
b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 + c^9)*cosh(x))*sinh(
x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.761 $\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=294

$$\frac{16ia\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(}}{15\sqrt{}}$$

```
[Out] (16*(a*c*Cosh[x] + a*b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c
*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^(3/2))/5 - (((2*I)/15)*(2
3*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 -
 c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*
Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + (((16*I)/15)*a*(a^2 - b^2 + c
^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^
2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a
+ b*Cosh[x] + c*Sinh[x]]
```

Rubi [A] time = 0.482871, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3120, 3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{16ia\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sin}}{15\sqrt{\frac{a+b\cosh(x}{a+\sqrt{}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```


```
[Out] (16*(a*c*Cosh[x] + a*b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c
*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^(3/2))/5 - (((2*I)/15)*(2
3*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 -
 c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*
Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + (((16*I)/15)*a*(a^2 - b^2 + c
^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^
2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a
+ b*Cosh[x] + c*Sinh[x]]
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
```

```
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2}\,dx = \frac{2}{5}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a + b\cosh(x) + c\,\mathrm{si}$$

$$= \frac{16}{15}(ac\cosh(x) + ab\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} + \frac{2}{5}(c\cosh(x) + b\sinh(x))($$

$$= \frac{16}{15}(ac\cosh(x) + ab\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} + \frac{2}{5}(c\cosh(x) + b\sinh(x))($$

$$= \frac{16}{15}(ac\cosh(x) + ab\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} + \frac{2}{5}(c\cosh(x) + b\sinh(x))($$

$$= \frac{16}{15}(ac\cosh(x) + ab\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} + \frac{2}{5}(c\cosh(x) + b\sinh(x))($$

Mathematica [C] time = 6.33537, size = 3775, normalized size = 12.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((2*b*(23*a^2 + 9*b^2 - 9*c^2))/(15*c) + (2
2*a*c*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (22*a*b*Sinh[x])/15 + ((b^2 + c^2
```

)*Sinh[2*x])/5) + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b
^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 -
b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(S
qrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/
c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*
c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)
/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
+ ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh
[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + S
qrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(
Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b
/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + A
rcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/
c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b
^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2
]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*S
qrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*Sqrt
[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1
 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
 ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*
Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b
^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a +
c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c
^2]*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1
 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x
 + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sin
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2]
)]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b
^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sq
rt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt
[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(15*c) - (3*b^4*((c*Appe
llF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
)/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/
b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b
^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^
2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2
- c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt
[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])

```
/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x
+ ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/
b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c) + (23*a
^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x +
ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*
Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*S
qrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*S
qrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a +
 b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTan
h[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + A
rcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*
Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/
15 + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*
Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2])))
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1
 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x
 + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*C
osh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqr
t[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[
c/b]]]))/5 - (3*c^3*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b
^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^
2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcT
anh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*
Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a +
 b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + Arc
Tanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar
cTanh[c/b]]]))/5
```

Maple [B] time = 0.879, size = 1036, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(5/2),x)
```

```
[Out] 1/6/(-b^2*sinh(x)^2+c^2*sinh(x)^2+2*sinh(x)*a*(b^2-c^2)^(1/2)-a^2)/sinh(x)*
(-6*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2
-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*s
inh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*a
^4+3*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(
b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^
2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*
sinh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*
a^2*b^2-3*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c
^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2
)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(
1/2)*sinh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(
1/2)*a^2*c^2+5*sinh(x)^3*cosh(x)*(b^2-c^2)^(3/2)*((-b^2+c^2)/(b^2-c^2)^(1/2
)*sinh(x)+a)^(1/2)*a-2*(b^4-2*b^2*c^2+c^4)*cosh(x)*sinh(x)^4*((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)+a)^(1/2)+3*(2*a^2-b^2+c^2)*sinh(x)*ln((cosh(x)*sinh(x
)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^
3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)
^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))*(b^2-
c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*a+2*sin
h(x)*cosh(x)*(9*a^2-2*b^2+2*c^2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2
)*sinh(x)+a)^(1/2)*a-(21*a^2*b^2-21*a^2*c^2-4*b^4+8*b^2*c^2-4*c^4)*cosh(x)*
sinh(x)^2*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh (x) + c\sinh (x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cosh (x)^2 + c^2\sinh (x)^2 + 2\,ab\cosh (x) + a^2 + 2\,(bc\cosh (x) + ac)\sinh (x)\right)\sqrt{b\cosh (x) + c\sinh (x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(x)^2 + c^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*(b*c*cosh
(x) + a*c)*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

3.762 $\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{2i\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(\right.\right.}{3\sqrt{\frac{a+b\cosh(x)+}{a+\sqrt{b^2}}}}$$

```
[Out] (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 - (((8*I)/3)*
a*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2
- c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/
(a + Sqrt[b^2 - c^2])] + (((2*I)/3)*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcT
an[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Co
sh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]]
```

Rubi [A] time = 0.270464, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2i\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan\right.\right.}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```


```
[Out] (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 - (((8*I)/3)*
a*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2
- c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/
(a + Sqrt[b^2 - c^2])] + (((2*I)/3)*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcT
an[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Co
sh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]]
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
```

```
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2-c^2\right)+2a}{\sqrt{a+b\cosh}}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{1}{3}(4a)\int \sqrt{a+b\cosh(x)+}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{\left(4a\sqrt{a+b\cosh(x)+c\sinh}\right.}{}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}-\frac{8iaE\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right.\right.}{3\sqrt{\frac{a}{}}}$$

Mathematica [C] time = 6.14134, size = 2292, normalized size = 9.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] ((8*a*b)/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[a + b*Cosh[x] + c*
Sinh[x]] + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]
*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2
]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 -
 b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqr
t[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTa
nh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh[x + Ar
cTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^
2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b
^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sq
rt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c
]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/
c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
 ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
```

```
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[
1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt
[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]
))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTa
nh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2]
 - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 +
 c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Si
nh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*Sqrt[I*(I +
 Sinh[x + ArcTanh[b/c]])]) - (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a
/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/
(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2
])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2)
 + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^
2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*c) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1
/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b
^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTa
nh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b
^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[
a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2
)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2
- c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(
b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*S
qrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/3
```

Maple [A] time = 0.362, size = 318, normalized size = 1.3

$$2\,{\frac {a \left(-{b}^{2}+{c}^{2} \right) \cosh \left(x \right) }{\sqrt {{b}^{2}-{c}^{2}}}}{\frac {1}{\sqrt {{\frac {-\sinh \left(x \right) {b}^{2}+\sinh \left(x \right) {c}^{2}+a\sqrt {{b}^{2}-{c}^{2}}}{\sqrt {{b}^{2}-{c}^{2}}}}}}}}+{\frac {{a}^{2}}{\sinh \left(x \right) }\sqrt { \left(\sinh \left(x \right) \right) ^{2} \left(-\sinh \left(x \right) {b}^{2}+\sinh \left(x \right) {c}^{2}+a\sqrt {{b}^{2}-{c}^{2}} \right) {\frac {1}{\sqrt {b^{2}}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(3/2),x)
```

```
[Out] 2*a/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2)
)/(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a^2*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cos
h(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(
1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2
)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2
))/(-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + c\sinh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\cosh(x) + c\sinh(x) + a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + c\sinh(x) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

3.763 $\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sin
h[x])/(a + Sqrt[b^2 - c^2])]
```

Rubi [A] time = 0.0714839, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3119, 2653}

$$-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```


```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sin
h[x])/(a + Sqrt[b^2 - c^2])]
```


Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}\,dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$= -\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Mathematica [C] time = 6.10253, size = 1401, normalized size = 13.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/c + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2,
 ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]
*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[
x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c
)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[
(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a
 + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b
^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c
^2]*c*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-1/2, -1/2,
 -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^
2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x +
 ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b
^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])
```

```
)/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/c + c*((c*AppellF1[-1/2, -1/2
, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c
^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + A
rcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x
+ ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqr
t[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
 c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(
b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
 b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]])
```

Maple [B] time = 0.373, size = 314, normalized size = 3.1

$$\left(-b^2+c^2\right)\cosh(x)\frac{1}{\sqrt{b^2-c^2}}\frac{1}{\sqrt{\left(-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\frac{1}{\sqrt{b^2-c^2}}}}+\frac{a}{\sinh(x)}\sqrt{(\sinh(x))^2\left(-\sinh(x)b^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b
^2-c^2)^(1/2))^(1/2)*cosh(x)+((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(
b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(
b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2
)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-s
inh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cosh(x)+c\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cosh\left(x\right)+c\sinh\left(x\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\cosh\left(x\right)+c\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x)+c*sinh(x))**(1/2),x)

[Out] Integral(sqrt(a + b*cosh(x) + c*sinh(x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cosh\left(x\right)+c\sinh\left(x\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)

3.764 $\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=102

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqr
t[a + b*Cosh[x] + c*Sinh[x]]
```

Rubi [A] time = 0.0713375, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3127, 2661}

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqr
t[a + b*Cosh[x] + c*Sinh[x]]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}\,dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$= -\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 0.482625, size = 237, normalized size = 2.32

$$\frac{2\text{sech}\left(\tanh^{-1}\left(\frac{b}{c}\right)+x\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\sqrt{-\frac{-ic\sqrt{1-\frac{b^2}{c^2}}+b\cosh(x)+c\sinh(x)}{a+ic\sqrt{1-\frac{b^2}{c^2}}}}\sqrt{-\frac{ic\sqrt{1-\frac{b^2}{c^2}}+b\cosh(x)+c\sinh(x)}{a-ic\sqrt{1-\frac{b^2}{c^2}}}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2}\right.}{c\sqrt{1-\frac{b^2}{c^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 -
 b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c)]*Sech
[x + ArcTanh[b/c]]*Sqrt[a + b*Cosh[x] + c*Sinh[x]]*Sqrt[-(((-I)*Sqrt[1 - b^
2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*Sqrt[-((I*S
qrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c))])/
(Sqrt[1 - b^2/c^2]*c)
```

Maple [A] time = 0.244, size = 248, normalized size = 2.4

$$\frac{1}{\sinh(x)}\sqrt{(\sinh(x))^2\left(-\sinh(x)\,b^2+\sinh(x)\,c^2+a\sqrt{b^2-c^2}\right)\frac{1}{\sqrt{b^2-c^2}}}\ln\left(\left(\cosh(x)\sinh(x)\left(-b^2+c^2\right)+\cosh(x)\sqrt{b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x)
```

```
[Out] ((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1
/2)*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2
-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*
(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c
^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\cosh(x) + c\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

3.765 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2(b\sinh(x)+c\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh
[x]]) - ((2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(
a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)*S
qrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])
```

Rubi [A] time = 0.0979996, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3128, 3119, 2653}

$$-\frac{2(b\sinh(x)+c\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh
[x]]) - ((2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(
a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)*S
qrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int\sqrt{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{\ldots}{a+\sqrt{\ldots}}}}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\ldots}{\ldots}}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\ldots}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\,c\ldots}{\ldots}}}$$

Mathematica [C] time = 6.19654, size = 1522, normalized size = 9.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(b^2 - c^2))/(b*c*(-a^2 + b^2 - c^2))
- (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x
] + c*Sinh[x]))) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^
2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b
^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sq
rt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c
```

```
]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/
c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
 ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)
*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-1/2, -1/2, -1/2
, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2
]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh
[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcT
anh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2
 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a
+ b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/
b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 -
c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^
2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqr
t[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(c*(a^2 - b^2 + c^2)) + (c*((c*App
ellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/
b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c
^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2
 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqr
t[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x
 + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(a^2 - b^2 +
 c^2)
```

Maple [B] time = 0.958, size = 1430, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x)
```

```
[Out] 1/2*(2*(b^2-c^2)^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a^2+b^2-c^2)*(b^2-c^2))^
(1/2))*(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((a^2+b^2-c^2)*(b
-c)*(b+c))^(1/2)*sinh(x)-a*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^
2)^(1/2)*((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*ln((cosh(x)*sinh(x)*(2*((a^2+b^2-c
^2)*(b-c)*(b+c))^(1/2)*b^4-4*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*b^2*c^2+2*((
a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*c^4)+cosh(x)*(-2*(b^2-c^2)^(1/2)*((a^2+b^2-
```

```
c^2)*(b-c)*(b+c))^(1/2)*a*b^2+2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))
^(1/2)*a*c^2)+sinh(x)*(2*b^6-6*b^4*c^2+6*b^2*c^4-2*c^6)+2*(-a^2/(b^2-c^2)^(
1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*s
inh(x)^2)^(1/2)*(b^2-c^2)^(3/2)*b^2-2*(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^
2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2
-c^2)^(3/2)*c^2+2*(b^2-c^2)^(3/2)*a^3-2*a^3*b^2*(b^2-c^2)^(1/2)+2*a^3*c^2*(
b^2-c^2)^(1/2)-2*(b^2-c^2)^(1/2)*a*b^4+4*(b^2-c^2)^(1/2)*a*b^2*c^2-2*(b^2-c
^2)^(1/2)*a*c^4)/(b^2*cosh(x)-c^2*cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)
)/(b^2-c^2)^(3/2))+a*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/
2)*((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*ln((cosh(x)*sinh(x)*(2*((a^2+b^2-c^2)*(b
-c)*(b+c))^(1/2)*b^4-4*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*b^2*c^2+2*((a^2+b^
2-c^2)*(b-c)*(b+c))^(1/2)*c^4)+cosh(x)*(-2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(
b-c)*(b+c))^(1/2)*a*b^2+2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)
*a*c^2)+sinh(x)*(-2*b^6+6*b^4*c^2-6*b^2*c^4+2*c^6)-2*(-a^2/(b^2-c^2)^(1/2)*
sinh(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x
)^2)^(1/2)*(b^2-c^2)^(3/2)*b^2+2*(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2
))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)
^(3/2)*c^2-2*(b^2-c^2)^(3/2)*a^3+2*a^3*b^2*(b^2-c^2)^(1/2)-2*a^3*c^2*(b^2-c
^2)^(1/2)+2*(b^2-c^2)^(1/2)*a*b^4-4*(b^2-c^2)^(1/2)*a*b^2*c^2+2*(b^2-c^2)^(
1/2)*a*c^4)/(-b^2*cosh(x)+c^2*cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2))/(b
^2-c^2)^(3/2)))/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2)/((a^2+b^2-c^2)
*(b^2-c^2))^(1/2)/(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)/((a^2+
b^2-c^2)*(b-c)*(b+c))^(1/2)/sinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cosh(x) + c\sinh(x) + a}}{b^2\cosh(x)^2 + c^2\sinh(x)^2 + 2\,ab\cosh(x) + a^2 + 2\,(bc\cosh(x) + ac)\sinh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^2*cosh(x)^2 + c^2*sinh(x)^2 + 2
*a*b*cosh(x) + a^2 + 2*(b*c*cosh(x) + a*c)*sinh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x) + c*sinh(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

3.766 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=322

$$\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-i\right.\right.}{3\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(3/2)) - (8*(a*c*Cosh[x] + a*b*Sinh[x]))/(3*(a^2 - b^2 + c^2)^2*Sqrt[a +
 b*Cosh[x] + c*Sinh[x]]) - (((8*I)/3)*a*EllipticE[(I*x - ArcTan[b, (-I)*c])
/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[
x]])/((a^2 - b^2 + c^2)^2*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 -
c^2])]) + (((2*I)/3)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c
^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2
- c^2])])/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

Rubi [A] time = 0.340195, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-\ldots}}{a+\sqrt{b^2\ldots}}\right.}{3\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(3/2)) - (8*(a*c*Cosh[x] + a*b*Sinh[x]))/(3*(a^2 - b^2 + c^2)^2*Sqrt[a +
 b*Cosh[x] + c*Sinh[x]]) - (((8*I)/3)*a*EllipticE[(I*x - ArcTan[b, (-I)*c])
/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[
x]])/((a^2 - b^2 + c^2)^2*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 -
c^2])]) + (((2*I)/3)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c
^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2
- c^2])])/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)+\frac{1}{2}c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx}{3\left(a^2-b^2+c^2\right)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)}}$$

Mathematica [C] time = 6.23447, size = 2492, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((8*a*(b^2 - c^2))/(3*b*c*(a^2 - b^2 + c^2)
^2) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x])^2) - (2*(-3*a^2*c - b^2*c + c^3 + 4*a*b^2*Sinh[x] - 4*
a*c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))) + (
2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-
1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sin
h[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqr
t[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((
-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
 + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqr
t[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sq
rt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c
]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + Arc
Tanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^
2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2
 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*
Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqr
t[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*(a^2 -
b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2
, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1
 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c
^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2
/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sq
rt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[
b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I
*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*S
qrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])])
- (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqr
t[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh
[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
```

```
]]]))/(3*c*(a^2 - b^2 + c^2)^2) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1
+ a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^
2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*S
qrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2]
+ b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/
b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c
^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 -
 c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*(a^2 - b^2 + c^2)^2)
```

Maple [B] time = 2.066, size = 6019, normalized size = 18.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cosh(x) + c\sinh(x) + a}}{b^3\cosh(x)^3 + c^3\sinh(x)^3 + 3\,ab^2\cosh(x)^2 + 3\,a^2b\cosh(x) + a^3 + 3\left(bc^2\cosh(x) + ac^2\right)\sinh(x)^2 + 3\left(b^2c\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^3*cosh(x)^3 + c^3*sinh(x)^3 + 3
*a*b^2*cosh(x)^2 + 3*a^2*b*cosh(x) + a^3 + 3*(b*c^2*cosh(x) + a*c^2)*sinh(x
)^2 + 3*(b^2*c*cosh(x)^2 + 2*a*b*c*cosh(x) + a^2*c)*sinh(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

3.767 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

Optimal. Leaf size=411

$$\frac{16ia\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}E}{15\left(a^2-b^2+c^2\right)^3\sqrt{\frac{a+b\cosh}{a+}}}$$

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(5*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(5/2)) - (16*(a*c*Cosh[x] + a*b*Sinh[x]))/(15*(a^2 - b^2 + c^2)^2*(a + b
*Cosh[x] + c*Sinh[x])^(3/2)) - (((2*I)/15)*(23*a^2 + 9*b^2 - 9*c^2)*Ellipti
cE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*
Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)^3*Sqrt[(a + b*Cosh[x] +
 c*Sinh[x])/(a + Sqrt[b^2 - c^2])]) + (((16*I)/15)*a*EllipticF[(I*x - ArcTa
n[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cos
h[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/((a^2 - b^2 + c^2)^2*Sqrt[a + b*C
osh[x] + c*Sinh[x]]) - (2*(c*(23*a^2 + 9*b^2 - 9*c^2)*Cosh[x] + b*(23*a^2 +
 9*b^2 - 9*c^2)*Sinh[x]))/(15*(a^2 - b^2 + c^2)^3*Sqrt[a + b*Cosh[x] + c*Si
nh[x]])
```

Rubi [A] time = 0.543733, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{16ia\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\right.\right.}{15\left(a^2-b^2+c^2\right)^3\sqrt{\frac{a+b\cosh(x)+c\sinh}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(5*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(5/2)) - (16*(a*c*Cosh[x] + a*b*Sinh[x]))/(15*(a^2 - b^2 + c^2)^2*(a + b
*Cosh[x] + c*Sinh[x])^(3/2)) - (((2*I)/15)*(23*a^2 + 9*b^2 - 9*c^2)*Ellipti
cE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*
Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)^3*Sqrt[(a + b*Cosh[x] +
 c*Sinh[x])/(a + Sqrt[b^2 - c^2])]) + (((16*I)/15)*a*EllipticF[(I*x - ArcTa
n[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cos
```

```
h[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/((a^2 - b^2 + c^2)^2*Sqrt[a + b*C
osh[x] + c*Sinh[x]]) - (2*(c*(23*a^2 + 9*b^2 - 9*c^2)*Cosh[x] + b*(23*a^2 +
 9*b^2 - 9*c^2)*Sinh[x]))/(15*(a^2 - b^2 + c^2)^3*Sqrt[a + b*Cosh[x] + c*Si
nh[x]])
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)+\frac{3}{2}c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx}{5\left(a^2-b^2+c^2\right)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\,\mathrm{si}}{15\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\,\mathrm{si}}{15\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\,\mathrm{si}}{15\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\,\mathrm{si}}{15\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\,\mathrm{si}}{15\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

Mathematica [C] time = 6.53114, size = 4093, normalized size = 9.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(23*a^2 + 9*b^2 - 9*c^2)*(b^2 - c^2))/
(15*b*c*(-a^2 + b^2 - c^2)^3) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(5*b*
(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (2*(-5*a^2*c - 3*b^2*c
+ 3*c^3 + 8*a*b^2*Sinh[x] - 8*a*c^2*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x])^2) + (2*(-15*a^3*c - 17*a*b^2*c + 17*a*c^3 + 23*a
^2*b^2*Sinh[x] + 9*b^4*Sinh[x] - 23*a^2*c^2*Sinh[x] - 18*b^2*c^2*Sinh[x] +
9*c^4*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^3*(a + b*Cosh[x] + c*Sinh[x]))) +
(2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
```

ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(15*c*(a^2 - b^2 + c^2)^3) - (3*b^4*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt

```
[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a
 + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c*(a^2 - b^2 + c^2)^3)
+ (23*a^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Co
sh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))),
(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
 a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sq
rt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]
])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
 ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cos
h[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[
1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b
]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]]]))/(15*(a^2 - b^2 + c^2)^3) + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1
/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(
1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh
[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 -
c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2
] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2
)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 -
 c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 - b^2 + c^2)^3) - (3*c^3*((c*
AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 -
c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c
^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2
- c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(
b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 -
c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 -
 b^2 + c^2)^3)
```

Maple [B] time = 10.031, size = 58437, normalized size = 142.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\,\cdots}}{b^4\cosh(x)^4 + c^4\sinh(x)^4 + 4\,ab^3\cosh(x)^3 + 6\,a^2b^2\cosh(x)^2 + 4\,a^3b\cosh(x) + a^4 + 4\left(bc^3\cosh(x) + ac^3\right)\sin\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^4*cosh(x)^4 + c^4*sinh(x)^4 + 4
*a*b^3*cosh(x)^3 + 6*a^2*b^2*cosh(x)^2 + 4*a^3*b*cosh(x) + a^4 + 4*(b*c^3*c
osh(x) + a*c^3)*sinh(x)^3 + 6*(b^2*c^2*cosh(x)^2 + 2*a*b*c^2*cosh(x) + a^2*
c^2)*sinh(x)^2 + 4*(b^3*c*cosh(x)^3 + 3*a*b^2*c*cosh(x)^2 + 3*a^2*b*c*cosh(
x) + a^3*c)*sinh(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(7/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

3.768 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

Optimal. Leaf size=140

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(}$$

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[
x] + c*Sinh[x]]) + (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rubi [A] time = 0.123173, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3113, 3112}

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[
x] + c*Sinh[x]]) + (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
```

```
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx = \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{1}{5}\Big(8\sqrt{b$$
$$= \frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}+$$
$$= \frac{64\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh($$

Mathematica [C] time = 74.0124, size = 10223, normalized size = 73.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.638, size = 518, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*(-1/3*(b^2-c^2
)^(3/2)*cosh(x)^3-(-2*b^2+2*c^2)*(-b^2+c^2)/(b^2-c^2)^(1/2)*cosh(x))-1/2*(c
osh(x)*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*(sinh(x
)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(b^2-c^2)-sinh(x)*(b^2-c^2)^(3/2)*
arctan((sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^
(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))+(b^2-c^2)^(1/2)*arctan((s
inh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*sin
```

```
h(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))*b^2-(b^2-c^2)^(1/2)*arctan((sinh(x
)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*sinh(x)^
3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))*c^2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh
(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)/(sinh(x)-1)/sinh(x)/(-(sin
h(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 15.5859, size = 2407, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e
^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 -
 c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^
2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt
(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*
e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/12*sqrt(2)*(b^3 +
b^2*c - b*c^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) +
 b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
 - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(
-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*
x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b
^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x)
+ (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sqrt
(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x
) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2
+ 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^
2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqr
t(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt
(b - c)*c^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/2*sqr
t(2)*(b^3 - b^2*c - b*c^2 + c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c
)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(
b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c
^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c
)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*
```

```
sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(sq
rt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqr
t(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*
b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) +
10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 1
0*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2
*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3
 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 1/20*sqrt(2)*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(5/
2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c
+ sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) + 10*(
sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 10*(b
^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqr
t(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3 - 3
*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

Fricas [B] time = 2.61326, size = 2079, normalized size = 14.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) + 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
```

```
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45515, size = 887, normalized size = 6.34

$$-\frac{\sqrt{2}\left(3\left(\sqrt{b^2-c^2}b^2\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)+2\sqrt{b^2-c^2}bc\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)+\sqrt{b^2-c^2}c^2\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 2
*sqrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b^2 - c^2)*c^
2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(5/2*x) + 25*(b^3*sgn(-sqrt(b^2 - c^
2)*e^x - b + c) + b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - b*c^2*sgn(-sqrt
(b^2 - c^2)*e^x - b + c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(3/2*x)
 + 150*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 -
c^2)*c^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(1/2*x) - 150*(b^3*sgn(-sqrt(
b^2 - c^2)*e^x - b + c) - b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - b*c^2*s
gn(-sqrt(b^2 - c^2)*e^x - b + c) + c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e
^(-1/2*x) - 25*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - 2*s
qrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b^2 - c^2)*c^2*
sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-3/2*x) - 3*(b^3*sgn(-sqrt(b^2 - c^2)
```

```
*e^x - b + c) - 3*b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 3*b*c^2*sgn(-sq
rt(b^2 - c^2)*e^x - b + c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-5/2
*x))/sqrt(b - c)
```

3.769 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal. Leaf size=92

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] (8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[Sqrt[b^2 - c^2] + b*Cos
h[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*Co
sh[x] + c*Sinh[x]])/3
```

Rubi [A] time = 0.0774329, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3113, 3112}

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[Sqrt[b^2 - c^2] + b*Cos
h[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*Co
sh[x] + c*Sinh[x]])/3
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

2 - b^2 - c^2, 0]

Rubi steps

$$\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}+\frac{1}{3}\left(4\sqrt{b^2-\right.$$
$$=\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-}}$$

Mathematica [C] time = 70.9472, size = 4392, normalized size = 47.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b^2 - c^2]*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c + ((2
*b*Sqrt[b^2 - c^2])/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]] + (32*b*(-b + c)*(b + c)^2*(EllipticF[Arc
Sin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2
 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c
 + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh
[x/2])))]], 1])*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]]*(-1 + T
anh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sq
rt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-c + (-b + Sqrt[b^2 - c^2])*Tanh[x/2]))
/(3*(b + c - Sqrt[b^2 - c^2])^2*(b + c + Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqr
t[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*Sqrt[(-1
 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) - b*(1
 + Tanh[x/2]^2))]) + (16*(b - c)*(b + c)*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cos
h[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 - 2*b^2*c^2*Sqrt[b^2 - c^2]
- 3*b*c^3*Sqrt[b^2 - c^2] - c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b^
3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/2]
 - 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] - 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2]
 - 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] + 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2] + 8*
b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11*b^2*c^3
*Tanh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 - 8*b^4*Sqrt[b^2 - c^2
]*Tanh[x/2]^2 - 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 + 5*b*c^3*Sqrt[b^2 - c
^2]*Tanh[x/2]^2 + c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*EllipticPi[-1,
ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[
```

b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^4*c*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 20*b^3*c^2*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/

```
((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^2*EllipticPi[-1,
 ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
+ 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tan
h[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*S
qrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^
2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt
[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])
*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1
 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 12*b^2*c^2
*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(
1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x
/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b
^2 - c^2])*(-1 + Tanh[x/2])))] + 2*c*EllipticE[ArcSin[Sqrt[-(((-b - c + Sqr
t[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])
))]], 1]*(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]
))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + c^3*(
Sqrt[b^2 - c^2] + c*Tanh[x/2]) - b^2*c*(2*Sqrt[b^2 - c^2] + 5*c*Tanh[x/2])
+ b^3*(2*c - 4*Sqrt[b^2 - c^2]*Tanh[x/2]) + b*c^2*(-2*c + 3*Sqrt[b^2 - c^2]
*Tanh[x/2])) + 2*b*EllipticF[ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 +
 Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 + Tan
h[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-4*b^4*Tanh[x/2] - c^3*(Sqrt[b^2 - c^2] +
 c*Tanh[x/2]) + b^2*c*(2*Sqrt[b^2 - c^2] + 5*c*Tanh[x/2]) + b*c^2*(2*c - 3*
Sqrt[b^2 - c^2]*Tanh[x/2]) + b^3*(-2*c + 4*Sqrt[b^2 - c^2]*Tanh[x/2]))))/(3
*c*(b + c - Sqrt[b^2 - c^2])^2*(-b + Sqrt[b^2 - c^2])*(-b + c + Sqrt[b^2 -
c^2])*(1 + Cosh[x])*Sqrt[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1
 + Cosh[x])^2]*Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-
1 + Tanh[x/2]^2) - b*(1 + Tanh[x/2]^2))]])
```

Maple [B] time = 0.513, size = 190, normalized size = 2.1

$$\left(-2\,b^{2}+2\,c^{2}\right)\cosh\left(x\right)\frac{1}{\sqrt{-\left(\sinh\left(x\right)b^{2}-\sinh\left(x\right)c^{2}-b^{2}+c^{2}\right)\frac{1}{\sqrt{b^{2}-c^{2}}}}}+\frac{b^{2}-c^{2}}{\sinh\left(x\right)}\sqrt{-\sqrt{b^{2}-c^{2}}\left(\sinh\left(x\right)-1\right)\left(\sinh\left(x\right)\right)^{2}}\ \mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] (-2*b^2+2*c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*c
osh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*arctan(((b^2-c^2)^(1/
2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2
))*(b^2-c^2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sin
h(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 3.21733, size = 864, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(2*sqrt
(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sqr
t(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) +
 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2
- 2*b*c + c^2)*e^(-3*x)) + 3/2*sqrt(2)*(b^2 - c^2)*(2*sqrt(b + c)*sqrt(b -
c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b -
 c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*s
qrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2 - 2*b*c + c^2)*e^
(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*
c)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-
1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^
2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2
*x) + (b^2 - 2*b*c + c^2)*e^(-3*x)) - 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(2*sq
rt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*x)/(
sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x
) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b
^2 - 2*b*c + c^2)*e^(-3*x))
```

Fricas [B] time = 2.42817, size = 967, normalized size = 10.51

$$\frac{\sqrt{\frac{1}{2}}\left(\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)^4+4\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)\sinh\left(x\right)^3+\left(b^2+2\,bc+c^2\right)\sinh\left(x\right)^4-18\left(b^2-c^2\right)\cosh\left(x\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) + 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26444, size = 409, normalized size = 4.45

$$-\frac{\sqrt{2}\left(\left(\sqrt{b^2-c^2}b\text{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)+\sqrt{b^2-c^2}c\text{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)\right)e^{\left(\frac{3}{2}x\right)}+9\left(b^2\text{sgn}\left(-\sqrt{b^2-c^2}e^x-b+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b
^2 - c^2)*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(3/2*x) + 9*(b^2*sgn(-sqrt
(b^2 - c^2)*e^x - b + c) - c^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(1/2*x)
 - 9*(sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 - c^2)
*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-1/2*x) - (b^2*sgn(-sqrt(b^2 - c^2
```

```
)*e^x - b + c) - 2*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + c^2*sgn(-sqrt(b^
2 - c^2)*e^x - b + c))*e^(-3/2*x))/sqrt(b - c)
```

3.770 $\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rubi [A] time = 0.0386524, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3112}

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 68.2719, size = 455, normalized size = 12.3

$$\frac{4(b-c)(b+c)^2\left(2bc\sqrt{b^2-c^2}\sinh(x)+b\left(2b\sqrt{b^2-c^2}-2b^2+c^2\right)\cosh(x)+\left(\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)\left(\left(2b^2\left(2\sqrt{b^2-c^2}+c\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (4*(b - c)*(b + c)^2*(2*b^3 - 2*b*c^2 - 2*b^2*Sqrt[b^2 - c^2] + c^2*Sqrt[b^
2 - c^2] + b*(-2*b^2 + c^2 + 2*b*Sqrt[b^2 - c^2])*Cosh[x] - 2*b^2*c*Sinh[x]
 + c^3*Sinh[x] + 2*b*c*Sqrt[b^2 - c^2]*Sinh[x] + EllipticE[ArcSin[Sqrt[((-b
 - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 - c^2])]],
1]*(-Cosh[x/2] + Sinh[x/2])*(c*(-2*b^2 + c*(c - Sqrt[b^2 - c^2]) + b*(c + 2
*Sqrt[b^2 - c^2]))*Cosh[x/2] + (-4*b^3 + b*c*(3*c - 2*Sqrt[b^2 - c^2]) - c^
2*(c + Sqrt[b^2 - c^2]) + 2*b^2*(c + 2*Sqrt[b^2 - c^2]))*Sinh[x/2])*Sqrt[((
-b - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 - c^2])])
)/(Sqrt[b^2 - c^2]*(b + c - Sqrt[b^2 - c^2])^2*(-b^2 + c^2 + b*Sqrt[b^2 - c
^2])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])
```

Maple [B] time = 0.442, size = 201, normalized size = 5.4

$$\left(-b^2+c^2\right)\cosh(x)\frac{1}{\sqrt{b^2-c^2}}\frac{1}{\sqrt{-(\sinh(x)\,b^2-\sinh(x)\,c^2-b^2+c^2)\frac{1}{\sqrt{b^2-c^2}}}}+\frac{1}{\sinh(x)}\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\,(\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1
/2))^(1/2)*cosh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)
^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)
-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/
(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 1.88173, size = 207, normalized size = 5.59

$$\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}(b-c)e^{\left(-\frac{1}{2}x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)*s
qrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
 - sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c
)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
```

Fricas [B] time = 2.39423, size = 468, normalized size = 12.65

$$\frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+b-c\right)\sqrt{\frac{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+b-c}{\cosh(x)+\sinh(x)}}}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-b+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

Giac [B] time = 1.195, size = 140, normalized size = 3.78

$$-\frac{\sqrt{2}\left(\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)-\left(b\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)-c\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)\right)e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - (b*
sgn(-sqrt(b^2 - c^2)*e^x - b + c) - c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^
(-1/2*x))/sqrt(b - c)
```

3.771 $\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] (Sqrt[2]*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*
Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(b
^2 - c^2)^(1/4)
```

Rubi [A] time = 0.112632, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*
Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(b
^2 - c^2)^(1/4)
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$= 2i\,\text{Subst}\left(\int \frac{1}{2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic\right.}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}\right.}}\right.$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] time = 29.1132, size = 211, normalized size = 2.13

$$-\frac{\sqrt{2}\left(c\sqrt{b^2-c^2}\sinh(x)+b\sqrt{b^2-c^2}\cosh(x)+b^2-c^2\right)\sqrt{-\frac{c\sqrt{b^2-c^2}\sinh(x)+b\sqrt{b^2-c^2}\cosh(x)-b^2+c^2}{b^2-c^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-c^2}-b\cos}{\sqrt{}}}}{\sqrt{}}\right.\right.}{\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*EllipticF[ArcSin[Sqrt[(Sqrt[b^2 - c^2] - b*Cosh[x] - c*Sinh[x])/
Sqrt[b^2 - c^2]]/Sqrt[2]], 1]*(b^2 - c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*Sq
rt[b^2 - c^2]*Sinh[x])*Sqrt[-((-b^2 + c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*S
qrt[b^2 - c^2]*Sinh[x])/(b^2 - c^2))])/(Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]))
```

Maple [A] time = 0.279, size = 129, normalized size = 1.3

$$\frac{1}{\sinh(x)}\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\,(\sinh(x))^2}\arctan\left(\cosh(x)\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\frac{1}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\,(\sin}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + \sqrt{b^2-c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + \sqrt{b^2 - c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.772 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

```
[Out] ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[Sqrt
[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]*(
b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rubi [A] time = 0.132054, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3116, 3115, 2649, 206}

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```


```
[Out] ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[Sqrt
[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]*(
b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int\frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh}}}{4\sqrt{b^2-c^2}}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int\frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+}}}{4\sqrt{b^2-c}}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{i\,\mathrm{Subst}\left(\int\frac{1}{2\sqrt{b^2-c^2}-x^2}\,d\right.}{}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}+\frac{c\cosh(x)+b\,s}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cos\right.}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.785, size = 417, normalized size = 2.7

$$\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)\frac{1}{\sqrt{b^2-c^2}}\frac{1}{\sqrt{-(\sinh(x)b^2-\sinh(x)c^2-b^2+c^2)\frac{1}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{2}}{(4b-4c)(b+c)\sinh(x)}\sqrt{-\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))+1/4*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2)*2^(1/2)*(b^2-c^2)^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-
1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh\left(x\right) + c\sinh\left(x\right) + \sqrt{b^2 - c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.773 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=205

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{b\sin}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-\right.}}$$

```
[Out] (3*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[S
qrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*Sqrt
[2]*(b^2 - c^2)^(5/4)) + (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(Sqrt[b
^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))/(16
*(b^2 - c^2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rubi [A] time = 0.175753, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3116, 3115, 2649, 206}

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{b\sin}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```


```
[Out] (3*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[S
qrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*Sqrt
[2]*(b^2 - c^2)^(5/4)) + (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(Sqrt[b
^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))/(16
*(b^2 - c^2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
```

```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\,s\right.}}{8\sqrt{b^2-c^2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}\right.}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}} + \frac{c\cosh(x)+}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\right.}$$

Mathematica [F] time = 180.009, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]

[Out] $Aborted
```

Maple [B] time = 1.046, size = 954, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/8/(sinh(x)-1)/sinh(x)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+(b^2-c^2)^(1/2)
)^(1/2)/(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)/(b^2-c^2)*(2*2^(1/
2)*(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*arctanh(1/2*cosh(x)*2^(
1/2))*sinh(x)^2+(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2
)*ln(-2/(-cosh(x)+2^(1/2))*(-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)*sinh(x)+cosh(x
)*(b^2-c^2)^(1/2)*2^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*
(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+sinh(x)*(b^2-c
^2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)*sinh(x)-(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^
2-c^2)^(1/2)*sinh(x)^2)^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/
2)*2^(1/2)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2
)+(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x
)^2)^(1/2)+sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)*sinh(x)-2*2^(1
/2)*(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*arctanh(1/2*cosh(x)*2^
(1/2))*sinh(x)-(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)
*ln(-2/(-cosh(x)+2^(1/2))*(-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)*sinh(x)+cosh(x)
*(b^2-c^2)^(1/2)*2^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*(
-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+sinh(x)*(b^2-c^
2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)+(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1
/2)*sinh(x)^2)^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/2
)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^
2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2
)+sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)-4*(-sinh(x)*(b^2-c^2)^(
1/2)+(b^2-c^2)^(1/2))^(1/2)*cosh(x)*sinh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)+\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.774 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cos}$$

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh
[x] + c*Sinh[x]]) - (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(-Sqrt
[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rubi [A] time = 0.122441, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3113, 3112}

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh
[x] + c*Sinh[x]]) - (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(-Sqrt
[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
```

```
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{1}{5}\left(8\right.$$

$$= -\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x}$$

$$= \frac{64\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}-\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sin$$

Mathematica [C] time = 75.4769, size = 9943, normalized size = 68.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.574, size = 288, normalized size = 2.

$$\left(-\frac{(\cosh(x))^3}{3}\left(b^2-c^2\right)^{\frac{3}{2}}-\left(2\,b^2-2\,c^2\right)\sqrt{b^2-c^2}\cosh(x)\right)\frac{1}{\sqrt{-(\sinh(x)\,b^2-\sinh(x)\,c^2+b^2-c^2)\frac{1}{\sqrt{b^2-c^2}}}}+\frac{1}{\sinh(x)}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*(-1/3*(b^2-c^2
)^(3/2)*cosh(x)^3-(2*b^2-2*c^2)*(b^2-c^2)^(1/2)*cosh(x))+(-(b^2-c^2)^(1/2)*
(sinh(x)+1)*sinh(x)^2)^(1/2)*(1/2*(b^2-c^2)^2*cosh(x)/(sinh(x)*b^2-sinh(x)*
c^2+b^2-c^2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)-1/2*(b^2-c^2)^(
3/2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1
```

```
))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)))/sinh(x)/(
-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 16.2856, size = 2415, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*
e^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
 c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2
- c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqr
t(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)
*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(b^3 +
 b^2*c - b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x)
 + b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt
(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e
^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-
2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)
*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x
) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sq
rt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^
(-x) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b
^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 +
 b^2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c
)*sqrt(b - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(
sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*
sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b
 - c)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*s
qrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c -
 b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b
 - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b +
 c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)
*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)
```

```
*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
 + c)^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
- c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-
x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x
) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^
2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) -
 (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 1/20*sqrt(2)*(b^3 - 3*b^2*c +
3*b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b +
c)^(5/2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c
)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x)
+ 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) -
 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 -
 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) - (b
^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

Fricas [B] time = 2.48302, size = 2079, normalized size = 14.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) - 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
```

```
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46866, size = 887, normalized size = 6.08

$$-\frac{\sqrt{2}\Big(3\left(\sqrt{b^2-c^2}b^2\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+2\sqrt{b^2-c^2}bc\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+\sqrt{b^2-c^2}c^2\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + 2
*sqrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2 - c^2)*c^
2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(5/2*x) - 25*(b^3*sgn(-sqrt(b^2 - c^
2)*e^x + b - c) + b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - b*c^2*sgn(-sqrt
(b^2 - c^2)*e^x + b - c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(3/2*x)
 + 150*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - sqrt(b^2 -
c^2)*c^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(1/2*x) + 150*(b^3*sgn(-sqrt(
b^2 - c^2)*e^x + b - c) - b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - b*c^2*s
gn(-sqrt(b^2 - c^2)*e^x + b - c) + c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e
^(-1/2*x) - 25*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 2*s
qrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2 - c^2)*c^2*
sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-3/2*x) + 3*(b^3*sgn(-sqrt(b^2 - c^2)
*e^x + b - c) - 3*b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + 3*b*c^2*sgn(-sq
rt(b^2 - c^2)*e^x + b - c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-5/2
*x))/sqrt(b - c)
```

3.775 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] (-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[-Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[b^2 - c^2] + b
*Cosh[x] + c*Sinh[x]])/3
```

Rubi [A] time = 0.0787381, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3113, 3112}

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[-Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[b^2 - c^2] + b
*Cosh[x] + c*Sinh[x]])/3
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

```
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{1}{3}\left(4\sqrt{b^2\right.$$
$$= -\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{}}$$

Mathematica [C] time = 73.3534, size = 9861, normalized size = 102.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.436, size = 190, normalized size = 2.

$$\left(2\,b^2-2\,c^2\right)\cosh(x)\,\frac{1}{\sqrt{-\left(\sinh(x)\,b^2-\sinh(x)\,c^2+b^2-c^2\right)\frac{1}{\sqrt{b^2-c^2}}}}+\frac{b^2-c^2}{\sinh(x)}\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,(\sinh(x))^2}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] (2*b^2-2*c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*co
sh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*arctan(((b^2-c^2)^(1/2
)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)
)*(b^2-c^2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh
(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 3.21121, size = 869, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(-2*sqr
t(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sq
rt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x)
+ 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2
 - 2*b*c + c^2)*e^(-3*x)) - 3/2*sqrt(2)*(b^2 - c^2)*(-2*sqrt(b + c)*sqrt(b
- c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b
 - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)
*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2 - 2*b*c + c^2)*
e^(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c
)*c)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e
^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 -
 c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^
(-2*x) - (b^2 - 2*b*c + c^2)*e^(-3*x)) + 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(-
2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*
x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e
^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x)
- (b^2 - 2*b*c + c^2)*e^(-3*x))
```

Fricas [B] time = 2.53898, size = 967, normalized size = 10.07

$$\frac{\sqrt{\frac{1}{2}}\left(\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)^4+4\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)\sinh\left(x\right)^3+\left(b^2+2\,bc+c^2\right)\sinh\left(x\right)^4-18\left(b^2-c^2\right)\cosh\left(x\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
```

```
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) - 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28982, size = 408, normalized size = 4.25

$$-\frac{\sqrt{2}\left(\left(\sqrt{b^2-c^2}b\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+\sqrt{b^2-c^2}c\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)\right)e^{\left(\frac{3}{2}x\right)}-9\left(b^2\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b
^2 - c^2)*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(3/2*x) - 9*(b^2*sgn(-sqrt
(b^2 - c^2)*e^x + b - c) - c^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(1/2*x)
 - 9*(sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - sqrt(b^2 - c^2)
*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-1/2*x) + (b^2*sgn(-sqrt(b^2 - c^2
)*e^x + b - c) - 2*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + c^2*sgn(-sqrt(b^
2 - c^2)*e^x + b - c))*e^(-3/2*x))/sqrt(b - c)
```

3.776 $\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rubi [A] time = 0.036615, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3112}

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 73.3303, size = 9771, normalized size = 250.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.444, size = 202, normalized size = 5.2

$$\left(-b^2+c^2\right)\cosh(x)\frac{1}{\sqrt{b^2-c^2}}\frac{1}{\sqrt{-(\sinh(x)\,b^2-\sinh(x)\,c^2+b^2-c^2)\frac{1}{\sqrt{b^2-c^2}}}}-\frac{1}{\sinh(x)}\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,(\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1
/2))^(1/2)*cosh(x)-(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*arctan(((
b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sin
h(x)^2)^(1/2))*(b^2-c^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)/sinh(x)/
(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [B] time = 2.12198, size = 211, normalized size = 5.41

$$-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
*sqrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c
)) - sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
+ c)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c))
```

Fricas [B] time = 2.61099, size = 468, normalized size = 12.

$$\frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+b-c\right)\sqrt{\frac{(b+c)}{}}}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

Giac [B] time = 1.20489, size = 139, normalized size = 3.56

$$-\frac{\sqrt{2}\left(\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+\left(b\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)-c\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)\right)e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + (b*
sgn(-sqrt(b^2 - c^2)*e^x + b - c) - c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^
(-1/2*x))/sqrt(b - c)
```

3.777 $\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] -((Sqrt[2]*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[
2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]
)/(b^2 - c^2)^(1/4))
```

Rubi [A] time = 0.0938029, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[
2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]
)/(b^2 - c^2)^(1/4))
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$= 2i\,\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,\right.}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\,\text{t}\right.}}\right.$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] time = 30.6454, size = 52609, normalized size = 515.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.323, size = 129, normalized size = 1.3

$$\frac{1}{\sinh(x)}\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,(\sinh(x))^2}\,\arctan\left(\cosh(x)\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\frac{1}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,(\sinh\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) - \sqrt{b^2 - c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) - \sqrt{b^2 - c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.778 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

```
[Out] ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[-Sq
rt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]
*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(-Sqrt[b^2
 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rubi [A] time = 0.12143, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3116, 3115, 2649, 204}

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[-Sq
rt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]
*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(-Sqrt[b^2
 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x}}}{4\sqrt{b^2-c}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,c}}}{4\sqrt{b}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{i\,\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2-c^2}}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}} - \frac{c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+\right.}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.783, size = 415, normalized size = 2.6

$$\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)\frac{1}{\sqrt{b^2-c^2}}\frac{1}{\sqrt{-(\sinh(x)\,b^2-\sinh(x)\,c^2+b^2-c^2)\frac{1}{\sqrt{b^2-c^2}}}}-\frac{\sqrt{2}}{(4\,b-4\,c)\,(b+c)\sinh(x)}\sqrt{-\sqrt{b^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))-1/4*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2)*2^(1/2)*(b^2-c^2)^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+
1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh\left(x\right) + c\sinh\left(x\right) - \sqrt{b^{2} - c^{2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.779 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{}{4\sqrt{b^2-c^2}\left(-\right.}$$

```
[Out] (-3*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt
[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*S
qrt[2]*(b^2 - c^2)^(5/4)) - (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(-Sq
rt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))
/(16*(b^2 - c^2)*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rubi [A] time = 0.166733, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{}{4\sqrt{b^2-c^2}\left(-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt
[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*S
qrt[2]*(b^2 - c^2)^(5/4)) - (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(-Sq
rt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))
/(16*(b^2 - c^2)*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
```

```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} - \frac{3\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cos\right.}}{8\sqrt{b^2}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c}{16\left(b^2-c^2\right)\left(-\sqrt{\right.}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c}{16\left(b^2-c^2\right)\left(-\sqrt{\right.}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c}{16\left(b^2-c^2\right)\left(-\sqrt{\right.}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}} - \frac{c\cos}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-\right.}}$$

Mathematica [F] time = 180.009, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 1.115, size = 984, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/8/(sinh(x)+1)/sinh(x)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+(-b^2+c^2)/(b^2
-c^2)^(1/2))^(1/2)/(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)/(b^2-c^
2)*(-2*(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*arctanh(1/2*cosh(x)
*2^(1/2))*2^(1/2)*sinh(x)^2+ln(2/(-cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2
)*2^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x)*(b^2-c^2)^(1/2)-(
-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3
-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)*(-(b^2-c^2)^(1/
2)*sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*sinh(x)-2*(-sinh(x)*(b^2-c^2)
^(1/2)-(b^2-c^2)^(1/2))^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))*2^(1/2)*sinh(x)-
2^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*ln(2/(
cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)
^(1/2)*2^(1/2)+sinh(x)*(b^2-c^2)^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^
(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+(
b^2-c^2)^(1/2)))*sinh(x)+ln(2/(-cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2
^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x)*(b^2-c^2)^(1/2)-(-si
nh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b
^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))*2^(1/2)*(-(b^2-c^2)^(1/2)*
sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-4*(-sinh(x)*(b^2-c^2)^(1/2)-(b^2
-c^2)^(1/2))^(1/2)*cosh(x)*sinh(x)-2^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2
-c^2)^(1/2)*sinh(x)^2)^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2
)*2^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+sinh(x)*(b^2-c^2)^(1/2)+(
-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3
-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+(b^2-c^2)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)

[Out] Exception raised: TypeError
```

3.780 $\int \frac{1}{a+c\,\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=107

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 -
c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 - c^2]) - (b*Log[c + a*Cosh[x] + b*Sinh[
x]])/(a^2 - b^2)
```

Rubi [A] time = 0.136864, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3159, 3138, 3124, 618, 204}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 -
c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 - c^2]) - (b*Log[c + a*Cosh[x] + b*Sinh[
x]])/(a^2 - b^2)
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
```

```
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a + c\,\mathrm{sech}(x) + b\tanh(x)}\,dx = \int \frac{\cosh(x)}{c + a\cosh(x) + b\sinh(x)}\,dx$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(c + a\cosh(x) + b\sinh(x))}{a^2-b^2} - \frac{(ac)\int \frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(c + a\cosh(x) + b\sinh(x))}{a^2-b^2} - \frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{a+c+2bx-(-a+c)x^2}\,dx, x, \mathrm{ta}\right.}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(c + a\cosh(x) + b\sinh(x))}{a^2-b^2} + \frac{(4ac)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx, x, 2b\right.}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{2ac\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}} - \frac{b\log(c + a\cosh(x) + b\sinh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.178435, size = 86, normalized size = 0.8

$$\frac{-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-b\log(a\cosh(x)+b\sinh(x)+c)+ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a
^2 - b^2 - c^2] - b*Log[c + a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [B] time = 0.062, size = 422, normalized size = 3.9

$$2\,\frac{\ln(\tanh(x/2)+1)}{2\,a-2\,b}-2\,\frac{\ln(\tanh(x/2)-1)}{2\,b+2\,a}-\frac{ab}{(a+b)\,(a-b)\,(a-c)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2c+2\,\tanh(x/2)\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] 2/(2*a-2*b)*ln(tanh(1/2*x)+1)-2/(2*b+2*a)*ln(tanh(1/2*x)-1)-1/(a+b)/(a-b)/(
a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*a*b+1/(a+b)/(a
-b)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*c*b-2/(a+
b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-
c^2)^(1/2))*a*c-2/(a+b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(
1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2+2/(a+b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arct
an(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2/(a-c)*a-2/(a+b)/(
a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)
^(1/2))*b^2/(a-c)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57903, size = 1077, normalized size = 10.07

$$\left[\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{\cdots}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)+a-b}\cdots\right.}{a^4-2\,a^2b^2+b^4\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a +
b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + (a^3 + a^2*b - a
*b^2 - b^3 - (a + b)*c^2)*x - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*si
nh(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2),
 (2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) +
c)/sqrt(a^2 - b^2 - c^2)) + (a^3 + a^2*b - a*b^2 - b^3 - (a + b)*c^2)*x - (
a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x)))
)/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\tanh(x)+c\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(1/(a + b*tanh(x) + c*sech(x)), x)
```

Giac [A] time = 1.18451, size = 143, normalized size = 1.34

$$-\frac{2\,ac\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}\left(a^{2}-b^{2}\right)}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^{x}+a-b\right)}{a^{2}-b^{2}}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(a^2 - b^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x + a - b)/(a^2 - b
^2) + x/(a - b)
```

3.781 $\int \frac{1}{a+b\coth(x)+c\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=113

$$\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ia\sinh(x)+ib\cosh(x)+ic)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) + (2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 +
 c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 + c^2]) - (b*Log[I*c + I*b*Cosh[x] + I*
a*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.156473, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3160, 3137, 3124, 618, 204}

$$\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ia\sinh(x)+ib\cosh(x)+ic)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 +
 c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 + c^2]) - (b*Log[I*c + I*b*Cosh[x] + I*
a*Sinh[x]])/(a^2 - b^2)
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
```

```
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\coth(x)+c\,\mathrm{csch}(x)}\,dx = i\int \frac{\sinh(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2} - \frac{(iac)\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2} - \frac{(2iac)\,\mathrm{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)x^2}\,dx, x\right.}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2} + \frac{(4iac)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2i\right.}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} + \frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.197327, size = 86, normalized size = 0.76

$$\frac{-\frac{2ac\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}-b\log(a\sinh(x)+b\cosh(x)+c)+ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

```
[Out] (a*x - (2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[
-a^2 + b^2 - c^2] - b*Log[c + b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Maple [B] time = 0.047, size = 421, normalized size = 3.7

$$-\frac{b^2}{(a+b)(a-b)(b-c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b-\left(\tanh\left(\frac{x}{2}\right)\right)^2c+2\,a\tanh(x/2)+b+c\right)+\frac{cb}{(a+b)(a-b)(b-c)}\ln\left(\left(\tanh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] -1/(a+b)/(a-b)/(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x)+b+c
)*b^2+1/(a+b)/(a-b)/(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x
)+b+c)*c*b-2/(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*
x)+2*a)/(-a^2+b^2-c^2)^(1/2))*a*b-2/(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan
(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^(1/2))*a*c+2/(a+b)/(a-b)/(-a^
2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^(1/2))
*a/(b-c)*b^2-2/(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/
2*x)+2*a)/(-a^2+b^2-c^2)^(1/2))*a/(b-c)*c*b+4/(4*a-4*b)*ln(tanh(1/2*x)+1)-4
/(4*a+4*b)*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59453, size = 1103, normalized size = 9.76

$$\left[-\frac{\sqrt{a^2 - b^2 + c^2} a c \log\left(\frac{2\,(a+b)c\cosh(x) + \left(a^2+2\,ab+b^2\right)\cosh(x)^2 + \left(a^2+2\,ab+b^2\right)\sinh(x)^2 + a^2 - b^2 + 2\,c^2 + 2\left((a+b)c + \left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x) - 2\,\sqrt{a^2\cdots}}{(a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + 2\,c\cosh(x) + 2\,((a+b)\cosh(x)+c)\sinh(x) - a + b}\right.}{a^4 - 2\,a^2 b^2 + b^4 + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) - (a^3 + a^2*b - a*
b^2 - b^3 + (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sin
h(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2),
-(2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(
x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^3 + a^2*b - a*b^2 - b^3 +
 (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(
cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\coth(x) + c\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(1/(a + b*coth(x) + c*csch(x)), x)
```

Giac [A] time = 1.17044, size = 143, normalized size = 1.27

$$-\frac{2\,ac\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^{x}-a+b\right)}{a^{2}-b^{2}}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2)*sqrt
(-a^2 + b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(a^2 -
 b^2) + x/(a - b)
```

3.782 $\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cx}{b^2-c^2}$$

```
[Out] -((c*x)/(b^2 - c^2)) - (2*a*c*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^
2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*Log[a + b*Cosh[x] + c*S
inh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.114731, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3137, 3124, 618, 206}

$$-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*x)/(b^2 - c^2)) - (2*a*c*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^
2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*Log[a + b*Cosh[x] + c*S
inh[x]])/(b^2 - c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(ac)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(2ac)\,\mathrm{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\mathrm{ta}\right.}{b^2-c^2}$$

$$= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{(4ac)\,\mathrm{Subst}\left(\int\frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx,x,2c\right.}{b^2-c^2}$$

$$= -\frac{cx}{b^2-c^2}-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.186858, size = 86, normalized size = 0.83

$$\frac{\frac{2ac\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a+b\cosh(x)+c\sinh(x))-cx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (-(c*x) + (2*a*c*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/S
qrt[-a^2 + b^2 - c^2] + b*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Maple [B] time = 0.036, size = 429, normalized size = 4.1

$$-4\,\frac{\ln(\tanh(x/2)+1)}{4\,b-4\,c}-4\,\frac{\ln(\tanh(x/2)-1)}{4\,b+4\,c}+\frac{ab}{(b-c)\,(b+c)\,(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-2\,c\tanh(x/2)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -4/(4*b-4*c)*ln(tanh(1/2*x)+1)-4/(4*b+4*c)*ln(tanh(1/2*x)-1)+1/(b-c)/(b+c)/
(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b-1/(b-c)/(
b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b^2-2/(b
-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b
^2-c^2)^(1/2))*a*c-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*t
anh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c*b+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b-2/
(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2
+b^2-c^2)^(1/2))*c/(a-b)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.64441, size = 1110, normalized size = 10.67

$$\left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+2\,bc+c^2)\cosh(x))\sinh(x)+2\,\sqrt{a^2-}}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)+b-c}\right.}{a^2b^2-b^4-c^4-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*
b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b +
 a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b +
 c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2
+ 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + (a^2*b - b^3 +
b*c^2 + c^3 + (a^2 - b^2)*c)*x - (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c
*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^
2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*co
sh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2*b - b^3 + b*c^2 + c^
3 + (a^2 - b^2)*c)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c*sinh(x) +
 a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14693, size = 143, normalized size = 1.38

$$\frac{2\,ac\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}\left(b^{2}-c^{2}\right)}+\frac{b\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}{b^{2}-c^{2}}-\frac{x}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*c*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) + b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 -
c^2) - x/(b - c)
```

3.783 $\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{x}{2}-\frac{\sinh(x)}{2}+\frac{\cosh(x)}{2}$$

```
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Rubi [A] time = 0.0253834, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3131}

$$\frac{x}{2}-\frac{\sinh(x)}{2}+\frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx=\frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

Mathematica [A] time = 0.039632, size = 18, normalized size = 1.

$$\frac{x}{2}-\frac{\sinh(x)}{2}+\frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Maple [B] time = 0.024, size = 28, normalized size = 1.6

$$\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] 1/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.16206, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*e^(-x)
```

Fricas [A] time = 2.438, size = 72, normalized size = 4.

$$\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.702367, size = 34, normalized size = 1.89

$$\frac{x\tanh\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2}+\frac{x}{2\tanh\left(\frac{x}{2}\right)+2}+\frac{2}{2\tanh\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] x*tanh(x/2)/(2*tanh(x/2) + 2) + x/(2*tanh(x/2) + 2) + 2/(2*tanh(x/2) + 2)
```

Giac [A] time = 1.2229, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*e^(-x)
```

3.784 $\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Rubi [A] time = 0.0817428, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3165, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx &= \int \frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx \\
&= 2\operatorname{Subst}\left(\int \frac{1}{a+c+2bx-(-a+c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= -\left(4\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx, x, 2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)\right) \\
&= \frac{2\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}
\end{aligned}$$

Mathematica [A] time = 0.0411778, size = 54, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Maple [A] time = 0.05, size = 53, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2-b^2-c^2}}\arctan\left(1/2\,\frac{2\,(a-c)\tanh(x/2)+2\,b}{\sqrt{a^2-b^2-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] 2/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46489, size = 637, normalized size = 11.8

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{-}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)+a-b}\right.}{a^2-b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2 + c^2)*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cos
h(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)*c +
 (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a + b)*c
osh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c
*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b))/(a^2 - b^2 - c^2), -2*
arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a^2 - b^2 - c^2))/sqrt
(a^2 - b^2 - c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*tanh(x) + c*sech(x)), x)
```

Giac [A] time = 1.1844, size = 62, normalized size = 1.15

$$\frac{2\,\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/sqrt(a^2 - b^2 - c^2)
```

3.785 $\int \frac{\mathbf{sech}^2(x)}{a+c\,\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=146

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)}{b^2+c^2}-\frac{b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}+\frac{2c\tan^{-1}\left(\tan\right.}{b^2+c^2}$$

```
[Out] (2*c*ArcTan[Tanh[x/2]])/(b^2 + c^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])
/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[1 + T
anh[x/2]^2])/(b^2 + c^2) + (b*Log[a + c + 2*b*Tanh[x/2] + (a - c)*Tanh[x/2]
^2])/(b^2 + c^2)
```

Rubi [A] time = 0.481288, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4397, 1075, 634, 618, 204, 628, 635, 203, 260}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)}{b^2+c^2}-\frac{b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}+\frac{2c\tan^{-1}\left(\tan\right.}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*c*ArcTan[Tanh[x/2]])/(b^2 + c^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])
/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[1 + T
anh[x/2]^2])/(b^2 + c^2) + (b*Log[a + c + 2*b*Tanh[x/2] + (a - c)*Tanh[x/2]
^2])/(b^2 + c^2)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
```

```
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{c+a\cosh(x)+b\sinh(x)}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)\left(a+c+2bx+(a-c)x^2\right)}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{4c-4bx}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{2(b^2+c^2)} + \frac{\text{Subst}\left(\int \frac{4b^2+(a-c)^2-(a+c)^2+4b(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right.}{2(b^2+c^2)}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{2b+2(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{(2b)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \cdots$$

$$= \frac{2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a-c)\,\mathrm{t}\cdots\right.}{b^2+c^2}$$

$$= \frac{2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{2ac\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \frac{b\log\left(a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.235881, size = 96, normalized size = 0.66

$$\frac{-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}} + b(\log(a\cosh(x)+b\sinh(x)+c)-\log(\cosh(x))) + 2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*c*ArcTan[Tanh[x/2]] - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b
^2 - c^2]])/Sqrt[a^2 - b^2 - c^2] + b*(-Log[Cosh[x]] + Log[c + a*Cosh[x] +
b*Sinh[x]]))/(b^2 + c^2)
```

Maple [B] time = 0.049, size = 406, normalized size = 2.8

$$-\frac{b}{b^2+c^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+2\frac{c\arctan(\tanh(x/2))}{b^2+c^2}+\frac{ab}{(b^2+c^2)(a-c)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2c+2\tan\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] -b*ln(tanh(1/2*x)^2+1)/(b^2+c^2)+2*c*arctan(tanh(1/2*x))/(b^2+c^2)+1/(b^2+c
^2)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*a*b-1/(b^
2+c^2)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*c*b-2/
(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2
-c^2)^(1/2))*a*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1
/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(
1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2/(a-c)*a+2/(b^2+c^2)/
(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2
))*b^2/(a-c)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 14.2307, size = 1254, normalized size = 8.59

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x)+2\,\sqrt{}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)+a-b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b
)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(-a^2 + b^2 + c^2)*((a +
 b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
```

```
+ 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + 2*(c^3 - (a^2 -
 b^2)*c)*arctan(cosh(x) + sinh(x)) - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x)
 + b*sinh(x) + c)/(cosh(x) - sinh(x))) + (a^2*b - b^3 - b*c^2)*log(2*cosh(x
)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), (2*sqrt(
a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a
^2 - b^2 - c^2)) - 2*(c^3 - (a^2 - b^2)*c)*arctan(cosh(x) + sinh(x)) + (a^2
*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x))) -
(a^2*b - b^3 - b*c^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 -
c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x) + c*sech(x)), x)
```

Giac [A] time = 1.14126, size = 170, normalized size = 1.16

$$-\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)} + \frac{2\,c\arctan\left(e^x\right)}{b^2+c^2} + \frac{b\log\left(ae^{(2\,x)} + be^{(2\,x)} + 2\,ce^x + a - b\right)}{b^2+c^2} - \frac{b\log\left(e^{(2\,x)}+1\right)}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(b^2 + c^2)) + 2*c*arctan(e^x)/(b^2 + c^2) + b*log(a*e^(2*x) + b*e^(2*
x) + 2*c*e^x + a - b)/(b^2 + c^2) - b*log(e^(2*x) + 1)/(b^2 + c^2)
```

3.786 $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

```
[Out] (-2*ArcTanh[(2 - Tanh[x/2])/3])/3
```

Rubi [A] time = 0.0502114, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(2 - Tanh[x/2])/3])/3
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx &= i\int \frac{1}{3i+2i\cosh(x)+2i\sinh(x)}\,dx\\
&= 2i\,\text{Subst}\left(\int \frac{1}{5i+4ix-ix^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= -\left(4i\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 4i-2i\tanh\left(\frac{x}{2}\right)\right)\right)\\
&= -\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0685395, size = 28, normalized size = 1.47

$$\frac{x}{6}-\frac{1}{3}\log\left(5\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

```
[Out] x/6 - Log[5*Cosh[x/2] - Sinh[x/2]]/3
```

Maple [A] time = 0.037, size = 20, normalized size = 1.1

$$\frac{1}{3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{3}\ln\left(\tanh\left(\frac{x}{2}\right)-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(2+2*coth(x)+3*csch(x)),x)
```

```
[Out] 1/3*ln(tanh(1/2*x)+1)-1/3*ln(tanh(1/2*x)-5)
```

Maxima [A] time = 1.09324, size = 15, normalized size = 0.79

$$-\frac{1}{3}\log\left(3e^{(-x)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/3*log(3*e^(-x) + 2)
```

Fricas [A] time = 2.31758, size = 59, normalized size = 3.11

$$\frac{1}{3}x-\frac{1}{3}\log\left(2\cosh\left(x\right)+2\sinh\left(x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x - 1/3*log(2*cosh(x) + 2*sinh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}\left(x\right)}{2\coth\left(x\right)+3\operatorname{csch}\left(x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x)
```

```
[Out] Integral(csch(x)/(2*coth(x) + 3*csch(x) + 2), x)
```

Giac [A] time = 1.15271, size = 18, normalized size = 0.95

$$\frac{1}{3}x-\frac{1}{3}\log\left(2e^{x}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="giac")
```

```
[Out] 1/3*x - 1/3*log(2*e^x + 3)
```

3.787 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] (-2*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rubi [A] time = 0.0900589, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx &= i\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx\\ &= 2i\,\mathrm{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= -\left(4i\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2ia+2(ib-ic)\tanh\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}\end{aligned}$$

Mathematica [A] time = 0.0401134, size = 54, normalized size = 1.08

$$\frac{2\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (2*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
- c^2]
```

Maple [A] time = 0.039, size = 53, normalized size = 1.1

$$2\,\frac{1}{\sqrt{-a^2+b^2-c^2}}\arctan\left(1/2\,\frac{2\,(b-c)\tanh(x/2)+2\,a}{\sqrt{-a^2+b^2-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] 2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4323, size = 657, normalized size = 13.14

$$\left[\frac{\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2+a^2-b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{a^2-b^2+c^2}((a+b)\cosh(x)+(}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)-a+b}\right.}{\sqrt{a^2-b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cosh(x)^2 + (a^2 + 2*a*b +
b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)*c + (a^2 + 2*a*b + b^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b)*cosh(x) + (a + b)*sinh(x)
+ c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c*cosh(x) + 2*((a + b)*cos
h(x) + c)*sinh(x) - a + b))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^2)
*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2
 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x) + c*csch(x)), x)
```

Giac [A] time = 1.1714, size = 62, normalized size = 1.24

$$\frac{2\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

3.788 $\int \frac{\mathbf{csch}^2(x)}{a+b\coth(x)+c\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=118

$$-\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

```
[Out] (-2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2
)*Sqrt[a^2 - b^2 + c^2]) + Log[Tanh[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tanh
[x/2] + (b - c)*Tanh[x/2]^2])/(b^2 - c^2)
```

Rubi [A] time = 0.584307, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4397, 12, 1628, 634, 618, 206, 628}

$$-\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2
)*Sqrt[a^2 - b^2 + c^2]) + Log[Tanh[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tanh
[x/2] + (b - c)*Tanh[x/2]^2])/(b^2 - c^2)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{csch}(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx$$
$$= -\left(2\operatorname{Subst}\left(\int \frac{-1+x^2}{2x\left(b+c+2ax+(b-c)x^2\right)}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\right)$$
$$= -\operatorname{Subst}\left(\int \frac{-1+x^2}{x\left(b+c+2ax+(b-c)x^2\right)}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{1}{(b+c)x}+\frac{2(a+bx)}{(b+c)\left(b+c+2ax+(b-c)x^2\right)}\right)dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} - \frac{2\operatorname{Subst}\left(\int \frac{a+bx}{b+c+2ax+(b-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b+c}$$
$$= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\operatorname{Subst}\left(\int \frac{2a+2(b-c)x}{b+c+2ax+(b-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2} + \frac{(2ac)\operatorname{Subst}\left(\int \frac{1}{b+c+2ax\cdots}\right.}{b^2\cdots}$$
$$= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2} - \frac{(4ac)\operatorname{Subst}\left(\int \frac{\cdots}{4(a\cdots}\right.}{\cdots}$$
$$= -\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}} + \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\cdots\right.}{b^2-c^2}$$

Mathematica [A] time = 0.193392, size = 97, normalized size = 0.82

$$\frac{-\frac{2ac\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a\sinh(x)+b\cosh(x)+c)-b\log(\sinh(x))+c\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] ((-2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2
+ b^2 - c^2] - b*Log[Sinh[x]] + b*Log[c + b*Cosh[x] + a*Sinh[x]] + c*Log[Ta
nh[x/2]])/(-b^2 + c^2)
```

Maple [A] time = 0.038, size = 180, normalized size = 1.5

$$-\frac{b}{(b-c)(b+c)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 b-\left(\tanh\left(\frac{x}{2}\right)\right)^2 c+2\,a\tanh(x/2)+b+c\right)-2\,\frac{a}{(b+c)\sqrt{-a^2+b^2-c^2}}\arctan\left(1/2\,\frac{2\,(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] -1/(b+c)*b/(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x)+b+c)-2/
(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c
^2)^(1/2))*a+2/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2
*a)/(-a^2+b^2-c^2)^(1/2))*b*a/(b-c)+ln(tanh(1/2*x))/(b+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.5817, size = 1372, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) + (a^2*b - b^3 + b*
c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) - (a^2*b - b^3
+ b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) - (a^2*b - b^3 +
```

```
b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 - b^4 - c
^4 - (a^2 - 2*b^2)*c^2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b
^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^2
*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) +
(a^2*b - b^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) + (a
^2*b - b^3 + b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x) + c*csch(x)), x)
```

Giac [A] time = 1.15592, size = 165, normalized size = 1.4

$$\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)} - \frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x-a+b\right)}{b^2-c^2} + \frac{\log\left(e^x+1\right)}{b-c} + \frac{\log\left(|e^x-1|\right)}{b+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] 2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(b^2 -
c^2) + log(e^x + 1)/(b - c) + log(abs(e^x - 1))/(b + c)
```

3.789 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=120

$$-\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

```
[Out] -((c*C*x)/(b^2 - c^2)) - (2*(A*(b^2 - c^2) + a*c*C)*ArcTanh[(c - (a - b)*Ta
nh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*C
*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.11779, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3137, 3124, 618, 206}

$$-\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) - (2*(A*(b^2 - c^2) + a*c*C)*ArcTanh[(c - (a - b)*Ta
nh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*C
*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(A+\frac{acC}{b^2-c^2}\right)\int \frac{1}{a+b\cosh(x)+c\sin\ldots}$$

$$= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(2\left(A+\frac{acC}{b^2-c^2}\right)\right)\text{Subst}\left(\int \frac{}{a+b+\ldots}\right.$$

$$= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\left(4\left(A+\frac{acC}{b^2-c^2}\right)\right)\text{Subst}\left(\int \frac{}{4\left(a^2-b\ldots\right.}\right.$$

$$= -\frac{cCx}{b^2-c^2}-\frac{2\left(A+\frac{acC}{b^2-c^2}\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.200776, size = 104, normalized size = 0.87

$$\frac{\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+C(b\log(a+b\cosh(x)+c\sinh(x))-cx)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((2*(A*(b^2 - c^2) + a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2
 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + C*(-(c*x) + b*Log[a + b*Cosh[x] + c*Sinh
```

```
[x]]))/((b - c)*(b + c))
```

Maple [B] time = 0.053, size = 573, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)+1/(b-c)/(b
+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b*C-1/(
b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*C*
b^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)
/(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*
(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2-2/(b-c)/(b+c)/(-a^2+b
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*
c*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)
/(-a^2+b^2-c^2)^(1/2))*C*c*b+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*
(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/(b+c)
/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(
1/2))*c/(a-b)*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38677, size = 1212, normalized size = 10.1

$$\left[-\frac{\left(Ab^2+Cac-Ac^2\right)\sqrt{a^2-b^2+c^2}\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+2\,bc+c^2\right)\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*b^2 + C*a*c - A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) + (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^2 - C*b^2)*c)*x - (C*a^2*b -
 C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), (2*(A*b^2 + C*a*c - A*c^2)*sqrt(-
a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*s
inh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^
2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x)
+ a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15594, size = 165, normalized size = 1.38

$$\frac{Cb\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}-\frac{Cx}{b-c}+\frac{2\left(Ab^2+Cac-Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) - C*x/(b - c)
+ 2*(A*b^2 + C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
```

```
2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.790 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}$$

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) + (b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.115556, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3154, 3124, 618, 206}

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) + (b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA+cC)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$=\frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2(aA+cC))\,\mathrm{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\right.}{a^2-b^2+c^2}$$

$$=\frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4(aA+cC))\,\mathrm{Subst}\left(\int\frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\right.}{a^2-b^2+}$$

$$=-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}+\frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.306712, size = 130, normalized size = 1.2

$$\frac{a^2C+\sinh(x)\left(acC+A\left(b^2-c^2\right)\right)-aAc-b^2C}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C - b^2*C + (A*(b^2 - c^2) + a*c*C
)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] time = 0.074, size = 287, normalized size = 2.7

$$-2\frac{1}{a(\tanh(x/2))^2-(\tanh(x/2))^2b-2c\tanh(x/2)-a-b}\left(-\frac{\left(aAb-Ab^2+Ac^2-acC+Ccb\right)\tanh(x/2)}{a^3-a^2b-ab^2+ac^2+b^3-bc^2}-\frac{aAc}{a^3-a^2b-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh
(1/2*x)-(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1
/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*A-2/(a^2-b
^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2
-c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52015, size = 4805, normalized size = 44.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(2*A*a^2*b^2 - 2*A*b^4 + 2*C*a*c^3 - 2*A*c^4 - 2*(A*a^2 - 2*A*b^2)*c^2 + (
A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2
+ (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (
2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c)*c
osh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A*a*b^2 + C*c^3 + (A*a
 + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2
)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^
2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*
cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(
x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*
cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^3 - C*a*b^2)*c - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^
4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*
b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)
*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^
2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c
^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*
b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
- 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^
2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3
 + b^5)*c)*cosh(x))*sinh(x)), 2*(A*a^2*b^2 - A*b^4 + C*a*c^3 - A*c^4 - (A*a
^2 - 2*A*b^2)*c^2 + (A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3
 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 +
(A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 +
 (A*a^2 + C*a*b)*c)*cosh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A
*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (C*a^3 - C*a*b^2)*c - (C*a^4 - A*a^
3*b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*
b^2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - (C*a^4 - A*a^3*
b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^
2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b
^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4
*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3
)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b
^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*
c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
 - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a
*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a
*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
```

```
 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a
^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^
3 + b^5)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15126, size = 242, normalized size = 2.24

$$\frac{2\,(Aa+Cc)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{-a^2+b^2-c^2}}-\frac{2\left(Ca^2e^x-Aabe^x-Cb^2e^x-Aace^x-Cbce^x-Ab^2-Cac+Ac^2\right)}{\left(a^2b-b^3+a^2c-b^2c+bc^2+c^3\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(C*a^2*e^x - A*a*b*e^x - C*b^2*e^x - A
*a*c*e^x - C*b*c*e^x - A*b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.791 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) + (b*C - (A*c - a*C)*Cosh[x] -
 A*b*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (a*b*C
- (3*a*A*c - a^2*C + 2*c^2*C)*Cosh[x] - b*(3*a*A + 2*c*C)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.291343, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3157, 3153, 3124, 618, 206}

$$-\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) + (b*C - (A*c - a*C)*Cosh[x] -
 A*b*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (a*b*C
- (3*a*A*c - a^2*C + 2*c^2*C)*Cosh[x] - b*(3*a*A + 2*c*C)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA+cC)+Ab\cosh(x)+(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}}{2\left(a^2-b^2+c^2\right)}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right)\cosh(}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right)\cosh(}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right)\cosh(}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= -\frac{\left(2a^2A+A\left(b^2-c^2\right)+3acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} + \frac{bC-(Ac-aC)\cosh}{2\left(a^2-b^2+c^2\right)(a+b\,c}$$

Mathematica [A] time = 0.76863, size = 373, normalized size = 1.88

$$\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{2bc\cosh(x)\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)+c\cosh(2x)\left(a^2(-c)C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 3*
a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c
^4*C + 2*b*c*(2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 +
 c^2) - a^2*c*C + 2*c*(-b^2 + c^2)*C)*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A
*b^4*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 4*a^3*c*C*Sinh[
x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Sinh[x] - 3*a*A*b^3*Sinh[2*x] + 3*a*A*
b*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sin
h[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

Maple [B] time = 0.099, size = 1091, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^
2+b^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a
^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5-2*C*
a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+
13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^
2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a
^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a
*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*
c^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a
^2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*
b^2*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2
+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(
-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/
2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+1/(a^4-2
*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a
-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2+2*a^2*c^2
+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.26035, size = 25911, normalized size = 130.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*A*a^3*b^3 - 6*A*a*b^5 + 4*C*c^6 + 2*(3*A*a - 2*C*b)*c^5 + 2*(C*a^2
- 3*A*a*b - 4*C*b^2)*c^4 + 2*(3*A*a^3 - C*a^2*b - 6*A*a*b^2 + 4*C*b^3)*c^3
+ 2*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2
 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 +
6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3
 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^3 + 2*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 -
 A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2
*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (
4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 -
 2*(C*a^4 + 3*A*a^3*b + C*a^2*b^2 - 6*A*a*b^3 - 2*C*b^4)*c^2 - 2*(2*C*a^6 -
 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6
+ 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^
2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4
)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh
(x)^2 - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 +
3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b + 2*C*b^2)*c^4
 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b + C*a^2*b^2 +
 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^
3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*
b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^
3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^
2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3
 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*
a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 +
 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b
^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3
+ (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)
*c)*sinh(x)^4 + (2*A*a^2 - 3*C*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 + A*a*b^
4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^
3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*
a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b +
3*C*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c
^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*
c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b
+ 3*C*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (
3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b
```

+ 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^
3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C
*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b +
3*C*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c
^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*
c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 + 6*C*a^3*b
+ 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4
+ (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^
2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*C*a*b^3 + A*b^4
)*c + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 - 2*
(A*a^3 + A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^2*c -
3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 +
(2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2
+ 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^3 - 2*(A*a^3 + A*a*b^2)*c^2
+ 3*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 +
3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^2 + (4*A
*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a
*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*
a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqr
t(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)
*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^
2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x
) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh
(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^4*b - 3*A*a^3*b^2
+ C*a^2*b^3 + 3*A*a*b^4 - 2*C*b^5)*c + 2*(10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b
^6 - 5*C*a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^
3 - (10*A*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + (4*C*a^5 + C*a^3*b^2 - 5*C*a*
b^4)*c)*cosh(x) + 2*(10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b^6 - 5*C*a*c^5 - A*c^
6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^3 - (10*A*a^4 - 22*A*
a^2*b^2 + 3*A*b^4)*c^2 + 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*
a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A
*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*
C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 + (4*C*a^5 + C*
a^3*b^2 - 5*C*a*b^4)*c - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3
+ 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a
*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^
3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3
*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3
*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6
+ 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^
6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b
- 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b
^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3
+ a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 +
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2

*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 -

c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c
^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b
^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)), (3*A*a^3*b^3 - 3*A*a*b
^5 + 2*C*c^6 + (3*A*a - 2*C*b)*c^5 + (C*a^2 - 3*A*a*b - 4*C*b^2)*c^4 + (3*A
*a^3 - C*a^2*b - 6*A*a*b^2 + 4*C*b^3)*c^3 + (2*A*a^4*b^2 - A*a^2*b^4 - A*b^
6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3
+ 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2
+ (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^
3 + (2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2
 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 +
6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3
 - 3*C*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 - (C*a^4 + 3*A*a^3*b + C*a^2*b^2 - 6*A
*a*b^3 - 2*C*b^4)*c^2 - (2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 +
6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b
+ 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b
 + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^
2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 - (2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^
2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 -
 3*(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 -
3*(C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*
C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - A*a^2*b^4
 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3
*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^
4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*c
osh(x))*sinh(x)^2 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A
*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2
)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 +
A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2
*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3
*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*C*a*b + 2*A*b^
2)*c^3 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(
A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3
 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3
+ 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 +
 A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6
*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*
cosh(x))*sinh(x)^3 - (2*A*a^2*b + 3*C*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b +
 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A
*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*
C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b + 4*
A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^
2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 +
 A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6
*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*

cosh(x)^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c + 6*(
2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*
a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 -
(2*A*a^2*b^2 - 3*C*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^
2*c - 3*C*a^2*c^3 + A*a*c^4 - 2*(A*a^3 + A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3
*b^2 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 + A*b
^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a
^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh
(x)^3 - 2*(A*a^3 + A*a*b^2)*c^2 + 3*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C
*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2
+ 2*A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C
*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b +
3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 +
A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2
- c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (C*a^4*
b - 3*A*a^3*b^2 + C*a^2*b^3 + 3*A*a*b^4 - 2*C*b^5)*c + (10*A*a^4*b^2 - 11*A
*a^2*b^4 + A*b^6 - 5*C*a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 -
10*C*a*b^2)*c^3 - (10*A*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + (4*C*a^5 + C*a^
3*b^2 - 5*C*a*b^4)*c)*cosh(x) + (10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b^6 - 5*C*
a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^3 - (10*A
*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A
*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A
*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*
A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 + (
4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 +
3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*
(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(
C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a
^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3
*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b -
4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^
5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9
+ (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a
^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b
- 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*
cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 +
c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b -
5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^
6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*
(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)
*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c
^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b

^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c
^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3
*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4
 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^
4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a
^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*
b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4
+ a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b
^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2
 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*
b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7
 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3
 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*
a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2
*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2
*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a
*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^
4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2
*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^
7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4
+ 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8
 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b
 - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*
(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6
 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a
*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^
2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^
8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22714, size = 844, normalized size = 4.26

$$\frac{\left(2\,Aa^2+Ab^2+3\,Cac-Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}+3\,Cab^2ce^{(3\,x)}+2\,Ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) + A*b^4*e^(3*x) + 4*A*a^2*b*c*e^(3*x
) + 3*C*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) + 6*C*a*b
*c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*C*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 4*C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*C*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*C*c^4*e^(2*x) + 10*A*a^2*b^2*e^x - A*b^4*e^x + 4*C*a^3
*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e
^x - A*c^4*e^x + 3*A*a*b^3 + C*a^2*b*c - 3*A*a*b^2*c + 2*C*b^3*c - C*a^2*c^
2 - 3*A*a*b*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

3.792 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=121

$$\frac{2\left(abB-A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

```
[Out] (b*B*x)/(b^2 - c^2) + (2*(a*b*B - A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - (B*c*Lo
g[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.1179, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3138, 3124, 618, 206}

$$\frac{2\left(abB-A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (2*(a*b*B - A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - (B*c*Lo
g[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{a+b\cosh(x)+c\sin\cdots}\\ &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(2\left(A-\frac{abB}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{a+b+2\cdots}\right.\\ &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\left(4\left(A-\frac{abB}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{4\left(a^2-b^{\cdots}\right.}\right.\\ &= \frac{bBx}{b^2-c^2}-\frac{2\left(A-\frac{abB}{b^2-c^2}\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.179444, size = 104, normalized size = 0.86

$$\frac{B(bx-c\log(a+b\cosh(x)+c\sinh(x)))-\frac{2\left(abB+A\left(c^2-b^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((-2*(a*b*B + A*(-b^2 + c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + B*(b*x - c*Log[a + b*Cosh[x] + c*Sinh[
```

```
x]]))/((b - c)*(b + c))
```

Maple [B] time = 0.051, size = 574, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b+
c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(b
-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*B
*c-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/
(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(
2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2+2/(b-c)/(b+c)/(-a^2+b^
2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*b
*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/
(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(
2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b-c)/(b+c)/
(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1
/2))*c^2/(a-b)*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38994, size = 1214, normalized size = 10.03

$$\left[-\frac{\left(Bab - Ab^2 + Ac^2\right)\sqrt{a^2 - b^2 + c^2}\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+2\,bc+c^2\right)\cosh\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a*b - A*b^2 + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a^2 - B*b^2)*c)*x + (B*c^3 + (
B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 + A*c^2)*sqrt(
-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*
sinh(x) + a)/(a^2 - b^2 + c^2)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a
^2 - B*b^2)*c)*x + (B*c^3 + (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x)
 + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14437, size = 165, normalized size = 1.36

$$-\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}+\frac{Bx}{b-c}-\frac{2\left(Bab-Ab^2+Ac^2\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) + B*x/(b - c)
 - 2*(B*a*b - A*b^2 + A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c
```

```
^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.793 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] (-2*(a*A - b*B)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) - (B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.128281, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3155, 3124, 618, 206}

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) - (B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2(aA-bB))\,\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\right.}{a^2-b^2+c^2}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4(aA-bB))\,\text{Subst}\left(\int\frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,d\right.}{a^2-b^2+}$$

$$= -\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.240954, size = 125, normalized size = 1.16

$$\frac{\sinh(x)\left(A\left(b^2-c^2\right)-abB\right)-aAc+bBc}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA-bB)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + b*B*c + (-(a*b*B) + A*(b^2 - c^2))*Sin
h[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] time = 0.072, size = 287, normalized size = 2.7

$$-2\,\frac{1}{a\left(\tanh\left(x/2\right)\right)^{2}-\left(\tanh\left(x/2\right)\right)^{2}b-2\,c\tanh\left(x/2\right)-a-b}\left(-\frac{\left(aAb-Ab^{2}+Ac^{2}-a^{2}B+abB-Bc^{2}\right)\tanh\left(x/2\right)}{a^{3}-a^{2}b-ab^{2}+ac^{2}+b^{3}-bc^{2}}-\frac{}{a^{3}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2
)*tanh(1/2*x)-(A*a-B*b)*c/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x)
^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*A+2/(a^2-b^2
+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c
^2)^(1/2))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.59228, size = 4775, normalized size = 44.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 + 2*A*c^4 + 2*(A*a^2 + B*a
*b - 2*A*b^2)*c^2 + (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 +
 (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + (A*a
 - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + (A*a^
2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c + (A*a*b^2
 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^
2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sin
h(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 +
2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) +
(b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x)
+ 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c
^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2
*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4
 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 -
 b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3
 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 +
(a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c
^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2
 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^
4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*
cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4 + A*c^4 + (A*a
^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 -
 B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^
3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
 + (A*a^2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c +
(A*a*b^2 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*
b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*
a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^
3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*a^
2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^
2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6
 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^
2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 +
 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2
+ 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 +
 a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a
^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5
 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*
```

```
c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2
*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*s
inh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15334, size = 239, normalized size = 2.21

$$\frac{2\,(Aa-Bb)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{-a^2+b^2-c^2}}-\frac{2\left(Ba^2e^x-Aabe^x-Aace^x+Bbce^x+Bc^2e^x+Bab-Ab^2+Ac^2\right)}{\left(a^2b-b^3+a^2c-b^2c+bc^2+c^3\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x - A*a*b*e^x - A*a*c*e^x + B
*b*c*e^x + B*c^2*e^x + B*a*b - A*b^2 + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.794 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cosh(x)(3aA-2bB)+aBc}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c + A*c*Cosh[x] + (A*b -
a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*B*c
+ (3*a*A - 2*b*B)*c*Cosh[x] + (3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.276005, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3158, 3153, 3124, 618, 206}

$$-\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cosh(x)(3aA-2bB)+aBc}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c + A*c*Cosh[x] + (A*b -
a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*B*c
+ (3*a*A - 2*b*B)*c*Cosh[x] + (3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(x)+Ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x)+\left(3a\right.}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x)+\left(3a\right.}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x)+\left(3a\right.}{2\left(a^2-b^2+c^2\right)^2(a+b\cos}$$

$$= -\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc+Ac\cosh(x)+(Ab}{2\left(a^2-b^2+c^2\right)(a+b\cos}$$

Mathematica [A] time = 0.614169, size = 336, normalized size = 1.73

$$\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{2bc\cosh(x)\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)+c\cosh(2x)\left(a^2bB+3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 9*
a^2*b*B*c - 3*a*A*c^3 + 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*Cosh[x] +
 c*(a^2*b*B + 2*b*B*(b^2 - c^2) + 3*a*A*(-b^2 + c^2))*Cosh[2*x] - 8*a^2*A*b
^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Sinh[x] + 12*a
^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8*a*b*B*c^2*Sinh[x] - 3*a*A*b^3*Si
nh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a*A*b*c^2*Sinh[2*x] -
 2*b^2*B*c^2*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]
)^2)
```

Maple [B] time = 0.096, size = 1112, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*ta
nh(1/2*x)^3-1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2-7*A*a^2*c^2-6*A*a*b^3+6*
A*a*b*c^2+A*b^4+A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2+4*B*a^2*c^
2-9*B*a*b^3+2*B*b^4-4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2
*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3
*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^
3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b
*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5+4*B*b^3*c^2-2*B*b*c^4)/(a^4-
2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*c*(4
*A*a^4-3*A*a^2*b^2+A*a^2*c^2-A*b^4+A*b^2*c^2-5*B*a^3*b+5*B*a*b^3-2*B*a*b*c^
2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4
-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(
-a^2+b^2-c^2)^(1/2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-
a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2
))*A*b^2+1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2+3/(a^4-2*
a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.22518, size = 26118, normalized size = 134.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(3
*A*a - 2*B*b)*c^5 + 2*(3*A*a*b - 2*B*b^2)*c^4 - 2*(3*A*a^3 - B*a^2*b - 6*A*
a*b^2 + 4*B*b^3)*c^3 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5
 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b -
 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*
b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*cosh(x)^3 - 2*(2*A*a^4*
b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*
a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a
^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*
a*b^4 - A*b^5)*c)*sinh(x)^3 + 2*(3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^
4)*c^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 +
 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*
c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*
b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a
^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A
*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B
*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*
B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a
^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3
 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*
b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A
*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*
cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 - A*c^5 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b -
2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B
*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4
 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A
*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3
*B*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^
3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*
b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 -
A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)
*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*
a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2
 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*
b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A
```

```
*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2
 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c
)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*
b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5
 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^
2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 - 6*B*a^3*b
 + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*
b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*
B*a^2*b^2 + A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*B*a*b^3 + A*b
^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A*a*c^4 - (2*A*a^3 - 3*B*a
^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A
*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 -
3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2
*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*
c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A
*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x)^2
+ (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*
c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)
*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*
sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 +
2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b
 + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^
2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(B*a^4*b -
3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*c + 2*(4*B*a^5*b - 10*A*a^4*
b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*
B*a*b - 3*A*b^2)*c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*
A*b^4)*c^2)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^
4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (10*A*
a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2*A*a^4*b^2 -
 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 -
 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 -
3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4
 - A*b^5)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3
 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A
*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 -
A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5
 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*
c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 +
 (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^
2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 +
 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^
```

4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9
*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^
3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*
a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(
x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 +
 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b -
 a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (
a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c
^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4
+ 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(
a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b
^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 -
15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 +
 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^
2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^
8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^
6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*
b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5
)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3
 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2
*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^
5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8
*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 +
3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 -
b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (
3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a
^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b
 - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^
8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 +
 (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2
*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b -
 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*co
sh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^
3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^
3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*

b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^
8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^
4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 +
 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^
6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)), -(B*a^4*b^2 - 3*A*
a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (3*A*a - 2*B*b)*c^5 + (3*A*a*b
 - 2*B*b^2)*c^4 - (3*A*a^3 - B*a^2*b - 6*A*a*b^2 + 4*B*b^3)*c^3 - (2*A*a^4*b
^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a
^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^
4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a
*b^4 - A*b^5)*c)*cosh(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a
*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2
*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*
a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*sinh(x)^3 + (3*A*a^
3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^2 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4
*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^
6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c
^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a
^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + (2*B*a^6
 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^
6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*
B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2
*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c
 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5
 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3
)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*
a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5 + A*b*c^4 - A*c^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4
 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A
*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3
 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^
3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*
b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*
B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2
*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^
2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*
A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A
*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2
 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3
 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A
*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b
^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*
A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 +
4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*
A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a

*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A
*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*
cosh(x)^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 6*(
2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*
B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x))*sinh(x)^2
- (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*
b^4 + A*a*c^4 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3
*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 -
3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a
*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2
*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^
4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*
a^2*b^2 + A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 -
3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (
4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 -
3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt
(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2
)) - (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*c + (4*B*a^5
*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 +
(11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 1
0*B*a*b^3 + 3*A*b^4)*c^2)*cosh(x) + (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 +
11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*
c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^
6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 +
(2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3
+ 3*B*a*b^4 - A*b^5)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 +
3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*
(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3
*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3
*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3
- 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2
*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 +
2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 +
c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b -
5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a
^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^
8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*
b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3
*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b
^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 -
3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6
+ 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*
b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b

^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a
*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a
^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^
6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5
 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*
(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 +
 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b -
 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 +
a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(
a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a
^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)
*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*
a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)
*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b
- 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
 b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2
 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15
*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 -
3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4
+ a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7
 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2
)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2
 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 -
a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 -
 (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^
2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b
*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a
^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
+ 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*
b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)
c^6 + 6(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3
 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*
a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7
- b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^
2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 -
 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (
2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19855, size = 844, normalized size = 4.35

$$\frac{\left(2 Aa^2 - 3 Bab + Ab^2 - Ac^2\right)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\left(a^4 - 2 a^2b^2 + b^4 + 2 a^2c^2 - 2 b^2c^2 + c^4\right)\sqrt{-a^2 + b^2 - c^2}} + \frac{2 Aa^2b^2e^{(3 x)} - 3 Bab^3e^{(3 x)} + Ab^4e^{(3 x)} + 4 Aa^2bce^{(3 x)} - 6 Bab^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^4*e^(3*x)
+ 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c
^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) - 4*B*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*B*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*B*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B
*a*b^3*e^x - A*b^4*e^x - 10*A*a^2*c^2*e^x + 5*B*a*b*c^2*e^x + 2*A*b^2*c^2*e
^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4 + B*a^2*b*c - 3*A*a*b^2*c
+ 2*B*b^3*c - 3*A*a*b*c^2 + 2*B*b^2*c^2 + 3*A*a*c^3 - 2*B*b*c^3)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

3.795 $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=125

$$\frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (2*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/
2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - ((B*c - b
*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.142734, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3136, 3124, 618, 206}

$$\frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (2*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/
2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - ((B*c - b
*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{(a(bB-cC))\int \frac{1}{a+b\cosh(x)+c\sin}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{(2a(bB-cC))\,\mathrm{Subst}\left(\int \frac{}{a+b+}\right.}{b^2-}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{(4a(bB-cC))\,\mathrm{Subst}\left(\int \frac{}{4(a^2-}\right.}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} + \frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\,\mathrm{s}}{b^2-c^2}$$

Mathematica [A] time = 0.262715, size = 107, normalized size = 0.86

$$\frac{-\frac{2a(bB-cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(bC-Bc)\log(a+b\cosh(x)+c\sinh(x))+x(bB-cC)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x - (2*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2
 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*Log[a + b*Cosh[x] +
 c*Sinh[x]])/((b - c)*(b + c))
```

Maple [B] time = 0.056, size = 873, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*B/(2*b+2*
c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b+c)/(a-b)*ln
(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(b-c)/(b+c)/(
a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*B*c+1/(b-c)/
(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b*C-1
/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*
C*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*a*b*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(b-c)/(b+c)/(-a^2
+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*
a*c*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*C*c*b-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b-c)/(b+
c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)
^(1/2))*c^2/(a-b)*b*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/(b+c)/(-a^2+
b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c
/(a-b)*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30363, size = 1364, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="frica
s")
```

```
[Out] [((B*a*b - C*a*c)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2
+ (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x)
 + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 +
 c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)
*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + ((B
- C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B
- C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*lo
g(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4
- (a^2 - 2*b^2)*c^2), -(2*(B*a*b - C*a*c)*sqrt(-a^2 + b^2 - c^2)*arctan(sqr
t(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^
2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)
*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*
b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 -
b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18471, size = 169, normalized size = 1.35

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}-\frac{2\,(Bab-Cac)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^
2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.796 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$\frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] (2*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2
 - b^2 + c^2)^(3/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a^2 - b^2 +
 c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.126024, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 3124, 618, 206}

$$\frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2
 - b^2 + c^2)^(3/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a^2 - b^2 +
 c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(bB-cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(2(bB-cC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x}\right.}{a^2-b^2+c^2}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(4(bB-cC))\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\right.}{a^2-b^2}$$

$$= \frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.302242, size = 123, normalized size = 1.14

$$\frac{2(bB-cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}+\frac{a^2C+a\sinh(x)(cC-bB)-b^2C+bBc}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a
^2 + b^2 - c^2)^(3/2) + (b*B*c + a^2*C - b^2*C + a*(-(b*B) + c*C)*Sinh[x])/
(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] time = 0.077, size = 287, normalized size = 2.7

$$2\frac{1}{a(\tanh(x/2))^2-(\tanh(x/2))^2b-2c\tanh(x/2)-a-b}\left(-\frac{\left(a^2B-abB+Bc^2+acC-Ccb\right)\tanh(x/2)}{a^3-a^2b-ab^2+ac^2+b^3-bc^2}-\frac{bBc+}{a^3-a^2b-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(-(B*a^2-B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(
1/2*x)-(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*b-2/(a^2-b^
2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3605, size = 4648, normalized size = 43.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*B*a^3*b - 2*B*a*b^3 + 2*B*a*b*c^2 - 2*C*a*c^3 + (B*b^3 - C*b^2*c - B*b
*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^
2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*
b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c
^2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*
sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c
^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c +
(b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cos
h(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*c
osh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c
+ 2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*
B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*cosh(x) +
2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*B
+ C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*sinh(x))/(a^
4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*
b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 +
 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cos
h(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4
*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(
x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3
 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c
^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a
^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - B*a*b^3 + B*a*b
*c^2 - C*a*c^3 + (B*b^3 - C*b^2*c - B*b*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^
2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(
x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-
a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2))
 - (C*a^3 - C*a*b^2)*c + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)
*b*c^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B -
C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c
^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b
^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a
^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 +
(2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^
2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2
```

```
*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*
b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 +
2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*
cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*
c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^
2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^
4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14547, size = 242, normalized size = 2.24

$$-\frac{2\,(Bb-Cc)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\left(Ba^{2}e^{x}+Ca^{2}e^{x}-Cb^{2}e^{x}+Bbce^{x}-Cbce^{x}+Bc^{2}e^{x}+Bab-Cac\right)}{\left(a^{2}b-b^{3}+a^{2}c-b^{2}c+bc^{2}+c^{3}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] -2*(B*b - C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b
^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - C*b^2*e^x +
B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - C*a*c)/((a^2*b - b^3 + a^2*c -
b^2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.797 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=194

$$\frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}-$$

```
[Out] (3*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a
^2 - b^2 + c^2)^(5/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/(2*(a^2 - b
^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*(B*c - b*C) - (2*b*B*c + (a^2
 - 2*c^2)*C)*Cosh[x] - (a^2*B + 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c
^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.259858, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3156, 3153, 3124, 618, 206}

$$\frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}-$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```


```
[Out] (3*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a
^2 - b^2 + c^2)^(5/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/(2*(a^2 - b
^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*(B*c - b*C) - (2*b*B*c + (a^2
 - 2*c^2)*C)*Cosh[x] - (a^2*B + 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c
^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```


Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
```

```
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{2(bB-cC)-aB\cosh(x)-aC\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right.}{2\left(a^2-b^2+c^2\right)^2(}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right.}{2\left(a^2-b^2+c^2\right)^2(}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right.}{2\left(a^2-b^2+c^2\right)^2(}$$

$$= \frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.639675, size = 319, normalized size = 1.64

$$\frac{c\cosh(2x)\left(a^2+2b^2-2c^2\right)(bB-cC)+a^2b^2B\sinh(2x)+4a^2b^2C-9a^2bBc+4a^3bB\sinh(x)-a^2bcC\sinh(2x)+5a^2c^2C}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (-3*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/
(-a^2 + b^2 - c^2)^(5/2) + (-9*a^2*b*B*c - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C
+ 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c^4*C - 6*a*b*c*(b*B - c*C)*Cosh[x] + c*(a^
2 + 2*b^2 - 2*c^2)*(b*B - c*C)*Cosh[2*x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Si
nh[x] - 8*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a
*c^3*C*Sinh[x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] - 2*b^2*B*c^2*Sinh
[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4
*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

Maple [B] time = 0.105, size = 885, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2+4*B*a^2*c^2-3*B*a*b^3+2*B*b^4-4*B*b^
2*c^2+2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b
^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3+1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2
+4*B*a^3*c^2+B*a^2*b^3+8*B*a^2*b*c^2-3*B*a*b^4-8*B*a*b^2*c^2+2*B*a*c^4+2*B*
b^5-4*B*b^3*c^2+2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c-4*C*a^2*c^3+5
*C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*
a*b+b^2)*tanh(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c+2*B*b*c^3+2*C*a^4-4*C*a^2
*b^2-C*a^2*c^2+2*C*b^4+C*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^
4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2
+3/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(
1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*a^2*b^2+
2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(
1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.3288, size = 21315, normalized size = 109.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")

[Out] [-1/2*(2*B*a^4*b^2 + 2*B*a^2*b^4 - 4*B*b^6 + 4*(B + C)*b*c^5 - 4*C*c^6 - 2*
(C*a^2 + 2*(B - 2*C)*b^2)*c^4 + 2*((B + C)*a^2*b - 4*(B + C)*b^3)*c^3 + 6*(
B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3
 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4
)*c)*cosh(x)^3 + 6*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*
a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b
^2 - (2*B - C)*a*b^4)*c)*sinh(x)^3 + 2*(C*a^4 - (B - C)*a^2*b^2 + 2*(2*B -
C)*b^4)*c^2 + 2*(2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4
- 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 -
2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^
2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c)*cosh(x)^2 + 2*(2*(B + C)*a^6 + 3
*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*
c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*
a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2
*b^3)*c + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2
*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2
*B - C)*a*b^4)*c)*cosh(x))*sinh(x)^2 + 3*(B*a*b^4 - (B + C)*a*b^3*c - (B -
C)*a*b^2*c^2 + (B + C)*a*b*c^3 - C*a*c^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3
*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^4 + (B*a*b^4 + (3
*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*sinh(x
)^4 + 4*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)
*cosh(x)^3 + 4*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a
^2*c^3 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b
*c^3 - C*a*c^4)*cosh(x))*sinh(x)^3 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b
*c^3 + C*a*c^4 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)
*a*b^3)*c)*cosh(x)^2 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4
 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + 3*(B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B -
C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^2 + (2*(B - C)*a^3*b +
(B - C)*a*b^3)*c + 6*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2
 - C*a^2*c^3)*cosh(x))*sinh(x)^2 + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2
 + C*a^2*c^3)*cosh(x) + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^
3 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3
- C*a*c^4)*cosh(x)^3 + 3*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b
*c^2 - C*a^2*c^3)*cosh(x)^2 + (2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*
a*c^4 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c
)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^
2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(
x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2
 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b +
c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*
((B + C)*a^4*b + (B + C)*a^2*b^3 - 2*(B + C)*b^5)*c + 2*(4*B*a^5*b + B*a^3*
```

b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a
^3*b - 10*B*a*b^3)*c^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + 2*(
4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C
*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^2 + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*
C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b
^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*cosh(x)^2 - (4*C*a^5 + C
*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2
*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B
 - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B
+ C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c)*cosh(x))*sinh(x))/(a
^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 -
(3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*
b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b
*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a
^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
+ 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 +
3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(
3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c
^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2
)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^
3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a
^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 +
2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^
6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (
a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)
c^2 + 3(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a
^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*
b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6
 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6
 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 +
4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 +
 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 -
4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b
 + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*
a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2
*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a
^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^
2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^

4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a *b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a ^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^ 6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3* b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4) *c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3* a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^ 2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3 *(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2* a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a ^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^ 3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^ 2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^ 4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4 *b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c ^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)), -(B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6 + 2*(B + C)*b*c^5 - 2*C*c^6 - (C*a^2 + 2* (B - 2*C)*b^2)*c^4 + ((B + C)*a^2*b - 4*(B + C)*b^3)*c^3 + 3*(B*a^3*b^3 - B *a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C) *a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*cosh(x)^ 3 + 3*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b ^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*sinh(x)^3 + (C*a^4 - (B - C)*a^2*b^2 + 2*(2*B - C)*b^4)*c^2 + (2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^ 4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^ 4*b - (B - C)*a^2*b^3)*c)*cosh(x)^2 + (2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b ^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c + 9*(B*a^3* b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*c osh(x))*sinh(x)^2 + 3*(B*a*b^4 - (B + C)*a*b^3*c - (B - C)*a*b^2*c^2 + (B + C)*a*b*c^3 - C*a*c^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3 *(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*sinh(x)^4 + 4*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*cosh(x)^3 + 4*(B*a ^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cos h(x))*sinh(x)^3 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2

*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c)*cosh(x)^
2 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2*C*a^3 + (B +
C)*a*b^2)*c^2 + 3*(B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B -
 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c +
6*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*cosh(
x))*sinh(x)^2 + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^3)*cosh(
x) + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^3 + (B*a*b^4 + (3*B
 - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^
3 + 3*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*c
osh(x)^2 + (2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2*C*a^3 +
(B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B + C)*a^4*b + (B + C)*a^2*b^3 -
2*(B + C)*b^5)*c + (4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a
*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^2 - (4*C*a^5 + C
*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + (4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B
*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^
2 + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b
^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C
)*a*b^4)*c)*cosh(x)^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*
a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)
*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*
B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B -
 C)*a^2*b^3)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 -
b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*
b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5
 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*
(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 +
 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 +
 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6
 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^
2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b
^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 +
2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5
 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a
*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2
)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^
3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 +
3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(
3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c
^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b

```
^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c
^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^
2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a
^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 +
 (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4
*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a
^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a
^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^
4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*
c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9
+ (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^
2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b
- 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*c
osh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 -
 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6
+ 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*
b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)
*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4
*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 -
 (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^
6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (
a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)
*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*
(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4
+ 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (
2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^
2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a
^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*
b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a
^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18384, size = 779, normalized size = 4.02

$$-\frac{3\,(Bab-Cac)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}-\frac{3\,Bab^3e^{(3\,x)}+6\,Bab^2ce^{(3\,x)}-3\,Cab^2ce^{(3\,x)}+3\,Babc^2e^{(3\,x)}-6\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] -3*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^4
 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2)) -
 (3*B*a*b^3*e^(3*x) + 6*B*a*b^2*c*e^(3*x) - 3*C*a*b^2*c*e^(3*x) + 3*B*a*b*c
^2*e^(3*x) - 6*C*a*b*c^2*e^(3*x) - 3*C*a*c^3*e^(3*x) + 2*B*a^4*e^(2*x) + 2*
C*a^4*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 4*C*a^2*b^2*e^(2*x) + 2*B*b^4*e^(2*x)
 + 2*C*b^4*e^(2*x) + 9*B*a^2*b*c*e^(2*x) - 9*C*a^2*b*c*e^(2*x) + 4*B*a^2*c^
2*e^(2*x) - 5*C*a^2*c^2*e^(2*x) - 4*B*b^2*c^2*e^(2*x) - 4*C*b^2*c^2*e^(2*x)
 + 2*B*c^4*e^(2*x) + 2*C*c^4*e^(2*x) + 4*B*a^3*b*e^x + 5*B*a*b^3*e^x - 4*C*
a^3*c*e^x - 5*C*a*b^2*c*e^x - 5*B*a*b*c^2*e^x + 5*C*a*c^3*e^x + B*a^2*b^2 +
 2*B*b^4 - B*a^2*b*c - C*a^2*b*c - 2*B*b^3*c - 2*C*b^3*c + C*a^2*c^2 - 2*B*
b^2*c^2 + 2*C*b^2*c^2 + 2*B*b*c^3 + 2*C*b*c^3 - 2*C*c^4)/((a^4*b - 2*a^2*b^
3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3
 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

3.798 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(-abB+acC+Ab^2-Ac^2\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTanh[(c
 - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 +
 c^2]) - ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rubi [A] time = 0.219307, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3136, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(-abB+acC+Ab^2-Ac^2\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTanh[(c
 - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 +
 c^2]) - ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{\left(Ab^2-abB-Ac^2+acC\right)\ldots}{b^2-c\ldots}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{\left(2\left(Ab^2-abB-Ac^2+ac\ldots\right.\right.}{\ldots}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{\left(4\left(Ab^2-abB-Ac^2+ac\ldots\right.\right.}{\ldots}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{2\left(Ab^2-abB-Ac^2+acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a\ldots}{\ldots}$$

Mathematica [A] time = 0.270228, size = 119, normalized size = 0.87

$$\frac{\frac{2\left(-abB+acC+Ab^2-Ac^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(bC-Bc)\log(a+b\cosh(x)+c\sinh(x))+x(bB-cC)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTan[(c + (-a + b)*Ta
nh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*L
og[a + b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c))
```

Maple [B] time = 0.056, size = 1009, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] 2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*B/(2*b+2*
c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b+c)/(a-b)*ln
(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(b-c)/(b+c)/(
a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*B*c+1/(b-c)/
(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b*C-1
/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*
C*b^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2+2/(b-c)/(b+c)/(-a^2
+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*
a*b*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C-2/(b-c)/(b+c)/(-a^2
+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*
C*c*b-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*a
rctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(
b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73085, size = 1407, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] [((B*a*b - A*b^2 - C*a*c + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c +
 c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*
b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*
sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cos
h(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x)
+ b - c)) + ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((
B - C)*a^2 - (B - C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^
2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 - C*a*c + A*c^2)*s
qrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b +
 c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C
)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3
 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(
cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15951, size = 184, normalized size = 1.34

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^{2}-c^{2}}-\frac{2\left(Bab-Ab^{2}-Cac+Ac^{2}\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}\left(b^{2}-c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="gia
c")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - A*b^2 - C*a*c + A*c^2)*arctan((b*e^x + c*e^x + a
)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.799 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)(aA-bB+cC)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] (-2*(a*A - b*B + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]
])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C + (A*c - a*C)*Cosh[x] + (A*b - a*B)
*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.150261, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3153, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)(aA-bB+cC)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]
])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C + (A*c - a*C)*Cosh[x] + (A*b - a*B)
*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB+cC)\int \frac{1}{a+b\cosh(x}$$
$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2(aA-bB+cC))\,\mathrm{Subst}\Big(}{a^2}$$
$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4(aA-bB+cC))\,\mathrm{Subst}\Big(}{}$$
$$= -\frac{2(aA-bB+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c}$$

Mathematica [A] time = 0.372806, size = 143, normalized size = 1.18

$$\frac{a^2C+\sinh(x)\left(-abB+acC+A\left(b^2-c^2\right)\right)-aAc+b(Bc-bC)}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA-bB+cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2
]])/(-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C + b*(B*c - b*C) + (-(a*b*B
) + A*(b^2 - c^2) + a*c*C)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] +
c*Sinh[x]))
```

Maple [B] time = 0.081, size = 376, normalized size = 3.1

$$-2\frac{1}{a(\tanh(x/2))^2-(\tanh(x/2))^2b-2c\tanh(x/2)-a-b}\left(-\frac{\left(aAb-Ab^2+Ac^2-a^2B+abB-Bc^2-acC+Ccb\right)\tanh}{a^3-a^2b-ab^2+ac^2+b^3-bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c
^2+b^3-b*c^2)*tanh(1/2*x)-(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+
b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^
2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*a*A+2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*b-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.82662, size = 5477, normalized size = 45.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="f
ricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - 2*C*a*c^3 + 2*A*c^4 + 2*
(A*a^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a -
 B*b)*c^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b -
 (2*B - C)*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*
b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*
a*c^2 + (A*a^2 - (B - C)*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 +
 (A*a^2 - (B - C)*a*b)*c + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c
^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*l
og(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 -
 b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x)
+ a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cos
h(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c + 2*((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*cosh(x) + 2*((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2
 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b
 - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*si
nh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^
2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 -
b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b
^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^
2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5
)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 -
a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) +
 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(
a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b
^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*
c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2
*b^2 - B*a*b^3 + A*b^4 - C*a*c^3 + A*c^4 + (A*a^2 + B*a*b - 2*A*b^2)*c^2 -
(A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 + C
*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x)^2 + (
A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^
2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c)
*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c + (A*a*
b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c
)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((
b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^3 - C*a*b^2
)*c + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4
```

```
- (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^
3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3
*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*
b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2
*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*
c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(
a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*
c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^
2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*
(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (
a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b -
b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19316, size = 279, normalized size = 2.31

$$\frac{2\,(Aa-Bb+Cc)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{-a^2+b^2-c^2}}-\frac{2\left(Ba^2e^x+Ca^2e^x-Aabe^x-Cb^2e^x-Aace^x+Bbce^x-Cbce^x+Bc^2e^x+Bab-A\right.}{\left(a^2b-b^3+a^2c-b^2c+bc^2+c^3\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="g
iac")
```

```
[Out] 2*(A*a - B*b + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
2 - b^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - A*a*b*e
```

```
^x - C*b^2*e^x - A*a*c*e^x + B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - A*
b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c + b*c^2 + c^3)*(b*e^(2*x
) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.800 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=233

$$-\frac{\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\cosh(x)\left(a^2(-\right.}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

```
[Out] -(((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c - b*C + (A*c
- a*C)*Cosh[x] + (A*b - a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] +
 c*Sinh[x])^2) - (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B - c*C))*Cosh[
x] + (3*a*A*b - a^2*B - 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x]))
```

Rubi [A] time = 0.511454, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3156, 3153, 3124, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\cosh(x)\left(a^2(-\right.}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```


```
[Out] -(((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c - b*C + (A*c
- a*C)*Cosh[x] + (A*b - a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] +
 c*Sinh[x])^2) - (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B - c*C))*Cosh[
x] + (3*a*A*b - a^2*B - 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x]))
```


Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
```

```
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA-bB+cC)+(Ab-aB)\cosh(x)+}{(a+b\cosh(x)+c\sinh(}}{2\left(a^2-b^2+c^2\right.}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aAc-a^2C-\right.}{2\left(a^2\right.}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aAc-a^2C-\right.}{2\left(a^2\right.}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aAc-a^2C-\right.}{2\left(a^2\right.}$$

$$= -\frac{\left(2a^2A+Ab^2-3abB-Ac^2+3acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc-bC+(Ac-a}{2\left(a^2-b^2+c^2\right)}$$

Mathematica [A] time = 0.934203, size = 465, normalized size = 2.

$$\frac{\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{2bc\cosh(x)\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)+c\cosh(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/
2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*
b^2*c - 9*a^2*b*B*c - 3*a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c
^2*C + 4*b^2*c^2*C - 2*c^4*C + 2*b*c*(2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3
*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 + c^2) + a^2*(b*B - c*C) + 2*(b^2 - c^2)*(
b*B - c*C))*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*S
inh[x] + 2*a*b^3*B*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8
*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Si
nh[x] - 3*a*A*b^3*Sinh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a
*A*b*c^2*Sinh[2*x] - 2*b^2*B*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*
C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x]
+ c*Sinh[x])^2)
```

Maple [B] time = 0.109, size = 1425, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b
^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+1
3*A*a^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5
+2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B
*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C
*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4
)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2
+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^
4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3
*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-
2*B*b^5+4*B*b^3*c^2-2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*c
^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^
2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*b
^2*c^3-5*B*a^3*b*c+5*B*a*b^3*c-2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*
C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b
+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2
*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*
tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4
-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(
-a^2+b^2-c^2)^(1/2))*A*b^2+1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-
a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2
))*A*c^2+3/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*
a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.1836, size = 29412, normalized size = 126.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="f
ricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 4*C*
c^6 - 2*(3*A*a - 2*(B + C)*b)*c^5 - 2*(C*a^2 - 3*A*a*b + 2*(B - 2*C)*b^2)*c
^4 - 2*(3*A*a^3 - (B + C)*a^2*b - 6*A*a*b^2 + 4*(B + C)*b^3)*c^3 - 2*(2*A*a
^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A
*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*
B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*
c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 -
2*A*b^5)*c)*cosh(x)^3 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^
5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)
*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B -
2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*
A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 + 2*(C*a^4 + 3*A*a^3*
b - (B - C)*a^2*b^2 - 6*A*a*b^3 + 2*(2*B - C)*b^4)*c^2 + 2*(2*(B + C)*a^6 -
 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*
a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*
b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((
2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 -
 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)
*cosh(x)^2 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b
^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C
)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C
)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2
*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 +
3*(B - C)*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b
 + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4
- 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3
*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 + (
```

2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*(B + C)*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*(B + C)*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*((B + C)*a^

4*b - 3*A*a^3*b^2 + (B + C)*a^2*b^3 + 3*A*a*b^4 - 2*(B + C)*b^5)*c + 2*(4*B
*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*
a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c
^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - (4*C*
a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a
^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*a*c^5 + A*c^6 + (11*A*a^2 -
 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c^3 + (10*A*a^4 - B*a^3*b -
22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A
*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B
- 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 +
 (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*
B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 -
(4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B -
 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b
^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*
c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^
3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B
- C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7
- (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^
2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3
*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3
*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^
3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7
+ 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3
*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)
c^3 + 3(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a
^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b
^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*
b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 +
 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*
(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4
+ 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3
*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*
c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 +
 (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^
5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^
4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c
^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a
^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5
+ 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2

```
+ 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2
- 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4
*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (
5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a
^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 +
 a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*
b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*
a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2
*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 -
 a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*
b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2
- (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^
4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c
^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5
+ 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b
^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*
a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 +
 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 -
2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2
*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 +
a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(
a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a
^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)
*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))
, -(B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 2*C*c^6 - (
3*A*a - 2*(B + C)*b)*c^5 - (C*a^2 - 3*A*a*b + 2*(B - 2*C)*b^2)*c^4 - (3*A*a
^3 - (B + C)*a^2*b - 6*A*a*b^2 + 4*(B + C)*b^3)*c^3 - (2*A*a^4*b^2 - 3*B*a^
3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^
2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b
^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*
b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cos
h(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6
 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 +
 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*
a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*
B - C)*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 + (C*a^4 + 3*A*a^3*b - (B - C)*a^2*b^2
 - 6*A*a*b^3 + 2*(2*B - C)*b^4)*c^2 + (2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2
*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6
 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^
4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*
```

b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + (2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*(B + C)*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*(B + C)*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A

*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2 + 3*(2*A*a^3*b^2
 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3
*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*co
sh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^
5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3
 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b +
 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^
2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) +
 a)/(a^2 - b^2 + c^2)) - ((B + C)*a^4*b - 3*A*a^3*b^2 + (B + C)*a^2*b^3 + 3
*A*a*b^4 - 2*(B + C)*b^5)*c + (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A
*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 +
 10*B*a*b^3 + 3*A*b^4)*c^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) +
 (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 +
 5*C*a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b
^2)*c^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 -
3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C
*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^
2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 +
 A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*
a*b^4 - 2*A*b^5)*c)*cosh(x)^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*
(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a
^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B -
C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^
2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B +
C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3
 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*
c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^
2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 +
 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a
^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*
a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3
*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^
5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3
*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2
 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2
 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*
a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 -
a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b
^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b
^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*
c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3
 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*

```
b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*
a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2
 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2
- 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4
*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (
5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b
^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2
*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4
 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*
b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^
2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 +
(9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*
b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b -
2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cos
h(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3
*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 +
6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^
5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5
 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c
 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c
^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a
*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (
3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4
*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6
+ 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^
6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c
^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*
b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a
^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 +
2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*
a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)
*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b
- 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31871, size = 1106, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="g
iac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sq
rt(-a^2 + b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4
)*sqrt(-a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^
4*e^(3*x) + 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 3*C*a*b^2*c*e^(3*x)
 + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) + 6*C*a*b*
c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*a
^4*e^(2*x) - 2*C*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 4*
C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 2*C*b^4*e^(2*x) +
 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*
c*e^(2*x) - 4*B*a^2*c^2*e^(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x)
 + 4*B*b^2*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3*A*a*c^3*e^(2*x) - 2*B*c^4*
e^(2*x) - 2*C*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^
x - A*b^4*e^x + 4*C*a^3*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 5*B*a*
b*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a
*b^3 - 2*B*b^4 + B*a^2*b*c + C*a^2*b*c - 3*A*a*b^2*c + 2*B*b^3*c + 2*C*b^3*
c - C*a^2*c^2 - 3*A*a*b*c^2 + 2*B*b^2*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*B*b
*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c
 + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(
b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

3.801 $\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=22

$$\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

```
[Out] (c*Cosh[x] + b*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])
```

Rubi [A] time = 0.0829528, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3150}

$$\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x
]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])
```

Rule 3150

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{c\cosh(x)+b\sinh(x)}{a+b\cosh(x)+c\sinh(x)}$$

Mathematica [A] time = 0.0831417, size = 34, normalized size = 1.55

$$\frac{-ac+b^2\sinh(x)-c^2\sinh(x)}{b(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x
])^2,x]
```

```
[Out] (-(a*c) + b^2*Sinh[x] - c^2*Sinh[x])/(b*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] time = 0.082, size = 73, normalized size = 3.3

$$2\frac{1}{a(\tanh(x/2))^2-(\tanh(x/2))^2b-2c\tanh(x/2)-a-b}\left(-\frac{(ab-b^2+c^2)\tanh(x/2)}{a-b}-\frac{ac}{a-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(-(a*b-b^2+c^2)/(a-b)*tanh(1/2*x)-a*c/(a-b))/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-2*c*tanh(1/2*x)-a-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3252, size = 178, normalized size = 8.09

$$-\frac{2(a\cosh(x)+a\sinh(x)+b-c)}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2a\cosh(x)+2((b+c)\cosh(x)+a)\sinh(x)+b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + a*sinh(x) + b - c)/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 +
 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2-c**2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1841, size = 47, normalized size = 2.14

$$-\frac{2\,(ae^{x} + b - c)}{be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^{x} + b - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="giac")
```

```
[Out] -2*(a*e^x + b - c)/(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)
```

3.802 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=71

$$\frac{x(2aA+bC)}{2a^2}-\frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x))-\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[Out] ((2*a*A + b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) - (((2*A)/a - C/b + (b*C)/a^2
)*Log[a + b*Cosh[x] + b*Sinh[x]])/2 - (C*Sinh[x])/(2*a)
```

Rubi [A] time = 0.0560363, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3131}

$$\frac{x(2aA+bC)}{2a^2}-\frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x))-\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) - (((2*A)/a - C/b + (b*C)/a^2
)*Log[a + b*Cosh[x] + b*Sinh[x]])/2 - (C*Sinh[x])/(2*a)
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA+bC)x}{2a^2}+\frac{C\cosh(x)}{2a}-\frac{1}{2}\left(\frac{2A}{a}-\frac{C}{b}+\frac{bC}{a^2}\right)\log(a+b\cosh(x)+b\sinh(x))-\frac{C\sinh(x)}{2a}$$

Mathematica [A] time = 0.210844, size = 86, normalized size = 1.21

$$\frac{x\left(a^2C+2aAb+b^2C\right)+2\left(a^2C-2aAb-b^2C\right)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)-2abC\sinh(x)+2abC\cosh(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*C + b^2*C)*x + 2*a*b*C*Cosh[x] + 2*(-2*a*A*b + a^2*C - b^2*
C)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]] - 2*a*b*C*Sinh[x])/(4*a^2*b)
```

Maple [B] time = 0.053, size = 136, normalized size = 1.9

$$\frac{C}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{bC}{2\,a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{C}{2\,b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{A}{a}\ln\left(a\tanh\left(\frac{x}{2}\right)-\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] C/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A+1/2/a^2*ln(tanh(1/2*x)+1)*b*C-1
/2*C/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*A+1/2/b*ln
(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*C-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)*
b-a-b)*C
```

Maxima [A] time = 1.12256, size = 78, normalized size = 1.1

$$\frac{1}{2}C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{\left(a^2-b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

Fricas [A] time = 2.29658, size = 284, normalized size = 4.

$$\frac{Cab + \left(2\,Aab + Cb^2\right)x\cosh\left(x\right) + \left(2\,Aab + Cb^2\right)x\sinh\left(x\right) + \left(\left(Ca^2 - 2\,Aab - Cb^2\right)\cosh\left(x\right) + \left(Ca^2 - 2\,Aab - Cb^2\right)\sin}{2\left(a^2b\cosh\left(x\right) + a^2b\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(C*a*b + (2*A*a*b + C*b^2)*x*cosh(x) + (2*A*a*b + C*b^2)*x*sinh(x) + ((
C*a^2 - 2*A*a*b - C*b^2)*cosh(x) + (C*a^2 - 2*A*a*b - C*b^2)*sinh(x))*log(b
*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13367, size = 78, normalized size = 1.1

$$\frac{Ce^{(-x)}}{2\,a} + \frac{(2\,Aa + Cb)x}{2\,a^2} + \frac{\left(Ca^2 - 2\,Aab - Cb^2\right)\log\left(|be^x + a|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*C*e^(-x)/a + 1/2*(2*A*a + C*b)*x/a^2 + 1/2*(C*a^2 - 2*A*a*b - C*b^2)*lo
g(abs(b*e^x + a))/(a^2*b)
```

3.803 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}-\frac{B\cosh(x)}{2a}$$

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - (B*Cosh[x])/(2*a) - ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rubi [A] time = 0.049617, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3132}

$$-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}-\frac{B\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - (B*Cosh[x])/(2*a) - ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA-bB)x}{2a^2}-\frac{B\cosh(x)}{2a}-\frac{\left(2aAb-a^2B-b^2B\right)\log(a+b\cosh(x)+b\sinh(x))}{2a^2b}+\frac{B\,\mathrm{s}}{}$$

Mathematica [A] time = 0.168832, size = 84, normalized size = 1.09

$$\frac{\frac{2(a^2B-2aAb+b^2B)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(\frac{a^2B}{b}+2aA-bB\right)+2aB\sinh(x)-2aB\cosh(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + (a^2*B)/b - b*B)*x - 2*a*B*Cosh[x] + (2*(-2*a*A*b + a^2*B + b^2*B
)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]])/b + 2*a*B*Sinh[x])/(4*a^2)
```

Maple [A] time = 0.052, size = 137, normalized size = 1.8

$$-\frac{B}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{Bb}{2a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{2b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{A}{a}\ln\left(a\tanh\left(\frac{x}{2}\right)-\text{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] -B/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A-1/2/a^2*ln(tanh(1/2*x)+1)*B*b-
1/2*B/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*A+1/2/b*l
n(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*B+1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)
*b-a-b)*B
```

Maxima [A] time = 1.13735, size = 77, normalized size = 1.

$$\frac{1}{2}B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{(a^2+b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*B*(x/b - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

Fricas [A] time = 2.25281, size = 285, normalized size = 3.7

$$-\frac{Bab-\left(2\,Aab-Bb^{2}\right)x\cosh\left(x\right)-\left(2\,Aab-Bb^{2}\right)x\sinh\left(x\right)-\left(\left(Ba^{2}-2\,Aab+Bb^{2}\right)\cosh\left(x\right)+\left(Ba^{2}-2\,Aab+Bb^{2}\right)\sinh}{2\left(a^{2}b\cosh\left(x\right)+a^{2}b\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(B*a*b - (2*A*a*b - B*b^2)*x*cosh(x) - (2*A*a*b - B*b^2)*x*sinh(x) - (
(B*a^2 - 2*A*a*b + B*b^2)*cosh(x) + (B*a^2 - 2*A*a*b + B*b^2)*sinh(x))*log(
b*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13103, size = 78, normalized size = 1.01

$$-\frac{Be^{(-x)}}{2\,a}+\frac{(2\,Aa-Bb)x}{2\,a^{2}}+\frac{\left(Ba^{2}-2\,Aab+Bb^{2}\right)\log\left(|be^{x}+a|\right)}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*B*e^(-x)/a + 1/2*(2*A*a - B*b)*x/a^2 + 1/2*(B*a^2 - 2*A*a*b + B*b^2)*l
og(abs(b*e^x + a))/(a^2*b)
```

3.804 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(a^2(-(B+C))+2aAb-b^2(B-C)\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B-C))}{2a^2}-\frac{(B-C)(\cosh(x)-\sinh(x)}{2a}$$

```
[Out] ((2*a*A - b*(B - C))*x)/(2*a^2) - ((2*a*A*b - b^2*(B - C) - a^2*(B + C))*Lo
g[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) - ((B - C)*(Cosh[x] - Sinh[x]))/(2*
a)
```

Rubi [A] time = 0.0830199, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3130}

$$-\frac{\left(a^2(-(B+C))+2aAb-b^2(B-C)\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B-C))}{2a^2}-\frac{(B-C)(\cosh(x)-\sinh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*(B - C))*x)/(2*a^2) - ((2*a*A*b - b^2*(B - C) - a^2*(B + C))*Lo
g[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) - ((B - C)*(Cosh[x] - Sinh[x]))/(2*
a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA-b(B-C))x}{2a^2}-\frac{\left(2aAb-b^2(B-C)-a^2(B+C)\right)\log(a+b\cosh(x)+b\sinh(x}{2a^2b}$$

Mathematica [A] time = 0.283727, size = 103, normalized size = 1.2

$$\frac{\frac{2\left(a^2(B+C)-2aAb+b^2(B-C)\right)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(\frac{a^2(B+C)}{b}+2aA+b(C-B)\right)+2a(B-C)\sinh(x)-2a(B-C)\cosh(}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*(-B + C) + (a^2*(B + C))/b)*x - 2*a*(B - C)*Cosh[x] + (2*(-2*a*
A*b + b^2*(B - C) + a^2*(B + C))*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]
])/b + 2*a*(B - C)*Sinh[x])/(4*a^2)
```

Maple [B] time = 0.055, size = 232, normalized size = 2.7

$$-\frac{B}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{C}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{Bb}{2a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{bC}{2a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] -B/a/(tanh(1/2*x)+1)+C/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A-1/2/a^2*ln
(tanh(1/2*x)+1)*B*b+1/2/a^2*ln(tanh(1/2*x)+1)*b*C-1/2*B/b*ln(tanh(1/2*x)-1)
-1/2*C/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*A+1/2/b*
ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*B+1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x
)*b-a-b)*B+1/2/b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*C-1/2/a^2*b*ln(a*tanh(
1/2*x)-tanh(1/2*x)*b-a-b)*C
```

Maxima [A] time = 1.26165, size = 134, normalized size = 1.56

$$\frac{1}{2}C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{\left(a^2-b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)+\frac{1}{2}B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{\left(a^2+b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) + 1/2*B*(x/b
 - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^(-x) + b)/
a
```

Fricas [A] time = 2.46166, size = 342, normalized size = 3.98

$$-\frac{(B-C)ab-\left(2\,Aab-(B-C)b^2\right)x\cosh(x)-\left(2\,Aab-(B-C)b^2\right)x\sinh(x)-\left(\left((B+C)a^2-2\,Aab+(B-C)b^2\right)\cosh(x)\right.}{2\left(a^2b\cosh(x)+a^2b\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] -1/2*((B - C)*a*b - (2*A*a*b - (B - C)*b^2)*x*cosh(x) - (2*A*a*b - (B - C)*
b^2)*x*sinh(x) - (((B + C)*a^2 - 2*A*a*b + (B - C)*b^2)*cosh(x) + ((B + C)*
a^2 - 2*A*a*b + (B - C)*b^2)*sinh(x))*log(b*cosh(x) + b*sinh(x) + a))/(a^2*
b*cosh(x) + a^2*b*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11438, size = 107, normalized size = 1.24

$$\frac{(2\,Aa-Bb+Cb)x}{2\,a^2}-\frac{(Ba-Ca)e^{(-x)}}{2\,a^2}+\frac{\left(Ba^2+Ca^2-2\,Aab+Bb^2-Cb^2\right)\log\left(|be^x+a|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(2*A*a - B*b + C*b)*x/a^2 - 1/2*(B*a - C*a)*e^(-x)/a^2 + 1/2*(B*a^2 + C
*a^2 - 2*A*a*b + B*b^2 - C*b^2)*log(abs(b*e^x + a))/(a^2*b)
```

3.805 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=77

$$\frac{(a^2C+2aAb-b^2C)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[Out] ((2*a*A - b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) + ((2*a*A*b + a^2*C - b^2*C)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (C*Sinh[x])/(2*a)
```

Rubi [A] time = 0.0512851, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3131}

$$\frac{(a^2C+2aAb-b^2C)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) + ((2*a*A*b + a^2*C - b^2*C)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (C*Sinh[x])/(2*a)
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-bC)x}{2a^2}+\frac{C\cosh(x)}{2a}+\frac{(2aAb+a^2C-b^2C)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}+$$

Mathematica [A] time = 0.233741, size = 86, normalized size = 1.12

$$\frac{\frac{2\left(a^2C+2aAb-b^2C\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(-\frac{a^2C}{b}+2aA-bC\right)+2aC\sinh(x)+2aC\cosh(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - (a^2*C)/b - b*C)*x + 2*a*C*Cosh[x] + (2*(2*a*A*b + a^2*C - b^2*C)
*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]])/b + 2*a*C*Sinh[x])/(4*a^2)
```

Maple [A] time = 0.048, size = 125, normalized size = 1.6

$$-\frac{C}{2b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{C}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{bC}{2a^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{A}{a}\ln\left(a\tanh\left(\frac{x}{2}\right)-\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] -1/2*C/b*ln(tanh(1/2*x)+1)-C/a/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/
a^2*ln(tanh(1/2*x)-1)*b*C+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A+1/2/b*l
n(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*C-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)
*b+a+b)*C
```

Maxima [A] time = 1.12609, size = 88, normalized size = 1.14

$$A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right)-\frac{1}{2}C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{\left(a^2-b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

Fricas [A] time = 2.3295, size = 155, normalized size = 2.01

$$-\frac{Ca^2x - Cab\cosh(x) - Cab\sinh(x) - (Ca^2 + 2\,Aab - Cb^2)\log(a\cosh(x) + a\sinh(x) + b)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a^2*x - C*a*b*cosh(x) - C*a*b*sinh(x) - (C*a^2 + 2*A*a*b - C*b^2)*l
og(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13059, size = 66, normalized size = 0.86

$$-\frac{Cx}{2\,b} + \frac{Ce^x}{2\,a} + \frac{(Ca^2 + 2\,Aab - Cb^2)\log(|ae^x + b|)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*C*x/b + 1/2*C*e^x/a + 1/2*(C*a^2 + 2*A*a*b - C*b^2)*log(abs(a*e^x + b)
)/(a^2*b)
```

3.806 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + (B*Cosh[x])/(2*a) + ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rubi [A] time = 0.0455736, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3132}

$$\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + (B*Cosh[x])/(2*a) + ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-bB)x}{2a^2}+\frac{B\cosh(x)}{2a}+\frac{\left(2aAb-a^2B-b^2B\right)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}+\frac{B\,\mathrm{s}}{}$$

Mathematica [A] time = 0.157305, size = 86, normalized size = 1.1

$$\frac{x\left(a^2B+2aAb-b^2B\right)-2\left(a^2B-2aAb+b^2B\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)+2abB\sinh(x)+2abB\cosh(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*B - b^2*B)*x + 2*a*b*B*Cosh[x] - 2*(-2*a*A*b + a^2*B + b^2*
B)*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]] + 2*a*b*B*Sinh[x])/(4*a^2*b)
```

Maple [A] time = 0.044, size = 125, normalized size = 1.6

$$\frac{B}{2\,b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{Bb}{2\,a^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{A}{a}\ln\left(a\tanh\left(\frac{x}{2}\right)-\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] 1/2*B/b*ln(tanh(1/2*x)+1)-B/a/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/a
^2*ln(tanh(1/2*x)-1)*B*b+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A-1/2/b*ln
(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*B-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)*
b+a+b)*B
```

Maxima [A] time = 1.07242, size = 84, normalized size = 1.08

$$A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right)-\frac{1}{2}B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{\left(a^2+b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

Fricas [A] time = 2.30002, size = 154, normalized size = 1.97

$$\frac{Ba^2x + Bab\cosh(x) + Bab\sinh(x) - \left(Ba^2 - 2\,Aab + Bb^2\right)\log(a\cosh(x) + a\sinh(x) + b)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*x + B*a*b*cosh(x) + B*a*b*sinh(x) - (B*a^2 - 2*A*a*b + B*b^2)*lo
g(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15686, size = 65, normalized size = 0.83

$$\frac{Bx}{2\,b} + \frac{Be^x}{2\,a} - \frac{\left(Ba^2 - 2\,Aab + Bb^2\right)\log(|ae^x + b|)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*B*x/b + 1/2*B*e^x/a - 1/2*(B*a^2 - 2*A*a*b + B*b^2)*log(abs(a*e^x + b))
/(a^2*b)
```

3.807 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a^2(-(B-C))+2aAb-b^2(B+C)\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cosh(x))}{2a}$$

```
[Out] ((2*a*A - b*(B + C))*x)/(2*a^2) + ((2*a*A*b - a^2*(B - C) - b^2*(B + C))*Lo
g[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + ((B + C)*(Cosh[x] + Sinh[x]))/(2*
a)
```

Rubi [A] time = 0.0802853, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3130}

$$\frac{\left(a^2(-(B-C))+2aAb-b^2(B+C)\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cosh(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```


```
[Out] ((2*a*A - b*(B + C))*x)/(2*a^2) + ((2*a*A*b - a^2*(B - C) - b^2*(B + C))*Lo
g[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + ((B + C)*(Cosh[x] + Sinh[x]))/(2*
a)
```


Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-b(B+C))x}{2a^2}+\frac{\left(2aAb-a^2(B-C)-b^2(B+C)\right)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}$$

Mathematica [A] time = 0.281354, size = 102, normalized size = 1.26

$$\frac{x\left(a^2(B-C)+2aAb-b^2(B+C)\right)-2\left(a^2(B-C)-2aAb+b^2(B+C)\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)+2ab(B}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - C) - b^2*(B + C))*x + 2*a*b*(B + C)*Cosh[x] - 2*(-2*a*
A*b + a^2*(B - C) + b^2*(B + C))*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]]
 + 2*a*b*(B + C)*Sinh[x])/(4*a^2*b)
```

Maple [B] time = 0.05, size = 213, normalized size = 2.6

$$\frac{B}{2b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{C}{2b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-\frac{C}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-\frac{A}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{Bb}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] 1/2*B/b*ln(tanh(1/2*x)+1)-1/2*C/b*ln(tanh(1/2*x)+1)-B/a/(tanh(1/2*x)-1)-C/a
/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/a^2*ln(tanh(1/2*x)-1)*B*b+1/2/
a^2*ln(tanh(1/2*x)-1)*b*C+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A-1/2/b*l
n(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*B-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)
*b+a+b)*B+1/2/b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*C-1/2/a^2*b*ln(a*tanh(1
/2*x)-tanh(1/2*x)*b+a+b)*C
```

Maxima [A] time = 1.18393, size = 142, normalized size = 1.75

$$A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right)-\frac{1}{2}B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{\left(a^2+b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)-\frac{1}{2}C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{\left(a^2-b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b)) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*e^(-x) +
a)/(a^2*b))
```

Fricas [A] time = 2.45155, size = 194, normalized size = 2.4

$$\frac{(B-C)a^2x + (B+C)ab\cosh(x) + (B+C)ab\sinh(x) - \left((B-C)a^2 - 2\,Aab + (B+C)b^2\right)\log\left(a\cosh(x) + a\sinh(x) + \right.}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] 1/2*((B - C)*a^2*x + (B + C)*a*b*cosh(x) + (B + C)*a*b*sinh(x) - ((B - C)*a
^2 - 2*A*a*b + (B + C)*b^2)*log(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18218, size = 93, normalized size = 1.15

$$\frac{(B-C)x}{2\,b} + \frac{Be^x + Ce^x}{2\,a} - \frac{\left(Ba^2 - Ca^2 - 2\,Aab + Bb^2 + Cb^2\right)\log\left(|ae^x + b|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(B - C)*x/b + 1/2*(B*e^x + C*e^x)/a - 1/2*(B*a^2 - C*a^2 - 2*A*a*b + B*
b^2 + C*b^2)*log(abs(a*e^x + b))/(a^2*b)
```

3.808 $\int \frac{1}{\cosh^2(x)+\sinh^2(x)} \, dx$

Optimal. Leaf size=3

$$\tan^{-1}(\tanh(x))$$

```
[Out] ArcTan[Tanh[x]]
```

Rubi [A] time = 0.0172277, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {203}

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^2(x)+\sinh^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \tanh(x)\right)\\ &= \tan^{-1}(\tanh(x))\end{aligned}$$

Mathematica [A] time = 0.0047023, size = 3, normalized size = 1.

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Maple [B] time = 0.039, size = 116, normalized size = 38.7

$$-2\,\frac{\sqrt{2}}{2+2\sqrt{2}}\arctan\left(2\,\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)-2\,\frac{1}{2+2\sqrt{2}}\arctan\left(2\,\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)+2\,\frac{\sqrt{2}}{-2+2\sqrt{2}}\arctan\left(2\,\frac{\tanh(x/2)}{-2+2\sqrt{2}}\right)-2\,\frac{1}{-2+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2+sinh(x)^2),x)
```

```
[Out] -2*2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))-2/(2+2*2^(1/2)
)*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))+2*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tan
h(1/2*x)/(-2+2*2^(1/2)))-2/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2
)))
```

Maxima [B] time = 1.61923, size = 47, normalized size = 15.67

$$\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-x)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

Fricas [B] time = 2.29802, size = 69, normalized size = 23.

$$-\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [B] time = 11.3414, size = 117, normalized size = 39.

$$\frac{\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{\sqrt{3-2\sqrt{2}}+\sqrt{2}\sqrt{3-2\sqrt{2}}}-\frac{\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{\sqrt{3-2\sqrt{2}}+\sqrt{2}\sqrt{3-2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2),x)
```

```
[Out] atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(sqrt(3 - 2*sqrt(2)) + sqrt(2)*sqrt(3 -
 2*sqrt(2))) - sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*atan(tanh(x/2)/sqrt(
2*sqrt(2) + 3))/(sqrt(3 - 2*sqrt(2)) + sqrt(2)*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.10793, size = 7, normalized size = 2.33

$$\arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(e^(2*x))
```

3.809 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{\tanh^2(x)+1}$$

```
[Out] Tanh[x]/(1 + Tanh[x]^2)
```

Rubi [A] time = 0.0249508, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {383}

$$\frac{\tanh(x)}{\tanh^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]

[Out] Tanh[x]/(1 + Tanh[x]^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} dx = \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2} dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{1+\tanh^2(x)}$$

Mathematica [A] time = 0.0027112, size = 8, normalized size = 0.73

$$\frac{1}{2}\tanh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[2*x]/2
```

Maple [B] time = 0.023, size = 36, normalized size = 3.3

$$-2\,\frac{-\left(\tanh\left(x/2\right)\right)^{3}-\tanh\left(x/2\right)}{\left(\tanh\left(x/2\right)\right)^{4}+6\,\left(\tanh\left(x/2\right)\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2+sinh(x)^2)^2,x)
```

```
[Out] -2*(-tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^4+6*tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.10095, size = 11, normalized size = 1.

$$\frac{1}{e^{(-4\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/(e^(-4*x) + 1)
```

Fricas [B] time = 2.19323, size = 135, normalized size = 12.27

$$-\frac{1}{\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)^{3}\sinh\left(x\right)+6\,\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 1)
```

Sympy [B] time = 5.28396, size = 48, normalized size = 4.36

$$\frac{2\tanh^3\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**2,x)
```

```
[Out] 2*tanh(x/2)**3/(tanh(x/2)**4 + 6*tanh(x/2)**2 + 1) + 2*tanh(x/2)/(tanh(x/2)
**4 + 6*tanh(x/2)**2 + 1)
```

Giac [A] time = 1.09845, size = 14, normalized size = 1.27

$$-\frac{1}{e^{(4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/(e^(4*x) + 1)
```

3.810 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\tanh(x)\text{sech}^2(x)}{2\left(\tanh^2(x)+1\right)^2}$$

```
[Out] ArcTan[Tanh[x]]/2 + (Sech[x]^2*Tanh[x])/(2*(1 + Tanh[x]^2)^2)
```

Rubi [A] time = 0.0287418, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {413, 21, 203}

$$\frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\tanh(x)\text{sech}^2(x)}{2\left(\tanh^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

```
[Out] ArcTan[Tanh[x]]/2 + (Sech[x]^2*Tanh[x])/(2*(1 + Tanh[x]^2)^2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^3}\,dx,x,\tanh(x)\right)\\ &= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{2+2x^2}{\left(1+x^2\right)^2}\,dx,x,\tanh(x)\right)\\ &= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tanh(x)\right)\\ &= \frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}\end{aligned}$$

Mathematica [A] time = 0.006351, size = 22, normalized size = 0.85

$$\frac{1}{4}\tan^{-1}(\sinh(2x))+\frac{1}{4}\tanh(2x)\operatorname{sech}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

```
[Out] ArcTan[Sinh[2*x]]/4 + (Sech[2*x]*Tanh[2*x])/4
```

Maple [B] time = 0.042, size = 166, normalized size = 6.4

$$-2\,\frac{-1/2\,(\tanh(x/2))^7+1/2\,(\tanh(x/2))^5+1/2\,(\tanh(x/2))^3-1/2\,\tanh(x/2)}{\left((\tanh(x/2))^4+6\,(\tanh(x/2))^2+1\right)^2}-\frac{\sqrt{2}}{2+2\sqrt{2}}\arctan\left(2\,\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)-\frac{}{2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2+sinh(x)^2)^3,x)
```

```
[Out] -2*(-1/2*tanh(1/2*x)^7+1/2*tanh(1/2*x)^5+1/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))
/(tanh(1/2*x)^4+6*tanh(1/2*x)^2+1)^2-2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/
2*x)/(2+2*2^(1/2)))-1/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))+2^(
1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-1/(-2+2*2^(1/2))*a
rctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))
```

Maxima [B] time = 1.7561, size = 86, normalized size = 3.31

$$\frac{e^{(-2x)} - e^{(-6x)}}{2\left(2e^{(-4x)} + e^{(-8x)} + 1\right)} + \frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) - \frac{1}{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2*x) - e^(-6*x))/(2*e^(-4*x) + e^(-8*x) + 1) + 1/2*arctan(1/2*sqrt
(2)*(sqrt(2) + 2*e^(-x))) - 1/2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

Fricas [B] time = 2.24376, size = 1022, normalized size = 39.31

$$\frac{\cosh(x)^6 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6 + \left(15\cosh(x)^4 - 1\right)\sinh}{2\left(\cosh(x)^8 + 56\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^6 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x
)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^4 - 1)*sinh(x)^2 - (cosh(x)^8 + 56*co
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh
(x)^3)*sinh(x) + 1)*arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x))) - cosh
(x)^2 + 2*(3*cosh(x)^5 - cosh(x))*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x
)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh
(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*sinh(x)^3 +
4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x)^3)*sinh(x)
 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16879, size = 62, normalized size = 2.38

$$\frac{e^{(2x)} - e^{(-2x)}}{2\left(\left(e^{(2x)} - e^{(-2x)}\right)^2 + 4\right)} + \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(4x)} - 1\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e^(2*x) - e^(-2*x))/((e^(2*x) - e^(-2*x))^2 + 4) + 1/4*arctan(1/2*(e^(
4*x) - 1)*e^(-2*x))
```

3.811 $\int \frac{1}{\cosh^2(x)-\sinh^2(x)} dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0147339, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cosh^2(x)-\sinh^2(x)} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0004396, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [C] time = 0.01, size = 8, normalized size = 8.

$$2\,\text{Artanh}\,(\tanh\,(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2),x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.10203, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.06439, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 0.503864, size = 10, normalized size = 10.

$$\frac{x}{-\sinh^2(x)+\cosh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2),x)
```

```
[Out] x/(-sinh(x)**2 + cosh(x)**2)
```

Giac [A] time = 1.15502, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="giac")
```

```
[Out] x
```

3.812 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0134379, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} dx &= \int 1 \, dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0004241, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] time = 0.011, size = 8, normalized size = 8.

$$2\,\mathrm{Artanh}\,(\tanh\,(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2)^2,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.06141, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.17843, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 1.35754, size = 22, normalized size = 22.

$$\frac{x}{\sinh^4(x) - 2\sinh^2(x)\cosh^2(x) + \cosh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**2,x)
```

```
[Out] x/(sinh(x)**4 - 2*sinh(x)**2*cosh(x)**2 + cosh(x)**4)
```

Giac [A] time = 1.19096, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.813 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0135388, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0003531, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [C] time = 0.013, size = 8, normalized size = 8.

$$2 \operatorname{Artanh}(\tanh(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2)^3,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.15499, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.22723, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 4.73535, size = 34, normalized size = 34.

$$\frac{x}{-\sinh^6(x)+3\sinh^4(x)\cosh^2(x)-3\sinh^2(x)\cosh^4(x)+\cosh^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**3,x)
```

```
[Out] x/(-sinh(x)**6 + 3*sinh(x)**4*cosh(x)**2 - 3*sinh(x)**2*cosh(x)**4 + cosh(x
)**6)
```

Giac [A] time = 1.11921, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

3.814 $\int \frac{1}{\mathbf{sech}^2(x)+\tanh^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01261, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-1),x]

[Out] x
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\operatorname{sech}^2(x)+\tanh^2(x)}\,dx &= \int 1\,dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0003844, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [C] time = 0.019, size = 8, normalized size = 8.

$$2 \operatorname{Artanh}(\tanh(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2),x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.05006, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.09045, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\tanh^2(x) + \operatorname{sech}^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2),x)
```

```
[Out] Integral(1/(tanh(x)**2 + sech(x)**2), x)
```

Giac [A] time = 1.18324, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="giac")
```

```
[Out] x
```

3.815 $\int \frac{1}{\left(\mathbf{sech}^2(x)+\tanh^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0123049, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0003646, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] time = 0.024, size = 8, normalized size = 8.

$$2\,\mathrm{Artanh}\left(\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2)^2,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.06034, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.12061, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\tanh^2(x) + \operatorname{sech}^2(x)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**2,x)
```

```
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-2), x)
```

Giac [A] time = 1.15356, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.816 $\int \frac{1}{\left(\mathbf{sech}^2(x)+\tanh^2(x)\right)^3}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.0129531, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-3),x]

[Out] x

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0003534, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [C] time = 0.024, size = 8, normalized size = 8.

$$2\,\mathrm{Artanh}\left(\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2)^3,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.10592, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.2931, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\tanh^2(x) + \operatorname{sech}^2(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**3,x)
```

```
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-3), x)
```

Giac [A] time = 1.14536, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

3.817 $\int \frac{1}{\mathbf{sech}^2(x)-\tanh^2(x)}\,dx$

Optimal. Leaf size=19

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rubi [A] time = 0.0282499, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1093, 207}

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-1),x]

[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \tanh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \tanh(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tanh(x)\right)$$
$$= -x + \sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)$$

Mathematica [A] time = 0.0894322, size = 19, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Maple [B] time = 0.041, size = 54, normalized size = 2.8

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\,\tanh\left(x/2\right)+2\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\,\tanh\left(x/2\right)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+ln(tanh(1
/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))
```

Maxima [B] time = 1.67006, size = 86, normalized size = 4.53

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Fricas [B] time = 2.40961, size = 220, normalized size = 11.58

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\tanh(x)+\operatorname{sech}(x))(\tanh(x)+\operatorname{sech}(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2),x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))*(tanh(x) + sech(x))), x)
```

Giac [B] time = 1.15631, size = 55, normalized size = 2.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

3.818 $\int \frac{1}{\left(\mathbf{sech}^2(x)-\tanh^2(x)\right)^2} dx$

Optimal. Leaf size=31

$$x-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

```
[Out] x - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2] + Tanh[x]/(1 - 2*Tanh[x]^2)
```

Rubi [A] time = 0.054305, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {414, 12, 481, 206}

$$x-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

```
[Out] x - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2] + Tanh[x]/(1 - 2*Tanh[x]^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx &= \operatorname{Subst}\left(\int \frac{1}{\left(1-2x^2\right)^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)}+\frac{1}{2}\operatorname{Subst}\left(\int -\frac{2x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)}-\operatorname{Subst}\left(\int \frac{x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)}-\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)+\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= x-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.156086, size = 42, normalized size = 1.35

$$\frac{-3x-\sinh(2x)+x\cosh(2x)}{\cosh(2x)-3}-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]) + (-3*x + x*Cosh[2*x] - Sinh[2*x])/(-3
+ Cosh[2*x])
```

Maple [B] time = 0.046, size = 108, normalized size = 3.5

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\left(2-2\tanh\left(x/2\right)\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+2\tanh\left(x/2\right)-1\right)^{-1}-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)+2\right)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2)^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-1/2*(2-2*tanh(1/2*x))/(tanh(1/2*x)^2+2*tanh(1/2*x)-1)-1/2
*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+1/2*(2*tanh(1/2*x)+2)/(tanh
(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2
))-ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.76692, size = 119, normalized size = 3.84

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+x-\frac{2\left(3e^{(-2x)}-1\right)}{6e^{(-2x)}-e^{(-4x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) + 1/4*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + x - 2*(3*e^(-2*x)
 - 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] time = 2.28395, size = 883, normalized size = 28.48

$$\frac{4x\cosh(x)^{4}+16x\cosh(x)\sinh(x)^{3}+4x\sinh(x)^{4}-24(x+1)\cosh(x)^{2}+24\left(x\cosh(x)^{2}-x-1\right)\sinh(x)^{2}+\left(\sqrt{2}\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 - 24*(x + 1)*co
sh(x)^2 + 24*(x*cosh(x)^2 - x - 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(
2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*
sinh(x)^2 - 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))
*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*co
sh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 - 2*sqrt(2) - 3)/(cosh(x)^2 + s
inh(x)^2 - 3)) + 16*(x*cosh(x)^3 - 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 -
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\tanh(x)+\operatorname{sech}(x))^2(\tanh(x)+\operatorname{sech}(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**2,x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))**2*(tanh(x) + sech(x))**2), x)
```

Giac [B] time = 1.1434, size = 85, normalized size = 2.74

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)+x-\frac{2\left(3e^{(2x)}-1\right)}{e^{(4x)}-6e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) -
 6)) + x - 2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

3.819 $\int \frac{1}{\left(\mathbf{sech}^2(x)-\tanh^2(x)\right)^3} dx$

Optimal. Leaf size=54

$$-x+\frac{7\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

```
[Out] -x + (7*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + Tanh[x]/(2*(1 - 2*Tanh[x]^2
)^2) - Tanh[x]/(4*(1 - 2*Tanh[x]^2))
```

Rubi [A] time = 0.0642893, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {414, 527, 522, 206}

$$-x+\frac{7\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] -x + (7*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + Tanh[x]/(2*(1 - 2*Tanh[x]^2
)^2) - Tanh[x]/(4*(1 - 2*Tanh[x]^2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)^3\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{2-6x^2}{\left(1-2x^2\right)^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{1}{8}\text{Subst}\left(\int \frac{6+2x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)-\text{Subs}\\
&= -x+\frac{7\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.191805, size = 66, normalized size = 1.22

$$\frac{-76x-2\sinh(2x)+3\sinh(4x)+48x\cosh(2x)-4x\cosh(4x)+7\sqrt{2}(\cosh(2x)-3)^2\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{8(\cosh(2x)-3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] (-76*x + 7*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]*(-3 + Cosh[2*x])^2 + 48*x*Cosh[
2*x] - 4*x*Cosh[4*x] - 2*Sinh[2*x] + 3*Sinh[4*x])/(8*(-3 + Cosh[2*x])^2)
```

Maple [B] time = 0.058, size = 140, normalized size = 2.6

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\,\frac{-1/8\,\left(\tanh\left(x/2\right)\right)^{3}+1/8\,\left(\tanh\left(x/2\right)\right)^{2}-5/8\,\tanh\left(x/2\right)+1/8}{\left(\left(\tanh\left(x/2\right)\right)^{2}+2\,\tanh\left(x/2\right)-1\right)^{2}}+\frac{7\sqrt{2}}{8}\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2)^3,x)
```

```
[Out] -ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)-1)-2*(-1/8*tanh(1/2*x)^3+1/8*tanh(1/2*x)^
2-5/8*tanh(1/2*x)+1/8)/(tanh(1/2*x)^2+2*tanh(1/2*x)-1)^2+7/8*2^(1/2)*arctan
h(1/4*(2*tanh(1/2*x)+2)*2^(1/2))-2*(-1/8*tanh(1/2*x)^3-1/8*tanh(1/2*x)^2-5/
8*tanh(1/2*x)-1/8)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)^2+7/8*2^(1/2)*arctanh(1/
4*(2*tanh(1/2*x)-2)*2^(1/2))
```

Maxima [B] time = 1.68986, size = 154, normalized size = 2.85

$$\frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x+\frac{19\,e^{(-2\,x)}-57\,e^{(-4\,x)}+17\,e^{(-6\,x)}-3}{2\left(12\,e^{(-2\,x)}-38\,e^{(-4\,x)}+12\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 7/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x + 1/2*(19*e^(-
2*x) - 57*e^(-4*x) + 17*e^(-6*x) - 3)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(-6
*x) - e^(-8*x) - 1)
```

Fricas [B] time = 2.13029, size = 2365, normalized size = 43.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 - 8*(24*x
+ 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 - 24*x - 17)*sinh(x)^6 + 16*(56*x*cosh(
x)^3 - 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*
x*cosh(x)^4 - 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*co
sh(x)^5 - 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 - 8*(24
*x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 - 15*(24*x + 17)*cosh(x)^4 + 114*(4*
x + 3)*cosh(x)^2 - 24*x - 19)*sinh(x)^2 - 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*
cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))
*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh
(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 - 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2
))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*c
osh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt
(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^2 - 12*sqrt(2)*co
sh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3
 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2
- 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(
2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16*(8*x*cosh(x)^7 - 3*(24*x + 17)*co
sh(x)^5 + 38*(4*x + 3)*cosh(x)^3 - (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24
)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x
)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^
4 - 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(
x)^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2
 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3
- 3*cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\tanh(x) + \operatorname{sech}(x))^3 (\tanh(x) + \operatorname{sech}(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**3,x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))**3*(tanh(x) + sech(x))**3), x)
```

<u>Giac</u> [A] time = 1.1718, size = 104, normalized size = 1.93

$$-\frac{7}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x+\frac{17e^{(6x)}-57e^{(4x)}+19e^{(2x)}-3}{2\left(e^{(4x)}-6e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x + 1/2*(17*e^(6*x) - 57*e^(4*x) + 19*e^(2*x) - 3)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

3.820 $\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx$

Optimal. Leaf size=18

$$x-\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rubi [A] time = 0.0302392, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1130, 207}

$$x-\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

```
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2}{2-3x^2+x^4}\,dx, x, \tanh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)$$
$$= x - \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.0806825, size = 18, normalized size = 1.

$$x - \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

```
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Maple [B] time = 0.034, size = 102, normalized size = 5.7

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{\sqrt{2}}{4}\ln\left(\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \sqrt{2}\tanh\left(\frac{x}{2}\right)+1\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \sqrt{2}\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)-1/4*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*t
anh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))+1/4*2^(1/2)*ln((tanh(1
/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

Maxima [B] time = 1.69103, size = 49, normalized size = 2.72

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
```

Fricas [B] time = 2.27656, size = 219, normalized size = 12.17

$$\frac{1}{2}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(x\right)^{2}-4\left(3\sqrt{2}+4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}+3\right)\sinh\left(x\right)^{2}+2\sqrt{2}+3}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*si
nh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2
 + 3)) + x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\coth^{2}\left(x\right)+\operatorname{csch}^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2),x)
```

```
[Out] Integral(1/(coth(x)**2 + csch(x)**2), x)
```

Giac [B] time = 1.13117, size = 49, normalized size = 2.72

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(2x\right)}-3}{2\sqrt{2}+e^{\left(2x\right)}+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
```

3.821 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^2} dx$

Optimal. Leaf size=32

$$x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

```
[Out] x - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2] - Tanh[x]/(2 - Tanh[x]^2)
```

Rubi [A] time = 0.0467893, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 12, 391, 206}

$$x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

```
[Out] x - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2] - Tanh[x]/(2 - Tanh[x]^2)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(2-x^2\right)^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{2}{\left(1-x^2\right)\left(2-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(2-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \tanh(x)\right)\\
&= x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.130939, size = 64, normalized size = 2.

$$\frac{(\cosh(2x)+3)\operatorname{csch}^4(x)\left(6x-2\sinh(2x)+2x\cosh(2x)-\sqrt{2}(\cosh(2x)+3)\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\right)}{8\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

```
[Out] ((3 + Cosh[2*x])*Csch[x]^4*(6*x + 2*x*Cosh[2*x] - Sqrt[2]*ArcTanh[Tanh[x]/S
qrt[2]]*(3 + Cosh[2*x]) - 2*Sinh[2*x]))/(8*(Coth[x]^2 + Csch[x]^2)^2)
```

Maple [B] time = 0.048, size = 129, normalized size = 4.

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{-1/2\,\left(\tanh\left(x/2\right)\right)^{3}-1/2\,\tanh\left(x/2\right)}{\left(\tanh\left(x/2\right)\right)^{4}+1}-\frac{\sqrt{2}}{8}\ln\left(\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\sqrt{2}\tanh\left(\frac{x}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2)^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)+2*(-1/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/
(tanh(1/2*x)^4+1)-1/8*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tan
h(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))+1/8*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*ta
nh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))
```

Maxima [B] time = 1.69692, size = 81, normalized size = 2.53

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{2\left(3e^{(-2x)}+1\right)}{6e^{(-2x)}+e^{(-4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
 - 2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

Fricas [B] time = 2.35679, size = 883, normalized size = 27.59

$$\frac{4\,x\cosh\left(x\right)^{4}+16\,x\cosh\left(x\right)\sinh\left(x\right)^{3}+4\,x\sinh\left(x\right)^{4}+24\,(x+1)\cosh\left(x\right)^{2}+24\left(x\cosh\left(x\right)^{2}+x+1\right)\sinh\left(x\right)^{2}+\left(\sqrt{2}\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 + 24*(x + 1)*co
sh(x)^2 + 24*(x*cosh(x)^2 + x + 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(
2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2))*
sinh(x)^2 + 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))
*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*co
sh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + s
inh(x)^2 + 3)) + 16*(x*cosh(x)^3 + 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\coth^2(x) + \operatorname{csch}^2(x)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**2,x)
```

```
[Out] Integral((coth(x)**2 + csch(x)**2)**(-2), x)
```

Giac [B] time = 1.15796, size = 81, normalized size = 2.53

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2} - e^{(2x)} - 3}{2\sqrt{2} + e^{(2x)} + 3}\right) + x + \frac{2\left(3e^{(2x)} + 1\right)}{e^{(4x)} + 6e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
+ 2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

3.822 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^3} \, dx$

Optimal. Leaf size=54

$$x-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{7\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] x - (7*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Tanh[x]^3/(2*(2 - Tanh[x]^2)
^2) - Tanh[x]/(4*(2 - Tanh[x]^2))
```

Rubi [A] time = 0.0884717, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 578, 522, 206}

$$x-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{7\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```



```
[Out] x - (7*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Tanh[x]^3/(2*(2 - Tanh[x]^2)
^2) - Tanh[x]/(4*(2 - Tanh[x]^2))
```


Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)\left(2-x^2\right)^3}\,dx,x,\tanh(x)\right)$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{x^2\left(6-2x^2\right)}{\left(1-x^2\right)\left(2-x^2\right)^2}\,dx,x,\tanh(x)\right)$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{1}{8}\,\text{Subst}\left(\int \frac{-2-6x^2}{\left(1-x^2\right)\left(2-x^2\right)}\,dx,x,\tanh(x)\right.$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{7}{4}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\tanh(x)\right)+\text{Subst}$$

$$= x-\frac{7\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}$$

Mathematica [A] time = 0.187356, size = 66, normalized size = 1.22

$$\frac{76x-2\sinh(2x)-3\sinh(4x)+48x\cosh(2x)+4x\cosh(4x)-7\sqrt{2}(\cosh(2x)+3)^2\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{8(\cosh(2x)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

```
[Out] (76*x + 48*x*Cosh[2*x] - 7*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]*(3 + Cosh[2*x])
^2 + 4*x*Cosh[4*x] - 2*Sinh[2*x] - 3*Sinh[4*x])/(8*(3 + Cosh[2*x])^2)
```

Maple [B] time = 0.049, size = 145, normalized size = 2.7

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{-1/8\,(\tanh(x/2))^7-5/8\,(\tanh(x/2))^5-5/8\,(\tanh(x/2))^3-1/8\,\tanh(x/2)}{\left((\tanh(x/2))^4+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2)^3,x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)+2*(-1/8*tanh(1/2*x)^7-5/8*tanh(1/2*x)^5
-5/8*tanh(1/2*x)^3-1/8*tanh(1/2*x))/(tanh(1/2*x)^4+1)^2-7/32*2^(1/2)*ln((ta
nh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))+7
/32*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)
*tanh(1/2*x)+1))
```

Maxima [B] time = 1.77191, size = 113, normalized size = 2.09

$$\frac{7}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{19\,e^{(-2x)}+57\,e^{(-4x)}+17\,e^{(-6x)}+3}{2\left(12\,e^{(-2x)}+38\,e^{(-4x)}+12\,e^{(-6x)}+e^{(-8x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
x - 1/2*(19*e^(-2*x) + 57*e^(-4*x) + 17*e^(-6*x) + 3)/(12*e^(-2*x) + 38*e^(
```

```
-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

Fricas [B] time = 2.41457, size = 2363, normalized size = 43.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 + 8*(24*x +
 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 + 24*x + 17)*sinh(x)^6 + 16*(56*x*cosh(x
)^3 + 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*x
*cosh(x)^4 + 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*cos
h(x)^5 + 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 + 8*(24*
x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 + 15*(24*x + 17)*cosh(x)^4 + 114*(4*x
 + 3)*cosh(x)^2 + 24*x + 19)*sinh(x)^2 + 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*c
osh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*
sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 + 9*sqrt(2)*cosh(
x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 + 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2)
)*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + 30*sqrt(2)*co
sh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 45*sqrt(
2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^2 + 12*sqrt(2)*cos
h(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3
+ 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 -
4*(3*sqrt(2) + 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2)
 + 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*(8*x*cosh(x)^7 + 3*(24*x + 17)*cosh
(x)^5 + 38*(4*x + 3)*cosh(x)^3 + (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24)/
(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 3)*sinh(x)^
6 + 12*cosh(x)^6 + 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4
+ 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 + 30*cosh(x)
^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 45*cosh(x)^4 + 57*cosh(x)^2 +
 3)*sinh(x)^2 + 12*cosh(x)^2 + 8*(cosh(x)^7 + 9*cosh(x)^5 + 19*cosh(x)^3 +
3*cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13614, size = 97, normalized size = 1.8

$$-\frac{7}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x+\frac{17\,e^{(6x)}+57\,e^{(4x)}+19\,e^{(2x)}+3}{2\left(e^{(4x)}+6\,e^{(2x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
 + 1/2*(17*e^(6*x) + 57*e^(4*x) + 19*e^(2*x) + 3)/(e^(4*x) + 6*e^(2*x) + 1)
^2
```

3.823 $\int \frac{1}{\coth^2(x)-\mathbf{csch}^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0155451, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\coth^2(x)-\mathrm{csch}^2(x)}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0003987, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [C] time = 0.02, size = 8, normalized size = 8.

$$2\,\mathrm{Artanh}\left(\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2),x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.11078, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.97177, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) - \operatorname{csch}(x))(\coth(x) + \operatorname{csch}(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2),x)
```

```
[Out] Integral(1/((coth(x) - csch(x))*(coth(x) + csch(x))), x)
```

Giac [A] time = 1.1481, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="giac")
```

```
[Out] x
```

3.824 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0154925, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\coth^2(x)-\mathrm{csch}^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0004863, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] time = 0.023, size = 8, normalized size = 8.

$$2 \operatorname{Artanh}(\tanh(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2)^2,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.1373, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.79049, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) - \operatorname{csch}(x))^2 (\coth(x) + \operatorname{csch}(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**2,x)
```

```
[Out] Integral(1/((coth(x) - csch(x))**2*(coth(x) + csch(x))**2), x)
```

Giac [A] time = 1.13707, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.825 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0160287, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3} \, dx &= \int 1 \, dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0004154, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [C] time = 0.027, size = 8, normalized size = 8.

$$2\,\mathrm{Artanh}\left(\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2)^3,x)
```

```
[Out] 2*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.10683, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.8898, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="fricas")
```

[Out] x

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1213, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

3.826 $\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=271

$$\frac{2\sqrt{2}c\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}c\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] (-2*Sqrt[2]*c*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2
])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2
+ 4*a*c]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) + (2*Sqrt[2]*c*A
rcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 -
2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 + 4*a*c]*Sqrt[b^2 - 2*(
a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rubi [A] time = 0.915188, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3248, 2660, 618, 204}

$$\frac{2\sqrt{2}c\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}c\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```


```
[Out] (-2*Sqrt[2]*c*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2
])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2
+ 4*a*c]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) + (2*Sqrt[2]*c*A
rcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 -
2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 + 4*a*c]*Sqrt[b^2 - 2*(
a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```


Rule 3248

```
Int[((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*
(x_)]^(n2_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c
)/q, Int[1/(b - q + 2*c*Sin[d + e*x]^n), x], x] - Dist[(2*c)/q, Int[1/(b +
```

```
q + 2*c*Sin[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{(2c)\int \frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}}+\frac{(2c)\int \frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}}$$

$$= -\frac{(4c)\,\text{Subst}\left(\int \frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}}+\frac{(4c)\,\text{Subst}\left(\int \frac{}{-i}\right.}{}$$

$$= \frac{(8c)\,\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic+2\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right.}{\sqrt{-b^2+4ac}}$$

$$= -\frac{2\sqrt{2}c\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}+\frac{2\sqrt{2}c\tan^{-1}\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tan}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2}}}\right.}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c-ib\sqrt{-}}}$$

Mathematica [A] time = 0.712136, size = 217, normalized size = 0.8

$$\frac{2\sqrt{2}c\left(\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}-\frac{\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b
^2 - 4*a*c]] - ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sq
rt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])]/Sqrt[-b^2 + 2*(a - c)*c - b*
Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] time = 0.037, size = 74, normalized size = 0.3

$$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{-\_R^2+1}{2\_R^3a-3b\_R^2-2\_Ra+4c\_R+b}\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] sum((-_R^2+1)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=Roo
tOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Fricas [B] time = 3.68895, size = 7035, normalized size = 25.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 +
 b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 +
 (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 + b^6 - 8*a*c^5 + 2
*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2
+ (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqr
t(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3
 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a
*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^
2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*
c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 +
3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^
2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^
4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)
*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))
)/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*lo
g(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c -
(a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 +
 (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^
4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)
*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (1
6*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 +
b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*
```

```
a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6
 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b
^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c +
 sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4
 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^
2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 -
4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*
a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(
8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(4*a*c^4
 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^
2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a
*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4
)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 +
 (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
 b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 -
4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*
a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
 b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
 + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*
b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)) - 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2
- (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^
2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c\sinh(x)^2 + b\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

3.827 $\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{2}\left(\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(-\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] (Sqrt[2]*(I + b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[
-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 +
4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] + (Sqrt[2]*(I -
 b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/
2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 -
 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rubi [A] time = 0.723667, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3256, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(-\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```


```
[Out] (Sqrt[2]*(I + b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[
-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 +
4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] + (Sqrt[2]*(I -
 b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/
2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 -
 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```


Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
```

```
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\left(i\int\left(\frac{1+\frac{ib}{\sqrt{-b^2+4ac}}}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}+\frac{1-\frac{ib}{\sqrt{-b^2+4ac}}}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\right)dx\right)$$

$$= -\left(\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx\right)-\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\int$$

$$= -\left(\left(2\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)}\right.\right.$$

$$= \left(4\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic\right.$$

$$= \frac{\sqrt{2}\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}+\frac{\sqrt{2}\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-(ib}{\sqrt{2}\sqrt{b^2}}\right.}{\sqrt{b^2-2(a-c)c-ib\sqrt{}}}$$

Mathematica [A] time = 0.480967, size = 244, normalized size = 0.87

$$\frac{\sqrt{2}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c
- (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

Maple [C] time = 0.035, size = 70, normalized size = 0.3

$$2\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\_R\ln\left(\tanh\left(x/2\right)-\_R\right)}{2\_R^3a-3b\_R^2-2\_Ra+4c\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] 2*sum(_R/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a
*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(x\right)}{c\sinh\left(x\right)^2+b\sinh\left(x\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Fricas [B] time = 4.15167, size = 7015, normalized size = 25.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2
+ b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4
*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3
+ (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c + (a^3*
b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (
4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5
 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c + (a^
2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*
b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)
) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 +
 b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b
^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^
3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (
4*a^2*b + b^3)*c + (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*
a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2
*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2
```

```
)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 +
b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*
a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (
4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*
(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*
a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6
 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*
c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(a^3*b^2 + a*b^4 - 4*
a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
+ 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 +
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b
^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (
8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3
 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a
^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 +
(16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*
c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b
^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2
)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) -
 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*
b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{c\sinh(x)^2 + b\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

3.828 $\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{x}{c}$$

```
[Out] x/c - (Sqrt[2]*(I*b + (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (
I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*
Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])
 - (Sqrt[2]*(I*b - (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b
 + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqr
t[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rubi [A] time = 1.08056, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3256, 3292, 2660, 618, 204}

$$-\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] x/c - (Sqrt[2]*(I*b + (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (
I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*
Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])
 - (Sqrt[2]*(I*b - (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b
 + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqr
t[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
```

```
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\int \left(-\frac{1}{c}+\frac{-a-b\sinh(x)}{c\left(-a-b\sinh(x)-c\sinh^2(x)\right)}\right)dx$$

$$= \frac{x}{c}-\frac{\int \frac{-a-b\sinh(x)}{-a-b\sinh(x)-c\sinh^2(x)}\,dx}{c}$$

$$= \frac{x}{c}-\frac{\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{ib+\sqrt{-b^2+4ac}+2ic\sinh(x)}\,dx}{c}-\frac{\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{ib-\sqrt{-b^2+4ac}+2ic\sinh(x}}{c}$$

$$= \frac{x}{c}-\frac{\left(2\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{ib+\sqrt{-b^2+4ac}+4icx-\left(ib+\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{(2}{}$$

$$= \frac{x}{c}+\frac{\left(4\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, 4ic+2\left(-ib-\sqrt{-b^2}\right.\right.}{c}$$

$$= \frac{x}{c}-\frac{\sqrt{2}\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}-\frac{\sqrt{2}\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\right.}{c\sqrt{b^2-2(a-c)c-}}$$

Mathematica [A] time = 0.512405, size = 283, normalized size = 0.92

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (x - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt
[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]]) - (S
qrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])
/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/c
```

Maple [C] time = 0.046, size = 108, normalized size = 0.4

$$\frac{1}{c}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{c}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\_R^2a-2\_Rb-a}{2\_R^3a-3b\_R^2-2\_Ra+4c\_R+b}\ln\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] 1/c*ln(tanh(1/2*x)+1)+1/c*sum((_R^2*a-2*_R*b-a)/(2*_R^3*a-3*_R^2*b-2*_R*a+4
*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_
Z+a))-1/c*ln(tanh(1/2*x)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c}-\frac{1}{4}\int\frac{8\left(be^{(3x)}+2ae^{(2x)}-be^{x}\right)}{c^2e^{(4x)}+2bce^{(3x)}-2bce^{x}+c^2+2\left(2ac-c^2\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) - b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) - 2*b*c*e^x + c^2 + 2*(2*a*c - c^2)*e^(2*x)), x)
```

Fricas [B] time = 6.56755, size = 9954, normalized size = 32.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*
a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*
sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(
4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b
^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
```

b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) + sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c

```
- (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*
c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)
*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*
a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b
^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (
a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*
a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7
- 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^
3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 +
2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2
 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4
*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(
-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2
)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a
^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 +
b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a
^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5
 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*
(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2
*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*
b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^
2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^
3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*
a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2
+ b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*
(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b
^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 +
 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3
*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2
+ b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2
 - 2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^
4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 +
 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 +
 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4
*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^
7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 -
(a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^
3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (
a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a
^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 -
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^
4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3
*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)
*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (
8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)
```

```
*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/
(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^4))) + 2*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 9.31595, size = 7, normalized size = 0.02

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] x/c
```

3.829 $\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=363

$$\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}+i\left(\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}-i\left(-\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\cdots\right.}{c^2\sqrt{-ib\sqrt{4ac-b^2}-2c(a-\cdots}}$$

```
[Out] -((b*x)/c^2) + (Sqrt[2]*(b^3/Sqrt[-b^2 + 4*a*c] + I*(b^2 - a*c + ((3*I)*a*b
*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*
c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/
(c^2*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) - (Sqrt[2]*(b^3/Sqrt
[-b^2 + 4*a*c] - I*(b^2 - a*c - ((3*I)*a*b*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[(
(2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a -
c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(c^2*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b
^2 + 4*a*c]]) + Cosh[x]/c
```

Rubi [A] time = 4.67652, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3256, 2638, 3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}+i\left(\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}-i\left(-\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\cdots\right.}{c^2\sqrt{-ib\sqrt{4ac-b^2}-2c(a-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```


```
[Out] -((b*x)/c^2) + (Sqrt[2]*(b^3/Sqrt[-b^2 + 4*a*c] + I*(b^2 - a*c + ((3*I)*a*b
*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*
c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/
(c^2*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) - (Sqrt[2]*(b^3/Sqrt
[-b^2 + 4*a*c] - I*(b^2 - a*c - ((3*I)*a*b*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[(
(2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a -
c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(c^2*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b
^2 + 4*a*c]]) + Cosh[x]/c
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = i\int\left(\frac{ib}{c^2}-\frac{i\sinh(x)}{c}+\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{c^2\left(a+b\sinh(x)+c\sinh^2(x)\right)}\right)dx$$

$$= -\frac{bx}{c^2}+\frac{i\int\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx}{c^2}+\frac{\int\sinh(x)\,dx}{c}$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}-\frac{\left(i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{c^2}+ \ldots$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}-\frac{\left(2i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-(\ldots}\ldots\right.}{c^2}\ldots$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}+\frac{\left(4i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+\ldots}\right.}\ldots\right.}{c^2}\ldots$$

$$= -\frac{bx}{c^2}+\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}+i\left(b^2-ac+\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}-\frac{\sqrt{2}\ldots}{\ldots}$$

Mathematica [A] time = 0.804816, size = 326, normalized size = 0.9

$$\frac{\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}-bx+\ldots}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (-(b*x) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*
(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a -
c)*c + b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2
])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/(Sqrt[b^2 - 4
*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]) + c*Cosh[x])/c^2
```

Maple [C] time = 0.049, size = 144, normalized size = 0.4

$$\frac{1}{c}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{c^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{c^2}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{-\_R^2ab+2\left(-ac+b^2\right)\_R}{2\_R^3a-3b\_R^2-2\_Ra+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] 1/c/(tanh(1/2*x)+1)-b/c^2*ln(tanh(1/2*x)+1)+1/c^2*sum((-_R^2*a*b+2*(-a*c+b^
2)*_R+a*b)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf
(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))-1/c/(tanh(1/2*x)-1)+b/c^2*ln(ta
nh(1/2*x)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{x}-ce^{(2\,x)}-c\right)e^{(-x)}}{2\,c^2}-\frac{1}{8}\int-\frac{16\left(2\,abe^{(2\,x)}+\left(b^2-ac\right)e^{(3\,x)}-\left(b^2-ac\right)e^{x}\right)}{c^3e^{(4\,x)}+2\,bc^2e^{(3\,x)}-2\,bc^2e^{x}+c^3+2\left(2\,ac^2-c^3\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) - c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
*x) + (b^2 - a*c)*e^(3*x) - (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 - 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 - c^3)*e^(2*x)), x)
```

Fricas [B] time = 9.06185, size = 13673, normalized size = 37.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*cosh(x) - c*cosh(x)^2 - sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqr
t(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*
```

a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^
2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b
^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*
b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 +
 a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a
*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6
 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^4 + 2*a^3*b^
6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 - (20
*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a
^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)*c + (12*a^2
*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5)*c^7 - (4*a
^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*c^5)*sqrt(-(
a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 +
 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a
*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a
^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2
*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2
*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^1
0 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^
4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*
a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c
^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*
cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*
c)*sinh(x) + 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 + 3*a^4*b^2)*c
^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2
c^4 - 12(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^
6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^
12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 +
 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8))) + sqrt(2)*(c
^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2
*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2
*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^1
0 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^
4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*
a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c
^4))*log(2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c
- sqrt(2)*(12*a^4*b*c^5 - (20*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b
^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 +
 2*a^2*b^7 + b^9)*c + (12*a^2*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b +
 27*a^2*b^3 + b^5)*c^7 - (4*a^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 +

```
2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 1
2*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 -
 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*
(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6
 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c
^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqr
t(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c
^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 +
 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a
*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*
c^2 + (a^5*b^3 + a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^
3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*sinh(x) + 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*
c^6 + 2*(2*a^6 + 3*a^4*b^2)*c^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 +
 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6
*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/
(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a
^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b
^4 + b^6)*c^8))) - sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 + b^6
 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c - (4*a*c
^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqr
t(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c
^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 +
 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a
*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2
 - 4*(a^6*b^2 + 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 - (20*a^5*b + 31*a^3*b
^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 + 5*a^3*b^
5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)*c - (12*a^2*b*c^9 - 7*(4*a^3
*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5)*c^7 - (4*a^5*b + 13*a^3*b^3
 + 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 + 2*a^2*b
^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 1
1*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^1
3 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6
)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*
a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c
^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4
 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c
^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 +
 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^
```

2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)) + 4*(3*a^5*b
c^3 - 2(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*cosh(x) + 4*(3*a^
5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*sinh(x) - 2*(4
*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 + 3*a^4*b^2)*c^5 - (a^5*b^2 + a
^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^
2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b
^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 +
a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*
b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8))) + sqrt(2)*(c^2*cosh(x) + c^2*
sinh(x))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*
a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c
^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4
 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c
^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 +
 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^
2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^
4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c - sqrt(2)*(12*a^4
*b*c^5 - (20*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*
c^3 - 3*(2*a^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)
*c - (12*a^2*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5
)*c^7 - (4*a^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*
c^5)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^
3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a
^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c
^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9
 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2
*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2
)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a
^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2
 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a
*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b
^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*
b^2 + b^4)*c^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 +
a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3
 + a^3*b^5)*c)*sinh(x) - 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 +
3*a^4*b^2)*c^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10
 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4
 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a
^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^1
0 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
- c*sinh(x)^2 + 2*(b*x - c*cosh(x))*sinh(x) - c)/(c^2*cosh(x) + c^2*sinh(x)
)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.830 $\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\cosh(x)}{b-a\sinh(x)}$$

```
[Out] Cosh[x]/(b - a*Sinh[x])
```

Rubi [A] time = 0.0906189, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3288, 2754, 8}

$$\frac{\cosh(x)}{b-a\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

```
[Out] Cosh[x]/(b - a*Sinh[x])
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = -\left((4a^2)\int \frac{a+b\sinh(x)}{\left(2iab-2ia^2\sinh(x)\right)^2}\,dx\right)$$
$$= \frac{\cosh(x)}{b-a\sinh(x)} - \frac{\int 0\,dx}{a^2+b^2}$$
$$= \frac{\cosh(x)}{b-a\sinh(x)}$$

Mathematica [A] time = 0.0337063, size = 14, normalized size = 1.17

$$-\frac{\cosh(x)}{a\sinh(x)-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

```
[Out] -(Cosh[x]/(-b + a*Sinh[x]))
```

Maple [B] time = 0.046, size = 36, normalized size = 3.

$$-2\,\frac{1}{\left(\tanh(x/2)\right)^2 b+2\,a\tanh(x/2)-b}\left(-\frac{a\tanh(x/2)}{b}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x)
```

```
[Out] -2*(-a/b*tanh(1/2*x)+1)/(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74691, size = 159, normalized size = 13.25

$$-\frac{2\,(b\cosh\left(x\right) + b\sinh\left(x\right) + a)}{a^{2}\cosh\left(x\right)^{2} + a^{2}\sinh\left(x\right)^{2} - 2\,ab\cosh\left(x\right) - a^{2} + 2\,\left(a^{2}\cosh\left(x\right) - ab\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 - 2*a*b*cosh(
x) - a^2 + 2*(a^2*cosh(x) - a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b**2-2*a*b*sinh(x)+a**2*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19291, size = 38, normalized size = 3.17

$$-\frac{2\,(be^{x} + a)}{\left(ae^{(2\,x)} - 2\,be^{x} - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) - 2*b*e^x - a)*a)
```

3.831 $\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] (Sqrt[2]*(I*e - (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tan
h[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] +
 (Sqrt[2]*(I*e + (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b +
 Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[
-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rubi [A] time = 0.757776, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```


```
[Out] (Sqrt[2]*(I*e - (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tan
h[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] +
 (Sqrt[2]*(I*e + (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b +
 Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[
-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```


Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
```

```
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \left(-ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx + \left(-ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\int$$

$$= -\left(\left(2\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b^2+4ac}\right)x\right.\right.$$

$$= \left(4\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic\right.$$

$$= \frac{\sqrt{2}\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(i\right.}{\sqrt{2}\sqrt{b}}\right.}{\sqrt{b^2-2(a-c)c-ib\sqrt{}}}$$

Mathematica [A] time = 0.512162, size = 258, normalized size = 0.86

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2*c + (-b + Sqrt[b^
2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])
/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e)*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt
[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] time = 0.056, size = 79, normalized size = 0.3

$$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{-\_R^2d+2\_Re+d}{2\_R^3a-3b\_R^2-2\_Ra+4c\_R+b}\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] sum((-_R^2*d+2*_R*e+d)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_
R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e\sinh(x)+d}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*sinh(x) + d)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Fricas [B] time = 32.5258, size = 14202, normalized size = 47.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 +
 b^2 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b
^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b
^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*
a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*d^4 + 2*a*
b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d^2*e^2 + 2
*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^
3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a^3*c - 8*a
*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)
*e^3 - ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^
2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*d -
(a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^
2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*
c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(
a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^
2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c
)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 + b^2 - 2*
a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*
c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (
8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^
2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^
2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c + 2*a*c^2 -
2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^2)*c)*d*e^
3)*sinh(x) - 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a
^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^3)*c^2 - 2
*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2
)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*
```

c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(
a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^
2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c
))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*
a^2 + b^2 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(
2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^
2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^
4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4
+ 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4
 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*d^4 +
 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d^2*e^
2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*
(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a^3*c
- 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^
3)*c)*e^3 - ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3
*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)
*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^
3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e
^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 +
 a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b
^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 + b^2
 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3
*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 -
 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 -
 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^
3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c*e^4
- (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 -
(2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c + 2*a*c
^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^2)*c)
*d*e^3)*sinh(x) - 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2
 - (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^3)*c^
2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 +
a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e
^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 +
 a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b
^4)*c))) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e
+ (2*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*
(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a
^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8
*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
+ b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*

d^4 + 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d
^2*e^2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3
 - 3*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a
^3*c - 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b
 + b^3)*c)*e^3 + ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^
3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^
4)*c)*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5
*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c
)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*
a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 +
2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2
+ b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 -
b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 +
b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4
*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c
*e^4 - (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c
^2 - (2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c +
2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^
2)*c)*d*e^3)*sinh(x) + 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2
)*c^2 - (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^
3)*c^2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a
^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c
)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*
a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 +
2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)
*d*e + (2*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)
c^2 - 2(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e
 + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2
+ 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 +
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2
*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b
^2*c*d^4 + 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2
*c)*d^2*e^2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c
)*d^3 - 3*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4
- 8*a^3*c - 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*
a^2*b + b^3)*c)*e^3 + ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*
(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4
*a*b^4)*c)*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*
b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^
4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b
- b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 1

```
2*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2
*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*
(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a
^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b
^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4
 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^
4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 -
a*b*c*e^4 - (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (
2*a*c^2 - (2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2
*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2
 - b^2)*c)*d*e^3)*sinh(x) + 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*
a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b
 + b^3)*c^2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 +
 (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^
4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b
- b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 1
2*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.832 $\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=223

$$\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] (4*c*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - S
qrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (4*c*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 -
4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.629001, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3249, 2659, 208}

$$\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (4*c*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - S
qrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (4*c*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 -
4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 3249

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b_.) + cos[(d_.) + (e_.)*(x_)]^
(n2_.)*(c_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c
)/q, Int[1/(b - q + 2*c*Cos[d + e*x]^n), x], x] - Dist[(2*c)/q, Int[1/(b +
q + 2*c*Cos[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
```

```
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx &= \frac{(2c)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{b^2-4ac}} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+2c+\sqrt{b}\ldots}\right.}{}\\
&= \frac{4c\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{4c\tanh^{-1}\left(\frac{\sqrt{b-2c+\ldots}}{\sqrt{b-\ldots}}\right.}{\sqrt{b^2-4ac}\sqrt{b-2c+\sqrt{b^2-\ldots}}}\end{aligned}$$

Mathematica [A] time = 0.553789, size = 198, normalized size = 0.89

$$\frac{2\sqrt{2}c\left(\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
+ 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^
2 - 4*a*c]] + ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b
^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.058, size = 1264, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arcta
n((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b-2*a/(-4*a
*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+
c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c-a/(a-b+c)/(((-4*
a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2
)^(1/2)-a+c)*(a-b+c))^(1/2))-a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a
-c)*(a-b+c))^(1/2))*b+2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a
-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(
a-b+c))^(1/2))*c-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh
((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))-1/(-4*a*c+b
^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*t
anh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+3*b/(-4*a*c+b^2)^(
1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1
/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c+b/(a-b+c)/(((-4*a*c+b^2)^
(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a
+c)*(a-b+c))^(1/2))+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*
(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+
c))^(1/2))*b^2-3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-
b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))
^(1/2))*c+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-
c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))-2/(-4*a*c+b^2)^(1/
2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2
*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c^2-c/(a-b+c)/(((-4*a*c+b^2)^
(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a
+c)*(a-b+c))^(1/2))+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*
(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+
c))^(1/2))*c^2-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((
```

```
-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c\cosh(x)^2 + b\cosh(x) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

Fricas [B] time = 2.72808, size = 7035, normalized size = 31.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
 b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 - b^6 + 8*a*c^5 + 2
*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2
+ (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqr
t(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a
*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^
2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*
c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^
2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^
```

4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
c^3 - 2(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*lo
g(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c -
(a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 +
 (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
c^3 - 2(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*
a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b
^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c +
 sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4
 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^
2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*
a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(
8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(4*a*c^4
 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^
2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a
*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*
a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
 - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -

```
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)) - 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2
- (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c \cosh(x)^2 + b \cosh(x) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

3.833 $\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=230

$$\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] (2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tan
h[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rubi [A] time = 0.58312, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3257, 2659, 208}

$$\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tan
h[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
```

```
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx=\int\left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{b-\sqrt{b^2-4ac}+2c\cosh(x)}+\frac{1+\frac{b}{\sqrt{b^2-4ac}}}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\right)dx$$

$$=\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{}{b+\sqrt{b^2-}}$$

$$=\left(2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\right.$$

$$=\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c+\sqrt{b^2-}}}{\sqrt{b+2c+}}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+}}$$

Mathematica [A] time = 0.466472, size = 227, normalized size = 0.99

$$\frac{\sqrt{2}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((b + Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[((-
b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqr
t[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2
 - 4*a*c]
```

Maple [B] time = 0.03, size = 1262, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] -2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2+3*a/(-
4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a
-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b+a/(a-b+c)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c)
)^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2))+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*a
rctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2-3
*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arct
anh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b-1/(-4*
a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b
+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2-b/(a-b+c)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2))-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c)
)^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2))+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*a
rctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-2
*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c+b/(-4*a*
c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c
)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c+c/(a-b+c)/(((-4*a
*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)
^(1/2)-a+c)*(a-b+c))^(1/2))+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+
```

```
2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arc
tanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c-b/(-4
*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-
a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

Fricas [B] time = 2.79751, size = 7015, normalized size = 30.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
- b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4
*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3
- (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c - (a^3*
b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (
4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c + (a^
2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
```

```
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 -
 b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b
^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^
3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (
4*a^2*b - b^3)*c - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*
a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2
*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2
)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 -
b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*
a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (
4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*
(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*
c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(a^3*b^2 - a*b^4 - 4*
a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 -
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b
^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (
8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3
 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a
^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
(16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*
```

```
c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b
^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2
)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) +
 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

3.834 $\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=255

$$-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{x}{c}$$

```
[Out] x/c - (2*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt
[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2
*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (2*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[
x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.29051, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3257, 3293, 2659, 208}

$$-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] x/c - (2*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt
[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2
*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (2*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[
x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
```

```
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \left(\frac{1}{c}+\frac{-a-b\cosh(x)}{c\left(a+b\cosh(x)+c\cosh^2(x)\right)}\right)dx$$

$$= \frac{x}{c}+\frac{\int \frac{-a-b\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx}{c}$$

$$= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c}$$

$$= \frac{x}{c}-\frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\operatorname{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{\left(2\left(\right.\right.}{}$$

$$= \frac{x}{c}-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c}}{\sqrt{b}}\right.}{c\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2}}$$

Mathematica [A] time = 0.564417, size = 264, normalized size = 1.04

$$\frac{\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (x + (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/c
```

Maple [B] time = 0.043, size = 1957, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*
x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)
+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)
*(a-b+c))^(1/2))-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan(
(a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))-b/(a-b+c)/(((
-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*
c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)-a+c)*(a-b+c))^(1/2))*a^2*b-2/c*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2))*b^2-1/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(
1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+
a-c)*(a-b+c))^(1/2))*a^2*b+2/c*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^
(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)
+a-c)*(a-b+c))^(1/2))*b^2-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)
```

```
+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)
*(a-b+c))^(1/2))*a^2+1/c*ln(tanh(1/2*x)+1)-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)-a+c)*(a-b+c))^(1/2))*a^2-1/c*ln(tanh(1/2*x)-1)+1/(-4*a*c+b^2)^(1/2)/(a-
b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/
(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2+1/c/(a-b+c)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a
-c)*(a-b+c))^(1/2))*b^2+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2
)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+
1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/
2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2+1/c/(a-b+c)/(((-4*a*c+b^
2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2))*a^2+1/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^
(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a
+c)*(a-b+c))^(1/2))*b^3-a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a
+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-
b+c))^(1/2))*b+2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-
b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(
1/2))*c+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b
-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*ar
ctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c-2/c*
a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*
x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b-2/c*a/(a-b+c)/(((-4*a*c+b^2)
^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2
)+a-c)*(a-b+c))^(1/2))*b-1/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2
)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c
)*(a-b+c))^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} - \frac{1}{4}\int \frac{8\left(be^{(3\,x)} + 2\,ae^{(2\,x)} + be^{x}\right)}{c^{2}e^{(4\,x)} + 2\,bce^{(3\,x)} + 2\,bce^{x} + c^{2} + 2\left(2\,ac + c^{2}\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) + b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) + 2*b*c*e^x + c^2 + 2*(2*a*c + c^2)*e^(2*x)), x)
```

Fricas [B] time = 4.40243, size = 9950, normalized size = 39.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4
*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)
*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/
(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*
b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 +
 b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*
b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b
^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2 - (a^2*b^4 - b^6)*c - (8*a^2*c^7 + 6*(
4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2
 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^
2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2
*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4
 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^
4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*
c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8
*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^
3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)
*c)*sinh(x) + 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*
c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2
*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a
*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 - b^4
 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^
3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 +
4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(
2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2
+ 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2
)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a
^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2
 - (a^2*b^4 - b^6)*c - (8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^
```

2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2
*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*
b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b
^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*
a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^
2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2
 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2
- 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2
)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(
x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 + (8*a^4
- a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(
-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c
^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 +
b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*
c^4))) + sqrt(2)*c*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c -
 (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c
^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*
c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a
^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^
4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a
^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^
2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2 - (a^2*b^4 - b^6)*c + (8*a^2*c^7 +
6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*
b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*
a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 -
 b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(
a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(
a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*
c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^
2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 +
 (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2
*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b
^3)*c)*sinh(x) - 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^
2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*
b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3
- a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 -
b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2
*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6
 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 1
2*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b

```
^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 -
b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 -
2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*
c^2 - (a^2*b^4 - b^6)*c + (8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22
*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*
a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a
^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 +
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^
4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 -
 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2
*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*
b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b
^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*
b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*co
sh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 + (8*a
^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sq
rt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*
a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^
6)*c^4))) - 2*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.94562, size = 7, normalized size = 0.03

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] x/c
```

3.835 $\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=299

$$\frac{2\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4}}}{\sqrt{\sqrt{b^2-4ac+}}}\right.}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] -((b*x)/c^2) + (2*(b^2 - a*c - b^3/Sqrt[b^2 - 4*a*c] + (3*a*b*c)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]) + (2*(b^2 - a*c + b^3/Sqrt[b^2 - 4*a*c] - (3*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]) + Sinh[x]/c
```

Rubi [A] time = 6.38888, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3257, 2637, 3293, 2659, 208}

$$\frac{2\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4}}}{\sqrt{\sqrt{b^2-4ac+}}}\right.}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] -((b*x)/c^2) + (2*(b^2 - a*c - b^3/Sqrt[b^2 - 4*a*c] + (3*a*b*c)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]) + (2*(b^2 - a*c + b^3/Sqrt[b^2 - 4*a*c] - (3*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]) + Sinh[x]/c
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \left(-\frac{b}{c^2}+\frac{\cosh(x)}{c}+\frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{c^2\left(a+b\cosh(x)+c\cosh^2(x)\right)}\right)dx$$

$$= -\frac{bx}{c^2}+\frac{\int \frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx}{c^2}+\frac{\int \cosh(x)\,dx}{c}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c^2}+\frac{\left(b^2-ac-\right.}{}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4\right.}}\right.}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(b^2-ac+\right.}{c^2\sqrt{}}$$

Mathematica [A] time = 0.766211, size = 309, normalized size = 1.03

$$\frac{-\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-bx+c}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (-(b*x) - (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*
(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a +
c) - b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*
c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2
])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*
Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]) + c*Sinh[x])/c^2
```

Maple [B] time = 0.053, size = 2530, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] -1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh
(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b+1/c^2/(-4*a*c+b^2)^
(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan
h(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b^2-2/c^2*a/(-4*a*c+
b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c
)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3-1/c^2/(-4*a*c+b
^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*t
anh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b^2+2/c^2*a/(-4*a*
c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c
)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3-a/(a-b+c)/(((-4
*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2))-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^
(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)
)-5/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*a
rctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b+2
/c*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*ar
ctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+5/c/
(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh
((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b-2/c*a
/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan
h((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-2/(-4*
a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a
+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2+2/c/(-4*a*c
+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)
*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^3-2/c/(-4*a*c+b^2)
^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^3-1/c/(tanh(1/2*x)+1)
-1/c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*
arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^4+1/
c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arc
tanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^4+2/(
-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((
a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2-1/c^2/(a-b
+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3-1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/
2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-
c)*(a-b+c))^(1/2))*b^3+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)
*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2
-1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b+1/c/(a-b+c)/(((-4
```

```
*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2-1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-
b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(
1/2))*a^2-1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+
b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2+1/c/(-4*a*c+
b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3-1/c/(tanh(1/2*x)-
1)-b/c^2*ln(tanh(1/2*x)+1)+2/c^2*a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c
))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2
))*b^2-3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1
/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b+
3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arc
tanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b-1/c/(
-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh(
(-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3+b/c^2*ln
(tanh(1/2*x)-1)+2/c^2*a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*ar
ctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,bxe^{x}-ce^{(2\,x)}+c\right)e^{(-x)}}{2\,c^{2}}-\frac{1}{8}\int -\frac{16\left(2\,abe^{(2\,x)}+\left(b^{2}-ac\right)e^{(3\,x)}+\left(b^{2}-ac\right)e^{x}\right)}{c^{3}e^{(4\,x)}+2\,bc^{2}e^{(3\,x)}+2\,bc^{2}e^{x}+c^{3}+2\left(2\,ac^{2}+c^{3}\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) + c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
*x) + (b^2 - a*c)*e^(3*x) + (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 + 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 + c^3)*e^(2*x)), x)
```

Fricas [B] time = 7.91034, size = 13678, normalized size = 45.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

[Out] -1/2*(2*b*x*cosh(x) - c*cosh(x)^2 + sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqr
t(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*
a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^
2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b
^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*
b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 -
 a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a
*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6
 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^5*b^4 + 2*a^3*b
^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 + (2
0*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*
a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c - (12*a^
2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*
a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-
(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3
+ 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*
a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2
 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^
2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(
2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^
10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b
^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16
*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c
^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))
)/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*
c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)
cosh(x) + 4(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)
*c)*sinh(x) - 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*
c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^
2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b
^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c
^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5
- 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) - sqrt(2)*(
c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^
2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(
2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^
10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b
^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16
*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c
^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))
)/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*
c^4))*log(-2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*
c - sqrt(2)*(12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4
*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5

- 2*a^2*b^7 + b^9)*c - (12*a^2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b
 - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3
- 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 +
 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2
 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 1
2*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3
*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b
^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a
*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*s
qrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)
c^3 + 2(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6
 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 +
2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^
4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3
*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3
)*c^2 - (a^5*b^3 - a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*
b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*sinh(x) - 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2
)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6
 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a
^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c
)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11
*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2
*b^4 + b^6)*c^8))) + sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 - b
^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a
*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*s
qrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)
c^3 + 2(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6
 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 +
2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^
4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3
*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*
c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 + (20*a^5*b - 31*a^
3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3
*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c + (12*a^2*b*c^9 + 7*(4*
a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*
b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^
2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2
- 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*
c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 +
b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*
(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2
)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*
c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6
)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^1

2 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*sinh(x) + 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) - sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c - sqrt(2)*(12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c + (12*a^2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*sinh(x) + 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) - c*sinh(x)^2 + 2*(b*x - c*cosh(x))*sinh(x) + c)/(c^2*cosh(x) + c^2*sin

```
h(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.14931, size = 32, normalized size = 0.11

$$-\frac{bx}{c^2} - \frac{e^{(-x)}}{2c} + \frac{e^x}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/c^2 - 1/2*e^(-x)/c + 1/2*e^x/c
```

3.836 $\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rubi [A] time = 0.086682, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3289, 2754, 8}

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rule 3289

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = \left(4a^2\right)\int \frac{a+b\cosh(x)}{\left(2ab+2a^2\cosh(x)\right)^2}\,dx$$
$$= \frac{\sinh(x)}{b+a\cosh(x)} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= \frac{\sinh(x)}{b+a\cosh(x)}$$

Mathematica [A] time = 0.0577431, size = 11, normalized size = 1.

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Maple [B] time = 0.026, size = 29, normalized size = 2.6

$$2\,\frac{\tanh(x/2)}{a\,(\tanh(x/2))^2-(\tanh(x/2))^2\,b+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x)
```

```
[Out] 2*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53732, size = 159, normalized size = 14.45

$$-\frac{2\left(b\cosh\left(x\right)+b\sinh\left(x\right)+a\right)}{a^{2}\cosh\left(x\right)^{2}+a^{2}\sinh\left(x\right)^{2}+2\,ab\cosh\left(x\right)+a^{2}+2\left(a^{2}\cosh\left(x\right)+ab\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b**2+2*a*b*cosh(x)+a**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13539, size = 35, normalized size = 3.18

$$-\frac{2\left(be^{x}+a\right)}{\left(ae^{\left(2\,x\right)}+2\,be^{x}+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

3.837 $\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 -
 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.682112, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3293, 2659, 208}

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 -
 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
```

```
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{b-\sqrt{b^2-4}}$$

$$=\left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx,x,\text{t}\right.$$

$$=\frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4a}}}{\sqrt{b+2c+\sqrt{}}}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{}}}$$

Mathematica [A] time = 0.445301, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.039, size = 2556, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arcta
n((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*d-2*c/(-4
*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-
b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e*a+c/(-4*a*c+b^
2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+2*c/(-4*a*c+b^2)^(1
/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e*a-a/(-4*a*c+b^2)^(1/2)/(
a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x
)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*d-3*a/(-4*a*c+b^2)^(1/2)/(a-b
+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(
((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)
/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d-c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a
*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)
^(1/2)-a+c)*(a-b+c))^(1/2))*c*d-3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^
2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2))*c*d+3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^
(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a
+c)*(a-b+c))^(1/2))*b*e-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)
-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(
a-b+c))^(1/2))*c*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arcta
nh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d+a/(a-b+
c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-
4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)
```

```
*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b
+c))^(1/2))*d+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-
b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-a/(a-b+c)/(((-
4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c
))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2))*e-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-c/(a-b+c)/(((-4*a*c
+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(
1/2)-a+c)*(a-b+c))^(1/2))*d-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*
d+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-2/(-4*a*c+b^2)^(1/2)/(a-
b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/((
(-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e*a^2+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/
(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4
*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e*a^2-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2*d-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*
c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^
(1/2)-a+c)*(a-b+c))^(1/2))*b^2*e+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2
)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/
2)+a-c)*(a-b+c))^(1/2))*b^2*d+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(
1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+
a-c)*(a-b+c))^(1/2))*b^2*e-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2
)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*
(a-b+c))^(1/2))*c^2*d+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c
)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-
b+c))^(1/2))*c^2*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arcta
n((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-b/(a-b+c)
/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*
a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e\cosh(x) + d}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*cosh(x) + d)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

Fricas [B] time = 33.0205, size = 14195, normalized size = 57.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
- 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^
2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^
6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a
^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a
*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4 + 2*a*b^
2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e^2 - 2*(
2*a^2*b + b^3 + 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3
- 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c
^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e
^3 - ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)
*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a
^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2
- (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*
c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)
*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c
+ 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^
5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b
^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*
c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2
+ b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*c^2 + 2*
c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3)
*sinh(x) + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2
*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(
2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*
c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)
```

```
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^
2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*
a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2
+ b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4
+ b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 -
11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 -
 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4 + 2*
a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e^2 -
 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*
b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8
*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*
c)*e^3 - ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*
b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d
- (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*
c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 +
2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*
b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*
a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4 - (
b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*
a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*c^2
+ 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c)*d*
e^3)*sinh(x) + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 -
(a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 -
 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b
^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (
2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2
*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*
a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*
b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^
4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b
^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4
```

+ 2*a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e
^2 - 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3
*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c
 - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b
^3)*c)*e^3 + ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 -
3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c
)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b
^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*
b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*
e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^
2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 -
3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2
+ 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c
^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4
 - (b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 +
 (2*a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*
c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c
)*d*e^3)*sinh(x) - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^
2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c
^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 -
 a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*
b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*
e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c))) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e
 + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2
*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*
a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(
8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*
d^4 + 2*a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d
^2*e^2 - 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3
 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a
^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b
 - b^3)*c)*e^3 + ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^
3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^
4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5
*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c
)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*

```
a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2
- b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4
*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c
*e^4 - (b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c
^2 + (2*a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c +
2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^
2)*c)*d*e^3)*sinh(x) - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2
)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^
3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a
^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c
)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*
a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)**2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")

[Out] Timed out
```

3.838 $\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{x}{a+b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[b]*(a + b))
```

Rubi [A] time = 0.151323, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {481, 207, 205}

$$\frac{x}{a+b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[b]*(a + b))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)\left(a+bx^2\right)}\,dx,x,\tanh(x)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(x)\right)}{a+b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tanh(x)\right)}{a+b}\\ &= \frac{x}{a+b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}\end{aligned}$$

Mathematica [A] time = 0.0946278, size = 34, normalized size = 0.87

$$\frac{x-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[b])/(a + b)
```

Maple [B] time = 0.045, size = 414, normalized size = 10.6

$$8\,\frac{\ln(\tanh(x/2)+1)}{8\,a+8\,b}-8\,\frac{\ln(\tanh(x/2)-1)}{8\,a+8\,b}+4\,\frac{a^2}{(4\,a+4\,b)\,\sqrt{b\,(a+b)}\sqrt{\left(2\,\sqrt{b\,(a+b)}+a+2\,b\right)a}}\arctan\left(\frac{a\tanh}{\sqrt{\left(2\,\sqrt{b\,(a+b\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x)
```

```
[Out] 8/(8*a+8*b)*ln(tanh(1/2*x)+1)-8/(8*a+8*b)*ln(tanh(1/2*x)-1)+4*a^2/(4*a+4*b)
/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/(
```

```
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+4*a/(4*a+4*b)/((2*(b*(a+b))^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+4*a/(4
*a+4*b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b+4*a^2/(4*a+4*b)/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2))-4*a/(4*a+4*b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+4*a/(4*a+4*b)/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91539, size = 1007, normalized size = 25.82

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(x)^4+4\left(a^2+2\,ab+b^2\right)\cosh(x)\sinh(x)^3+\left(a^2+2\,ab+b^2\right)\sinh(x)^4+2\left(a^2-b^2\right)\cosh(x)^2+2\left(3\left(a^2+2\,ab+b^2\right)\cosh(x)^2+a^2-b^2\right)\sinh}{(a+b)\cosh(x)^4+4\,(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4+2\,(a-b)\cosh(x)^2}\right.}{2\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) -
4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh
(x)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(
x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
 - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))
```

```
 + 2*x)/(a + b), -(sqrt(a/b)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(a/b)/a) - x)/(a + b)]
```

Sympy [A] time = 2.82976, size = 258, normalized size = 6.62

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x - \frac{\sinh(x)}{\cosh(x)}}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} + \frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} - \frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a = -b \\ \frac{2i\sqrt{a}bx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2\sqrt{\frac{1}{b}}} - \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}\cosh(x)+\sinh(x)\right)}{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2\sqrt{\frac{1}{b}}} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}\cosh(x)+\sinh(x)\right)}{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((x - sinh(x)/cosh(x))/a, Eq(b, 0))
, (x/b, Eq(a, 0)), (-x*sinh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) + x*co
sh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(x)
**2 + 2*b*cosh(x)**2), Eq(a, -b)), (2*I*sqrt(a)*b*x*sqrt(1/b)/(2*I*a**(3/2)
*b*sqrt(1/b) + 2*I*sqrt(a)*b**2*sqrt(1/b)) - a*log(-I*sqrt(a)*sqrt(1/b)*cos
h(x) + sinh(x))/(2*I*a**(3/2)*b*sqrt(1/b) + 2*I*sqrt(a)*b**2*sqrt(1/b)) + a
*log(I*sqrt(a)*sqrt(1/b)*cosh(x) + sinh(x))/(2*I*a**(3/2)*b*sqrt(1/b) + 2*I
*sqrt(a)*b**2*sqrt(1/b)), True))
```

Giac [A] time = 1.15839, size = 62, normalized size = 1.59

$$-\frac{a\arctan\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} + \frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)
) + x/(a + b)
```

3.839 $\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a+b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))
```

Rubi [A] time = 0.1121, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {391, 206, 205}

$$\frac{x}{a+b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)}{a+b} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(x)\right)}{a+b}$$
$$= \frac{x}{a+b} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

Mathematica [A] time = 0.0539545, size = 33, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}}+x}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] (x + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[a])/(a + b)
```

Maple [B] time = 0.036, size = 389, normalized size = 10.2

$$2\,\frac{\ln(\tanh(x/2)+1)}{2\,b+2\,a}-2\,\frac{\ln(\tanh(x/2)-1)}{2\,b+2\,a}-\frac{ab}{a+b}\arctan\left(a\tanh\left(\frac{x}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}+a+2\,b\right)a}}\right)\frac{1}{\sqrt{b(a+b)}}\frac{1}{\sqrt{(2\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x)
```

```
[Out] 2/(2*b+2*a)*ln(tanh(1/2*x)+1)-2/(2*b+2*a)*ln(tanh(1/2*x)-1)-b/(a+b)*a/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2))-b/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-b^2/(a+b)/(b*(a+b)
)^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a+
```

```
b))^(1/2)+a+2*b)*a)^(1/2))-b/(a+b)*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+b/
(a+b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2))-b^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88787, size = 1006, normalized size = 26.47

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(x)^4+4\left(a^2+2\,ab+b^2\right)\cosh(x)\sinh(x)^3+\left(a^2+2\,ab+b^2\right)\sinh(x)^4+2\left(a^2-b^2\right)\cosh(x)^2+2\left(3\left(a^2+2\,ab+b^2\right)\cosh(x)^2+a^2-b^2\right)\sinh}{(a+b)\cosh(x)^4+4\,(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4+2\,(a-b)\cosh(x)^2}\right.}{2\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) +
4*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh
(x)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(
x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
 - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))
 + 2*x)/(a + b), (sqrt(b/a)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(
x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(b/a)/b) + x)/(a + b)]
```

Sympy [A] time = 2.63476, size = 250, normalized size = 6.58

$$\begin{cases} \tilde{\infty}\left(x-\frac{\cosh(x)}{\sinh(x)}\right) & \text{for } a=0 \wedge b=0 \\ \frac{x-\frac{\cosh(x)}{\sinh(x)}}{b} & \text{for } a=0 \\ \frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a=-b \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2i\sqrt{a}x\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}+2i\sqrt{a}b\sqrt{\frac{1}{b}}}+\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}\cosh(x)+\sinh(x)\right)}{2ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}+2i\sqrt{a}b\sqrt{\frac{1}{b}}}-\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}\cosh(x)+\sinh(x)\right)}{2ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}+2i\sqrt{a}b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

```
[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/s
inh(x))/b, Eq(a, 0)), (x*sinh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - x*
cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(
x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), (x/a, Eq(b, 0)), (2*I*sqrt(a)*x*sqrt(1
/b)/(2*I*a**(3/2)*sqrt(1/b) + 2*I*sqrt(a)*b*sqrt(1/b)) + log(-I*sqrt(a)*sqr
t(1/b)*cosh(x) + sinh(x))/(2*I*a**(3/2)*sqrt(1/b) + 2*I*sqrt(a)*b*sqrt(1/b)
) - log(I*sqrt(a)*sqrt(1/b)*cosh(x) + sinh(x))/(2*I*a**(3/2)*sqrt(1/b) + 2*
I*sqrt(a)*b*sqrt(1/b)), True))
```

Giac [A] time = 1.1596, size = 61, normalized size = 1.61

$$\frac{b\arctan\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b))
 + x/(a + b)
```

3.840 $\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{2}+\frac{1}{6(\tanh(x)+1)}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x/2 + (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + 1/(6*(1 + Tanh[x]))
```

Rubi [A] time = 0.153718, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2074, 207, 618, 204}

$$\frac{x}{2}+\frac{1}{6(\tanh(x)+1)}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 + (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + 1/(6*(1 + Tanh[x]))
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{x^3}{-1+x^2-x^3+x^5}\,dx, x, \tanh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(\frac{1}{6(1+x)^2}+\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)dx, x, \tanh(x)\right)\\ &= \frac{1}{6(1+\tanh(x))}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tan\right.\\ &= \frac{x}{2}+\frac{1}{6(1+\tanh(x))}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\ &= \frac{x}{2}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.131348, size = 40, normalized size = 1.05

$$\frac{1}{36}\left(18x-3\sinh(2x)+3\cosh(2x)-8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (18*x - 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] + 3*Cosh[2*x] - 3*Sinh[2
*x])/36
```

Maple [C] time = 0.071, size = 96, normalized size = 2.5

$$\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{i}{9}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)-\frac{i}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] 1/3/(tanh(1/2*x)+1)^2-1/3/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)+1/9*I*3^(1/
2)*ln(tanh(1/2*x)^2+(I*3^(1/2)-1)*tanh(1/2*x)+1)-1/9*I*3^(1/2)*ln(tanh(1/2*
x)^2+(-I*3^(1/2)-1)*tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.63195, size = 99, normalized size = 2.61

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x+\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) + 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqr
t(2))) + 1/2*x + 1/12*e^(-2*x)
```

Fricas [B] time = 1.7515, size = 340, normalized size = 8.95

$$\frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2+8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2\right)\mathrm{arcta}}{36\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 + 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 2.42531, size = 136, normalized size = 3.58

$$\frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)}-\frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)}-\frac{\sqrt{3}}{3}\right)}{18\sinh(x)+18\cosh(x)}-\frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{2\sqrt{3}}{3\,\mathrm{co}}\right.}{18\sinh(x)+18\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) - 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(18*
sinh(x) + 18*cosh(x)) - 4*sqrt(3)*cosh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)
) - sqrt(3)/3)/(18*sinh(x) + 18*cosh(x)) + 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

Giac [A] time = 1.16556, size = 45, normalized size = 1.18

$$-\frac{1}{12}\left(3e^{(2x)}-1\right)e^{(-2x)}-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) - 1)*e^(-2*x) - 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

3.841 $\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)} dx$

Optimal. Leaf size=38

$$\frac{x}{2}-\frac{1}{6(\tanh(x)+1)}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rubi [A] time = 0.0924765, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2058, 207, 618, 204}

$$\frac{x}{2}-\frac{1}{6(\tanh(x)+1)}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2+x^3-x^5}\,dx,x,\tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)dx,x,\tanh(x)\right)\\ &= -\frac{1}{6(1+\tanh(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\tanh(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\text{ta}\right.\\ &= \frac{x}{2}-\frac{1}{6(1+\tanh(x))}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\tanh(x)\right)\\ &= \frac{x}{2}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.0956132, size = 40, normalized size = 1.05

$$\frac{1}{36}\left(18x+3\sinh(2x)-3\cosh(2x)+8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (18*x + 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2
*x])/36
```

Maple [C] time = 0.062, size = 96, normalized size = 2.5

$$-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{i}{9}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] -1/3/(tanh(1/2*x)+1)^2+1/3/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)+1/9*I*3^(1
/2)*ln(tanh(1/2*x)^2+(-I*3^(1/2)-1)*tanh(1/2*x)+1)-1/9*I*3^(1/2)*ln(tanh(1/
2*x)^2+(I*3^(1/2)-1)*tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.63032, size = 99, normalized size = 2.61

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/2*x - 1/12*e^(-2*x)
```

Fricas [B] time = 2.03123, size = 340, normalized size = 8.95

$$\frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2-8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2\right)\text{arcta}}{36\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 2.50059, size = 136, normalized size = 3.58

$$\frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)}+\frac{4\sqrt{3}\sinh(x)\,\text{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)}-\frac{\sqrt{3}}{3}\right)}{18\sinh(x)+18\cosh(x)}+\frac{4\sqrt{3}\cosh(x)\,\text{atan}\left(\frac{2\sqrt{3}}{3\text{c}}\right.}{18\sinh(x)+18\text{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) + 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(18*
sinh(x) + 18*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)
) - sqrt(3)/3)/(18*sinh(x) + 18*cosh(x)) - 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

Giac [A] time = 1.11437, size = 45, normalized size = 1.18

$$-\frac{1}{12}\left(3\,e^{(2\,x)}+1\right)e^{(-2\,x)}+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}e^{(2\,x)}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) + 1)*e^(-2*x) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

3.842 $\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\operatorname{sech}(x)\operatorname{PolyLog}(2,-e^x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{\operatorname{sech}(x)\operatorname{PolyLog}(2,e^x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{2x\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

```
[Out] (-2*x*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (PolyLog[2, -E^x]*Sech[x])/
Sqrt[a*Sech[x]^2] + (PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.701356, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 4182, 2279, 2391}

$$-\frac{\operatorname{sech}(x)\operatorname{PolyLog}(2,-e^x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{\operatorname{sech}(x)\operatorname{PolyLog}(2,e^x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{2x\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (PolyLog[2, -E^x]*Sech[x])/
Sqrt[a*Sech[x]^2] + (PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx &= \frac{\operatorname{sech}(x)\int x\operatorname{csch}(x)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\ &= -\frac{2x\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{\operatorname{sech}(x)\int \log(1-e^x)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{\operatorname{sech}(x)\int \log(1+e^x)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\ &= -\frac{2x\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{\operatorname{sech}(x)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^x\right)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{\operatorname{sech}(x)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,\right.}{\sqrt{a\operatorname{sech}^2(x)}}\\ &= -\frac{2x\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{\operatorname{Li}_2(-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{\operatorname{Li}_2(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0542851, size = 55, normalized size = 0.93

$$\frac{\operatorname{sech}(x)\left(\operatorname{PolyLog}(2,-e^{-x})-\operatorname{PolyLog}(2,e^{-x})+x\left(\log(1-e^{-x})-\log(e^{-x}+1)\right)\right)}{\sqrt{a\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^(-x)] - PolyLog[2,
E^(-x)])*Sech[x])/Sqrt[a*Sech[x]^2]
```

Maple [B] time = 0.079, size = 136, normalized size = 2.3

$$-\frac{x\mathrm{e}^{x}\ln\left(\mathrm{e}^{x}+1\right)}{\mathrm{e}^{2x}+1}\frac{1}{\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}-\frac{\mathrm{e}^{x}\mathrm{polylog}\left(2,-\mathrm{e}^{x}\right)}{\mathrm{e}^{2x}+1}\frac{1}{\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}+\frac{x\mathrm{e}^{x}\ln\left(1-\mathrm{e}^{x}\right)}{\mathrm{e}^{2x}+1}\frac{1}{\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}+\frac{\mathrm{e}^{x}\mathrm{polylog}\left(2,\mathrm{e}^{x}\right)}{\mathrm{e}^{2x}+1}\frac{1}{\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*ln(exp(x)+1)-1/(
a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(2,-exp(x))+1/(
a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*ln(1-exp(x))+1/(a*ex
p(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(2,exp(x))
```

Maxima [A] time = 1.81995, size = 49, normalized size = 0.83

$$-\frac{x\log\left(e^{x}+1\right)+\mathrm{Li}_{2}\left(-e^{x}\right)}{\sqrt{a}}+\frac{x\log\left(-e^{x}+1\right)+\mathrm{Li}_{2}\left(e^{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))/sq
rt(a)
```

Fricas [A] time = 2.09782, size = 285, normalized size = 4.83

$$\frac{\left(\left(e^{(2x)}+1\right)\mathrm{Li}_{2}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-\left(e^{(2x)}+1\right)\mathrm{Li}_{2}\left(-\cosh\left(x\right)-\sinh\left(x\right)\right)-\left(xe^{(2x)}+x\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) + 1)*dilog(cosh(x) + sinh(x)) - (e^(2*x) + 1)*dilog(-cosh(x) - si
nh(x)) - (x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) + (x*e^(2*x) + x)*log(-
```

```
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.843 $\int \frac{x^2\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=104

$$-\frac{2x\text{sech}(x)\text{PolyLog}(2,-e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{2x\text{sech}(x)\text{PolyLog}(2,e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{2\text{sech}(x)\text{PolyLog}(3,-e^x)}{\sqrt{a\text{sech}^2(x)}}-\frac{2\text{sech}(x)\text{PolyLog}(3,e^x)}{\sqrt{a\text{sech}^2(x)}}-$$

```
[Out] (-2*x^2*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*x*PolyLog[2, -E^x]*Sec
h[x])/Sqrt[a*Sech[x]^2] + (2*x*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2] +
 (2*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*PolyLog[3, E^x]*Sech[x
])/Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.813223, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 4182, 2531, 2282, 6589}

$$-\frac{2x\text{sech}(x)\text{PolyLog}(2,-e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{2x\text{sech}(x)\text{PolyLog}(2,e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{2\text{sech}(x)\text{PolyLog}(3,-e^x)}{\sqrt{a\text{sech}^2(x)}}-\frac{2\text{sech}(x)\text{PolyLog}(3,e^x)}{\sqrt{a\text{sech}^2(x)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```


```
[Out] (-2*x^2*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*x*PolyLog[2, -E^x]*Sec
h[x])/Sqrt[a*Sech[x]^2] + (2*x*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2] +
 (2*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*PolyLog[3, E^x]*Sech[x
])/Sqrt[a*Sech[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\int x^2\text{csch}(x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(2\text{sech}(x))\int x\log(1-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int x\log(1+e^x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int \text{Li}_2(-}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\,\text{Subst}\Big(}{\sqrt{asec}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2\text{Li}_3(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] time = 0.0659689, size = 83, normalized size = 0.8

$$\frac{\text{sech}(x)\left(2x\text{PolyLog}(2,-e^{-x}) - 2x\text{PolyLog}(2,e^{-x}) + 2\text{PolyLog}(3,-e^{-x}) - 2\text{PolyLog}(3,e^{-x}) + x^2\log(1-e^{-x}) - x^2\text{lo}\right.}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*PolyLog[2, -E^(-x)] - 2*x
*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyLog[3, E^(-x)])*Sech[x]
)/Sqrt[a*Sech[x]^2]
```

Maple [B] time = 0.069, size = 209, normalized size = 2.

$$-\frac{x^2e^x\ln(e^x+1)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} - 2\frac{xe^x\text{polylog}(2,-e^x)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} + 2\frac{e^x\text{polylog}(3,-e^x)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} + \frac{x^2e^x\ln(1-e^x)}{e^{2x}+1}\frac{1}{\sqrt{\frac{}{(e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^2*ln(exp(x)+1)-2
/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*polylog(2,-exp(x))
+2/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(3,-exp(x))
+1/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^2*ln(1-exp(x))+2
/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*polylog(2,exp(x))-
2/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(3,exp(x))
```

Maxima [A] time = 1.65807, size = 81, normalized size = 0.78

$$-\frac{x^2\log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)}{\sqrt{a}}+\frac{x^2\log\left(-e^x+1\right)+2\,x\mathrm{Li}_2\left(e^x\right)-2\,\mathrm{Li}_3(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))/sqrt(a) + (x^2*l
og(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a)
```

Fricas [C] time = 2.09906, size = 559, normalized size = 5.38

$$-\frac{\left(2\sqrt{\frac{a}{e^{(4x)}+2\,e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}\left(3,\cosh\left(x\right)+\sinh\left(x\right)\right)-2\sqrt{\frac{a}{e^{(4x)}+2\,e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}\left(3,-\cosh\left(x\right)-\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(3, cosh(x)
+ sinh(x)) - 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(
3, -cosh(x) - sinh(x)) - (2*(x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 2*(x
*e^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*e^(2*x) + x^2)*log(cosh(x) +
 sinh(x) + 1) + (x^2*e^(2*x) + x^2)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^
(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.844 $\int \frac{x^3\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=150

$$-\frac{3x^2\text{sech}(x)\text{PolyLog}(2,-e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{3x^2\text{sech}(x)\text{PolyLog}(2,e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{6x\text{sech}(x)\text{PolyLog}(3,-e^x)}{\sqrt{a\text{sech}^2(x)}}-\frac{6x\text{sech}(x)\text{PolyLog}(3,e^x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] (-2*x^3*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (3*x^2*PolyLog[2, -E^x]*S
ech[x])/Sqrt[a*Sech[x]^2] + (3*x^2*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*x*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*x*PolyLog[3, E^x
]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*PolyLog[4, -E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*PolyLog[4, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.833424, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{sech}(x)\text{PolyLog}(2,-e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{3x^2\text{sech}(x)\text{PolyLog}(2,e^x)}{\sqrt{a\text{sech}^2(x)}}+\frac{6x\text{sech}(x)\text{PolyLog}(3,-e^x)}{\sqrt{a\text{sech}^2(x)}}-\frac{6x\text{sech}(x)\text{PolyLog}(3,e^x)}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (3*x^2*PolyLog[2, -E^x]*S
ech[x])/Sqrt[a*Sech[x]^2] + (3*x^2*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*x*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*x*PolyLog[3, E^x
]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*PolyLog[4, -E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*PolyLog[4, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\int x^3\text{csch}(x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(3\text{sech}(x))\int x^2\log(1-e^x)\ dx}{\sqrt{a\text{sech}^2(x)}} + \frac{(3\text{sech}(x))\int x^2\log(1+e^x)\ dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(6\text{sech}(x))\int x\text{Li}_2}{\sqrt{a\text{sech}^2(x}}$$

$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)\,\text{sech}(x}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)\,\text{sech}(x}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)\,\text{sech}(x}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] time = 0.0803852, size = 113, normalized size = 0.75

$$\frac{\text{sech}(x)\left(24x^2\text{PolyLog}(2,-e^{-x}) + 24x^2\text{PolyLog}(2,e^x) + 48x\text{PolyLog}(3,-e^{-x}) - 48x\text{PolyLog}(3,e^x) + 48\text{PolyLog}(4,\right.}{8\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((Pi^4 - 2*x^4 - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[1 - E^x] + 24*x^2*PolyLo
g[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyLog[3, -E^(-x)] - 48*x*Po
lyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog[4, E^x])*Sech[x])/(8*Sq
rt[a*Sech[x]^2])
```

Maple [B] time = 0.072, size = 281, normalized size = 1.9

$$-\frac{x^3e^x\ln(e^x+1)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} - 3\frac{x^2e^x\text{polylog}(2,-e^x)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} + 6\frac{xe^x\text{polylog}(3,-e^x)}{e^{2x}+1}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}} - 6\frac{e^x\text{polylog}(4,-e^x}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^3*ln(exp(x)+1)-3
/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^2*polylog(2,-exp(x
))+6/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*polylog(3,-exp
(x))-6/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(4,-exp
(x))+1/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^3*ln(1-exp(x
))+3/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x^2*polylog(2,ex
p(x))-6/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*x*polylog(3,e
xp(x))+6/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(x)*polylog(4,ex
p(x))
```

Maxima [A] time = 1.81711, size = 108, normalized size = 0.72

$$-\frac{x^3\log\left(e^x+1\right)+3\,x^2\mathrm{Li}_2\left(-e^x\right)-6\,x\mathrm{Li}_3(-e^x)+6\,\mathrm{Li}_4(-e^x)}{\sqrt{a}}+\frac{x^3\log\left(-e^x+1\right)+3\,x^2\mathrm{Li}_2\left(e^x\right)-6\,x\mathrm{Li}_3(e^x)+6\,\mathrm{Li}_4(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4
, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e
^x) + 6*polylog(4, e^x))/sqrt(a)
```

Fricas [C] time = 2.10084, size = 807, normalized size = 5.38

$$\frac{\left(6\,\sqrt{\frac{a}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}\left(e^{(2\,x)}+1\right)e^x\mathrm{polylog}\left(4,\cosh\left(x\right)+\sinh\left(x\right)\right)-6\,\sqrt{\frac{a}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}\left(e^{(2\,x)}+1\right)e^x\mathrm{polylog}\left(4,-\cosh\left(x\right)-\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4, cosh(x) +
 sinh(x)) - 6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4
, -cosh(x) - sinh(x)) - 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))
```

```
*e^x*polylog(3, cosh(x) + sinh(x)) + 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*
e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) + (3*(x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 3*(x^2*e^(2*x) + x^2)*dilog(-cosh(x) - sinh(x)) -
 (x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(x) + 1) + (x^3*e^(2*x) + x^3)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.845 $\int \frac{x\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^4(x)}}\,dx$

Optimal. Leaf size=73

$$\frac{\operatorname{sech}^2(x)\operatorname{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\operatorname{sech}^4(x)}}-\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] -(x^2*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (x*Log[1 - E^(2*x)]*Sech[x]^2)/Sqr
t[a*Sech[x]^4] + (PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.555619, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6720, 3716, 2190, 2279, 2391}

$$\frac{\operatorname{sech}^2(x)\operatorname{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\operatorname{sech}^4(x)}}-\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -(x^2*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (x*Log[1 - E^(2*x)]*Sech[x]^2)/Sqr
t[a*Sech[x]^4] + (PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx &= \frac{\operatorname{sech}^2(x)\int x\coth(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} - \frac{\left(2\operatorname{sech}^2(x)\right)\int \frac{e^{2x}x}{1-e^{2x}}\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\int \log\left(1-e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} + \frac{\operatorname{Li}_2\left(e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.048376, size = 44, normalized size = 0.6

$$\frac{\operatorname{sech}^2(x)\left(x\left(x+2\log\left(1-e^{-2x}\right)\right)-\operatorname{PolyLog}\left(2,e^{-2x}\right)\right)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] ((x*(x + 2*Log[1 - E^(-2*x)]) - PolyLog[2, E^(-2*x)])*Sech[x]^2)/(2*Sqrt[a*
Sech[x]^4])
```

Maple [B] time = 0.077, size = 175, normalized size = 2.4

$$-\frac{e^{2x}x^2}{2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+\frac{xe^{2x}\ln\left(e^x+1\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+\frac{e^{2x}\mathrm{polylog}\left(2,-e^x\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+\frac{xe^{2x}\ln\left(1-e^x\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^2+1/(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x*ln(exp(x)+1)+1/(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*polylog(2,-exp(x))+1/(a
*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x*ln(1-exp(x))+1/(a
*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*polylog(2,exp(x))
```

Maxima [A] time = 1.93777, size = 58, normalized size = 0.79

$$-\frac{x^2}{2\sqrt{a}}+\frac{x\log\left(e^x+1\right)+\mathrm{Li}_2\left(-e^x\right)}{\sqrt{a}}+\frac{x\log\left(-e^x+1\right)+\mathrm{Li}_2\left(e^x\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/sqrt(a) + (x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1
) + dilog(e^x))/sqrt(a)
```

Fricas [B] time = 2.0882, size = 454, normalized size = 6.22

$$-\frac{\left(x^2e^{(4\,x)} + 2\,x^2e^{(2\,x)} + x^2 - 2\left(e^{(4\,x)} + 2\,e^{(2\,x)} + 1\right)\mathrm{Li}_2\left(\cosh\left(x\right) + \sinh\left(x\right)\right) - 2\left(e^{(4\,x)} + 2\,e^{(2\,x)} + 1\right)\mathrm{Li}_2\left(-\cosh\left(x\right) - \sinh\left(x\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2 - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog
(cosh(x) + sinh(x)) - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog(-cosh(x) - sinh(x))
 - 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) - 2*(x*e^(4*x
) + 2*x*e^(2*x) + x)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*
x) + 6*e^(4*x) + 4*e^(2*x) + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)}{\sqrt{a\operatorname{sech}^{4}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)}{\sqrt{a\operatorname{sech}\left(x\right)^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.846 $\int \frac{x^2\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^4(x)}}\,dx$

Optimal. Leaf size=98

$$\frac{x\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}}+\frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -(x^3*Sech[x]^2)/(3*Sqrt[a*Sech[x]^4]) + (x^2*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (x*PolyLog[2, E^(2*x)]*Sech[x]^2)/Sqrt[a*Sech[x]^4] - (P
olyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.608181, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}}+\frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```


```
[Out] -(x^3*Sech[x]^2)/(3*Sqrt[a*Sech[x]^4]) + (x^2*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (x*PolyLog[2, E^(2*x)]*Sech[x]^2)/Sqrt[a*Sech[x]^4] - (P
olyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int x^2\coth(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int x\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{sech}^2(x)\int \text{Li}_2\left(e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{sech}^2(x)\,\text{Subst}\left(\int \frac{\text{Li}_2(x}{x}\right.}{2\sqrt{a\text{sech}^4(x}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

Mathematica [A] time = 0.0485789, size = 57, normalized size = 0.58

$$\frac{\text{sech}^2(x)\left(6x\text{PolyLog}\left(2,e^{2x}\right)-3\text{PolyLog}\left(3,e^{2x}\right)-2x^2\left(x-3\log\left(1-e^{2x}\right)\right)\right)}{6\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] ((-2*x^2*(x - 3*Log[1 - E^(2*x)]) + 6*x*PolyLog[2, E^(2*x)] - 3*PolyLog[3,
E^(2*x)])*Sech[x]^2)/(6*Sqrt[a*Sech[x]^4])
```

Maple [B] time = 0.072, size = 253, normalized size = 2.6

$$-\frac{e^{2x}x^3}{3\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+\frac{e^{2x}x^2\ln\left(e^x+1\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+2\frac{xe^{2x}\text{polylog}\left(2,-e^x\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}-2\frac{e^{2x}\text{polylog}\left(3,-e^x\right)}{\left(e^{2x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/3/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^3+1/(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^2*ln(exp(x)+1)+2/(a*e
xp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x*polylog(2,-exp(x))-
2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*polylog(3,-exp(
x))+1/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^2*ln(1-ex
p(x))+2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x*polylog
(2,exp(x))-2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*poly
log(3,exp(x))
```

Maxima [A] time = 1.98089, size = 90, normalized size = 0.92

$$-\frac{x^3}{3\sqrt{a}}+\frac{x^2\log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)}{\sqrt{a}}+\frac{x^2\log\left(-e^x+1\right)+2\,x\mathrm{Li}_2\left(e^x\right)-2\,\mathrm{Li}_3(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^3/sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x)
)/sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a
)
```

Fricas [C] time = 2.11378, size = 848, normalized size = 8.65

$$-\frac{\left(6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\mathrm{polylog}\left(3,\cosh\left(x\right)+\sinh\left(x\right)\right)+6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*sqrt(a/(e^(8*x)
+ 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x)
*polylog(3, -cosh(x) - sinh(x)) + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3 - 6*(x
```

```
*e^(4*x) + 2*x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 6*(x*e^(4*x) + 2*x*e
^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^
2)*log(cosh(x) + sinh(x) + 1) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x)
 + 1))*e^(2*x))*e^(-2*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^4(x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.847 $\int \frac{x^3\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^4(x)}}\,dx$

Optimal. Leaf size=129

$$\frac{3x^2\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\text{sech}^2(x)\text{PolyLog}\left(4,e^{2x}\right)}{4\sqrt{a\text{sech}^4(x)}}-\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log\left(1-\right.}{\sqrt{a}}$$

```
[Out] -(x^4*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4]) + (x^3*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (3*x^2*PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^
4]) - (3*x*PolyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (3*PolyLo
g[4, E^(2*x)]*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.553772, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6720, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\text{sech}^2(x)\text{PolyLog}\left(4,e^{2x}\right)}{4\sqrt{a\text{sech}^4(x)}}-\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log\left(1-\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -(x^4*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4]) + (x^3*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (3*x^2*PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^
4]) - (3*x*PolyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (3*PolyLo
g[4, E^(2*x)]*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int x^3\coth(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int \frac{e^{2x}x^3}{1-e^{2x}}\,dx}{\sqrt{a\text{sech}^4(x)}}$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\left(3\text{sech}^2(x)\right)\int x^2\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{\left(3\text{sech}^2(x)\right)\int x\text{Li}_2\left(e^2\right.}{\sqrt{a\text{sech}^4(x)}}$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} +$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} +$$

$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} +$$

Mathematica [A] time = 0.0542898, size = 68, normalized size = 0.53

$$-\frac{\text{sech}^2(x)\left(-6x^2\text{PolyLog}\left(2,e^{2x}\right)+6x\text{PolyLog}\left(3,e^{2x}\right)-3\text{PolyLog}\left(4,e^{2x}\right)+x^4-4x^3\log\left(1-e^{2x}\right)\right)}{4\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -((x^4 - 4*x^3*Log[1 - E^(2*x)] - 6*x^2*PolyLog[2, E^(2*x)] + 6*x*PolyLog[3
, E^(2*x)] - 3*PolyLog[4, E^(2*x)])*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4])
```

Maple [B] time = 0.072, size = 329, normalized size = 2.6

$$-\frac{e^{2x}x^4}{4\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+\frac{e^{2x}x^3\ln\left(e^x+1\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}+3\frac{e^{2x}x^2\mathrm{polylog}\left(2,-e^x\right)}{\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}+1\right)^4}}}-6\frac{xe^{2x}\mathrm{polylog}\left(3,-e^x\right)}{\left(e^{2x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/4/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^4+1/(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^3*ln(exp(x)+1)+3/(a*e
xp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^2*polylog(2,-exp(x)
)-6/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x*polylog(3,-
exp(x))+6/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*polylog
(4,-exp(x))+1/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x^3
*ln(1-exp(x))+3/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x
^2*polylog(2,exp(x))-6/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp
(2*x)*x*polylog(3,exp(x))+6/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^
2*exp(2*x)*polylog(4,exp(x))
```

Maxima [A] time = 1.91817, size = 117, normalized size = 0.91

$$-\frac{x^4}{4\sqrt{a}}+\frac{x^3\log\left(e^x+1\right)+3\,x^2\mathrm{Li}_2\left(-e^x\right)-6\,x\mathrm{Li}_3(-e^x)+6\,\mathrm{Li}_4(-e^x)}{\sqrt{a}}+\frac{x^3\log\left(-e^x+1\right)+3\,x^2\mathrm{Li}_2\left(e^x\right)-6\,x\mathrm{Li}_3(e^x)+6\,\mathrm{Li}_4}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^4/sqrt(a) + (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -
e^x) + 6*polylog(4, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x)
- 6*x*polylog(3, e^x) + 6*polylog(4, e^x))/sqrt(a)
```

Fricas [C] time = 2.17801, size = 1223, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(24*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(4, cosh(x) + sinh(x)) + 24*sqrt(a/(e^(8*x)
 + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x
)*polylog(4, -cosh(x) - sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/
(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(
x) + sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (x
^4*e^(4*x) + 2*x^4*e^(2*x) + x^4 - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*dilog(-cos
h(x) - sinh(x)) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(
x) + 1) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(-cosh(x) - sinh(x) + 1)
)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x))*e^(-2*
x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.848 $\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx$

Optimal. Leaf size=88

$$-\cosh(x)\mathrm{PolyLog}\left(2,-e^x\right)\sqrt{a\mathrm{sech}^2(x)}+\cosh(x)\mathrm{PolyLog}\left(2,e^x\right)\sqrt{a\mathrm{sech}^2(x)}+x\sqrt{a\mathrm{sech}^2(x)}-2x\cosh(x)\tanh^{-1}\left(e^x\right.$$

```
[Out] x*Sqrt[a*Sech[x]^2] - ArcTan[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*ArcTa
nh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - Cosh[x]*PolyLog[2, -E^x]*Sqrt[a*Sech[x]
^2] + Cosh[x]*PolyLog[2, E^x]*Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.359617, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6720, 2622, 321, 207, 5462, 6271, 4182, 2279, 2391, 3770}

$$-\cosh(x)\mathrm{PolyLog}\left(2,-e^x\right)\sqrt{a\mathrm{sech}^2(x)}+\cosh(x)\mathrm{PolyLog}\left(2,e^x\right)\sqrt{a\mathrm{sech}^2(x)}+x\sqrt{a\mathrm{sech}^2(x)}-2x\cosh(x)\tanh^{-1}\left(e^x\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```


```
[Out] x*Sqrt[a*Sech[x]^2] - ArcTan[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*ArcTa
nh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - Cosh[x]*PolyLog[2, -E^x]*Sqrt[a*Sech[x]
^2] + Cosh[x]*PolyLog[2, E^x]*Sqrt[a*Sech[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{csch}(x)\text{sech}^2(x)\,dx$$
$$= x\sqrt{a\text{sech}^2(x)} - x\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)$$
$$= x\sqrt{a\text{sech}^2(x)} - x\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh(x)\sqrt{as}$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh(x)\sqrt{as}$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh(x)\sqrt{as}$$

Mathematica [A] time = 0.056472, size = 74, normalized size = 0.84

$$\sqrt{a\text{sech}^2(x)}\left(\cosh(x)\textbf{PolyLog}(2,-e^{-x}) - \cosh(x)\textbf{PolyLog}(2,e^{-x}) + x + x\log(1-e^{-x})\cosh(x) - x\log(e^{-x}+1)\cosh(x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x - 2*ArcTan[Tanh[x/2]]*Cosh[x] + x*Cosh[x]*Log[1 - E^(-x)] - x*Cosh[x]*Lo
g[1 + E^(-x)] + Cosh[x]*PolyLog[2, -E^(-x)] - Cosh[x]*PolyLog[2, E^(-x)])*S
qrt[a*Sech[x]^2]
```

Maple [A] time = 0.092, size = 150, normalized size = 1.7

$$2\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}x - 2\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}e^{-x}(e^{2x}+1)\arctan(e^x) - \sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}e^{-x}(e^{2x}+1)\,\text{dilog}(e^x+1) - \sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}e^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*x-2*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*e
xp(-x)*(exp(2*x)+1)*arctan(exp(x))-(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*exp(-x
)*(exp(2*x)+1)*dilog(exp(x)+1)-(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(e
xp(2*x)+1)*x*ln(exp(x)+1)-(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(exp(2*
x)+1)*dilog(exp(x))
```

Maxima [A] time = 1.78432, size = 81, normalized size = 0.92

$$-(x\log(e^x+1)+\mathrm{Li}_2(-e^x))\sqrt{a}+(x\log(-e^x+1)+\mathrm{Li}_2(e^x))\sqrt{a}-2\sqrt{a}\arctan(e^x)+\frac{2\sqrt{a}xe^x}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x*log(e^x + 1) + dilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sq
rt(a) - 2*sqrt(a)*arctan(e^x) + 2*sqrt(a)*x*e^x/(e^(2*x) + 1)
```

Fricas [B] time = 2.12159, size = 1173, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)*e^(2*x) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*arctan(cosh(x) +
sinh(x)) + 2*x*cosh(x) + ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + s
inh(x)) - ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) +
2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x) - sinh(x)) - (x*c
osh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cos
h(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(cosh(x) + sinh(x) + 1) + (x*cosh
(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x
```

```
)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(-cosh(x) - sinh(x) + 1) + 2*(x*e^(2
*x) + x)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(2*cosh(x)*e^x*sinh
(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\,\text{sech}^2\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\text{sech}\,(x)^2}x\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x*csch(x)*sech(x), x)
```

3.849 $\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}^2(x)} \, dx$

Optimal. Leaf size=187

$$-2x \cosh(x) \text{PolyLog}(2, -e^x) \sqrt{a \text{sech}^2(x)} + 2x \cosh(x) \text{PolyLog}(2, e^x) \sqrt{a \text{sech}^2(x)} + 2i \cosh(x) \text{PolyLog}(2, -ie^x) \sqrt{a\text{se}}$$

```
[Out] x^2*Sqrt[a*Sech[x]^2] - 4*x*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^2*A
rcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*Cosh[x]*PolyLog[2, -E^x]*Sqrt[a
*Sech[x]^2] + (2*I)*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] - (2*I)*
Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 2*x*Cosh[x]*PolyLog[2, E^x]*S
qrt[a*Sech[x]^2] + 2*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2] - 2*Cosh[x]
*PolyLog[3, E^x]*Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.512386, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6720, 2622, 321, 207, 5462, 14, 6273, 4182, 2531, 2282, 6589, 4180, 2279, 2391}

$$-2x \cosh(x) \text{PolyLog}(2, -e^x) \sqrt{a \text{sech}^2(x)} + 2x \cosh(x) \text{PolyLog}(2, e^x) \sqrt{a \text{sech}^2(x)} + 2i \cosh(x) \text{PolyLog}(2, -ie^x) \sqrt{a\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sech[x]^2] - 4*x*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^2*A
rcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*Cosh[x]*PolyLog[2, -E^x]*Sqrt[a
*Sech[x]^2] + (2*I)*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] - (2*I)*
Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 2*x*Cosh[x]*PolyLog[2, E^x]*S
qrt[a*Sech[x]^2] + 2*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2] - 2*Cosh[x]
*PolyLog[3, E^x]*Sqrt[a*Sech[x]^2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{csch}(x)\text{sech}^2(x)\,dx$$
$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(2\cosh(x)\sqrt{a\text{sech}^2(}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(2\cosh(x)\sqrt{a\text{sech}^2(}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \Big(2\cosh(x)\sqrt{a\text{sech}^2(}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(2i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int$$
$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cosh(x)\sqrt{a}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cosh(x)\sqrt{a}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cosh(x)\sqrt{a}$$
$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cosh(x)\sqrt{a}$$

Mathematica [A] time = 0.186668, size = 154, normalized size = 0.82

$$\sqrt{a\text{sech}^2(x)}\left(2x\cosh(x)\left(\text{PolyLog}\left(2,-e^{-x}\right)-\text{PolyLog}\left(2,e^{-x}\right)\right)+2i\cosh(x)\left(\text{PolyLog}\left(2,-ie^{-x}\right)-\text{PolyLog}\left(2,ie^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x^2 + (2*I)*x*Cosh[x]*(Log[1 - I/E^x] - Log[1 + I/E^x]) + x^2*Cosh[x]*(Log
[1 - E^(-x)] - Log[1 + E^(-x)]) + (2*I)*Cosh[x]*(PolyLog[2, (-I)/E^x] - Pol
yLog[2, I/E^x]) + 2*x*Cosh[x]*(PolyLog[2, -E^(-x)] - PolyLog[2, E^(-x)]) +
2*Cosh[x]*(PolyLog[3, -E^(-x)] - PolyLog[3, E^(-x)]))*Sqrt[a*Sech[x]^2]
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a (\operatorname{sech}(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{a}x^2e^x}{e^{(2x)}+1} - \left(x^2\log\left(e^x+1\right) + 2x\mathrm{Li}_2\left(-e^x\right) - 2\,\mathrm{Li}_3(-e^x)\right)\sqrt{a} + \left(x^2\log\left(-e^x+1\right) + 2x\mathrm{Li}_2\left(e^x\right) - 2\,\mathrm{Li}_3(e^x)\right)\sqrt{a} - 4\sqrt{a}\int \frac{x}{e^{(2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*x^2*e^x/(e^(2*x) + 1) - (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*p
olylog(3, -e^x))*sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(
3, e^x))*sqrt(a) - 4*sqrt(a)*integrate(x*e^x/(e^(2*x) + 1), x)
```

Fricas [C] time = 2.46694, size = 2506, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*
polylog(3, cosh(x) + sinh(x)) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (c
osh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(
e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) - (2*x^2*cosh(
x)*e^(2*x) + 2*x^2*cosh(x) + 2*(x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (
x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*d
```

```
ilog(cosh(x) + sinh(x)) + ((-2*I*e^(2*x) - 2*I)*sinh(x)^2 - 2*I*cosh(x)^2 +
 (-2*I*cosh(x)^2 - 2*I)*e^(2*x) + (-4*I*cosh(x)*e^(2*x) - 4*I*cosh(x))*sinh
(x) - 2*I)*dilog(I*cosh(x) + I*sinh(x)) + ((2*I*e^(2*x) + 2*I)*sinh(x)^2 +
2*I*cosh(x)^2 + (2*I*cosh(x)^2 + 2*I)*e^(2*x) + (4*I*cosh(x)*e^(2*x) + 4*I*
cosh(x))*sinh(x) + 2*I)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^2 + (x
*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x)
+ x*cosh(x))*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*cosh(x)^2 + (x^2
*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*co
sh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + (2*I*x*c
osh(x)^2 + (2*I*x*e^(2*x) + 2*I*x)*sinh(x)^2 + (2*I*x*cosh(x)^2 + 2*I*x)*e^
(2*x) + (4*I*x*cosh(x)*e^(2*x) + 4*I*x*cosh(x))*sinh(x) + 2*I*x)*log(I*cosh
(x) + I*sinh(x) + 1) + (-2*I*x*cosh(x)^2 + (-2*I*x*e^(2*x) - 2*I*x)*sinh(x)
^2 + (-2*I*x*cosh(x)^2 - 2*I*x)*e^(2*x) + (-4*I*x*cosh(x)*e^(2*x) - 4*I*x*c
osh(x))*sinh(x) - 2*I*x)*log(-I*cosh(x) - I*sinh(x) + 1) + (x^2*cosh(x)^2 +
 (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x
^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*
(x^2*e^(2*x) + x^2)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)/(2*cosh
(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a\,\text{sech}^2\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\text{sech}\,(x)^2}x^2\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x^2*csch(x)*sech(x), x)
```

3.850 $\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=287

$$-3x^2\cosh(x)\text{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}+3x^2\cosh(x)\text{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)}+6ix\cosh(x)\text{PolyLog}\left(2,-ie^x\right)\sqrt{}$$

```
[Out] x^3*Sqrt[a*Sech[x]^2] - 6*x^2*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^3
*ArcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 3*x^2*Cosh[x]*PolyLog[2, -E^x]*Sq
rt[a*Sech[x]^2] + (6*I)*x*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] -
(6*I)*x*Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 3*x^2*Cosh[x]*PolyLog
[2, E^x]*Sqrt[a*Sech[x]^2] + 6*x*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2]
 - (6*I)*Cosh[x]*PolyLog[3, (-I)*E^x]*Sqrt[a*Sech[x]^2] + (6*I)*Cosh[x]*Pol
yLog[3, I*E^x]*Sqrt[a*Sech[x]^2] - 6*x*Cosh[x]*PolyLog[3, E^x]*Sqrt[a*Sech[
x]^2] - 6*Cosh[x]*PolyLog[4, -E^x]*Sqrt[a*Sech[x]^2] + 6*Cosh[x]*PolyLog[4,
 E^x]*Sqrt[a*Sech[x]^2]
```

Rubi [A] time = 0.673415, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2622, 321, 207, 5462, 14, 6273, 4182, 2531, 6609, 2282, 6589, 4180}

$$-3x^2\cosh(x)\text{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}+3x^2\cosh(x)\text{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)}+6ix\cosh(x)\text{PolyLog}\left(2,-ie^x\right)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```


```
[Out] x^3*Sqrt[a*Sech[x]^2] - 6*x^2*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^3
*ArcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 3*x^2*Cosh[x]*PolyLog[2, -E^x]*Sq
rt[a*Sech[x]^2] + (6*I)*x*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] -
(6*I)*x*Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 3*x^2*Cosh[x]*PolyLog
[2, E^x]*Sqrt[a*Sech[x]^2] + 6*x*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2]
 - (6*I)*Cosh[x]*PolyLog[3, (-I)*E^x]*Sqrt[a*Sech[x]^2] + (6*I)*Cosh[x]*Pol
yLog[3, I*E^x]*Sqrt[a*Sech[x]^2] - 6*x*Cosh[x]*PolyLog[3, E^x]*Sqrt[a*Sech[
x]^2] - 6*Cosh[x]*PolyLog[4, -E^x]*Sqrt[a*Sech[x]^2] + 6*Cosh[x]*PolyLog[4,
 E^x]*Sqrt[a*Sech[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
```

```
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^3\text{csch}(x)\text{sech}^2(x)\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(3\cosh(x)\sqrt{a\text{sech}^2(\right.$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(3\cosh(x)\sqrt{a\text{sech}^2(\right.$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(3\cosh(x)\sqrt{a\text{sech}^2(\right.$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(6i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\cosh(x)\sqrt{}$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\cosh(x)\sqrt{}$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\cosh(x)\sqrt{}$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\cosh(x)\sqrt{}$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\cosh(x)\sqrt{}$$

Mathematica [A] time = 0.631798, size = 249, normalized size = 0.87

$$\frac{1}{8}\sqrt{a\text{sech}^2(x)}\left(24x^2\cosh(x)\text{PolyLog}(2,-e^{-x}) + 24x^2\cosh(x)\text{PolyLog}(2,e^x) + 48ix\cosh(x)\text{PolyLog}(2,-ie^{-x}) - 48ix\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((8*x^3 + Pi^4*Cosh[x] - 2*x^4*Cosh[x] + (24*I)*x^2*Cosh[x]*Log[1 - I/E^x]
- (24*I)*x^2*Cosh[x]*Log[1 + I/E^x] - 8*x^3*Cosh[x]*Log[1 + E^(-x)] + 8*x^3
*Cosh[x]*Log[1 - E^x] + 24*x^2*Cosh[x]*PolyLog[2, -E^(-x)] + (48*I)*x*Cosh[
x]*PolyLog[2, (-I)/E^x] - (48*I)*x*Cosh[x]*PolyLog[2, I/E^x] + 24*x^2*Cosh[
```

```
x]*PolyLog[2, E^x] + 48*x*Cosh[x]*PolyLog[3, -E^(-x)] + (48*I)*Cosh[x]*Poly
Log[3, (-I)/E^x] - (48*I)*Cosh[x]*PolyLog[3, I/E^x] - 48*x*Cosh[x]*PolyLog[
3, E^x] + 48*Cosh[x]*PolyLog[4, -E^(-x)] + 48*Cosh[x]*PolyLog[4, E^x])*Sqrt
[a*Sech[x]^2])/8
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a(\operatorname{sech}(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{a}x^3e^x}{e^{(2x)}+1} - \left(x^3\log(e^x+1) + 3x^2\mathrm{Li}_2(-e^x) - 6x\mathrm{Li}_3(-e^x) + 6\,\mathrm{Li}_4(-e^x)\right)\sqrt{a} + \left(x^3\log(-e^x+1) + 3x^2\mathrm{Li}_2(e^x) - 6x\mathrm{Li}_3(e^x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*x^3*e^x/(e^(2*x) + 1) - (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6
*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) + (x^3*log(-e^x + 1) + 3*
x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4, e^x))*sqrt(a) - 12*sqrt
(a)*integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Fricas [C] time = 2.51009, size = 3722, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh
(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*p
olylog(4, cosh(x) + sinh(x)) - 6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (co
sh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cosh(x)^
2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^
(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polyl
og(3, cosh(x) + sinh(x)) + ((6*I*e^(2*x) + 6*I)*sinh(x)^2 + 6*I*cosh(x)^2 +
 (6*I*cosh(x)^2 + 6*I)*e^(2*x) + (12*I*cosh(x)*e^(2*x) + 12*I*cosh(x))*sinh
(x) + 6*I)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, I*cosh(x) + I*s
inh(x)) + ((-6*I*e^(2*x) - 6*I)*sinh(x)^2 - 6*I*cosh(x)^2 + (-6*I*cosh(x)^2
 - 6*I)*e^(2*x) + (-12*I*cosh(x)*e^(2*x) - 12*I*cosh(x))*sinh(x) - 6*I)*sqr
t(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -I*cosh(x) - I*sinh(x)) + 6*(
x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))
*e^x*polylog(3, -cosh(x) - sinh(x)) + (2*x^3*cosh(x)*e^(2*x) + 2*x^3*cosh(x
) + 3*(x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2
 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog(cosh
(x) + sinh(x)) + (-6*I*x*cosh(x)^2 + (-6*I*x*e^(2*x) - 6*I*x)*sinh(x)^2 + (
-6*I*x*cosh(x)^2 - 6*I*x)*e^(2*x) + (-12*I*x*cosh(x)*e^(2*x) - 12*I*x*cosh(
x))*sinh(x) - 6*I*x)*dilog(I*cosh(x) + I*sinh(x)) + (6*I*x*cosh(x)^2 + (6*I
*x*e^(2*x) + 6*I*x)*sinh(x)^2 + (6*I*x*cosh(x)^2 + 6*I*x)*e^(2*x) + (12*I*x
*cosh(x)*e^(2*x) + 12*I*x*cosh(x))*sinh(x) + 6*I*x)*dilog(-I*cosh(x) - I*si
nh(x)) - 3*(x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh
(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog
(-cosh(x) - sinh(x)) - (x^3*cosh(x)^2 + x^3 + (x^3*e^(2*x) + x^3)*sinh(x)^2
 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*si
nh(x))*log(cosh(x) + sinh(x) + 1) + (3*I*x^2*cosh(x)^2 + (3*I*x^2*e^(2*x) +
 3*I*x^2)*sinh(x)^2 + 3*I*x^2 + (3*I*x^2*cosh(x)^2 + 3*I*x^2)*e^(2*x) + (6*
I*x^2*cosh(x)*e^(2*x) + 6*I*x^2*cosh(x))*sinh(x))*log(I*cosh(x) + I*sinh(x)
 + 1) + (-3*I*x^2*cosh(x)^2 + (-3*I*x^2*e^(2*x) - 3*I*x^2)*sinh(x)^2 - 3*I*
x^2 + (-3*I*x^2*cosh(x)^2 - 3*I*x^2)*e^(2*x) + (-6*I*x^2*cosh(x)*e^(2*x) -
6*I*x^2*cosh(x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) + (x^3*cosh(x)^2
+ x^3 + (x^3*e^(2*x) + x^3)*sinh(x)^2 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(
x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2
*(x^3*e^(2*x) + x^3)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)/(2*cos
h(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{a \operatorname{sech}^2(x)} \operatorname{csch}(x) \operatorname{sech}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{sech}(x)^2} x^3 \operatorname{csch}(x) \operatorname{sech}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x^3*csch(x)*sech(x), x)
```

3.851 $\int x\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}x\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}-\frac{1}{2}$$

```
[Out] (x*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*S
ech[x]^4] - (Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (Cosh[x]
^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[x]*Sqrt[a*Sech[x]^4]*Si
nh[x])/2 - (x*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rubi [A] time = 0.407189, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6720, 2620, 14, 5462, 2548, 5461, 4182, 2279, 2391, 3473, 8}

$$-\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}x\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}-\frac{1}{2}x$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*S
ech[x]^4] - (Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (Cosh[x]
^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[x]*Sqrt[a*Sech[x]^4]*Si
nh[x])/2 - (x*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\text{csch}(x)\text{sech}^3(x)\,dx$$

$$= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \left(\cosh^2(x)\sqrt{a\text{se}\right.$$

$$= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\right.$$

$$= -\frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}\right.$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh(x)\sqrt{as$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh(x)\sqrt{as$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh^2(x)\text{Li}_2$$

Mathematica [A] time = 0.220251, size = 71, normalized size = 0.54

$$\frac{1}{2}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(\text{PolyLog}\left(2,-e^{-2x}\right) - \text{PolyLog}\left(2,e^{-2x}\right) + 2x\log\left(1-e^{-2x}\right) - 2x\log\left(e^{-2x}+1\right) - \tanh(x) + x\text{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2*x*Log[1 - E^(-2*x)] - 2*x*Log[1 + E^(-2*x)]
 + PolyLog[2, -E^(-2*x)] - PolyLog[2, E^(-2*x)] + x*Sech[x]^2 - Tanh[x]))/2
```

Maple [B] time = 0.071, size = 252, normalized size = 1.9

$$\sqrt{\frac{\mathrm{e}^{4x}a}{\left(\mathrm{e}^{2x}+1\right)^{4}}}\,\mathrm{e}^{-2x}\left(2x\mathrm{e}^{2x}+\mathrm{e}^{2x}+1\right)+\sqrt{\frac{\mathrm{e}^{4x}a}{\left(\mathrm{e}^{2x}+1\right)^{4}}}\,\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}+1\right)^{2}x\ln\left(\mathrm{e}^{x}+1\right)+\sqrt{\frac{\mathrm{e}^{4x}a}{\left(\mathrm{e}^{2x}+1\right)^{4}}}\,\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}+1\right)^{2}\operatorname{polylog}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(2*x*exp(2*x)+exp(2*x)+1)+(a*ex
p(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*ln(exp(x)+1)+(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(2,-exp(x))+(a*
exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*ln(1-exp(x))+(a*e
xp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(2,exp(x))-(a
*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*ln(exp(2*x)+1)-1
/2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(2,-ex
p(2*x))
```

Maxima [A] time = 1.70466, size = 124, normalized size = 0.94

$$-\frac{1}{2}\left(2x\log\left(e^{(2x)}+1\right)+\mathrm{Li}_2\left(-e^{(2x)}\right)\right)\sqrt{a}+\left(x\log\left(e^{x}+1\right)+\mathrm{Li}_2\left(-e^{x}\right)\right)\sqrt{a}+\left(x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(e^{x}\right)\right)\sqrt{a}+\frac{\left(2\sqrt{a}x+\sqrt{a}\right.}{e^{(4x)}+2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (x*log(e^x + 1) + d
ilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sqrt(a) + ((2*sqrt(a)*
x + sqrt(a))*e^(2*x) + sqrt(a))/(e^(4*x) + 2*e^(2*x) + 1)
```

Fricas [C] time = 2.53856, size = 5553, normalized size = 42.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

[Out] ((2*x + 1)*cosh(x)^2 + ((2*x + 1)*e^(4*x) + 2*(2*x + 1)*e^(2*x) + 2*x + 1)*
sinh(x)^2 + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e
^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(
x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^
2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)
*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + co
sh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + sinh(x)) - ((e^(4*x)
 + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^
(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2
*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*co
sh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)
^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh
(x))*sinh(x) + 1)*dilog(I*cosh(x) + I*sinh(x)) - ((e^(4*x) + 2*e^(2*x) + 1)
sinh(x)^4 + cosh(x)^4 + 4(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*
sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1
)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4
x) + 2(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 +
 cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)
*dilog(-I*cosh(x) - I*sinh(x)) + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cos
h(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3
*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*s
inh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)
^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x
) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x)
 - sinh(x)) + ((2*x + 1)*cosh(x)^2 + 1)*e^(4*x) + 2*((2*x + 1)*cosh(x)^2 + 1
)*e^(2*x) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*c
osh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2
 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e
^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh
(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) +
 x)*log(cosh(x) + sinh(x) + 1) - (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) +
x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh
(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*
(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 +
 x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3
 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x
))*e^(2*x))*sinh(x) + x)*log(I*cosh(x) + I*sinh(x) + 1) - (x*cosh(x)^4 + (x
*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*
e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*co
sh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*
e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) +
2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*log(-I*cosh(x) - I*sinh(x
) + 1) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh

```
(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2
*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(
x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)
^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)
*log(-cosh(x) - sinh(x) + 1) + 2*((2*x + 1)*cosh(x)*e^(4*x) + 2*(2*x + 1)*c
osh(x)*e^(2*x) + (2*x + 1)*cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2
*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*e^(2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\,\mathrm{sech}^4(x)}\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\mathrm{sech}(x)^4}\,x\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x*csch(x)*sech(x), x)
```

3.852 $\int x^2\mathbf{csch}(x)\mathbf{sech}(x)\sqrt{a\mathbf{sech}^4(x)}\,dx$

Optimal. Leaf size=204

$$-x\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+x\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(3,-e^{2x}\right)$$

```
[Out] (x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x^2*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt
[a*Sech[x]^4] + Cosh[x]^2*Log[Cosh[x]]*Sqrt[a*Sech[x]^4] - x*Cosh[x]^2*Poly
Log[2, -E^(2*x)]*Sqrt[a*Sech[x]^4] + x*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a
*Sech[x]^4] + (Cosh[x]^2*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[
x]^2*PolyLog[3, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - x*Cosh[x]*Sqrt[a*Sech[x]^4]
*Sinh[x] - (x^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rubi [A] time = 0.638415, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2620, 14, 5462, 2551, 5461, 4182, 2531, 2282, 6589, 3720, 3475, 30}

$$-x\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+x\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(3,-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x^2*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt
[a*Sech[x]^4] + Cosh[x]^2*Log[Cosh[x]]*Sqrt[a*Sech[x]^4] - x*Cosh[x]^2*Poly
Log[2, -E^(2*x)]*Sqrt[a*Sech[x]^4] + x*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a
*Sech[x]^4] + (Cosh[x]^2*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[
x]^2*PolyLog[3, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - x*Cosh[x]*Sqrt[a*Sech[x]^4]
*Sinh[x] - (x^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\text{csch}(x)\text{sech}^3(x)\,dx$$

$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \left(2\cosh^2(x)\sqrt{\right.$$

$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \left(2\cosh^2(x)\sqrt{\right.$$

$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(\cosh^2(x)\sqrt{as\right.$$

$$= -x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(\cosh^2(x)\sqrt{a\text{sech}^4(\right.$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sec}}$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log$$

Mathematica [C] time = 0.619928, size = 120, normalized size = 0.59

$$\frac{1}{24}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(24x\text{PolyLog}\left(2,-e^{-2x}\right) + 24x\text{PolyLog}\left(2,e^{2x}\right) + 12\text{PolyLog}\left(3,-e^{-2x}\right) - 12\text{PolyLog}\left(3,e^{2x}\right) -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(I*Pi^3 - 16*x^3 - 24*x^2*Log[1 + E^(-2*x)] +
24*x^2*Log[1 - E^(2*x)] + 24*Log[Cosh[x]] + 24*x*PolyLog[2, -E^(-2*x)] + 24
*x*PolyLog[2, E^(2*x)] + 12*PolyLog[3, -E^(-2*x)] - 12*PolyLog[3, E^(2*x)]
+ 12*x^2*Sech[x]^2 - 24*x*Tanh[x]))/24
```

Maple [B] time = 0.083, size = 441, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] 2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*x*(x*exp(2*x)+exp(2*x)+1)-2*(
a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*ln(exp(x))+(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*ln(exp(2*x)+1)+(a*exp(
4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^2*ln(exp(x)+1)+2*(a*e
xp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*polylog(2,-exp(x))
-2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(3,-ex
p(x))+(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^2*ln(1-e
xp(x))+2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*polyl
og(2,exp(x))-2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*p
olylog(3,exp(x))-(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2
*x^2*ln(exp(2*x)+1)-(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1
)^2*x*polylog(2,-exp(2*x))+1/2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*
(exp(2*x)+1)^2*polylog(3,-exp(2*x))
```

Maxima [A] time = 1.8728, size = 208, normalized size = 1.02

$$-\frac{1}{2}\left(2\,x^2\log\left(e^{(2\,x)}+1\right)+2\,x\mathrm{Li}_2\left(-e^{(2\,x)}\right)-\mathrm{Li}_3(-e^{(2\,x)})\right)\sqrt{a}+\left(x^2\log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)\right)\sqrt{a}+\left(x^2\log\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2*log(e^(2*x) + 1) + 2*x*dilog(-e^(2*x)) - polylog(3, -e^(2*x)))*
sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))*sqrt(a)
 + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))*sqrt(a) - 2*sqr
t(a)*x + sqrt(a)*log(e^(2*x) + 1) + 2*((sqrt(a)*x^2 + sqrt(a)*x)*e^(2*x) +
sqrt(a)*x)/(e^(4*x) + 2*e^(2*x) + 1)
```

Fricas [C] time = 3.07512, size = 10301, normalized size = 50.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1
)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x)
 + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e
^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*
e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) - 2*((e^(4*x) + 2*e^(2*
x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x
)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 +
1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh
(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(
x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*p
olylog(3, I*cosh(x) + I*sinh(x)) - 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 +
 cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 +
2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) +
1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^
(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(
8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -I*cosh(x
) - I*sinh(x)) + 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(co
sh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (
3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*c
osh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)
^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)
^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) +
 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) + (2*x*
cosh(x)^4 + 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 8*(x*cosh(x)*e^(4*x
) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 - 2*(x^2 - x)*cosh(x)^2 + 2*
(6*x*cosh(x)^2 - x^2 + (6*x*cosh(x)^2 - x^2 + x)*e^(4*x) + 2*(6*x*cosh(x)^2
 - x^2 + x)*e^(2*x) + x)*sinh(x)^2 - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2
*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x)
)*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x
) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(
x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh
(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*
cosh(x))*e^(2*x))*sinh(x) + x)*dilog(cosh(x) + sinh(x)) + 2*(x*cosh(x)^4 +
(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x
)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*
cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*
cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x
)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x)
+ 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*dilog(I*cosh(x) + I*sin
h(x)) + 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cos
h(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
```

2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(
2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh
(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x
)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x
)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x)
 + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*s
inh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) +
 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^
2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)
^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cos
h(x))*e^(2*x))*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) + 2*(x*cosh(x)^4 - (x
^2 - x)*cosh(x)^2)*e^(4*x) + 4*(x*cosh(x)^4 - (x^2 - x)*cosh(x)^2)*e^(2*x)
- (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*co
sh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sin
h(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh
(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) +
4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*
(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1)
- ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*
e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) +
 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(
2*x) + cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + I) - ((e^(4*x) + 2*e^(
2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) +
cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2
+ 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (co
sh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sin
h(x) + 1)*log(cosh(x) + sinh(x) - I) + (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^
2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x
^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3
*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^
2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^
4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*
cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))
*sinh(x))*log(I*cosh(x) + I*sinh(x) + 1) + (x^2*cosh(x)^4 + (x^2*e^(4*x) +
2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) +
 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2
+ (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh
(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh
(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (
x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2
*x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) - (x^2*cosh(x)^4 + (x^2*e^(4*

```
x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4
*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 +
 x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))
*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2
*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x
) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))
*e^(2*x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 4*(2*x*cosh(x)^3 - (x^2 -
x)*cosh(x) + (2*x*cosh(x)^3 - (x^2 - x)*cosh(x))*e^(4*x) + 2*(2*x*cosh(x)^3
 - (x^2 - x)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(
4*x) + 4*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh
(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(
2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a\operatorname{sech}^4(x)}\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\operatorname{sech}(x)^4}x^2\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x^2*csch(x)*sech(x), x)
```

3.853 $\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=326

$$-\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x\cosh^2(x)\text{PolyLog}\left(3,\right.$$

```
[Out] (-3*x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 + (x^3*Cosh[x]^2*Sqrt[a*Sech[x]^4])/
2 - 2*x^3*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*Sech[x]^4] + 3*x*Cosh[x]^2*Log[
1 + E^(2*x)]*Sqrt[a*Sech[x]^4] + (3*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*S
ech[x]^4])/2 - (3*x^2*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 +
 (3*x^2*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (3*x*Cosh[x]^2
*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (3*x*Cosh[x]^2*PolyLog[3, E^(2
*x)]*Sqrt[a*Sech[x]^4])/2 - (3*Cosh[x]^2*PolyLog[4, -E^(2*x)]*Sqrt[a*Sech[x
]^4])/4 + (3*Cosh[x]^2*PolyLog[4, E^(2*x)]*Sqrt[a*Sech[x]^4])/4 - (3*x^2*Co
sh[x]*Sqrt[a*Sech[x]^4]*Sinh[x])/2 - (x^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rubi [A] time = 0.682779, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {6720, 2620, 14, 5462, 2551, 5461, 4182, 2531, 6609, 2282, 6589, 3720, 3718, 2190, 2279, 2391, 30}

$$-\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x\cosh^2(x)\text{PolyLog}\left(3,\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```


```
[Out] (-3*x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 + (x^3*Cosh[x]^2*Sqrt[a*Sech[x]^4])/
2 - 2*x^3*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*Sech[x]^4] + 3*x*Cosh[x]^2*Log[
1 + E^(2*x)]*Sqrt[a*Sech[x]^4] + (3*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*S
ech[x]^4])/2 - (3*x^2*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 +
 (3*x^2*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (3*x*Cosh[x]^2
*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (3*x*Cosh[x]^2*PolyLog[3, E^(2
*x)]*Sqrt[a*Sech[x]^4])/2 - (3*Cosh[x]^2*PolyLog[4, -E^(2*x)]*Sqrt[a*Sech[x
]^4])/4 + (3*Cosh[x]^2*PolyLog[4, E^(2*x)]*Sqrt[a*Sech[x]^4])/4 - (3*x^2*Co
sh[x]*Sqrt[a*Sech[x]^4]*Sinh[x])/2 - (x^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^3\text{csch}(x)\text{sech}^3(x)\,dx$$

$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(3\cosh^2(x)$$

$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(3\cosh^2(x)$$

$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \frac{1}{2}\Big(3\cosh^2($$

$$= -\frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \Big(\cosh^2(x)\sqrt{ase$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4($$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\left(e^{2x}\right)\cosh$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\left(e^{2x}\right)\cosh$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\left(e^{2x}\right)\cosh$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\left(e^{2x}\right)\cosh$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\left(e^{2x}\right)\cosh$$

Mathematica [A] time = 0.99359, size = 157, normalized size = 0.48

$$\frac{1}{64}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(96x^2\text{PolyLog}\left(2,e^{2x}\right) + 96\left(x^2-1\right)\text{PolyLog}\left(2,-e^{-2x}\right) + 96x\text{PolyLog}\left(3,-e^{-2x}\right) - 96x\text{PolyL}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(Pi^4 + 96*x^2 - 32*x^4 + 192*x*Log[1 + E^(-2*
x)] - 64*x^3*Log[1 + E^(-2*x)] + 64*x^3*Log[1 - E^(2*x)] + 96*(-1 + x^2)*Po
lyLog[2, -E^(-2*x)] + 96*x^2*PolyLog[2, E^(2*x)] + 96*x*PolyLog[3, -E^(-2*x
)] - 96*x*PolyLog[3, E^(2*x)] + 48*PolyLog[4, -E^(-2*x)] + 48*PolyLog[4, E^
(2*x)] + 32*x^3*Sech[x]^2 - 96*x^2*Tanh[x]))/64
```

Maple [B] time = 0.085, size = 602, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*x^2*(2*x*exp(2*x)+3*exp(2*x)+3)
-3*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^2+3*(a*exp(
4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*ln(exp(2*x)+1)+3/2*(a
*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(2,-exp(2*x
))+(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^3*ln(exp(x)
+1)+3*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^2*polylo
g(2,-exp(x))-6*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x
*polylog(3,-exp(x))+6*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)
+1)^2*polylog(4,-exp(x))+(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2
*x)+1)^2*x^3*ln(1-exp(x))+3*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(ex
p(2*x)+1)^2*x^2*polylog(2,exp(x))-6*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-
2*x)*(exp(2*x)+1)^2*x*polylog(3,exp(x))+6*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)
*exp(-2*x)*(exp(2*x)+1)^2*polylog(4,exp(x))-(a*exp(4*x)/(exp(2*x)+1)^4)^(1/
2)*exp(-2*x)*(exp(2*x)+1)^2*x^3*ln(exp(2*x)+1)-3/2*(a*exp(4*x)/(exp(2*x)+1)
^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x^2*polylog(2,-exp(2*x))+3/2*(a*exp(4*x)
/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*x*polylog(3,-exp(2*x))-3/4*
(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)^2*polylog(4,-exp(2
*x))
```

Maxima [A] time = 1.81104, size = 279, normalized size = 0.86

$$-3\sqrt{a}x^2-\frac{1}{3}\left(4x^3\log\left(e^{(2x)}+1\right)+6x^2\mathrm{Li}_2\left(-e^{(2x)}\right)-6x\mathrm{Li}_3(-e^{(2x)})+3\mathrm{Li}_4(-e^{(2x)})\right)\sqrt{a}+\left(x^3\log\left(e^x+1\right)+3x^2\mathrm{Li}_2\left(-e^x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*x^2 - 1/3*(4*x^3*log(e^(2*x) + 1) + 6*x^2*dilog(-e^(2*x)) - 6*x*
polylog(3, -e^(2*x)) + 3*polylog(4, -e^(2*x)))*sqrt(a) + (x^3*log(e^x + 1)
+ 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) +
(x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4,
e^x))*sqrt(a) + 3/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (3*s
qrt(a)*x^2 + (2*sqrt(a)*x^3 + 3*sqrt(a)*x^2)*e^(2*x))/(e^(4*x) + 2*e^(2*x)
```

```
+ 1)
```

Fricas [C] time = 3.39832, size = 13428, normalized size = 41.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)
*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh
(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x)
+ 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^
(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e
^(2*x) + 1))*e^(2*x)*polylog(4, cosh(x) + sinh(x)) - 6*((e^(4*x) + 2*e^(2*x
) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cos
h(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)
^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1
)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(
x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x
) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*po
lylog(4, I*cosh(x) + I*sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 +
cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2
*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1
)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh
(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(
4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8
*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -I*cosh(x)
 - I*sinh(x)) + 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cos
h(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3
*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*co
sh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^
2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^
3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) +
6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -cosh(x) - sinh(x)) - 6*(x*c
osh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) +
 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x
)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh
(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*co
sh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x
```

))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, I*cosh(x) + I*sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -I*cosh(x) - I*sinh(x)) - 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (3*x^2*cosh(x)^4 + 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 12*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 - (2*x^3 - 3*x^2)*cosh(x)^2 + (18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2 + (18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2)*e^(4*x) + 2*(18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2)*e^(2*x))*sinh(x)^2 - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e^(2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*e^(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2 - 1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(x)^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(I*cosh(x) + I*sinh(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e

^(2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*
e^(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2
- 1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^
2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(
x)^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 +
2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 -
1)*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 -
1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(-I*cosh(x) -
I*sinh(x)) - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)
^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2
*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^
(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4
 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^
2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))
*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(-cosh(x)
 - sinh(x)) + (3*x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(2*x) - (x^3*cosh(x)^4 + 2*x^3
*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)
*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*
cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^
3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2
*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*c
osh(x) + (x^3*cosh(x)^3 + x^3*cosh(x))*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cos
h(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4
 + (x^3 + (x^3 - 3*x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*
((x^3 - 3*x)*cosh(x)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*
cosh(x))*sinh(x)^3 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)
cosh(x)^2 + (x^3 + 3(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^
3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4
 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 +
 (x^3 - 3*x)*cosh(x) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x
) + 2*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)
*log(I*cosh(x) + I*sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4 + (x^3 + (x^3 - 3*
x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*((x^3 - 3*x)*cosh(x
)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*cosh(x))*sinh(x)^3
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2 + (x^3 +
 3*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2
- 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3
*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3*
x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x
) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x) + 2*((x^3 - 3*x)*
cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)*log(-I*cosh(x) - I
*sinh(x) + 1) - (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(
2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^

```
3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 +
x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^
4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x
^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*cosh(x) + (x^3*cosh(x)^3 + x^3*cosh(x)
)*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cosh(x))*e^(2*x))*sinh(x))*log(-cosh(x)
- sinh(x) + 1) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*x^2)*cosh(x) + (6*x^2*cosh
(x)^3 - (2*x^3 - 3*x^2)*cosh(x))*e^(4*x) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*
x^2)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4
*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(2*x)*sin
h(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a\,\mathrm{sech}^4(x)}\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\mathrm{sech}(x)^4}x^3\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x^3*csch(x)*sech(x), x)
```

3.854 $\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$

Optimal. Leaf size=147

$$\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

```
[Out] (I*AppellF1[1/2, 1/2, -m, 3/2, (1 - I*Sinh[2*c + 2*d*x])/2, (b*(1 - I*Sinh[
2*c + 2*d*x]))/((2*I)*a + b)]*Cosh[2*c + 2*d*x]*(a + (b*Sinh[2*c + 2*d*x])/
2)^m)/(Sqrt[2]*d*Sqrt[1 + I*Sinh[2*c + 2*d*x]]*((2*a + b*Sinh[2*c + 2*d*x])
/(2*a - I*b))^m)
```

Rubi [A] time = 0.129911, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2665, 139, 138}

$$\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

```
[Out] (I*AppellF1[1/2, 1/2, -m, 3/2, (1 - I*Sinh[2*c + 2*d*x])/2, (b*(1 - I*Sinh[
2*c + 2*d*x]))/((2*I)*a + b)]*Cosh[2*c + 2*d*x]*(a + (b*Sinh[2*c + 2*d*x])/
2)^m)/(Sqrt[2]*d*Sqrt[1 + I*Sinh[2*c + 2*d*x]]*((2*a + b*Sinh[2*c + 2*d*x])
/(2*a - I*b))^m)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
```

```
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m dx$$

$$= -\frac{(i\cosh(2c+2dx))\,\text{Subst}\left(\int \frac{\left(a-\frac{ibx}{2}\right)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, i\sinh(2c+2dx)\right)}{2d\sqrt{1-i\sinh(2c+2dx)}\sqrt{1+i\sinh(2c+2dx)}}$$

$$= -\frac{\left(i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(-\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{-a+\frac{ib}{2}}\right)^{-m}\right)\text{Subst}\left(\int \right.}{2d\sqrt{1-i\sinh(2c+2dx)}\sqrt{1+i\sinh(2c+}}$$

$$= \frac{iF_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)\cosh(2c+2dx)\left(a+\right.}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

Mathematica [A] time = 0.64647, size = 162, normalized size = 1.1

$$\frac{\operatorname{sech}(2(c+dx))\sqrt{\frac{b(1-i\sinh(2(c+dx)))}{b+2ia}}\sqrt{\frac{b(1+i\sinh(2(c+dx)))}{b-2ia}}\left(a+\frac{1}{2}b\sinh(2(c+dx))\right)^{m+1}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2a+b\sinh(2(c+dx))}{2a+ib}\right.}{bd(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sinh[2*(c + d*x)])/(2*a + I*b),
(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)]*Sech[2*(c + d*x)]*Sqrt[(b*(1 - I*S
inh[2*(c + d*x)]))/((2*I)*a + b)]*Sqrt[(b*(1 + I*Sinh[2*(c + d*x)]))/((-2*I
)*a + b)]*(a + (b*Sinh[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (a+b\cosh(dx+c)\sinh(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

```
[Out] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cosh(dx+c)\sinh(dx+c)+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

3.855 $\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$

Optimal. Leaf size=109

$$\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{1}{8}ax(8a^2-3b^2)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh}{48d}$$

```
[Out] (a*(8*a^2 - 3*b^2)*x)/8 + (b*(16*a^2 - b^2)*Cosh[2*c + 2*d*x])/(24*d) + (5*
a*b^2*Cosh[2*c + 2*d*x]*Sinh[2*c + 2*d*x])/(48*d) + (b*Cosh[2*c + 2*d*x]*(2
*a + b*Sinh[2*c + 2*d*x])^2)/(48*d)
```

Rubi [A] time = 0.0988748, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2666, 2656, 2734}

$$\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{1}{8}ax(8a^2-3b^2)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 - 3*b^2)*x)/8 + (b*(16*a^2 - b^2)*Cosh[2*c + 2*d*x])/(24*d) + (5*
a*b^2*Cosh[2*c + 2*d*x]*Sinh[2*c + 2*d*x])/(48*d) + (b*Cosh[2*c + 2*d*x]*(2
*a + b*Sinh[2*c + 2*d*x])^2)/(48*d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3\,dx$$
$$= \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d} + \frac{1}{3}\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)\left(\frac{1}{2}\right.$$
$$= \frac{1}{8}a\left(8a^2-3b^2\right)x + \frac{b\left(16a^2-b^2\right)\cosh(2c+2dx)}{24d} + \frac{5ab^2\cosh(2c+2dx)\sinh(}{48d}$$

Mathematica [A] time = 0.259059, size = 77, normalized size = 0.71

$$\frac{6a\left(4\left(8a^2-3b^2\right)(c+dx)+3b^2\sinh(4(c+dx))\right)+9\left(16a^2b-b^3\right)\cosh(2(c+dx))+b^3\cosh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

```
[Out] (9*(16*a^2*b - b^3)*Cosh[2*(c + d*x)] + b^3*Cosh[6*(c + d*x)] + 6*a*(4*(8*a
^2 - 3*b^2)*(c + d*x) + 3*b^2*Sinh[4*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.047, size = 124, normalized size = 1.1

$$\frac{1}{d}\left(b^3\left(\frac{(\sinh(dx+c))^2(\cosh(dx+c))^4}{6}-\frac{(\sinh(dx+c))^2(\cosh(dx+c))^2}{12}-\frac{(\cosh(dx+c))^2}{12}\right)+3\,ab^2\left(1/4\,\sinh(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/6*sinh(d*x+c)^2*cosh(d*x+c)^4-1/12*sinh(d*x+c)^2*cosh(d*x+c)^2-
1/12*cosh(d*x+c)^2)+3*a*b^2*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*
```

```
sinh(d*x+c)-1/8*d*x-1/8*c)+3/2*a^2*cosh(d*x+c)^2*b+a^3*(d*x+c))
```

Maxima [A] time = 1.24572, size = 170, normalized size = 1.56

$$a^3x - \frac{1}{384}b^3\left(\frac{\left(9e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d} + \frac{9e^{(-2dx-2c)}-e^{(-6dx-6c)}}{d}\right) - \frac{3}{64}ab^2\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right) + \frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/384*b^3*((9*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + (9*e^(-2*d*
x - 2*c) - e^(-6*d*x - 6*c))/d) - 3/64*a*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*
c)/d + e^(-4*d*x - 4*c)/d) + 3/2*a^2*b*cosh(d*x + c)^2/d
```

Fricas [A] time = 2.01423, size = 409, normalized size = 3.75

$$\frac{b^3\cosh(dx+c)^6 + 15b^3\cosh(dx+c)^2\sinh(dx+c)^4 + b^3\sinh(dx+c)^6 + 72ab^2\cosh(dx+c)^3\sinh(dx+c) + 72ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(b^3*cosh(d*x + c)^6 + 15*b^3*cosh(d*x + c)^2*sinh(d*x + c)^4 + b^3*s
inh(d*x + c)^6 + 72*a*b^2*cosh(d*x + c)^3*sinh(d*x + c) + 72*a*b^2*cosh(d*x
 + c)*sinh(d*x + c)^3 + 24*(8*a^3 - 3*a*b^2)*d*x + 9*(16*a^2*b - b^3)*cosh(
d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 48*a^2*b - 3*b^3)*sinh(d*x + c)^2)/
d
```

Sympy [A] time = 5.90162, size = 190, normalized size = 1.74

$$\begin{cases} a^3x + \frac{3a^2b\cosh^2(c+dx)}{2d} - \frac{3ab^2x\sinh^4(c+dx)}{8} + \frac{3ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{3ab^2x\cosh^4(c+dx)}{8} + \frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{3ab^2\sin}{} \\ x(a+b\sinh(c)\cosh(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)**2/(2*d) - 3*a*b**2*x*sinh(c + d
*x)**4/8 + 3*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a*b**2*x*cosh
(c + d*x)**4/8 + 3*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**3/(8*d) - b**3*sinh(c + d*x)**6/(12*d) + b**3*s
inh(c + d*x)**4*cosh(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c
))**3, True))
```

Giac [A] time = 1.14285, size = 232, normalized size = 2.13

$$\frac{b^3 e^{(6\,dx+6\,c)} + 18\,ab^2 e^{(4\,dx+4\,c)} + 144\,a^2 b e^{(2\,dx+2\,c)} - 9\,b^3 e^{(2\,dx+2\,c)} + 48\left(8\,a^3 - 3\,ab^2\right)(dx+c) - \left(352\,a^3 e^{(6\,dx+6\,c)} - 132\,ab^2 e^{(6\,d}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/384*(b^3*e^(6*d*x + 6*c) + 18*a*b^2*e^(4*d*x + 4*c) + 144*a^2*b*e^(2*d*x
+ 2*c) - 9*b^3*e^(2*d*x + 2*c) + 48*(8*a^3 - 3*a*b^2)*(d*x + c) - (352*a^3*
e^(6*d*x + 6*c) - 132*a*b^2*e^(6*d*x + 6*c) - 144*a^2*b*e^(4*d*x + 4*c) + 9
*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) - b^3)*e^(-6*d*x - 6*c))/d
```

3.856 $\int (a + b\cosh(c + dx)\sinh(c + dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{1}{8}x\left(8a^2-b^2\right)+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\sinh(2c+2dx)\cosh(2c+2dx)}{16d}$$

```
[Out] ((8*a^2 - b^2)*x)/8 + (a*b*Cosh[2*c + 2*d*x])/(2*d) + (b^2*Cosh[2*c + 2*d*x
]*Sinh[2*c + 2*d*x])/(16*d)
```

Rubi [A] time = 0.0354427, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2666, 2644}

$$\frac{1}{8}x\left(8a^2-b^2\right)+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\sinh(2c+2dx)\cosh(2c+2dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

```
[Out] ((8*a^2 - b^2)*x)/8 + (a*b*Cosh[2*c + 2*d*x])/(2*d) + (b^2*Cosh[2*c + 2*d*x
]*Sinh[2*c + 2*d*x])/(16*d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx=\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2\,dx$$
$$=\frac{1}{8}\left(8a^2-b^2\right)x+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\cosh(2c+2dx)\sinh(2c+2dx)}{16d}$$

Mathematica [A] time = 0.113222, size = 50, normalized size = 0.79

$$\frac{4\left(8a^2-b^2\right)(c+dx)+16ab\cosh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

```
[Out] (4*(8*a^2 - b^2)*(c + d*x) + 16*a*b*Cosh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x
)])/(32*d)
```

Maple [A] time = 0.039, size = 68, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{\sinh(dx+c)(\cosh(dx+c))^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+ab(\cosh(dx+c))^2+a^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x
-1/8*c)+a*b*cosh(d*x+c)^2+a^2*(d*x+c))
```

Maxima [A] time = 1.16773, size = 85, normalized size = 1.35

$$a^2x-\frac{1}{64}b^2\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)+\frac{ab\cosh(dx+c)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/64*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d) +
 a*b*cosh(d*x + c)^2/d
```

Fricas [A] time = 2.17857, size = 198, normalized size = 3.14

$$\frac{b^2\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+4\,ab\cosh\left(dx+c\right)^2+4\,ab\sinh\left(dx+c\right)^2+\left(8\,a^2-b^2\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3*sinh(d*x + c) + b^2*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^2 + (8*a^2 - b^2)*d*x)/d
```

Sympy [A] time = 1.46069, size = 129, normalized size = 2.05

$$\begin{cases} a^2x+\frac{ab\cosh^2\left(c+dx\right)}{d}-\frac{b^2x\sinh^4\left(c+dx\right)}{8}+\frac{b^2x\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{4}-\frac{b^2x\cosh^4\left(c+dx\right)}{8}+\frac{b^2\sinh^3\left(c+dx\right)\cosh\left(c+dx\right)}{8d}+\frac{b^2\sinh\left(c+dx\right)\cos}{8d} \\ x\left(a+b\sinh\left(c\right)\cosh\left(c\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*cosh(c + d*x)**2/d - b**2*x*sinh(c + d*x)**4/8 + b*
*2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b**2*x*cosh(c + d*x)**4/8 + b**2
*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b**2*sinh(c + d*x)*cosh(c + d*x)**3
/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c))**2, True))
```

Giac [A] time = 1.18512, size = 143, normalized size = 2.27

$$\frac{b^2e^{(4\,dx+4\,c)}+16\,abe^{(2\,dx+2\,c)}+8\left(8\,a^2-b^2\right)(dx+c)-\left(48\,a^2e^{(4\,dx+4\,c)}-6\,b^2e^{(4\,dx+4\,c)}-16\,abe^{(2\,dx+2\,c)}+b^2\right)e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(b^2*e^(4*d*x + 4*c) + 16*a*b*e^(2*d*x + 2*c) + 8*(8*a^2 - b^2)*(d*x +
 c) - (48*a^2*e^(4*d*x + 4*c) - 6*b^2*e^(4*d*x + 4*c) - 16*a*b*e^(2*d*x + 2
*c) + b^2)*e^(-4*d*x - 4*c))/d
```

3.857 $\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx$

Optimal. Leaf size=20

$$ax+\frac{b\sinh^2(c+dx)}{2d}$$

```
[Out] a*x + (b*Sinh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0187967, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2564, 30}

$$ax+\frac{b\sinh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
```

```
[Out] a*x + (b*Sinh[c + d*x]^2)/(2*d)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax+b\int \cosh(c+dx)\sinh(c+dx)\,dx$$
$$= ax - \frac{b\,\mathrm{Subst}\left(\int x\,dx, x, i\sinh(c+dx)\right)}{d}$$
$$= ax + \frac{b\sinh^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0074473, size = 38, normalized size = 1.9

$$ax + \frac{b\sinh(2c)\sinh(2dx)}{4d} + \frac{b\cosh(2c)\cosh(2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
```

```
[Out] a*x + (b*Cosh[2*c]*Cosh[2*d*x])/(4*d) + (b*Sinh[2*c]*Sinh[2*d*x])/(4*d)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$ax + \frac{b\left(\cosh\left(dx+c\right)\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(d*x+c)*sinh(d*x+c),x)
```

```
[Out] a*x+1/2*b*cosh(d*x+c)^2/d
```

Maxima [A] time = 1.23071, size = 24, normalized size = 1.2

$$ax + \frac{b\cosh\left(dx+c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*b*cosh(d*x + c)^2/d
```

Fricas [A] time = 2.48052, size = 77, normalized size = 3.85

$$\frac{4\,adx + b\cosh\left(dx + c\right)^{2} + b\sinh\left(dx + c\right)^{2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*d*x + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2)/d
```

Sympy [A] time = 0.288004, size = 24, normalized size = 1.2

$$ax + b\left(\begin{cases} \frac{\sinh^{2}\left(c + dx\right)}{2d} & \text{for}\: d \neq 0 \\ x\sinh\left(c\right)\cosh\left(c\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)**2/(2*d), Ne(d, 0)), (x*sinh(c)*cosh(c), T
rue))
```

Giac [A] time = 1.13355, size = 39, normalized size = 1.95

$$ax + \frac{b\left(e^{\left(2\,dx + 2\,c\right)} + e^{\left(-2\,dx - 2\,c\right)}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/8*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))/d
```

3.858 $\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

```
[Out] (-2*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/(Sqrt[4*a^2 + b^2]*
d)
```

Rubi [A] time = 0.123194, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/(Sqrt[4*a^2 + b^2]*
d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx &= \int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\ &= -\frac{i\,\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx, x, -ib+2a\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}\,d}\end{aligned}$$

Mathematica [A] time = 0.0795756, size = 48, normalized size = 1.09

$$\frac{2\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{d\sqrt{-4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(Sqrt[-4*a^2 - b^2]*
d)
```

Maple [B] time = 0.087, size = 207, normalized size = 4.7

$$\frac{1}{d}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a+\sqrt{4\,a^2+b^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)\frac{1}{\sqrt{4\,a^2+b^2}}-4\,\frac{a^2\ln\left(-(\tanh(1/2\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
```

```
[Out] 1/d/(4*a^2+b^2)^(1/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2
*d*x+1/2*c)+b*tanh(1/2*d*x+1/2*c)+a)-4/d*a^2/(4*a^2+b^2)^(3/2)*ln(-tanh(1/2
*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)-b*tanh(1/2*d*x+1/2*c)
-a)-1/d/(4*a^2+b^2)^(3/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tan
h(1/2*d*x+1/2*c)-b*tanh(1/2*d*x+1/2*c)-a)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3383, size = 772, normalized size = 17.55

$$\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab\right)\sinh(dx+c)^2+8\,a^2+b^2+4\left(b^2\cosh(dx+c)^3+2\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+4\,a\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2+2\,a\right)\sinh}\right.}{\sqrt{4\,a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d
*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(
d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*si
```

```
nh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh
(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x +
c))*sinh(d*x + c) - b))/(sqrt(4*a^2 + b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18213, size = 107, normalized size = 2.43

$$\frac{\log\left(\frac{\left|2\, be^{\left(2\, dx+2\, c\right)}+4\, a-2\, \sqrt{4\, a^{2}+b^{2}}\right|}{\left|2\, be^{\left(2\, dx+2\, c\right)}+4\, a+2\, \sqrt{4\, a^{2}+b^{2}}\right|}\right)}{\sqrt{4\, a^{2}+b^{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*d*x +
 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

3.859 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2} \, dx$

Optimal. Leaf size=89

$$-\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{3/2}}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))}$$

```
[Out] (-8*a*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a^2 + b^2)^(3
/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x
]))
```

Rubi [A] time = 0.103961, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2666, 2664, 12, 2660, 618, 204}

$$-\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{3/2}}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-8*a*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a^2 + b^2)^(3
/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x
]))
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a + b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \int \frac{1}{\left(a + \frac{1}{2}b\sinh(2c+2dx)\right)^2}\,dx$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{4\int \frac{a}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} - \frac{(4ia)\,\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\right.\right.}{\left(4a^2+b^2\right)d}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{(8ia)\,\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx, x, -ib +\right.}{\left(4a^2+b^2\right)d}$$

$$= -\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\left(4a^2+b^2\right)^{3/2}d} - \frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))}$$

Mathematica [A] time = 0.336885, size = 90, normalized size = 1.01

$$\frac{2\left(-\frac{4a\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\left(-4a^2-b^2\right)^{3/2}} - \frac{b\cosh(2(c+dx))}{\left(4a^2+b^2\right)(2a+b\sinh(2(c+dx)))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (2*((-4*a*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(-4*a^2 - b^2
)^(3/2) - (b*Cosh[2*(c + d*x)])/((4*a^2 + b^2)*(2*a + b*Sinh[2*(c + d*x)]))
))/d
```

Maple [B] time = 0.119, size = 469, normalized size = 5.3

$$2\,\frac{b^2\left(\tanh(1/2\,dx+c/2)\right)^3}{d\left(\left(\tanh(1/2\,dx+c/2)\right)^4 a + 2\,b\left(\tanh(1/2\,dx+c/2)\right)^3 - 2\left(\tanh(1/2\,dx+c/2)\right)^2 a + 2\,b\tanh(1/2\,dx+c/2) + a\right)a\left(4\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c
)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)*b^2/a/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)^3-8/
d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^
2*a+2*b*tanh(1/2*d*x+1/2*c)+a)*b/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)^2+2/d/(tan
h(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*
b*tanh(1/2*d*x+1/2*c)+a)*b^2/a/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)+16/d*a^3/(4*
a^2+b^2)^(5/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/
2*c)+b*tanh(1/2*d*x+1/2*c)+a)+4/d*a/(4*a^2+b^2)^(5/2)*ln(tanh(1/2*d*x+1/2*c
)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+b*tanh(1/2*d*x+1/2*c)+a)*b^2-4/
d*a/(4*a^2+b^2)^(3/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/
2*d*x+1/2*c)-b*tanh(1/2*d*x+1/2*c)-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16257, size = 1895, normalized size = 21.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -4*(4*a^2*b + b^3 - 2*(4*a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(4*a^3 + a*b^2)*c
osh(d*x + c)*sinh(d*x + c) - 2*(4*a^3 + a*b^2)*sinh(d*x + c)^2 - (a*b*cosh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 4*
a^2*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2)*sinh(d*x + c)^2 - a
*b + 4*(a*b*cosh(d*x + c)^3 + 2*a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^
```

```
2 + b^2)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b)*sinh(d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x
 + c))*sinh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*co
sh(d*x + c))*sinh(d*x + c) - b)))/((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x
+ c)^4 + 4*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
16*a^4*b + 8*a^2*b^3 + b^5)*d*sinh(d*x + c)^4 + 4*(16*a^5 + 8*a^3*b^2 + a*b
^4)*d*cosh(d*x + c)^2 + 2*(3*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^2
 + 2*(16*a^5 + 8*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 - (16*a^4*b + 8*a^2*b^
3 + b^5)*d + 4*((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^3 + 2*(16*a^5
+ 8*a^3*b^2 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19051, size = 208, normalized size = 2.34

$$-\frac{4\,a\log\left(\frac{\left|-2\,be^{(2\,dx+2\,c)}-4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|-2\,be^{(2\,dx+2\,c)}-4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{\left(4\,a^2d+b^2d\right)\sqrt{4\,a^2+b^2}}+\frac{4\left(2\,ae^{(2\,dx+2\,c)}-b\right)}{\left(4\,a^2d+b^2d\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -4*a*log(abs(-2*b*e^(2*d*x + 2*c) - 4*a - 2*sqrt(4*a^2 + b^2))/abs(-2*b*e^(
2*d*x + 2*c) - 4*a + 2*sqrt(4*a^2 + b^2)))/((4*a^2*d + b^2*d)*sqrt(4*a^2 +
b^2)) + 4*(2*a*e^(2*d*x + 2*c) - b)/((4*a^2*d + b^2*d)*(b*e^(4*d*x + 4*c) +
 4*a*e^(2*d*x + 2*c) - b))
```

3.860 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{5/2}}-\frac{12ab\cosh(2c+2dx)}{d\left(4a^2+b^2\right)^2(2a+b\sinh(2c+2dx))}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

```
[Out] (-4*(8*a^2 - b^2)*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a
^2 + b^2)^(5/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh
[2*c + 2*d*x])^2) - (12*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)^2*d*(2*a + b*
Sinh[2*c + 2*d*x]))
```

Rubi [A] time = 0.171853, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2666, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{5/2}}-\frac{12ab\cosh(2c+2dx)}{d\left(4a^2+b^2\right)^2(2a+b\sinh(2c+2dx))}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

```
[Out] (-4*(8*a^2 - b^2)*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a
^2 + b^2)^(5/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh
[2*c + 2*d*x])^2) - (12*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)^2*d*(2*a + b*
Sinh[2*c + 2*d*x]))
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3}\,dx$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{2\int \frac{-2a+\frac{1}{2}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(2c+2dx}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(2c+2dx}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(2c+2dx}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(2c+2dx}$$

$$= -\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\left(4a^2+b^2\right)^{5/2}d} - \frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx)}$$

Mathematica [A] time = 0.640178, size = 121, normalized size = 0.85

$$\frac{2\left(\frac{2\left(8a^2-b^2\right)\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}} - \frac{b\cosh(2(c+dx))\left(16a^2+6ab\sinh(2(c+dx))+b^2\right)}{(2a+b\sinh(2(c+dx)))^2}\right)}{d\left(4a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

```
[Out] (2*((2*(8*a^2 - b^2)*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/Sq
rt[-4*a^2 - b^2] - (b*Cosh[2*(c + d*x)]*(16*a^2 + b^2 + 6*a*b*Sinh[2*(c + d
*x)]))/(2*a + b*Sinh[2*(c + d*x)])^2))/((4*a^2 + b^2)^2*d)
```

Maple [B] time = 0.158, size = 2082, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x)
```

```
[Out] 20/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*
c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d
*x+1/2*c)^7+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1
/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)
*tanh(1/2*d*x+1/2*c)^7-64/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c
)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*b*a^2/(16*a^4+8*
a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6+28/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1
/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*b^3/
(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2
*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+
a)^2*b^5/a^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6-116/d/(tanh(1/2*d
*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(
1/2*d*x+1/2*c)+a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^5-2/d/
(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*
a+2*b*tanh(1/2*d*x+1/2*c)+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/
2*c)^5+128/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*
d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*a^2*b/(16*a^4+8*a^2*b^2+b^4)*ta
nh(1/2*d*x+1/2*c)^4+72/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3
-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*b^3/(16*a^4+8*a^2*b
^2+b^4)*tanh(1/2*d*x+1/2*c)^4+4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x
+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2/a^2*b^5/(1
6*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^4-116/d/(tanh(1/2*d*x+1/2*c)^4*a+2
*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+
a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/2*d*x+1
/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*
d*x+1/2*c)+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3-64/d/(ta
nh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2
*b*tanh(1/2*d*x+1/2*c)+a)^2*b*a^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c
)^2+28/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+
1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*b^3/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2
*d*x+1/2*c)^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1/2*c)^3-2*tanh
(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*b^5/a^2/(16*a^4+8*a^2*b^2+
b^4)*tanh(1/2*d*x+1/2*c)^2+20/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(1/2*d*x+1
/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2*a*b^2/(16*a^
4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*b*tanh(
```

```
1/2*d*x+1/2*c)^3-2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c)+a)^2/a*b
^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)+64/d/(16*a^4+8*a^2*b^2+b^4)*a
^4/(4*a^2+b^2)^(3/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*
d*x+1/2*c)+b*tanh(1/2*d*x+1/2*c)+a)+8/d/(16*a^4+8*a^2*b^2+b^4)*a^2/(4*a^2+b
^2)^(3/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+
b*tanh(1/2*d*x+1/2*c)+a)*b^2-2/d/(16*a^4+8*a^2*b^2+b^4)/(4*a^2+b^2)^(3/2)*l
n(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+b*tanh(1/2*
d*x+1/2*c)+a)*b^4-16/d/(16*a^4+8*a^2*b^2+b^4)*a^2/(4*a^2+b^2)^(1/2)*ln(-tan
h(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)-b*tanh(1/2*d*x+1
/2*c)-a)+2/d/(16*a^4+8*a^2*b^2+b^4)/(4*a^2+b^2)^(1/2)*ln(-tanh(1/2*d*x+1/2*
c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)-b*tanh(1/2*d*x+1/2*c)-a)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07796, size = 5688, normalized size = 39.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(2*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^6 + 12*(32*a^4*b + 4*a^2*b^
3 - b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(32*a^4*b + 4*a^2*b^3 - b^5)*sin
h(d*x + c)^6 + 48*a^3*b^2 + 12*a*b^4 + 12*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh
(d*x + c)^4 + 6*(64*a^5 + 8*a^3*b^2 - 2*a*b^4 + 5*(32*a^4*b + 4*a^2*b^3 - b
^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(32*a^4*b + 4*a^2*b^3 - b^5)*co
sh(d*x + c)^3 + 6*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(160*a^4*b + 44*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 2*(160*a^4*b + 44*a
^2*b^3 + b^5 - 15*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36*(32*a^5
 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((8*a^2*b^2 - b^4)
*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a
```

```
^2*b^2 - b^4)*sinh(d*x + c)^8 + 8*(8*a^3*b - a*b^3)*cosh(d*x + c)^6 + 4*(16
*a^3*b - 2*a*b^3 + 7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^3 + 6*(8*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*(35*(8*
a^2*b^2 - b^4)*cosh(d*x + c)^4 + 64*a^4 - 16*a^2*b^2 + b^4 + 60*(8*a^3*b -
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - b^4 + 8*(7*(8*a^2*b^2
 - b^4)*cosh(d*x + c)^5 + 20*(8*a^3*b - a*b^3)*cosh(d*x + c)^3 + (64*a^4 -
16*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a^3*b - a*b^3)*cosh
(d*x + c)^2 + 4*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^6 + 30*(8*a^3*b - a*b^3)
*cosh(d*x + c)^4 - 16*a^3*b + 2*a*b^3 + 3*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - b^4)*cosh(d*x + c)^7 + 6*(8*a
^3*b - a*b^3)*cosh(d*x + c)^5 + (64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^3
 - 2*(8*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^2 + b^2)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
 c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(d*x
+ c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*sinh(d
*x + c) + 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d
*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x + c))*
sinh(d*x + c) - b)) + 4*(3*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^5 + 1
2*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^3 - (160*a^4*b + 44*a^2*b^3 +
b^5)*cosh(d*x + c))*sinh(d*x + c))/((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 +
 b^8)*d*cosh(d*x + c)^8 + 8*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8
)*d*sinh(d*x + c)^8 + 8*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh
(d*x + c)^6 + 4*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x
+ c)^2 + 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^6
+ 2*(512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^
4 + 8*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^3 + 6
*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^4
+ 60*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + (512*
a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d)*sinh(d*x + c)^4 - 8*(6
4*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(64*a^6
*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^5 + 20*(64*a^7*b + 48
*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + (512*a^8 + 320*a^6*b^2 +
 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(64*
a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^6 + 30*(64*a^7*b +
 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^4 + 3*(512*a^8 + 320*a^6*
b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 - 2*(64*a^7*b + 48*a^
5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^2 + (64*a^6*b^2 + 48*a^4*b^4 +
 12*a^2*b^6 + b^8)*d + 8*((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*co
sh(d*x + c)^7 + 6*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x +
 c)^5 + (512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x +
```

```
c)^3 - 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.57034, size = 351, normalized size = 2.45

$$-\frac{2\left(8a^2-b^2\right)\log\left(\frac{\left|-2be^{(2dx+2c)}-4a-2\sqrt{4a^2+b^2}\right|}{\left|-2be^{(2dx+2c)}-4a+2\sqrt{4a^2+b^2}\right|}\right)}{\left(16a^4d+8a^2b^2d+b^4d\right)\sqrt{4a^2+b^2}}+\frac{4\left(8a^2be^{(6dx+6c)}-b^3e^{(6dx+6c)}+48a^3e^{(4dx+4c)}-6ab^2e^{(4dx+4c)}-40a^2b\right.}{\left(16a^4d+8a^2b^2d+b^4d\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(8*a^2 - b^2)*log(abs(-2*b*e^(2*d*x + 2*c) - 4*a - 2*sqrt(4*a^2 + b^2))/
abs(-2*b*e^(2*d*x + 2*c) - 4*a + 2*sqrt(4*a^2 + b^2)))/((16*a^4*d + 8*a^2*b
^2*d + b^4*d)*sqrt(4*a^2 + b^2)) + 4*(8*a^2*b*e^(6*d*x + 6*c) - b^3*e^(6*d*
x + 6*c) + 48*a^3*e^(4*d*x + 4*c) - 6*a*b^2*e^(4*d*x + 4*c) - 40*a^2*b*e^(2
*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + 6*a*b^2)/((16*a^4*d + 8*a^2*b^2*d + b^4
*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b)^2)
```

3.861 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=301

$$\frac{2i\sqrt{2}a\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{15d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx\right.\right.}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] (2*Sqrt[2]*a*b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(15*d) +
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) - ((
I/60)*(92*a^2 - 9*b^2)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*
I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2
*c + 2*d*x])/(2*a - I*b)]) + (((2*I)/15)*Sqrt[2]*a*(4*a^2 + b^2)*EllipticF[
((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c
 + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rubi [A] time = 0.39189, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2666, 2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\sqrt{2}a\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{15d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx\right.\right.}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```


```
[Out] (2*Sqrt[2]*a*b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(15*d) +
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) - ((
I/60)*(92*a^2 - 9*b^2)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*
I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2
*c + 2*d*x])/(2*a - I*b)]) + (((2*I)/15)*Sqrt[2]*a*(4*a^2 + b^2)*EllipticF[
((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c
 + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```


Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

```
x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}\,dx$$

$$= \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{2}{5}\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sin}{20\sqrt{2}d}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sin}{20\sqrt{2}d}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sin}{20\sqrt{2}d}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sin}{20\sqrt{2}d}$$

Mathematica [A] time = 1.37497, size = 239, normalized size = 0.79

$$\frac{-32ia\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)+2\left(92a^2b+184ia^3-18iab^2-9b^3\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}E}{120d\sqrt{4a+2b\sinh(2(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*((184*I)*a^3 + 92*a^2*b - (18*I)*a*b^2 - 9*b^3)*EllipticE[((-4*I)*c + Pi
 - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(
2*a - I*b)] - (32*I)*a*(4*a^2 + b^2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/
4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] +
b*(88*a^2*Cosh[2*(c + d*x)] + b*(28*a + 3*b*Sinh[2*(c + d*x)])*Sinh[4*(c +
d*x)]))/(120*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

Maple [B] time = 0.542, size = 1260, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x)
```

```
[Out] 1/60*(64*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)
*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*
a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3*b+1
6*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b
+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^3+240*(-(2
*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(
1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^4+24*(-(2*a+b*sinh(2
*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+si
nh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-
2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b^2-9*(-(2*a+b*sinh(2*d*x+2*c
))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x
+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1
/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^4-368*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^4-56*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*(
(-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(
1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+
2*a))^(1/2))*a^2*b^2+9*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2
*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*Ell
ipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1
/2))*b^4+3*b^4*sinh(2*d*x+2*c)^4+28*a*b^3*sinh(2*d*x+2*c)^3+44*a^2*b^2*sinh
(2*d*x+2*c)^2+3*b^4*sinh(2*d*x+2*c)^2+28*a*b^3*sinh(2*d*x+2*c)+44*a^2*b^2)/
b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cosh(dx+c)^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)\sinh(dx+c)+a^2\right)\sqrt{b\cosh(dx+c)\sinh(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*
x + c) + a^2)*sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(5/2), x)
```

3.862 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=248

$$\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}-\frac{2i\sqrt{2}a\sqrt{2a+b\sinh(2c+2dx)}\ldots}{\ldots}$$

```
[Out] (b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(6*Sqrt[2]*d) - (((2*
I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a +
b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2
*a - I*b)]) + ((I/6)*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2
, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(Sqrt
[2]*d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rubi [A] time = 0.253067, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2666, 2656, 2752, 2663, 2661, 2655, 2653}

$$\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}-\frac{2i\sqrt{2}a\sqrt{2a+b\sinh(2c+2dx)}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```


```
[Out] (b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(6*Sqrt[2]*d) - (((2*
I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a +
b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2
*a - I*b)]) + ((I/6)*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2
, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(Sqrt
[2]*d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```


Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2} dx$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} + \frac{2}{3}\int \frac{\frac{1}{8}\left(12a^2-b^2\right)+ab\sinh(2c}{\sqrt{a+\frac{1}{2}b\sinh(2c+2d}}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} + \frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sinh(2c+2d}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} + \frac{\left(4a\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int}{3\sqrt{\frac{a+\frac{1}{2}b\sinh(2c}{a-\frac{ib}{2}}}}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} - \frac{2i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+}\right.}{3d\sqrt{\frac{2a+b\sinh(}{2a-}}}$$

Mathematica [A] time = 0.803819, size = 202, normalized size = 0.81

$$\frac{-2i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)+b(4a\cosh(2(c+dx))+b\sinh(4(c+dx)))+16a(b+2i}{12d\sqrt{4a+2b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (16*a*((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*
a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] - (2*I)*(4*a^2 + b^
2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] + b*(4*a*Cosh[2*(c + d*x)] + b*Sinh[
4*(c + d*x)]))/(12*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

Maple [B] time = 0.49, size = 935, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(4*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b
/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+
b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b+I*(
-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a)
)^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d
*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^3+24*(-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I
+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I
*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3+6*(-(2*a+b*sinh(2*d*x+2*c)
)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+
2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/
2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^2-32*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^3-8*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((
-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1
/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2
*a))^(1/2))*a*b^2+b^3*sinh(2*d*x+2*c)^3+2*a*b^2*sinh(2*d*x+2*c)^2+b^3*sinh(
2*d*x+2*c)+2*a*b^2)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

3.863 $\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{i\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] ((-I)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2
*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a
 - I*b)])
```

Rubi [A] time = 0.0726453, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2666, 2655, 2653}

$$-\frac{i\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2
*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a
 - I*b)])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx &= \int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\ &= \frac{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\int \sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}}\\ &= -\frac{iE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}\end{aligned}$$

Mathematica [A] time = 0.108877, size = 94, normalized size = 0.98

$$\frac{(b+2ia)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}E\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)}{d\sqrt{4a+2b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] (((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I
*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[4*a + 2*b*Sinh[
2*(c + d*x)]])
```

Maple [B] time = 0.396, size = 351, normalized size = 3.7

$$\frac{ib-2\,a}{b\cosh(2\,dx+2\,c)\,d}\sqrt{-\frac{2\,a+b\sinh(2\,dx+2\,c)}{ib-2\,a}}\sqrt{\frac{(-\sinh(2\,dx+2\,c)+i)\,b}{ib+2\,a}}\sqrt{\frac{(i+\sinh(2\,dx+2\,c))\,b}{ib-2\,a}}\left(i\mathrm{EllipticE}\left(\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x)
```

```
[Out] 1/b*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)
+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*(I*EllipticE
((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b
-I*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*
a))^(1/2))*b+2*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-
2*a)/(I*b+2*a))^(1/2))*a-2*a*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))
^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2)))/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2
*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.864 $\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] ((-I)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)
]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c +
 2*d*x]])
```

Rubi [A] time = 0.0784631, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2666, 2663, 2661}

$$-\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)
]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c +
 2*d*x]])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx$$

$$= \frac{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}$$

$$= -\frac{i\sqrt{2}F\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] time = 0.134803, size = 90, normalized size = 0.94

$$\frac{i\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)}{d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] (I*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2
])
```

Maple [A] time = 0.362, size = 181, normalized size = 1.9

$$-2\,\frac{ib-2\,a}{b\cosh(2\,dx+2\,c)\,\sqrt{4\,a+2\,b\sinh(2\,dx+2\,c)}d}\sqrt{-\frac{2\,a+b\sinh(2\,dx+2\,c)}{ib-2\,a}}\sqrt{\frac{(-\sinh(2\,dx+2\,c)+i)\,b}{ib+2\,a}}\sqrt{\frac{(i+\sinh(2}{ib-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x)
```

```
[Out] -2*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+
I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(
2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))/b/cos
h(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh{(dx+c)}\sinh{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cosh{(dx+c)}\sinh{(dx+c)}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sinh{(c+dx)}\cosh{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.865 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] (-2*Sqrt[2]*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqrt[2*a + b*Sinh[2*c + 2
*d*x]]) - ((2*I)*Sqrt[2]*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((
2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((4*a^2 + b^2)*d*Sqrt[(2*a +
b*Sinh[2*c + 2*d*x])/(2*a - I*b)])
```

Rubi [A] time = 0.113346, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2666, 2664, 21, 2655, 2653}

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```


```
[Out] (-2*Sqrt[2]*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqrt[2*a + b*Sinh[2*c + 2
*d*x]]) - ((2*I)*Sqrt[2]*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((
2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((4*a^2 + b^2)*d*Sqrt[(2*a +
b*Sinh[2*c + 2*d*x])/(2*a - I*b)])
```


Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{4\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int \sqrt{\cdots}}{\left(4a^2+b^2\right)\sqrt{\frac{a+\frac{1}{2}b\sinh\cdots}{a\cdots}}}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{2i\sqrt{2}E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\cdots}{\left(4a^2+b^2\right)d\sqrt{\frac{2a+b\,\mathrm{s}\cdots}{\cdots}}}$$

Mathematica [A] time = 0.473617, size = 119, normalized size = 0.75

$$\frac{2\left(-b\cosh(2(c+dx))+(b+2ia)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}E\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)\right)}{d\left(4a^2+b^2\right)\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*(-(b*Cosh[2*(c + d*x)]) + ((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)
*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*
b)]))/((4*a^2 + b^2)*d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2])
```

Maple [B] time = 0.452, size = 630, normalized size = 4.

$$4\,\frac{1}{\left(4\,a^2+b^2\right)b\cosh\left(2\,dx+2\,c\right)\sqrt{4\,a+2\,b\sinh\left(2\,dx+2\,c\right)}d}\left(4\sqrt{-\frac{2\,a+b\sinh\left(2\,dx+2\,c\right)}{ib-2\,a}}\sqrt{\frac{\left(-\sinh\left(2\,dx+2\,c\right)+i\right)b\cdots}{ib+2\,a}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x)
```

```
[Out] 4*(4*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*
b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*si
nh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2+(-(2*a+b*
sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*
((I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))
/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-4*(-(2*a+b*sinh(2*d*x+2
*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d
*x+2*c))*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^
(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2-(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a)
)^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d*x+2*c))*b/(I*
b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2
*a)/(I*b+2*a))^(1/2))*b^2-b^2*sinh(2*d*x+2*c)^2-b^2)/(4*a^2+b^2)/b/cosh(2*d
*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cosh(dx+c)\sinh(dx+c)+a}}{b^2\cosh(dx+c)^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)\sinh(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a)/(b^2*cosh(d*x + c)^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-3/2), x)
```

3.866 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{32\sqrt{2}ab\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)^2\sqrt{2a+b\sinh(2c+2dx)}}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2\right.\right.}{3d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(}}$$

```
[Out] (-4*Sqrt[2]*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d
*x])^(3/2)) - (32*Sqrt[2]*a*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)^2*d*Sqrt[
2*a + b*Sinh[2*c + 2*d*x]]) - (((32*I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi
/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((
4*a^2 + b^2)^2*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)]) + (((4*I)/3
)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sq
rt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/((4*a^2 + b^2)*d*Sqrt[2*a + b*
Sinh[2*c + 2*d*x]])
```

Rubi [A] time = 0.38078, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2666, 2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{32\sqrt{2}ab\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)^2\sqrt{2a+b\sinh(2c+2dx)}}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}F\left(\frac{1}{2}\left(2ic+2\right.\right.}{3d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```


```
[Out] (-4*Sqrt[2]*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d
*x])^(3/2)) - (32*Sqrt[2]*a*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)^2*d*Sqrt[
2*a + b*Sinh[2*c + 2*d*x]]) - (((32*I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi
/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((
4*a^2 + b^2)^2*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)]) + (((4*I)/3
)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sq
rt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/((4*a^2 + b^2)*d*Sqrt[2*a + b*
Sinh[2*c + 2*d*x]])
```


Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}}\,dx$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}\,dx}{3\left(4a^2+b^2\right)}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b\sinh(2c}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b\sinh(2c}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b\sinh(2c}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b\sinh(2c}$$

Mathematica [A] time = 1.59987, size = 237, normalized size = 0.73

$$\frac{4\sqrt{2}\left(-b\cosh(2(c+dx))\left(20a^2+8ab\sinh(2(c+dx))+b^2\right)+(b-2ia)(2a-ib)^2\left(\frac{2a+b\sinh(2(c+dx))}{2a-ib}\right)^{3/2}F\left(\frac{1}{4}(-4ic-4idx+\right.\right.}{3d\left(4a^2+b^2\right)^2(2a+b\sinh(2(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*((8*I)*a*(2*a - I*b)^2*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4,
((-2*I)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) + (
2*a - I*b)^2*((-2*I)*a + b)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I
)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) - b*Cosh[
2*(c + d*x)]*(20*a^2 + b^2 + 8*a*b*Sinh[2*(c + d*x)])))/(3*(4*a^2 + b^2)^2*
d*(2*a + b*Sinh[2*(c + d*x)])^(3/2))
```

Maple [A] time = 0.98, size = 641, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x)
```

```
[Out] 4*((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/2)*(-2/3/b/(4*a^2+b^2)*((2
*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/2)/(sinh(2*d*x+2*c)+2*a/b)^2-16
/3*b*cosh(2*d*x+2*c)^2/(4*a^2+b^2)^2*a/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+
2*c)^2)^(1/2)+2*(12*a^2-b^2)/(48*a^4+24*a^2*b^2+3*b^4)*(2*a/b-I)*((-b*sinh(
2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*(
(I+sinh(2*d*x+2*c))*b/(I*b-2*a))^(1/2)/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+
2*c)^2)^(1/2)*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I*
b)/(I*b+2*a))^(1/2))+16/3*a*b/(4*a^2+b^2)^2*(2*a/b-I)*((-b*sinh(2*d*x+2*c)-
2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((I+sinh(2*d
*x+2*c))*b/(I*b-2*a))^(1/2)/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/
2)*((-2*a/b-I)*EllipticE(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I
*b)/(I*b+2*a))^(1/2))+I*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2
),((2*a-I*b)/(I*b+2*a))^(1/2))))/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}}{b^3\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3+3\,ab^2\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2+3\,a^2b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a)/(b^3*cosh(d*x + c)^3*sinh(
d*x + c)^3 + 3*a*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 3*a^2*b*cosh(d*x + c
)*sinh(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.867 $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=386

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}$$

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^3*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (3*x
^2*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^
2]) - (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[
4*a^2 + b^2]) - (3*x*PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/
(2*Sqrt[4*a^2 + b^2]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(2*Sqrt[4*a^2 + b^2]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - Sqrt[4*a
^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a + Sq
rt[4*a^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Rubi [A] time = 0.604468, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5628, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^3*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (3*x
^2*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^
2]) - (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[
4*a^2 + b^2]) - (3*x*PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/
(2*Sqrt[4*a^2 + b^2]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(2*Sqrt[4*a^2 + b^2]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - Sqrt[4*a
^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a + Sq
rt[4*a^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{a+\frac{1}{2}b\sinh(2x)}\,dx$$

$$= 2\int \frac{e^{2x}x^3}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x^3}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{3\int x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} + \frac{3\int}{}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a}\right.}{2\sqrt{4a^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a}\right.}{2\sqrt{4a^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a}\right.}{2\sqrt{4a^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a}\right.}{2\sqrt{4a^2}}$$

Mathematica [A] time = 0.366706, size = 279, normalized size = 0.72

$$\frac{6x^2\text{PolyLog}\left(2,\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right)-6x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)-6x\text{PolyLog}\left(3,\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right)+6x\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right.}{4\sqrt{4a^2+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 4*x^3*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 6*x^2*PolyLog[2, (b*E^(2*x))/(-2*a + Sq
rt[4*a^2 + b^2])] - 6*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]
))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + 6*x*PolyLog[
3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))] + 3*PolyLog[4, (b*E^(2*x))/(-2
*a + Sqrt[4*a^2 + b^2])] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Maple [B] time = 0.059, size = 687, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x^3+2/
(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2
)^(1/2)))*a*x^3-1/2/(-2*a-(4*a^2+b^2)^(1/2))*x^4-1/(4*a^2+b^2)^(1/2)/(-2*a-
(4*a^2+b^2)^(1/2))*a*x^4+3/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/
(-2*a-(4*a^2+b^2)^(1/2)))*x^2+3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x^2-3/2/(-2*a-(4*a^2+b^2)^
(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-3/(4*a^2+b^2)^(1/2)
/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*
x+3/4/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)
))+3/2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*
a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x^3*ln(1-b*exp(2*x)/((4*a^2+b^2
)^(1/2)-2*a))-1/2/(4*a^2+b^2)^(1/2)*x^4+3/2/(4*a^2+b^2)^(1/2)*x^2*polylog(2
,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-3/2/(4*a^2+b^2)^(1/2)*x*polylog(3,b*ex
p(2*x)/((4*a^2+b^2)^(1/2)-2*a))+3/4/(4*a^2+b^2)^(1/2)*polylog(4,b*exp(2*x)/
((4*a^2+b^2)^(1/2)-2*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cosh{(x)}\sinh{(x)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

Fricas [C] time = 2.00767, size = 3650, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^3*sqrt((4*a^2 + b^2)/
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 3*b*x^2*s
qrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*
sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b
) + b)/b + 1) + 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*s
inh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*
a^2 + b^2)/b^2) + 2*a)/b) - b)/b + 1) - 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dil
og(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2
)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 3*b*x^2*sqr
t((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sin
h(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) -
b)/b + 1) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sin
h(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^
2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a
```

```
*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 6*b*x*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a
^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 6*b*x*sqrt((
4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*s
inh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/
b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, (2*a*cosh(x) + 2*a*sinh(x) - (b
*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/
b^2) + 2*a)/b)/b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, -(2*a*cosh(x) +
2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqr
t((4*a^2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4,
(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2
))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^
2)*polylog(4, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4
*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cosh(x)\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

3.868 $\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=281

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}+$$

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^2*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (x*P
olyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] - (x
*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] -
PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
+ PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]
)
```

Rubi [A] time = 0.515022, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5628, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```


```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^2*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (x*P
olyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] - (x
*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] -
PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
+ PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]
)
```


Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a+\frac{1}{2}b\sinh(2x)}\,dx$$
$$= 2\int \frac{e^{2x}x^2}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^2}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x^2}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{2\int x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} + \frac{2\int x}{}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a}}\right.}{\sqrt{4a^2+b^2}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a}}\right.}{\sqrt{4a^2+b^2}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a}}\right.}{\sqrt{4a^2+b^2}}$$

Mathematica [A] time = 0.25076, size = 210, normalized size = 0.75

$$\frac{2x\text{PolyLog}\left(2,\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right) - 2x\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right) - \text{PolyLog}\left(3,\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right) + \text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right) + 2x}{2\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 2*x^2*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 2*x*PolyLog[2, (b*E^(2*x))/(-2*a + Sqrt
[4*a^2 + b^2])] - 2*x*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]
- PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + PolyLog[3, -((b*E^(2
*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^2])
```

Maple [B] time = 0.053, size = 530, normalized size = 1.9

$$-\frac{2x^3}{3}\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}+x^2\ln\left(1-be^{2x}\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}\right)\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}+x\mathrm{polylog}\left(2,be^{2x}\left(-2a-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] -2/3/(-2*a-(4*a^2+b^2)^(1/2))*x^3+1/(-2*a-(4*a^2+b^2)^(1/2))*x^2*ln(1-b*exp
(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+1/(-2*a-(4*a^2+b^2)^(1/2))*x*polylog(2,b*ex
p(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*e
xp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-4/3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(
1/2))*a*x^3+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*a*x^2*ln(1-b*exp(2
*x)/(-2*a-(4*a^2+b^2)^(1/2)))+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
a*x*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/(4*a^2+b^2)^(1/2)/(-2*
a-(4*a^2+b^2)^(1/2))*a*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-2/3/(
4*a^2+b^2)^(1/2)*x^3+1/(4*a^2+b^2)^(1/2)*x^2*ln(1-b*exp(2*x)/((4*a^2+b^2)^(
1/2)-2*a))+1/(4*a^2+b^2)^(1/2)*x*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*
a))-1/2/(4*a^2+b^2)^(1/2)*polylog(3,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

Fricas [C] time = 1.98174, size = 2738, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^2*sqrt((4*a^2 + b^2)/
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 2*b*x*sqr
t((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*si
nh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)
+ b)/b + 1) + 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(
x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2
+ b^2)/b^2) + 2*a)/b) - b)/b + 1) - 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(-((
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 2*b*x*sqrt((4*a^2
 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sq
rt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1
) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) - (b*
cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b
^2) + 2*a)/b)/b) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2
*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt
((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*
a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\sinh(x)\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Integral(x**2/(a + b*sinh(x)*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b \cosh(x) \sinh(x) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

3.869 $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=186

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}+\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}}-\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x*L
og[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + PolyLog[
2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]) - PolyLo
g[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
```

Rubi [A] time = 0.30264, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5628, 3322, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}+\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}}-\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x*L
og[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + PolyLog[
2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]) - PolyLo
g[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
```

Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{a+\frac{1}{2}b\sinh(2x)}\,dx$$
$$= 2\int \frac{e^{2x}x}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\int \log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} + \frac{\int \log\left(1+\right.}{\sqrt{}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{2a-\sqrt{4a^2+b^2}}\right)}{x}\,dx, x, e^{2x}\right)}{2\sqrt{4a^2+b^2}} +$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

Mathematica [C] time = 1.47401, size = 956, normalized size = 5.14

$$\frac{1}{2}\left(-\frac{i\pi\tanh^{-1}\left(\frac{2a\tanh(x)-b}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{2\cos^{-1}\left(-\frac{2ia}{b}\right)\tanh^{-1}\left(\frac{(2a+ib)\cot\left(\frac{1}{4}(4ix+\pi)\right)}{\sqrt{-4a^2-b^2}}\right) + (\pi-4ix)\tanh^{-1}\left(\frac{(2a-ib)\tan\left(\frac{1}{4}(4ix+\pi)\right)}{\sqrt{-4a^2-b^2}}\right) - \left(\mathrm{co}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (((-I)*Pi*ArcTanh[(-b + 2*a*Tanh[x])/Sqrt[4*a^2 + b^2]])/Sqrt[4*a^2 + b^2]
- (2*ArcCos[((-2*I)*a)/b]*ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[
-4*a^2 - b^2]] + (Pi - (4*I)*x)*ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I)*x)/4])
/Sqrt[-4*a^2 - b^2]] - (ArcCos[((-2*I)*a)/b] + (2*I)*ArcTanh[((2*a + I*b)*C
ot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]])*Log[(((2*I)*a + b)*((-2*I)*a + b
 + Sqrt[-4*a^2 - b^2])*(1 + I*Cot[(Pi + (4*I)*x)/4]))/(b*((2*I)*a + b + I*S
qrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] - (ArcCos[((-2*I)*a)/b] - (2*I)*
ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]])*Log[(((2*I
)*a + b)*((2*I)*a - b + Sqrt[-4*a^2 - b^2])*(I + Cot[(Pi + (4*I)*x)/4]))/(b
*(2*a - I*b + Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] + (ArcCos[((-2*I)
```

```
*a)/b] - (2*I)*ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^
2]] - (2*I)*ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]]
)*Log[-((-1)^(3/4)*Sqrt[-4*a^2 - b^2])/(2*Sqrt[(-I)*b]*E^x*Sqrt[a + b*Cosh[
x]*Sinh[x]])] + (ArcCos[((-2*I)*a)/b] + (2*I)*(ArcTanh[((2*a + I*b)*Cot[(Pi
 + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]] + ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I)*
x)/4])/Sqrt[-4*a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 - b^2]*E^x)/(2*Sqr
t[(-I)*b]*Sqrt[a + b*Cosh[x]*Sinh[x]])] + I*(PolyLog[2, (((2*I)*a + Sqrt[-4
*a^2 - b^2])*((2*I)*a + b - I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))/(b
*((2*I)*a + b + I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] - PolyLog[2,
((2*a + I*Sqrt[-4*a^2 - b^2])*(-2*a + I*b + Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4
*I)*x)/4]))/(b*((2*I)*a + b + I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))]
))/Sqrt[-4*a^2 - b^2])/2
```

Maple [B] time = 0.05, size = 376, normalized size = 2.

$$x\ln\left(1-b{e}^{2x}\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}\right)\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}-x^2\left(-2a-\sqrt{4a^2+b^2}\right)^{-1}+2\frac{ax}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-1/(-
2*a-(4*a^2+b^2)^(1/2))*x^2+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(
1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x-2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2
+b^2)^(1/2))*a*x^2+1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/(-2*a-
(4*a^2+b^2)^(1/2)))+1/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,
b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x*ln(1-b*exp(2*x
)/((4*a^2+b^2)^(1/2)-2*a))-1/(4*a^2+b^2)^(1/2)*x^2+1/2/(4*a^2+b^2)^(1/2)*po
lylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

Fricas [B] time = 1.87469, size = 1814, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
+ b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*
a)/b) + b)/b) + b*x*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x
) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 +
 b^2)/b^2) + 2*a)/b) - b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x
) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*
sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log
(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/
b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + b*sqrt((4*a^2 + b
^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt(
(4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) + b)/b + 1)
+ b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
+ b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*
a)/b) - b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*si
nh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^
2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a
*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1))/(4*a^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b \sinh(x) \cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Integral(x/(a + b*sinh(x)*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

3.870 $\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + (b*Sinh[2*x])/2)), x]
```

Rubi [A] time = 0.0939793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + (b*Sinh[2*x])/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)}\,dx$$

Mathematica [A] time = 0.964546, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
```

```
[Out] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])), x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cosh\left(x\right)\sinh\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b\cosh\left(x\right)\sinh\left(x\right)+a\right)}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{bx\cosh\left(x\right)\sinh\left(x\right)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*cosh(x)*sinh(x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sinh\left(x\right)\cosh\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Integral(1/(x*(a + b*sinh(x)*cosh(x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cosh\left(x\right)\sinh\left(x\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

3.871 $\int F^{c(a+bx)} \sinh^n(d+ex)\,dx$

Optimal. Leaf size=95

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \sinh^n(d+ex)\, {}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);e^{2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -(e*n - b*c*Log[F])/(2*e), (2 - n
+ (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d + e*x)
))^n*(e*n - b*c*Log[F])))
```

Rubi [A] time = 0.148316, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5482, 2259}

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \sinh^n(d+ex)\, {}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);e^{2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -(e*n - b*c*Log[F])/(2*e), (2 - n
+ (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d + e*x)
))^n*(e*n - b*c*Log[F])))
```

Rule 5482

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Sinh[d + e*x]^n)/(-1 + E^(2*(d + e*x)))^n, Int
[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = \left(e^{n(d+ex)}\left(-1+e^{2(d+ex)}\right)^{-n}\sinh^n(d+ex)\right)\int e^{-n(d+ex)}\left(-1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$= -\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)\sinh^n(d+ex)}{en-bc\log(F)}$$

Mathematica [A] time = 0.069411, size = 96, normalized size = 1.01

$$\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)}\sinh^n(d+ex)\,{}_2F_1\left(-n,\frac{bc\log(F)-en}{2e};\frac{bc\log(F)-en}{2e}+1;e^{2(d+ex)}\right)}{bc\log(F)-en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F])/(2*e), 1 + (-(
e*n) + b*c*Log[F])/(2*e), E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d +
 e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\sinh(ex+d)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sinh(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sinh\left(ex+d\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sinh(e*x + d)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sinh\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

3.872 $\int e^{2(a+bx)} \sinh^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

```
[Out] E^(-a - b*x)/(8*b) + (3*E^(a + b*x))/(8*b) - E^(3*a + 3*b*x)/(8*b) + E^(5*a
 + 5*b*x)/(40*b)
```

Rubi [A] time = 0.0360092, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 270}

$$\frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-a - b*x)/(8*b) + (3*E^(a + b*x))/(8*b) - E^(3*a + 3*b*x)/(8*b) + E^(5*a
 + 5*b*x)/(40*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(3-\frac{1}{x^2}-3x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

Mathematica [A] time = 0.0342435, size = 50, normalized size = 0.76

$$\frac{e^{-a-bx}\left(15e^{2(a+bx)}-5e^{4(a+bx)}+e^{6(a+bx)}+5\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(-a - b*x)*(5 + 15*E^(2*(a + b*x)) - 5*E^(4*(a + b*x)) + E^(6*(a + b*x))
))/(40*b)
```

Maple [A] time = 0.022, size = 80, normalized size = 1.2

$$\frac{\sinh(bx+a)}{4\,b}-\frac{\sinh(3\,bx+3\,a)}{8\,b}+\frac{\sinh(5\,bx+5\,a)}{40\,b}+\frac{\cosh(bx+a)}{2\,b}-\frac{\cosh(3\,bx+3\,a)}{8\,b}+\frac{\cosh(5\,bx+5\,a)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*sinh(b*x+a)/b-1/8/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)+1/2*cosh(b*x
+a)/b-1/8*cosh(3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

Maxima [A] time = 1.23995, size = 72, normalized size = 1.09

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}-1\right)e^{(5\,bx+5\,a)}}{40\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) - 1)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

Fricas [A] time = 1.75249, size = 289, normalized size = 4.38

$$\frac{3\,\cosh\left(bx+a\right)^{3}+9\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-2\,\sinh\left(bx+a\right)^{3}-2\left(3\,\cosh\left(bx+a\right)^{2}+5\right)\sinh\left(bx+a\right)+5\,\cosh\left(\right.}{20\left(b\cosh\left(bx+a\right)^{2}-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - 2*sinh(b*x + a)
^3 - 2*(3*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x
 + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 38.2161, size = 124, normalized size = 1.88

$$\begin{cases}\frac{2e^{2a}e^{2bx}\sinh^{3}(a+bx)}{5b}+\frac{e^{2a}e^{2bx}\sinh^{2}(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{2a}e^{2bx}\sinh(a+bx)\cosh^{2}(a+bx)}{5b}+\frac{2e^{2a}e^{2bx}\cosh^{3}(a+bx)}{5b} & \text{for } b\neq 0\\ xe^{2a}\sinh^{3}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(5*b) + exp(2*a)*exp(2*b*
x)*sinh(a + b*x)**2*cosh(a + b*x)/(5*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*
```

```
x)*cosh(a + b*x)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(5*b), N
e(b, 0)), (x*exp(2*a)*sinh(a)**3, True))
```

Giac [A] time = 1.13422, size = 72, normalized size = 1.09

$$\frac{\left(e^{(5\,bx+10\,a)} - 5\,e^{(3\,bx+8\,a)} + 15\,e^{(bx+6\,a)}\right)e^{(-5\,a)} + 5\,e^{(-bx-a)}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/40*((e^(5*b*x + 10*a) - 5*e^(3*b*x + 8*a) + 15*e^(b*x + 6*a))*e^(-5*a) +
5*e^(-b*x - a))/b
```

3.873 $\int e^{2(a+bx)} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}$$

```
[Out] -E^(2*a + 2*b*x)/(4*b) + E^(4*a + 4*b*x)/(16*b) + x/4
```

Rubi [A] time = 0.035378, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 266, 43}

$$-\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(2*a + 2*b*x)/(4*b) + E^(4*a + 4*b*x)/(16*b) + x/4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{4x}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{8b}\\
&= -\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}
\end{aligned}$$

Mathematica [A] time = 0.0191898, size = 32, normalized size = 0.8

$$\frac{-4e^{2(a+bx)}+e^{4(a+bx)}+4bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*E^(2*(a + b*x)) + E^(4*(a + b*x)) + 4*b*x)/(16*b)
```

Maple [A] time = 0.016, size = 61, normalized size = 1.5

$$\frac{x}{4}-\frac{\sinh(2\,bx+2\,a)}{4\,b}+\frac{\sinh(4\,bx+4\,a)}{16\,b}-\frac{\cosh(2\,bx+2\,a)}{4\,b}+\frac{\cosh(4\,bx+4\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/4*x-1/4*sinh(2*b*x+2*a)/b+1/16/b*sinh(4*b*x+4*a)-1/4*cosh(2*b*x+2*a)/b+1/
16*cosh(4*b*x+4*a)/b
```

Maxima [A] time = 1.25528, size = 50, normalized size = 1.25

$$\frac{1}{4}\,x - \frac{\left(4\,e^{(-2\,bx-2\,a)} - 1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{a}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/16*(4*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + 1/4*a/b
```

Fricas [B] time = 1.80609, size = 254, normalized size = 6.35

$$\frac{(4\,bx+1)\cosh{(bx+a)}^2 - 2\,(4\,bx-1)\cosh{(bx+a)}\sinh{(bx+a)} + (4\,bx+1)\sinh{(bx+a)}^2 - 4}{16\left(b\cosh{(bx+a)}^2 - 2\,b\cosh{(bx+a)}\sinh{(bx+a)} + b\sinh{(bx+a)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*b*x + 1)*cosh(b*x + a)^2 - 2*(4*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a) + (4*b*x + 1)*sinh(b*x + a)^2 - 4)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 7.28974, size = 139, normalized size = 3.48

$$\begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2{(a+bx)}}{4} - \frac{xe^{2a}e^{2bx}\sinh{(a+bx)}\cosh{(a+bx)}}{2} + \frac{xe^{2a}e^{2bx}\cosh^2{(a+bx)}}{4} + \frac{e^{2a}e^{2bx}\sinh^2{(a+bx)}}{2b} - \frac{e^{2a}e^{2bx}\sinh{(a+bx)}\cosh{(a+bx)}}{4b} & \text{for } b \\ xe^{2a}\sinh^2{(a)} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)*cosh(a + b*x)/2 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4 +
 exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/(2*b) - exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2, True))
```

Giac [A] time = 1.14819, size = 41, normalized size = 1.02

$$\frac{4\,bx + e^{(4\,bx+4\,a)} - 4\,e^{(2\,bx+2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x + e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a))/b
```

3.874 $\int e^{2(a+bx)} \sinh(a+bx)\,dx$

Optimal. Leaf size=32

$$\frac{e^{3a+3bx}}{6b}-\frac{e^{a+bx}}{2b}$$

```
[Out] -E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(6*b)
```

Rubi [A] time = 0.0173545, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12}

$$\frac{e^{3a+3bx}}{6b}-\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

```
[Out] -E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(6*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1}{2}\left(-1+x^2\right)\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= -\frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b}$$

Mathematica [A] time = 0.0127617, size = 25, normalized size = 0.78

$$\frac{e^{a+bx}\left(e^{2(a+bx)}-3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

```
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(6*b)
```

Maple [A] time = 0.005, size = 52, normalized size = 1.6

$$-\frac{\sinh(bx+a)}{2\,b} + \frac{\sinh(3\,bx+3\,a)}{6\,b} - \frac{\cosh(bx+a)}{2\,b} + \frac{\cosh(3\,bx+3\,a)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a),x)
```

```
[Out] -1/2*sinh(b*x+a)/b+1/6/b*sinh(3*b*x+3*a)-1/2*cosh(b*x+a)/b+1/6*cosh(3*b*x+3
*a)/b
```

Maxima [A] time = 1.28741, size = 35, normalized size = 1.09

$$\frac{e^{(3\,bx+3\,a)}}{6\,b} - \frac{e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

Fricas [B] time = 1.69578, size = 153, normalized size = 4.78

$$\frac{\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 3}{6\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3)
/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 2.51426, size = 54, normalized size = 1.69

$$\begin{cases} \frac{2e^{2a}e^{2bx}\sinh(a+bx)}{3b} - \frac{e^{2a}e^{2bx}\cosh(a+bx)}{3b} & \text{for } b \neq 0 \\ xe^{2a}\sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)/(3*b) - exp(2*a)*exp(2*b*x)*
cosh(a + b*x)/(3*b), Ne(b, 0)), (x*exp(2*a)*sinh(a), True))
```

Giac [A] time = 1.13691, size = 31, normalized size = 0.97

$$\frac{e^{(3\,bx+3\,a)} - 3\,e^{(bx+a)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(e^(3*b*x + 3*a) - 3*e^(b*x + a))/b
```

3.875 $\int e^{2(a+bx)}\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{2e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (2*E^(a + b*x))/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0180458, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 321, 207}

$$\frac{2e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\text{csch}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{2x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0173468, size = 23, normalized size = 0.88

$$\frac{2\left(e^{a+bx}-\tanh^{-1}\left(e^{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x],x]
```

```
[Out] (2*(E^(a + b*x) - ArcTanh[E^(a + b*x)]))/b
```

Maple [A] time = 0.033, size = 40, normalized size = 1.5

$$2\,\frac{e^{bx+a}}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(1+e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a),x)
```

```
[Out] 2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(1+exp(b*x+a))
```

Maxima [A] time = 1.28773, size = 61, normalized size = 2.35

$$\frac{2e^{(bx+a)}}{b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 1.82856, size = 163, normalized size = 6.27

$$\frac{2\cosh\left(bx+a\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+2\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x +
a) + sinh(b*x + a) - 1) + 2*sinh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{2a}\int e^{2bx}\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x), x)
```

Giac [B] time = 1.12418, size = 68, normalized size = 2.62

$$-\frac{\left(e^{(-2a)}\log\left(e^{(bx+a)}+1\right)-e^{(-2a)}\log\left(\left|e^{(bx+a)}-1\right|\right)-2\,e^{(bx-a)}\right)e^{(2a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -(e^(-2*a)*log(e^(b*x + a) + 1) - e^(-2*a)*log(abs(e^(b*x + a) - 1)) - 2*e^
(b*x - a))*e^(2*a)/b
```

3.876 $\int e^{2(a+bx)}\mathrm{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.0427926, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 266, 43}

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^3}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{4\operatorname{Subst}\left(\int \frac{x^3}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2\operatorname{Subst}\left(\int \frac{x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\ &= \frac{2\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\ &= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0489636, size = 37, normalized size = 0.88

$$\frac{2\left(\frac{1}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

```
[Out] (2*((1 - E^(2*a + 2*b*x))^(-1) + Log[1 - E^(2*a + 2*b*x)]))/b
```

Maple [A] time = 0.032, size = 43, normalized size = 1.

$$-4\,\frac{a}{b}-2\,\frac{1}{b\left(e^{2\,bx+2\,a}-1\right)}+2\,\frac{\ln\left(e^{2\,bx+2\,a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^2,x)
```

```
[Out] -4*a/b-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

Maxima [A] time = 1.45041, size = 84, normalized size = 2.

$$4x + \frac{4a}{b} + \frac{2\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{2\log\left(e^{(-bx-a)} - 1\right)}{b} + \frac{2}{b\left(e^{(-2bx-2a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 2/(b*
(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 1.77485, size = 286, normalized size = 6.81

$$\frac{2\left(\left(\cosh\left(bx+a\right)^2 + 2\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sinh\left(bx+a\right)^2 - 1\right)\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right) - 1\right)}{b\cosh\left(bx+a\right)^2 + 2b\cosh\left(bx+a\right)\sinh\left(bx+a\right) + b\sinh\left(bx+a\right)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 1)/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{2a}\int e^{2bx}\operatorname{csch}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**2,x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**2, x)
```

Giac [A] time = 1.15745, size = 65, normalized size = 1.55

$$\frac{2\left(e^{(-2a)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)-\frac{e^{(2bx)}}{e^{(2bx+2a)}-1}\right)e^{(2a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*(e^(-2*a)*log(abs(e^(2*b*x + 2*a) - 1)) - e^(2*b*x)/(e^(2*b*x + 2*a) - 1)
)*e^(2*a)/b
```

3.877 $\int e^{2(a+bx)}\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (-2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 -
E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.044398, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 288, 207}

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 -
E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\operatorname{csch}^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{8x^4}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{8\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{6\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0583474, size = 61, normalized size = 0.84

$$\frac{3e^{a+bx} - 5e^{3(a+bx)} - 3\left(e^{2(a+bx)}-1\right)^2\tanh^{-1}\left(e^{a+bx}\right)}{b\left(e^{2(a+bx)}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

```
[Out] (3*E^(a + b*x) - 5*E^(3*(a + b*x)) - 3*(-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(
a + b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

Maple [A] time = 0.037, size = 67, normalized size = 0.9

$$-\frac{e^{bx+a}\left(5\,e^{2\,bx+2\,a}-3\right)}{b\left(e^{2\,bx+2\,a}-1\right)^{2}}-\frac{3\,\ln\left(1+e^{bx+a}\right)}{2\,b}+\frac{3\,\ln\left(e^{bx+a}-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-3/2/b*ln(1+exp(b*x+
a))+3/2/b*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.05462, size = 119, normalized size = 1.63

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{5\,e^{(-bx-a)}-3\,e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1)/b + (5*e^(-b*x - a
) - 3*e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.72627, size = 1098, normalized size = 15.04

$$-\frac{10\,\cosh\left(bx+a\right)^{3}+30\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+10\,\sinh\left(bx+a\right)^{3}+3\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(10*cosh(b*x + a)^3 + 30*cosh(b*x + a)*sinh(b*x + a)^2 + 10*sinh(b*x +
 a)^3 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a
)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(co
```

```
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) + 1) - 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 6*cosh(b*
x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{2a} \int e^{2bx} \operatorname{csch}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**3,x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**3, x)
```

Giac [A] time = 1.15136, size = 105, normalized size = 1.44

$$-\frac{\left(3\,e^{(-2\,a)}\log\left(e^{(bx+a)}+1\right)-3\,e^{(-2\,a)}\log\left(\left|e^{(bx+a)}-1\right|\right)+\frac{2\left(5\,e^{(3\,bx+2\,a)}-3\,e^{(bx)}\right)e^{(-a)}}{\left(e^{(2\,bx+2\,a)}-1\right)^2}\right)e^{(2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*e^(-2*a)*log(e^(b*x + a) + 1) - 3*e^(-2*a)*log(abs(e^(b*x + a) - 1)
) + 2*(5*e^(3*b*x + 2*a) - 3*e^(b*x))*e^(-a)/(e^(2*b*x + 2*a) - 1)^2)*e^(2*
a)/b
```

3.878 $\int e^{a+bx}\sinh^3(c+dx)\,dx$

Optimal. Leaf size=139

$$\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{6d^3e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh^2(c+dx)\cosh(c+dx)}{b^2-9d^2}$$

```
[Out] (-6*d^3*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (6*b*d^2*E^
(a + b*x)*Sinh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh
[c + d*x]*Sinh[c + d*x]^2)/(b^2 - 9*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^3)/
(b^2 - 9*d^2)
```

Rubi [A] time = 0.0630272, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5476, 5474}

$$\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{6d^3e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh^2(c+dx)\cosh(c+dx)}{b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (-6*d^3*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (6*b*d^2*E^
(a + b*x)*Sinh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh
[c + d*x]*Sinh[c + d*x]^2)/(b^2 - 9*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^3)/
(b^2 - 9*d^2)
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = -\frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2}+\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}+\frac{\left(6d^2\right)\int e^{a+bx}\sinh(c+dx)\,dx}{b^2-9d^2}$$

$$= -\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}-\frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2}+\frac{be}{}$$

Mathematica [A] time = 0.488643, size = 108, normalized size = 0.78

$$\frac{e^{a+bx}\left(3d\left(b^2-9d^2\right)\cosh(c+dx)+\left(3d^3-3b^2d\right)\cosh(3(c+dx))+2b\sinh(c+dx)\left(\left(b^2-d^2\right)\cosh(2(c+dx))-b^2+13d\right.\right.}{4\left(-10b^2d^2+b^4+9d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*d*(b^2 - 9*d^2)*Cosh[c + d*x] + (-3*b^2*d + 3*d^3)*Cosh[3*(
c + d*x)] + 2*b*(-b^2 + 13*d^2 + (b^2 - d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*
x]))/(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

Maple [A] time = 0.028, size = 166, normalized size = 1.2

$$-\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8b-8d}-\frac{3\sinh(a+c+(b+d)x)}{8b+8d}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}-\frac{\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^3,x)
```

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/8*cosh(a-3*c+(b-3*d)*x)
/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)-3/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(a
+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79787, size = 752, normalized size = 5.41

$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)^3-\left(\left(b^3-bd^2\right)\cosh\left(bx+a\right)+\left(b^3-bd^2\right)\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)^3-3\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^3 - ((b^3 - b*d^2)*cosh(b
*x + a) + (b^3 - b*d^2)*sinh(b*x + a))*sinh(d*x + c)^3 - 3*(b^2*d - 9*d^3)*
cosh(b*x + a)*cosh(d*x + c) + 9*((b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)
+ (b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((b^2*d -
d^3)*cosh(d*x + c)^3 - (b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b
^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a) -
 (b^3 - 9*b*d^2 - (b^3 - b*d^2)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x +
c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13631, size = 113, normalized size = 0.81

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)}-\frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)}-\frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) - 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)
```

3.879 $\int e^{a+bx} \sinh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}+\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

```
[Out] (2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh
[c + d*x])/(b^2 - 4*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^2)/(b^2 - 4*d^2)
```

Rubi [A] time = 0.0361949, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5476, 2194}

$$\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}+\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

```
[Out] (2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh
[c + d*x])/(b^2 - 4*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^2)/(b^2 - 4*d^2)
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{a+bx}\sinh^2(c+dx)\,dx=-\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}+\frac{(2d^2)\int e^{a+bx}\,dx}{b^2-4d^2}$$
$$=\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}-\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}$$

Mathematica [A] time = 0.151616, size = 58, normalized size = 0.66

$$\frac{e^{a+bx}\left(b^2\cosh(2(c+dx))-b^2-2bd\sinh(2(c+dx))+4d^2\right)}{2\left(b^3-4bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(-b^2 + 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)
]))/(2*(b^3 - 4*b*d^2))
```

Maple [A] time = 0.016, size = 112, normalized size = 1.3

$$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^2,x)
```

```
[Out] -1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)
*x)/(b+2*d)-1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+
2*c+(b+2*d)*x)/(b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52869, size = 382, normalized size = 4.34

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2+\left(b^2\cosh(bx+a)+b^2\sinh(bx+a)\right)\sinh(dx+c)^2-\left(b^2-4\,d^2\right)\cosh(bx+a)+\left(b^2\,\mathrm{c}\right.}{2\left(b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
- b^2 + 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

Sympy [A] time = 14.8118, size = 476, normalized size = 5.41

$$\begin{cases} xe^a\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2}-\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^a \\ \frac{xe^ae^{-2dx}\sinh^2(c+dx)}{4}+\frac{xe^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{-2dx}\cosh^2(c+dx)}{4}-\frac{e^ae^{-2dx}\sinh^2(c+dx)}{8d}+\frac{e^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3e^ae^{-\cdots}}{} \\ \frac{xe^ae^{2dx}\sinh^2(c+dx)}{4}-\frac{xe^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{2dx}\cosh^2(c+dx)}{4}+\frac{e^ae^{2dx}\sinh^2(c+dx)}{8d}+\frac{e^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3e^ae^{2dx}\cos\cdots}{8d} \\ \frac{b^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2bde^ae^{bx}\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2}-\frac{2d^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2}+\frac{2d^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2/
2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b,
 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh(
c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 - exp(a)
*exp(-2*d*x)*sinh(c + d*x)**2/(8*d) + exp(a)*exp(-2*d*x)*sinh(c + d*x)*cosh
(c + d*x)/(2*d) + 3*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, -2*d))
```

```
, (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 - x*exp(a)*exp(2*d*x)*sinh(c + d*
x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*cosh(c + d*x)**2/4 + exp(a)*exp(2*
d*x)*sinh(c + d*x)**2/(8*d) + exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)
/(2*d) - 3*exp(a)*exp(2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, 2*d)), (b**2*exp
(a)*exp(b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sin
h(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*sinh(c
+ d*x)**2/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3
 - 4*b*d**2), True))
```

Giac [A] time = 1.15619, size = 76, normalized size = 0.86

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)}-\frac{e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/2*e^(b*x + a)/b
```

3.880 $\int e^{a+bx}\sinh(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

```
[Out] -((d*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x]
)/(b^2 - d^2)
```

Rubi [A] time = 0.0186246, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5474}

$$\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x]
)/(b^2 - d^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\sinh(c+dx)\,dx=-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}+\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] time = 0.0694216, size = 38, normalized size = 0.7

$$\frac{e^{a+bx}(b\sinh(c+dx)-d\cosh(c+dx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(-(d*Cosh[c + d*x]) + b*Sinh[c + d*x]))/((b - d)*(b + d))
```

Maple [A] time = 0.004, size = 78, normalized size = 1.4

$$-\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}-\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c),x)
```

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)-1/2*cosh(a-c+(b-d)
*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51805, size = 176, normalized size = 3.26

$$-\frac{d\cosh(bx+a)\cosh(dx+c)+d\cosh(dx+c)\sinh(bx+a)-(b\cosh(bx+a)+b\sinh(bx+a))\sinh(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) + d*cosh(d*x + c)*sinh(b*x + a) - (b*cosh(b
*x + a) + b*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

Sympy [A] time = 3.03425, size = 167, normalized size = 3.09

$$\begin{cases} xe^{a}\sinh{(c)} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{xe^{a}e^{-dx}\sinh{(c+dx)}}{2} + \frac{xe^{a}e^{-dx}\cosh{(c+dx)}}{2} + \frac{e^{a}e^{-dx}\cosh{(c+dx)}}{2d} & \text{for}\: b = -d \\ \frac{xe^{a}e^{dx}\sinh{(c+dx)}}{2} - \frac{xe^{a}e^{dx}\cosh{(c+dx)}}{2} + \frac{e^{a}e^{dx}\cosh{(c+dx)}}{2d} & \text{for}\: b = d \\ \frac{be^{a}e^{bx}\sinh{(c+dx)}}{b^{2}-d^{2}} - \frac{de^{a}e^{bx}\cosh{(c+dx)}}{b^{2}-d^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*cosh(c
+ d*x)/(2*d), Eq(b, -d)), (x*exp(a)*exp(d*x)*sinh(c + d*x)/2 - x*exp(a)*exp
(d*x)*cosh(c + d*x)/2 + exp(a)*exp(d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*
exp(a)*exp(b*x)*sinh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*cosh(c + d*
x)/(b**2 - d**2), True))
```

Giac [A] time = 1.2214, size = 54, normalized size = 1.

$$\frac{e^{(bx+dx+a+c)}}{2\,(b+d)} - \frac{e^{(bx-dx+a-c)}}{2\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(b*x - d*x + a - c)/(b - d)
```

3.881 $\int e^{a+bx}\operatorname{csch}(c+dx)\,dx$

Optimal. Leaf size=50

$$-\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);e^{2(c+dx)}\right)}{b+d}$$

```
[Out] (-2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
E^(2*(c + d*x))])/(b + d)
```

Rubi [A] time = 0.0191584, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5493}

$$-\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[c + d*x],x]
```

```
[Out] (-2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
E^(2*(c + d*x))])/(b + d)
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\operatorname{csch}(c+dx)\,dx=-\frac{2e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] time = 0.132807, size = 59, normalized size = 1.18

$$-\frac{2(\sinh(c)+\cosh(c))e^{a+x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{b+3d}{2d};e^{2dx}(\cosh(c)+\sinh(c))^2\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[c + d*x],x]
```

```
[Out] (-2*E^(a + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (b + 3*d)/(2*d),
E^(2*d*x)*(Cosh[c] + Sinh[c])^2]*(Cosh[c] + Sinh[c]))/(b + d)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int e^{bx+a}\operatorname{csch}(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(d*x+c),x)
```

```
[Out] int(exp(b*x+a)*csch(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(dx+c)\,e^{(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{csch}(dx+c)\,e^{(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)*e^(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}\left(dx+c\right)e^{(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

3.882 $\int e^{c+dx}\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{4e^{2(a+bx)+c+dx}\,{}_2F_1\left(2,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] (4*E^(c + d*x + 2*(a + b*x))*Hypergeometric2F1[2, 1 + d/(2*b), 2 + d/(2*b),
 E^(2*(a + b*x))])/(2*b + d)
```

Rubi [A] time = 0.0294736, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5493}

$$\frac{4e^{2(a+bx)+c+dx}\,{}_2F_1\left(2,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(c + d*x + 2*(a + b*x))*Hypergeometric2F1[2, 1 + d/(2*b), 2 + d/(2*b),
 E^(2*(a + b*x))])/(2*b + d)
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=\frac{4e^{c+dx+2(a+bx)}\,{}_2F_1\left(2,1+\frac{d}{2b};2+\frac{d}{2b};e^{2(a+bx)}\right)}{2b+d}$$

Mathematica [B] time = 3.22928, size = 131, normalized size = 2.43

$$\frac{e^c\left(\operatorname{csch}(a)e^{dx}\sinh(bx)\operatorname{csch}(a+bx)-\frac{2e^{2a}d\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{e^{2a}-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (E^c*((-2*d*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^
(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 +
d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a)) + E^(d*x)*Csch[a]*Csc
h[a + b*x]*Sinh[b*x]))/b
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int e^{dx+c}\left(\operatorname{csch}\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,bd\int-\frac{e^{(dx+c)}}{8\,b^2-6\,bd+d^2-\left(8\,b^2-6\,bd+d^2\right)e^{(6\,bx+6\,a)}+3\left(8\,b^2-6\,bd+d^2\right)e^{(4\,bx+4\,a)}-3\left(8\,b^2-6\,bd+d^2\right)e^{(2\,bx+2\,a)}}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
```

```
d + d^2)*e^(2*b*x + 2*a)), x) - 4*((4*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 4*b*
e^c)*e^(d*x)/(8*b^2 - 6*b*d + d^2 + (8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) -
 2*(8*b^2 - 6*b*d + d^2)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{csch}\,(bx+a)^2\,e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{dx}\,\text{csch}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{csch}\,(bx+a)^2\,e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*e^(d*x + c), x)
```

3.883 $\int e^{c+dx}\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=100

$$\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{1}{2}\left(\frac{d}{b}+3\right);e^{2(a+bx)}\right)}{b^2}-\frac{de^{c+dx}\mathrm{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\mathrm{csch}(a+bx)}{2b}$$

```
[Out] -(d*E^(c + d*x)*Csch[a + b*x])/(2*b^2) - (E^(c + d*x)*Coth[a + b*x]*Csch[a
+ b*x])/(2*b) + ((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)
/(2*b), (3 + d/b)/2, E^(2*(a + b*x))])/b^2
```

Rubi [A] time = 0.0489449, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5491, 5493}

$$\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{1}{2}\left(\frac{d}{b}+3\right);e^{2(a+bx)}\right)}{b^2}-\frac{de^{c+dx}\mathrm{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\mathrm{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -(d*E^(c + d*x)*Csch[a + b*x])/(2*b^2) - (E^(c + d*x)*Coth[a + b*x]*Csch[a
+ b*x])/(2*b) + ((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)
/(2*b), (3 + d/b)/2, E^(2*(a + b*x))])/b^2
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
```

```
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = -\frac{de^{c+dx}\operatorname{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{1}{2}\left(1-\frac{d^2}{b^2}\right)\int e^{c+dx}\operatorname{csch}(a+bx)\,d$$

$$= -\frac{de^{c+dx}\operatorname{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)}{2b} + \frac{(b-d)e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{1}{2}\left(3\right.\right.}{b^2}$$

Mathematica [A] time = 2.354, size = 94, normalized size = 0.94

$$\frac{e^c\left(\frac{2\operatorname{csch}(a)(b-d)e^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)}{\coth(a)-1}-e^{dx}\operatorname{csch}(a+bx)(b\coth(a+bx)+d)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] (E^c*(-(E^(d*x)*(d + b*Coth[a + b*x])*Csch[a + b*x]) + (2*(b - d)*E^((b + d
)*x)*Csch[a]*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*b*x)
*(Cosh[a] + Sinh[a])^2])/(-1 + Coth[a])))/(2*b^2)
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int e^{dx+c}\left(\operatorname{csch}(bx+a)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*csch(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$48\left(b^2e^c+bde^c\right)\int\frac{e^{(bx+dx+a)}}{15\,b^2-8\,bd+d^2+\left(15\,b^2-8\,bd+d^2\right)e^{(8\,bx+8\,a)}-4\left(15\,b^2-8\,bd+d^2\right)e^{(6\,bx+6\,a)}+6\left(15\,b^2-8\,bd+d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 48*(b^2*e^c + b*d*e^c)*integrate(e^(b*x + d*x + a)/(15*b^2 - 8*b*d + d^2 +
(15*b^2 - 8*b*d + d^2)*e^(8*b*x + 8*a) - 4*(15*b^2 - 8*b*d + d^2)*e^(6*b*x
+ 6*a) + 6*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 4*(15*b^2 - 8*b*d + d^2
)*e^(2*b*x + 2*a)), x) + 8*((5*b*e^c - d*e^c)*e^(3*b*x + 3*a) - 6*b*e^(b*x
+ a + c))*e^(d*x)/(15*b^2 - 8*b*d + d^2 - (15*b^2 - 8*b*d + d^2)*e^(6*b*x +
 6*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 3*(15*b^2 - 8*b*d + d^2)
*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{csch}\left(bx+a\right)^{3}e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{dx}\operatorname{csch}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)**3,x)
```

```
[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}\left(bx+a\right)^{3}e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*e^(d*x + c), x)
```

3.884 $\int F^{c(a+bx)}\cosh^n(d+ex)\,dx$

Optimal. Leaf size=95

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);-e^{2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -((F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, -(e*n - b*c*Log[F]
)/(2*e), (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d + e*x
)))^n*(e*n - b*c*Log[F])))
```

Rubi [A] time = 0.117063, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5483, 2259}

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);-e^{2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, -(e*n - b*c*Log[F]
)/(2*e), (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d + e*x
)))^n*(e*n - b*c*Log[F])))
```

Rule 5483

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Cosh[d + e*x]^n)/(1 + E^(2*(d + e*x)))^n, Int[
(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; FreeQ[
{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \left(e^{n(d+ex)}\left(1+e^{2(d+ex)}\right)^{-n}\cosh^n(d+ex)\right)\int e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$= -\frac{\left(1+e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)}{en-bc\log(F)}$$

Mathematica [A] time = 0.0618008, size = 96, normalized size = 1.01

$$\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,\frac{bc\log(F)-en}{2e};\frac{bc\log(F)-en}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)-en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F]
)/(2*e), 1 + (-(e*n) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/((1 + E^(2*(d
+ e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\cosh(ex+d)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\cosh(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\cosh\left(ex+d\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*cosh(e*x + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\cosh^{n}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*cosh(d + e*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\cosh\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

3.885 $\int e^{a+bx}\cosh^3(c+dx)\,dx$

Optimal. Leaf size=139

$$\frac{6d^3e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}$$

```
[Out] (-6*b*d^2*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (b*E^(a +
 b*x)*Cosh[c + d*x]^3)/(b^2 - 9*d^2) + (6*d^3*E^(a + b*x)*Sinh[c + d*x])/(b
^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(b^2 - 9*d^2)
```

Rubi [A] time = 0.0495677, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 5475}

$$\frac{6d^3e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^3,x]
```


```
[Out] (-6*b*d^2*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (b*E^(a +
 b*x)*Cosh[c + d*x]^3)/(b^2 - 9*d^2) + (6*d^3*E^(a + b*x)*Sinh[c + d*x])/(b
^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(b^2 - 9*d^2)
```


Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} - \frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2} - \frac{(6d^2)\int e^{a+bx}\cosh(c+dx)\,dx}{b^2-9d^2}$$
$$= -\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} + \frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\cosh^2(c+\ldots}{b^2-\ldots}$$

Mathematica [A] time = 0.496639, size = 106, normalized size = 0.76

$$\frac{e^{a+bx}\left(3b\left(b^2-9d^2\right)\cosh(c+dx)+\left(b^3-bd^2\right)\cosh(3(c+dx))+6d\sinh(c+dx)\left(\left(d^2-b^2\right)\cosh(2(c+dx))-b^2+5d^2\right)\right)}{4\left(-10b^2d^2+b^4+9d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*b*(b^2 - 9*d^2)*Cosh[c + d*x] + (b^3 - b*d^2)*Cosh[3*(c + d
*x)] + 6*d*(-b^2 + 5*d^2 + (-b^2 + d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/
(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

Maple [A] time = 0.007, size = 166, normalized size = 1.2

$$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d} + \frac{3\sinh(a-c+(b-d)x)}{8b-8d} + \frac{3\sinh(a+c+(b+d)x)}{8b+8d} + \frac{\sinh(a+3c+(b+3d)x)}{8b+24d} + \frac{\cosh\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c)^3,x)
```

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/8*cosh(a-3*c+(b-3*d)*x)/
(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(a+
3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.40101, size = 756, normalized size = 5.44

$$\left(b^{3}-bd^{2}\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)^{3}-3\left(\left(b^{2}d-d^{3}\right)\cosh\left(bx+a\right)+\left(b^{2}d-d^{3}\right)\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)^{3}+3\left(b^{3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 - 3*((b^2*d - d^3)*cosh(b*
x + a) + (b^2*d - d^3)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*c
osh(b*x + a)*cosh(d*x + c) + 3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c) +
 (b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + ((b^3 - b*d^2
)*cosh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(
b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a)
+ (b^2*d - 9*d^3 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x
 + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12845, size = 113, normalized size = 0.81

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) + 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)
```

3.886 $\int e^{a+bx}\cosh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}-\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}$$

```
[Out] (-2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) + (b*E^(a + b*x)*Cosh[c + d*x]^2)/(b
^2 - 4*d^2) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x])/(b^2 - 4*d^2)
```

Rubi [A] time = 0.030614, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 2194}

$$\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}-\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

```
[Out] (-2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) + (b*E^(a + b*x)*Cosh[c + d*x]^2)/(b
^2 - 4*d^2) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x])/(b^2 - 4*d^2)
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} - \frac{(2d^2)\int e^{a+bx}\,dx}{b^2-4d^2}$$
$$= -\frac{2d^2e^{a+bx}}{b(b^2-4d^2)} + \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}$$

Mathematica [A] time = 0.153333, size = 56, normalized size = 0.64

$$\frac{e^{a+bx}\left(b^2\cosh(2(c+dx)) + b^2 - 2bd\sinh(2(c+dx)) - 4d^2\right)}{2(b^3-4bd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(b^2 - 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)]
))/(2*(b^3 - 4*b*d^2))
```

Maple [A] time = 0.008, size = 112, normalized size = 1.3

$$\frac{\sinh(bx+a)}{2b} + \frac{\sinh(a-2c+(b-2d)x)}{4b-8d} + \frac{\sinh(a+2c+(b+2d)x)}{4b+8d} + \frac{\cosh(bx+a)}{2b} + \frac{\cosh(a-2c+(b-2d)x)}{4b-8d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c)^2,x)
```

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)+1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2
*c+(b+2*d)*x)/(b+2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30202, size = 382, normalized size = 4.34

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2+\left(b^2\cosh(bx+a)+b^2\sinh(bx+a)\right)\sinh(dx+c)^2+\left(b^2-4\,d^2\right)\cosh(bx+a)+\left(b^2\,c\right.}{2\left(b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
+ b^2 - 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

Sympy [A] time = 13.1498, size = 476, normalized size = 5.41

$$\begin{cases} xe^a\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^a \\ \frac{xe^ae^{-2dx}\sinh^2(c+dx)}{4}+\frac{xe^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{-2dx}\cosh^2(c+dx)}{4}+\frac{3e^ae^{-2dx}\sinh^2(c+dx)}{8d}+\frac{e^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{e^ae^{-}}{} \\ \frac{xe^ae^{2dx}\sinh^2(c+dx)}{4}-\frac{xe^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{2dx}\cosh^2(c+dx)}{4}-\frac{3e^ae^{2dx}\sinh^2(c+dx)}{8d}+\frac{e^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{e^ae^{2dx}\cos}{8d} \\ \frac{b^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2}-\frac{2bde^ae^{bx}\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2}+\frac{2d^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2d^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**2
/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b
, 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh
(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 + 3*exp
(a)*exp(-2*d*x)*sinh(c + d*x)**2/(8*d) + exp(a)*exp(-2*d*x)*sinh(c + d*x)*c
osh(c + d*x)/(2*d) - exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, -2*d)
```

```
), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 - x*exp(a)*exp(2*d*x)*sinh(c + d
*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*cosh(c + d*x)**2/4 - 3*exp(a)*exp
(2*d*x)*sinh(c + d*x)**2/(8*d) + exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d
*x)/(2*d) + exp(a)*exp(2*d*x)*cosh(c + d*x)**2/(8*d), Eq(b, 2*d)), (b**2*ex
p(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*si
nh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sinh(c
 + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**
3 - 4*b*d**2), True))
```

Giac [A] time = 1.13823, size = 76, normalized size = 0.86

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)}+\frac{e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/2*e^(b*x + a)/b
```

3.887 $\int e^{a+bx}\cosh(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

```
[Out] (b*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/(
b^2 - d^2)
```

Rubi [A] time = 0.0175259, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5475}

$$\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (b*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/(
b^2 - d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] time = 0.0694697, size = 38, normalized size = 0.7

$$\frac{e^{a+bx}(b\cosh(c+dx)-d\sinh(c+dx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(b*Cosh[c + d*x] - d*Sinh[c + d*x]))/((b - d)*(b + d))
```

Maple [A] time = 0.004, size = 78, normalized size = 1.4

$$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}+\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)+1/2*cosh(a-c+(b-d)*
x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25044, size = 174, normalized size = 3.22

$$\frac{b\cosh(bx+a)\cosh(dx+c)+b\cosh(dx+c)\sinh(bx+a)-(d\cosh(bx+a)+d\sinh(bx+a))\sinh(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(d*x + c) + b*cosh(d*x + c)*sinh(b*x + a) - (d*cosh(b*
x + a) + d*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

Sympy [A] time = 3.51693, size = 201, normalized size = 3.72

$$\begin{cases} xe^{a}\cosh(c) & \text{for } b=0 \wedge d=0 \\ \frac{xe^{a}e^{-dx}\sinh(c+dx)}{2}+\frac{xe^{a}e^{-dx}\cosh(c+dx)}{2}+\frac{e^{a}e^{-dx}\sinh(c+dx)}{d}+\frac{e^{a}e^{-dx}\cosh(c+dx)}{2d} & \text{for } b=-d \\ -\frac{xe^{a}e^{dx}\sinh(c+dx)}{2}+\frac{xe^{a}e^{dx}\cosh(c+dx)}{2}+\frac{e^{a}e^{dx}\sinh(c+dx)}{d}-\frac{e^{a}e^{dx}\cosh(c+dx)}{2d} & \text{for } b=d \\ \frac{be^{a}e^{bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a}e^{bx}\sinh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(c
+ d*x)/d + exp(a)*exp(-d*x)*cosh(c + d*x)/(2*d), Eq(b, -d)), (-x*exp(a)*exp
(d*x)*sinh(c + d*x)/2 + x*exp(a)*exp(d*x)*cosh(c + d*x)/2 + exp(a)*exp(d*x)
*sinh(c + d*x)/d - exp(a)*exp(d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*exp(a
)*exp(b*x)*cosh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*sinh(c + d*x)/(b
**2 - d**2), True))
```

Giac [A] time = 1.16257, size = 54, normalized size = 1.

$$\frac{e^{(bx+dx+a+c)}}{2(b+d)}+\frac{e^{(bx-dx+a-c)}}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) + 1/2*e^(b*x - d*x + a - c)/(b - d)
```

3.888 $\int e^{a+bx}\operatorname{sech}(c+dx)\,dx$

Optimal. Leaf size=52

$$\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

```
[Out] (2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Rubi [A] time = 0.0178902, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5492}

$$\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x],x]
```

```
[Out] (2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}(c+dx)\,dx=\frac{2e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);-e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] time = 0.0182925, size = 51, normalized size = 0.98

$$\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x],x]
```

```
[Out] (2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \mathrm{e}^{bx+a}\mathrm{sech}\,(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c),x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)}\,\mathrm{sech}\,(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(bx+a)}\,\mathrm{sech}\,(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(e^(b*x + a)*sech(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

3.889 $\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Rubi [A] time = 0.026573, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5492}

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^2,x]
```

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,1+\frac{b}{2d};2+\frac{b}{2d};-e^{2(c+dx)}\right)}{b+2d}$$

Mathematica [A] time = 0.0167047, size = 56, normalized size = 1.

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^2,x]
```

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int e^{bx+a}\left(\operatorname{sech}\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c)^2,x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,b\int\frac{e^{(bx+a)}}{2\left(de^{(2\,dx+2\,c)}+d\right)}\,dx-\frac{2\,e^{(bx+a)}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*b*integrate(1/2*e^(b*x + a)/(d*e^(2*d*x + 2*c) + d), x) - 2*e^(b*x + a)/(
d*e^(2*d*x + 2*c) + d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(b*x + a)*sech(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c)^2, x)
```

3.890 $\int e^{a+bx}\mathbf{sech}^3(c+dx)\,dx$

Optimal. Leaf size=103

$$-\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{d^2}+\frac{be^{a+bx}\mathrm{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

```
[Out] -(((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b
/d)/2, -E^(2*(c + d*x))])/d^2) + (b*E^(a + b*x)*Sech[c + d*x])/(2*d^2) + (E
^(a + b*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0500383, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5490, 5492}

$$-\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{d^2}+\frac{be^{a+bx}\mathrm{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^3,x]
```

```
[Out] -(((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b
/d)/2, -E^(2*(c + d*x))])/d^2) + (b*E^(a + b*x)*Sech[c + d*x])/(2*d^2) + (E
^(a + b*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
```

```
+ b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = \frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\text{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{1}{2}\left(1-\frac{b^2}{d^2}\right)\int e^{a+bx}\text{sech}(c+dx)\,dx$$

$$= -\frac{(b-d)e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);-e^{2(c+dx)}\right)}{d^2}+\frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\text{sech}(c+d}{2}$$

Mathematica [A] time = 0.146712, size = 80, normalized size = 0.78

$$\frac{e^{a+bx}\left(\text{sech}(c+dx)(b+d\tanh(c+dx))-2(b-d)e^{c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(-2*(b - d)*E^(c + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3
 + b/d)/2, -E^(2*(c + d*x))] + Sech[c + d*x]*(b + d*Tanh[c + d*x])))/(2*d^2
)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int e^{bx+a}\left(\text{sech}\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c)^3,x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8\left(b^2e^c-d^2e^c\right)\int\frac{e^{(bx+dx+a)}}{8\left(d^2e^{(2\,dx+2\,c)}+d^2\right)}\,dx+\frac{\left(be^{(3\,c)}+de^{(3\,c)}\right)e^{(bx+3\,dx+a)}+\left(be^c-de^c\right)e^{(bx+dx+a)}}{d^2e^{(4\,dx+4\,c)}+2\,d^2e^{(2\,dx+2\,c)}+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -8*(b^2*e^c - d^2*e^c)*integrate(1/8*e^(b*x + d*x + a)/(d^2*e^(2*d*x + 2*c)
 + d^2), x) + ((b*e^(3*c) + d*e^(3*c))*e^(b*x + 3*d*x + a) + (b*e^c - d*e^c
)*e^(b*x + d*x + a))/(d^2*e^(4*d*x + 4*c) + 2*d^2*e^(2*d*x + 2*c) + d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*x + a)*sech(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c)^3, x)
```

3.891 $\int F^{c(a+bx)}\mathbf{sech}^n(d+ex)\,dx$

Optimal. Leaf size=90

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

Rubi [A] time = 0.137385, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5494, 2259}

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```


```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```


Rule 5494

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Dist[((1 + E^(2*(d + e*x)))^n*Sech[d + e*x]^n)/E^(n*(d + e*x)), Int
[SimplifyIntegrand[(F^(c*(a + b*x))*E^(n*(d + e*x)))/(1 + E^(2*(d + e*x)))^
n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \left(e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n\text{sech}^n(d+ex)\right)\int e^{dn+enx}\left(1+e^{2(d+ex)}\right)^{-n}F^{ac+bcx}\,dx$$

$$= \frac{\left(1+e^{2(d+ex)}\right)^n F^{ac+bcx}\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};1+\frac{en+bc\log(F)}{2e};-e^{2(d+ex)}\right)\text{sech}^n(d+ex)}{en+bc\log(F)}$$

Mathematica [A] time = 0.0737386, size = 89, normalized size = 0.99

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{c(a+bx)}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\text{sech}\left(ex+d\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\text{sech}\left(ex+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F^{bcx+ac}\,\mathrm{sech}\,(ex+d)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\,\mathrm{sech}^n\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\mathrm{sech}\,(ex+d)^n\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

3.892 $\int F^{c(a+bx)}\mathbf{csch}^n(d+ex)\,dx$

Optimal. Leaf size=91

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*x)*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Rubi [A] time = 0.167155, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5495, 2259}

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*x)*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Rule 5495

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Dist[((1 - E^(-2*(d + e*x)))^n*Csch[d + e*x]^n)/E^(-(n*(d + e*x))),
 Int[SimplifyIntegrand[F^(c*(a + b*x))/(E^(n*(d + e*x))*(1 - E^(-2*(d + e*x
)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \left(e^{n(d+ex)}\left(1-e^{-2(d+ex)}\right)^n\text{csch}^n(d+ex)\right)\int e^{-dn-enx}\left(1-e^{-2(d+ex)}\right)^{-n}F^{ac+bcx}\,dx$$
$$= -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\text{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

Mathematica [A] time = 0.0944063, size = 90, normalized size = 0.99

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{c(a+bx)}\text{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(c*(a + b*x))*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\text{csch}\left(ex+d\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\text{csch}\left(ex+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\operatorname{csch}(ex+d)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\operatorname{csch}(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

3.893 $\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$

Optimal. Leaf size=254

$$\frac{bcf^2\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ief^2\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2\sinh(d+ex)}{4e^2-b^2}$$

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + ((2*I)*e*f^2*F^(a*c + b*c*x)*Cosh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4
*e^2 - b^2*c^2*Log[F]^2)) - ((2*I)*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sinh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sin
h[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Si
nh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.406945, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6741, 12, 6742, 2194, 5474, 5476}

$$\frac{bcf^2\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ief^2\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2\sinh(d+ex)}{4e^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + ((2*I)*e*f^2*F^(a*c + b*c*x)*Cosh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4
*e^2 - b^2*c^2*Log[F]^2)) - ((2*I)*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sinh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sin
h[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Si
nh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2iF^{ac+bcx}\sinh(d+ex)-F^{ac+bcx}\sinh^2(d+ex)\right)dx \\
&= \left(2if^2\right)\int F^{ac+bcx}\sinh(d+ex)\,dx+f^2\int F^{ac+bcx}\,dx-f^2\int F^{ac+bcx}\sinh^2(d+ex)\,dx \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2 \ldots}{} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{2ibcf^2F^{\ldots}}{e\ldots}
\end{aligned}$$

Mathematica [A] time = 1.70937, size = 196, normalized size = 0.77

$$\frac{F^{c(a+bx)}(f+if\sinh(d+ex))^2\left(-\frac{2e\sinh(2(d+ex))}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(2(d+ex))}{b^2c^2\log^2(F)-4e^2}+\frac{4ibc\log(F)\sinh(d+ex)}{(bc\log(F)-e)(bc\log(F)+e)}+\frac{4ie\cosh(d+ex)}{(e-bc\log(F))(bc\log(F)+e)}+\frac{3}{bc\log\cdots}\right.}{2\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2*(3/(b*c*Log[F]) + ((4*I)*e*Cosh[
d + e*x])/((e - b*c*Log[F])*(e + b*c*Log[F])) - (b*c*Cosh[2*(d + e*x)]*Log[
F])/(-4*e^2 + b^2*c^2*Log[F]^2) + ((4*I)*b*c*Log[F]*Sinh[d + e*x])/((-e + b
*c*Log[F])*(e + b*c*Log[F])) - (2*e*Sinh[2*(d + e*x)])/(4*e^2 - b^2*c^2*Log
[F]^2)))/(2*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^4)
```

Maple [A] time = 0.128, size = 434, normalized size = 1.7

$$\frac{f^2\left(16i\ln(F)bce^3e^{3ex+3d}-(\ln(F))^4b^4c^4e^{4ex+4d}+16i\ln(F)bce^3e^{ex+d}+6(\ln(F))^4b^4c^4e^{2ex+2d}-16i(\ln(F))^2b^2c^2e^2e^{3\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x)
```

```
[Out] 1/4*f^2*(16*I*ln(F)*b*c*e^3*exp(3*e*x+3*d)-ln(F)^4*b^4*c^4*exp(4*e*x+4*d)+1
6*I*ln(F)*b*c*e^3*exp(e*x+d)+6*ln(F)^4*b^4*c^4*exp(2*e*x+2*d)-16*I*ln(F)^2*
b^2*c^2*e^2*exp(3*e*x+3*d)+2*ln(F)^3*b^3*c^3*e*exp(4*e*x+4*d)-ln(F)^4*b^4*c
^4-4*I*ln(F)^4*b^4*c^4*exp(e*x+d)-4*I*ln(F)^3*b^3*c^3*e*exp(3*e*x+3*d)+ln(F
)^2*b^2*c^2*e^2*exp(4*e*x+4*d)-2*ln(F)^3*b^3*c^3*e+16*I*ln(F)^2*b^2*c^2*e^2
*exp(e*x+d)-30*ln(F)^2*b^2*c^2*e^2*exp(2*e*x+2*d)+4*I*ln(F)^4*b^4*c^4*exp(3
*e*x+3*d)-2*ln(F)*b*c*e^3*exp(4*e*x+4*d)+ln(F)^2*b^2*c^2*e^2-4*I*ln(F)^3*b^
3*c^3*e*exp(e*x+d)+2*ln(F)*b*c*e^3+24*e^4*exp(2*e*x+2*d))/b/c/ln(F)/(e-b*c*
ln(F))*exp(-2*e*x-2*d)/(-b*c*ln(F)+2*e)/(e+b*c*ln(F))/(b*c*ln(F)+2*e)*F^(c*
(b*x+a))
```

Maxima [A] time = 1.089, size = 255, normalized size = 1.

$$-\frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2ex+2d)}}{bc\log(F)+2e}+\frac{F^{ac}e^{(bcx\log(F)-2ex)}}{bce^{(2d)}\log(F)-2ee^{(2d)}}-\frac{2F^{bcx+ac}}{bc\log(F)}\right)+if^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^d\log(F)-ee^d}\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*
c)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 2*F^(b*c*x
 + a*c)/(b*c*log(F))) + I*f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(
F) + e) - F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c
*x + a*c)*f^2/(b*c*log(F))
```

Fricas [A] time = 1.37817, size = 999, normalized size = 3.93

$$\frac{\left(24\,e^4 f^2 e^{(2\,ex+2\,d)} - \left(b^4 c^4 f^2 e^{(4\,ex+4\,d)} - 4i\,b^4 c^4 f^2 e^{(3\,ex+3\,d)} - 6\,b^4 c^4 f^2 e^{(2\,ex+2\,d)} + 4i\,b^4 c^4 f^2 e^{(ex+d)} + b^4 c^4 f^2\right)\log\left(F\right)^4 + \left(2\,b^3 c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(24*e^4*f^2*e^(2*e*x + 2*d) - (b^4*c^4*f^2*e^(4*e*x + 4*d) - 4*I*b^4*c^
4*f^2*e^(3*e*x + 3*d) - 6*b^4*c^4*f^2*e^(2*e*x + 2*d) + 4*I*b^4*c^4*f^2*e^(
e*x + d) + b^4*c^4*f^2)*log(F)^4 + (2*b^3*c^3*e*f^2*e^(4*e*x + 4*d) - 4*I*b
^3*c^3*e*f^2*e^(3*e*x + 3*d) - 4*I*b^3*c^3*e*f^2*e^(e*x + d) - 2*b^3*c^3*e*
f^2)*log(F)^3 + (b^2*c^2*e^2*f^2*e^(4*e*x + 4*d) - 16*I*b^2*c^2*e^2*f^2*e^(
3*e*x + 3*d) - 30*b^2*c^2*e^2*f^2*e^(2*e*x + 2*d) + 16*I*b^2*c^2*e^2*f^2*e^
(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 - (2*b*c*e^3*f^2*e^(4*e*x + 4*d) - 16
*I*b*c*e^3*f^2*e^(3*e*x + 3*d) - 16*I*b*c*e^3*f^2*e^(e*x + d) - 2*b*c*e^3*f
^2)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*e^(2*e*x + 2*d)*log(F)^5 - 5*b^3*c^3*e
^2*e^(2*e*x + 2*d)*log(F)^3 + 4*b*c*e^4*e^(2*e*x + 2*d)*log(F))
```

Sympy [A] time = 128.612, size = 1731, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d))**2,x)
```

```
[Out] Piecewise((-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x
- f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(F,
1)), (zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d
+ e*x)**2 + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*s
inh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/
(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**4*f**2*e
xp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**4*f**2*ex
p(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)
))), (zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x)
 + zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), E
q(F, exp(e/(b*c)))), (zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(
b*c*x)*sinh(d + e*x)**2 + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c)
)**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c
)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(2*e/(b*c)))), (F**(a*
c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**2*
sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e), Eq(b, 0)), (
-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**2*sinh
(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(c, 0)), (-F**(
a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)**2/(b**5*c**5*log(F)
**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*I*F**(a*c)*F**(b*
c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c
**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c**4*f**
2*log(F)**4/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) + 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)*co
sh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) - 2*I*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*cosh(d + e*x)/
(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*
F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)**2/(b**5*c*
*5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*I*F**(a*
c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)/(b**5*c**5*log(F)
**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*
x)*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*
b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F**(a*c)*F**(b*c*x)*b**2*
c**2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3
+ 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d +
e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*
c*e**4*log(F)) + 8*I*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*cosh(d + e*x)
/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2
*F**(a*c)*F**(b*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b*
*3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f*
*2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b
*c*e**4*log(F)) + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 - 5*
b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)), True))
```

Giac [B] time = 1.35772, size = 2125, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-6*I*f^2*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 6*I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) - 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b
*c*log(abs(F)) + 2*e)*x + 2*d) - 1/2*I*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) -
1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4
*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) +
 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(
F)) + 2*e)*x + 2*d) - 2*((pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + e)^2) + 2*(b*c*log(abs(F)) + e)*f^2*sin(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sg
n(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*lo
g(abs(F)) + e)*x + d) + 1/2*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*
c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b
*c*log(abs(F)) + 2*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*
log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 2*(
(pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*
pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)
) - e)^2) + 2*(b*c*log(abs(F)) - e)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1
/2*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
```

```
) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*e) -
2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*(2*(b*c*log(abs(F)) - 2
*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c
*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*
sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e
)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I*(2*I*f^
2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) - 2*I*f^
2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*
c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

3.894 $\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$

Optimal. Leaf size=106

$$-\frac{ibcf\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ief\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (I*e*f*F^(a*c + b*c*x)*Cosh[d + e*x])/(e
^2 - b^2*c^2*Log[F]^2) - (I*b*c*f*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^
2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.179155, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6741, 12, 6742, 2194, 5474}

$$-\frac{ibcf\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ief\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (I*e*f*F^(a*c + b*c*x)*Cosh[d + e*x])/(e
^2 - b^2*c^2*Log[F]^2) - (I*b*c*f*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^
2 - b^2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx &= \int fF^{ac+bcx}(1+i\sinh(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+i\sinh(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+iF^{ac+bcx}\sinh(d+ex)\right)\,dx \\
&= (if)\int F^{ac+bcx}\sinh(d+ex)\,dx + f\int F^{ac+bcx}\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)} + \frac{iefF^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)} - \frac{ibcfF^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] time = 0.612028, size = 93, normalized size = 0.88

$$\frac{fF^{c(a+bx)}\left(ib^2c^2\log^2(F)\sinh(d+ex)+b^2c^2\log^2(F)-ibce\log(F)\cosh(d+ex)-e^2\right)}{bc\log(F)(bc\log(F)-e)(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 - I*b*c*e*Cosh[d + e*x]*Log[F] + b^2*c^2*Log[F]^2
+ I*b^2*c^2*Log[F]^2*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c
*Log[F]))
```

Maple [A] time = 0.046, size = 141, normalized size = 1.3

$$\frac{f\left(-i(\ln(F))^2b^2c^2e^{2ex+2d}+i(\ln(F))^2b^2c^2-2(\ln(F))^2b^2c^2e^{ex+d}+i\ln(F)bce\,e^{2ex+2d}+i\ln(F)bce+2e^2e^{ex+d}\right)e^{-ex-d}F}{2bc\ln(F)(e-bc\ln(F))(e+bc\ln(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

```
[Out] 1/2*f*(-I*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)+I*ln(F)^2*b^2*c^2-2*ln(F)^2*b^2*c^
2*exp(e*x+d)+I*ln(F)*b*c*e*exp(2*e*x+2*d)+I*ln(F)*b*c*e+2*e^2*exp(e*x+d))/b
/c/ln(F)/(e-b*c*ln(F))*exp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A] time = 1.03752, size = 119, normalized size = 1.12

$$\frac{1}{2} i f \left(\frac{F^{ac} e^{(bcx \log(F) + ex + d)}}{bc \log(F) + e} - \frac{F^{ac} e^{(bcx \log(F) - ex)}}{bce^d \log(F) - ee^d} \right) + \frac{F^{bcx+ac} f}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*I*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F
))
```

Fricas [A] time = 1.33784, size = 317, normalized size = 2.99

$$-\frac{\left(2 e^2 f e^{(ex+d)} - \left(i b^2 c^2 f e^{(2 ex+2 d)} + 2 b^2 c^2 f e^{(ex+d)} - i b^2 c^2 f\right) \log(F)^2 - \left(-i bcef e^{(2 ex+2 d)} - i bcef\right) \log(F)\right) F^{bcx+ac}}{2 \left(b^3 c^3 e^{(ex+d)} \log(F)^3 - bce^2 e^{(ex+d)} \log(F)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*e^2*f*e^(e*x + d) - (I*b^2*c^2*f*e^(2*e*x + 2*d) + 2*b^2*c^2*f*e^(e
*x + d) - I*b^2*c^2*f)*log(F)^2 - (-I*b*c*e*f*e^(2*e*x + 2*d) - I*b*c*e*f)*
log(F))*F^(b*c*x + a*c)/(b^3*c^3*e^(e*x + d)*log(F)^3 - b*c*e^2*e^(e*x + d)
*log(F))
```

Sympy [A] time = 17.4982, size = 400, normalized size = 3.77

$$\begin{cases} fx + \frac{if\cosh(d+ex)}{e} & \text{for } F = 1 \\ \tilde{\infty} e^2 f \left(e^{-\frac{e}{bc}}\right)^{ac} \left(e^{-\frac{e}{bc}}\right)^{bcx} \sinh(d+ex) + \tilde{\infty} e^2 f \left(e^{-\frac{e}{bc}}\right)^{ac} \left(e^{-\frac{e}{bc}}\right)^{bcx} \cosh(d+ex) & \text{for } F = e^{-\frac{e}{bc}} \\ \tilde{\infty} e^2 f \left(e^{\frac{e}{bc}}\right)^{ac} \left(e^{\frac{e}{bc}}\right)^{bcx} \sinh(d+ex) + \tilde{\infty} e^2 f \left(e^{\frac{e}{bc}}\right)^{ac} \left(e^{\frac{e}{bc}}\right)^{bcx} \cosh(d+ex) & \text{for } F = e^{\frac{e}{bc}} \\ F^{ac}\left(fx + \frac{if\cosh(d+ex)}{e}\right) & \text{for } b = 0 \\ fx + \frac{if\cosh(d+ex)}{e} & \text{for } c = 0 \\ \frac{iF^{ac}F^{bcx}b^2c^2f\log(F)^2\sinh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} + \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{iF^{ac}F^{bcx}bcef\log(F)\cosh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log(F)^3 - bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

```
[Out] Piecewise((f*x + I*f*cosh(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-e/(b*c))*
*(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(-e/(b*c))**(a*
c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))), (zoo*e**2*f
*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(e
/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(b*c)))), (
F**(a*c)*(f*x + I*f*cosh(d + e*x)/e), Eq(b, 0)), (f*x + I*f*cosh(d + e*x)/e
, Eq(c, 0)), (I*F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*sinh(d + e*x)/(b*
*3*c**3*log(F)**3 - b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(
F)**2/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - I*F**(a*c)*F**(b*c*x)*b*c*e
*f*log(F)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c)*
F**(b*c*x)*e**2*f/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

Giac [B] time = 1.25489, size = 1214, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f*e^(1/2*I
```

```
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f*e^(-1/2*I*pi*b*c*x*sg
n(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(
F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 - ((pi*b*c*sgn(F) - pi*b*c)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + e)^2) + 2*(b*c*log(abs(F)) + e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1
/2*(2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e) +
 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1
/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e))*e
^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + ((pi*b*c*sgn(F) - pi*b*c
)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) + 2*(b*c*log(ab
s(F)) - e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*
c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*(-2*I*f*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e) - 2*I*f*e^(-1/2*I*pi*b*c*x*
sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*
sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (b*c*l
og(abs(F)) - e)*x - d)
```

3.895 $\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx$

Optimal. Leaf size=85

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)])/(f*(e + b*c*L
og[F]))
```

Rubi [A] time = 0.0752858, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5496, 5492}

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)])/(f*(e + b*c*L
og[F]))
```

Rule 5496

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
 - (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)dx}{2f}$$
$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)}{f(e+bc\log(F))}$$

Mathematica [A] time = 3.36968, size = 104, normalized size = 1.22

$$\frac{2F^{c(a+bx)}\left({}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;-ie^{d+ex}\right)+\frac{\cosh\left(\frac{ex}{2}\right)-\sinh\left(\frac{ex}{2}\right)}{\left(1-ie^d\right)\sinh\left(\frac{ex}{2}\right)+\left(-1-ie^d\right)\cosh\left(\frac{ex}{2}\right)}\right)}{ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (2*F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e
, (-I)*E^(d + e*x)] + (Cosh[(e*x)/2] - Sinh[(e*x)/2])/((-1 - I*E^d)*Cosh[(e
*x)/2] + (1 - I*E^d)*Sinh[(e*x)/2])))/(e*f)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{f+if\sinh(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4F^{ac}bce\int \frac{}{ib^2c^2f\log(F)^2-3ibcef\log(F)+2ie^2f+\left(b^2c^2fe^{(3d)}\log(F)^2-3bcefe^{(3d)}\log(F)+2e^2fe^{(3d)}\right)e^{(3ex)}+\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

```
[Out] -4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(I*b^2*c^2*f*log(F)^2 - 3*I*b*c*e*f*lo
g(F) + 2*I*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) +
 2*e^2*f*e^(3*d))*e^(3*e*x) + (-3*I*b^2*c^2*f*e^(2*d)*log(F)^2 + 9*I*b*c*e*
f*e^(2*d)*log(F) - 6*I*e^2*f*e^(2*d))*e^(2*e*x) - 3*(b^2*c^2*f*e^d*log(F)^2
 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) + (4*I*F^(a*c)*e
 + 2*(F^(a*c)*b*c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(-I*b^2*
c^2*f*log(F)^2 + 3*I*b*c*e*f*log(F) - 2*I*e^2*f + (I*b^2*c^2*f*e^(2*d)*log(
F)^2 - 3*I*b*c*e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c
^2*f*e^d*log(F)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(efe^{(ex+d)} - i\,ef\right)\text{integral}\left(-\frac{2i\,F^{bcx+ac}bc\log(F)}{efe^{(ex+d)}-i\,ef}, x\right) + 2i\,F^{bcx+ac}}{efe^{(ex+d)} - i\,ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

```
[Out] ((e*f*e^(e*x + d) - I*e*f)*integral(-2*I*F^(b*c*x + a*c)*b*c*log(F)/(e*f*e^
(e*x + d) - I*e*f), x) + 2*I*F^(b*c*x + a*c))/(e*f*e^(e*x + d) - I*e*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{i\sinh(d+ex)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(I*sinh(d + e*x) + 1), x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{i f \sinh (ex + d) + f} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f), x)
```

3.896 $\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$

Optimal. Leaf size=196

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3e^2f^2}+\frac{bc\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)F^{c(a+}}{6e^2f^2}$$

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)]*(e - b*c*Log[F
]))/(3*e^2*f^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d/2 + (I/4)*Pi + (e*x)/2
]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[d/2 + (I/4)*Pi + (e*x)/2]^2*Tanh[d
/2 + (I/4)*Pi + (e*x)/2])/(6*e*f^2)
```

Rubi [A] time = 0.114658, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5496, 5490, 5492}

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3e^2f^2}+\frac{bc\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)F^{c(a+}}{6e^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```


```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)]*(e - b*c*Log[F
]))/(3*e^2*f^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d/2 + (I/4)*Pi + (e*x)/2
]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[d/2 + (I/4)*Pi + (e*x)/2]^2*Tanh[d
/2 + (I/4)*Pi + (e*x)/2])/(6*e*f^2)
```


Rule 5496

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
 - (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\operatorname{sech}^4\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= \frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6ef^2} +$$

$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c}}{}$$

Mathematica [A] time = 3.01077, size = 255, normalized size = 1.3

$$\frac{F^{c(a+bx)}\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)\left((1-i)\left(e^2-b^2c^2\log^2(F)\right)\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)^3\left(-1+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])*(e*(I*e + b*c*Lo
g[F])*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2]) + (1 - I)*(-1 + (1 + I)*Hyp
ergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, (-I)*(Cosh[d + e*x] +
 Sinh[d + e*x])])*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I*Sinh[(d +
```

```
 e*x)/2])^3 + 2*e^2*Sinh[(d + e*x)/2] + 2*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d
 + e*x)/2] + I*Sinh[(d + e*x)/2])^2*Sinh[(d + e*x)/2]))/(3*e^3*(f + I*f*Sin
h[d + e*x])^2)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{\left(f + if \sinh\left(ex+d\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] (16*I*F^(a*c)*b^2*c^2*e*log(F)^2 + 16*I*F^(a*c)*b*c*e^2*log(F))*integrate(F
^(b*c*x)/(-I*b^3*c^3*f^2*log(F)^3 + 9*I*b^2*c^2*e*f^2*log(F)^2 - 26*I*b*c*e
^2*f^2*log(F) + 24*I*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*
f^2*e^(5*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*
e^(5*e*x) + (-5*I*b^3*c^3*f^2*e^(4*d)*log(F)^3 + 45*I*b^2*c^2*e*f^2*e^(4*d)
*log(F)^2 - 130*I*b*c*e^2*f^2*e^(4*d)*log(F) + 120*I*e^3*f^2*e^(4*d))*e^(4*
e*x) - 10*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2
+ 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + (10*I*b^3
*c^3*f^2*e^(2*d)*log(F)^3 - 90*I*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 260*I*b*c
*e^2*f^2*e^(2*d)*log(F) - 240*I*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2
*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) -
24*e^3*f^2*e^d)*e^(e*x)), x) + (16*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 -
4*(F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F
^(a*c)*e^2*e^(2*d))*e^(2*e*x) - (16*I*F^(a*c)*b*c*e*e^d*log(F) - 64*I*F^(a*
c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(
F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(F)^3 -
```

```
 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*
f^2*e^(4*d))*e^(4*e*x) + (-4*I*b^3*c^3*f^2*e^(3*d)*log(F)^3 + 36*I*b^2*c^2*
e*f^2*e^(3*d)*log(F)^2 - 104*I*b*c*e^2*f^2*e^(3*d)*log(F) + 96*I*e^3*f^2*e^
(3*d))*e^(3*e*x) - 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d
)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e^(2*d))*e^(2*e*x)
+ (4*I*b^3*c^3*f^2*e^d*log(F)^3 - 36*I*b^2*c^2*e*f^2*e^d*log(F)^2 + 104*I*b
*c*e^2*f^2*e^d*log(F) - 96*I*e^3*f^2*e^d)*e^(e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(6\,e^2e^{(ex+d)} + \left(-2i\,b^2c^2e^{(2\,ex+2\,d)} - 4\,b^2c^2e^{(ex+d)} + 2i\,b^2c^2\right)\log\left(F\right)^2 - 2i\,e^2 + \left(-2i\,bcee^{(2\,ex+2\,d)} - 2\,bcee^{(ex+d)}\right)\log\left(F\right)\right)F^{bcx+ac}}{3\,e^3f^2e^{(3\,ex+3\,d)} - 9i\,e^3f^2e^{(2\,ex+2\,d)} - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] ((6*e^2*e^(e*x + d) + (-2*I*b^2*c^2*e^(2*e*x + 2*d) - 4*b^2*c^2*e^(e*x + d)
 + 2*I*b^2*c^2)*log(F)^2 - 2*I*e^2 + (-2*I*b*c*e*e^(2*e*x + 2*d) - 2*b*c*e*
e^(e*x + d))*log(F))*F^(b*c*x + a*c) + (3*e^3*f^2*e^(3*e*x + 3*d) - 9*I*e^3
*f^2*e^(2*e*x + 2*d) - 9*e^3*f^2*e^(e*x + d) + 3*I*e^3*f^2)*integral(1/3*(2
*I*b^3*c^3*log(F)^3 - 2*I*b*c*e^2*log(F))*F^(b*c*x + a*c)/(e^3*f^2*e^(e*x +
 d) - I*e^3*f^2), x))/(3*e^3*f^2*e^(3*e*x + 3*d) - 9*I*e^3*f^2*e^(2*e*x + 2
*d) - 9*e^3*f^2*e^(e*x + d) + 3*I*e^3*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{\left(if \sinh\left(ex+d\right)+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f)^2, x)
```

3.897 $\int F^{c(a+bx)}(f + f\cosh(d + ex))^2\,dx$

Optimal. Leaf size=251

$$\frac{2ef^2\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)} - \frac{bcf^2\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)} - \frac{2bcf^2\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)} + \frac{2ef^2\sinh(d+ex)\cos}{4e^2-b^2c^2\mathrm{l}}$$

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*
x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[
F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]^2*L
og[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sinh[d + e*x])
/(e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d +
 e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.312521, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6741, 12, 6742, 2194, 5475, 5477}

$$\frac{2ef^2\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)} - \frac{bcf^2\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)} - \frac{2bcf^2\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)} + \frac{2ef^2\sinh(d+ex)\cos}{4e^2-b^2c^2\mathrm{l}}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*
x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[
F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]^2*L
og[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sinh[d + e*x])
/(e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d +
 e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+\cosh(d+ex))^2\,dx\\
&= f^2\int F^{ac+bcx}(1+\cosh(d+ex))^2\,dx\\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\cosh(d+ex)+F^{ac+bcx}\cosh^2(d+ex)\right)dx\\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\cosh^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\cosh(d+ex)\,d\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}-\frac{bcf^2F^{ac+bcx}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)}\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-
\end{aligned}$$

Mathematica [A] time = 0.572814, size = 230, normalized size = 0.92

$$\frac{f^2F^{c(a+bx)}\left(4\cosh(d+ex)\left(b^4c^4\log^4(F)-4b^2c^2e^2\log^2(F)\right)+\cosh(2(d+ex))\left(b^4c^4\log^4(F)-b^2c^2e^2\log^2(F)\right)-4b^3c^3e\log^3\right.}{2\left(-5b^3c^3\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 - 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + 4*Cosh[d + e*x]*(-4*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + Cosh[2*(d
 + e*x)]*(-(b^2*c^2*e^2*Log[F]^2) + b^4*c^4*Log[F]^4) + 16*b*c*e^3*Log[F]*S
inh[d + e*x] - 4*b^3*c^3*e*Log[F]^3*Sinh[d + e*x] + 2*b*c*e^3*Log[F]*Sinh[2
*(d + e*x)] - 2*b^3*c^3*e*Log[F]^3*Sinh[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F]
 - 5*b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Maple [A] time = 0.073, size = 426, normalized size = 1.7

$$f^2\left((\ln(F))^4 b^4c^4e^{4ex+4d}+4(\ln(F))^4 b^4c^4e^{3ex+3d}+6(\ln(F))^4 b^4c^4e^{2ex+2d}-2(\ln(F))^3 b^3c^3ee^{4ex+4d}+4(\ln(F))^4 b^4c^4e^{e\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x)
```

```
[Out] 1/4*f^2*(ln(F)^4*b^4*c^4*exp(4*e*x+4*d)+4*ln(F)^4*b^4*c^4*exp(3*e*x+3*d)+6*
ln(F)^4*b^4*c^4*exp(2*e*x+2*d)-2*ln(F)^3*b^3*c^3*e*exp(4*e*x+4*d)+4*ln(F)^4
*b^4*c^4*exp(e*x+d)-4*ln(F)^3*b^3*c^3*e*exp(3*e*x+3*d)+ln(F)^4*b^4*c^4-ln(F
)^2*b^2*c^2*e^2*exp(4*e*x+4*d)+4*ln(F)^3*b^3*c^3*e*exp(e*x+d)-16*ln(F)^2*b^
2*c^2*e^2*exp(3*e*x+3*d)+2*ln(F)^3*b^3*c^3*e-30*ln(F)^2*b^2*c^2*e^2*exp(2*e
*x+2*d)+2*ln(F)*b*c*e^3*exp(4*e*x+4*d)-16*ln(F)^2*b^2*c^2*e^2*exp(e*x+d)+16
*ln(F)*b*c*e^3*exp(3*e*x+3*d)-ln(F)^2*b^2*c^2*e^2-16*ln(F)*b*c*e^3*exp(e*x+
d)-2*ln(F)*b*c*e^3+24*e^4*exp(2*e*x+2*d))/b/c/ln(F)/(b*c*ln(F)-e)*exp(-2*e*
x-2*d)/(b*c*ln(F)-2*e)/(e+b*c*ln(F))/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

Maxima [A] time = 1.06969, size = 252, normalized size = 1.

$$\frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2ex+2d)}}{bc\log(F)+2e}+\frac{F^{ac}e^{(bcx\log(F)-2ex)}}{bce^{(2d)}\log(F)-2ee^{(2d)}}+\frac{2F^{bcx+ac}}{bc\log(F)}\right)+f^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx\ldots}}{bc\log\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 2*F^(b*c*x
+ a*c)/(b*c*log(F))) + f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F)
+ e) + F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x
+ a*c)*f^2/(b*c*log(F))
```

Fricas [B] time = 1.73146, size = 5389, normalized size = 21.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((24*e^4*f^2*cosh(e*x + d)^2 + (b^4*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4
*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*cosh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e
*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*l
og(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*b*c*e^3*f^2*log(F))*sinh(e*x + d)^4
- 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^
3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(
e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3
*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(
F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^
3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 +
 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^
2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2 + (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b
^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*
x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*
x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x + d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 +
2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*e^3*f^2*cosh(e*x + d))*log(F))*sinh(
e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3
 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e
*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3 + 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*
b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e
*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x + d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^
2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2
*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*c
osh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x + d)^2 - 2*b*c*e^3*f^2)*log(F))*sin
```

```
h(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (24*e^4*f^2*cosh(e*x + d)^2 + (b^4
*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*co
sh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*
c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*log(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*
b*c*e^3*f^2*log(F))*sinh(e*x + d)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*
b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f
^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b
^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cos
h(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2
*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 +
16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 1
6*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2
+ (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)
*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))
*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x +
 d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*
e^3*f^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x +
d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^
3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3
+ 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2
)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x +
d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*
c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*
e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x
 + d)^2 - 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))
/(b^5*c^5*cosh(e*x + d)^2*log(F)^5 - 5*b^3*c^3*e^2*cosh(e*x + d)^2*log(F)^3
 + 4*b*c*e^4*cosh(e*x + d)^2*log(F) + (b^5*c^5*log(F)^5 - 5*b^3*c^3*e^2*log
(F)^3 + 4*b*c*e^4*log(F))*sinh(e*x + d)^2 + 2*(b^5*c^5*cosh(e*x + d)*log(F)
^5 - 5*b^3*c^3*e^2*cosh(e*x + d)*log(F)^3 + 4*b*c*e^4*cosh(e*x + d)*log(F))
*sinh(e*x + d))
```

Sympy [A] time = 112.801, size = 1719, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d))**2,x)
```

```
[Out] Piecewise((-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x
+ f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(F, 1)
), (zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d +
```

```
e*x)**2 + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sin
h(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b
*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**4*f**2*exp
(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**4*f**2*exp(
-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))
), (zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) +
 zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(
F, exp(e/(b*c)))), (zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*
c*x)*sinh(d + e*x)**2 + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))*
*(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*
exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(2*e/(b*c)))), (F**(a*c)
*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*si
nh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e), Eq(b, 0)), (-f**
2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*sinh(d +
 e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(c, 0)), (F**(a*c)*F*
*(b*c*x)*b**4*c**4*f**2*log(F)**4*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5
*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*b**4
*c**4*f**2*log(F)**4*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)*
*4/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) -
 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)*cosh(d + e*
x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) -
 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)/(b**5*c**5*
log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F*
*(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**
5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 3*F**(a*c)*F**(b*c*x)
*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b*
*3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*F**(a*c)*F**(b*c*x)*b**2*c*
*2*e**2*f**2*log(F)**2*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**
2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**
2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) + 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d + e*x)*cosh(d +
 e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)
) + 8*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d + e*x)/(b**5*c**5*log
(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b
*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*lo
g(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f**2*cosh(d + e*x
)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F))
 + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)), True))
```

Giac [C] time = 1.29641, size = 2128, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-6*I*f^2*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 6*I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b
*c*log(abs(F)) + 2*e)*x + 2*d) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) -
 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) -
4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F)
+ 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs
(F)) + 2*e)*x + 2*d) + 2*(2*(b*c*log(abs(F)) + e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*
pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*l
og(abs(F)) + e)*x + d) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*p
i*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c +
 2*b*c*log(abs(F)) + 2*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*
c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*
b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) +
 2*(2*(b*c*log(abs(F)) - e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
 - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2
```

```
*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*(2*(b*c*log(abs(F
)) - 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (
pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
 - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I*(
-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) +
 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

3.898 $\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$

Optimal. Leaf size=101

$$\frac{ef\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (b*c*f*F^(a*c + b*c*x)*Cosh[d + e*x]*Log
[F])/(e^2 - b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 -
b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.14612, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6741, 12, 6742, 2194, 5475}

$$\frac{ef\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (b*c*f*F^(a*c + b*c*x)*Cosh[d + e*x]*Log
[F])/(e^2 - b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 -
b^2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx &= \int fF^{ac+bcx}(1+\cosh(d+ex))\,dx\\
&= f\int F^{ac+bcx}(1+\cosh(d+ex))\,dx\\
&= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cosh(d+ex)\right)\,dx\\
&= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\cosh(d+ex)\,dx\\
&= \frac{fF^{ac+bcx}}{bc\log(F)}-\frac{bcfF^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}
\end{aligned}
$$

Mathematica [A] time = 0.19586, size = 88, normalized size = 0.87

$$
\frac{fF^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(d+ex)+b^2c^2\log^2(F)-bce\log(F)\sinh(d+ex)-e^2\right)}{bc\log(F)(bc\log(F)-e)(bc\log(F)+e)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[d + e*x]*Log[F]^
2 - b*c*e*Log[F]*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c*Log
[F]))
```

Maple [A] time = 0.036, size = 135, normalized size = 1.3

$$
\frac{f\left((\ln(F))^2b^2c^2e^{2ex+2d}+2(\ln(F))^2b^2c^2e^{ex+d}+b^2c^2(\ln(F))^2-\ln(F)bce e^{2ex+2d}+\ln(F)bce-2e^2e^{ex+d}\right)e^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)(bc\ln(F)-e)(e+bc\ln(F))}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

```
[Out] 1/2*f*(ln(F)^2*b^2*c^2*exp(2*e*x+2*d)+2*ln(F)^2*b^2*c^2*exp(e*x+d)+b^2*c^2*
ln(F)^2-ln(F)*b*c*e*exp(2*e*x+2*d)+ln(F)*b*c*e-2*e^2*exp(e*x+d))/b/c/ln(F)/
(b*c*ln(F)-e)*exp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A] time = 1.03593, size = 117, normalized size = 1.16

$$\frac{1}{2}f\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f}{bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + F^(a*c)*e^(b*c
*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F))
```

Fricas [B] time = 1.54361, size = 1080, normalized size = 10.69

$$-\frac{\left(2e^{2}f\cosh(ex+d)-\left(b^{2}c^{2}f\cosh(ex+d)^{2}+2b^{2}c^{2}f\cosh(ex+d)+b^{2}c^{2}f\right)\log(F)^{2}-\left(b^{2}c^{2}f\log(F)^{2}-bcef\log(F)\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*((2*e^2*f*cosh(e*x + d) - (b^2*c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cos
h(e*x + d) + b^2*c^2*f)*log(F)^2 - (b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*si
nh(e*x + d)^2 + (b*c*e*f*cosh(e*x + d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cos
h(e*x + d)*log(F) + e^2*f - (b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)
*sinh(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (2*e^2*f*cosh(e*x + d) - (b^2*
c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2 - (
b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*sinh(e*x + d)^2 + (b*c*e*f*cosh(e*x +
d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cosh(e*x + d)*log(F) + e^2*f - (b^2*c^2
*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*l
og(F)))/(b^3*c^3*cosh(e*x + d)*log(F)^3 - b*c*e^2*cosh(e*x + d)*log(F) + (b
```

```
^3*c^3*log(F)^3 - b*c*e^2*log(F))*sinh(e*x + d))
```

Sympy [A] time = 11.1659, size = 391, normalized size = 3.87

$$\begin{cases} fx + \frac{f\sinh(d+ex)}{e} & \text{for } F = 1 \\ \tilde{\infty}e^2f\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e^2f\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = e^{-\frac{e}{bc}} \\ \tilde{\infty}e^2f\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e^2f\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = e^{\frac{e}{bc}} \\ F^{ac}\left(fx + \frac{f\sinh(d+ex)}{e}\right) & \text{for } b = 0 \\ fx + \frac{f\sinh(d+ex)}{e} & \text{for } c = 0 \\ \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2\cosh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} + \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{F^{ac}F^{bcx}bcef\log(F)\sinh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log(F)^3 - bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

```
[Out] Piecewise((f*x + f*sinh(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-e/(b*c))**(
a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(-e/(b*c))**(a*c)
*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))), (zoo*e**2*f*e
xp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(e/(
b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(b*c)))), (F*
*(a*c)*(f*x + f*sinh(d + e*x)/e), Eq(b, 0)), (f*x + f*sinh(d + e*x)/e, Eq(c
, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*cosh(d + e*x)/(b**3*c**3*
log(F)**3 - b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2/(b
**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)*b*c*e*f*log(F)*
sinh(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)
*e**2*f/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

Giac [C] time = 1.24201, size = 1215, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
```

```
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f*e^(-1/2*I*pi*b*c*x*sg
n(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(
F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 + (2*(b*c*log(abs(F)) + e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1
/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e
) + 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
+ 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) -
 e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*
a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn
(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F)
 + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^
(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-2*I*f*e^(1/2*I*pi
*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*p
i*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e) + 2*I*f*e^(-1/2*I*pi*b
*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi
*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) - e)*x - d)
```

3.899 $\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx$

Optimal. Leaf size=61

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(f*(e + b*c*Log[F]))
```

Rubi [A] time = 0.0627048, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5497, 5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(f*(e + b*c*Log[F]))
```

Rule 5497

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{2f}$$
$$= \frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{d+ex}\right)}{f(e+bc\log(F))}$$

Mathematica [A] time = 0.0461783, size = 61, normalized size = 1.

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{bcf\log(F)+ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(e*f + b*c*f*Log[F])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{f+f\cosh(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4F^{ac}bce\int \frac{F^{bcx}}{b^2c^2f\log(F)^2-3bcef\log(F)+2e^2f+\left(b^2c^2fe^{(3d)}\log(F)^2-3bcefe^{(3d)}\log(F)+2e^2fe^{(3d)}\right)e^{(3ex)}+3\left(b^2c^2f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*b*c*e*f*log(F)
+ 2*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) + 2*e^2*
f*e^(3*d))*e^(3*e*x) + 3*(b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*lo
g(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 3*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e
^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(2*F^(a*c)*e - (F^(a*c)*b*
c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*
b*c*e*f*log(F) + 2*e^2*f + (b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*
log(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f
*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bcx+ac}}{f\cosh(ex+d)+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f*cosh(e*x + d) + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cosh(d+ex)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cosh(d + e*x) + 1), x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{f\cosh(ex+d)+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f), x)
```

3.900 $\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx$

Optimal. Leaf size=151

$$\frac{2e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{3e^2f^2}+\frac{bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{}$$

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]))/(3*e^2*f^2) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sech[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[
d/2 + (e*x)/2]^2*Tanh[d/2 + (e*x)/2])/(6*e*f^2)
```

Rubi [A] time = 0.102074, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5497, 5490, 5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{3e^2f^2}+\frac{bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]))/(3*e^2*f^2) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sech[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[
d/2 + (e*x)/2]^2*Tanh[d/2 + (e*x)/2])/(6*e*f^2)
```

Rule 5497

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
```

```
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\mathrm{sech}^4\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= \frac{bcF^{c(a+bx)}\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2} + \frac{\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right.}{}$$

$$= \frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{d+ex}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}\log(F)\mathrm{se}}{6e^2f^2}$$

Mathematica [A] time = 0.332149, size = 127, normalized size = 0.84

$$\frac{2\cosh\left(\frac{1}{2}(d+ex)\right)F^{c(a+bx)}\left(4e^{d+ex}\cosh^3\left(\frac{1}{2}(d+ex)\right)(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)+bc\log(F)\mathrm{co}\right.}{3e^2f^2(\cosh(d+ex)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (2*F^(c*(a + b*x))*Cosh[(d + e*x)/2]*(b*c*Cosh[(d + e*x)/2]*Log[F] + 4*E^(d
 + e*x)*Cosh[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 + (b
*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]) + e*Sinh[(d + e*x)/2]))/(3*e^2
*f^2*(1 + Cosh[d + e*x])^2)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{\left(f + f\cosh\left(ex+d\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)

[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="maxima")

[Out] -16*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*b*c*e^2*log(F))*integrate(F^(b*c*
x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F)
 - 24*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(5*d)*log
(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*e^(5*e*x) + 5*(
b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^
2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 10*(b^3*c^3*f^2*e^(3
*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*lo
g(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 10*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9
*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^
2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*lo
g(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x)), x) + 4*(4*F^
(a*c)*b*c*e*log(F) + 4*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*
F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F^(a*c)*e^2*e^(2*d))*e^(2*e*x) - 4*(F^(a*
c)*b*c*e*e^d*log(F) - 4*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*lo
g(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b
^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2
*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*f^2*e^(3*d
)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*log(
F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^
2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e
^(2*d))*e^(2*e*x) + 4*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F
)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bcx+ac}}{f^2\cosh\left(ex+d\right)^2+2\,f^2\cosh\left(ex+d\right)+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f^2*cosh(e*x + d)^2 + 2*f^2*cosh(e*x + d) + f^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cosh^2\left(d+ex\right)+2\cosh\left(d+ex\right)+1}\,dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cosh(d + e*x)**2 + 2*cosh(d + e*x) + 1), x)/f
**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{\left(f\cosh\left(ex+d\right)+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f)^2, x)
```

3.901 $\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rubi [A] time = 0.0510697, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{2}{x^2}-2x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}\end{aligned}$$

Mathematica [A] time = 0.0618103, size = 51, normalized size = 0.74

$$\frac{e^{-3(a+bx)}\left(-30e^{2(a+bx)}-10e^{6(a+bx)}+3e^{8(a+bx)}+5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (5 - 30*E^(2*(a + b*x)) - 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

Maple [A] time = 0.01, size = 90, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^2(\sinh(bx+a))^3}{5}-\frac{\sinh(bx+a)(\cosh(bx+a))^2}{5}+\frac{\sinh(bx+a)}{5}+\frac{(\cosh(bx+a))^3(\sinh(bx+a))^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^2*sinh(b*x+a)^3-1/5*sinh(b*x+a)*cosh(b*x+a)^2+1/5*sinh
(b*x+a)+1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15
```

```
*cosh(b*x+a))
```

Maxima [A] time = 0.994325, size = 76, normalized size = 1.1

$$-\frac{\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}-10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
 - 10*e^(3*b*x + 3*a))/b
```

Fricas [A] time = 1.56103, size = 302, normalized size = 4.38

$$\frac{\cosh\left(bx+a\right)^{4}-\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+\left(6\,\cosh\left(bx+a\right)^{2}-5\right)\sinh\left(bx+a\right)^{2}-5\,\cosh\left(bx+a\right.}{30\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^2 - 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 - 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 92.6981, size = 139, normalized size = 2.01

$$\begin{cases}\frac{e^{a}e^{bx}\sinh^{4}\left(a+bx\right)}{5b}-\frac{e^{a}e^{bx}\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{5b}+\frac{e^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{5b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{15b}-\frac{2e^{a}e^{bx}\cosh^{4}\left(a+bx\right)}{15b} & \text{fo}\\ xe^{a}\sinh^{3}\left(a\right)\cosh\left(a\right) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)**3*cosh(a + b*x)/(5*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b
*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b) - 2*
exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3*co
sh(a), True))
```

Giac [A] time = 1.14918, size = 70, normalized size = 1.01

$$-\frac{5\left(6e^{(2bx+2a)}-1\right)e^{(-3bx-3a)}-3e^{(5bx+5a)}+10e^{(3bx+3a)}}{240b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) + 10
*e^(3*b*x + 3*a))/b
```

3.902 $\int e^{a+bx} \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=57

$$-\frac{e^{-2a-2bx}}{16b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

```
[Out] -E^(-2*a - 2*b*x)/(16*b) - E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b)
- x/8
```

Rubi [A] time = 0.052599, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 75}

$$-\frac{e^{-2a-2bx}}{16b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-2*a - 2*b*x)/(16*b) - E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b)
- x/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2(1+x)}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\ &= -\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}\end{aligned}$$

Mathematica [A] time = 0.0661357, size = 45, normalized size = 0.79

$$-\frac{2e^{-2(a+bx)}+2e^{2(a+bx)}-e^{4(a+bx)}+4bx}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -(2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) - E^(4*(a + b*x)) + 4*b*x)/(32*b)
```

Maple [A] time = 0.01, size = 71, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^2(\sinh(bx+a))^2}{4}-\frac{(\cosh(bx+a))^2}{4}+\frac{(\cosh(bx+a))^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^2*sinh(b*x+a)^2-1/4*cosh(b*x+a)^2+1/4*cosh(b*x+a)^3*si
nh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a)
```

Maxima [A] time = 1.03779, size = 68, normalized size = 1.19

$$-\frac{1}{8}x-\frac{a}{8b}+\frac{e^{(4bx+4a)}-2e^{(2bx+2a)}}{32b}-\frac{e^{(-2bx-2a)}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a))/b - 1/16*e^(-
2*b*x - 2*a)/b
```

Fricas [B] time = 1.53401, size = 261, normalized size = 4.58

$$-\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2-3\sinh(bx+a)^3+2(2bx+1)\cosh(bx+a)-\left(4bx+9\cosh(bx+\right.}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 + 2*(2*b*x + 1)*cosh(b*x + a) - (4*b*x + 9*cosh(b*x + a)^2 - 2)*sinh(b*x
+ a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 26.8278, size = 177, normalized size = 3.11

$$\begin{cases}-\frac{xe^{a}e^{bx}\sinh^{3}(a+bx)}{8}+\frac{xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh(a+bx)}{8}+\frac{xe^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{8}-\frac{xe^{a}e^{bx}\cosh^{3}(a+bx)}{8}+\frac{3e^{a}e^{bx}\sinh^{3}(a+bx)}{8b}-\frac{e^{a}e^{bx}\sinh}{} \\ xe^{a}\sinh^{2}(a)\cosh(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 + 3*exp(a)*exp(b*x)*sinh(a + b*
x)**3/(8*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)
*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(a),
True))
```

Giac [A] time = 1.13002, size = 77, normalized size = 1.35

$$-\frac{4\,bx - 2\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) + 2*e^(2*b*x + 2*a))/b
```

3.903 $\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{e^{-a-bx}}{4b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] E^(-a - b*x)/(4*b) + E^(3*a + 3*b*x)/(12*b)
```

Rubi [A] time = 0.0267362, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2282, 12, 14}

$$\frac{e^{-a-bx}}{4b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

Mathematica [A] time = 0.0098237, size = 28, normalized size = 0.8

$$\frac{e^{-a-bx}\left(e^{4(a+bx)}+3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - b*x)*(3 + E^(4*(a + b*x))))/(12*b)
```

Maple [A] time = 0.005, size = 54, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\sinh(bx+a)(\cosh(bx+a))^2}{3} - \frac{\sinh(bx+a)}{3} + \frac{\cosh(bx+a)(\sinh(bx+a))^2}{3} + \frac{\cosh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)*cosh(b*x+a)^2-1/3*sinh(b*x+a)+1/3*cosh(b*x+a)*sinh(b*x
+a)^2+1/3*cosh(b*x+a))
```

Maxima [A] time = 1.01073, size = 39, normalized size = 1.11

$$\frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/4*e^(-b*x - a)/b
```

Fricas [A] time = 1.45966, size = 144, normalized size = 4.11

$$\frac{\cosh(bx+a)^2 - \cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2}{3\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*co
sh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 8.2502, size = 76, normalized size = 2.17

$$\begin{cases} \frac{e^a e^{bx}\sinh^2(a+bx)}{3b} - \frac{e^a e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b} + \frac{e^a e^{bx}\cosh^2(a+bx)}{3b} & \text{for } b \neq 0 \\ xe^a\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne(b,
0)), (x*exp(a)*sinh(a)*cosh(a), True))
```

Giac [A] time = 1.12734, size = 35, normalized size = 1.

$$\frac{e^{(3\,bx+3\,a)} + 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")

[Out] 1/12*(e^(3*b*x + 3*a) + 3*e^(-b*x - a))/b
```

3.904 $\int e^{a+bx}\coth(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0177557, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 388, 206}

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{-1-x^2}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.0162444, size = 22, normalized size = 0.88

$$\frac{e^{a+bx}-2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] (E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] time = 0.014, size = 27, normalized size = 1.1

$$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\text{Artanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] 1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [A] time = 1.08431, size = 51, normalized size = 2.04

$$\frac{e^{(bx+a)}}{b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Fricas [B] time = 1.56986, size = 158, normalized size = 6.32

$$\frac{\cosh(bx+a) - \log(\cosh(bx+a) + \sinh(bx+a) + 1) + \log(\cosh(bx+a) + \sinh(bx+a) - 1) + \sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14439, size = 53, normalized size = 2.12

$$\frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(\left|e^{(bx+a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(abs(e^(b*x + a) - 1))/b
```

3.905 $\int e^{a+bx} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rubi [A] time = 0.0448145, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2282, 12, 444, 43}

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{2x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0466463, size = 34, normalized size = 0.83

$$
\frac{\log\left(1-e^{2(a+bx)}\right)-\frac{2}{e^{2(a+bx)}-1}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-2/(-1 + E^(2*(a + b*x))) + Log[1 - E^(2*(a + b*x))])/b
```

Maple [A] time = 0.017, size = 30, normalized size = 0.7

$$x+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}-\frac{\coth\left(bx+a\right)}{b}+\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] x+ln(sinh(b*x+a))/b-coth(b*x+a)/b+a/b
```

Maxima [A] time = 1.02518, size = 61, normalized size = 1.49

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1
))
```

Fricas [B] time = 1.47199, size = 284, normalized size = 6.93

$$\frac{\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-2}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 2)/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2552, size = 63, normalized size = 1.54

$$\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}-\frac{e^{(2\,bx+2\,a)}+1}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] log(abs(e^(2*b*x + 2*a) - 1))/b - (e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a)
 - 1))
```

3.906 $\int e^{a+bx}\coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (-2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 - E^(2
*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/b
```

Rubi [A] time = 0.0602406, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2282, 12, 455, 385, 206}

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 - E^(2
*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\operatorname{Subst}\left(\int \frac{x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-2-4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0802675, size = 59, normalized size = 0.84

$$\frac{e^{a+bx}-3e^{3(a+bx)}+\left(e^{2(a+bx)}-1\right)^2\left(-\tanh^{-1}\left(e^{a+bx}\right)\right)}{b\left(e^{2(a+bx)}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x) - 3*E^(3*(a + b*x)) - (-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(a +
b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

Maple [A] time = 0.017, size = 69, normalized size = 1.

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{\sinh(bx+a)}+\sinh(bx+a)-\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)*cosh(b*x+a)^2+sinh(b*x+a)-1/sinh(b*x+a)^2*cosh(b*x+a)+1
/2*csch(b*x+a)*coth(b*x+a)-arctanh(exp(b*x+a)))
```

Maxima [A] time = 1.01084, size = 105, normalized size = 1.5

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b}-\frac{3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(b*x + a) + 1)/b + 1/2*log(e^(b*x + a) - 1)/b - (3*e^(3*b*x + 3*
a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.5487, size = 1089, normalized size = 15.56

$$-\frac{6\,\cosh\left(bx+a\right)^{3}+18\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+6\,\sinh\left(bx+a\right)^{3}+\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 + 6*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b
*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x
+ a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sin
h(b*x + a) - 1) + 2*(9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 2*cosh(b*x + a)
)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(
b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37044, size = 92, normalized size = 1.31

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}+\frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}-\frac{3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*log(e^(b*x + a) + 1)/b + 1/2*log(abs(e^(b*x + a) - 1))/b - (3*e^(3*b*x
 + 3*a) - e^(b*x + a))/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.907 $\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}$$

```
[Out] E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b)
 - E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) + x/16
```

Rubi [A] time = 0.0743978, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 88}

$$\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b)
 - E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) + x/16
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3(1+x)^2}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2-\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-x+x^2\right)dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}
\end{aligned}$$

Mathematica [A] time = 0.0900048, size = 67, normalized size = 0.74

$$\frac{3e^{-4(a+bx)}-6e^{-2(a+bx)}-12e^{2(a+bx)}-3e^{4(a+bx)}+2e^{6(a+bx)}+24bx}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (3/E^(4*(a + b*x)) - 6/E^(2*(a + b*x)) - 12*E^(2*(a + b*x)) - 3*E^(4*(a + b
*x)) + 2*E^(6*(a + b*x)) + 24*b*x)/(384*b)
```

Maple [A] time = 0.012, size = 107, normalized size = 1.2

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3(\sinh(bx+a))^3}{6} - \frac{(\cosh(bx+a))^3\sinh(bx+a)}{8} + \frac{\cosh(bx+a)\sinh(bx+a)}{16} + \frac{bx}{16} + \frac{a}{16} + \frac{(\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a+1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*c
osh(b*x+a)^2*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2)
```

Maxima [A] time = 1.00758, size = 104, normalized size = 1.14

$$-\frac{\left(2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}}{128\,b}+\frac{bx+a}{16\,b}+\frac{2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/128*(2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4*a)/b + 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

Fricas [B] time = 1.49023, size = 456, normalized size = 5.01

$$\frac{5\,\cosh(bx+a)^5+25\,\cosh(bx+a)\sinh(bx+a)^4-\sinh(bx+a)^5-\left(10\,\cosh(bx+a)^2-3\right)\sinh(bx+a)^3-9\,\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*x + a)^5 + 25*cosh(b*x + a)*sinh(b*x + a)^4 - sinh(b*x + a)
^5 - (10*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^3 - 9*cosh(b*x + a)^3 + (50*cos
h(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x - 1)*cosh(b*x
+ a) - (5*cosh(b*x + a)^4 + 24*b*x - 9*cosh(b*x + a)^2 + 12)*sinh(b*x + a))
```

```
/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25396, size = 109, normalized size = 1.2

$$\frac{24\,bx - 3\left(6\,e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-4\,bx-4\,a)} + 24\,a + 2\,e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} - 12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4
*a) + 24*a + 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

3.908 $\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(a + b*x)/(8*b) + E^(5*a + 5*b*x)/(80*b)
```

Rubi [A] time = 0.0530814, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(a + b*x)/(8*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-3a-3bx}}{48b} - \frac{e^{a+bx}}{8b} + \frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.025766, size = 40, normalized size = 0.82

$$\frac{e^{-3(a+bx)}\left(-30e^{4(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-5 - 30*E^(4*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [B] time = 0.01, size = 84, normalized size = 1.7

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3(\sinh(bx+a))^2}{5} - \frac{2\cosh(bx+a)(\sinh(bx+a))^2}{15} - \frac{2\cosh(bx+a)}{15} + \frac{(\cosh(bx+a))^4\sinh(bx+a}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)*sinh(b*x+a)^2-2/15*co
sh(b*x+a)+1/5*cosh(b*x+a)^4*sinh(b*x+a)-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*
x+a))
```

Maxima [A] time = 0.992282, size = 51, normalized size = 1.04

$$\frac{e^{(5\,bx+5\,a)} - 10\,e^{(bx+a)}}{80\,b} - \frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/80*(e^(5*b*x + 5*a) - 10*e^(b*x + a))/b - 1/48*e^(-3*b*x - 3*a)/b
```

Fricas [B] time = 1.55048, size = 258, normalized size = 5.27

$$-\frac{\cosh\left(bx+a\right)^{4} - 16\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right) + 6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2} - 16\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3} +}{120\left(b\cosh\left(bx+a\right) - b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a
)^2*sinh(b*x + a)^2 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
15)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 137.372, size = 144, normalized size = 2.94

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^{4}\left(a+bx\right)}{15b} + \frac{2e^{a}e^{bx}\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{15b} + \frac{e^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{5b} + \frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{15b} - \frac{2e^{a}e^{bx}\cosh^{4}\left(a+bx\right)}{15b} \\ xe^{a}\sinh^{2}\left(a\right)\cosh^{2}\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*
b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a
)**2*cosh(a)**2, True))
```

Giac [A] time = 1.14842, size = 49, normalized size = 1.

$$\frac{3\,e^{(5\,bx+5\,a)} - 30\,e^{(bx+a)} - 5\,e^{(-3\,bx-3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/240*(3*e^(5*b*x + 5*a) - 30*e^(b*x + a) - 5*e^(-3*b*x - 3*a))/b
```

3.909 $\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

```
[Out] E^(-2*a - 2*b*x)/(16*b) + E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b) -
 x/8
```

Rubi [A] time = 0.0469278, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) + E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b) -
 x/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^2}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^2}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}\end{aligned}$$

Mathematica [A] time = 0.0442352, size = 43, normalized size = 0.75

$$\frac{2e^{-2(a+bx)}+2e^{2(a+bx)}+e^{4(a+bx)}-4bx}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x)/(32*b)
```

Maple [A] time = 0.007, size = 71, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}+\frac{(\cosh(bx+a))^2(\sinh(bx+a))^2}{4}+\frac{(\cosh(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a+1/4*cosh(b*x+a)^2*sinh(b*x+a)^2+1/4*cosh(b*x+a)^2)
```

Maxima [A] time = 1.019, size = 68, normalized size = 1.19

$$-\frac{1}{8}x - \frac{a}{8b} + \frac{e^{(4bx+4a)} + 2e^{(2bx+2a)}}{32b} + \frac{e^{(-2bx-2a)}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a))/b + 1/16*e^(-
2*b*x - 2*a)/b
```

Fricas [B] time = 1.5585, size = 259, normalized size = 4.54

$$\frac{3\cosh(bx+a)^3 + 9\cosh(bx+a)\sinh(bx+a)^2 - \sinh(bx+a)^3 - 2(2bx-1)\cosh(bx+a) + \left(4bx - 3\cosh(bx+a\right.}{32(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 - 2*(2*b*x - 1)*cosh(b*x + a) + (4*b*x - 3*cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 35.0223, size = 175, normalized size = 3.07

$$\begin{cases} -\frac{xe^a e^{bx}\sinh^3(a+bx)}{8} + \frac{xe^a e^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8} + \frac{xe^a e^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8} - \frac{xe^a e^{bx}\cosh^3(a+bx)}{8} - \frac{e^a e^{bx}\sinh^3(a+bx)}{8b} + \frac{e^a e^{bx}\sinh^2}{} \\ xe^a\sinh(a)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sinh(a + b*x)
**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)*e
xp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh(a)**2, Tr
ue))
```

Giac [A] time = 1.12852, size = 77, normalized size = 1.35

$$-\frac{4\,bx - 2\left(e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) - 2*e^(2*b*x + 2*a))/b
```

3.910 $\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{e^{2a+2bx}}{4b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{2}$$

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2 + Log[1 - E^(2*a + 2*b*x)]/b
```

Rubi [A] time = 0.0433302, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2282, 12, 446, 72}

$$\frac{e^{2a+2bx}}{4b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2 + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{2x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{4}{-1+x}-\frac{1}{x}\right)dx, x, e^{2a+2bx}\right)}{4b}\\ &= \frac{e^{2a+2bx}}{4b}-\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0293295, size = 39, normalized size = 0.93

$$\frac{e^{2a+2bx}+4\log\left(1-e^{2a+2bx}\right)-2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (E^(2*a + 2*b*x) - 2*b*x + 4*Log[1 - E^(2*a + 2*b*x)])/(4*b)
```

Maple [A] time = 0.018, size = 52, normalized size = 1.2

$$\frac{\cosh(bx+a)\sinh(bx+a)}{2b}+\frac{x}{2}+\frac{a}{2b}+\frac{(\cosh(bx+a))^2}{2b}+\frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)*sinh(b*x+a)/b+1/2*x+1/2*a/b+1/2*cosh(b*x+a)^2/b+ln(sinh(b*x
+a))/b
```

Maxima [A] time = 1.02406, size = 68, normalized size = 1.62

$$-\frac{1}{2}x-\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}+\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(
b*x + a) - 1)/b
```

Fricas [A] time = 1.65233, size = 190, normalized size = 4.52

$$-\frac{2\,bx-\cosh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\sinh\left(bx+a\right)^{2}-4\,\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2 - 4*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

[Out] Timed out

Giac [A] time = 1.17022, size = 57, normalized size = 1.36

$$-\frac{bx+a}{2\,b}+\frac{e^{(2\,bx+2\,a)}}{4\,b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)/b + 1/4*e^(2*b*x + 2*a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.911 $\int e^{a+bx} \coth^2(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rubi [A] time = 0.0381733, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 390, 288, 206}

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\coth^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{4x^2}{\left(1-x^2\right)^2}\right)dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 2.54259, size = 179, normalized size = 3.38

$$\frac{e^{a+bx}\left(\frac{4}{105}\left(e^{a+bx}+e^{3(a+bx)}\right)^2 {}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2(a+bx)}\right)+\frac{1}{48}e^{-4(a+bx)}\left(-713e^{2(a+bx)}-181e^{4(a+bx)}+61e^{6(a+bx)}+\frac{3\left(196e^{2(a+\cdots}{\cdots}\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*(
a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*(a
 + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a +
b*x))]])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b
```

Maple [A] time = 0.019, size = 47, normalized size = 0.9

$$\frac{1}{b}\left(\cosh\left(bx+a\right)-2\,\mathrm{Artanh}\left(\mathrm{e}^{bx+a}\right)-\frac{\left(\cosh\left(bx+a\right)\right)^{2}}{\sinh\left(bx+a\right)}+2\,\sinh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))-1/sinh(b*x+a)*cosh(b*x+a)^2+2*sinh(b
*x+a))
```

Maxima [A] time = 0.999177, size = 84, normalized size = 1.58

$$\frac{e^{(bx+a)}}{b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x
+ a)/(b*(e^(2*b*x + 2*a) - 1))
```

Fricas [B] time = 1.59806, size = 585, normalized size = 11.04

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cos
h(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(
cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1461, size = 85, normalized size = 1.6

$$\frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(abs(e^(b*x + a) - 1))/b - 2*e^
(b*x + a)/(b*(e^(2*b*x + 2*a) - 1))
```

3.912 $\int e^{a+bx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{4}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 4/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2
*a + 2*b*x)]/b
```

Rubi [A] time = 0.0653165, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 444, 43}

$$\frac{4}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 4/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2
*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{x(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{(-1+x)^3}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{4}{(-1+x)^3}+\frac{4}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{4}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0814723, size = 46, normalized size = 0.74

$$
\frac{\frac{2-4e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+\log\left(1-e^{2(a+bx)}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] ((2 - 4*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*x))
])/b
```

Maple [A] time = 0.024, size = 43, normalized size = 0.7

$$x+\frac{a}{b}-\frac{\coth(bx+a)}{b}+\frac{\ln(\sinh(bx+a))}{b}-\frac{(\coth(bx+a))^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] x+a/b-coth(b*x+a)/b+ln(sinh(b*x+a))/b-1/2*coth(b*x+a)^2/b
```

Maxima [A] time = 1.01903, size = 93, normalized size = 1.5

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*(2*e^(2*b*x + 2*a) - 1)
/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.62851, size = 711, normalized size = 11.47

$$-\frac{4\cosh(bx+a)^{2}-\left(\cosh(bx+a)^{4}+4\cosh(bx+a)\sinh(bx+a)^{3}+\sinh(bx+a)^{4}+2\left(3\cosh(bx+a)^{2}-1\right)\sinh(\cdots}{b\cosh(bx+a)^{4}+4\,b\cosh(bx+a)\sinh(bx+a)^{3}+b\sinh(bx+a)^{4}-2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(4*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 8*cosh(b*x + a)*sinh(b*x + a) + 4
```

```
*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14078, size = 81, normalized size = 1.31

$$\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}-\frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}-1}{2\,b\left(e^{(2\,bx+2\,a)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(abs(e^(2*b*x + 2*a) - 1))/b - 1/2*(3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a
) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.913 $\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

```
[Out] E^(-5*a - 5*b*x)/(320*b) - (3*E^(-a - b*x))/(64*b) - E^(3*a + 3*b*x)/(64*b)
 + E^(7*a + 7*b*x)/(448*b)
```

Rubi [A] time = 0.0589133, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 270}

$$\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-5*a - 5*b*x)/(320*b) - (3*E^(-a - b*x))/(64*b) - E^(3*a + 3*b*x)/(64*b)
 + E^(7*a + 7*b*x)/(448*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^4\right)^3}{64x^6}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^4\right)^3}{x^6}\,dx,x,e^{a+bx}\right)}{64b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^6}+\frac{3}{x^2}-3x^2+x^6\right)dx,x,e^{a+bx}\right)}{64b}\\ &= \frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}\end{aligned}$$

Mathematica [A] time = 0.0375875, size = 51, normalized size = 0.74

$$\frac{e^{-5(a+bx)}\left(-105e^{4(a+bx)}-35e^{8(a+bx)}+5e^{12(a+bx)}+7\right)}{2240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (7 - 105*E^(4*(a + b*x)) - 35*E^(8*(a + b*x)) + 5*E^(12*(a + b*x)))/(2240*b
*E^(5*(a + b*x)))
```

Maple [B] time = 0.013, size = 120, normalized size = 1.7

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4(\sinh(bx+a))^3}{7}-\frac{3\,(\cosh(bx+a))^4\sinh(bx+a)}{35}+\frac{3\,\sinh(bx+a)}{35}\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)+\frac{(\sinh}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/7*cosh(b*x+a)^4*sinh(b*x+a)^3-3/35*cosh(b*x+a)^4*sinh(b*x+a)+3/35*(2
/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/7*sinh(b*x+a)^2*cosh(b*x+a)^5-2/35*cosh
```

```
(b*x+a)^3*sinh(b*x+a)^2-2/35*cosh(b*x+a)*sinh(b*x+a)^2-2/35*cosh(b*x+a))
```

Maxima [A] time = 1.04281, size = 73, normalized size = 1.06

$$-\frac{\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}}{320\,b}+\frac{e^{(7\,bx+7\,a)}-7\,e^{(3\,bx+3\,a)}}{448\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/320*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a)/b + 1/448*(e^(7*b*x + 7*a)
 - 7*e^(3*b*x + 3*a))/b
```

Fricas [B] time = 1.53266, size = 420, normalized size = 6.09

$$\frac{3\,\cosh\left(bx+a\right)^{6}-10\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{3}+45\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}-3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}}{560\,(b\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/560*(3*cosh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x
 + a)^2*sinh(b*x + a)^4 - 3*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)
^6 + 5*(9*cosh(b*x + a)^4 - 7)*sinh(b*x + a)^2 - 35*cosh(b*x + a)^2 - (3*co
sh(b*x + a)^5 - 35*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(
b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18104, size = 70, normalized size = 1.01

$$-\frac{7\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}-5\,e^{(7\,bx+7\,a)}+35\,e^{(3\,bx+3\,a)}}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2240*(7*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a) - 5*e^(7*b*x + 7*a) +
35*e^(3*b*x + 3*a))/b
```

3.914 $\int e^{a+bx} \cosh^3(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=91

$$-\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}$$

```
[Out] -E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b
) + E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) - x/16
```

Rubi [A] time = 0.0793523, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 88}

$$-\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b
) + E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) - x/16
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)^3}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)^3}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2(1+x)^3}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{1}{x^2}-\frac{2}{x}+x+x^2\right)dx, x, e^{2a+2bx}\right)}{64b}\\
&= -\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}
\end{aligned}$$

Mathematica [A] time = 0.102667, size = 67, normalized size = 0.74

$$-\frac{3e^{-4(a+bx)}+6e^{-2(a+bx)}+12e^{2(a+bx)}-3e^{4(a+bx)}-2e^{6(a+bx)}+24bx}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -(3/E^(4*(a + b*x)) + 6/E^(2*(a + b*x)) + 12*E^(2*(a + b*x)) - 3*E^(4*(a +
b*x)) - 2*E^(6*(a + b*x)) + 24*b*x)/(384*b)
```

Maple [A] time = 0.01, size = 102, normalized size = 1.1

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4(\sinh(bx+a))^2}{6}-\frac{(\cosh(bx+a))^2(\sinh(bx+a))^2}{12}-\frac{(\cosh(bx+a))^2}{12}+\frac{\sinh(bx+a)(\cosh(bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^2*sinh(b*x+a)^2-1/12*
cosh(b*x+a)^2+1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh
(b*x+a))*sinh(b*x+a)-1/16*b*x-1/16*a)
```

Maxima [A] time = 1.03309, size = 104, normalized size = 1.14

$$-\frac{\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}}{128\,b}-\frac{bx+a}{16\,b}+\frac{2\,e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a)/b - 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

Fricas [B] time = 1.47983, size = 458, normalized size = 5.03

$$-\frac{\cosh(bx+a)^5+5\,\cosh(bx+a)\sinh(bx+a)^4-5\,\sinh(bx+a)^5-\left(50\,\cosh(bx+a)^2+9\right)\sinh(bx+a)^3+3\,\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 - 5*sinh(b*x + a)
^5 - (50*cosh(b*x + a)^2 + 9)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + (10*cos
h(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x + 1)*cosh(b*x +
 a) - (25*cosh(b*x + a)^4 + 24*b*x + 27*cosh(b*x + a)^2 - 12)*sinh(b*x + a)
```

```
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17899, size = 109, normalized size = 1.2

$$-\frac{24\,bx - 3\left(6\,e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-4\,bx-4\,a)} + 24\,a - 2\,e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} + 12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x -
4*a) + 24*a - 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a))/b
```

3.915 $\int e^{a+bx} \cosh^3(a+bx) \sinh(a+bx)\, dx$

Optimal. Leaf size=69

$$\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rubi [A] time = 0.0466618, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(1+x^2\right)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(1+x^2\right)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}-\frac{2}{x^2}+2x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.0510072, size = 51, normalized size = 0.74

$$\frac{e^{-3(a+bx)}\left(30e^{2(a+bx)}+10e^{6(a+bx)}+3e^{8(a+bx)}+5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (5 + 30*E^(2*(a + b*x)) + 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

Maple [A] time = 0.008, size = 84, normalized size = 1.2

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^4\sinh(bx+a)}{5}-\frac{\sinh(bx+a)}{5}\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)+\frac{(\cosh(bx+a))^3(\sinh(bx+a))^2}{5}+\frac{\cosh(bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^4*sinh(b*x+a)-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+
1/5*cosh(b*x+a)^3*sinh(b*x+a)^2+1/5*cosh(b*x+a)*sinh(b*x+a)^2+1/5*cosh(b*x+
```

```
a))
```

Maxima [A] time = 1.01224, size = 76, normalized size = 1.1

$$\frac{\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/48*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
+ 10*e^(3*b*x + 3*a))/b
```

Fricas [A] time = 1.65127, size = 302, normalized size = 4.38

$$\frac{\cosh\left(bx+a\right)^{4}-\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+\left(6\,\cosh\left(bx+a\right)^{2}+5\right)\sinh\left(bx+a\right)^{2}+5\,\cosh\left(bx+a\right.}{30\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^2 + 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 + 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 107.192, size = 139, normalized size = 2.01

$$\begin{cases}-\frac{2e^{a}e^{bx}\sinh^{4}(a+bx)}{15b}+\frac{2e^{a}e^{bx}\sinh^{3}(a+bx)\cosh(a+bx)}{15b}+\frac{e^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{5b}-\frac{e^{a}e^{bx}\sinh(a+bx)\cosh^{3}(a+bx)}{5b}+\frac{e^{a}e^{bx}\cosh^{4}(a+bx)}{5b}\\ xe^{a}\sinh(a)\cosh^{3}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*b)
+ exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh
(a)**3, True))
```

Giac [A] time = 1.14272, size = 70, normalized size = 1.01

$$\frac{5\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/240*(5*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) + 10*
e^(3*b*x + 3*a))/b
```

3.916 $\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(-a - b*x)/(4*b) + E^(a + b*x)/b + E^(3*a + 3*b*x)/(12*b) - (2*ArcTanh[E^
(a + b*x)])/b
```

Rubi [A] time = 0.0537115, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 461, 207}

$$\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(a + b*x)/b + E^(3*a + 3*b*x)/(12*b) - (2*ArcTanh[E^
(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{4x^2(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^2(-1+x^2)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\operatorname{Subst}\left(\int \left(4-\frac{1}{x^2}+x^2+\frac{8}{-1+x^2}\right)dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}+\frac{2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.347232, size = 68, normalized size = 1.15

$$\frac{e^{-a-bx}\left(12e^{2(a+bx)}+e^{4(a+bx)}-24\sqrt{e^{2(a+bx)}}\tanh^{-1}\left(\sqrt{e^{2(a+bx)}}\right)+3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (E^(-a - b*x)*(3 + 12*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 24*Sqrt[E^(2*(a +
 b*x))]*ArcTanh[Sqrt[E^(2*(a + b*x))]]))/(12*b)
```

Maple [A] time = 0.02, size = 50, normalized size = 0.9

$$\frac{1}{b}\left(\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)\sinh(bx+a)+\frac{(\cosh(bx+a))^3}{3}+\cosh(bx+a)-2\,\mathrm{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/b*((2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*ar
ctanh(exp(b*x+a)))
```

Maxima [A] time = 0.984571, size = 88, normalized size = 1.49

$$\frac{e^{(3bx+3a)}+12e^{(bx+a)}}{12b}+\frac{e^{(-bx-a)}}{4b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a))/b + 1/4*e^(-b*x - a)/b - log(e^(b*x
 + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Fricas [B] time = 1.83499, size = 520, normalized size = 8.81

$$\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+6\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+12\cosh(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 - 12*(cosh(b*
x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 12*(cosh(b
*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 4*(cosh(b
*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) + 3)/(b*cosh(b*x + a) + b*sinh(b
```

```
*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17613, size = 99, normalized size = 1.68

$$\frac{e^{(-bx-a)}}{4\,b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}+\frac{b^2e^{(3\,bx+3\,a)}+12\,b^2e^{(bx+a)}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(-b*x - a)/b - log(e^(b*x + a) + 1)/b + log(abs(e^(b*x + a) - 1))/b +
 1/12*(b^2*e^(3*b*x + 3*a) + 12*b^2*e^(b*x + a))/b^3
```

3.917 $\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x}{2}$$

```
[Out] E^(2*a + 2*b*x)/(4*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + x/2 + Log[1 - E^(2*a
+ 2*b*x)]/b
```

Rubi [A] time = 0.0645684, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 88}

$$\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + x/2 + Log[1 - E^(2*a
+ 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{2x(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(1-x)^2x}\,dx, x, e^{2a+2bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^2}+\frac{4}{-1+x}+\frac{1}{x}\right)dx, x, e^{2a+2bx}\right)}{4b}\\ &= \frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.081858, size = 52, normalized size = 0.83

$$\frac{e^{2(a+bx)}-\frac{8}{e^{2(a+bx)}-1}+4\log\left(1-e^{2(a+bx)}\right)+2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^(2*(a + b*x)) - 8/(-1 + E^(2*(a + b*x))) + 2*b*x + 4*Log[1 - E^(2*(a + b
*x))])/(4*b)
```

Maple [A] time = 0.02, size = 67, normalized size = 1.1

$$\frac{(\cosh(bx+a))^2}{2b}+\frac{\ln(\sinh(bx+a))}{b}+\frac{(\cosh(bx+a))^3}{2b\sinh(bx+a)}+\frac{3x}{2}+\frac{3a}{2b}-\frac{3\coth(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b+1/2/b*cosh(b*x+a)^3/sinh(b*x+a)+3/2*x
+3/2*a/b-3/2*coth(b*x+a)/b
```

Maxima [A] time = 1.02303, size = 92, normalized size = 1.46

$$\frac{1}{2}x+\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}+\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2}{b\left(e^{(2bx+2a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(b
*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

Fricas [B] time = 1.92155, size = 594, normalized size = 9.43

$$\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+(2bx-1)\cosh(bx+a)^2+\left(2bx+6\cosh(bx+a)^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(2*b*x - 1)*cosh(b*x + a)^2 + (2*b*x + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 2*b*x + 4*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cos
```

```
h(b*x + a)^3 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 8)/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15194, size = 96, normalized size = 1.52

$$\frac{bx+a}{2\,b}+\frac{e^{(2\,bx+2\,a)}}{4\,b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}-\frac{e^{(2\,bx+2\,a)}+1}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x + a)/b + 1/4*e^(2*b*x + 2*a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b -
 (e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1))
```

3.918 $\int e^{a+bx}\coth^3(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0515906, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1158, 12, 288, 207}

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\coth^3(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1+x^2)^3}{(-1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{2\,\text{Subst}\left(\int\frac{1+3x^4}{(-1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\text{Subst}\left(\int\frac{12x^2}{(-1+x^2)^2}\,dx,x,e^{a+bx}\right)}{2b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6\,\text{Subst}\left(\int\frac{x^2}{(-1+x^2)^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{3\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

<u>Mathematica</u> [C] time = 4.42973, size = 286, normalized size = 3.53

$$-\frac{e^{-5(a+bx)}\left(256e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^3{}_6F_5\left(\frac{3}{2},2,2,2,2,2,2;1,1,1,1,\frac{11}{2};e^{2(a+bx)}\right)+384e^{8(a+bx)}\left(5e^{2(a+bx)}+7\right)\left(e^{2(a+bx)}+1\right)^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] -(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) - 154296*E^
(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) - (315*(-168
07 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(a + b*x)) +
 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*x)))*ArcTanh
[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a + b*x))*(1 + E
^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2,
```

```
2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a +
b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*(
a + b*x))])/(60480*b*E^(5*(a + b*x)))
```

Maple [A] time = 0.025, size = 88, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{\sinh(bx+a)}+2\sinh(bx+a)+\frac{(\cosh(bx+a))^3}{(\sinh(bx+a))^2}-3\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{3\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\operatorname{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)*cosh(b*x+a)^2+2*sinh(b*x+a)+cosh(b*x+a)^3/sinh(b*x+a)^2
-3/sinh(b*x+a)^2*cosh(b*x+a)+3/2*csch(b*x+a)*coth(b*x+a)-3*arctanh(exp(b*x+
a)))
```

Maxima [A] time = 1.00041, size = 119, normalized size = 1.47

$$\frac{e^{(bx+a)}}{b}-\frac{3\log\left(e^{(bx+a)}+1\right)}{2b}+\frac{3\log\left(e^{(bx+a)}-1\right)}{2b}-\frac{3e^{(3bx+3a)}-e^{(bx+a)}}{b\left(e^{(4bx+4a)}-2e^{(2bx+2a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - (
3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) +
1))
```

Fricas [B] time = 1.86649, size = 1291, normalized size = 15.94

$$\frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\sinh(bx+a)^5+10\left(2\cosh(bx+a)^2-1\right)\sinh(bx+a)^3-10\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a)
^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10*(
2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 +
 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(5
*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13996, size = 105, normalized size = 1.3

$$\frac{e^{(bx+a)}}{b} - \frac{3 \log\left(e^{(bx+a)} + 1\right)}{2 b} + \frac{3 \log\left(\left|e^{(bx+a)} - 1\right|\right)}{2 b} - \frac{3 e^{(3 bx+3 a)} - e^{(bx+a)}}{b\left(e^{(2 bx+2 a)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(abs(e^(b*x + a) - 1))/
b - (3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.919 $\int e^{2(a+bx)} \cosh(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{32b} - \frac{e^{4a+4bx}}{32b} + \frac{e^{6a+6bx}}{96b} + \frac{x}{8}$$

```
[Out] E^(-2*a - 2*b*x)/(32*b) - E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) +
 x/8
```

Rubi [A] time = 0.053828, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{32b} - \frac{e^{4a+4bx}}{32b} + \frac{e^{6a+6bx}}{96b} + \frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) - E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) +
 x/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1-x)(1-x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2}{x}-2x+x^2\right)dx, x, e^{2a+2bx}\right)}{32b}\\
&= \frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}
\end{aligned}$$

Mathematica [A] time = 0.0725796, size = 43, normalized size = 0.75

$$\frac{3e^{-2(a+bx)}-3e^{4(a+bx)}+e^{6(a+bx)}+12bx}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (3/E^(2*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) + 12*b*x)/(96*b)
```

Maple [A] time = 0.016, size = 89, normalized size = 1.6

$$\frac{x}{8}-\frac{\sinh(2\,bx+2\,a)}{32\,b}-\frac{\sinh(4\,bx+4\,a)}{32\,b}+\frac{\sinh(6\,bx+6\,a)}{96\,b}+\frac{\cosh(2\,bx+2\,a)}{32\,b}-\frac{\cosh(4\,bx+4\,a)}{32\,b}+\frac{\cosh(6\,bx+6\,a)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/8*x-1/32*sinh(2*b*x+2*a)/b-1/32/b*sinh(4*b*x+4*a)+1/96/b*sinh(6*b*x+6*a)+
1/32*cosh(2*b*x+2*a)/b-1/32*cosh(4*b*x+4*a)/b+1/96*cosh(6*b*x+6*a)/b
```

Maxima [A] time = 1.03007, size = 70, normalized size = 1.23

$$-\frac{\left(3\,e^{(-2\,bx-2\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b}+\frac{bx+a}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*e^(-2*b*x - 2*a) - 1)*e^(6*b*x + 6*a)/b + 1/8*(b*x + a)/b + 1/32*e
^(-2*b*x - 2*a)/b
```

Fricas [B] time = 1.88637, size = 413, normalized size = 7.25

$$\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+3\,(4\,bx-1)\cosh(bx+a)^2+3\left(4\,bx+8\cosh(bx\right.}{96\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 + 3*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(4*b*x + 8*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 + 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

Sympy [A] time = 90.0155, size = 241, normalized size = 4.23

$$\begin{cases}-\frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8}+\frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4}-\frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4}+\frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8}+\frac{e^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a}{8b} \\ xe^{2a}\sinh^3(a)\cosh(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)**3*cosh(a + b*x)/4 - x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*co
sh(a + b*x)**3/4 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + exp(2*a)*exp(2*b*x)*sinh(a +
b*x)**2*cosh(a + b*x)**2/(4*b) - 7*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a
 + b*x)**3/(24*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(12*b), Ne(b, 0)),
 (x*exp(2*a)*sinh(a)**3*cosh(a), True))
```

Giac [A] time = 1.16822, size = 81, normalized size = 1.42

$$\frac{12\,bx - 3\left(2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + \left(e^{(6\,bx+12\,a)} - 3\,e^{(4\,bx+10\,a)}\right)e^{(-6\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*(12*b*x - 3*(2*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + (e^(6*b*x + 12*
a) - 3*e^(4*b*x + 10*a))*e^(-6*a))/b
```

3.920 $\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

```
[Out] -E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(5*a +
 5*b*x)/(40*b)
```

Rubi [A] time = 0.049802, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 448}

$$-\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(5*a +
 5*b*x)/(40*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}-x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= -\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Mathematica [A] time = 0.0802082, size = 51, normalized size = 0.77

$$\frac{3e^{-a-bx}\left(e^{6(a+bx)}-5\right)-5e^{a+bx}\left(e^{2(a+bx)}+3\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*E^(a + b*x)*(3 + E^(2*(a + b*x))) + 3*E^(-a - b*x)*(-5 + E^(6*(a + b*x)
)))/(120*b)
```

Maple [A] time = 0.01, size = 69, normalized size = 1.1

$$-\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}-\frac{\cosh(bx+a)}{4b}-\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] -1/24/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)-1/4*cosh(b*x+a)/b-1/24*cosh(
3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

Maxima [A] time = 1.02501, size = 72, normalized size = 1.09

$$-\frac{\left(5\,e^{\left(-2\,bx-2\,a\right)}+15\,e^{\left(-4\,bx-4\,a\right)}-3\right)e^{\left(5\,bx+5\,a\right)}}{120\,b}-\frac{e^{\left(-bx-a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/120*(5*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/8*e^(-b*x - a)/b
```

Fricas [A] time = 1.87815, size = 292, normalized size = 4.42

$$-\frac{6\,\cosh\left(bx+a\right)^{3}+18\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\,\sinh\left(bx+a\right)^{3}-\left(27\,\cosh\left(bx+a\right)^{2}+5\right)\sinh\left(bx+a\right)+10\,\cosh}{60\left(b\cosh\left(bx+a\right)^{2}-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/60*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 - 9*sinh(b*x +
a)^3 - (27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x + a))/(b*cosh(b
*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 30.2939, size = 128, normalized size = 1.94

$$\begin{cases}\frac{e^{2a}e^{2bx}\sinh^{3}\left(a+bx\right)}{15b}-\frac{2e^{2a}e^{2bx}\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{15b}+\frac{8e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{15b}-\frac{4e^{2a}e^{2bx}\cosh^{3}\left(a+bx\right)}{15b} & \text{for}\: b\neq 0\\xe^{2a}\sinh^{2}\left(a\right)\cosh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) - 2*exp(2*a)*exp(2*b
*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a +
```

```
b*x)*cosh(a + b*x)**2/(15*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*b
), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a), True))
```

Giac [A] time = 1.12355, size = 74, normalized size = 1.12

$$\frac{\left(3\,e^{(5\,bx+10\,a)}-5\,e^{(3\,bx+8\,a)}-15\,e^{(bx+6\,a)}\right)e^{(-5\,a)}-15\,e^{(-bx-a)}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/120*((3*e^(5*b*x + 10*a) - 5*e^(3*b*x + 8*a) - 15*e^(b*x + 6*a))*e^(-5*a)
 - 15*e^(-b*x - a))/b
```

3.921 $\int e^{2(a+bx)} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=23

$$\frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

```
[Out] E^(4*a + 4*b*x)/(16*b) - x/4
```

Rubi [A] time = 0.0229656, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 14}

$$\frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(4*a + 4*b*x)/(16*b) - x/4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x^3\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

Mathematica [A] time = 0.0151823, size = 25, normalized size = 1.09

$$\frac{1}{4}\left(\frac{e^{4a+4bx}}{4b} - x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(4*a + 4*b*x)/(4*b) - x)/4
```

Maple [A] time = 0.006, size = 33, normalized size = 1.4

$$-\frac{x}{4} + \frac{\sinh(4\,bx+4\,a)}{16\,b} + \frac{\cosh(4\,bx+4\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*x+1/16/b*sinh(4*b*x+4*a)+1/16*cosh(4*b*x+4*a)/b
```

Maxima [A] time = 1.00212, size = 32, normalized size = 1.39

$$-\frac{1}{4}\,x - \frac{a}{4\,b} + \frac{e^{(4\,bx+4\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x - 1/4*a/b + 1/16*e^(4*b*x + 4*a)/b
```

Fricas [B] time = 1.74641, size = 250, normalized size = 10.87

$$-\frac{(4\,bx-1)\cosh\left(bx+a\right)^{2}-2\,(4\,bx+1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(4\,bx-1)\sinh\left(bx+a\right)^{2}}{16\left(b\cosh\left(bx+a\right)^{2}-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/16*((4*b*x - 1)*cosh(b*x + a)^2 - 2*(4*b*x + 1)*cosh(b*x + a)*sinh(b*x +
 a) + (4*b*x - 1)*sinh(b*x + a)^2)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 9.81299, size = 117, normalized size = 5.09

$$\begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^{2}\left(a+bx\right)}{4}+\frac{xe^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2}-\frac{xe^{2a}e^{2bx}\cosh^{2}\left(a+bx\right)}{4}+\frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b} & \text{for}\ b\neq 0\\ xe^{2a}\sinh\left(a\right)\cosh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)*cosh(a + b*x)/2 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4
+ exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(
2*a)*sinh(a)*cosh(a), True))
```

Giac [A] time = 1.1319, size = 24, normalized size = 1.04

$$-\frac{1}{4}\,x+\frac{e^{(4\,bx+4\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*x + 1/16*e^(4*b*x + 4*a)/b
```

3.922 $\int e^{2(a+bx)}\coth(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{e^{2a+2bx}}{2b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] E^(2*a + 2*b*x)/(2*b) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rubi [A] time = 0.032322, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x],x]

[Out] E^(2*a + 2*b*x)/(2*b) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x(-1-x^2)}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1-x}{1-x}\,dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{2}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.0245513, size = 35, normalized size = 0.95

$$\frac{e^{2a+2bx}+2\log\left(1-e^{2a+2bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x],x]
```

```
[Out] (E^(2*a + 2*b*x) + 2*Log[1 - E^(2*a + 2*b*x)])/(2*b)
```

Maple [A] time = 0.034, size = 38, normalized size = 1.

$$\frac{e^{2bx+2a}}{2b} - 2\frac{a}{b} + \frac{\ln\left(e^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] 1/2*exp(2*b*x+2*a)/b-2*a/b+1/b*ln(exp(2*b*x+2*a)-1)
```

Maxima [A] time = 1.0186, size = 77, normalized size = 2.08

$$\frac{2(bx+a)}{b} + \frac{e^{(2bx+2a)}}{2b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*(b*x + a)/b + 1/2*e^(2*b*x + 2*a)/b + log(e^(-b*x - a) + 1)/b + log(e^(-b
*x - a) - 1)/b
```

Fricas [A] time = 1.85139, size = 178, normalized size = 4.81

$$\frac{\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 + 2\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 2*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16356, size = 41, normalized size = 1.11

$$\frac{e^{(2\,bx+2\,a)} + 2\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) + 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.923 $\int e^{2(a+bx)} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (2*E^(a + b*x))/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (4*ArcTanh[
E^(a + b*x)])/b
```

Rubi [A] time = 0.0434778, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2282, 12, 455, 388, 206}

$$\frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (4*ArcTanh[
E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x^2(1+x^2)}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{x^2(1+x^2)}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{\operatorname{Subst}\left(\int \frac{2+2x^2}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.116034, size = 62, normalized size = 1.15

$$\frac{2\left(\frac{e^{a+bx}\left(e^{2(a+bx)}-2\right)}{e^{2(a+bx)}-1}+\log\left(1-e^{a+bx}\right)-\log\left(e^{a+bx}+1\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (2*((E^(a + b*x)*(-2 + E^(2*(a + b*x))))/(-1 + E^(2*(a + b*x))) + Log[1 - E
^(a + b*x)] - Log[1 + E^(a + b*x)]))/b
```

Maple [A] time = 0.039, size = 65, normalized size = 1.2

$$2\,{\frac {{{\rm e}^{bx+a}}}{b}}-2\,{\frac {{{\rm e}^{bx+a}}}{b \left({{\rm e}^{2\,bx+2\,a}}-1 \right) }}-2\,{\frac {\ln \left(1+{{\rm e}^{bx+a}} \right) }{b}}+2\,{\frac {\ln \left({{\rm e}^{bx+a}}-1 \right) }{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] 2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b*ln(1+exp(b*x+a))+2/b*l
n(exp(b*x+a)-1)
```

Maxima [A] time = 1.05406, size = 103, normalized size = 1.91

$$-\frac{2 \, \log\left(e^{\left(-b x - a\right)} + 1\right)}{b} + \frac{2 \, \log\left(e^{\left(-b x - a\right)} - 1\right)}{b} - \frac{2 \, {\left(2 \, e^{\left(-2 \, b x - 2 \, a\right)} - 1\right)}}{b {\left(e^{\left(-b x - a\right)} - e^{\left(-3 \, b x - 3 \, a\right)}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(2*e^(-2*b*x - 2
*a) - 1)/(b*(e^(-b*x - a) - e^(-3*b*x - 3*a)))
```

Fricas [B] time = 1.85101, size = 587, normalized size = 10.87

$$2 \, {\left(\cosh\left(b x + a\right)^{3} + 3 \, \cosh\left(b x + a\right) \sinh\left(b x + a\right)^{2} + \sinh\left(b x + a\right)^{3} - {\left(\cosh\left(b x + a\right)^{2} + 2 \, \cosh\left(b x + a\right) \sinh\left(b x + a\right) + \sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (
3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 2*cosh(b*x + a))/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15969, size = 74, normalized size = 1.37

$$-\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)} + 1\right) - \log\left(\left|e^{(bx+a)} - 1\right|\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

3.924 $\int e^{2(a+bx)}\coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E
^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.0721251, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 77}

$$\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E
^(2*a + 2*b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\operatorname{Subst}\left(\int \frac{x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{(-1-x)x}{(1-x)^3}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \left(\frac{2}{(-1+x)^3}+\frac{3}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.079961, size = 47, normalized size = 0.75

$$\frac{2\left(\frac{2-3e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+\log\left(1-e^{2(a+bx)}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (2*((2 - 3*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*
x))]))/b
```

Maple [A] time = 0.04, size = 56, normalized size = 0.9

$$-4\frac{a}{b}-2\frac{3\,\mathrm{e}^{2\,bx+2\,a}-2}{b\left(\mathrm{e}^{2\,bx+2\,a}-1\right)^{2}}+2\frac{\ln\left(\mathrm{e}^{2\,bx+2\,a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -4*a/b-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+2/b*ln(exp(2*b*x+2*a)-
1)
```

Maxima [A] time = 1.03539, size = 116, normalized size = 1.84

$$4\,x+\frac{4\,a}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{2\left(e^{(-2\,bx-2\,a)}-2\right)}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(e^
(-2*b*x - 2*a) - 2)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.85458, size = 714, normalized size = 11.33

$$-\frac{2\left(3\,\cosh\left(bx+a\right)^{2}-\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\right.\right.}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
```

```
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 6*cosh(b*x + a)*sinh(b*x + a) +
 3*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a
)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) +
b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18718, size = 65, normalized size = 1.03

$$-\frac{\frac{3\,e^{(4\,bx+4\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -((3*e^(4*b*x + 4*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*
a) - 1)))/b
```

3.925 $\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=100

$$\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}+\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}-\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

```
[Out] E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) + E^(a + b*x)/(16*b) - E^(3*a
 + 3*b*x)/(48*b) - E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rubi [A] time = 0.0778496, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}+\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}-\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) + E^(a + b*x)/(16*b) - E^(3*a
 + 3*b*x)/(48*b) - E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{32x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{x^4}\,dx, x, e^{a+bx}\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \left(2-\frac{1}{x^4}+\frac{1}{x^2}-2x^2-x^4+x^6\right)dx, x, e^{a+bx}\right)}{32b}$$
$$= \frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}+\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}-\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Mathematica [A] time = 0.102981, size = 73, normalized size = 0.73

$$\frac{e^{-3(a+bx)}\left(-105e^{2(a+bx)}+210e^{4(a+bx)}-70e^{6(a+bx)}-21e^{8(a+bx)}+15e^{10(a+bx)}+35\right)}{3360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (35 - 105*E^(2*(a + b*x)) + 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) - 21*E
^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

Maple [A] time = 0.016, size = 108, normalized size = 1.1

$$\frac{3\sinh(bx+a)}{32b}-\frac{\sinh(3bx+3a)}{32b}-\frac{\sinh(5bx+5a)}{160b}+\frac{\sinh(7bx+7a)}{224b}+\frac{\cosh(bx+a)}{32b}-\frac{\cosh(3bx+3a)}{96b}-\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 3/32*sinh(b*x+a)/b-1/32/b*sinh(3*b*x+3*a)-1/160/b*sinh(5*b*x+5*a)+1/224/b*s
inh(7*b*x+7*a)+1/32*cosh(b*x+a)/b-1/96*cosh(3*b*x+3*a)/b-1/160*cosh(5*b*x+5
```

```
*a)/b+1/224*cosh(7*b*x+7*a)/b
```

Maxima [A] time = 1.00432, size = 105, normalized size = 1.05

$$-\frac{\left(21\,e^{(-2\,bx-2\,a)}+70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}-15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3360*(21*e^(-2*b*x - 2*a) + 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) -
 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) - e^(-3*b*x - 3*a))/b
```

Fricas [B] time = 1.76445, size = 501, normalized size = 5.01

$$\frac{25\,\cosh\left(bx+a\right)^{5}+125\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-10\,\sinh\left(bx+a\right)^{5}-2\left(50\,\cosh\left(bx+a\right)^{2}-21\right)\sinh\left(bx+a\right)^{3}-63}{1680\left(b\cosh\left(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1680*(25*cosh(b*x + a)^5 + 125*cosh(b*x + a)*sinh(b*x + a)^4 - 10*sinh(b*
x + a)^5 - 2*(50*cosh(b*x + a)^2 - 21)*sinh(b*x + a)^3 - 63*cosh(b*x + a)^3
 + (250*cosh(b*x + a)^3 - 189*cosh(b*x + a))*sinh(b*x + a)^2 - 2*(25*cosh(b
*x + a)^4 - 63*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 70*cosh(b*x + a))/(b*c
osh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

[Out] Timed out

Giac [A] time = 1.21009, size = 108, normalized size = 1.08

$$-\frac{35\left(3e^{(2bx+2a)}-1\right)e^{(-3bx-3a)}-\left(15e^{(7bx+28a)}-21e^{(5bx+26a)}-70e^{(3bx+24a)}+210e^{(bx+22a)}\right)e^{(-21a)}}{3360b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - (15*e^(7*b*x + 28*a)
 - 21*e^(5*b*x + 26*a) - 70*e^(3*b*x + 24*a) + 210*e^(b*x + 22*a))*e^(-21*a
))/b
```

3.926 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx)\, dx$

Optimal. Leaf size=52

$$-\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

```
[Out] -E^(-2*a - 2*b*x)/(32*b) - E^(2*a + 2*b*x)/(16*b) + E^(6*a + 6*b*x)/(96*b)
```

Rubi [A] time = 0.0592756, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 270}

$$-\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-2*a - 2*b*x)/(32*b) - E^(2*a + 2*b*x)/(16*b) + E^(6*a + 6*b*x)/(96*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{16x^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{x^3}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}-2x+x^5\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

Mathematica [A] time = 0.0378905, size = 38, normalized size = 0.73

$$\frac{e^{-2(a+bx)}\left(-6e^{4(a+bx)}+e^{8(a+bx)}-3\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-3 - 6*E^(4*(a + b*x)) + E^(8*(a + b*x)))/(96*b*E^(2*(a + b*x)))
```

Maple [A] time = 0.008, size = 58, normalized size = 1.1

$$-\frac{\sinh(2bx+2a)}{32b}+\frac{\sinh(6bx+6a)}{96b}-\frac{3\cosh(2bx+2a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/32*sinh(2*b*x+2*a)/b+1/96/b*sinh(6*b*x+6*a)-3/32*cosh(2*b*x+2*a)/b+1/96*
cosh(6*b*x+6*a)/b
```

Maxima [A] time = 1.01396, size = 57, normalized size = 1.1

$$-\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b}-\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(6*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

Fricas [B] time = 1.75634, size = 304, normalized size = 5.85

$$-\frac{\cosh\left(bx+a\right)^{4}-8\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}-8\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sin}{48\left(b\cosh\left(bx+a\right)^{2}-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(cosh(b*x + a)^4 - 8*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^
2*sinh(b*x + a)^2 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 3)/
(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 111.514, size = 70, normalized size = 1.35

$$\begin{cases} \frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{3b}-\frac{e^{2a}e^{2bx}\cosh^{4}\left(a+bx\right)}{6b} & \text{for}\ b\neq 0 \\ xe^{2a}\sinh^{2}\left(a\right)\cosh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(3*b) - exp(2
*a)*exp(2*b*x)*cosh(a + b*x)**4/(6*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*co
sh(a)**2, True))
```

Giac [A] time = 1.1451, size = 58, normalized size = 1.12

$$\frac{\left(e^{(6\,bx+12\,a)} - 6\,e^{(2\,bx+8\,a)}\right)e^{(-6\,a)} - 3\,e^{(-2\,bx-2\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/96*((e^(6*b*x + 12*a) - 6*e^(2*b*x + 8*a))*e^(-6*a) - 3*e^(-2*b*x - 2*a))
/b
```

3.927 $\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

```
[Out] E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(5*a +
5*b*x)/(40*b)
```

Rubi [A] time = 0.0466779, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 448}

$$\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(5*a +
5*b*x)/(40*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(-1-\frac{1}{x^2}+x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Mathematica [A] time = 0.0562757, size = 54, normalized size = 0.82

$$\frac{e^{a+bx}\left(e^{2(a+bx)}-3\right)}{24b}+\frac{e^{-a-bx}\left(e^{6(a+bx)}+5\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(24*b) + (E^(-a - b*x)*(5 + E^(6*(a +
b*x))))/(40*b)
```

Maple [A] time = 0.007, size = 69, normalized size = 1.1

$$-\frac{\sinh(bx+a)}{4\,b}+\frac{\sinh(3\,bx+3\,a)}{24\,b}+\frac{\sinh(5\,bx+5\,a)}{40\,b}+\frac{\cosh(3\,bx+3\,a)}{24\,b}+\frac{\cosh(5\,bx+5\,a)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -1/4*sinh(b*x+a)/b+1/24/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)+1/24*cosh(
3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

Maxima [A] time = 1.01299, size = 72, normalized size = 1.09

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}+3\right)e^{(5\,bx+5\,a)}}{120\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/120*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

Fricas [A] time = 1.77927, size = 290, normalized size = 4.39

$$\frac{9\,\cosh\left(bx+a\right)^3+27\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-6\,\sinh\left(bx+a\right)^3-2\left(9\,\cosh\left(bx+a\right)^2-5\right)\sinh\left(bx+a\right)-5\,\cosh\left(bx+a\right)}{60\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(9*cosh(b*x + a)^3 + 27*cosh(b*x + a)*sinh(b*x + a)^2 - 6*sinh(b*x + a
)^3 - 2*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(b*
x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [A] time = 65.1969, size = 128, normalized size = 1.94

$$\begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^3(a+bx)}{15b}+\frac{8e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b}-\frac{2e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b}+\frac{e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{2a}\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) + 8*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) - 2*exp(2*a)*exp(2*b*x)*sinh(a
```

```
 + b*x)*cosh(a + b*x)**2/(15*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*
b), Ne(b, 0)), (x*exp(2*a)*sinh(a)*cosh(a)**2, True))
```

Giac [A] time = 1.15405, size = 74, normalized size = 1.12

$$\frac{\left(3\,e^{(5\,bx+10\,a)}+5\,e^{(3\,bx+8\,a)}-15\,e^{(bx+6\,a)}\right)e^{(-5\,a)}+15\,e^{(-bx-a)}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/120*((3*e^(5*b*x + 10*a) + 5*e^(3*b*x + 8*a) - 15*e^(b*x + 6*a))*e^(-5*a)
 + 15*e^(-b*x - a))/b
```

3.928 $\int e^{2(a+bx)} \cosh(a+bx) \coth(a+bx) \, dx$

Optimal. Leaf size=45

$$\frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (3*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) - (2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0341525, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 390, 207}

$$\frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (3*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{2\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+x^2+\frac{4}{-1+x^2}\right)dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.262552, size = 58, normalized size = 1.29

$$-\frac{e^{a+bx}\left(-\frac{1}{3}e^{2(a+bx)}+\frac{4\tanh^{-1}\left(\sqrt{e^{2(a+bx)}}\right)}{\sqrt{e^{2(a+bx)}}}-3\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(E^(a + b*x)*(-3 - E^(2*(a + b*x))/3 + (4*ArcTanh[Sqrt[E^(2*(a + b*x))]])/
Sqrt[E^(2*(a + b*x))]))/(2*b)
```

Maple [A] time = 0.073, size = 54, normalized size = 1.2

$$\frac{e^{3\,bx+3\,a}}{6\,b}+\frac{3\,e^{bx+a}}{2\,b}-\frac{\ln\left(1+e^{bx+a}\right)}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+3/2*exp(b*x+a)/b-1/b*ln(1+exp(b*x+a))+1/b*ln(exp(b*x+a
)-1)
```

Maxima [A] time = 1.00814, size = 82, normalized size = 1.82

$$\frac{\left(9\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{6\,b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(9*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b - log(e^(-b*x - a) + 1)/b +
log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 1.84004, size = 298, normalized size = 6.62

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}+3\left(\cosh\left(bx+a\right)^{2}+3\right)\sinh\left(bx+a\right)+9\,\cosh\left(bx+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 9*cosh(b*x + a) - 6*log(cosh(b*x +
a) + sinh(b*x + a) + 1) + 6*log(cosh(b*x + a) + sinh(b*x + a) - 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15038, size = 73, normalized size = 1.62

$$\frac{\left(e^{(3\,bx+9\,a)} + 9\,e^{(bx+7\,a)}\right)e^{(-6\,a)} - 6\,\log\left(e^{(bx+a)} + 1\right) + 6\,\log\left(\left|e^{(bx+a)} - 1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*((e^(3*b*x + 9*a) + 9*e^(b*x + 7*a))*e^(-6*a) - 6*log(e^(b*x + a) + 1)
+ 6*log(abs(e^(b*x + a) - 1)))/b
```

3.929 $\int e^{2(a+bx)} \coth^2(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] E^(2*a + 2*b*x)/(2*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2
*b*x)])/b
```

Rubi [A] time = 0.0496781, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2
*b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{4}{(-1+x)^2}+\frac{4}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.0620764, size = 48, normalized size = 0.81

$$\frac{e^{2(a+bx)}-\frac{4}{e^{2(a+bx)}-1}+4\log\left(1-e^{2(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

```
[Out] (E^(2*(a + b*x)) - 4/(-1 + E^(2*(a + b*x))) + 4*Log[1 - E^(2*(a + b*x))])/(
2*b)
```

Maple [A] time = 0.069, size = 57, normalized size = 1.

$$\frac{e^{2\,bx+2\,a}}{2\,b}-4\,\frac{a}{b}-2\,\frac{1}{b\left(e^{2\,bx+2\,a}-1\right)}+2\,\frac{\ln\left(e^{2\,bx+2\,a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/2*exp(2*b*x+2*a)/b-4*a/b-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

Maxima [A] time = 1.01709, size = 116, normalized size = 1.97

$$\frac{4\,(bx+a)}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4*(b*x + a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 1/2
*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

Fricas [B] time = 1.87767, size = 540, normalized size = 9.15

$$\frac{\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+\left(6\,\cosh\left(bx+a\right)^2-1\right)\sinh\left(bx+a\right)^2-\cosh\left(bx+a\right)^2}{2\left(b\cosh\left(bx+a\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*x +
 a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) +
 b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20936, size = 76, normalized size = 1.29

$$-\frac{\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}-e^{(2\,bx+2\,a)}-4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - e^(2*b*x + 2*a) - 4*log(abs
(e^(2*b*x + 2*a) - 1)))/b
```

3.930 $\int e^{2(a+bx)} \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{2e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (2*E^(a + b*x))/b - (2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^
(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (5*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0684383, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 463, 455, 388, 207}

$$\frac{2e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^
(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (5*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
```

```
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2(a+bx)}\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = \frac{\mathrm{Subst}\left(\int \frac{2x^2(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2\,\mathrm{Subst}\left(\int \frac{x^2(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{x^2(8+4x^2)}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\mathrm{Subst}\left(\int \frac{-12-8x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] time = 4.02851, size = 247, normalized size = 2.91

$$-\frac{e^{-3(a+bx)}\left(128e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^2 {}_5F_4\left(2,2,2,2,\frac{5}{2};1,1,1,\frac{11}{2};e^{2(a+bx)}\right)+128e^{8(a+bx)}\left(16e^{2(a+bx)}+7e^{4(a+bx)}+9\right) {}_4F_3\left(2,2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -(-21*(56595 + 62725*E^(2*(a + b*x)) - 12071*E^(4*(a + b*x)) - 19353*E^(6*(
a + b*x)) + 768*E^(8*(a + b*x))) + (315*(3773 + 2924*E^(2*(a + b*x)) - 2534
*E^(4*(a + b*x)) - 1548*E^(6*(a + b*x)) + 297*E^(8*(a + b*x)))*ArcTanh[Sqrt
[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(9 + 16*E^(
2*(a + b*x)) + 7*E^(4*(a + b*x)))*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1,
11/2}, E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(1 + E^(2*(a + b*x)))^2*Hyper
geometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))])/(10080*b
*E^(3*(a + b*x)))
```

Maple [A] time = 0.08, size = 78, normalized size = 0.9

$$2\,\frac{e^{bx+a}}{b}-\frac{e^{bx+a}\left(5\,e^{2\,bx+2\,a}-3\right)}{b\left(e^{2\,bx+2\,a}-1\right)^{2}}-\frac{5\,\ln\left(1+e^{bx+a}\right)}{2\,b}+\frac{5\,\ln\left(e^{bx+a}-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 2*exp(b*x+a)/b-exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-5/2/b
*ln(1+exp(b*x+a))+5/2/b*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.23156, size = 130, normalized size = 1.53

$$-\frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}-\frac{9\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-2}{b\left(e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -5/2*log(e^(-b*x - a) + 1)/b + 5/2*log(e^(-b*x - a) - 1)/b - (9*e^(-2*b*x -
 2*a) - 5*e^(-4*b*x - 4*a) - 2)/(b*(e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + e^(
-5*b*x - 5*a)))
```

Fricas [B] time = 1.84851, size = 1295, normalized size = 15.24

$$\frac{4\,\cosh\left(bx+a\right)^{5}+20\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+4\,\sinh\left(bx+a\right)^{5}+2\left(20\,\cosh\left(bx+a\right)^{2}-9\right)\sinh\left(bx+a\right)^{3}-18\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + a)^5 + 20*cosh(b*x + a)*sinh(b*x + a)^4 + 4*sinh(b*x + a)
^5 + 2*(20*cosh(b*x + a)^2 - 9)*sinh(b*x + a)^3 - 18*cosh(b*x + a)^3 + 2*(2
```

```
0*cosh(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 5*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(
10*cosh(b*x + a)^4 - 27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x +
a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4
*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18629, size = 97, normalized size = 1.14

$$-\frac{\frac{2\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-4\,e^{(bx+a)}+5\,\log\left(e^{(bx+a)}+1\right)-5\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 4*e^(
b*x + a) + 5*log(e^(b*x + a) + 1) - 5*log(abs(e^(b*x + a) - 1)))/b
```

3.931 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

```
[Out] E^(-4*a - 4*b*x)/(256*b) - (3*E^(4*a + 4*b*x))/(256*b) + E^(8*a + 8*b*x)/(5
12*b) + (3*x)/64
```

Rubi [A] time = 0.0624658, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 12, 266, 43}

$$\frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(256*b) - (3*E^(4*a + 4*b*x))/(256*b) + E^(8*a + 8*b*x)/(5
12*b) + (3*x)/64
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^4\right)^3}{64x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(-1+x^4\right)^3}{x^5}\,dx, x, e^{a+bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{4a+4bx}\right)}{256b}\\
&= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{4a+4bx}\right)}{256b}\\
&= \frac{e^{-4a-4bx}}{256b}-\frac{3e^{4a+4bx}}{256b}+\frac{e^{8a+8bx}}{512b}+\frac{3x}{64}
\end{aligned}$$

Mathematica [A] time = 0.0435625, size = 45, normalized size = 0.79

$$\frac{e^{-4(a+bx)}-3e^{4(a+bx)}+\frac{1}{2}e^{8(a+bx)}+12bx}{256b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(-4*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(8*(a + b*x))/2 + 12*b*x)/(256*b)
```

Maple [A] time = 0.014, size = 61, normalized size = 1.1

$$\frac{3\,x}{64}-\frac{\sinh\left(4\,bx+4\,a\right)}{64\,b}+\frac{\sinh\left(8\,bx+8\,a\right)}{512\,b}-\frac{\cosh\left(4\,bx+4\,a\right)}{128\,b}+\frac{\cosh\left(8\,bx+8\,a\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 3/64*x-1/64/b*sinh(4*b*x+4*a)+1/512/b*sinh(8*b*x+8*a)-1/128*cosh(4*b*x+4*a)
/b+1/512*cosh(8*b*x+8*a)/b
```

Maxima [A] time = 1.14565, size = 70, normalized size = 1.23

$$-\frac{\left(6 e^{(-4 bx-4 a)}-1\right)e^{(8 bx+8 a)}}{512 b}+\frac{3 (bx+a)}{64 b}+\frac{e^{(-4 bx-4 a)}}{256 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/512*(6*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/64*(b*x + a)/b + 1/25
6*e^(-4*b*x - 4*a)/b
```

Fricas [B] time = 1.73983, size = 513, normalized size = 9.

$$\frac{3 \cosh(bx+a)^6-20 \cosh(bx+a)^3 \sinh(bx+a)^3+45 \cosh(bx+a)^2 \sinh(bx+a)^4-6 \cosh(bx+a) \sinh(bx+a)^5}{512 \left(b \cosh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(3*cosh(b*x + a)^6 - 20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x
 + a)^2*sinh(b*x + a)^4 - 6*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)
^6 + 6*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(15*cosh(b*x + a)^4 + 8*b*x - 2)*sin
h(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 2*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x +
 a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20434, size = 81, normalized size = 1.42

$$\frac{24\,bx-2\left(3\,e^{(4\,bx+4\,a)}-1\right)e^{(-4\,bx-4\,a)}+\left(e^{(8\,bx+16\,a)}-6\,e^{(4\,bx+12\,a)}\right)e^{(-8\,a)}}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/512*(24*b*x - 2*(3*e^(4*b*x + 4*a) - 1)*e^(-4*b*x - 4*a) + (e^(8*b*x + 16
*a) - 6*e^(4*b*x + 12*a))*e^(-8*a))/b
```

3.932 $\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=100

$$-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

```
[Out] -E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) - E^(a + b*x)/(16*b) - E^(3*
a + 3*b*x)/(48*b) + E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rubi [A] time = 0.0735327, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 448}

$$-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) - E^(a + b*x)/(16*b) - E^(3*
a + 3*b*x)/(48*b) + E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1-x^2)^2(1+x^2)^3}{32x^4}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\frac{(1-x^2)^2(1+x^2)^3}{x^4}\,dx,x,e^{a+bx}\right)}{32b}$$
$$=\frac{\text{Subst}\left(\int\left(-2+\frac{1}{x^4}+\frac{1}{x^2}-2x^2+x^4+x^6\right)dx,x,e^{a+bx}\right)}{32b}$$
$$=-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Mathematica [A] time = 0.0949638, size = 73, normalized size = 0.73

$$\frac{e^{-3(a+bx)}\left(-105e^{2(a+bx)}-210e^{4(a+bx)}-70e^{6(a+bx)}+21e^{8(a+bx)}+15e^{10(a+bx)}-35\right)}{3360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-35 - 105*E^(2*(a + b*x)) - 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) + 21*
E^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

Maple [A] time = 0.01, size = 108, normalized size = 1.1

$$-\frac{\sinh(bx+a)}{32\,b}-\frac{\sinh(3\,bx+3\,a)}{96\,b}+\frac{\sinh(5\,bx+5\,a)}{160\,b}+\frac{\sinh(7\,bx+7\,a)}{224\,b}-\frac{3\,\cosh(bx+a)}{32\,b}-\frac{\cosh(3\,bx+3\,a)}{32\,b}+\frac{\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/32*sinh(b*x+a)/b-1/96/b*sinh(3*b*x+3*a)+1/160/b*sinh(5*b*x+5*a)+1/224/b*
sinh(7*b*x+7*a)-3/32*cosh(b*x+a)/b-1/32*cosh(3*b*x+3*a)/b+1/160*cosh(5*b*x+
```

```
5*a)/b+1/224*cosh(7*b*x+7*a)/b
```

Maxima [A] time = 1.04958, size = 103, normalized size = 1.03

$$\frac{\left(21\,e^{(-2\,bx-2\,a)}-70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}+15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3360*(21*e^(-2*b*x - 2*a) - 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) +
15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) + e^(-3*b*x - 3*a))/b
```

Fricas [B] time = 1.75935, size = 501, normalized size = 5.01

$$-\frac{10\,\cosh\left(bx+a\right)^{5}+50\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-25\,\sinh\left(bx+a\right)^{5}-\left(250\,\cosh\left(bx+a\right)^{2}+63\right)\sinh\left(bx+a\right)^{3}+}{1680\left(b\cosh\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/1680*(10*cosh(b*x + a)^5 + 50*cosh(b*x + a)*sinh(b*x + a)^4 - 25*sinh(b*
x + a)^5 - (250*cosh(b*x + a)^2 + 63)*sinh(b*x + a)^3 + 42*cosh(b*x + a)^3
+ 2*(50*cosh(b*x + a)^3 + 63*cosh(b*x + a))*sinh(b*x + a)^2 - (125*cosh(b*x
 + a)^4 + 189*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 140*cosh(b*x + a))/(b*c
osh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21889, size = 108, normalized size = 1.08

$$-\frac{35\left(3e^{(2bx+2a)}+1\right)e^{(-3bx-3a)}-\left(15e^{(7bx+28a)}+21e^{(5bx+26a)}-70e^{(3bx+24a)}-210e^{(bx+22a)}\right)e^{(-21a)}}{3360b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - (15*e^(7*b*x + 28*a)
 + 21*e^(5*b*x + 26*a) - 70*e^(3*b*x + 24*a) - 210*e^(b*x + 22*a))*e^(-21*a
))/b
```

3.933 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

```
[Out] E^(-2*a - 2*b*x)/(32*b) + E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) -
 x/8
```

Rubi [A] time = 0.0522093, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) + E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) -
 x/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2}{x}+2x+x^2\right)dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}\end{aligned}$$

Mathematica [A] time = 0.0579636, size = 43, normalized size = 0.75

$$\frac{3e^{-2(a+bx)}+3e^{4(a+bx)}+e^{6(a+bx)}-12bx}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (3/E^(2*(a + b*x)) + 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) - 12*b*x)/(96*b)
```

Maple [A] time = 0.007, size = 89, normalized size = 1.6

$$-\frac{x}{8}-\frac{\sinh(2bx+2a)}{32b}+\frac{\sinh(4bx+4a)}{32b}+\frac{\sinh(6bx+6a)}{96b}+\frac{\cosh(2bx+2a)}{32b}+\frac{\cosh(4bx+4a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/8*x-1/32*sinh(2*b*x+2*a)/b+1/32/b*sinh(4*b*x+4*a)+1/96/b*sinh(6*b*x+6*a)
+1/32*cosh(2*b*x+2*a)/b+1/32*cosh(4*b*x+4*a)/b+1/96*cosh(6*b*x+6*a)/b
```

Maxima [A] time = 1.06912, size = 70, normalized size = 1.23

$$\frac{\left(3\,e^{(-2\,bx-2\,a)}+1\right)e^{(6\,bx+6\,a)}}{96\,b}-\frac{bx+a}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*b*x - 2*a) + 1)*e^(6*b*x + 6*a)/b - 1/8*(b*x + a)/b + 1/32*e^
(-2*b*x - 2*a)/b
```

Fricas [B] time = 1.88129, size = 413, normalized size = 7.25

$$\frac{4\,\cosh\left(bx+a\right)^{4}-8\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+4\,\sinh\left(bx+a\right)^{4}-3\left(4\,bx-1\right)\cosh\left(bx+a\right)^{2}-3\left(4\,bx-8\,\cosh\left(bx\right.\right.}{96\left(b\cosh\left(bx+a\right)^{2}-2\,b\cosh\left(bx+a\right)\sinh\left(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 - 3*(4*b*x - 1)*cosh(b*x + a)^2 - 3*(4*b*x - 8*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 - 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

Sympy [A] time = 103.65, size = 240, normalized size = 4.21

$$\begin{cases}\frac{xe^{2a}e^{2bx}\sinh^{4}(a+bx)}{8}-\frac{xe^{2a}e^{2bx}\sinh^{3}(a+bx)\cosh(a+bx)}{4}+\frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^{3}(a+bx)}{4}-\frac{xe^{2a}e^{2bx}\cosh^{4}(a+bx)}{8}-\frac{e^{2a}e^{2bx}\sinh^{3}(a+bx)\cosh(a}{8b} \\ xe^{2a}\sinh(a)\cosh^{3}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)**3*cosh(a + b*x)/4 + x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cos
h(a + b*x)**3/4 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 - exp(2*a)*exp(2
*b*x)*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + exp(2*a)*exp(2*b*x)*sinh(a + b
*x)**2*cosh(a + b*x)**2/(4*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a +
b*x)**3/(24*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(12*b), Ne(b, 0)), (x
*exp(2*a)*sinh(a)*cosh(a)**3, True))
```

Giac [A] time = 1.17915, size = 82, normalized size = 1.44

$$-\frac{12\,bx-3\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}-\left(e^{(6\,bx+12\,a)}+3\,e^{(4\,bx+10\,a)}\right)e^{(-6\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/96*(12*b*x - 3*(2*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) - (e^(6*b*x + 12
*a) + 3*e^(4*b*x + 10*a))*e^(-6*a))/b
```

3.934 $\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{4}$$

```
[Out] E^(2*a + 2*b*x)/(2*b) + E^(4*a + 4*b*x)/(16*b) - x/4 + Log[1 - E^(2*a + 2*b
*x)]/b
```

Rubi [A] time = 0.0603323, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 72}

$$\frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + E^(4*a + 4*b*x)/(16*b) - x/4 + Log[1 - E^(2*a + 2*b
*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{4x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{8}{-1+x}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}-\frac{x}{4}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0424042, size = 48, normalized size = 0.81

$$\frac{8e^{2(a+bx)}+e^{4(a+bx)}+16\log\left(1-e^{2(a+bx)}\right)-4bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (8*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x + 16*Log[1 - E^(2*(a + b*x))])
/(16*b)
```

Maple [A] time = 0.115, size = 55, normalized size = 0.9

$$-\frac{x}{4}+\frac{e^{4bx+4a}}{16b}+\frac{e^{2bx+2a}}{2b}-2\frac{a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/4*x+1/16*exp(4*b*x+4*a)/b+1/2*exp(2*b*x+2*a)/b-2*a/b+1/b*ln(exp(2*b*x+2*
a)-1)
```

Maxima [A] time = 1.0337, size = 95, normalized size = 1.61

$$\frac{\left(8\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{16\,b}+\frac{7\,(bx+a)}{4\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(8*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + 7/4*(b*x + a)/b + log(e^(
-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Fricas [B] time = 1.94596, size = 354, normalized size = 6.

$$\frac{\cosh(bx+a)^4+4\,\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\,\cosh(bx+a)^2+4\right)\sinh(bx+a)^2-4\,bx+8\,\text{co}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^2 - 4*b*x + 8*cosh(b*x + a)^2 + 4*
(cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a) + 16*log(2*sinh(b*x + a)/
(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21171, size = 70, normalized size = 1.19

$$-\frac{4\,bx - \left(e^{(4\,bx+8\,a)} + 8\,e^{(2\,bx+6\,a)}\right)e^{(-4\,a)} - 16\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*b*x - (e^(4*b*x + 8*a) + 8*e^(2*b*x + 6*a))*e^(-4*a) - 16*log(abs(
e^(2*b*x + 2*a) - 1)))/b
```

3.935 $\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (5*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) + (2*E^(a + b*x))/(b*(1 - E^(
2*a + 2*b*x))) - (4*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0537078, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2282, 12, 390, 385, 206}

$$\frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (5*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) + (2*E^(a + b*x))/(b*(1 - E^(
2*a + 2*b*x))) - (4*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{2\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(5+x^2-\frac{4\left(1-3x^2\right)}{\left(1-x^2\right)^2}\right)dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\,\text{Subst}\left(\int \frac{1-3x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 2.07236, size = 220, normalized size = 3.01

$$e^{-5(a+bx)}\left(256e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^3 {}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};e^{2(a+bx)}\right)-21\left(91925e^{2(a+bx)}+61158e^{4(a+bx)}-20166e^{6(a+b}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-21*(36015 + 91925*E^(2*(a + b*x)) + 61158*E^(4*(a + b*x)) - 20166*E^(6*(a
 + b*x)) - 15061*E^(8*(a + b*x)) + 753*E^(10*(a + b*x))) - (315*(-2401 - 53
28*E^(2*(a + b*x)) - 1821*E^(4*(a + b*x)) + 3264*E^(6*(a + b*x)) + 1149*E^(
8*(a + b*x)) - 240*E^(10*(a + b*x)) + E^(12*(a + b*x)))*ArcTanh[Sqrt[E^(2*(
a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a + b*x
)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))
])/(60480*b*E^(5*(a + b*x)))
```

Maple [A] time = 0.119, size = 79, normalized size = 1.1

$$\frac{e^{3bx+3a}}{6b}+\frac{5e^{bx+a}}{2b}-2\frac{e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}-2\frac{\ln\left(1+e^{bx+a}\right)}{b}+2\frac{\ln\left(e^{bx+a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+5/2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b
*ln(1+exp(b*x+a))+2/b*ln(exp(b*x+a)-1)
```

Maxima [A] time = 1.04947, size = 117, normalized size = 1.6

$$-\frac{2\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{14e^{(-2bx-2a)}-27e^{(-4bx-4a)}+1}{6b\left(e^{(-3bx-3a)}-e^{(-5bx-5a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 1/6*(14*e^(-2*b*x - 2*a) - 27*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))

Fricas [B] time = 1.86758, size = 797, normalized size = 10.92

$$\frac{\cosh(bx+a)^5 + 5\cosh(bx+a)\sinh(bx+a)^4 + \sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^2 + 7\right)\sinh(bx+a)^3 + 14\cosh(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")

[Out] 1/6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 21*cosh(b*x + a))*sinh(b*x + a)^2 - 12*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 12*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (5*cosh(b*x + a)^4 + 42*cosh(b*x + a)^2 - 27)*sinh(b*x + a) - 27*cosh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)

[Out] Timed out

Giac [A] time = 1.19178, size = 101, normalized size = 1.38

$$\frac{\left(e^{(3\,bx+15\,a)} + 15\,e^{(bx+13\,a)}\right)e^{(-12\,a)} - \frac{12\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - 12\,\log\left(e^{(bx+a)} + 1\right) + 12\,\log\left(\left|e^{(bx+a)} - 1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*((e^(3*b*x + 15*a) + 15*e^(b*x + 13*a))*e^(-12*a) - 12*e^(b*x + a)/(e^(
2*b*x + 2*a) - 1) - 12*log(e^(b*x + a) + 1) + 12*log(abs(e^(b*x + a) - 1)))
/b
```

3.936 $\int e^{2(a+bx)} \coth^3(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{e^{2a+2bx}}{2b}+\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] E^(2*a + 2*b*x)/(2*b) - 2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a +
2*b*x))) + (3*Log[1 - E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.06267, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) - 2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a +
2*b*x))) + (3*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\coth^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x\left(1+x^2\right)^3}{\left(-1+x^2\right)^3}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)^3}\,dx,x,e^{2a+2bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^3}+\frac{12}{(-1+x)^2}+\frac{6}{-1+x}\right)dx,x,e^{2a+2bx}\right)}{2b}\\
&= \frac{e^{2a+2bx}}{2b}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.109898, size = 60, normalized size = 0.75

$$\frac{\frac{8-12e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+e^{2(a+bx)}+6\log\left(1-e^{2(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

```
[Out] (E^(2*(a + b*x)) + (8 - 12*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 6*Lo
g[1 - E^(2*(a + b*x))])/(2*b)
```

Maple [A] time = 0.127, size = 70, normalized size = 0.9

$$\frac{e^{2bx+2a}}{2b}-6\frac{a}{b}-2\frac{3e^{2bx+2a}-2}{b\left(e^{2bx+2a}-1\right)^2}+3\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/2*exp(2*b*x+2*a)/b-6*a/b-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+3/
b*ln(exp(2*b*x+2*a)-1)
```

Maxima [A] time = 1.0164, size = 143, normalized size = 1.79

$$\frac{6\,(bx+a)}{b}+\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{10\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-1}{2\,b\left(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 6*(b*x + a)/b + 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 1/2
*(10*e^(-2*b*x - 2*a) - 5*e^(-4*b*x - 4*a) - 1)/(b*(e^(-2*b*x - 2*a) - 2*e^
(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Fricas [B] time = 1.867, size = 1099, normalized size = 13.74

$$\frac{\cosh\left(bx+a\right)^{6}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sinh\left(bx+a\right)^{6}+\left(15\,\cosh\left(bx+a\right)^{2}-2\right)\sinh\left(bx+a\right)^{4}-2\,\cosh\left(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
(15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*cosh
(b*x + a)^2 - 11)*sinh(b*x + a)^2 - 11*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^4
 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))
) + 2*(3*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x +
 a) + 8)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*
x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
```

```
2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23137, size = 95, normalized size = 1.19

$$-\frac{\frac{9\,e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}+1}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-e^{(2\,bx+2\,a)}-6\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*((9*e^(4*b*x + 4*a) - 6*e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)^2 -
 e^(2*b*x + 2*a) - 6*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.937 $\int e^x \mathbf{sech}(2x)\tanh(2x)\,dx$

Optimal. Leaf size=113

$$-\frac{e^{3x}}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

```
[Out] -(E^(3*x)/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Lo
g[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0820164, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {2282, 12, 457, 297, 1162, 617, 204, 1165, 628}

$$-\frac{e^{3x}}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x],x]
```

```
[Out] -(E^(3*x)/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Lo
g[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \mathbf{sech}(2x)\tanh(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(-1+x^4\right)}{\left(1+x^4\right)^2}\,dx, x, e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2\left(-1+x^4\right)}{\left(1+x^4\right)^2}\,dx, x, e^x\right)\\ &= -\frac{e^{3x}}{1+e^{4x}} + \text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)\\ &= -\frac{e^{3x}}{1+e^{4x}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\ &= -\frac{e^{3x}}{1+e^{4x}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) + \text{S}\\ &= -\frac{e^{3x}}{1+e^{4x}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}}\\ &= -\frac{e^{3x}}{1+e^{4x}} - \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x\right.}{4\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0275269, size = 42, normalized size = 0.37

$$\frac{2}{3}e^{3x}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{4x}\right)-2\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{4x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x],x]
```

```
[Out] (2*E^(3*x)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)] - 2*Hypergeometric2F1[
3/4, 2, 7/4, -E^(4*x)]))/3
```

Maple [C] time = 0.052, size = 40, normalized size = 0.4

$$-\frac{e^{3x}}{1+e^{4x}}+2\left(\sum_{\_R=\mathrm{RootOf}\left(4096\_Z^4+1\right)}\_R\ln\left(512\_R^3+e^x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)*tanh(2*x),x)
```

```
[Out] -exp(3*x)/(1+exp(4*x))+2*sum(_R*ln(512*_R^3+exp(x)),_R=RootOf(4096*_Z^4+1))
```

Maxima [A] time = 1.49453, size = 122, normalized size = 1.08

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

Fricas [B] time = 1.99851, size = 500, normalized size = 4.42

$$-\frac{4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^x+\cdots}\right.}{8\left(e^{(4x)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) + (sqrt(2)*e^(
4*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - (sqrt(2)*e^(4*x) + sqr
```

```
t(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^(3*x))/(e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \tanh(2x)\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x), x)
```

Giac [A] time = 1.16349, size = 122, normalized size = 1.08

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

3.938 $\int e^x \mathbf{sech}^2(2x)\tanh(2x)\,dx$

Optimal. Leaf size=129

$$-\frac{e^x}{4\left(e^{4x}+1\right)}-\frac{e^{5x}}{\left(e^{4x}+1\right)^2}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

```
[Out] -(E^(5*x)/(1 + E^(4*x))^2) - E^x/(4*(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x
]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*E^x +
 E^(2*x)]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rubi [A] time = 0.0960216, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2282, 12, 457, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{e^x}{4\left(e^{4x}+1\right)}-\frac{e^{5x}}{\left(e^{4x}+1\right)^2}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x],x]
```


```
[Out] -(E^(5*x)/(1 + E^(4*x))^2) - E^x/(4*(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x
]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*E^x +
 E^(2*x)]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \mathbf{sech}^2(2x)\tanh(2x)\,dx = \text{Subst}\left(\int \frac{4x^4\left(-1+x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x^4\left(-1+x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} + \text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} - \frac{e^x}{4\left(1+e^{4x}\right)} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} - \frac{e^x}{4\left(1+e^{4x}\right)} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} - \frac{e^x}{4\left(1+e^{4x}\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{1}{16}\,\text{Subst}\left(\int \frac{}{1+\sqrt{}}\right.$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} - \frac{e^x}{4\left(1+e^{4x}\right)} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{}$$

$$= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2} - \frac{e^x}{4\left(1+e^{4x}\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right.}{16\sqrt{2}}$$

Mathematica [A] time = 0.118189, size = 120, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{40e^x}{e^{4x}+1}+\frac{32e^x}{\left(e^{4x}+1\right)^2}-\sqrt{2}\log\left(-\sqrt{2}e^x+e^{2x}+1\right)+\sqrt{2}\log\left(\sqrt{2}e^x+e^{2x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}e^x\right)+2\sqrt{2}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

```
[Out] ((32*E^x)/(1 + E^(4*x))^2 - (40*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E
^x + E^(2*x)] + Sqrt[2]*Log[1 + Sqrt[2]*E^x + E^(2*x)])/32
```

Maple [C] time = 0.05, size = 44, normalized size = 0.3

$$-\frac{e^x\left(5e^{4x}+1\right)}{4\left(1+e^{4x}\right)^2}+4\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^4+1\right)}\_R\ln\left(e^x+64\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x),x)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)+1)/(1+exp(4*x))^2+4*sum(_R*ln(exp(x)+64*_R),_R=Root
Of(16777216*_Z^4+1))
```

Maxima [A] time = 1.69146, size = 136, normalized size = 1.05

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{32}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
```

```
(8*x) + 2*e^(4*x) + 1)
```

Fricas [B] time = 1.93438, size = 637, normalized size = 4.94

$$-\frac{4\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^
x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) + 2*s
qrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)
*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2)
)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x)
 + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 40*e^(5*x) + 8*e^x)/(e^(8
*x) + 2*e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh(2x)\operatorname{sech}^{2}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x)**2, x)
```

Giac [A] time = 1.18154, size = 128, normalized size = 0.99

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{32}\sqrt{2}\log\left(-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
(4*x) + 1)^2
```

3.939 $\int e^x \textbf{sech}(2x)\tanh^2(2x)\,dx$

Optimal. Leaf size=130

$$-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}+\frac{e^{3x}}{\left(e^{4x}+1\right)^2}+\frac{5\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}e^x+\right.}{8\sqrt{2}}$$

```
[Out] E^(3*x)/(1 + E^(4*x))^2 - (3*E^(3*x))/(4*(1 + E^(4*x))) - (5*ArcTan[1 - Sqr
t[2]*E^x])/(8*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*E^x])/(8*Sqrt[2]) + (5*Log[1
 - Sqrt[2]*E^x + E^(2*x)])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*E^x + E^(2*x)]
)/(16*Sqrt[2])
```

Rubi [A] time = 0.106234, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2282, 12, 463, 457, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}+\frac{e^{3x}}{\left(e^{4x}+1\right)^2}+\frac{5\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}e^x+\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

```
[Out] E^(3*x)/(1 + E^(4*x))^2 - (3*E^(3*x))/(4*(1 + E^(4*x))) - (5*ArcTan[1 - Sqr
t[2]*E^x])/(8*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*E^x])/(8*Sqrt[2]) + (5*Log[1
 - Sqrt[2]*E^x + E^(2*x)])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*E^x + E^(2*x)]
)/(16*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \mathbf{sech}(2x)\tanh^2(2x)\,dx = \text{Subst}\left(\int \frac{2x^2\left(1-x^4\right)^2}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x^2\left(1-x^4\right)^2}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2\left(4-8x^4\right)}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5}{4}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} - \frac{5}{8}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{5}{8}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5}{16}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{5}{16}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x\right.$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{5\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} - \frac{5\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\log\left(1-\sqrt{2}e^x+\right.}{16\sqrt{2}}$$

Mathematica [C] time = 0.0603184, size = 58, normalized size = 0.45

$$\frac{e^{3x}-3e^{7x}}{4\left(e^{4x}+1\right)^2}-\frac{5}{16}\text{RootSum}\left[\#1^4+1\&,\frac{x-\log\left(e^x-\#1\right)}{\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

```
[Out] (E^(3*x) - 3*E^(7*x))/(4*(1 + E^(4*x))^2) - (5*RootSum[1 + #1^4 & , (x - Lo
g[E^x - #1])/#1 & ])/16
```

Maple [C] time = 0.061, size = 48, normalized size = 0.4

$$-\frac{e^{3x}\left(3e^{4x}-1\right)}{4\left(1+e^{4x}\right)^2}+2\sum_{\_R=\text{RootOf}\left(1048576\_Z^4+625\right)}\_R\ln\left(e^x+\frac{32768\_R^3}{125}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)*tanh(2*x)^2,x)
```

```
[Out] -1/4*exp(3*x)*(3*exp(4*x)-1)/(1+exp(4*x))^2+2*sum(_R*ln(exp(x)+32768/125*_R
^3),_R=RootOf(1048576*_Z^4+625))
```

Maxima [A] time = 1.623, size = 142, normalized size = 1.09

$$\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{5}{32}\sqrt{2}\log\left(-\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(8*x) + 2*e^(4*x) + 1)
```

Fricas [B] time = 2.10761, size = 651, normalized size = 5.01

$$-\frac{20\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+20\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(20*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e
^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 20*(sqrt(2)*e^(8*x) + 2
*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(
2)*e^x + 4*e^(2*x) + 4) + 1) + 5*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqr
t(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 5*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e
^(4*x) + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 24*e^(7*x) - 8*e^(3
*x))/(e^(8*x) + 2*e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh^{2}(2x)\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x), x)
```

Giac [A] time = 1.15151, size = 134, normalized size = 1.03

$$\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{5}{32}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(4*x) + 1)^2
```

3.940 $\int e^x \mathbf{sech}^2(2x)\tanh^2(2x)\,dx$

Optimal. Leaf size=149

$$-\frac{3e^x}{8\left(e^{4x}+1\right)}-\frac{5e^{5x}}{6\left(e^{4x}+1\right)^2}+\frac{4e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{3\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}}+\frac{3\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}}+$$

```
[Out] (4*E^(5*x))/(3*(1 + E^(4*x))^3) - (5*E^(5*x))/(6*(1 + E^(4*x))^2) - (3*E^x)
/(8*(1 + E^(4*x))) - (3*ArcTan[1 - Sqrt[2]*E^x])/(16*Sqrt[2]) + (3*ArcTan[1
 + Sqrt[2]*E^x])/(16*Sqrt[2]) - (3*Log[1 - Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt
[2]) + (3*Log[1 + Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt[2])
```

Rubi [A] time = 0.127663, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {2282, 12, 463, 457, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3e^x}{8\left(e^{4x}+1\right)}-\frac{5e^{5x}}{6\left(e^{4x}+1\right)^2}+\frac{4e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{3\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}}+\frac{3\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```


```
[Out] (4*E^(5*x))/(3*(1 + E^(4*x))^3) - (5*E^(5*x))/(6*(1 + E^(4*x))^2) - (3*E^x)
/(8*(1 + E^(4*x))) - (3*ArcTan[1 - Sqrt[2]*E^x])/(16*Sqrt[2]) + (3*ArcTan[1
 + Sqrt[2]*E^x])/(16*Sqrt[2]) - (3*Log[1 - Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt
[2]) + (3*Log[1 + Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt[2])
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \mathbf{sech}^2(2x)\tanh^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4\left(1-x^4\right)^2}{\left(1+x^4\right)^4}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4\left(1-x^4\right)^2}{\left(1+x^4\right)^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4\left(8-12x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{16}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{3}{16}\,\text{Subst}\left(\int\right.$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{32}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{3}{32}\,\text{Su}$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} - \frac{3\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}e^x+e^{2x}\right.}{32\sqrt{2}}$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}e^x\right)}{16\sqrt{2}} - \frac{3}{}$$

Mathematica [C] time = 0.0621056, size = 64, normalized size = 0.43

$$\frac{1}{96}\left(-9\text{RootSum}\left[\#1^4+1\&, \frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right] - \frac{4e^x\left(6e^{4x}+29e^{8x}+9\right)}{\left(e^{4x}+1\right)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

```
[Out] ((-4*E^x*(9 + 6*E^(4*x) + 29*E^(8*x)))/(1 + E^(4*x))^3 - 9*RootSum[1 + #1^4
 & , (x - Log[E^x - #1])/#1^3 & ])/96
```

Maple [C] time = 0.067, size = 50, normalized size = 0.3

$$-\frac{e^{x} \left(29 e^{8 x}+6 e^{4 x}+9\right)}{24 \left(1+e^{4 x}\right)^{3}}+4 \sum_{\_R=\operatorname{RootOf}\left(268435456 \_Z^{4}+81\right)} \_R \ln\left(e^{x}+\frac{128 \_R}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x)
```

```
[Out] -1/24*exp(x)*(29*exp(8*x)+6*exp(4*x)+9)/(1+exp(4*x))^3+4*sum(_R*ln(exp(x)+1
28/3*_R),_R=RootOf(268435456*_Z^4+81))
```

Maxima [A] time = 1.6337, size = 155, normalized size = 1.04

$$\frac{3}{32} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}\left(\sqrt{2}+2 e^{x}\right)\right)+\frac{3}{32} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}\left(\sqrt{2}-2 e^{x}\right)\right)+\frac{3}{64} \sqrt{2} \log\left(\sqrt{2} e^{x}+e^{(2 x)}+1\right)-\frac{3}{64} \sqrt{2} \log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

Fricas [B] time = 2.24074, size = 798, normalized size = 5.36

$$-\frac{36 \left(\sqrt{2} e^{(12 x)}+3 \sqrt{2} e^{(8 x)}+3 \sqrt{2} e^{(4 x)}+\sqrt{2}\right) \arctan\left(-\sqrt{2} e^{x}+\sqrt{2} \sqrt{\sqrt{2} e^{x}+e^{(2 x)}+1}-1\right)+36 \left(\sqrt{2} e^{(12 x)}+3 \sqrt{2} e^{(8 x)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(36*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt
(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 3
6*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt(2))*arct
an(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 9
*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt(2))*log(4
*sqrt(2)*e^x + 4*e^(2*x) + 4) + 9*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3
*sqrt(2)*e^(4*x) + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 232*e^(9*
x) + 48*e^(5*x) + 72*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \tanh^2(2x)\operatorname{sech}^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x)**2, x)
```

Giac [A] time = 1.13499, size = 139, normalized size = 0.93

$$\frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{3}{64}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(4*x) + 1)^3
```

3.941 $\int e^x \coth(2x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=34

$$\frac{e^{3x}}{1-e^{4x}}+\frac{1}{2}\tan^{-1}(e^x)-\frac{1}{2}\tanh^{-1}(e^x)$$

```
[Out] E^(3*x)/(1 - E^(4*x)) + ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rubi [A] time = 0.0283708, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 12, 457, 298, 203, 206}

$$\frac{e^{3x}}{1-e^{4x}}+\frac{1}{2}\tan^{-1}(e^x)-\frac{1}{2}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]*Csch[2*x],x]
```

```
[Out] E^(3*x)/(1 - E^(4*x)) + ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^x \coth(2x)\text{csch}(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(1+x^4\right)}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(1+x^4\right)}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} + \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)
\end{aligned}
$$

Mathematica [A] time = 0.0558284, size = 31, normalized size = 0.91

$$
\frac{1}{2}\left(-\frac{2e^{3x}}{e^{4x}-1} + \tan^{-1}\left(e^x\right) - \tanh^{-1}\left(e^x\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x],x]
```

```
[Out] ((-2*E^(3*x))/(-1 + E^(4*x)) + ArcTan[E^x] - ArcTanh[E^x])/2
```

Maple [C] time = 0.061, size = 48, normalized size = 1.4

$$-\frac{e^{3x}}{e^{4x}-1}-\frac{\ln\left(e^{x}+1\right)}{4}+\frac{\ln\left(e^{x}-1\right)}{4}+\frac{i}{4}\ln\left(e^{x}+i\right)-\frac{i}{4}\ln\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)*csch(2*x),x)
```

```
[Out] -exp(3*x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/4*I*ln(exp(x)+I)
-1/4*I*ln(exp(x)-I)
```

Maxima [A] time = 1.55826, size = 46, normalized size = 1.35

$$-\frac{e^{(3x)}}{e^{(4x)}-1}+\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x -
 1)
```

Fricas [B] time = 1.84365, size = 745, normalized size = 21.91

$$-\frac{4\cosh(x)^{3}+12\cosh(x)^{2}\sinh(x)+12\cosh(x)\sinh(x)^{2}+4\sinh(x)^{3}-2\left(\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(4*cosh(x)^3 + 12*cosh(x)^2*sinh(x) + 12*cosh(x)*sinh(x)^2 + 4*sinh(x)
^3 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cosh
(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1
)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x) -
 1))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\coth(2x)\operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x)
```

```
[Out] Integral(exp(x)*coth(2*x)*csch(2*x), x)
```

Giac [A] time = 1.16302, size = 47, normalized size = 1.38

$$-\frac{e^{(3x)}}{e^{(4x)}-1}+\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="giac")
```

```
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e
^x - 1))
```

3.942 $\int e^x \coth(2x)\mathbf{csch}^2(2x)\,dx$

Optimal. Leaf size=53

$$\frac{e^x}{4\left(1-e^{4x}\right)}-\frac{e^{5x}}{\left(1-e^{4x}\right)^2}-\frac{1}{8}\tan^{-1}\left(e^x\right)-\frac{1}{8}\tanh^{-1}\left(e^x\right)$$

```
[Out] -(E^(5*x)/(1 - E^(4*x))^2) + E^x/(4*(1 - E^(4*x))) - ArcTan[E^x]/8 - ArcTan
h[E^x]/8
```

Rubi [A] time = 0.0423016, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 12, 457, 288, 212, 206, 203}

$$\frac{e^x}{4\left(1-e^{4x}\right)}-\frac{e^{5x}}{\left(1-e^{4x}\right)^2}-\frac{1}{8}\tan^{-1}\left(e^x\right)-\frac{1}{8}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -(E^(5*x)/(1 - E^(4*x))^2) + E^x/(4*(1 - E^(4*x))) - ArcTan[E^x]/8 - ArcTan
h[E^x]/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^x \coth(2x)\mathrm{csch}^2(2x)\,dx &= \mathrm{Subst}\left(\int \frac{4x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right) \\
&= 4\,\mathrm{Subst}\left(\int \frac{x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right) \\
&= -\frac{e^{5x}}{(1-e^{4x})^2} + \mathrm{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right) \\
&= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) \\
&= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{8}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) \\
&= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x)
\end{aligned}
$$

Mathematica [A] time = 0.0910944, size = 54, normalized size = 1.02

$$
-\frac{-2e^x + 10e^{5x} + (e^{4x}-1)^2\tan^{-1}(e^x) + (e^{4x}-1)^2\tanh^{-1}(e^x)}{8(e^{4x}-1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -(-2*E^x + 10*E^(5*x) + (-1 + E^(4*x))^2*ArcTan[E^x] + (-1 + E^(4*x))^2*Arc
Tanh[E^x])/(8*(-1 + E^(4*x))^2)
```

Maple [C] time = 0.065, size = 54, normalized size = 1.

$$
-\frac{e^x(5e^{4x}-1)}{4(e^{4x}-1)^2} - \frac{\ln(e^x+1)}{16} + \frac{\ln(e^x-1)}{16} + \frac{i}{16}\ln(e^x-i) - \frac{i}{16}\ln(e^x+i)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)*csch(2*x)^2,x)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)-1)/(exp(4*x)-1)^2-1/16*ln(exp(x)+1)+1/16*ln(exp(x)-
1)+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x)+I)
```

Maxima [A] time = 1.50556, size = 63, normalized size = 1.19

$$-\frac{5\,e^{(5\,x)}-e^{x}}{4\left(e^{(8\,x)}-2\,e^{(4\,x)}+1\right)}-\frac{1}{8}\,\arctan\left(e^{x}\right)-\frac{1}{16}\,\log\left(e^{x}+1\right)+\frac{1}{16}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(8*x) - 2*e^(4*x) + 1) - 1/8*arctan(e^x) - 1/16*l
og(e^x + 1) + 1/16*log(e^x - 1)
```

Fricas [B] time = 1.89066, size = 1750, normalized size = 33.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(20*cosh(x)^5 + 200*cosh(x)^3*sinh(x)^2 + 200*cosh(x)^2*sinh(x)^3 + 1
00*cosh(x)*sinh(x)^4 + 20*sinh(x)^5 + 2*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cos
h(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*s
inh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(co
sh(x) + sinh(x) + 1) - (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*s
inh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4
 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*c
osh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(cosh(x) +
sinh(x) - 1) + 4*(25*cosh(x)^4 - 1)*sinh(x) - 4*cosh(x))/(cosh(x)^8 + 56*co
```

```
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh
(x)^3)*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x} \coth(2x) \operatorname{csch}^{2}(2x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)*csch(2*x)**2, x)
```

Giac [A] time = 1.19141, size = 57, normalized size = 1.08

$$-\frac{5\,e^{(5\,x)} - e^{x}}{4\left(e^{(4\,x)} - 1\right)^{2}} - \frac{1}{8}\,\arctan\left(e^{x}\right) - \frac{1}{16}\,\log\left(e^{x} + 1\right) + \frac{1}{16}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(4*x) - 1)^2 - 1/8*arctan(e^x) - 1/16*log(e^x + 1
) + 1/16*log(abs(e^x - 1))
```

3.943 $\int e^x \coth^2(2x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=55

$$\frac{3e^{3x}}{4\left(1-e^{4x}\right)}-\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{5}{8}\tan^{-1}\left(e^x\right)-\frac{5}{8}\tanh^{-1}\left(e^x\right)$$

```
[Out] -(E^(3*x)/(1 - E^(4*x))^2) + (3*E^(3*x))/(4*(1 - E^(4*x))) + (5*ArcTan[E^x]
)/8 - (5*ArcTanh[E^x])/8
```

Rubi [A] time = 0.0480443, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 12, 463, 457, 298, 203, 206}

$$\frac{3e^{3x}}{4\left(1-e^{4x}\right)}-\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{5}{8}\tan^{-1}\left(e^x\right)-\frac{5}{8}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] -(E^(3*x)/(1 - E^(4*x))^2) + (3*E^(3*x))/(4*(1 - E^(4*x))) + (5*ArcTan[E^x]
)/8 - (5*ArcTanh[E^x])/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
```

```
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(1+x^4\right)^2}{\left(-1+x^4\right)^3}\,dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(1+x^4\right)^2}{\left(-1+x^4\right)^3}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{x^2\left(4+8x^4\right)}{\left(-1+x^4\right)^2}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{3e^{3x}}{4\left(1-e^{4x}\right)}+\frac{5}{4}\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{3e^{3x}}{4\left(1-e^{4x}\right)}-\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)+\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{3e^{3x}}{4\left(1-e^{4x}\right)}+\frac{5}{8}\tan^{-1}\left(e^x\right)-\frac{5}{8}\tanh^{-1}\left(e^x\right)
\end{aligned}
$$

Mathematica [C] time = 3.15484, size = 161, normalized size = 2.93

$$
-\frac{16e^{7x}\left(e^{4x}+1\right)^2\,{}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{19}{4};e^{4x}\right)}{1155}-\frac{8e^{7x}\left(26e^{4x}+11e^{8x}+15\right)\,{}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{4x}\right)}{1155}+\frac{e^{-5x}\left(-7\left(2415\right.\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] (177023 + 244931*E^(4*x) + 43161*E^(8*x) - 26091*E^(12*x) - 7*(25289 + 2415
2*E^(4*x) - 10058*E^(8*x) - 9048*E^(12*x) + 513*E^(16*x))*Hypergeometric2F1
[3/4, 1, 7/4, E^(4*x)])/(10752*E^(5*x)) - (8*E^(7*x)*(15 + 26*E^(4*x) + 11*
E^(8*x))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(4*x)])/1155 - (
16*E^(7*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1, 1, 1
9/4}, E^(4*x)])/1155
```

Maple [C] time = 0.097, size = 56, normalized size = 1.

$$
-\frac{e^{3x}\left(3\,e^{4x}+1\right)}{4\left(e^{4x}-1\right)^2}-\frac{5\ln\left(e^x+1\right)}{16}+\frac{5i}{16}\ln\left(e^x+i\right)-\frac{5i}{16}\ln\left(e^x-i\right)+\frac{5\ln\left(e^x-1\right)}{16}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)^2*csch(2*x),x)
```

```
[Out] -1/4*exp(3*x)*(3*exp(4*x)+1)/(exp(4*x)-1)^2-5/16*ln(exp(x)+1)+5/16*I*ln(exp
(x)+I)-5/16*I*ln(exp(x)-I)+5/16*ln(exp(x)-1)
```

Maxima [A] time = 1.53406, size = 63, normalized size = 1.15

$$-\frac{3\,e^{(7\,x)}+e^{(3\,x)}}{4\left(e^{(8\,x)}-2\,e^{(4\,x)}+1\right)}+\frac{5}{8}\,\arctan\left(e^{x}\right)-\frac{5}{16}\,\log\left(e^{x}+1\right)+\frac{5}{16}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(8*x) - 2*e^(4*x) + 1) + 5/8*arctan(e^x) - 5/
16*log(e^x + 1) + 5/16*log(e^x - 1)
```

Fricas [B] time = 1.89445, size = 1868, normalized size = 33.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/16*(12*cosh(x)^7 + 420*cosh(x)^3*sinh(x)^4 + 252*cosh(x)^2*sinh(x)^5 + 8
4*cosh(x)*sinh(x)^6 + 12*sinh(x)^7 + 4*(105*cosh(x)^4 + 1)*sinh(x)^3 + 4*co
sh(x)^3 + 12*(21*cosh(x)^5 + cosh(x))*sinh(x)^2 - 10*(cosh(x)^8 + 56*cosh(x
)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 +
2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*si
nh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^
3)*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(cosh(x)^8 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*c
osh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3
 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
```

```
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*log(cosh(x) + sinh(x) - 1) + 12*(7*cosh(x)^6 + cosh(x)^2)*sinh(x))/(cosh
(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)
^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x
)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cos
h(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15557, size = 57, normalized size = 1.04

$$-\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\left(e^{(4\,x)} - 1\right)^2} + \frac{5}{8}\,\arctan\left(e^{x}\right) - \frac{5}{16}\,\log\left(e^{x} + 1\right) + \frac{5}{16}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(4*x) - 1)^2 + 5/8*arctan(e^x) - 5/16*log(e^x
 + 1) + 5/16*log(abs(e^x - 1))
```

3.944 $\int e^x \coth^2(2x)\mathbf{csch}^2(2x)\,dx$

Optimal. Leaf size=75

$$\frac{3e^x}{8(1-e^{4x})}-\frac{5e^{5x}}{6(1-e^{4x})^2}+\frac{4e^{5x}}{3(1-e^{4x})^3}-\frac{3}{16}\tan^{-1}(e^x)-\frac{3}{16}\tanh^{-1}(e^x)$$

```
[Out] (4*E^(5*x))/(3*(1 - E^(4*x))^3) - (5*E^(5*x))/(6*(1 - E^(4*x))^2) + (3*E^x)
/(8*(1 - E^(4*x))) - (3*ArcTan[E^x])/16 - (3*ArcTanh[E^x])/16
```

Rubi [A] time = 0.0669314, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 12, 463, 457, 288, 212, 206, 203}

$$\frac{3e^x}{8(1-e^{4x})}-\frac{5e^{5x}}{6(1-e^{4x})^2}+\frac{4e^{5x}}{3(1-e^{4x})^3}-\frac{3}{16}\tan^{-1}(e^x)-\frac{3}{16}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

```
[Out] (4*E^(5*x))/(3*(1 - E^(4*x))^3) - (5*E^(5*x))/(6*(1 - E^(4*x))^2) + (3*E^x)
/(8*(1 - E^(4*x))) - (3*ArcTan[E^x])/16 - (3*ArcTanh[E^x])/16
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
```

```
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4\left(1+x^4\right)^2}{\left(1-x^4\right)^4}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4\left(1+x^4\right)^2}{\left(1-x^4\right)^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4\left(8+12x^4\right)}{\left(1-x^4\right)^3}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{16}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{3}{16}\,\text{Subst}\left(\int\right.$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{16}\tan^{-1}\left(e^x\right) - \frac{3}{16}\tanh^{-1}\left(e^x\right)$$

Mathematica [C] time = 5.27065, size = 310, normalized size = 4.13

$$e^{-7x}\left(1280e^{16x}\left(1346e^{4x}+557e^{8x}+821\right)\,{}_4F_3\left(2,2,2,\tfrac{9}{4};1,1,\tfrac{21}{4};e^{4x}\right)+10240e^{16x}\left(42e^{4x}+19e^{8x}+23\right)\,{}_5F_4\left(2,2,2,2,\tfrac{9}{4};1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

```
[Out] (-1070609085 - 946471617*E^(4*x) + 369641285*E^(8*x) + 351173641*E^(12*x) -
 23818496*E^(16*x) + 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] + 7
32349800*E^(4*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 635067810*E^(8*x
)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 384831720*E^(12*x)*Hypergeometr
ic2F1[1/4, 1, 5/4, E^(4*x)] + 60913125*E^(16*x)*Hypergeometric2F1[1/4, 1, 5
/4, E^(4*x)] + 1280*E^(16*x)*(821 + 1346*E^(4*x) + 557*E^(8*x))*Hypergeomet
ricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(4*x)] + 10240*E^(16*x)*(23 + 42*E^(
4*x) + 19*E^(8*x))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(4*x)] + 20480*E^(16*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1
, 21/4}, E^(4*x)] + 40960*E^(20*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
```

```
{1, 1, 1, 1, 21/4}, E^(4*x)] + 20480*E^(24*x)*HypergeometricPFQ[{2, 2, 2, 2
, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)])/(3818880*E^(7*x))
```

Maple [C] time = 0.105, size = 60, normalized size = 0.8

$$-\frac{e^{x} \left(29 e^{8 x}-6 e^{4 x}+9\right)}{24 \left(e^{4 x}-1\right)^{3}}+\frac{3 \ln \left(e^{x}-1\right)}{32}-\frac{3 \ln \left(e^{x}+1\right)}{32}+\frac{3 i}{32} \ln \left(e^{x}-i\right)-\frac{3 i}{32} \ln \left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)^2*csch(2*x)^2,x)
```

```
[Out] -1/24*exp(x)*(29*exp(8*x)-6*exp(4*x)+9)/(exp(4*x)-1)^3+3/32*ln(exp(x)-1)-3/
32*ln(exp(x)+1)+3/32*I*ln(exp(x)-I)-3/32*I*ln(exp(x)+I)
```

Maxima [A] time = 1.51523, size = 80, normalized size = 1.07

$$-\frac{29 e^{(9 x)}-6 e^{(5 x)}+9 e^{x}}{24 \left(e^{(12 x)}-3 e^{(8 x)}+3 e^{(4 x)}-1\right)}-\frac{3}{16} \arctan \left(e^{x}\right)-\frac{3}{32} \log \left(e^{x}+1\right)+\frac{3}{32} \log \left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(12*x) - 3*e^(8*x) + 3*e^(4*x) -
1) - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(e^x - 1)
```

Fricas [B] time = 2.00212, size = 3366, normalized size = 44.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(116*cosh(x)^9 + 9744*cosh(x)^3*sinh(x)^6 + 4176*cosh(x)^2*sinh(x)^7
+ 1044*cosh(x)*sinh(x)^8 + 116*sinh(x)^9 + 24*(609*cosh(x)^4 - 1)*sinh(x)^5
```

```
 - 24*cosh(x)^5 + 24*(609*cosh(x)^5 - 5*cosh(x))*sinh(x)^4 + 48*(203*cosh(x
)^6 - 5*cosh(x)^2)*sinh(x)^3 + 48*(87*cosh(x)^7 - 5*cosh(x)^3)*sinh(x)^2 +
18*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 12*cos
h(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)^8 - 3*cosh(x)^
8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6 - cosh(x)^2)*s
inh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(165*cosh(x)^8 - 7
0*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 - 42*cosh(x)^5 +
 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*cosh(x)^2)*sinh
(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) - 1)*arctan(cosh(
x) + sinh(x)) + 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh
(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)
^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6
 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(16
5*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 -
 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*
cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) -
1)*log(cosh(x) + sinh(x) + 1) - 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 6
6*cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x
)^4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 +
84*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*s
inh(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4
*(55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 1
4*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(
x)^3)*sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) + 12*(87*cosh(x)^8 - 10*cosh(
x)^4 + 3)*sinh(x) + 36*cosh(x))/(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*
cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^
4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84
*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sin
h(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(
55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*
cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)
^3)*sinh(x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1748, size = 65, normalized size = 0.87

$$-\frac{29\,e^{(9\,x)}-6\,e^{(5\,x)}+9\,e^{x}}{24\,\left(e^{(4\,x)}-1\right)^{3}}-\frac{3}{16}\,\arctan\left(e^{x}\right)-\frac{3}{32}\,\log\left(e^{x}+1\right)+\frac{3}{32}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(4*x) - 1)^3 - 3/16*arctan(e^x) -
 3/32*log(e^x + 1) + 3/32*log(abs(e^x - 1))
```

3.945 $\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8\left(16b^2-d^2\right)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}$$

```
[Out] -(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[
4*a + 4*b*x])/(2*(16*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*
b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rubi [A] time = 0.101286, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8\left(16b^2-d^2\right)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[
4*a + 4*b*x])/(2*(16*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*
b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx$$
$$=\frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx-\frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$=-\frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}+\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c}}{}$$

Mathematica [A] time = 1.11985, size = 86, normalized size = 0.63

$$\frac{1}{8}e^{c+dx}\left(\frac{2d\sinh(2(a+bx))-4b\cosh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*((-4*b*Cosh[2*(a + b*x)] + 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2
) + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

Maple [A] time = 0.03, size = 202, normalized size = 1.5

$$\frac{\sinh(2a-c+(2b-d)x)}{16b-8d}-\frac{\sinh(2a+c+(2b+d)x)}{16b+8d}-\frac{\sinh((4b-d)x+4a-c)}{64b-16d}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}-\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(4
*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)-1/8*cosh(2*a
-c+(2*b-d)*x)/(2*b-d)-1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*x
+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95394, size = 1161, normalized size = 8.47

$$-\frac{\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(4\,b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^4+\left(16\,b^3-bd^2-6\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 + (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 - (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1639, size = 126, normalized size = 0.92

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)}-\frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)}-\frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)}+\frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

3.946 $\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{be^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(4*(9*b^2 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2)) +
(3*b*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rubi [A] time = 0.0907898, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5475}

$$-\frac{be^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(4*(9*b^2 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2)) +
(3*b*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{4}e^{c+dx}\cosh(a+bx)+\frac{1}{4}e^{c+dx}\cosh(3a+3bx)\right)dx$$
$$= -\left(\frac{1}{4}\int e^{c+dx}\cosh(a+bx)\,dx\right)+\frac{1}{4}\int e^{c+dx}\cosh(3a+3bx)\,dx$$
$$= \frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\text{si}}{4(9}$$

Mathematica [A] time = 0.953156, size = 80, normalized size = 0.63

$$\frac{1}{4}e^{c+dx}\left(\frac{3b\sinh(3(a+bx))-d\cosh(3(a+bx))}{9b^2-d^2}+\frac{d\cosh(a+bx)-b\sinh(a+bx)}{(b-d)(b+d)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*((d*Cosh[a + b*x] - b*Sinh[a + b*x])/((b - d)*(b + d)) + (-(d*
Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

Maple [A] time = 0.024, size = 178, normalized size = 1.4

$$-\frac{\sinh(a-c+(b-d)x)}{8b-8d}-\frac{\sinh(a+c+(b+d)x)}{8b+8d}+\frac{\sinh(3a-c+(3b-d)x)}{24b-8d}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(a}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)-1/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3*
b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-
d)-1/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3
*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82452, size = 887, normalized size = 6.98

$$-\frac{3\left(b^{2}d-d^{3}\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}-3\left(b^{3}-bd^{2}\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{3}+\left(9\,b^{3}-bd^{2}-9\,(b^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (9*b^3 - b*d^2 - 9*(b^3 - b*d^2)*c
osh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a)^
3 - (9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(b*x
+ a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2)*si
nh(b*x + a)^3 - (9*b^2*d - d^3)*cosh(b*x + a) + (9*b^3 - b*d^2 - 9*(b^3 - b
*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 +
 d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b
*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [A] time = 102.679, size = 1059, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp
(d*x)*sinh(a - d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)
/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 - x*exp(c)*exp(d*x)
*cosh(a - d*x)**3/8 - exp(c)*exp(d*x)*sinh(a - d*x)**3/(3*d) - 5*exp(c)*exp
(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(24*d) - exp(c)*exp(d*x)*sinh(a - d*x)
*cosh(a - d*x)**2/(24*d) + exp(c)*exp(d*x)*cosh(a - d*x)**3/(12*d), Eq(b, -
```

```
d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a -
 d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a -
 d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 + exp(c)*exp(d*x)*sin
h(a - d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3
)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*exp(c
)*exp(d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*co
sh(a + d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/8
+ x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x/3)*
*3/d - 21*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/(8*d) + 27*exp
(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/(8*d) - 5*exp(c)*exp(d*x)*c
osh(a + d*x/3)**3/(4*d), Eq(b, d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)**3/
8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 + x*exp(c)*exp(d*x)*
sinh(a + d*x)*cosh(a + d*x)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x)**3/8 + e
xp(c)*exp(d*x)*sinh(a + d*x)**3/(3*d) - 5*exp(c)*exp(d*x)*sinh(a + d*x)**2*
cosh(a + d*x)/(24*d) + exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(24*d
) + exp(c)*exp(d*x)*cosh(a + d*x)**3/(12*d), Eq(b, d)), (3*b**3*exp(c)*exp(
d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - 3*b**2*d*exp(c)*exp(
d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 2*b**2
*d*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - b*d**2
*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - 2*b*d**2
*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d*
*4) + d**3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2
*d**2 + d**4), True))
```

Giac [A] time = 1.20754, size = 116, normalized size = 0.91

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{8\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

3.947 $\int e^{c+dx} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=66

$$\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2(4b^2-d^2)}$$

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) - (d*E^(c + d*x)*Sinh[2*a +
 2*b*x])/(2*(4*b^2 - d^2))
```

Rubi [A] time = 0.0471871, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5509, 12, 5474}

$$\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2(4b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) - (d*E^(c + d*x)*Sinh[2*a +
 2*b*x])/(2*(4*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
```

```
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx &= \int \frac{1}{2}e^{c+dx}\sinh(2a+2bx)\,dx\\ &= \frac{1}{2}\int e^{c+dx}\sinh(2a+2bx)\,dx\\ &= \frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\left(4b^2-d^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0526181, size = 47, normalized size = 0.71

$$\frac{e^{c+dx}(2b\cosh(2(a+bx))-d\sinh(2(a+bx)))}{2\left(4b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*(2*b*Cosh[2*(a + b*x)] - d*Sinh[2*(a + b*x)]))/(2*(4*b^2 - d^2
))
```

Maple [A] time = 0.006, size = 102, normalized size = 1.6

$$-\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(2a-c+(2b-d)x)}{8b-4d}+\frac{\cosh(2a+c+(2b+d)x)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*sinh(2*a+c+(2*b+d)*x)/(2*b+d)+1/4*co
sh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*cosh(2*a+c+(2*b+d)*x)/(2*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76207, size = 360, normalized size = 5.45

$$\frac{b\cosh(bx+a)^2\cosh(dx+c) - d\cosh(bx+a)\cosh(dx+c)\sinh(bx+a) + b\cosh(dx+c)\sinh(bx+a)^2 + \left(b\cosh(b\right.}{\left(4\,b^2-d^2\right)\cosh(bx+a)^2 - \left(4\,b^2-d^2\right)\sinh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)^2*cosh(d*x + c) - d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x +
 a) + b*cosh(d*x + c)*sinh(b*x + a)^2 + (b*cosh(b*x + a)^2 - d*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)*sinh(d*x + c))/((4*b^2 - d^2)*cosh(b*x
 + a)^2 - (4*b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 16.925, size = 347, normalized size = 5.26

$$\begin{cases} xe^c\sinh(a)\cosh(a) \\ \frac{xe^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{e^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d} \\ -\frac{xe^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2} - \frac{xe^ce^{dx}\cosh^2\left(a+\frac{dx}{2}\right)}{4} + \frac{e^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{d} - \frac{3e^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2d} + \frac{e^ce^{dx}\mathrm{co}}{} \\ \frac{be^ce^{dx}\sinh^2(a+bx)}{4b^2-d^2} + \frac{be^ce^{dx}\cosh^2(a+bx)}{4b^2-d^2} - \frac{de^ce^{dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*
x)*sinh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)
/2 + x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d*x/
2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/2)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/2)
**2/4 + x*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)/2 - x*exp(c)*exp(
```

```
d*x)*cosh(a + d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a + d*x/2)**2/d - 3*exp(c)
*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)/(2*d) + exp(c)*exp(d*x)*cosh(a +
d*x/2)**2/d, Eq(b, d/2)), (b*exp(c)*exp(d*x)*sinh(a + b*x)**2/(4*b**2 - d**
2) + b*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2 - d**2) - d*exp(c)*exp(d*x)
*sinh(a + b*x)*cosh(a + b*x)/(4*b**2 - d**2), True))
```

Giac [A] time = 1.18206, size = 63, normalized size = 0.95

$$\frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)}+\frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d)
```

3.948 $\int e^{c+dx}\cosh(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

```
[Out] -((d*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x]
)/(b^2 - d^2)
```

Rubi [A] time = 0.0168503, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5475}

$$\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] -((d*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x]
)/(b^2 - d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\,dx=-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}+\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}$$

Mathematica [A] time = 0.030573, size = 38, normalized size = 0.7

$$\frac{e^{c+dx}(b\sinh(a+bx)-d\cosh(a+bx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*(-(d*Cosh[a + b*x]) + b*Sinh[a + b*x]))/((b - d)*(b + d))
```

Maple [A] time = 0.005, size = 78, normalized size = 1.4

$$\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{2b+2d}-\frac{\cosh\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\cosh\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)-1/2*cosh(a-c+(b-d)*
x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84209, size = 240, normalized size = 4.44

$$-\frac{d\cosh\left(bx+a\right)\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)\sinh\left(bx+a\right)+\left(d\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)}{\left(b^{2}-d^{2}\right)\cosh\left(bx+a\right)^{2}-\left(b^{2}-d^{2}\right)\sinh\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) - b*cosh(d*x + c)*sinh(b*x + a) + (d*cosh(b
*x + a) - b*sinh(b*x + a))*sinh(d*x + c))/((b^2 - d^2)*cosh(b*x + a)^2 - (b
^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [A] time = 4.72467, size = 201, normalized size = 3.72

$$\begin{cases} xe^c\cosh(a) & \text{for } b=0 \wedge d=0 \\ \frac{xe^ce^{dx}\sinh(a-dx)}{2}+\frac{xe^ce^{dx}\cosh(a-dx)}{2}-\frac{e^ce^{dx}\sinh(a-dx)}{d}-\frac{e^ce^{dx}\cosh(a-dx)}{2d} & \text{for } b=-d \\ -\frac{xe^ce^{dx}\sinh(a+dx)}{2}+\frac{xe^ce^{dx}\cosh(a+dx)}{2}+\frac{e^ce^{dx}\sinh(a+dx)}{d}-\frac{e^ce^{dx}\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{be^ce^{dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^ce^{dx}\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*sinh(
a - d*x)/2 + x*exp(c)*exp(d*x)*cosh(a - d*x)/2 - exp(c)*exp(d*x)*sinh(a - d
*x)/d - exp(c)*exp(d*x)*cosh(a - d*x)/(2*d), Eq(b, -d)), (-x*exp(c)*exp(d*x
)*sinh(a + d*x)/2 + x*exp(c)*exp(d*x)*cosh(a + d*x)/2 + exp(c)*exp(d*x)*sin
h(a + d*x)/d - exp(c)*exp(d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*exp(c)*ex
p(d*x)*sinh(a + b*x)/(b**2 - d**2) - d*exp(c)*exp(d*x)*cosh(a + b*x)/(b**2
- d**2), True))
```

Giac [A] time = 1.27937, size = 54, normalized size = 1.

$$\frac{e^{(bx+dx+a+c)}}{2\,(b+d)}-\frac{e^{(-bx+dx-a+c)}}{2\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(-b*x + d*x - a + c)/(b - d)
```

3.949 $\int e^{c+dx} \coth(a+bx) \, dx$

Optimal. Leaf size=53

$$\frac{e^{c+dx}}{d} - \frac{2e^{c+dx} {}_2F_1\left(1, \frac{d}{2b}; \frac{d}{2b}+1; e^{2(a+bx)}\right)}{d}$$

```
[Out] E^(c + d*x)/d - (2*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d
```

Rubi [A] time = 0.0645266, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5485, 2194, 2251}

$$\frac{e^{c+dx}}{d} - \frac{2e^{c+dx} {}_2F_1\left(1, \frac{d}{2b}; \frac{d}{2b}+1; e^{2(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] E^(c + d*x)/d - (2*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
```

```
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\coth(a+bx)\,dx &= \int\left(e^{c+dx}+\frac{2e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx\\ &= 2\int\frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx+\int e^{c+dx}\,dx\\ &= \frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}\end{aligned}$$

Mathematica [B] time = 1.92924, size = 120, normalized size = 2.26

$$\frac{\coth(a)e^{c+dx}}{d}-\frac{2e^{2a+c}\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{e^{2a}-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] (E^(c + d*x)*Coth[a])/d - (2*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/(
2*b), 1 + d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1
[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^{dx+c}\cosh(bx+a)\operatorname{csch}(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,b\int \frac{e^{(dx+c)}}{(2\,b-d)e^{(4\,bx+4\,a)}-2\,(2\,b-d)e^{(2\,bx+2\,a)}+2\,b-d}\,dx-\frac{\left((2\,be^{c}-de^{c})e^{(2\,bx+2\,a)}-2\,be^{c}-de^{c}\right)e^{(dx)}}{2\,bd-d^{2}-\left(2\,bd-d^{2}\right)e^{(2\,bx+2\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((2*b - d)*e^(4*b*x + 4*a) - 2*(2*b - d)*e^(2*b*
x + 2*a) + 2*b - d), x) - ((2*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 2*b*e^c - d*
e^c)*e^(d*x)/(2*b*d - d^2 - (2*b*d - d^2)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

3.950 $\int e^{c+dx} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{4e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (4*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d)
```

Rubi [A] time = 0.249677, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5511, 2251}

$$\frac{4e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (4*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx = \int\left(\frac{4e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\right)dx$$
$$= 2\int\frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\,dx+4\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx$$
$$= -\frac{2e^{a+c+(b+d)x}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{4e^{a+c+(b+d)x}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

Mathematica [A] time = 0.656673, size = 92, normalized size = 0.91

$$\frac{e^c\text{csch}(a)\left(-2de^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)-(b+d)e^{dx}(\cosh(a)-\sinh(a))\text{csch}(a+bx)\right)}{b(\coth(a)-1)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (E^c*Csch[a]*(-2*d*E^((b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b +
 d)/(2*b), E^(2*b*x)*(Cosh[a] + Sinh[a])^2] - (b + d)*E^(d*x)*Csch[a + b*x]
*(Cosh[a] - Sinh[a])))/(b*(b + d)*(-1 + Coth[a]))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int e^{dx+c}\cosh(bx+a)\left(\text{csch}(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,bd\int -\frac{e^{(bx+dx+a+c)}}{3\,b^2-4\,bd+d^2-\left(3\,b^2-4\,bd+d^2\right)e^{(6\,bx+6\,a)}+3\,\left(3\,b^2-4\,bd+d^2\right)e^{(4\,bx+4\,a)}-3\,\left(3\,b^2-4\,bd+d^2\right)e^{(2\,bx+2\,a)}}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(b*x + d*x + a + c)/(3*b^2 - 4*b*d + d^2 - (3*b^2 - 4*b
*d + d^2)*e^(6*b*x + 6*a) + 3*(3*b^2 - 4*b*d + d^2)*e^(4*b*x + 4*a) - 3*(3*
b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a)), x) - 2*((3*b*e^c - d*e^c)*e^(3*b*x + 3
*a) - (3*b*e^c + d*e^c)*e^(b*x + a))*e^(d*x)/(3*b^2 - 4*b*d + d^2 + (3*b^2
- 4*b*d + d^2)*e^(4*b*x + 4*a) - 2*(3*b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(bx+a\right)\mathrm{csch}\left(bx+a\right)^2e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)\operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

3.951 $\int e^{c+dx}\coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{4e^{2a+x(2b+d)+c}\,{}_2F_1\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+x(2b+d)+c}\,{}_2F_1\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] (4*E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[2, (2 + d/b)/2, (4 + d/b)/2,
 E^(2*(a + b*x))])/(2*b + d) - (8*E^(2*a + c + (2*b + d)*x)*Hypergeometric2
F1[3, (2 + d/b)/2, (4 + d/b)/2, E^(2*(a + b*x))])/(2*b + d)
```

Rubi [A] time = 0.272688, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5511, 2251}

$$\frac{4e^{2a+x(2b+d)+c}\,{}_2F_1\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+x(2b+d)+c}\,{}_2F_1\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[2, (2 + d/b)/2, (4 + d/b)/2,
 E^(2*(a + b*x))])/(2*b + d) - (8*E^(2*a + c + (2*b + d)*x)*Hypergeometric2
F1[3, (2 + d/b)/2, (4 + d/b)/2, E^(2*(a + b*x))])/(2*b + d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=\int\left(\frac{8e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}+\frac{4e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\right)dx$$
$$=4\int\frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+8\int\frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx$$
$$=\frac{4e^{2a+c+(2b+d)x}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{d}{b}\right);\frac{1}{2}\left(4+\frac{d}{b}\right);e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+c+(2b+d)x}\,{}_2F_1\left(3,\frac{1}{2}\left(\right.\right.}{2b}$$

Mathematica [A] time = 1.54049, size = 159, normalized size = 1.41

$$-\frac{e^{c-\frac{ad}{b}}\left(d^2e^{\left(\frac{d}{b}+2\right)(a+bx)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)+d(2b+d)e^{d\left(\frac{a}{b}+x\right)}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)+(2b+d)e^{d\left(\frac{a}{b}+x\right)}\left(d\,\text{co}\right.\right.}{2b^2(2b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -(E^(c - (a*d)/b)*((2*b + d)*E^(d*(a/b + x))*(d*Coth[a + b*x] + b*Csch[a +
b*x]^2) + d*(2*b + d)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/(2*b), 1 + d/(
2*b), E^(2*(a + b*x))] + d^2*E^((2 + d/b)*(a + b*x))*Hypergeometric2F1[1, 1
 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))]))/(2*b^2*(2*b + d))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int e^{dx+c}\cosh(bx+a)\,(\text{csch}\,(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-48\,bd^2\int\frac{e^{(dx+c)}}{48\,b^3-44\,b^2d+12\,bd^2-d^3+\left(48\,b^3-44\,b^2d+12\,bd^2-d^3\right)e^{(8\,bx+8\,a)}-4\left(48\,b^3-44\,b^2d+12\,bd^2-d^3\right)e^{(6\,bx+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -48*b*d^2*integrate(e^(d*x + c)/(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3 + (48*b
^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 - 44*b^2*d + 12
*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4
*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a)), x) +
 4*(12*b*d*e^c + (24*b^2*e^c - 10*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - (24*
b^2*e^c + 2*b*d*e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3 - 44*b^2*d
+ 12*b*d^2 - d^3 - (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3
*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(48*b^3 - 44*b^2*
d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)\operatorname{csch}(bx+a)^3 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

3.952 $\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=195

$$\frac{de^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}-\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}+$$

```
[Out] -(b*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (3*b*E^(c + d*x)*Cosh[3*a
+ 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*
b^2 - d^2)) + (d*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (d*E^(c + d*x
)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[5*a + 5*b*x])
/(16*(25*b^2 - d^2))
```

Rubi [A] time = 0.136203, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5474}

$$\frac{de^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}-\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -(b*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (3*b*E^(c + d*x)*Cosh[3*a
+ 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*
b^2 - d^2)) + (d*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (d*E^(c + d*x
)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[5*a + 5*b*x])
/(16*(25*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}e^{c+dx}\sinh(a+bx)-\frac{1}{16}e^{c+dx}\sinh(3a+3bx)+\frac{1}{16}e^{c+dx}\sinh(5a+5bx)\right.$$
$$= -\left(\frac{1}{16}\int e^{c+dx}\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int e^{c+dx}\sinh(5a+5bx)\,dx-\frac{1}{8}\int e^{c+d}$$
$$= -\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}+\frac{5be^{c+dx}\cosh(5a+5bx)}{16\left(25b^2-d^2\right)}+$$

Mathematica [A] time = 1.19026, size = 117, normalized size = 0.6

$$\frac{1}{16}e^{c+dx}\left(\frac{d\sinh(3(a+bx))-3b\cosh(3(a+bx))}{9b^2-d^2}+\frac{5b\cosh(5(a+bx))-d\sinh(5(a+bx))}{25b^2-d^2}+\frac{2d\sinh(a+bx)-2b\cosh}{(b-d)(b+d)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*((-2*b*Cosh[a + b*x] + 2*d*Sinh[a + b*x])/((b - d)*(b + d)) +
(-3*b*Cosh[3*(a + b*x)] + d*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (5*b*Cosh[5*
(a + b*x)] - d*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

Maple [A] time = 0.021, size = 278, normalized size = 1.4

$$\frac{\sinh(a-c+(b-d)x)}{16b-16d}-\frac{\sinh(a+c+(b+d)x)}{16b+16d}+\frac{\sinh(3a-c+(3b-d)x)}{96b-32d}-\frac{\sinh(3a+c+(3b+d)x)}{96b+32d}-\frac{\sinh((5}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+(
3*b-d)*x)/(3*b-d)-1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-1/32/(5*b-d)*sinh((5*b
-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)-1/16*cosh(a-c+(b-d)*x)/(b-d
)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-1/32*cosh
```

```
(3*a+c+(3*b+d)*x)/(3*b+d)+1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*b
+d)*x+5*a+c)/(5*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88115, size = 2201, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a)^4 - (9*b^4*d - 10*b^2*d^3 + d^5)*cosh(d*x + c)*sinh(b*x + a)^5 + (25*b
^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^2)*
cosh(d*x + c)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x +
 a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)*sinh(b
*x + a)^2 + (450*b^4*d - 68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5
)*cosh(b*x + a)^4 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*cosh(d
*x + c)*sinh(b*x + a) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^5 - 3
*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 - 2*(225*b^5 - 34*b^3*d^2 +
b*d^4)*cosh(b*x + a))*cosh(d*x + c) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(
b*x + a)^5 + 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*sinh(b*x + a)^4
- (9*b^4*d - 10*b^2*d^3 + d^5)*sinh(b*x + a)^5 - 3*(25*b^5 - 26*b^3*d^2 + b
*d^4)*cosh(b*x + a)^3 + (25*b^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2
*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*
d^4)*cosh(b*x + a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^5 - 34*b^3*d^2 + b*d^4)*cosh(b*x + a) + (450*b^4*d -
68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^4 + 3*(25
*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/(
(225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 - 2
```

```
59*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^6
 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*b
^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20741, size = 178, normalized size = 0.91

$$\frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)} - \frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{16\,(b+d)} - \frac{e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)} + \frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) - 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) - 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

3.953 $\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}-\frac{e^{c+dx}}{8d}$$

```
[Out] -E^(c + d*x)/(8*d) - (d*E^(c + d*x)*Cosh[4*a + 4*b*x])/(8*(16*b^2 - d^2)) +
 (b*E^(c + d*x)*Sinh[4*a + 4*b*x])/(2*(16*b^2 - d^2))
```

Rubi [A] time = 0.0792998, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5509, 2194, 5475}

$$\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}-\frac{e^{c+dx}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(c + d*x)/(8*d) - (d*E^(c + d*x)*Cosh[4*a + 4*b*x])/(8*(16*b^2 - d^2)) +
 (b*E^(c + d*x)*Sinh[4*a + 4*b*x])/(2*(16*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
```

```
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \int\left(-\frac{1}{8}e^{c+dx}+\frac{1}{8}e^{c+dx}\cosh(4a+4bx)\right)dx\\ &= -\left(\frac{1}{8}\int e^{c+dx}\,dx\right)+\frac{1}{8}\int e^{c+dx}\cosh(4a+4bx)\,dx\\ &= -\frac{e^{c+dx}}{8d}-\frac{de^{c+dx}\cosh(4a+4bx)}{8\left(16b^2-d^2\right)}+\frac{be^{c+dx}\sinh(4a+4bx)}{2\left(16b^2-d^2\right)}\end{aligned}$$

Mathematica [A] time = 0.355294, size = 58, normalized size = 0.7

$$\frac{e^{c+dx}\left(d^2\cosh(4(a+bx))-4bd\sinh(4(a+bx))+16b^2-d^2\right)}{8\left(d^3-16b^2d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(16*b^2 - d^2 + d^2*Cosh[4*(a + b*x)] - 4*b*d*Sinh[4*(a + b*x)
]))/(8*(-16*b^2*d + d^3))
```

Maple [A] time = 0.012, size = 124, normalized size = 1.5

$$-\frac{\sinh(dx+c)}{8d}+\frac{\sinh((4b-d)x+4a-c)}{64b-16d}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}-\frac{\cosh(dx+c)}{8d}-\frac{\cosh((4b-d)x+4a-c)}{64b-16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/8*sinh(d*x+c)/d+1/16/(4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*
b+d)*x+4*a+c)-1/8*cosh(d*x+c)/d-1/16*cosh((4*b-d)*x+4*a-c)/(4*b-d)+1/16*cos
h((4*b+d)*x+4*a+c)/(4*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98238, size = 770, normalized size = 9.28

$$\frac{16\,bd\cosh\left(bx+a\right)^{3}\cosh\left(dx+c\right)\sinh\left(bx+a\right)-6\,d^{2}\cosh\left(bx+a\right)^{2}\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}+16\,bd\cosh\left(bx+a\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(16*b*d*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a) - 6*d^2*cosh(b*x +
a)^2*cosh(d*x + c)*sinh(b*x + a)^2 + 16*b*d*cosh(b*x + a)*cosh(d*x + c)*sin
h(b*x + a)^3 - d^2*cosh(d*x + c)*sinh(b*x + a)^4 - (d^2*cosh(b*x + a)^4 + 1
6*b^2 - d^2)*cosh(d*x + c) - (d^2*cosh(b*x + a)^4 - 16*b*d*cosh(b*x + a)^3*
sinh(b*x + a) + 6*d^2*cosh(b*x + a)^2*sinh(b*x + a)^2 - 16*b*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 + 16*b^2 - d^2)*sinh(d*x + c))/((16
*b^2*d - d^3)*cosh(b*x + a)^4 - 2*(16*b^2*d - d^3)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (16*b^2*d - d^3)*sinh(b*x + a)^4)
```

Sympy [A] time = 150.693, size = 831, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*
exp(d*x)*sinh(a - d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(
```

```
a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8
+ x*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x
)*cosh(a - d*x/4)**4/16 + exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4
)/(4*d) + exp(c)*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/d + exp(c)*
exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/(4*d), Eq(b, -d/4)), (x*exp(c)*
exp(d*x)*sinh(a + d*x/4)**4/16 - x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(
a + d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**2/8
- x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4 + x*exp(c)*exp(d*x
)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a + d*x/4)**4/(8*d) + exp(c)
*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(4*d) + exp(c)*exp(d*x)*sinh(a
 + d*x/4)**2*cosh(a + d*x/4)**2/(4*d) + exp(c)*exp(d*x)*sinh(a + d*x/4)*cos
h(a + d*x/4)**3/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x/4)**4/(8*d), Eq(b, d/4
)), ((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 - x*cos
h(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)*cosh
(a + b*x)**3/(8*b))*exp(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh(a + b*
x)**4/(16*b**2*d - d**3) + 4*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a +
 b*x)**2/(16*b**2*d - d**3) - 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**4/(16*b
**2*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(16*b*
*2*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**
2*d - d**3) - d**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b*
*2*d - d**3), True))
```

Giac [A] time = 1.18579, size = 78, normalized size = 0.94

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)} - \frac{e^{(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/16*e^(-4*b*x + d*x - 4*a + c)/
(4*b - d) - 1/8*e^(d*x + c)/d
```

3.954 $\int e^{c+dx} \cosh^2(a+bx) \sinh(a+bx)\, dx$

Optimal. Leaf size=127

$$-\frac{de^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}+\frac{be^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}$$

```
[Out] (b*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[3*a +
 3*b*x])/(4*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2))
- (d*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rubi [A] time = 0.0901656, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$-\frac{de^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}+\frac{be^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[3*a +
 3*b*x])/(4*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2))
- (d*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \int\left(\frac{1}{4}e^{c+dx}\sinh(a+bx)+\frac{1}{4}e^{c+dx}\sinh(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int e^{c+dx}\sinh(a+bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(3a+3bx)\,dx$$
$$= \frac{be^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}}{4}$$

Mathematica [A] time = 0.524209, size = 80, normalized size = 0.63

$$\frac{1}{4}e^{c+dx}\left(\frac{3b\cosh(3(a+bx))-d\sinh(3(a+bx))}{9b^2-d^2}+\frac{b\cosh(a+bx)-d\sinh(a+bx)}{(b-d)(b+d)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*((b*Cosh[a + b*x] - d*Sinh[a + b*x])/((b - d)*(b + d)) + (3*b*
Cosh[3*(a + b*x)] - d*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

Maple [A] time = 0.007, size = 178, normalized size = 1.4

$$-\frac{\sinh(a-c+(b-d)x)}{8b-8d}+\frac{\sinh(a+c+(b+d)x)}{8b+8d}-\frac{\sinh(3a-c+(3b-d)x)}{24b-8d}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)+1/8*sinh(a+c+(b+d)*x)/(b+d)-1/8*sinh(3*a-c+(3*
b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-
d)+1/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3
*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13763, size = 886, normalized size = 6.98

$$\frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(9\,b^2d-d^3+3\left(b^2d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - (b^2*d -
 d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - (9*b^2*d - d^3 + 3*(b^2*d - d^3)*cosh
(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + (3*(b^3 - b*d^2)*cosh(b*x + a)^3
 + (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) + (3*(b^3 - b*d^2)*cosh(b*x
 + a)^3 + 9*(b^3 - b*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (b^2*d - d^3)*sin
h(b*x + a)^3 + (9*b^3 - b*d^2)*cosh(b*x + a) - (9*b^2*d - d^3 + 3*(b^2*d -
d^3)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 +
d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*
x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [A] time = 86.4629, size = 994, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*ex
p(d*x)*sinh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x
)/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 - exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(8*d)
 - exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/(8*d) - exp(c)*exp(d*x)*c
osh(a - d*x)**3/(4*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8
```

```
+ 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp
(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x
/3)**3/8 - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) - 3*exp(c)*exp(d*x)*s
inh(a - d*x/3)**2*cosh(a - d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**
3/(8*d), Eq(b, -d/3)), (x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 - 3*x*exp(c)
*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a
 + d*x/3)*cosh(a + d*x/3)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - 3
*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/(8*d) + 9*exp(c)*exp(d*
x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/(8*d) - exp(c)*exp(d*x)*cosh(a + d*x/
3)**3/(4*d), Eq(b, d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)
*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x
)*cosh(a + d*x)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*
x)*sinh(a + d*x)**2*cosh(a + d*x)/(8*d) - exp(c)*exp(d*x)*sinh(a + d*x)*cos
h(a + d*x)**2/(8*d) + exp(c)*exp(d*x)*cosh(a + d*x)**3/(4*d), Eq(b, d)), (3
*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + 2*b
**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - 3*b
**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2
 + d**4) - 2*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4
- 10*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b**4 -
10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/
(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Giac [A] time = 1.17696, size = 116, normalized size = 0.91

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}+\frac{e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{8\,(b-d)}+\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) + 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

3.955 $\int e^{c+dx}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}+\frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)}$$

```
[Out] (2*b^2*E^(c + d*x))/(d*(4*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[a + b*x]^2)/(4*
b^2 - d^2) + (2*b*E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2 - d^2)
```

Rubi [A] time = 0.0359479, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 2194}

$$-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}+\frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (2*b^2*E^(c + d*x))/(d*(4*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[a + b*x]^2)/(4*
b^2 - d^2) + (2*b*E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2 - d^2)
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = -\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}+\frac{\left(2b^2\right)\int e^{c+dx}\,dx}{4b^2-d^2}$$
$$=\frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)}-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}$$

Mathematica [A] time = 0.141704, size = 55, normalized size = 0.58

$$\frac{e^{c+dx}\left(d^2\cosh(2(a+bx))-2bd\sinh(2(a+bx))-4b^2+d^2\right)}{2d^3-8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(-4*b^2 + d^2 + d^2*Cosh[2*(a + b*x)] - 2*b*d*Sinh[2*(a + b*x)
]))/(-8*b^2*d + 2*d^3)
```

Maple [A] time = 0.007, size = 124, normalized size = 1.3

$$\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(dx+c)}{2d}-\frac{\cosh(2a-c+(2b-d)x)}{8b-4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2,x)
```

```
[Out] 1/2*sinh(d*x+c)/d+1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)+1/2*cosh(d*x+c)/d-1/4*cosh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*cosh(2*a
+c+(2*b+d)*x)/(2*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0804, size = 433, normalized size = 4.56

$$\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)-d^{2}\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}-\left(d^{2}\cosh\left(bx+a\right)^{2}-4\,b^{2}+d^{2}\right)\cosh\left(dx\right.}{2\left(\left(4\,b^{2}d-d^{3}\right)\cosh\left(bx+a\right)^{2}-\left(4\,b^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - d^2*cosh(d*x + c)*si
nh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 - 4*b^2 + d^2)*cosh(d*x + c) - (d^2*co
sh(b*x + a)^2 - 4*b*d*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - 4
*b^2 + d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(b*x + a)^2 - (4*b^2*d - d^
3)*sinh(b*x + a)^2)
```

Sympy [A] time = 16.198, size = 456, normalized size = 4.8

$$\begin{cases} xe^{c}\cosh^{2}\left(a\right) \\ \frac{xe^{c}e^{dx}\sinh^{2}\left(a-\frac{dx}{2}\right)}{4}+\frac{xe^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2}+\frac{xe^{c}e^{dx}\cosh^{2}\left(a-\frac{dx}{2}\right)}{4}+\frac{e^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d}+\frac{e^{c}e^{dx}\cosh^{2}\left(a-\frac{dx}{2}\right)}{d} \\ \frac{xe^{c}e^{dx}\sinh^{2}\left(a+\frac{dx}{2}\right)}{4}-\frac{xe^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2}+\frac{xe^{c}e^{dx}\cosh^{2}\left(a+\frac{dx}{2}\right)}{4}-\frac{3e^{c}e^{dx}\sinh^{2}\left(a+\frac{dx}{2}\right)}{4d}+\frac{e^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{d}+\frac{e^{c}e^{dx}\cosh^{2}}{4d} \\ \left(-\frac{x\sinh^{2}\left(a+bx\right)}{2}+\frac{x\cosh^{2}\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}\right)e^{c} \\ -\frac{2b^{2}e^{c}e^{dx}\sinh^{2}\left(a+bx\right)}{4b^{2}d-d^{3}}+\frac{2b^{2}e^{c}e^{dx}\cosh^{2}\left(a+bx\right)}{4b^{2}d-d^{3}}+\frac{2bde^{c}e^{dx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^{2}d-d^{3}}-\frac{d^{2}e^{c}e^{dx}\cosh^{2}\left(a+bx\right)}{4b^{2}d-d^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*si
nh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/2 +
x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d*x/2)*co
sh(a - d*x/2)/(2*d) + exp(c)*exp(d*x)*cosh(a - d*x/2)**2/d, Eq(b, -d/2)), (
```

```
x*exp(c)*exp(d*x)*sinh(a + d*x/2)**2/4 - x*exp(c)*exp(d*x)*sinh(a + d*x/2)*
cosh(a + d*x/2)/2 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**2/4 - 3*exp(c)*exp(d
*x)*sinh(a + d*x/2)**2/(4*d) + exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x
/2)/d + exp(c)*exp(d*x)*cosh(a + d*x/2)**2/(4*d), Eq(b, d/2)), ((-x*sinh(a
+ b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*exp
(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**2/(4*b**2*d - d**3)
 + 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3) + 2*b*d*exp(c)
*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b**2*d - d**3) - d**2*exp(c)*exp(d
*x)*cosh(a + b*x)**2/(4*b**2*d - d**3), True))
```

Giac [A] time = 1.17611, size = 78, normalized size = 0.82

$$\frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)}-\frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)}+\frac{e^{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d) + 1/2*e^(d*x + c)/d
```

3.956 $\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{2e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[Out] (-3*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (2*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d)
```

Rubi [A] time = 0.207046, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5511, 2194, 2227, 2251}

$$\frac{2e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-3*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (2*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx &= \int\left(\frac{3}{2}e^{-a+c-(b-d)x}+\frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)}+\frac{2e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\right)dx\\
&= \frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx+\frac{3}{2}\int e^{-a+c-(b-d)x}\,dx+2\int\frac{e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\,dx\\
&= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{2e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}+\frac{1}{2}\int e^{a+c+(b+d)x}\,dx\\
&= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{2e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}
\end{aligned}$$

Mathematica [A] time = 0.554539, size = 93, normalized size = 0.9

$$\frac{e^c\left(\frac{e^{dx}(b\cosh(a+bx)-d\sinh(a+bx))}{b-d}-2(\sinh(a)+\cosh(a))e^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (E^c*(-2*E^((b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b),
 E^(2*b*x)*(Cosh[a] + Sinh[a])^2]*(Cosh[a] + Sinh[a]) + (E^(d*x)*(b*Cosh[a
+ b*x] - d*Sinh[a + b*x]))/(b - d)))/(b + d)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \mathrm{e}^{dx+c} (\cosh (bx + a))^2 \operatorname{csch} (bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,b \int \frac{e^{(dx+c)}}{(3\,b-d)e^{(5\,bx+5\,a)} - 2\,(3\,b-d)e^{(3\,bx+3\,a)} + (3\,b-d)e^{(bx+a)}}\,dx + \frac{\left(5\,b^2e^c + 6\,bde^c + d^2e^c + \left(3\,b^2e^c - 4\,bde^c + d^2e^c\right)e^{(4\,b}}{2\left(\left(3\,b^3 - b^2d - 3\,bd^2 + d^3\right)e^{(3\,bx+3\,a)} - (}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((3*b - d)*e^(5*b*x + 5*a) - 2*(3*b - d)*e^(3*b*
x + 3*a) + (3*b - d)*e^(b*x + a)), x) + 1/2*(5*b^2*e^c + 6*b*d*e^c + d^2*e^
c + (3*b^2*e^c - 4*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(6*b^2*e^c + b*d*
e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^
(3*b*x + 3*a) - (3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^(b*x + a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\cosh (bx + a)^2 \operatorname{csch} (bx + a)\, e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right) e^{\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

3.957 $\int e^{c+dx}\coth^2(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[Out] E^(c + d*x)/d - (4*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (4*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2*
b), E^(2*(a + b*x))])/d
```

Rubi [A] time = 0.114463, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5485, 2194, 2251}

$$-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (4*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (4*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2*
b), E^(2*(a + b*x))])/d
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\coth^2(a+bx)\,dx &= \int\left(e^{c+dx}+\frac{4e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{4e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx\\ &= 4\int\frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+4\int\frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx+\int e^{c+dx}\,dx\\ &= \frac{e^{c+dx}}{d}-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.87354, size = 145, normalized size = 1.54

$$-\frac{2de^{2a+c}\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{\left(e^{2a}-1\right)b}+\frac{\text{csch}(a)\sinh(bx)e^{c+dx}\text{csch}(a+bx)}{b}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (2*d*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1
+ d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 +
d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(2*a))) + (E
^(c + d*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int e^{dx+c}\left(\cosh\left(bx+a\right)\right)^2\left(\text{csch}\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,bd\int -\frac{e^{(dx+c)}}{8\,b^2-6\,bd+d^2-\left(8\,b^2-6\,bd+d^2\right)e^{(6\,bx+6\,a)}+3\left(8\,b^2-6\,bd+d^2\right)e^{(4\,bx+4\,a)}-3\left(8\,b^2-6\,bd+d^2\right)e^{(2\,bx+2\,a)}}\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
d + d^2)*e^(2*b*x + 2*a)), x) + (8*b^2*e^c + 10*b*d*e^c + d^2*e^c + (8*b^2*
e^c - 6*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(8*b^2*e^c + 2*b*d*e^c - d^2
*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(8*b^2*d - 6*b*d^2 + d^3 + (8*b^2*d - 6*b*d^
2 + d^3)*e^(4*b*x + 4*a) - 2*(8*b^2*d - 6*b*d^2 + d^3)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

3.958 $\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(3,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (8*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d) - (8*E^(a + c
+ (b + d)*x)*Hypergeometric2F1[3, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a +
 b*x))])/(b + d)
```

Rubi [A] time = 0.343, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5511, 2251}

$$-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(3,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (8*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d) - (8*E^(a + c
+ (b + d)*x)*Hypergeometric2F1[3, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a +
 b*x))])/(b + d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= \int\left(\frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}+\frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\right)dx\\ &= 2\int\frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\,dx+8\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx+8\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx\\ &= -\frac{2e^{a+c+(b+d)x}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+c+(b+d)x}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}\end{aligned}$$

Mathematica [A] time = 1.32094, size = 111, normalized size = 0.74

$$-\frac{e^{c-\frac{ad}{b}}\left(2\left(b^2+d^2\right)e^{\frac{(b+d)(a+bx)}{b}}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)+(b+d)e^{d\left(\frac{a}{b}+x\right)}\operatorname{csch}(a+bx)(b\coth(a+bx)+d)\right)}{2b^2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -(E^(c - (a*d)/b)*((b + d)*E^(d*(a/b + x))*(d + b*Coth[a + b*x])*Csch[a + b
*x] + 2*(b^2 + d^2)*E^(((b + d)*(a + b*x))/b)*Hypergeometric2F1[1, (b + d)/
(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]))/(2*b^2*(b + d))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int e^{dx+c}\left(\cosh\left(bx+a\right)\right)^2\left(\operatorname{csch}\left(bx+a\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-48\left(b^{3}e^{c}+bd^{2}e^{c}\right)\int\frac{e^{(bx+dx+a}}{15\,b^{3}-23\,b^{2}d+9\,bd^{2}-d^{3}+\left(15\,b^{3}-23\,b^{2}d+9\,bd^{2}-d^{3}\right)e^{(8\,bx+8\,a)}-4\left(15\,b^{3}-23\,b^{2}d+9\,bd^{2}-d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -48*(b^3*e^c + b*d^2*e^c)*integrate(e^(b*x + d*x + a)/(15*b^3 - 23*b^2*d +
9*b*d^2 - d^3 + (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(15
*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(15*b^3 - 23*b^2*d + 9
*b*d^2 - d^3)*e^(4*b*x + 4*a) - 4*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*
b*x + 2*a)), x) + 2*((15*b^2*e^c - 8*b*d*e^c + d^2*e^c)*e^(5*b*x + 5*a) - 2
*(10*b^2*e^c + 3*b*d*e^c - d^2*e^c)*e^(3*b*x + 3*a) + (9*b^2*e^c + 14*b*d*e
^c + d^2*e^c)*e^(b*x + a))*e^(d*x)/(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3 - (15
*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3*(15*b^3 - 23*b^2*d + 9
*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*
b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{3}e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

3.959 $\int e^{c+dx} \cosh^3(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=137

$$\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}$$

```
[Out] (-3*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(16*(4*b^2 - d^2)) + (3*b*E^(c + d*x)*
Cosh[6*a + 6*b*x])/(16*(36*b^2 - d^2)) + (3*d*E^(c + d*x)*Sinh[2*a + 2*b*x]
)/(32*(4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[6*a + 6*b*x])/(32*(36*b^2 - d^2)
)
```

Rubi [A] time = 0.113393, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5474}

$$\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```


```
[Out] (-3*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(16*(4*b^2 - d^2)) + (3*b*E^(c + d*x)*
Cosh[6*a + 6*b*x])/(16*(36*b^2 - d^2)) + (3*d*E^(c + d*x)*Sinh[2*a + 2*b*x]
)/(32*(4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[6*a + 6*b*x])/(32*(36*b^2 - d^2)
)
```


Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}e^{c+dx}\sinh(2a+2bx)+\frac{1}{32}e^{c+dx}\sinh(6a+6bx)\right)dx$$
$$=\frac{1}{32}\int e^{c+dx}\sinh(6a+6bx)\,dx-\frac{3}{32}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$=-\frac{3be^{c+dx}\cosh(2a+2bx)}{16\left(4b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16\left(36b^2-d^2\right)}+\frac{3de^{c+dx}\sinh(2a+2bx)}{32\left(4b^2-d^2\right)}$$

Mathematica [A] time = 0.985342, size = 113, normalized size = 0.82

$$\frac{e^{c+dx}\left(6b\left(d^2-36b^2\right)\cosh(2(a+bx))+6\left(4b^3-bd^2\right)\cosh(6(a+bx))+2d\sinh(2(a+bx))\left(\left(d^2-4b^2\right)\cosh(4(a+bx))\right)\right)}{32\left(-40b^2d^2+144b^4+d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(6*b*(-36*b^2 + d^2)*Cosh[2*(a + b*x)] + 6*(4*b^3 - b*d^2)*Cos
h[6*(a + b*x)] + 2*d*(52*b^2 - d^2 + (-4*b^2 + d^2)*Cosh[4*(a + b*x)])*Sinh
[2*(a + b*x)]))/(32*(144*b^4 - 40*b^2*d^2 + d^4))
```

Maple [A] time = 0.023, size = 202, normalized size = 1.5

$$\frac{3\sinh(2a-c+(2b-d)x)}{128b-64d}-\frac{3\sinh(2a+c+(2b+d)x)}{128b+64d}-\frac{\sinh((6b-d)x+6a-c)}{384b-64d}+\frac{\sinh((6b+d)x+6a+c)}{384b+64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 3/64*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/64/
(6*b-d)*sinh((6*b-d)*x+6*a-c)+1/64/(6*b+d)*sinh((6*b+d)*x+6*a+c)-3/64*cosh(
2*a-c+(2*b-d)*x)/(2*b-d)-3/64*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/64*cosh((6*b-
d)*x+6*a-c)/(6*b-d)+1/64*cosh((6*b+d)*x+6*a+c)/(6*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02437, size = 1611, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(10*(4*b^2*d - d^3)*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a)^3 - 4
5*(4*b^3 - b*d^2)*cosh(b*x + a)^2*cosh(d*x + c)*sinh(b*x + a)^4 + 3*(4*b^2*
d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^5 - 3*(4*b^3 - b*d^2)*co
sh(d*x + c)*sinh(b*x + a)^6 - 3*(15*(4*b^3 - b*d^2)*cosh(b*x + a)^4 - 36*b^
3 + b*d^2)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((4*b^2*d - d^3)*cosh(b*x + a)
^5 - (36*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - 3*((4*b^
3 - b*d^2)*cosh(b*x + a)^6 - (36*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x + c
) - (3*(4*b^3 - b*d^2)*cosh(b*x + a)^6 - 10*(4*b^2*d - d^3)*cosh(b*x + a)^3
*sinh(b*x + a)^3 + 45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*sinh(b*x + a)^4 - 3*(
4*b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(4*b^3 - b*d^2)*sinh(b*x +
 a)^6 - 3*(36*b^3 - b*d^2)*cosh(b*x + a)^2 + 3*(15*(4*b^3 - b*d^2)*cosh(b*x
 + a)^4 - 36*b^3 + b*d^2)*sinh(b*x + a)^2 - 3*((4*b^2*d - d^3)*cosh(b*x + a
)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a))*sinh(d*x + c))/((144*b
^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^6 - 3*(144*b^4 - 40*b^2*d^2 + d^4)*cos
h(b*x + a)^4*sinh(b*x + a)^2 + 3*(144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)
^2*sinh(b*x + a)^4 - (144*b^4 - 40*b^2*d^2 + d^4)*sinh(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2048, size = 126, normalized size = 0.92

$$\frac{e^{(6\,bx+dx+6\,a+c)}}{64\,(6\,b+d)}-\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{64\,(2\,b+d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{64\,(2\,b-d)}+\frac{e^{(-6\,bx+dx-6\,a+c)}}{64\,(6\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*e^(6*b*x + d*x + 6*a + c)/(6*b + d) - 3/64*e^(2*b*x + d*x + 2*a + c)/(
2*b + d) - 3/64*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/64*e^(-6*b*x + d*x
 - 6*a + c)/(6*b - d)
```

3.960 $\int e^{c+dx} \cosh^3(a+bx) \sinh^2(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{be^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}$$

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*b^2 -
 d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (3*b*E^(c + d*x)*S
inh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Sinh[5*a + 5*b*x])/
(16*(25*b^2 - d^2))
```

Rubi [A] time = 0.132763, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5475}

$$-\frac{be^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*b^2 -
 d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (3*b*E^(c + d*x)*S
inh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Sinh[5*a + 5*b*x])/
(16*(25*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}e^{c+dx}\cosh(a+bx)+\frac{1}{16}e^{c+dx}\cosh(3a+3bx)+\frac{1}{16}e^{c+dx}\cosh(5a+5bx\right.$$

$$=\frac{1}{16}\int e^{c+dx}\cosh(3a+3bx)\,dx+\frac{1}{16}\int e^{c+dx}\cosh(5a+5bx)\,dx-\frac{1}{8}\int e^{c+dx}\text{c}$$

$$=\frac{de^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16\left(25b^2-d^2\right)}-\frac{be^{c+}}{}$$

Mathematica [A] time = 1.20979, size = 118, normalized size = 0.61

$$\frac{1}{16}e^{c+dx}\left(\frac{3b\sinh(3(a+bx))-d\cosh(3(a+bx))}{9b^2-d^2}+\frac{5b\sinh(5(a+bx))-d\cosh(5(a+bx))}{25b^2-d^2}+\frac{2d\cosh(a+bx)-2b\sinh}{(b-d)(b+d)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*((2*d*Cosh[a + b*x] - 2*b*Sinh[a + b*x])/((b - d)*(b + d)) + (
-(d*Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (-(d*Cosh[5
*(a + b*x)]) + 5*b*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

Maple [A] time = 0.009, size = 278, normalized size = 1.4

$$-\frac{\sinh(a-c+(b-d)x)}{16b-16d}-\frac{\sinh(a+c+(b+d)x)}{16b+16d}+\frac{\sinh(3a-c+(3b-d)x)}{96b-32d}+\frac{\sinh(3a+c+(3b+d)x)}{96b+32d}+\frac{\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+
(3*b-d)*x)/(3*b-d)+1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32/(5*b-d)*sinh((5*
b-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)+1/16*cosh(a-c+(b-d)*x)/(b-
d)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/32*cos
```

```
h(3*a+c+(3*b+d)*x)/(3*b+d)-1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*
b+d)*x+5*a+c)/(5*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11295, size = 2202, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a)^4 - 5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(d*x + c)*sinh(b*x + a)^5 - (75
*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)
^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)*si
nh(b*x + a)^2 + (450*b^5 - 68*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 +
b*d^4)*cosh(b*x + a)^4 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*c
osh(d*x + c)*sinh(b*x + a) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^5
+ (25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^3 - 2*(225*b^4*d - 34*b^2*d^3
 + d^5)*cosh(b*x + a))*cosh(d*x + c) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^5 + 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*sinh(b*x + a)^4 -
5*(9*b^5 - 10*b^3*d^2 + b*d^4)*sinh(b*x + a)^5 + (25*b^4*d - 26*b^2*d^3 + d
^5)*cosh(b*x + a)^3 - (75*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 +
d^5)*cosh(b*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^4*d - 34*b^2*d^3 + d^5)*cosh(b*x + a) + (450*b^5 - 68
*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^4 - 9*(2
5*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/
((225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 -
```

```
259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^
6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*
b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17169, size = 178, normalized size = 0.91

$$\frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)}+\frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)}-\frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) + 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) + 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

3.961 $\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8\left(16b^2-d^2\right)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}$$

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[4
*a + 4*b*x])/(2*(16*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*b
^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rubi [A] time = 0.0936001, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$-\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8\left(16b^2-d^2\right)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[4
*a + 4*b*x])/(2*(16*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*b
^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \int\left(\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$= \frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c}}{}$$

Mathematica [A] time = 0.830326, size = 86, normalized size = 0.63

$$\frac{1}{8}e^{c+dx}\left(\frac{4b\cosh(2(a+bx))-2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*((4*b*Cosh[2*(a + b*x)] - 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2)
 + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

Maple [A] time = 0.009, size = 202, normalized size = 1.5

$$-\frac{\sinh(2a-c+(2b-d)x)}{16b-8d}+\frac{\sinh(2a+c+(2b+d)x)}{16b+8d}-\frac{\sinh((4b-d)x+4a-c)}{64b-16d}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(
4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)+1/8*cosh(2*
a-c+(2*b-d)*x)/(2*b-d)+1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*
x+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07682, size = 1161, normalized size = 8.47

$$-\frac{\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(4\,b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^4-\left(16\,b^3-bd^2+6\left(4\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 - (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 + (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15977, size = 126, normalized size = 0.92

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)}+\frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)}+\frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)}+\frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) + 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) + 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

3.962 $\int e^{c+dx}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{6b^3e^{c+dx}\sinh(a+bx)}{-10b^2d^2+9b^4+d^4}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}-\frac{6b^2de^{c+dx}\cosh(a+bx)}{-10b^2d^2+9b^4+d^4}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}$$

```
[Out] (-6*b^2*d*E^(c + d*x)*Cosh[a + b*x])/(9*b^4 - 10*b^2*d^2 + d^4) - (d*E^(c +
 d*x)*Cosh[a + b*x]^3)/(9*b^2 - d^2) + (6*b^3*E^(c + d*x)*Sinh[a + b*x])/(9
*b^4 - 10*b^2*d^2 + d^4) + (3*b*E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/
(9*b^2 - d^2)
```

Rubi [A] time = 0.0566672, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 5475}

$$\frac{6b^3e^{c+dx}\sinh(a+bx)}{-10b^2d^2+9b^4+d^4}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}-\frac{6b^2de^{c+dx}\cosh(a+bx)}{-10b^2d^2+9b^4+d^4}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3,x]
```


```
[Out] (-6*b^2*d*E^(c + d*x)*Cosh[a + b*x])/(9*b^4 - 10*b^2*d^2 + d^4) - (d*E^(c +
 d*x)*Cosh[a + b*x]^3)/(9*b^2 - d^2) + (6*b^3*E^(c + d*x)*Sinh[a + b*x])/(9
*b^4 - 10*b^2*d^2 + d^4) + (3*b*E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/
(9*b^2 - d^2)
```


Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = -\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} + \frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2} + \frac{\left(6b^2\right)\int e^{c+dx}\cosh(a+bx)\,d}{9b^2-d^2}$$

$$= -\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4} - \frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} + \frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4} + \frac{3be^{c+dx}\cosh^2(}{9}$$

Mathematica [A] time = 0.444232, size = 106, normalized size = 0.74

$$\frac{e^{c+dx}\left(3d\left(d^2-9b^2\right)\cosh(a+bx)+\left(d^3-b^2d\right)\cosh(3(a+bx))+6b\sinh(a+bx)\left(\left(b^2-d^2\right)\cosh(2(a+bx))+5b^2-d^2\right)\right.}{4\left(-10b^2d^2+9b^4+d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(3*d*(-9*b^2 + d^2)*Cosh[a + b*x] + (-(b^2*d) + d^3)*Cosh[3*(a
 + b*x)] + 6*b*(5*b^2 - d^2 + (b^2 - d^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])
)/(4*(9*b^4 - 10*b^2*d^2 + d^4))
```

Maple [A] time = 0.006, size = 178, normalized size = 1.2

$$\frac{3\sinh(a-c+(b-d)x)}{8b-8d} + \frac{3\sinh(a+c+(b+d)x)}{8b+8d} + \frac{\sinh(3a-c+(3b-d)x)}{24b-8d} + \frac{\sinh(3a+c+(3b+d)x)}{24b+8d} - \frac{3\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3,x)
```

```
[Out] 3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3*b
-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-3/8*cosh(a-c+(b-d)*x)/(b-d
)+3/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3*
a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12696, size = 898, normalized size = 6.24

$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-3\left(9\,b^3-bd^2+3\left(b^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3 - b*d^2 + 3*(b^3 - b*d^2)
*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a
)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(
b*x + a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2
)*sinh(b*x + a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a) - 3*(9*b^3 - b*d^2 + 3*
(b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b
^2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^
2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [A] time = 82.6233, size = 1114, normalized size = 7.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp(c)*exp(d*x)
*sinh(a - d*x)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/8
```

```
+ 3*x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + 3*x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 + 2*exp(c)*exp(d*x)*sinh(a - d*x)**3/(3*d) + 7*exp(c)*
exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(24*d) - 25*exp(c)*exp(d*x)*sinh(a
- d*x)*cosh(a - d*x)**2/(24*d) - 5*exp(c)*exp(d*x)*cosh(a - d*x)**3/(12*d),
 Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*
sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*
cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - 11*exp(c)*e
xp(d*x)*sinh(a - d*x/3)**3/(8*d) - 15*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*co
sh(a - d*x/3)/(4*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**2/
d - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*exp(c)*exp(
d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a +
 d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/8 + x*ex
p(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - 2*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/d
 + 51*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/(8*d) - 57*exp(c)*
exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/(8*d) + 13*exp(c)*exp(d*x)*cosh
(a + d*x/3)**3/(4*d), Eq(b, d/3)), (3*x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8
- 3*x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 - 3*x*exp(c)*exp(d*x
)*sinh(a + d*x)*cosh(a + d*x)**2/8 + 3*x*exp(c)*exp(d*x)*cosh(a + d*x)**3/8
 - 2*exp(c)*exp(d*x)*sinh(a + d*x)**3/(3*d) + 7*exp(c)*exp(d*x)*sinh(a + d*
x)**2*cosh(a + d*x)/(24*d) + 25*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)
**2/(24*d) - 5*exp(c)*exp(d*x)*cosh(a + d*x)**3/(12*d), Eq(b, d)), (-6*b**3
*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + 9*b**3*e
xp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4
) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b*
*2*d**2 + d**4) - 7*b**2*d*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b**4 - 10*b*
*2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(
9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4), True))
```

Giac [A] time = 1.17066, size = 116, normalized size = 0.81

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}+\frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)}-\frac{3\,e^{(-bx+dx-a+c)}}{8\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 - 3/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

3.963 $\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=125

$$\frac{2e^{-2a-x(2b-d)+c}\,{}_2F_1\left(1,\frac{1}{2}\left(\frac{d}{b}-2\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d}-\frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)}+\frac{e^{2a+x(2b+d)+c}}{4(2b+d)}+\frac{e^{c+dx}}{d}$$

```
[Out] (-7*E^(-2*a + c - (2*b - d)*x))/(4*(2*b - d)) + E^(c + d*x)/d + E^(2*a + c
+ (2*b + d)*x)/(4*(2*b + d)) + (2*E^(-2*a + c - (2*b - d)*x)*Hypergeometric
2F1[1, (-2 + d/b)/2, d/(2*b), E^(2*(a + b*x))])/(2*b - d)
```

Rubi [A] time = 0.247492, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5511, 2194, 2227, 2251}

$$\frac{2e^{-2a-x(2b-d)+c}\,{}_2F_1\left(1,\frac{1}{2}\left(\frac{d}{b}-2\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d}-\frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)}+\frac{e^{2a+x(2b+d)+c}}{4(2b+d)}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (-7*E^(-2*a + c - (2*b - d)*x))/(4*(2*b - d)) + E^(c + d*x)/d + E^(2*a + c
+ (2*b + d)*x)/(4*(2*b + d)) + (2*E^(-2*a + c - (2*b - d)*x)*Hypergeometric
2F1[1, (-2 + d/b)/2, d/(2*b), E^(2*(a + b*x))])/(2*b - d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int\left(\frac{7}{4}e^{-2a+c-(2b-d)x} + e^{-2a+c-(2b-d)x+2(a+bx)} + \frac{1}{4}e^{-2a+c-(2b-d)x+4(a+bx)} + \frac{2e^{-2a+c-(}}{-1+e^{2(}}\right.$$

$$= \frac{1}{4}\int e^{-2a+c-(2b-d)x+4(a+bx)}\,dx + \frac{7}{4}\int e^{-2a+c-(2b-d)x}\,dx + 2\int \frac{e^{-2a+c-(2b-d)x}}{-1+e^{2(a+bx)}}\,dx +$$

$$= -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)} + \frac{2e^{-2a+c-(2b-d)x}\,{}_2F_1\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d} + \frac{1}{4}\int e^{2a+c}$$

$$= -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)} + \frac{e^{c+dx}}{d} + \frac{e^{2a+c+(2b+d)x}}{4(2b+d)} + \frac{2e^{-2a+c-(2b-d)x}\,{}_2F_1\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right);\frac{}{2}\right.}{2b-d}$$

Mathematica [A] time = 1.03722, size = 172, normalized size = 1.38

$$-\frac{e^{c-\frac{ad}{b}}\left(2\left(4b^2-d^2\right)e^{d\left(\frac{a}{b}+x\right)}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)+2d(2b-d)e^{\left(\frac{d}{b}+2\right)(a+bx)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)+de^{d\left(\frac{a}{b}+x\right)}(\right.}{8b^2d-2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] -((E^(c - (a*d)/b)*(2*(4*b^2 - d^2)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/
(2*b), 1 + d/(2*b), E^(2*(a + b*x))] + 2*(2*b - d)*d*E^((2 + d/b)*(a + b*x)
)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))] + d*E^(d*
(a/b + x))*(-2*b*Cosh[2*(a + b*x)] + d*Sinh[2*(a + b*x)])))/(8*b^2*d - 2*d^
3))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int e^{dx+c} (\cosh (bx+a))^3 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,b \int \frac{e^{(dx+c)}}{(4\,b-d)e^{(6\,bx+6\,a)} - 2\,(4\,b-d)e^{(4\,bx+4\,a)} + (4\,b-d)e^{(2\,bx+2\,a)}}\, dx + \frac{\left(24\,b^2de^c + 14\,bd^2e^c + d^3e^c + \left(8\,b^2de^c - 6\,bd^2e^c + \right.\right.}{4\left(\left(16\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((4*b - d)*e^(6*b*x + 6*a) - 2*(4*b - d)*e^(4*b*
x + 4*a) + (4*b - d)*e^(2*b*x + 2*a)), x) + 1/4*(24*b^2*d*e^c + 14*b*d^2*e^
c + d^3*e^c + (8*b^2*d*e^c - 6*b*d^2*e^c + d^3*e^c)*e^(6*b*x + 6*a) + (64*b
^3*e^c - 24*b^2*d*e^c - 10*b*d^2*e^c + 3*d^3*e^c)*e^(4*b*x + 4*a) - (64*b^3
*e^c + 40*b^2*d*e^c - 2*b*d^2*e^c - 3*d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((1
6*b^3*d - 4*b^2*d^2 - 4*b*d^3 + d^4)*e^(4*b*x + 4*a) - (16*b^3*d - 4*b^2*d^
2 - 4*b*d^3 + d^4)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\cosh (bx+a)^3 \operatorname{csch}(bx+a)\, e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^3 \operatorname{csch}(bx+a)\, e^{(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

3.964 $\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=160

$$\frac{6e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a-x(b-d)+c}\,{}_2F_1\left(2,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[Out] (-5*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (6*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d) - (4*E^(-a + c - (b - d)*x)*Hypergeometric2
F1[2, -(b - d)/(2*b), (b + d)/(2*b), E^(2*(a + b*x))])/(b - d)
```

Rubi [A] time = 0.305758, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5511, 2194, 2227, 2251}

$$\frac{6e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a-x(b-d)+c}\,{}_2F_1\left(2,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-5*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (6*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d) - (4*E^(-a + c - (b - d)*x)*Hypergeometric2
F1[2, -(b - d)/(2*b), (b + d)/(2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx=\int\left(\frac{5}{2}e^{-a+c-(b-d)x}+\frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)}+\frac{4e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{6e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\right)dx$$

$$=\frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx+\frac{5}{2}\int e^{-a+c-(b-d)x}\,dx+4\int\frac{e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+6$$

$$=-\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{6e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a+c-(b-d)x}\,{}_2F_1\left(2\right.}{b}$$

$$=-\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{6e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a+}}{}$$

Mathematica [A] time = 1.20762, size = 145, normalized size = 0.91

$$\frac{e^{c-\frac{ad}{b}}\operatorname{csch}(a+bx)\left(e^{d\left(\frac{a}{b}+x\right)}\left(b^2\cosh(2(a+bx))-bd\sinh(2(a+bx))-3b^2+2d^2\right)-4d(b-d)e^{\frac{(b+d)(a+bx)}{b}}\sinh(a+bx)\,{}_2F_1\left(\right.}{2b(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^(c - (a*d)/b)*Csch[a + b*x]*(-4*(b - d)*d*E^(((b + d)*(a + b*x))/b)*Hype
rgeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]*Sinh[a +
b*x] + E^(d*(a/b + x))*(-3*b^2 + 2*d^2 + b^2*Cosh[2*(a + b*x)] - b*d*Sinh[2
```

```
*(a + b*x)])))/(2*b*(b - d)*(b + d))
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int \mathrm{e}^{dx+c} \left(\cosh \left(bx+a\right)\right)^{3} \left(\mathrm{csch}\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,bd\int \frac{e^{(dx+c)}}{\left(15\,b^2-8\,bd+d^2\right)e^{(7\,bx+7\,a)}-3\left(15\,b^2-8\,bd+d^2\right)e^{(5\,bx+5\,a)}+3\left(15\,b^2-8\,bd+d^2\right)e^{(3\,bx+3\,a)}-\left(15\,b^2-8\,bd+d^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(e^(d*x + c)/((15*b^2 - 8*b*d + d^2)*e^(7*b*x + 7*a) - 3*(1
5*b^2 - 8*b*d + d^2)*e^(5*b*x + 5*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(3*b*x +
3*a) - (15*b^2 - 8*b*d + d^2)*e^(b*x + a)), x) - 1/2*(15*b^3*e^c + 39*b^2*d
*e^c + 25*b*d^2*e^c + d^3*e^c - (15*b^3*e^c - 23*b^2*d*e^c + 9*b*d^2*e^c -
d^3*e^c)*e^(6*b*x + 6*a) + (105*b^3*e^c - 11*b^2*d*e^c - 17*b*d^2*e^c + 3*d
^3*e^c)*e^(4*b*x + 4*a) - (105*b^3*e^c + 59*b^2*d*e^c - b*d^2*e^c - 3*d^3*e
^c)*e^(2*b*x + 2*a))*e^(d*x)/((15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^
4)*e^(5*b*x + 5*a) - 2*(15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(3
*b*x + 3*a) + (15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(b*x + a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh \left(bx+a\right)^{3} \mathrm{csch}\left(bx+a\right)^{2} e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

3.965 $\int e^{c+dx} \coth^3(a+bx)\,dx$

Optimal. Leaf size=135

$$-\frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,{}_2F_1\left(3,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[Out] E^(c + d*x)/d - (6*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (12*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2
*b), E^(2*(a + b*x))])/d - (8*E^(c + d*x)*Hypergeometric2F1[3, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))])/d
```

Rubi [A] time = 0.164018, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5485, 2194, 2251}

$$-\frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,{}_2F_1\left(3,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(c + d*x)/d - (6*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (12*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2
*b), E^(2*(a + b*x))])/d - (8*E^(c + d*x)*Hypergeometric2F1[3, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))])/d
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int\left(e^{c+dx}+\frac{8e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3}+\frac{12e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{6e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx$$
$$=6\int\frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx+8\int\frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx+12\int\frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+\int e^{c+dx}\,dx$$
$$=\frac{e^{c+dx}}{d}-\frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,{}_2F_1\left(\right.}{}$$

Mathematica [A] time = 3.72413, size = 176, normalized size = 1.3

$$\frac{1}{2}e^c\left(-\frac{2e^{2a}\left(2b^2+d^2\right)\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{\left(e^{2a}-1\right)b^2}+\frac{d\,\mathrm{csch}(a)e^{dx}\sinh(bx)\mathrm{csch}(a+bx)}{b^2}-\frac{e^{dx}\mathrm{c}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] (E^c*((2*E^(d*x)*Coth[a])/d - (E^(d*x)*Csch[a + b*x]^2)/b - (2*(2*b^2 + d^2
)*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*(a + b*
x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E
^(2*(a + b*x))])/(2*b + d)))/(b^2*(-1 + E^(2*a))) + (d*E^(d*x)*Csch[a]*Csch
[a + b*x]*Sinh[b*x])/b^2))/2
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int e^{dx+c}\left(\cosh(bx+a)\right)^3\left(\mathrm{csch}(bx+a)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-48\left(2\,b^3e^c + bd^2e^c\right)\int \frac{e}{48\,b^3 - 44\,b^2d + 12\,bd^2 - d^3 + \left(48\,b^3 - 44\,b^2d + 12\,bd^2 - d^3\right)e^{(8\,bx+8\,a)} - 4\left(48\,b^3 - 44\,b^2d + 12\,bd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -48*(2*b^3*e^c + b*d^2*e^c)*integrate(e^(d*x)/(48*b^3 - 44*b^2*d + 12*b*d^2
 - d^3 + (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 -
 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d
^2 - d^3)*e^(4*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x
 + 2*a)), x) + (48*b^3*e^c + 44*b^2*d*e^c + 36*b*d^2*e^c + d^3*e^c - (48*b^
3*e^c - 44*b^2*d*e^c + 12*b*d^2*e^c - d^3*e^c)*e^(6*b*x + 6*a) + 3*(48*b^3*
e^c + 4*b^2*d*e^c - 8*b*d^2*e^c + d^3*e^c)*e^(4*b*x + 4*a) - 3*(48*b^3*e^c
+ 28*b^2*d*e^c - d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3*d - 44*b^2*d^2 +
 12*b*d^3 - d^4 - (48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(6*b*x + 6*a)
+ 3*(48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(4*b*x + 4*a) - 3*(48*b^3*d
- 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(2*b*x + 2*a))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3 e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^3 e^{(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

3.966 $\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$

Optimal. Leaf size=73

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

```
[Out] (-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*b
*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

Rubi [A] time = 0.613906, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {5482, 2253, 2252, 2251, 5476}

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) + E^(a +
 b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*b
*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

Rule 5482

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Sinh[d + e*x]^n)/(-1 + E^(2*(d + e*x)))^n, Int
[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx = -\frac{\left(3d^2\right)\int \frac{e^{a+bx}}{\sqrt{\sinh(c+dx)}}\,dx}{4b^2-9d^2}+\int e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\,dx$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+}{4b^2-9d^2}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+}{4b^2-9d^2}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+}{4b^2-9d^2}$$

$$= -\frac{6d^2\exp\left(\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x+\frac{1}{2}(-c-dx)\right)\sqrt{1-e^{2(c+}}}{(2b+d)\left(4b^2-9d^2\right)\sqrt{\sinh(}}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+}{4b^2-9d^2}$$

Mathematica [C] time = 1.32062, size = 155, normalized size = 2.12

$$\frac{2e^{a+bx}\left(e^{2(c+dx)}-1\right)\left(\left(4b^2+8bd+3d^2\right)\sinh^2(c+dx)\,{}_2F_1\left(1,\frac{1}{4}\left(\frac{2b}{d}+7\right);\frac{2b+d}{4d};e^{2(c+dx)}\right)-3d^2\,{}_2F_1\left(1,\frac{1}{4}\left(\frac{2b}{d}+3\right);\frac{1}{4}\left(\frac{2b}{d}+5\right.\right.\right.}{(2b+d)(3d-2b)(2b+3d)\sqrt{\sinh(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) +
E^(a + b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (2*E^(a + b*x)*(-1 + E^(2*(c + d*x)))*(-3*d^2*Hypergeometric2F1[1, (3 + (2*
b)/d)/4, (5 + (2*b)/d)/4, E^(2*(c + d*x))] + (4*b^2 + 8*b*d + 3*d^2)*Hyperg
eometric2F1[1, (7 + (2*b)/d)/4, (2*b + d)/(4*d), E^(2*(c + d*x))]*Sinh[c +
d*x]^2))/((2*b + d)*(-2*b + 3*d)*(2*b + 3*d)*Sqrt[Sinh[c + d*x]])
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int e^{bx+a} (\sinh (dx+c))^{\frac{3}{2}} - \frac{3 d^2 e^{bx+a}}{4} \left(b^2 - \frac{9 d^2}{4}\right)^{-1} \frac{1}{\sqrt{\sinh (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

```
[Out] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)} \sinh (dx+c)^{\frac{3}{2}} - \frac{3 d^2 e^{(bx+a)}}{(4 b^2 - 9 d^2) \sqrt{\sinh (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**(3/2)-3/4*d**2*exp(b*x+a)/(b**2-9/4*d**2)
/sinh(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(bx+a)}\sinh(dx+c)^{\frac{3}{2}} - \frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

3.967 $\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

```
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Rubi [A] time = 0.015365, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

```
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{e^{n\cosh(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0405984, size = 17, normalized size = 1.

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

```
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Maple [A] time = 0.007, size = 17, normalized size = 1.

$$\frac{e^{n\cosh(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

```
[Out] exp(n*cosh(b*x+a))/b/n
```

Maxima [A] time = 1.02596, size = 22, normalized size = 1.29

$$\frac{e^{(n\cosh(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*x + a))/(b*n)
```

Fricas [A] time = 2.34454, size = 74, normalized size = 4.35

$$\frac{\cosh\left(n\cosh\left(bx+a\right)\right)+\sinh\left(n\cosh\left(bx+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*x + a)) + sinh(n*cosh(b*x + a)))/(b*n)
```

Sympy [A] time = 0.590437, size = 36, normalized size = 2.12

$$\begin{cases} x \sinh(a) & \text{for}\: b = 0 \wedge n = 0 \\ x e^{n \cosh(a)} \sinh(a) & \text{for}\: b = 0 \\ \frac{\cosh(a+bx)}{b} & \text{for}\: n = 0 \\ \frac{e^{n \cosh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cosh(a))*sinh(a), Eq(b
, 0)), (cosh(a + b*x)/b, Eq(n, 0)), (exp(n*cosh(a + b*x))/(b*n), True))
```

Giac [A] time = 1.15765, size = 36, normalized size = 2.12

$$\frac{e^{\left(\frac{1}{2} n\left(e^{(bx+a)}+e^{(-bx-a)}\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] e^(1/2*n*(e^(b*x + a) + e^(-b*x - a)))/(b*n)
```

3.968 $\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Rubi [A] time = 0.0167992, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\cosh(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.208852, size = 22, normalized size = 1.

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Maple [A] time = 0.088, size = 23, normalized size = 1.1

$$\frac{e^{n\cosh(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
```

```
[Out] exp(n*cosh(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 0.984368, size = 30, normalized size = 1.36

$$\frac{e^{(n\cosh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.3356, size = 88, normalized size = 4.

$$\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh(ac+bcx)}\sinh(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*cosh(a*c + b*c*x))*sinh(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh(bcx+ac))}\sinh((bx+a)c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*c*x + a*c))*sinh((b*x + a)*c), x)
```

3.969 $\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

```
[Out] E^(n*Cosh[a*c + b*c*x])/(b*c*n)
```

Rubi [A] time = 0.0151182, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

```
[Out] E^(n*Cosh[a*c + b*c*x])/(b*c*n)
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\cosh(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.0437569, size = 22, normalized size = 0.96

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Maple [A] time = 0.009, size = 23, normalized size = 1.

$$\frac{e^{n\cosh(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
```

```
[Out] exp(n*cosh(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 1.00429, size = 30, normalized size = 1.3

$$\frac{e^{(n\cosh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.26688, size = 88, normalized size = 3.83

$$\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

Sympy [A] time = 3.98863, size = 48, normalized size = 2.09

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \cosh{\left(a c \right)}} \sinh{\left(a c \right)} & \text{for}\: b = 0 \\ \frac{\cosh{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ 0 & \text{for}\: c = 0 \\ \frac{e^{n \cosh{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cosh(a*c))*sinh(a*c
), Eq(b, 0)), (cosh(a*c + b*c*x)/(b*c), Eq(n, 0)), (0, Eq(c, 0)), (exp(n*co
sh(a*c + b*c*x))/(b*c*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n \cosh\left((bx+a)c\right)\right)} \sinh\left(bcx+ac\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh((b*x + a)*c))*sinh(b*c*x + a*c), x)
```

3.970 $\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Rubi [A] time = 0.0235904, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\text{Ei}(n\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0602792, size = 13, normalized size = 1.

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Maple [A] time = 0.01, size = 17, normalized size = 1.3

$$-\frac{\text{Ei}\left(1,-n\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*cosh(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh(bx+a))}\tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

Fricas [A] time = 2.31028, size = 31, normalized size = 2.38

$$\frac{\text{Ei}\left(n\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh(bx+a))}\tanh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

3.971 $\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Rubi [A] time = 0.0231976, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.159525, size = 18, normalized size = 1.

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.046, size = 23, normalized size = 1.3

$$-\frac{\text{Ei}\left(1,-n\cosh\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

```
[Out] -1/c/b*Ei(1,-n*cosh(b*c*x+a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

Fricas [A] time = 2.1577, size = 42, normalized size = 2.33

$$\frac{\text{Ei}\left(n\cosh\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh(ac+bcx)}\tanh(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh(bcx+ac))}\tanh((bx+a)c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

3.972 $\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n\cosh(ac+bxc))}{bc}$$

```
[Out] ExpIntegralEi[n*Cosh[a*c + b*c*x]]/(b*c)
```

Rubi [A] time = 0.0238702, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a*c + b*c*x]]/(b*c)
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(ac+bcx)\right)}{bc}\\ &= \frac{\text{Ei}(n\cosh(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0613761, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.039, size = 23, normalized size = 1.2

$$-\frac{\text{Ei}(1,-n\cosh(bcx+ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

```
[Out] -1/c/b*Ei(1,-n*cosh(b*c*x+a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh((bx+a)c))}\tanh(bcx+ac)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

Fricas [A] time = 2.04753, size = 42, normalized size = 2.21

$$\frac{\text{Ei}(n\cosh(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh(ac+bcx)}\tanh(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

```
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cosh((bx+a)c))}\tanh(bcx+ac)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

3.973 $\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

```
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Rubi [A] time = 0.0135215, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

```
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{e^{n\sinh(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0160259, size = 17, normalized size = 1.

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

```
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Maple [A] time = 0.006, size = 17, normalized size = 1.

$$\frac{e^{n\sinh(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
```

```
[Out] exp(n*sinh(b*x+a))/b/n
```

Maxima [A] time = 1.00828, size = 22, normalized size = 1.29

$$\frac{e^{(n\sinh(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*sinh(b*x + a))/(b*n)
```

Fricas [A] time = 2.03225, size = 74, normalized size = 4.35

$$\frac{\cosh(n\sinh(bx+a)) + \sinh(n\sinh(bx+a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(n*sinh(b*x + a)) + sinh(n*sinh(b*x + a)))/(b*n)
```

Sympy [A] time = 1.14793, size = 36, normalized size = 2.12

$$\begin{cases} x \cosh(a) & \text{for}\ b = 0 \wedge n = 0 \\ x e^{n \sinh(a)} \cosh(a) & \text{for}\ b = 0 \\ \frac{\sinh(a+bx)}{b} & \text{for}\ n = 0 \\ \frac{e^{n \sinh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sinh(a))*cosh(a), Eq(b
, 0)), (sinh(a + b*x)/b, Eq(n, 0)), (exp(n*sinh(a + b*x))/(b*n), True))
```

Giac [A] time = 1.15056, size = 39, normalized size = 2.29

$$\frac{e^{\left(\frac{1}{2} n\left(e^{(bx+a)}-e^{(-bx-a)}\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] e^(1/2*n*(e^(b*x + a) - e^(-b*x - a)))/(b*n)
```

3.974 $\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

```
[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

Rubi [A] time = 0.0147721, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
```

```
[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx,x,\sinh(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\sinh(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.127983, size = 23, normalized size = 1.05

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
```

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Maple [A] time = 0.064, size = 23, normalized size = 1.1

$$\frac{e^{n\sinh(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
```

```
[Out] exp(n*sinh(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 0.990072, size = 30, normalized size = 1.36

$$\frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.04636, size = 88, normalized size = 4.

$$\frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh{(ac+bcx)}}\cosh{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*sinh(a*c + b*c*x))*cosh(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left((bx+a)c\right)e^{(n\sinh(bcx+ac))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cosh((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

3.975 $\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Rubi [A] time = 0.0135506, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx,x,\sinh(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\sinh(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.0416347, size = 23, normalized size = 1.

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Maple [A] time = 0.009, size = 23, normalized size = 1.

$$\frac{e^{n\sinh(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
```

```
[Out] exp(n*sinh(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 1.05436, size = 30, normalized size = 1.3

$$\frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.01133, size = 88, normalized size = 3.83

$$\frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

Sympy [A] time = 3.69556, size = 48, normalized size = 2.09

$$\begin{cases} x & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \sinh{(ac)}} \cosh{(ac)} & \text{for } b = 0 \\ \frac{\sinh{(ac+bcx)}}{bc} & \text{for } n = 0 \\ x & \text{for } c = 0 \\ \frac{e^{n \sinh{(ac+bcx)}}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
```

```
[Out] Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sinh(a*c))*cosh(a*c
), Eq(b, 0)), (sinh(a*c + b*c*x)/(b*c), Eq(n, 0)), (x, Eq(c, 0)), (exp(n*si
nh(a*c + b*c*x))/(b*c*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bcx + ac)\, e^{(n \sinh((bx+a)c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

3.976 $\int e^{n \sinh(a+bx)} \coth(a+bx)\, dx$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n\sinh(a+bx))}{b}$$

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0228231, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4340, 2178}

$$\frac{\text{Ei}(n\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\text{Ei}(n\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0334604, size = 13, normalized size = 1.

$$\frac{\text{Ei}(n\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Maple [A] time = 0.01, size = 17, normalized size = 1.3

$$-\frac{\text{Ei}\left(1,-n\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*sinh(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(bx+a\right)e^{\left(n\sinh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

Fricas [A] time = 1.97297, size = 31, normalized size = 2.38

$$\frac{\text{Ei}\left(n\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a + b*x))*coth(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(bx+a)\,e^{(n\sinh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

3.977 $\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\mathrm{Ei}(n\sinh(c(a+bx)))}{bc}$$

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Rubi [A] time = 0.0229849, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4340, 2178}

$$\frac{\mathrm{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx &= \frac{\mathrm{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(c(a+bx))\right)}{bc}\\ &= \frac{\mathrm{Ei}(n\sinh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0619603, size = 18, normalized size = 1.

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.051, size = 23, normalized size = 1.3

$$-\frac{\text{Ei}(1,-n\sinh(bcx+ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

```
[Out] -1/c/b*Ei(1,-n*sinh(b*c*x+a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((bx+a)c)\,e^{(n\sinh(bcx+ac))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

Fricas [A] time = 2.13657, size = 42, normalized size = 2.33

$$\frac{\text{Ei}(n\sinh(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh(ac+bcx)}\coth(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((bx+a)c)\,e^{(n\sinh(bcx+ac))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

3.978 $\int e^{n \sinh(c(a+bx))} \coth(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n\sinh(ac+bxc))}{bc}$$

```
[Out] ExpIntegralEi[n*Sinh[a*c + b*c*x]]/(b*c)
```

Rubi [A] time = 0.0228418, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4340, 2178}

$$\frac{\text{Ei}(n\sinh(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a*c + b*c*x]]/(b*c)
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(ac+bcx)\right)}{bc}\\ &= \frac{\text{Ei}(n\sinh(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0583053, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.042, size = 23, normalized size = 1.2

$$-\frac{\text{Ei}\left(1,-n\sinh\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

```
[Out] -1/c/b*Ei(1,-n*sinh(b*c*x+a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(bcx+ac\right)e^{\left(n\sinh\left((bx+a)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

Fricas [A] time = 2.02444, size = 42, normalized size = 2.21

$$\frac{\text{Ei}\left(n\sinh\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(bcx + ac\right)e^{\left(n\sinh\left((bx+a)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

3.979 $\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tanh(x))}{b}$$

```
[Out] Log[a + b*Tanh[x]]/b
```

Rubi [A] time = 0.0401233, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$\frac{\log(a+b\tanh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] Log[a + b*Tanh[x]]/b
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{\log(a+b\tanh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.055458, size = 20, normalized size = 1.82

$$\frac{\log(a\cosh(x)+b\sinh(x))-\log(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (-Log[Cosh[x]] + Log[a*Cosh[x] + b*Sinh[x]])/b
```

Maple [A] time = 0.023, size = 12, normalized size = 1.1

$$\frac{\ln(a+b\tanh(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*tanh(x)),x)
```

```
[Out] ln(a+b*tanh(x))/b
```

Maxima [A] time = 1.04408, size = 15, normalized size = 1.36

$$\frac{\log(b\tanh(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] log(b*tanh(x) + a)/b
```

Fricas [B] time = 2.06499, size = 126, normalized size = 11.45

$$\frac{\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] (log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x
) - sinh(x))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x)), x)
```

Giac [B] time = 1.17001, size = 61, normalized size = 5.55

$$\frac{(a+b)\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{ab + b^2} - \frac{\log\left(e^{(2x)} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x) +
 1)/b
```

3.980 $\int \frac{\text{sech}^2(x)}{1+\tanh^2(x)} \, dx$

Optimal. Leaf size=3

$$\tan^{-1}(\tanh(x))$$

```
[Out] ArcTan[Tanh[x]]
```

Rubi [A] time = 0.0322492, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)$$
$$= \tan^{-1}(\tanh(x))$$

Mathematica [A] time = 0.0033085, size = 3, normalized size = 1.

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Maple [B] time = 0.038, size = 116, normalized size = 38.7

$$-2\frac{\sqrt{2}}{2+2\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)-2\frac{1}{2+2\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{2+2\sqrt{2}}\right)+2\frac{\sqrt{2}}{-2+2\sqrt{2}}\arctan\left(2\frac{\tanh(x/2)}{-2+2\sqrt{2}}\right)-2\frac{}{-2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+tanh(x)^2),x)
```

```
[Out] -2*2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))-2/(2+2*2^(1/2)
)*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))+2*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tan
h(1/2*x)/(-2+2*2^(1/2)))-2/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2
)))
```

Maxima [B] time = 1.51455, size = 47, normalized size = 15.67

$$\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

Fricas [B] time = 1.9952, size = 69, normalized size = 23.

$$-\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x)**2 + 1), x)
```

Giac [A] time = 1.13488, size = 7, normalized size = 2.33

$$\arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(e^(2*x))
```

3.981 $\int \frac{\mathbf{sech}^2(x)}{9+\tanh^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)$$

```
[Out] ArcTan[Tanh[x]/3]/3
```

Rubi [A] time = 0.0340017, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(9 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]/3]/3
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)$$

Mathematica [F] time = 0.0196953, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(9 + Tanh[x]^2),x]
```

```
[Out] Integrate[Sech[x]^2/(9 + Tanh[x]^2), x]
```

Maple [B] time = 0.071, size = 116, normalized size = 10.6

$$-2\,\frac{\sqrt{10}}{6\sqrt{10}+6}\arctan\left(18\,\frac{\tanh(x/2)}{6\sqrt{10}+6}\right)-2\,\frac{1}{6\sqrt{10}+6}\arctan\left(18\,\frac{\tanh(x/2)}{6\sqrt{10}+6}\right)+2\,\frac{\sqrt{10}}{6\sqrt{10}-6}\arctan\left(18\,\frac{\tanh(x/2)}{6\sqrt{10}-6}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(9+tanh(x)^2),x)
```

```
[Out] -2*10^(1/2)/(6*10^(1/2)+6)*arctan(18*tanh(1/2*x)/(6*10^(1/2)+6))-2/(6*10^(1
/2)+6)*arctan(18*tanh(1/2*x)/(6*10^(1/2)+6))+2*10^(1/2)/(6*10^(1/2)-6)*arct
an(18*tanh(1/2*x)/(6*10^(1/2)-6))-2/(6*10^(1/2)-6)*arctan(18*tanh(1/2*x)/(6
*10^(1/2)-6))
```

Maxima [A] time = 1.57431, size = 15, normalized size = 1.36

$$-\frac{1}{3}\arctan\left(\frac{5}{3}\,e^{(-2\,x)}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(5/3*e^(-2*x) + 4/3)
```

Fricas [B] time = 2.33005, size = 82, normalized size = 7.45

$$-\frac{1}{3}\arctan\left(-\frac{9\cosh\left(x\right)+\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(-1/3*(9*cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\tanh^{2}\left(x\right)+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(9+tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x)**2 + 9), x)
```

Giac [A] time = 1.1588, size = 15, normalized size = 1.36

$$\frac{1}{3}\arctan\left(\frac{5}{3}e^{\left(2x\right)}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(5/3*e^(2*x) + 4/3)
```

3.982 $\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

```
[Out] (a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0432321, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

```
[Out] (a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^n\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{(a+b\tanh(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.156339, size = 18, normalized size = 0.95

$$\frac{(a+b\tanh(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

```
[Out] (a + b*Tanh[x])^(1 + n)/(b + b*n)
```

Maple [A] time = 0.018, size = 20, normalized size = 1.1

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^n,x)
```

```
[Out] (a+b*tanh(x))^(n+1)/b/(n+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.44807, size = 220, normalized size = 11.58

$$\frac{(a\cosh(x)+b\sinh(x))\cosh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)+(a\cosh(x)+b\sinh(x))\sinh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)}{(bn+b)\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="fricas")
```

```
[Out] ((a*cosh(x) + b*sinh(x))*cosh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))) + (a*
cosh(x) + b*sinh(x))*sinh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))))/((b*n +
b)*cosh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh(x) + a)^n \operatorname{sech}(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(x) + a)^n*sech(x)^2, x)
```

3.983 $\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

```
[Out] x + Tanh[x]
```

Rubi [A] time = 0.0523521, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {206}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

```
[Out] x + Tanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)dx &= \operatorname{Subst}\left(\int \left(1+\frac{1}{1-x^2}\right)dx, x, \tanh(x)\right)\\ &= \tanh(x)+\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= x+\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0063401, size = 4, normalized size = 1.

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

```
[Out] x + Tanh[x]
```

Maple [B] time = 0.032, size = 34, normalized size = 8.5

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{\tanh\left(x/2\right)}{\left(\tanh\left(x/2\right)\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+1/(1-tanh(x)^2)),x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [B] time = 1.02392, size = 16, normalized size = 4.

$$x+\frac{2}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="maxima")
```

```
[Out] x + 2/(e^(-2*x) + 1)
```

Fricas [B] time = 2.31436, size = 50, normalized size = 12.5

$$\frac{(x-1)\cosh\left(x\right)+\sinh\left(x\right)}{\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [B] time = 1.48378, size = 29, normalized size = 7.25

$$-\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1}-\frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+1/(1-tanh(x)**2)),x)
```

```
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

Giac [B] time = 1.15623, size = 16, normalized size = 4.

$$x-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="giac")
```

```
[Out] x - 2/(e^(2*x) + 1)
```

3.984 $\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

```
[Out] x + Tanh[x]
```

Rubi [A] time = 0.0742743, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3657, 3473, 8}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
```

```
[Out] x + Tanh[x]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = \int \left(2-\tanh^2(x)\right)\,dx$$
$$= 2x - \int \tanh^2(x)\,dx$$
$$= 2x + \tanh(x) - \int 1\,dx$$
$$= x + \tanh(x)$$

Mathematica [A] time = 0.0031483, size = 4, normalized size = 1.

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
```

```
[Out] x + Tanh[x]
```

Maple [B] time = 0.033, size = 34, normalized size = 8.5

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{\tanh\left(x/2\right)}{\left(\tanh\left(x/2\right)\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [B] time = 1.06472, size = 16, normalized size = 4.

$$x+\frac{2}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x + 2/(e^(-2*x) + 1)
```

Fricas [B] time = 2.10975, size = 50, normalized size = 12.5

$$\frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [B] time = 1.42465, size = 29, normalized size = 7.25

$$-\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1}-\frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(2-tanh(x)**2)/(1-tanh(x)**2),x)
```

```
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

Giac [B] time = 1.14009, size = 16, normalized size = 4.

$$x-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="giac")
```

```
[Out] x - 2/(e^(2*x) + 1)
```

3.985 $\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$

Optimal. Leaf size=5

$$\tan^{-1}(\tanh(x)+1)$$

```
[Out] ArcTan[1 + Tanh[x]]
```

Rubi [A] time = 0.0491681, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4342, 617, 204}

$$\tan^{-1}(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
```

```
[Out] ArcTan[1 + Tanh[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tanh(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tanh(x)\right)$$
$$= \tan^{-1}(1+\tanh(x))$$

Mathematica [F] time = 0.0386977, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
```

```
[Out] Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2), x]
```

Maple [C] time = 0.059, size = 42, normalized size = 8.4

$$\frac{i}{2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+(1-i)\tanh\left(\frac{x}{2}\right)+1\right)-\frac{i}{2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+(1+i)\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x)
```

```
[Out] 1/2*I*ln(tanh(1/2*x)^2+(1-I)*tanh(1/2*x)+1)-1/2*I*ln(tanh(1/2*x)^2+(1+I)*ta
nh(1/2*x)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{\tanh(x)^2+2\tanh(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(tanh(x)^2 + 2*tanh(x) + 2), x)
```

Fricas [B] time = 2.0324, size = 74, normalized size = 14.8

$$-\arctan\left(-\frac{3\cosh\left(x\right)+2\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(-(3*cosh(x) + 2*sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\tanh^{2}\left(x\right)+2\tanh\left(x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(2+2*tanh(x)+tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x)**2 + 2*tanh(x) + 2), x)
```

Giac [A] time = 1.16817, size = 12, normalized size = 2.4

$$\arctan\left(\frac{5}{2}\,e^{(2\,x)}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(5/2*e^(2*x) + 1/2)
```

3.986 $\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx$

Optimal. Leaf size=15

$$-\coth(x) - \log(\tanh(x)) + \log(\tanh(x) + 1)$$

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rubi [A] time = 0.0530338, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4342, 44}

$$-\coth(x) - \log(\tanh(x)) + \log(\tanh(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \tanh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \tanh(x)\right)$$
$$= -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))$$

Mathematica [A] time = 0.0302431, size = 11, normalized size = 0.73

$$x-\coth(x)-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] x - Coth[x] - Log[Sinh[x]]
```

Maple [B] time = 0.049, size = 32, normalized size = 2.1

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)+2\,\ln\left(\tanh\left(x/2\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(tanh(x)^2+tanh(x)^3),x)
```

```
[Out] -1/2*tanh(1/2*x)+2*ln(tanh(1/2*x)+1)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Maxima [A] time = 1.0052, size = 39, normalized size = 2.6

$$\frac{2}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.06623, size = 267, normalized size = 17.8

$$\frac{2 x \cosh(x)^2 + 4 x \cosh(x)\sinh(x) + 2 x \sinh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{(\tanh(x)+1)\tanh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(tanh(x)**2+tanh(x)**3),x)
```

```
[Out] Integral(sech(x)**2/((tanh(x) + 1)*tanh(x)**2), x)
```

Giac [A] time = 1.15948, size = 39, normalized size = 2.6

$$2x + \frac{e^{(2x)} - 3}{e^{(2x)} - 1} - \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.987 $\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx$

Optimal. Leaf size=15

$$\coth(x) + \log(1 - \tanh(x)) - \log(\tanh(x))$$

```
[Out] Coth[x] + Log[1 - Tanh[x]] - Log[Tanh[x]]
```

Rubi [A] time = 0.0576355, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4342, 44}

$$\coth(x) + \log(1 - \tanh(x)) - \log(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] Coth[x] + Log[1 - Tanh[x]] - Log[Tanh[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx, x, \tanh(x)\right)$$
$$= \coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

Mathematica [A] time = 0.0239386, size = 11, normalized size = 0.73

$$-x+\coth(x)-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] -x + Coth[x] - Log[Sinh[x]]
```

Maple [B] time = 0.05, size = 32, normalized size = 2.1

$$\frac{1}{2}\tanh\left(\frac{x}{2}\right)+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\,\ln\left(\tanh\left(x/2\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x)
```

```
[Out] 1/2*tanh(1/2*x)+1/2/tanh(1/2*x)-ln(tanh(1/2*x))+2*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.03875, size = 43, normalized size = 2.87

$$-2\,x-\frac{2}{e^{(-2\,x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] -2*x - 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.00154, size = 188, normalized size = 12.53

$$-\frac{\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)-2}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
sinh(x))) - 2)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\left(\tanh\left(x\right)-1\right)\tanh^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(-tanh(x)**2+tanh(x)**3),x)
```

```
[Out] Integral(sech(x)**2/((tanh(x) - 1)*tanh(x)**2), x)
```

Giac [A] time = 1.20419, size = 35, normalized size = 2.33

$$\frac{e^{(2\,x)}+1}{e^{(2\,x)}-1}-\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

```
[Out] (e^(2*x) + 1)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.988 $\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

Optimal. Leaf size=102

$$\frac{\log\left(2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}+\frac{\tan^{-1}\left(\frac{2\ 2^{2/3}\tanh(x)+\sqrt[3]{3}}{3^{5/6}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

```
[Out] ArcTan[(3^(1/3) + 2*2^(2/3)*Tanh[x])/3^(5/6)]/(3*2^(2/3)*3^(1/6)) - Log[3^(
1/3) - 2^(2/3)*Tanh[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tanh[x]
 + 2*2^(1/3)*Tanh[x]^2]/(6*6^(2/3))
```

Rubi [A] time = 0.112152, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3675, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}+\frac{\tan^{-1}\left(\frac{2\ 2^{2/3}\tanh(x)+\sqrt[3]{3}}{3^{5/6}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```


```
[Out] ArcTan[(3^(1/3) + 2*2^(2/3)*Tanh[x])/3^(5/6)]/(3*2^(2/3)*3^(1/6)) - Log[3^(
1/3) - 2^(2/3)*Tanh[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tanh[x]
 + 2*2^(1/3)*Tanh[x]^2]/(6*6^(2/3))
```


Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\mathrm{sech}^2(x)}{3-4\tanh^3(x)}\,dx = \mathrm{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tanh(x)\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}} + \frac{\mathrm{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{2\sqrt[3]{3}} + \frac{\mathrm{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{3}+4\sqrt[3]{\ldots}}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+\ldots}\ldots\right)}{6\ 6\ldots}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}} - \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\ldots\right)}{6^{2/3}\ldots}$$
$$= \frac{\tan^{-1}\left(\frac{3+2\ 6^{2/3}\tanh(x)}{3\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}\ldots}$$

Mathematica [A] time = 0.105728, size = 74, normalized size = 0.73

$$\frac{\log\left(2\sqrt[3]{6}\tanh^2(x)+6^{2/3}\tanh(x)+3\right)-2\log\left(3-6^{2/3}\tanh(x)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\ 6^{2/3}\tanh(x)+3}{3\sqrt{3}}\right)}{6\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tanh[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*
Tanh[x]] + Log[3 + 6^(2/3)*Tanh[x] + 2*6^(1/3)*Tanh[x]^2])/(6*6^(2/3))
```

Maple [C] time = 0.052, size = 34, normalized size = 0.3

$$\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(36\_Z^3+1\right)}\_R\ \ln\left(-24\ \tanh(x/2)\_R^2+\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(3-4*tanh(x)^3),x)
```

```
[Out] 1/3*sum(_R*ln(-24*tanh(1/2*x)*_R^2+tanh(1/2*x)^2+1),_R=RootOf(36*_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{sech}(x)^2}{4 \tanh(x)^3 - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="maxima")
```

```
[Out] -integrate(sech(x)^2/(4*tanh(x)^3 - 3), x)
```

Fricas [B] time = 2.1398, size = 1177, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/18*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/54*36^(1/6)*((36^(2/3)*sqrt(3)*(
-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)^2 + 2*(36^(2
/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)
*sinh(x) + (36^(2/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-
1)^(1/3))*sinh(x)^2 - 36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3)))
- 1/216*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(
2/3) + 3)*cosh(x)^2 - 2*(36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3))*cosh(
x)*sinh(x) + (36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) + 3)*sinh(x)^2 -
36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) - 21)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 1/108*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3)
 - 3*36^(1/3)*(-1)^(2/3) - 9)*cosh(x) - (36^(2/3)*(-1)^(1/3) - 3*36^(1/3)*(
-1)^(2/3) - 12)*sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{sech}^{2}(x)}{4\tanh^{3}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(3-4*tanh(x)**3),x)
```

```
[Out] -Integral(sech(x)**2/(4*tanh(x)**3 - 3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.989 $\int \frac{\text{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$

Optimal. Leaf size=22

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

```
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]
```

Rubi [A] time = 0.0680059, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4342, 618, 204}

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
```

```
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx, x, \tanh(x)\right)\\ &= -\left(2\operatorname{Subst}\left(\int \frac{1}{-195-x^2}\,dx, x, -5+10\tanh(x)\right)\right)\\ &= -\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}\end{aligned}$$

Mathematica [F] time = 0.0307312, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
```

```
[Out] Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2), x]
```

Maple [C] time = 0.069, size = 62, normalized size = 2.8

$$\frac{i}{195}\sqrt{195}\ln\left(11\,(\tanh(x/2))^2+\left(-i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)-\frac{i}{195}\sqrt{195}\ln\left(11\,(\tanh(x/2))^2+\left(i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x)
```

```
[Out] 1/195*I*195^(1/2)*ln(11*tanh(1/2*x)^2+(-I*195^(1/2)-5)*tanh(1/2*x)+11)-1/19
5*I*195^(1/2)*ln(11*tanh(1/2*x)^2+(I*195^(1/2)-5)*tanh(1/2*x)+11)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{5 \tanh(x)^2 - 5 \tanh(x) + 11} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(5*tanh(x)^2 - 5*tanh(x) + 11), x)
```

Fricas [A] time = 2.09234, size = 132, normalized size = 6.

$$-\frac{2}{195} \sqrt{195} \arctan\left(-\frac{17 \sqrt{195} \cosh(x) + 5 \sqrt{195} \sinh(x)}{195 (\cosh(x) - \sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -2/195*sqrt(195)*arctan(-1/195*(17*sqrt(195)*cosh(x) + 5*sqrt(195)*sinh(x))
/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{5 \tanh^2(x) - 5 \tanh(x) + 11} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(11-5*tanh(x)+5*tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(5*tanh(x)**2 - 5*tanh(x) + 11), x)
```

Giac [A] time = 1.15518, size = 26, normalized size = 1.18

$$\frac{2}{195} \sqrt{195} \arctan\left(\frac{1}{195} \sqrt{195} \left(11 e^{(2x)} + 6\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="giac")
```

```
[Out] 2/195*sqrt(195)*arctan(1/195*sqrt(195)*(11*e^(2*x) + 6))
```

3.990 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=28

$$\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

```
[Out] -(((b*c - a*d)*Log[c + d*Tanh[x]])/d^2) + (b*Tanh[x])/d
```

Rubi [A] time = 0.0988653, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4342, 43}

$$\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*Tanh[x]])/d^2) + (b*Tanh[x])/d
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \operatorname{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \tanh(x)\right)$$
$$= -\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}+\frac{b\tanh(x)}{d}$$

Mathematica [A] time = 0.328921, size = 54, normalized size = 1.93

$$\frac{\cosh(x)(a+b\tanh(x))((bc-ad)(\log(\cosh(x))-\log(c\cosh(x)+d\sinh(x)))+bd\tanh(x))}{d^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

```
[Out] (Cosh[x]*(a + b*Tanh[x])*((b*c - a*d)*(Log[Cosh[x]] - Log[c*Cosh[x] + d*Sin
h[x]]) + b*d*Tanh[x]))/(d^2*(a*Cosh[x] + b*Sinh[x]))
```

Maple [B] time = 0.042, size = 100, normalized size = 3.6

$$2\,\frac{\tanh(x/2)\,b}{d\left((\tanh(x/2))^2+1\right)}-\frac{a}{d}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+\frac{cb}{d^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+\frac{a}{d}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2c+2\,\tanh(x/2)\,d+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

```
[Out] 2/d*tanh(1/2*x)*b/(tanh(1/2*x)^2+1)-1/d*ln(tanh(1/2*x)^2+1)*a+1/d^2*ln(tanh
(1/2*x)^2+1)*c*b+1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a-1/d^2*ln(tanh(
1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c*b
```

Maxima [B] time = 1.56624, size = 89, normalized size = 3.18

$$-b\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right)+\frac{a\log(d\tanh(x)+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*(c*log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e
^(-2*x) + d)) + a*log(d*tanh(x) + c)/d
```

Fricas [B] time = 2.26192, size = 467, normalized size = 16.68

$$-\frac{2\,bd + \left((bc-ad)\cosh\left(x\right)^{2} + 2\,(bc-ad)\cosh\left(x\right)\sinh\left(x\right) + (bc-ad)\sinh\left(x\right)^{2} + bc - ad\right)\log\left(\frac{2\,(c\cosh(x)+d\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \left(\left(\right.\right.}{d^{2}\cosh\left(x\right)^{2} + 2\,d^{2}\cosh\left(x\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(2*b*d + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a
*d)*sinh(x)^2 + b*c - a*d)*log(2*(c*cosh(x) + d*sinh(x))/(cosh(x) - sinh(x)
)) - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*s
inh(x)^2 + b*c - a*d)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 +
2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 + d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\tanh\left(x\right))\operatorname{sech}^{2}\left(x\right)}{c+d\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))*sech(x)**2/(c + d*tanh(x)), x)
```

Giac [B] time = 1.15344, size = 153, normalized size = 5.46

$$-\frac{\left(bc^{2}-acd+bcd-ad^{2}\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}+c-d\right|\right)}{cd^{2}+d^{3}} + \frac{(bc-ad)\log\left(e^{(2\,x)}+1\right)}{d^{2}} - \frac{bce^{(2\,x)}-ade^{(2\,x)}+bc-ad+2\,bd}{d^{2}\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c
*d^2 + d^3) + (b*c - a*d)*log(e^(2*x) + 1)/d^2 - (b*c*e^(2*x) - a*d*e^(2*x)
 + b*c - a*d + 2*b*d)/(d^2*(e^(2*x) + 1))
```

3.991 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{b\tanh(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}+\frac{(a+b\tanh(x))^2}{2d}$$

```
[Out] ((b*c - a*d)^2*Log[c + d*Tanh[x]])/d^3 - (b*(b*c - a*d)*Tanh[x])/d^2 + (a +
 b*Tanh[x])^2/(2*d)
```

Rubi [A] time = 0.157541, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$-\frac{b\tanh(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}+\frac{(a+b\tanh(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tanh[x]])/d^3 - (b*(b*c - a*d)*Tanh[x])/d^2 + (a +
 b*Tanh[x])^2/(2*d)
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \tanh(x)\right)$$
$$= \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}-\frac{b(bc-ad)\tanh(x)}{d^2}+\frac{(a+b\tanh(x))^2}{2d}$$

Mathematica [A] time = 0.565627, size = 61, normalized size = 1.15

$$-\frac{2bd\tanh(x)(bc-2ad)+2(bc-ad)^2(\log(\cosh(x))-\log(c\cosh(x)+d\sinh(x)))+b^2d^2\operatorname{sech}^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

```
[Out] -(2*(b*c - a*d)^2*(Log[Cosh[x]] - Log[c*Cosh[x] + d*Sinh[x]]) + b^2*d^2*Sec
h[x]^2 + 2*b*d*(b*c - 2*a*d)*Tanh[x])/(2*d^3)
```

Maple [B] time = 0.063, size = 251, normalized size = 4.7

$$4\,\frac{(\tanh(x/2))^3ab}{d\left((\tanh(x/2))^2+1\right)^2}-2\,\frac{(\tanh(x/2))^3b^2c}{d^2\left((\tanh(x/2))^2+1\right)^2}+2\,\frac{b^2(\tanh(x/2))^2}{d\left((\tanh(x/2))^2+1\right)^2}+4\,\frac{a\tanh(x/2)\,b}{d\left((\tanh(x/2))^2+1\right)^2}-2\,\frac{\tanh(x}{d^2\left((\tanh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x)
```

```
[Out] 4/d/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a*b-2/d^2/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^3*b^2*c+2/d/(tanh(1/2*x)^2+1)^2*b^2*tanh(1/2*x)^2+4/d/(tanh(1/2*x)^2+1
)^2*tanh(1/2*x)*a*b-2/d^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^2*c-1/d*ln(tanh
(1/2*x)^2+1)*a^2+2/d^2*ln(tanh(1/2*x)^2+1)*c*b*a-1/d^3*ln(tanh(1/2*x)^2+1)*
c^2*b^2+1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a^2-2/d^2*ln(tanh(1/2*x)^
2*c+2*tanh(1/2*x)*d+c)*c*b*a+1/d^3*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c^
2*b^2
```

Maxima [B] time = 1.58313, size = 204, normalized size = 3.85

$$-b^2\left(\frac{2\left((c+d)e^{(-2x)}+c\right)}{2d^2e^{(-2x)}+d^2e^{(-4x)}+d^2}-\frac{c^2\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^3}+\frac{c^2\log\left(e^{(-2x)}+1\right)}{d^3}\right)-2ab\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^2*(2*((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*
log(-(c - d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 2*a*b*(c*
log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x
) + d)) + a^2*log(d*tanh(x) + c)/d
```

Fricas [B] time = 2.37237, size = 1667, normalized size = 31.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] (2*b^2*c*d - 4*a*b*d^2 + 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 + 4*(b^2
*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) + 2*(b^2*c*d - (2*a*b + b^2)*d^2)
*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(x
)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos
h(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 + (b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x
))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*c
osh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*cosh(x)/
(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(x
)^4 + 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 + d^3)*sinh(x)^2 + 4*(d^3*
cosh(x)^3 + d^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\tanh(x))^2\operatorname{sech}^2(x)}{c+d\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**2/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))**2*sech(x)**2/(c + d*tanh(x)), x)
```

Giac [B] time = 1.15526, size = 356, normalized size = 6.72

$$\frac{\left(b^2c^3-2\,abc^2d+b^2c^2d+a^2cd^2-2\,abcd^2+a^2d^3\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}+c-d\right|\right)}{cd^3+d^4}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(e^{(2\,x)}+1\right)}{d^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] (b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*log
(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c*d^3 + d^4) - (b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*log(e^(2*x) + 1)/d^3 + 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e^(4*x)
 + 3*a^2*d^2*e^(4*x) + 6*b^2*c^2*e^(2*x) - 12*a*b*c*d*e^(2*x) + 4*b^2*c*d*e
^(2*x) + 6*a^2*d^2*e^(2*x) - 8*a*b*d^2*e^(2*x) - 4*b^2*d^2*e^(2*x) + 3*b^2*
c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) + 1)^2)
```

3.992 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{b\tanh(x)(bc-ad)^2}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2} - \frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4} + \frac{(a+b\tanh(x))^3}{3d}$$

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tanh[x]])/d^4) + (b*(b*c - a*d)^2*Tanh[x])/d^3 -
 ((b*c - a*d)*(a + b*Tanh[x])^2)/(2*d^2) + (a + b*Tanh[x])^3/(3*d)
```

Rubi [A] time = 0.162722, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$\frac{b\tanh(x)(bc-ad)^2}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2} - \frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4} + \frac{(a+b\tanh(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tanh[x]])/d^4) + (b*(b*c - a*d)^2*Tanh[x])/d^3 -
 ((b*c - a*d)*(a + b*Tanh[x])^2)/(2*d^2) + (a + b*Tanh[x])^3/(3*d)
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right) dx, x, \tanh$$
$$= -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4} + \frac{b(bc-ad)^2\tanh(x)}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2} + $$

Mathematica [A] time = 0.785405, size = 134, normalized size = 1.72

$$\frac{(a+b\tanh(x))^3(c\cosh(x)+d\sinh(x))\left(bd^2\left(\sinh(2x)\left(9a^2d-9abc+b^2d\right)+b(-9ad+3bc-2bd\tanh(x))\right)+6\cosh^2(x)}{6d^4(c+d\tanh(x))(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] ((c*Cosh[x] + d*Sinh[x])*(a + b*Tanh[x])^3*(6*(b*c - a*d)^3*Cosh[x]^2*(Log[
Cosh[x]] - Log[c*Cosh[x] + d*Sinh[x]]) + 6*b^3*c^2*d*Cosh[x]*Sinh[x] + b*d^
2*((-9*a*b*c + 9*a^2*d + b^2*d)*Sinh[2*x] + b*(3*b*c - 9*a*d - 2*b*d*Tanh[x
]))))/(6*d^4*(a*Cosh[x] + b*Sinh[x])^3*(c + d*Tanh[x]))
```

Maple [B] time = 0.082, size = 542, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x)
```

```
[Out] 6/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*a^2*b-6/d^2/(tanh(1/2*x)^2+1)^3*tanh(
1/2*x)^5*a*b^2*c+2/d^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^3*c^2+6/d/(tanh(
1/2*x)^2+1)^3*tanh(1/2*x)^4*a*b^2-2/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^4*b
^3*c+12/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*a^2*b-12/d^2/(tanh(1/2*x)^2+1)^
3*tanh(1/2*x)^3*a*b^2*c+4/d^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^3*c^2+8/3
/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^3+6/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x
)^2*a*b^2-2/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^2*b^3*c+6/d/(tanh(1/2*x)^2+
1)^3*tanh(1/2*x)*a^2*b-6/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*a*b^2*c+2/d^3/
(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^3*c^2-1/d*ln(tanh(1/2*x)^2+1)*a^3+3/d^2*l
```

```
n(tanh(1/2*x)^2+1)*a^2*b*c-3/d^3*ln(tanh(1/2*x)^2+1)*c^2*b^2*a+1/d^4*ln(tan
h(1/2*x)^2+1)*c^3*b^3+1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a^3-3/d^2*l
n(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a^2*b*c+3/d^3*ln(tanh(1/2*x)^2*c+2*tan
h(1/2*x)*d+c)*c^2*b^2*a-1/d^4*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c^3*b^3
```

Maxima [B] time = 1.63636, size = 373, normalized size = 4.78

$$\frac{1}{3}b^3\left(\frac{2\left(3c^2+d^2+3\left(2c^2+cd\right)e^{(-2x)}+3\left(c^2+cd+d^2\right)e^{(-4x)}\right)}{3d^3e^{(-2x)}+3d^3e^{(-4x)}+d^3e^{(-6x)}+d^3}-\frac{3c^3\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^4}+\frac{3c^3\log\left(e^{(-2x)}+1\right)}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 + 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) + 3*d^3*e^(-4*x) + d^3*e^(-6*x) + d^3) - 3*c^3*log(
-(c - d)*e^(-2*x) - c - d)/d^4 + 3*c^3*log(e^(-2*x) + 1)/d^4) - 3*a*b^2*(2*
((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*log(-(c
- d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 3*a^2*b*(c*log(-(
c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x) + d)
) + a^3*log(d*tanh(x) + c)/d
```

Fricas [B] time = 2.70859, size = 4383, normalized size = 56.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 +
 (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)
*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d -
(3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2
*b + b^3)*d^3 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2
)*d^3)*cosh(x)^2 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*
b^2)*d^3 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)
*d^3)*cosh(x)^2)*sinh(x)^2 + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
```

```
cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sin
h(x)^6 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x))/
(cosh(x) - sinh(x))) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 + (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 + 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 + d^4)*sinh(x)^4
+ d^4 + 4*(5*d^4*cosh(x)^3 + 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
+ 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 + 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**3/(c+d*tanh(x)),x)
```

[Out] Timed out

Giac [B] time = 1.18165, size = 733, normalized size = 9.4

$$-\frac{\left(b^3c^4 - 3\,ab^2c^3d + b^3c^3d + 3\,a^2bc^2d^2 - 3\,ab^2c^2d^2 - a^3cd^3 + 3\,a^2bcd^3 - a^3d^4\right)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} + c - d\right|\right)}{cd^4 + d^5} + \frac{\left(b^3c^3 - 3\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
 a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d
))/(c*d^4 + d^5) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
e^(2*x) + 1)/d^4 - 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 33*a^
2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) + 33*b^3*c^3*e^(4*x) - 99*a*b^2*c^2*
d*e^(4*x) + 12*b^3*c^2*d*e^(4*x) + 99*a^2*b*c*d^2*e^(4*x) - 36*a*b^2*c*d^2*
e^(4*x) - 12*b^3*c*d^2*e^(4*x) - 33*a^3*d^3*e^(4*x) + 36*a^2*b*d^3*e^(4*x)
+ 36*a*b^2*d^3*e^(4*x) + 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a*b^2
*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b^2*c
*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3*e^(
2*x) + 36*a*b^2*d^3*e^(2*x) + 11*b^3*c^3 - 33*a*b^2*c^2*d + 12*b^3*c^2*d +
33*a^2*b*c*d^2 - 36*a*b^2*c*d^2 - 11*a^3*d^3 + 36*a^2*b*d^3 + 4*b^3*d^3)/(d
^4*(e^(2*x) + 1)^3)
```

3.993 $\int \frac{\text{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

[Out] -1/(3*(2 + Tanh[x]^3))

Rubi [A] time = 0.0857805, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4342, 261}

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

Antiderivative was successfully verified.

[In] Int[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]

[Out] -1/(3*(2 + Tanh[x]^3))

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = \operatorname{Subst}\left(\int \frac{x^2}{\left(2+x^3\right)^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3\left(2+\tanh^3(x)\right)}$$

Mathematica [A] time = 0.0418217, size = 12, normalized size = 1.

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

```
[Out] -1/(3*(2 + Tanh[x]^3))
```

Maple [A] time = 0.041, size = 11, normalized size = 0.9

$$-\frac{1}{6+3\left(\tanh(x)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x)
```

```
[Out] -1/3/(2+tanh(x)^3)
```

Maxima [A] time = 0.991249, size = 14, normalized size = 1.17

$$-\frac{1}{3\left(\tanh(x)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(tanh(x)^3 + 2)
```

Fricas [B] time = 1.99299, size = 250, normalized size = 20.83

$$-\frac{8\left(\cosh\left(x\right)^{2}+\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}{9\left(3\,\cosh\left(x\right)^{4}+12\,\cosh\left(x\right)\sinh\left(x\right)^{3}+3\,\sinh\left(x\right)^{4}+2\left(9\,\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)^{2}+4\left(3\,\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="fricas")
```

```
[Out] -8/9*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(3*cosh(x)^4 + 12*cosh(x)*si
nh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(3*
cosh(x)^3 + cosh(x))*sinh(x) + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*tanh(x)**2/(2+tanh(x)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51189, size = 43, normalized size = 3.58

$$-\frac{2\left(3\,e^{(4\,x)}+1\right)}{9\left(3\,e^{(6\,x)}+3\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="giac")
```

```
[Out] -2/9*(3*e^(4*x) + 1)/(3*e^(6*x) + 3*e^(4*x) + 9*e^(2*x) + 1)
```

3.994 $\int \mathbf{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rubi [A] time = 0.107892, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3657, 2607, 270}

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx=\int \operatorname{sech}^8(x)\tanh^6(x)\,dx$$
$$=i\,\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx,x,i\tanh(x)\right)$$
$$=i\,\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx,x,i\tanh(x)\right)$$
$$=\frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

Mathematica [B] time = 0.0277655, size = 67, normalized size = 2.03

$$\frac{16\tanh(x)}{3003}-\frac{1}{13}\tanh(x)\operatorname{sech}^{12}(x)+\frac{27}{143}\tanh(x)\operatorname{sech}^{10}(x)-\frac{53}{429}\tanh(x)\operatorname{sech}^8(x)+\frac{5\tanh(x)\operatorname{sech}^6(x)}{3003}+\frac{2\tanh(x)\mathrm{s}}{100}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

Maple [B] time = 0.088, size = 306, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x)
```

```
[Out] -1/2*sinh(x)^5/cosh(x)^7-5/8*sinh(x)^3/cosh(x)^7-5/16*sinh(x)/cosh(x)^7+5/1
6*(16/35+1/7*sech(x)^6+6/35*sech(x)^4+8/35*sech(x)^2)*tanh(x)+3/2*sinh(x)^7
/cosh(x)^9+21/8*sinh(x)^5/cosh(x)^9+35/16*sinh(x)^3/cosh(x)^9+105/128*sinh(
x)/cosh(x)^9-105/128*(128/315+1/9*sech(x)^8+8/63*sech(x)^6+16/105*sech(x)^4
+64/315*sech(x)^2)*tanh(x)-3/2*sinh(x)^9/cosh(x)^11-27/8*sinh(x)^7/cosh(x)^
11-63/16*sinh(x)^5/cosh(x)^11-315/128*sinh(x)^3/cosh(x)^11-189/256*sinh(x)/
cosh(x)^11+189/256*(256/693+1/11*sech(x)^10+10/99*sech(x)^8+80/693*sech(x)^
6+32/231*sech(x)^4+128/693*sech(x)^2)*tanh(x)+1/2*sinh(x)^11/cosh(x)^13+11/
```

```
8*sinh(x)^9/cosh(x)^13+33/16*sinh(x)^7/cosh(x)^13+231/128*sinh(x)^5/cosh(x)
^13+231/256*sinh(x)^3/cosh(x)^13+231/1024*sinh(x)/cosh(x)^13-231/1024*(1024
/3003+1/13*sech(x)^12+12/143*sech(x)^10+40/429*sech(x)^8+320/3003*sech(x)^6
+128/1001*sech(x)^4+512/3003*sech(x)^2)*tanh(x)
```

Maxima [A] time = 1.02054, size = 34, normalized size = 1.03

$$-\frac{1}{13}\,\tanh\left(x\right)^{13}+\frac{3}{11}\,\tanh\left(x\right)^{11}-\frac{1}{3}\,\tanh\left(x\right)^{9}+\frac{1}{7}\,\tanh\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/13*tanh(x)^13 + 3/11*tanh(x)^11 - 1/3*tanh(x)^9 + 1/7*tanh(x)^7
```

Fricas [B] time = 2.03391, size = 3032, normalized size = 91.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
```

```
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*tanh(x)**6*(1-tanh(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21079, size = 89, normalized size = 2.7

$$-\frac{32\left(3003\,e^{(18\,x)}-9009\,e^{(16\,x)}+18018\,e^{(14\,x)}-16302\,e^{(12\,x)}+10296\,e^{(10\,x)}-2288\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+1\right)}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

3.995 $\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx$

Optimal. Leaf size=26

$$\log(\tanh(x)+1)-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tanh[x]]
```

Rubi [A] time = 0.0906898, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4342, 1863, 31, 618, 204}

$$\log(\tanh(x)+1)-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tanh[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx &= \operatorname{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tanh(x)\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tanh(x)\right)+\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\ &= \log(1+\tanh(x))-2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+\tanh(x))\end{aligned}$$

Mathematica [A] time = 0.218905, size = 27, normalized size = 1.04

$$x+\frac{2\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
```

```
[Out] x + (2*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]])/Sqrt[3] - Log[Cosh[x]]
```

Maple [C] time = 0.075, size = 78, normalized size = 3.

$$2\ln(\tanh(x/2)+1)+\frac{i}{3}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)-\frac{i}{3}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x)
```

```
[Out] 2*ln(tanh(1/2*x)+1)+1/3*I*3^(1/2)*ln(tanh(1/2*x)^2+(-I*3^(1/2)-1)*tanh(1/2*
x)+1)-1/3*I*3^(1/2)*ln(tanh(1/2*x)^2+(I*3^(1/2)-1)*tanh(1/2*x)+1)-ln(tanh(1
/2*x)^2+1)
```

Maxima [B] time = 1.62797, size = 165, normalized size = 6.35

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{3}\log\left(\tanh(x)^3+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/3*log(tanh(x)^3 + 1) - 1/3*log(3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e
^(-2*x) + 1) - 1/3*log(-3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e^(-2*x) + 1)
```

Fricas [B] time = 2.441, size = 170, normalized size = 6.54

$$-\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+2x-\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sin
h(x))) + 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\tanh^2(x)+2\right)\operatorname{sech}^2(x)}{(\tanh(x)+1)\left(\tanh^2(x)-\tanh(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(2+tanh(x)**2)/(1+tanh(x)**3),x)
```

```
[Out] Integral((tanh(x)**2 + 2)*sech(x)**2/((tanh(x) + 1)*(tanh(x)**2 - tanh(x) +
 1)), x)
```

Giac [A] time = 1.22031, size = 38, normalized size = 1.46

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+2x-\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 2*x - log(e^(2*x) + 1)
```

3.996 $\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

```
[Out] x + Tanh[x]
```

Rubi [A] time = 0.0196341, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

```
[Out] x + Tanh[x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx &= \tanh(x)+\int 1\, dx\\ &= x+\tanh(x) \end{aligned}$$

Mathematica [A] time = 0.0022229, size = 4, normalized size = 1.

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

```
[Out] x + Tanh[x]
```

Maple [A] time = 0.014, size = 5, normalized size = 1.3

$$x + \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)*sech(x)^2,x)
```

```
[Out] x+tanh(x)
```

Maxima [B] time = 1.0527, size = 16, normalized size = 4.

$$x + \frac{2}{e^{(-2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="maxima")
```

```
[Out] x + 2/(e^(-2*x) + 1)
```

Fricas [B] time = 2.17168, size = 50, normalized size = 12.5

$$\frac{(x - 1)\cosh(x) + \sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\cosh^2(x)+1\right)\operatorname{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)*sech(x)**2,x)
```

```
[Out] Integral((cosh(x)**2 + 1)*sech(x)**2, x)
```

Giac [B] time = 1.15911, size = 16, normalized size = 4.

$$x-\frac{2}{e^{(2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="giac")
```

```
[Out] x - 2/(e^(2*x) + 1)
```

3.997 $\int \frac{\mathbf{sech}^2(x)}{1+\mathbf{sech}^2(x)-3\tanh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{2\tanh^{-1}\left(\frac{2\tanh(x)+3}{\sqrt{17}}\right)}{\sqrt{17}}$$

```
[Out] (2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

Rubi [A] time = 0.124094, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {618, 206}

$$\frac{2\tanh^{-1}\left(\frac{2\tanh(x)+3}{\sqrt{17}}\right)}{\sqrt{17}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

```
[Out] (2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \text{Subst}\left(\int \frac{1}{2-3x-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{17-x^2}\,dx, x, -3-2\tanh(x)\right)\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

Mathematica [F] time = 0.0291272, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

```
[Out] Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]), x]
```

Maple [B] time = 0.069, size = 63, normalized size = 3.2

$$\frac{\sqrt{17}}{17}\ln\left(\sqrt{17}\tanh\left(\frac{x}{2}\right)+2\,(\tanh(x/2))^2-3\,\tanh(x/2)+2\right)-\frac{\sqrt{17}}{17}\ln\left(-\sqrt{17}\tanh\left(\frac{x}{2}\right)+2\,(\tanh(x/2))^2-3\,\tanh(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x)
```

```
[Out] 1/17*17^(1/2)*ln(17^(1/2)*tanh(1/2*x)+2*tanh(1/2*x)^2-3*tanh(1/2*x)+2)-1/17
*17^(1/2)*ln(-17^(1/2)*tanh(1/2*x)+2*tanh(1/2*x)^2-3*tanh(1/2*x)+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{\operatorname{sech}(x)^2-3\,\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(sech(x)^2 - 3*tanh(x) + 1), x)
```

Fricas [B] time = 2.35321, size = 244, normalized size = 12.2

$$\frac{1}{17}\sqrt{17}\log\left(\frac{3\left(\sqrt{17}-5\right)\cosh\left(x\right)^{2}-2\left(3\sqrt{17}-11\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(\sqrt{17}-5\right)\sinh\left(x\right)^{2}-2\sqrt{17}+6}{\cosh\left(x\right)^{2}-6\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/17*sqrt(17)*log((3*(sqrt(17) - 5)*cosh(x)^2 - 2*(3*sqrt(17) - 11)*cosh(x)
*sinh(x) + 3*(sqrt(17) - 5)*sinh(x)^2 - 2*sqrt(17) + 6)/(cosh(x)^2 - 6*cosh
(x)*sinh(x) + sinh(x)^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{2}\left(x\right)}{-3\tanh\left(x\right)+\operatorname{sech}^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+sech(x)**2-3*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(-3*tanh(x) + sech(x)**2 + 1), x)
```

Giac [B] time = 1.15557, size = 47, normalized size = 2.35

$$-\frac{1}{17}\sqrt{17}\log\left(\frac{\left|-\sqrt{17}+2\,e^{(2\,x)}-3\right|}{\left|\sqrt{17}+2\,e^{(2\,x)}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="giac")
```

```
[Out] -1/17*sqrt(17)*log(abs(-sqrt(17) + 2*e^(2*x) - 3)/abs(sqrt(17) + 2*e^(2*x)
- 3))
```

3.998 $\int \frac{\mathbf{sech}^2(x)}{\sqrt{4-\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

```
[Out] ArcSinh[Tanh[x]/Sqrt[3]]
```

Rubi [A] time = 0.0494739, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4146, 215}

$$\sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]/Sqrt[3]]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

Mathematica [B] time = 0.0462612, size = 43, normalized size = 4.78

$$\frac{\sqrt{2\cosh(2x)+1}\operatorname{sech}(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{4\sinh^2(x)+3}}\right)}{\sqrt{4-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

```
[Out] (ArcTanh[Sinh[x]/Sqrt[3 + 4*Sinh[x]^2]]*Sqrt[1 + 2*Cosh[2*x]]*Sech[x])/Sqrt
[4 - Sech[x]^2]
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^2 \frac{1}{\sqrt{4-(\operatorname{sech}(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-\operatorname{sech}(x)^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/sqrt(-sech(x)^2 + 4), x)
```

Fricas [B] time = 2.40214, size = 367, normalized size = 40.78

$$-\log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \sqrt{\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}}\right) + \log\left(-\cosh(x)^2 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sin
h(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + log(-cosh(x)^2
- 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-(\operatorname{sech}(x) - 2)(\operatorname{sech}(x) + 2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(4-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sech(x)**2/sqrt(-(sech(x) - 2)*(sech(x) + 2)), x)
```

Giac [B] time = 1.16656, size = 59, normalized size = 6.56

$$-\log\left(\sqrt{e^{(4x)} + e^{(2x)} + 1} - e^{(2x)}\right) + \log\left(-\sqrt{e^{(4x)} + e^{(2x)} + 1} + e^{(2x)} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(e^(4*x) + e^(2*x) + 1) - e^(2*x)) + log(-sqrt(e^(4*x) + e^(2*x) +
 1) + e^(2*x) + 2)
```

3.999 $\int \frac{\text{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin^{-1}(2\tanh(x))$$

[Out] ArcSin[2*Tanh[x]]/2

Rubi [A] time = 0.0495234, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3675, 216}

$$\frac{1}{2}\sin^{-1}(2\tanh(x))$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]

[Out] ArcSin[2*Tanh[x]]/2

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sin^{-1}(2\tanh(x))$$

Mathematica [B] time = 0.053116, size = 47, normalized size = 5.22

$$\frac{\sqrt{3\cosh(2x)-5}\operatorname{sech}(x)\tanh^{-1}\left(\frac{2\sinh(x)}{\sqrt{3\sinh^2(x)-1}}\right)}{2\sqrt{2-8\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

```
[Out] (ArcTanh[(2*Sinh[x])/Sqrt[-1 + 3*Sinh[x]^2]]*Sqrt[-5 + 3*Cosh[2*x]]*Sech[x]
)/(2*Sqrt[2 - 8*Tanh[x]^2])
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^2 \frac{1}{\sqrt{1-4\,(\tanh(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-4\tanh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/sqrt(-4*tanh(x)^2 + 1), x)
```

Fricas [B] time = 2.55978, size = 398, normalized size = 44.22

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{2}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\sqrt{-\frac{3\cosh(x)^{2}+3\sinh(x)^{2}-5}{\cosh(x)^{2}-2\cosh(x)\sinh(x)+\sin}}}{3\cosh\left(x\right)^{4}+12\cosh\left(x\right)\sinh\left(x\right)^{3}+3\sinh\left(x\right)^{4}+2\left(9\cosh\left(x\right)^{2}-5\right)\sinh\left(x\right)^{2}-10\cosh\left(x\right)^{2}+4\left(3\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-(3*cosh(x)^2 + 3*sinh(x)^2 - 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(3*cosh(x)^3 - 5*cosh(x))*sinh(x) + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\sqrt{-\left(2\tanh\left(x\right)-1\right)\left(2\tanh\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1-4*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sech(x)**2/sqrt(-(2*tanh(x) - 1)*(2*tanh(x) + 1)), x)
```

Giac [B] time = 1.21861, size = 59, normalized size = 6.56

$$-\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{3}\sqrt{-3e^{(4x)}+10e^{(2x)}-3}-4\right)}{3e^{(2x)}-5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(1/3*sqrt(3)*(2*(sqrt(3)*sqrt(-3*e^(4*x) + 10*e^(2*x) - 3) - 4)/(3*e
^(2*x) - 5) - 1))
```

3.1000 $\int \frac{\mathbf{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

```
[Out] ArcTanh[Tanh[x]/Sqrt[-4 + Tanh[x]^2]]
```

Rubi [A] time = 0.051154, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3675, 217, 206}

$$\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[-4 + Tanh[x]^2]]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \operatorname{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$

Mathematica [B] time = 0.0448098, size = 46, normalized size = 3.29

$$\frac{\sqrt{3\cosh(2x)+5}\,\operatorname{sech}(x)\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{3\sinh^2(x)+4}}\right)}{\sqrt{2}\sqrt{\tanh^2(x)-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]/Sqrt[4 + 3*Sinh[x]^2]]*Sqrt[5 + 3*Cosh[2*x]]*Sech[x])/(Sqrt
[2]*Sqrt[-4 + Tanh[x]^2])
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^2 \frac{1}{\sqrt{-4+(\tanh(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{\tanh(x)^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/sqrt(tanh(x)^2 - 4), x)
```

Fricas [A] time = 2.21427, size = 4, normalized size = 0.29

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{(\tanh(x)-2)(\tanh(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(-4+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sech(x)**2/sqrt((tanh(x) - 2)*(tanh(x) + 2)), x)
```

Giac [B] time = 1.18857, size = 132, normalized size = 9.43

$$\log\left(\frac{\sqrt{\left(\sqrt{3}+\frac{4}{3\,e^{(2\,x)}+5}-2\right)^{2}+\frac{3\,\left(3\,e^{(4\,x)}+10\,e^{(2\,x)}+3\right)}{\left(3\,e^{(2\,x)}+5\right)^{2}}}}{\sqrt{\left(\sqrt{3}-\frac{4}{3\,e^{(2\,x)}+5}+2\right)^{2}+\frac{3\,\left(3\,e^{(4\,x)}+10\,e^{(2\,x)}+3\right)}{\left(3\,e^{(2\,x)}+5\right)^{2}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(sqrt((sqrt(3) + 4/(3*e^(2*x) + 5) - 2)^2 + 3*(3*e^(4*x) + 10*e^(2*x) +
3)/(3*e^(2*x) + 5)^2)/sqrt((sqrt(3) - 4/(3*e^(2*x) + 5) + 2)^2 + 3*(3*e^(4*
x) + 10*e^(2*x) + 3)/(3*e^(2*x) + 5)^2))
```

3.1001 $\int \sqrt{1+\coth^2(x)}\,\mathbf{sech}^2(x)\,dx$

Optimal. Leaf size=19

$$\tanh(x)\sqrt{\coth^2(x)+1}-\sinh^{-1}(\coth(x))$$

```
[Out] -ArcSinh[Coth[x]] + Sqrt[1 + Coth[x]^2]*Tanh[x]
```

Rubi [A] time = 0.0491916, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 277, 215}

$$\tanh(x)\sqrt{\coth^2(x)+1}-\sinh^{-1}(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

```
[Out] -ArcSinh[Coth[x]] + Sqrt[1 + Coth[x]^2]*Tanh[x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\coth^2(x)}\mathrm{sech}^2(x)\,dx &= -\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \coth(x)\right)\\ &= \sqrt{1+\coth^2(x)}\tanh(x) - \mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \coth(x)\right)\\ &= -\sinh^{-1}(\coth(x)) + \sqrt{1+\coth^2(x)}\tanh(x)\end{aligned}$$

Mathematica [B] time = 0.208612, size = 51, normalized size = 2.68

$$\sinh(x)\sqrt{\coth^2(x)+1}\mathrm{sech}(2x)\left(\cosh(x)+\sinh(x)\tanh(x)-\sqrt{-\cosh(2x)}\tan^{-1}\left(\frac{\cosh(x)}{\sqrt{-\cosh(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

```
[Out] Sqrt[1 + Coth[x]^2]*Sech[2*x]*Sinh[x]*(Cosh[x] - ArcTan[Cosh[x]/Sqrt[-Cosh[
2*x]]]*Sqrt[-Cosh[2*x]] + Sinh[x]*Tanh[x])
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int (\mathrm{sech}\,(x))^2\sqrt{1+(\coth\,(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+coth(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+coth(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth\,(x)^2+1}\,\mathrm{sech}\,(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x)^2 + 1)*sech(x)^2, x)
```

Fricas [B] time = 2.06866, size = 786, normalized size = 41.37

$$-\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+2\sqrt{\frac{\cosh(x)^2+\sinh(x)^2}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}+1}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}\right)-}{2\left(\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - 4*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth^2(x)+1}\,\mathrm{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+coth(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x)**2 + 1)*sech(x)**2, x)
```

Giac [B] time = 1.17636, size = 162, normalized size = 8.53

$$\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{e^{(4x)}+1}-2e^{(2x)}+2\right|}{2\left(\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}\right)-\frac{4\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(e^(4*x) + 1) - 2*e^(2*
x) + 2)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) + 1)) - 4*(sqrt(e^(4*x) + 1)
 - e^(2*x) + 1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*
e^(2*x) - 1))*sgn(e^(2*x) - 1)
```

3.1002 $\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}+\frac{1}{2}\sinh^{-1}(\tanh(x))$$

```
[Out] ArcSinh[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rubi [A] time = 0.0433874, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3675, 195, 215}

$$\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}+\frac{1}{2}\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx &= \operatorname{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\sinh^{-1}(\tanh(x)) + \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}\end{aligned}$$

Mathematica [B] time = 0.0917873, size = 55, normalized size = 2.29

$$\frac{1}{4}\sqrt{\tanh^2(x)+1}\operatorname{sech}(x)\operatorname{sech}(2x)\left(-\sinh(x)+\sinh(3x)+2\sqrt{\cosh(2x)}\cosh^2(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] (Sech[x]*Sech[2*x]*(2*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]]*Cosh[x]^2*Sqrt[Cosh[
2*x]] - Sinh[x] + Sinh[3*x])*Sqrt[1 + Tanh[x]^2])/4
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^2\sqrt{1+(\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+tanh(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh(x)^2+1}\,\operatorname{sech}(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^2 + 1)*sech(x)^2, x)
```

Fricas [B] time = 2.08352, size = 1181, normalized size = 49.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sin
h(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 +
 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt((
cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*
cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh^2(x)+1}\,\mathrm{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 + 1)*sech(x)**2, x)
```

Giac [B] time = 1.18325, size = 196, normalized size = 8.17

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{3}-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}-1\right)}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(
2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*x))
^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/((s
qrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2)
```

3.1003 $\int \mathbf{sech}^4(x)\left(-1+\mathbf{sech}^2(x)\right)^2 \tanh(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}$$

```
[Out] Tanh[x]^6/6 - Tanh[x]^8/8
```

Rubi [A] time = 0.0781993, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 2607, 14}

$$\frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

```
[Out] Tanh[x]^6/6 - Tanh[x]^8/8
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx &= \int \operatorname{sech}^4(x)\tanh^5(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, i\tanh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}\end{aligned}$$

Mathematica [A] time = 0.0152839, size = 25, normalized size = 1.47

$$-\frac{1}{8}\operatorname{sech}^8(x)+\frac{\operatorname{sech}^6(x)}{3}-\frac{\operatorname{sech}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

```
[Out] -Sech[x]^4/4 + Sech[x]^6/3 - Sech[x]^8/8
```

Maple [A] time = 0.013, size = 20, normalized size = 1.2

$$-\frac{(\operatorname{sech}(x))^8}{8}+\frac{(\operatorname{sech}(x))^6}{3}-\frac{(\operatorname{sech}(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x)
```

```
[Out] -1/8*sech(x)^8+1/3*sech(x)^6-1/4*sech(x)^4
```

Maxima [B] time = 1.02699, size = 46, normalized size = 2.71

$$-\frac{4}{\left(e^{(-x)}+e^x\right)^4}+\frac{64}{3\left(e^{(-x)}+e^x\right)^6}-\frac{32}{\left(e^{(-x)}+e^x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="maxima")
```

```
[Out] -4/(e^(-x) + e^x)^4 + 64/3/(e^(-x) + e^x)^6 - 32/(e^(-x) + e^x)^8
```

Fricas [B] time = 1.99374, size = 1157, normalized size = 68.06

$$-\frac{}{3\left(\cosh(x)^{10}+10\cosh(x)\sinh(x)^{9}+\sinh(x)^{10}+\left(45\cosh(x)^{2}+8\right)\sinh(x)^{8}+8\cosh(x)^{8}+8\left(15\cosh(x)^{3}+8\,\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 - 4)
*sinh(x)^4 - 4*cosh(x)^4 + 4*(15*cosh(x)^3 - 4*cosh(x))*sinh(x)^3 + (45*cos
h(x)^4 - 24*cosh(x)^2 + 13)*sinh(x)^2 + 13*cosh(x)^2 + 2*(9*cosh(x)^5 - 8*c
osh(x)^3 + 7*cosh(x))*sinh(x) - 4)/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sin
h(x)^10 + (45*cosh(x)^2 + 8)*sinh(x)^8 + 8*cosh(x)^8 + 8*(15*cosh(x)^3 + 8*
cosh(x))*sinh(x)^7 + (210*cosh(x)^4 + 224*cosh(x)^2 + 29)*sinh(x)^6 + 29*co
sh(x)^6 + 2*(126*cosh(x)^5 + 224*cosh(x)^3 + 81*cosh(x))*sinh(x)^5 + (210*c
osh(x)^6 + 560*cosh(x)^4 + 435*cosh(x)^2 + 64)*sinh(x)^4 + 64*cosh(x)^4 + 4
*(30*cosh(x)^7 + 112*cosh(x)^5 + 135*cosh(x)^3 + 48*cosh(x))*sinh(x)^3 + (4
5*cosh(x)^8 + 224*cosh(x)^6 + 435*cosh(x)^4 + 384*cosh(x)^2 + 98)*sinh(x)^2
 + 98*cosh(x)^2 + 2*(5*cosh(x)^9 + 32*cosh(x)^7 + 81*cosh(x)^5 + 96*cosh(x)
^3 + 42*cosh(x))*sinh(x) + 56)
```

Sympy [A] time = 18.3691, size = 19, normalized size = 1.12

$$-\frac{\operatorname{sech}^{8}(x)}{8}+\frac{\operatorname{sech}^{6}(x)}{3}-\frac{\operatorname{sech}^{4}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4*(-1+sech(x)**2)**2*tanh(x),x)
```

```
[Out] -sech(x)**8/8 + sech(x)**6/3 - sech(x)**4/4
```

Giac [B] time = 1.13431, size = 55, normalized size = 3.24

$$-\frac{4\left(3\,e^{(12\,x)} - 4\,e^{(10\,x)} + 10\,e^{(8\,x)} - 4\,e^{(6\,x)} + 3\,e^{(4\,x)}\right)}{3\left(e^{(2\,x)} + 1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(12*x) - 4*e^(10*x) + 10*e^(8*x) - 4*e^(6*x) + 3*e^(4*x))/(e^(2*x
) + 1)^8
```

3.1004 $\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rubi [A] time = 0.039057, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2176, 2194}

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx &= \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{bn}\\ &= -\frac{2e^{n\sinh(a+bx)}}{bn^2} + \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0557636, size = 28, normalized size = 0.65

$$\frac{2e^{n\sinh(a+bx)}(n\sinh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

Maple [A] time = 0.071, size = 61, normalized size = 1.4

$$\frac{n\mathrm{e}^{2\,bx+2\,a}-n-2\,\mathrm{e}^{bx+a}}{n^2b}\mathrm{e}^{-bx-a+\frac{n\mathrm{e}^{bx+a}}{2}-\frac{n\mathrm{e}^{-bx-a}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] (n*exp(2*b*x+2*a)-n-2*exp(b*x+a))/b/n^2*exp(-b*x-a+1/2*n*exp(b*x+a)-1/2*n*e
xp(-b*x-a))
```

Maxima [B] time = 1.32767, size = 140, normalized size = 3.26

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}-\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Fricas [A] time = 2.03058, size = 193, normalized size = 4.49

$$\frac{2\left(\left(n\sinh\left(bx+a\right)-1\right)\cosh\left(n\sinh\left(bx+a\right)\right)+\left(n\sinh\left(bx+a\right)-1\right)\sinh\left(n\sinh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh\left(a+bx\right)}\sinh\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sinh\left(bx+a\right)\right)}\sinh\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1005 $\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rubi [A] time = 0.0399517, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx &= \frac{\text{Subst}\left(\int 2e^{nx}x\,dx,x,\sinh(a+bx)\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\sinh(a+bx)\right)}{b}\\ &= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}-\frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\sinh(a+bx)\right)}{bn}\\ &= -\frac{2e^{n\sinh(a+bx)}}{bn^2}+\frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0264681, size = 28, normalized size = 0.65

$$\frac{2e^{n\sinh(a+bx)}(n\sinh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

Maple [A] time = 0.002, size = 61, normalized size = 1.4

$$\frac{n\mathrm{e}^{2\,bx+2\,a}-n-2\,\mathrm{e}^{bx+a}}{bn^2}\mathrm{e}^{-bx-a+\frac{n\mathrm{e}^{bx+a}}{2}-\frac{n\mathrm{e}^{-bx-a}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] (n*exp(2*b*x+2*a)-n-2*exp(b*x+a))/b/n^2*exp(-b*x-a+1/2*n*exp(b*x+a)-1/2*n*e
xp(-b*x-a))
```

Maxima [B] time = 1.31629, size = 140, normalized size = 3.26

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}-\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Fricas [A] time = 2.11555, size = 193, normalized size = 4.49

$$\frac{2\left(\left(n\sinh\left(bx+a\right)-1\right)\cosh\left(n\sinh\left(bx+a\right)\right)+\left(n\sinh\left(bx+a\right)-1\right)\sinh\left(n\sinh\left(bx+a\right)\right)\right)}{bn^{2}\cosh\left(bx+a\right)^{2}-bn^{2}\sinh\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh\left(a+bx\right)}\sinh\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sinh\left(bx+a\right)\right)}\sinh\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1006 $\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.038549, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx &= \frac{2\,\mathrm{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= \frac{4\,\mathrm{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}-\frac{4\,\mathrm{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\ &= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0603013, size = 36, normalized size = 0.56

$$\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(n\sinh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] time = 0.07, size = 65, normalized size = 1.

$$2\,\frac{\left(n\mathrm{e}^{bx+a}-n-2\,\mathrm{e}^{1/2\,bx+a/2}\right)\mathrm{e}^{-1/2\,bx-a/2+1/2\,n\mathrm{e}^{1/2\,bx+a/2}-1/2\,n\mathrm{e}^{-1/2\,bx-a/2}}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2*(n*exp(b*x+a)-n-2*exp(1/2*b*x+1/2*a))/b/n^2*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)-1/2*n*exp(-1/2*b*x-1/2*a))
```

Maxima [B] time = 1.41199, size = 158, normalized size = 2.47

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}-\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Fricas [A] time = 2.12055, size = 258, normalized size = 4.03

$$\frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Giac [B] time = 1.23528, size = 344, normalized size = 5.38

$$2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}-ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

3.1007 $\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0439971, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.028602, size = 36, normalized size = 0.56

$$\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(n\sinh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] time = 0., size = 65, normalized size = 1.

$$2\,\frac{\left(n\mathrm{e}^{bx+a}-n-2\,\mathrm{e}^{1/2\,bx+a/2}\right)\mathrm{e}^{-1/2\,bx-a/2+1/2\,n\mathrm{e}^{1/2\,bx+a/2}-1/2\,n\mathrm{e}^{-1/2\,bx-a/2}}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2*(n*exp(b*x+a)-n-2*exp(1/2*b*x+1/2*a))/b/n^2*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)-1/2*n*exp(-1/2*b*x-1/2*a))
```

Maxima [B] time = 1.37153, size = 158, normalized size = 2.47

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}-\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Fricas [A] time = 2.20184, size = 258, normalized size = 4.03

$$\frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^{2}\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}-bn^{2}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Giac [B] time = 1.17551, size = 344, normalized size = 5.38

$$2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}-ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

3.1008 $\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rubi [A] time = 0.0422842, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2176, 2194}

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{i\,\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

Mathematica [A] time = 0.116076, size = 28, normalized size = 0.65

$$\frac{2e^{n\cosh(a+bx)}(n\cosh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

Maple [A] time = 0.086, size = 59, normalized size = 1.4

$$\frac{n\mathrm{e}^{2\,bx+2\,a}+n-2\,\mathrm{e}^{bx+a}}{n^2 b}\mathrm{e}^{-bx-a+\frac{n\mathrm{e}^{bx+a}}{2}+\frac{n\mathrm{e}^{-bx-a}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] (n*exp(2*b*x+2*a)+n-2*exp(b*x+a))/b/n^2*exp(-b*x-a+1/2*n*exp(b*x+a)+1/2*n*e
xp(-b*x-a))
```

Maxima [B] time = 1.31576, size = 139, normalized size = 3.23

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}+\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Fricas [A] time = 2.02311, size = 193, normalized size = 4.49

$$\frac{2\left(\left(n\cosh\left(bx+a\right)-1\right)\cosh\left(n\cosh\left(bx+a\right)\right)+\left(n\cosh\left(bx+a\right)-1\right)\sinh\left(n\cosh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh\left(a+bx\right)}\sinh\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cosh\left(bx+a\right)\right)}\sinh\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1009 $\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rubi [A] time = 0.0388002, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{i\,\text{Subst}\left(\int -2ie^{nx}x\,dx,x,\cosh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\cosh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\cosh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

Mathematica [A] time = 0.0309611, size = 28, normalized size = 0.65

$$\frac{2e^{n\cosh(a+bx)}(n\cosh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

Maple [A] time = 0., size = 59, normalized size = 1.4

$$\frac{n\mathrm{e}^{2\,bx+2\,a}+n-2\,\mathrm{e}^{bx+a}}{bn^2}\mathrm{e}^{-bx-a+\frac{n\mathrm{e}^{bx+a}}{2}+\frac{n\mathrm{e}^{-bx-a}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] (n*exp(2*b*x+2*a)+n-2*exp(b*x+a))/b/n^2*exp(-b*x-a+1/2*n*exp(b*x+a)+1/2*n*e
xp(-b*x-a))
```

Maxima [B] time = 1.34352, size = 139, normalized size = 3.23

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}+\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Fricas [A] time = 2.06515, size = 193, normalized size = 4.49

$$\frac{2\left((n\cosh\left(bx+a\right)-1)\cosh\left(n\cosh\left(bx+a\right)\right)+\left(n\cosh\left(bx+a\right)-1\right)\sinh\left(n\cosh\left(bx+a\right)\right)\right)}{bn^{2}\cosh\left(bx+a\right)^{2}-bn^{2}\sinh\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh\left(a+bx\right)}\sinh\left(2a+2bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cosh\left(bx+a\right)\right)}\sinh\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1010 $\int e^{n \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)} \sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0487021, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx &= \frac{(2i)\,\mathrm{Subst}\left(\int -2ie^{nx}x\,dx,x,\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= \frac{4\,\mathrm{Subst}\left(\int e^{nx}x\,dx,x,\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}-\frac{4\,\mathrm{Subst}\left(\int e^{nx}\,dx,x,\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\ &= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.158314, size = 36, normalized size = 0.56

$$\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(n\cosh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] time = 0.091, size = 63, normalized size = 1.

$$2\,\frac{\left(n\mathrm{e}^{bx+a}+n-2\,\mathrm{e}^{1/2\,bx+a/2}\right)\mathrm{e}^{-1/2\,bx-a/2+1/2\,n\mathrm{e}^{1/2\,bx+a/2}+1/2\,n\mathrm{e}^{-1/2\,bx-a/2}}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2*(n*exp(b*x+a)+n-2*exp(1/2*b*x+1/2*a))/b/n^2*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)+1/2*n*exp(-1/2*b*x-1/2*a))
```

Maxima [B] time = 1.33967, size = 158, normalized size = 2.47

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}+\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Fricas [A] time = 2.02435, size = 258, normalized size = 4.03

$$\frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Giac [B] time = 1.18923, size = 343, normalized size = 5.36

$$2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

3.1011 $\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0422843, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{(2i)\operatorname{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\operatorname{Subst}\left(\int e^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\operatorname{Subst}\left(\int e^{nx}\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.0320554, size = 36, normalized size = 0.56

$$\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(n\cosh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] time = 0., size = 63, normalized size = 1.

$$2\,\frac{\left(n\mathrm{e}^{bx+a}+n-2\,\mathrm{e}^{1/2\,bx+a/2}\right)\mathrm{e}^{-1/2\,bx-a/2+1/2\,n\mathrm{e}^{1/2\,bx+a/2}+1/2\,n\mathrm{e}^{-1/2\,bx-a/2}}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2*(n*exp(b*x+a)+n-2*exp(1/2*b*x+1/2*a))/b/n^2*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)+1/2*n*exp(-1/2*b*x-1/2*a))
```

Maxima [B] time = 1.30947, size = 158, normalized size = 2.47

$$\frac{2 e^{\left(\frac{1}{2} bx+\frac{1}{2} ne^{\left(\frac{1}{2} bx+\frac{1}{2} a\right)}+\frac{1}{2} ne^{\left(-\frac{1}{2} bx-\frac{1}{2} a\right)}+\frac{1}{2} a\right)}}{bn}+\frac{2 e^{\left(-\frac{1}{2} bx+\frac{1}{2} ne^{\left(\frac{1}{2} bx+\frac{1}{2} a\right)}+\frac{1}{2} ne^{\left(-\frac{1}{2} bx-\frac{1}{2} a\right)}-\frac{1}{2} a\right)}}{bn}-\frac{4 e^{\left(\frac{1}{2} ne^{\left(\frac{1}{2} bx+\frac{1}{2} a\right)}+\frac{1}{2} ne^{\left(-\frac{1}{2} bx-\frac{1}{2} a\right)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Fricas [A] time = 2.078, size = 258, normalized size = 4.03

$$\frac{4 \left(\left(n \cosh \left(\frac{1}{2} bx+\frac{1}{2} a\right)-1\right) \cosh \left(n \cosh \left(\frac{1}{2} bx+\frac{1}{2} a\right)\right)+\left(n \cosh \left(\frac{1}{2} bx+\frac{1}{2} a\right)-1\right) \sinh \left(n \cosh \left(\frac{1}{2} bx+\frac{1}{2} a\right)\right)\right)}{bn^2 \cosh \left(\frac{1}{2} bx+\frac{1}{2} a\right)^2-bn^2 \sinh \left(\frac{1}{2} bx+\frac{1}{2} a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n \cosh \left(\frac{a}{2}+\frac{bx}{2}\right)} \sinh (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Giac [B] time = 1.17302, size = 343, normalized size = 5.36

$$\frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

3.1012 $\int \operatorname{csch}(x) \log(\tanh(x)) \operatorname{sech}(x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log^2(\tanh(x))$$

```
[Out] Log[Tanh[x]]^2/2
```

Rubi [A] time = 0.0260572, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2} \log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

```
[Out] Log[Tanh[x]]^2/2
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \operatorname{csch}(x) \log(\tanh(x)) \operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

Mathematica [A] time = 0.0058762, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

```
[Out] Log[Tanh[x]]^2/2
```

Maple [A] time = 0.017, size = 8, normalized size = 0.9

$$\frac{(\ln(\tanh(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)*ln(tanh(x))*sech(x),x)
```

```
[Out] 1/2*ln(tanh(x))^2
```

Maxima [B] time = 4.90216, size = 128, normalized size = 14.22

$$(\log(e^x+1)+\log(-e^x+1))\log\left(e^{(2x)}+1\right)-\frac{1}{2}\log\left(e^{(2x)}+1\right)^2-\frac{1}{2}\log(e^x+1)^2-\log(e^x+1)\log(-e^x+1)-\frac{1}{2}\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="maxima")
```

```
[Out] (log(e^x + 1) + log(-e^x + 1))*log(e^(2*x) + 1) - 1/2*log(e^(2*x) + 1)^2 -
1/2*log(e^x + 1)^2 - log(e^x + 1)*log(-e^x + 1) - 1/2*log(-e^x + 1)^2 + (lo
g(e^(-x) + 1) + log(e^(-x) - 1) - log(e^(-2*x) + 1))*log(tanh(x))
```

Fricas [A] time = 1.98701, size = 38, normalized size = 4.22

$$\frac{1}{2}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sinh(x)/cosh(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(\tanh(x))\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*ln(tanh(x))*sech(x),x)
```

```
[Out] Integral(log(tanh(x))*csch(x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="giac")
```

```
[Out] integrate(csch(x)*log(tanh(x))*sech(x), x)
```

3.1013 $\int \mathbf{csch}(2x)\log(\tanh(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{4}\log^2(\tanh(x))$$

```
[Out] Log[Tanh[x]]^2/4
```

Rubi [A] time = 0.0243586, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3770, 6686}

$$\frac{1}{4}\log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*x]*Log[Tanh[x]],x]
```

```
[Out] Log[Tanh[x]]^2/4
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \text{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

Mathematica [A] time = 0.0090192, size = 9, normalized size = 1.

$$\frac{1}{4}\log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*x]*Log[Tanh[x]],x]
```

```
[Out] Log[Tanh[x]]^2/4
```

Maple [A] time = 0.015, size = 8, normalized size = 0.9

$$\frac{(\ln(\tanh(x)))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*x)*ln(tanh(x)),x)
```

```
[Out] 1/4*ln(tanh(x))^2
```

Maxima [A] time = 1.01263, size = 9, normalized size = 1.

$$\frac{1}{4}\log(\tanh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/4*log(tanh(x))^2
```

Fricas [A] time = 2.11135, size = 38, normalized size = 4.22

$$\frac{1}{4}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*log(sinh(x)/cosh(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\tanh\left(x\right)\right)\operatorname{csch}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*ln(tanh(x)),x)
```

```
[Out] Integral(log(tanh(x))*csch(2*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(2\,x\right)\log\left(\tanh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(csch(2*x)*log(tanh(x)), x)
```

3.1014 $\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}(\cosh(a+bx)F(c,d,\sinh(a+bx),r,s),x)$$

```
[Out] CannotIntegrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s], x]
```

Rubi [A] time = 0.0156299, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sinh[a + b*x]]/b
```

Rubi steps

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\sinh(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0329499, size = 0, normalized size = 0.

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

```
[Out] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s], x]
```

Maple [A] time = 0.029, size = 0, normalized size = 0.

$$\int \cosh\left(bx+a\right) F\left(c,d,\sinh\left(bx+a\right),r,s\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

```
[Out] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\sinh\left(bx+a\right),r,s\right)\cosh\left(bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(c,d,\sinh\left(bx+a\right),r,s\right)\cosh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\sinh\left(a+bx\right),r,s)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, sinh(a + b*x), r, s)*cosh(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\sinh\left(bx+a\right),r,s\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

3.1015 $\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}(\sinh(a+bx)F(c,d,\cosh(a+bx),r,s),x)$$

```
[Out] CannotIntegrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x], x]
```

Rubi [A] time = 0.015913, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cosh[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\cosh(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0367883, size = 0, normalized size = 0.

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
```

```
[Out] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x], x]
```

Maple [A] time = 0.009, size = 0, normalized size = 0.

$$\int F\left(c,d,\cosh\left(bx+a\right),r,s\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

```
[Out] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\cosh\left(bx+a\right),r,s\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(c,d,\cosh\left(bx+a\right),r,s\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cosh\left(a+bx\right),r,s)\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

```
[Out] Integral(F(c, d, cosh(a + b*x), r, s)*sinh(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c, d, \cosh(bx + a), r, s) \sinh(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

3.1016 $\int F(c,d,\tanh(a+bx),r,s)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\text{sech}^2(a+bx)F(c,d,\tanh(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2, x]
```

Rubi [A] time = 0.0200352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tanh[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx = \frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.0757455, size = 0, normalized size = 0.

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
```

```
[Out] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2, x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int F\left(c,d,\tanh\left(bx+a\right),r,s\right)\left(\operatorname{sech}\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

```
[Out] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\tanh\left(bx+a\right),r,s\right)\operatorname{sech}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(F\left(c,d,\tanh\left(bx+a\right),r,s\right)\operatorname{sech}\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\tanh\left(a+bx\right),r,s)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)**2,x)
```

```
[Out] Integral(F(c, d, tanh(a + b*x), r, s)*sech(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c, d, \tanh(bx + a), r, s) \operatorname{sech}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

3.1017 $\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s], x]
```

Rubi [A] time = 0.0218718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
```

```
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Coth[a + b*x]]/b)
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx=-\frac{\operatorname{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\coth(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0698806, size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
```

```
[Out] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s], x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(bx+a))^2 F(c,d,\coth(bx+a),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

```
[Out] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\coth(a+bx),r,s)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*F(c,d,coth(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, coth(a + b*x), r, s)*csch(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

3.1018 $\int \mathbf{sech}(x)\left(5-11\mathbf{sech}^2(x)\right)\tanh(x)\,dx$

Optimal. Leaf size=13

$$\frac{11\mathrm{sech}^3(x)}{3}-5\mathrm{sech}(x)$$

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Rubi [A] time = 0.035788, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4339, 14}

$$\frac{11\mathrm{sech}^3(x)}{3}-5\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx = \text{Subst}\left(\int \frac{-11+5x^2}{x^4}\,dx, x, \cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{11}{x^4}+\frac{5}{x^2}\right)dx, x, \cosh(x)\right)$$
$$= -5\operatorname{sech}(x)+\frac{11\operatorname{sech}^3(x)}{3}$$

Mathematica [A] time = 0.007138, size = 13, normalized size = 1.

$$\frac{11\operatorname{sech}^3(x)}{3}-5\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Maple [A] time = 0.013, size = 12, normalized size = 0.9

$$-5\operatorname{sech}(x)+\frac{11(\operatorname{sech}(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*(5-11*sech(x)^2)*tanh(x),x)
```

```
[Out] -5*sech(x)+11/3*sech(x)^3
```

Maxima [B] time = 1.03603, size = 31, normalized size = 2.38

$$-\frac{10}{e^{(-x)}+e^x}+\frac{88}{3\left(e^{(-x)}+e^x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="maxima")
```

```
[Out] -10/(e^(-x) + e^x) + 88/3/(e^(-x) + e^x)^3
```

Fricas [B] time = 2.02602, size = 309, normalized size = 23.77

$$-\frac{2\left(15\,\cosh\left(x\right)^{3}+45\,\cosh\left(x\right)\sinh\left(x\right)^{2}+15\,\sinh\left(x\right)^{3}+\left(45\,\cosh\left(x\right)^{2}-29\right)\sinh\left(x\right)+\cosh\left(x\right)\right)}{3\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(x)^3 + 45*cosh(x)*sinh(x)^2 + 15*sinh(x)^3 + (45*cosh(x)^2 -
29)*sinh(x) + cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*
cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
3)
```

Sympy [A] time = 0.741429, size = 12, normalized size = 0.92

$$\frac{11\,\mathrm{sech}^{3}\left(x\right)}{3}-5\,\mathrm{sech}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)**2)*tanh(x),x)
```

```
[Out] 11*sech(x)**3/3 - 5*sech(x)
```

Giac [B] time = 1.11225, size = 32, normalized size = 2.46

$$-\frac{2\left(15\left(e^{\left(-x\right)}+e^{x}\right)^{2}-44\right)}{3\left(e^{\left(-x\right)}+e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="giac")
```

```
[Out] -2/3*(15*(e^(-x) + e^x)^2 - 44)/(e^(-x) + e^x)^3
```

3.1019 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\coth(x))}{b}$$

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rubi [A] time = 0.0484419, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\coth(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0536766, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x)) - \log(a\sinh(x) + b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Maple [A] time = 0.019, size = 13, normalized size = 1.1

$$-\frac{\ln(a + b\coth(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)),x)
```

```
[Out] -ln(a+b*coth(x))/b
```

Maxima [A] time = 1.00074, size = 16, normalized size = 1.33

$$-\frac{\log(b\coth(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -log(b*coth(x) + a)/b
```

Fricas [B] time = 2.16145, size = 127, normalized size = 10.58

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Giac [B] time = 1.09929, size = 62, normalized size = 5.17

$$-\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

3.1020 $\int (a+b\coth(x))^n \mathbf{csch}^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

```
[Out] -((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

Rubi [A] time = 0.0483491, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[x])^n*Csch[x]^2,x]
```

```
[Out] -((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(x))^n\operatorname{csch}^2(x)\,dx &= -\frac{\operatorname{Subst}\left(\int (a+x)^n\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{(a+b\coth(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.196076, size = 19, normalized size = 0.95

$$-\frac{(a+b\coth(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x])^n*Csch[x]^2,x]
```

```
[Out] -((a + b*Coth[x])^(1 + n)/(b + b*n))
```

Maple [A] time = 0.021, size = 21, normalized size = 1.1

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^n*csch(x)^2,x)
```

```
[Out] -(a+b*coth(x))^(n+1)/b/(n+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15537, size = 221, normalized size = 11.05

$$-\frac{(b\cosh(x)+a\sinh(x))\cosh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)+(b\cosh(x)+a\sinh(x))\sinh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)}{(bn+b)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="fricas")
```

```
[Out] -((b*cosh(x) + a*sinh(x))*cosh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))) + (b
*cosh(x) + a*sinh(x))*sinh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))))/((b*n +
 b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**n*csch(x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(x) + a)^n \operatorname{csch}(x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*coth(x) + a)^n*csch(x)^2, x)
```

3.1021 $\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx$

Optimal. Leaf size=4

$$x+\coth(x)$$

[Out] x + Coth[x]

Rubi [A] time = 0.0198524, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\coth(x)$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2*(-1 + Sinh[x]^2),x]

[Out] x + Coth[x]

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx &= \coth(x)+\int 1\,dx\\ &= x+\coth(x)\end{aligned}$$

Mathematica [A] time = 0.0031859, size = 4, normalized size = 1.

$$x+\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

```
[Out] x + Coth[x]
```

Maple [A] time = 0.013, size = 5, normalized size = 1.3

$$x + \coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2*(-1+sinh(x)^2),x)
```

```
[Out] x+coth(x)
```

Maxima [B] time = 1.02795, size = 16, normalized size = 4.

$$x - \frac{2}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x - 2/(e^(-2*x) - 1)
```

Fricas [B] time = 1.99655, size = 50, normalized size = 12.5

$$\frac{(x-1)\sinh(x) + \cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2*(-1+sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11156, size = 16, normalized size = 4.

$$x + \frac{2}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="giac")
```

```
[Out] x + 2/(e^(2*x) - 1)
```

3.1022 $\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx$

Optimal. Leaf size=4

$$x + \coth(x)$$

```
[Out] x + Coth[x]
```

Rubi [A] time = 0.0618662, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {453, 206}

$$x + \coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

```
[Out] x + Coth[x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx = -\operatorname{Subst}\left(\int \frac{1-2x^2}{x^2(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= \coth(x) + \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= x + \coth(x)$$

Mathematica [A] time = 0.0057326, size = 4, normalized size = 1.

$$x + \coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

```
[Out] x + Coth[x]
```

Maple [B] time = 0.03, size = 32, normalized size = 8.

$$\frac{1}{2}\tanh\left(\frac{x}{2}\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-1/(1-coth(x)^2))*csch(x)^2,x)
```

```
[Out] 1/2*tanh(1/2*x)+ln(tanh(1/2*x)+1)+1/2/tanh(1/2*x)-ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.06225, size = 16, normalized size = 4.

$$x - \frac{2}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="maxima")
```

```
[Out] x - 2/(e^(-2*x) - 1)
```

Fricas [B] time = 1.97203, size = 50, normalized size = 12.5

$$\frac{(x-1)\sinh(x)+\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="fricas")
```

```
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2\operatorname{csch}^2(x)}{\coth^2(x)-1}\,dx-\int \frac{\coth^2(x)\operatorname{csch}^2(x)}{\coth^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)**2))*csch(x)**2,x)
```

```
[Out] -Integral(-2*csch(x)**2/(coth(x)**2 - 1), x) - Integral(coth(x)**2*csch(x)*
*2/(coth(x)**2 - 1), x)
```

Giac [B] time = 1.11284, size = 16, normalized size = 4.

$$x+\frac{2}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="giac")
```

```
[Out] x + 2/(e^(2*x) - 1)
```

3.1023 $\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=28

$$\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

```
[Out] -((b*Coth[x])/d) + ((b*c - a*d)*Log[c + d*Coth[x]])/d^2
```

Rubi [A] time = 0.0967657, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4344, 43}

$$\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*Coth[x])/d) + ((b*c - a*d)*Log[c + d*Coth[x]])/d^2
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\coth(x))\mathrm{csch}^2(x)}{c+d\coth(x)}\,dx = -\mathrm{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \coth(x)\right)$$
$$= -\mathrm{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \coth(x)\right)$$
$$= -\frac{b\coth(x)}{d} + \frac{(bc-ad)\log(c+d\coth(x))}{d^2}$$

Mathematica [A] time = 0.300637, size = 56, normalized size = 2.

$$\frac{\sinh(x)(a+b\coth(x))(-(bc-ad)(\log(\sinh(x))-\log(c\sinh(x)+d\cosh(x)))-bd\coth(x))}{d^2(a\sinh(x)+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] ((a + b*Coth[x])*(-(b*d*Coth[x]) - (b*c - a*d)*(Log[Sinh[x]] - Log[d*Cosh[x
] + c*Sinh[x]]))*Sinh[x])/(d^2*(b*Cosh[x] + a*Sinh[x]))
```

Maple [B] time = 0.036, size = 94, normalized size = 3.4

$$-\frac{b}{2d}\tanh\left(\frac{x}{2}\right)-\frac{a}{d}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2d+2c\tanh(x/2)+d\right)+\frac{cb}{d^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2d+2c\tanh(x/2)+d\right)-\frac{b}{2d}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/2*b/d*tanh(1/2*x)-1/d*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a+1/d^2*ln(t
anh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*c*b-1/2*b/d/tanh(1/2*x)+1/d*ln(tanh(1/2*x
))*a-1/d^2*ln(tanh(1/2*x))*c*b
```

Maxima [B] time = 1.19942, size = 104, normalized size = 3.71

$$b\left(\frac{c\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^2}-\frac{c\log\left(e^{(-x)}+1\right)}{d^2}-\frac{c\log\left(e^{(-x)}-1\right)}{d^2}+\frac{2}{de^{(-2x)}-d}\right)-\frac{a\log(d\coth(x)+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(
-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a*log(d*coth(x) + c)/d
```

Fricas [B] time = 2.20974, size = 467, normalized size = 16.68

$$-\frac{2\,bd - \left((bc-ad)\cosh(x)^2 + 2\,(bc-ad)\cosh(x)\sinh(x) + (bc-ad)\sinh(x)^2 - bc + ad\right)\log\left(\frac{2\,(d\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right) + \left((b\cdots\right.}{d^2\cosh(x)^2 + 2\,d^2\cosh(x)\sinh(x) + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*b*d - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a
*d)*sinh(x)^2 - b*c + a*d)*log(2*(d*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x)
)) + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*s
inh(x)^2 - b*c + a*d)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 +
2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 - d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\coth(x))\operatorname{csch}^2(x)}{c + d\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)**2/(c+d*coth(x)),x)
```

```
[Out] Integral((a + b*coth(x))*csch(x)**2/(c + d*coth(x)), x)
```

Giac [B] time = 1.12092, size = 153, normalized size = 5.46

$$\frac{\left(bc^2 - acd + bcd - ad^2\right)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} - c + d\right|\right)}{cd^2 + d^3} - \frac{(bc-ad)\log\left(\left|e^{(2\,x)} - 1\right|\right)}{d^2} + \frac{bce^{(2\,x)} - ade^{(2\,x)} - bc + ad - 2\,bd}{d^2\left(e^{(2\,x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*
d^2 + d^3) - (b*c - a*d)*log(abs(e^(2*x) - 1))/d^2 + (b*c*e^(2*x) - a*d*e^(
2*x) - b*c + a*d - 2*b*d)/(d^2*(e^(2*x) - 1))
```

3.1024 $\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}-\frac{(a+b\coth(x))^2}{2d}$$

```
[Out] (b*(b*c - a*d)*Coth[x])/d^2 - (a + b*Coth[x])^2/(2*d) - ((b*c - a*d)^2*Log[
c + d*Coth[x]])/d^3
```

Rubi [A] time = 0.145078, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}-\frac{(a+b\coth(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] (b*(b*c - a*d)*Coth[x])/d^2 - (a + b*Coth[x])^2/(2*d) - ((b*c - a*d)^2*Log[
c + d*Coth[x]])/d^3
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \coth(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \coth(x)\right)$$
$$= \frac{b(bc-ad)\coth(x)}{d^2} - \frac{(a+b\coth(x))^2}{2d} - \frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}$$

Mathematica [A] time = 0.465753, size = 62, normalized size = 1.17

$$\frac{2bd\coth(x)(bc-2ad)+2(bc-ad)^2(\log(\sinh(x))-\log(c\sinh(x)+d\cosh(x)))-b^2d^2\operatorname{csch}^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] (2*b*d*(b*c - 2*a*d)*Coth[x] - b^2*d^2*Csch[x]^2 + 2*(b*c - a*d)^2*(Log[Sin
h[x]] - Log[d*Cosh[x] + c*Sinh[x]]))/(2*d^3)
```

Maple [B] time = 0.06, size = 203, normalized size = 3.8

$$-\frac{b^2}{8d}\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \frac{ab}{d}\tanh\left(\frac{x}{2}\right) + \frac{cb^2}{2d^2}\tanh\left(\frac{x}{2}\right) - \frac{a^2}{d}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 d + 2c\tanh(x/2) + d\right) + 2\frac{\ln\left((\tanh(x/2))^2 d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/8*b^2/d*tanh(1/2*x)^2-b/d*a*tanh(1/2*x)+1/2*b^2/d^2*tanh(1/2*x)*c-1/d*ln
(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a^2+2/d^2*ln(tanh(1/2*x)^2*d+2*c*tanh(1
/2*x)+d)*c*b*a-1/d^3*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*c^2*b^2-1/8*b^2/
d/tanh(1/2*x)^2+1/d*ln(tanh(1/2*x))*a^2-2/d^2*ln(tanh(1/2*x))*c*b*a+1/d^3*l
n(tanh(1/2*x))*c^2*b^2-b/d/tanh(1/2*x)*a+1/2*b^2/d^2/tanh(1/2*x)*c
```

Maxima [B] time = 1.22232, size = 239, normalized size = 4.51

$$b^2\left(\frac{2\left((c+d)e^{(-2x)}-c\right)}{2d^2e^{(-2x)}-d^2e^{(-4x)}-d^2} - \frac{c^2\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}+1\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}-1\right)}{d^3}\right) + 2ab\left(\frac{c\log\left(-(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-4*x) - d^2) - c^2*l
og(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/d^3 + c^2*log(e^(-x
) - 1)/d^3) + 2*a*b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) +
1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a^2*log(d*coth(x) +
c)/d
```

Fricas [B] time = 2.30032, size = 1669, normalized size = 31.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*b^2*c*d - 4*a*b*d^2 - 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 - 4*(b^
2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) - 2*(b^2*c*d - (2*a*b + b^2)*d^2
)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(
x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*co
sh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(
x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*cosh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
cosh(x)^3 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*sinh(x)
/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(
x)^4 - 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 - d^3)*sinh(x)^2 + 4*(d^3
*cosh(x)^3 - d^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(x))^2 \operatorname{csch}^2(x)}{c + d \coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**2*csch(x)**2/(c+d*coth(x)),x)
```

```
[Out] Integral((a + b*coth(x))**2*csch(x)**2/(c + d*coth(x)), x)
```

Giac [B] time = 1.15915, size = 358, normalized size = 6.75

$$-\frac{\left(b^{2}c^{3}-2\,abc^{2}d+b^{2}c^{2}d+a^{2}cd^{2}-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}-c+d\right|\right)}{cd^{3}+d^{4}}+\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(\left|e^{(2\,x)}-1\right.\right.}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*lo
g(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*d^3 + d^4) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(e^(2*x) - 1))/d^3 - 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e
^(4*x) + 3*a^2*d^2*e^(4*x) - 6*b^2*c^2*e^(2*x) + 12*a*b*c*d*e^(2*x) - 4*b^2
*c*d*e^(2*x) - 6*a^2*d^2*e^(2*x) + 8*a*b*d^2*e^(2*x) + 4*b^2*d^2*e^(2*x) +
3*b^2*c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) -
1)^2)
```

3.1025 $\int \frac{(a+b\coth(x))^3\mathbf{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{b\coth(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{(a+b\coth(x))^3}{3d}$$

```
[Out] -((b*(b*c - a*d)^2*Coth[x])/d^3) + ((b*c - a*d)*(a + b*Coth[x])^2)/(2*d^2)
- (a + b*Coth[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Coth[x]])/d^4
```

Rubi [A] time = 0.153484, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$-\frac{b\coth(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{(a+b\coth(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Coth[x])/d^3) + ((b*c - a*d)*(a + b*Coth[x])^2)/(2*d^2)
- (a + b*Coth[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Coth[x]])/d^4
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\coth(x))^3\mathbf{csch}^2(x)}{c+d\coth(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx,x,\coth(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx,x,\mathrm{co}$$
$$= -\frac{b(bc-ad)^2\coth(x)}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}-\frac{(a+b\coth(x))^3}{3d}+\frac{(bc-ad)^3\log}{}$$

Mathematica [A] time = 1.14586, size = 136, normalized size = 1.74

$$\frac{(a+b\coth(x))^3(c\sinh(x)+d\cosh(x))\left(-bd\left(\sinh(2x)\left(9a^2d^2-9abcd+b^2\left(3c^2+d^2\right)\right)-3bd(bc-3ad)\right)-6\sinh^2(x)(b\right.}{6d^4(c+d\coth(x))(a\sinh(x)+b\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] ((a + b*Coth[x])^3*(d*Cosh[x] + c*Sinh[x])*(-2*b^3*d^3*Coth[x] - 6*(b*c - a
*d)^3*(Log[Sinh[x]] - Log[d*Cosh[x] + c*Sinh[x]])*Sinh[x]^2 - b*d*(-3*b*d*(
b*c - 3*a*d) + (-9*a*b*c*d + 9*a^2*d^2 + b^2*(3*c^2 + d^2))*Sinh[2*x])))/(6
*d^4*(c + d*Coth[x])*(b*Cosh[x] + a*Sinh[x])^3)
```

Maple [B] time = 0.073, size = 378, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/24*b^3/d*tanh(1/2*x)^3-3/8*b^2/d*tanh(1/2*x)^2*a+1/8*b^3/d^2*tanh(1/2*x)
^2*c-3/2*b/d*a^2*tanh(1/2*x)+3/2*b^2/d^2*c*a*tanh(1/2*x)-1/2*b^3/d^3*c^2*ta
nh(1/2*x)-1/8*b^3/d*tanh(1/2*x)-1/d*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a
^3+3/d^2*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a^2*b*c-3/d^3*ln(tanh(1/2*x)
^2*d+2*c*tanh(1/2*x)+d)*c^2*b^2*a+1/d^4*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+
d)*c^3*b^3-1/24*b^3/d/tanh(1/2*x)^3+1/d*ln(tanh(1/2*x))*a^3-3/d^2*ln(tanh(1
/2*x))*a^2*b*c+3/d^3*ln(tanh(1/2*x))*c^2*b^2*a-1/d^4*ln(tanh(1/2*x))*c^3*b^
3-3/2*b/d/tanh(1/2*x)*a^2+3/2*b^2/d^2/tanh(1/2*x)*c*a-1/2*b^3/d^3/tanh(1/2*
x)*c^2-1/8*b^3/d/tanh(1/2*x)-3/8*b^2/d/tanh(1/2*x)^2*a+1/8*b^3/d^2/tanh(1/2
```

```
*x)^2*c
```

Maxima [B] time = 1.23211, size = 427, normalized size = 5.47

$$\frac{1}{3}\,b^3\left(\frac{2\left(3\,c^2+d^2-3\left(2\,c^2+cd\right)e^{(-2\,x)}+3\left(c^2+cd+d^2\right)e^{(-4\,x)}\right)}{3\,d^3e^{(-2\,x)}-3\,d^3e^{(-4\,x)}+d^3e^{(-6\,x)}-d^3}+\frac{3\,c^3\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^4}-\frac{3\,c^3\log\left(e^{(-x)}+1\right)}{d^4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 - 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) - 3*d^3*e^(-4*x) + d^3*e^(-6*x) - d^3) + 3*c^3*log(
-(c - d)*e^(-2*x) + c + d)/d^4 - 3*c^3*log(e^(-x) + 1)/d^4 - 3*c^3*log(e^(-
x) - 1)/d^4) + 3*a*b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-
4*x) - d^2) - c^2*log(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/
d^3 + c^2*log(e^(-x) - 1)/d^3) + 3*a^2*b*(c*log(-(c - d)*e^(-2*x) + c + d)/
d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) -
 a^3*log(d*coth(x) + c)/d
```

Fricas [B] time = 2.78804, size = 4383, normalized size = 56.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 +
 (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)
*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d -
(3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2
*b + b^3)*d^3 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2
)*d^3)*cosh(x)^2 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*
b^2)*d^3 - 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)
*d^3)*cosh(x)^2)*sinh(x)^2 - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sin
h(x)^6 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3
```

```
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(x))/
(cosh(x) - sinh(x))) + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 - (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 - 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 - d^4)*sinh(x)^4
- d^4 + 4*(5*d^4*cosh(x)^3 - 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
- 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 - 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\coth(x))^3 \operatorname{csch}^2(x)}{c + d\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**3*csch(x)**2/(c+d*coth(x)),x)
```

```
[Out] Integral((a + b*coth(x))**3*csch(x)**2/(c + d*coth(x)), x)
```

Giac [B] time = 1.20912, size = 734, normalized size = 9.41

$$\frac{\left(b^3c^4 - 3\,ab^2c^3d + b^3c^3d + 3\,a^2bc^2d^2 - 3\,ab^2c^2d^2 - a^3cd^3 + 3\,a^2bcd^3 - a^3d^4\right)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} - c + d\right|\right)}{cd^4 + d^5} - \frac{\left(b^3c^3 - 3\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d)
)/(c*d^4 + d^5) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(a
bs(e^(2*x) - 1))/d^4 + 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 3
3*a^2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) - 33*b^3*c^3*e^(4*x) + 99*a*b^2*
c^2*d*e^(4*x) - 12*b^3*c^2*d*e^(4*x) - 99*a^2*b*c*d^2*e^(4*x) + 36*a*b^2*c*
d^2*e^(4*x) + 12*b^3*c*d^2*e^(4*x) + 33*a^3*d^3*e^(4*x) - 36*a^2*b*d^3*e^(4
*x) - 36*a*b^2*d^3*e^(4*x) - 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a
*b^2*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b
^2*c*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3
*e^(2*x) + 36*a*b^2*d^3*e^(2*x) - 11*b^3*c^3 + 33*a*b^2*c^2*d - 12*b^3*c^2*
d - 33*a^2*b*c*d^2 + 36*a*b^2*c*d^2 + 11*a^3*d^3 - 36*a^2*b*d^3 - 4*b^3*d^3
)/(d^4*(e^(2*x) - 1)^3)
```

3.1026 $\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}$$

```
[Out] -(a*(a + b*Cosh[x]^2)^4)/(8*b^2) + (a + b*Cosh[x]^2)^5/(10*b^2)
```

Rubi [A] time = 0.0948361, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4335, 266, 43}

$$\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] -(a*(a + b*Cosh[x]^2)^4)/(8*b^2) + (a + b*Cosh[x]^2)^5/(10*b^2)
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx &= \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\cosh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\cosh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\cosh^2(x)\right)\\ &= -\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}\end{aligned}$$

Mathematica [B] time = 0.248408, size = 136, normalized size = 3.78

$$\frac{1}{32}\left(12a^2b\cosh^4(x)+4a^2b\cosh(3x)\cosh^3(x)+4a^3\cosh(2x)+a^3\cosh(4x)+8ab^2\cosh^6(x)+\frac{1}{32}ab^2(48\cosh(2x)+36\cosh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] (12*a^2*b*Cosh[x]^4 + 8*a*b^2*Cosh[x]^6 + 2*b^3*Cosh[x]^8 + 4*a^3*Cosh[2*x]
 + 4*a^2*b*Cosh[x]^3*Cosh[3*x] + a^3*Cosh[4*x] + (a*b^2*(48*Cosh[2*x] + 36*
Cosh[4*x] + 16*Cosh[6*x] + 3*Cosh[8*x]))/32 + (b^3*(140*Cosh[2*x] + 100*Cos
h[4*x] + 50*Cosh[6*x] + 15*Cosh[8*x] + 2*Cosh[10*x]))/320)/32
```

Maple [A] time = 0.007, size = 40, normalized size = 1.1

$$\frac{b^3(\cosh(x))^{10}}{10}+\frac{3ab^2(\cosh(x))^8}{8}+\frac{a^2b(\cosh(x))^6}{2}+\frac{a^3(\cosh(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x)
```

```
[Out] 1/10*b^3*cosh(x)^10+3/8*a*b^2*cosh(x)^8+1/2*a^2*b*cosh(x)^6+1/4*a^3*cosh(x)
^4
```

Maxima [A] time = 1.11638, size = 53, normalized size = 1.47

$$\frac{1}{10}\,b^3\cosh\left(x\right)^{10}+\frac{3}{8}\,ab^2\cosh\left(x\right)^8+\frac{1}{2}\,a^2b\cosh\left(x\right)^6+\frac{1}{4}\,a^3\cosh\left(x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*cosh(x)^10 + 3/8*a*b^2*cosh(x)^8 + 1/2*a^2*b*cosh(x)^6 + 1/4*a^3*c
osh(x)^4
```

Fricas [B] time = 2.20736, size = 1041, normalized size = 28.92

$$\frac{1}{5120}\,b^3\cosh\left(x\right)^{10}+\frac{1}{5120}\,b^3\sinh\left(x\right)^{10}+\frac{1}{1024}\left(3\,ab^2+2\,b^3\right)\cosh\left(x\right)^8+\frac{1}{1024}\left(9\,b^3\cosh\left(x\right)^2+3\,ab^2+2\,b^3\right)\sinh\left(x\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 + 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 + 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
+ 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 + 2*b^3)*cosh(x)^4 + 32*a^3 + 96*a^2*b + 84*a*b^2 + 24*b^3 + 15*(
16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 + 1/512*(64*a^3 + 120*a^2
*b + 84*a*b^2 + 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 +
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^4 + 128*a^3 +
240*a^2*b + 168*a*b^2 + 42*b^3 + 24*(8*a^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

Sympy [B] time = 24.0822, size = 100, normalized size = 2.78

$$\frac{a^3\cosh^4\left(x\right)}{4}+\frac{a^2b\cosh^6\left(x\right)}{2}-\frac{3ab^2\sinh^8\left(x\right)}{8}+\frac{3ab^2\sinh^6\left(x\right)\cosh^2\left(x\right)}{2}-\frac{9ab^2\sinh^4\left(x\right)\cosh^4\left(x\right)}{4}+\frac{3ab^2\sinh^2\left(x\right)\cosh^6\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*(a+b*cosh(x)**2)**3*sinh(x),x)
```

```
[Out] a**3*cosh(x)**4/4 + a**2*b*cosh(x)**6/2 - 3*a*b**2*sinh(x)**8/8 + 3*a*b**2*
sinh(x)**6*cosh(x)**2/2 - 9*a*b**2*sinh(x)**4*cosh(x)**4/4 + 3*a*b**2*sinh(
x)**2*cosh(x)**6/2 + b**3*cosh(x)**10/10
```

Giac [B] time = 1.16365, size = 302, normalized size = 8.39

$$\frac{1}{10240}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^5+\frac{3}{2048}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4+\frac{1}{1024}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4+\frac{1}{128}\,a^2b\left(e^{(2\,x)}+e^{(-2\,x)}\right)^3+\frac{3}{256}\,ab^2\left(e^{(2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 +
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 + 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 + 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 + 1/128*b^3*(e^(2*x) + e^(-2*x))^2 + 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) + 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

3.1027 $\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3 dx$

Optimal. Leaf size=36

$$\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}$$

```
[Out] -(a*(a + b*Sinh[x]^2)^4)/(8*b^2) + (a + b*Sinh[x]^2)^5/(10*b^2)
```

Rubi [A] time = 0.093683, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3198, 266, 43}

$$\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] -(a*(a + b*Sinh[x]^2)^4)/(8*b^2) + (a + b*Sinh[x]^2)^5/(10*b^2)
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx &= \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\sinh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\sinh^2(x)\right)\\ &= -\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}\end{aligned}$$

Mathematica [B] time = 0.583876, size = 114, normalized size = 3.17

$$\frac{-20\left(64a^3+24ab^2-7b^3\right)\cosh(2x)+20\left(16a^3+18ab^2-5b^3\right)\cosh(4x)+b\left(320\sinh^6(x)\left((b-4a)^2-b^2\cosh(2x)\right)-10b\right.}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 - 7*b^3)*Cosh[2*x] + 20*(16*a^3 + 18*a*b^2 - 5*b^3)
*Cosh[4*x] + b*(-10*(16*a - 5*b)*b*Cosh[6*x] + 15*(2*a - b)*b*Cosh[8*x] + 2
*b^2*Cosh[10*x] + 320*((-4*a + b)^2 - b^2*Cosh[2*x])*Sinh[x]^6))/10240
```

Maple [A] time = 0.01, size = 40, normalized size = 1.1

$$\frac{b^3(\sinh(x))^{10}}{10}+\frac{3\,ab^2(\sinh(x))^8}{8}+\frac{a^2b(\sinh(x))^6}{2}+\frac{a^3(\sinh(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x)
```

```
[Out] 1/10*b^3*sinh(x)^10+3/8*a*b^2*sinh(x)^8+1/2*a^2*b*sinh(x)^6+1/4*a^3*sinh(x)
^4
```

Maxima [A] time = 1.02097, size = 53, normalized size = 1.47

$$\frac{1}{10}\,b^{3}\sinh\left(x\right)^{10}+\frac{3}{8}\,ab^{2}\sinh\left(x\right)^{8}+\frac{1}{2}\,a^{2}b\sinh\left(x\right)^{6}+\frac{1}{4}\,a^{3}\sinh\left(x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sinh(x)^10 + 3/8*a*b^2*sinh(x)^8 + 1/2*a^2*b*sinh(x)^6 + 1/4*a^3*s
inh(x)^4
```

Fricas [B] time = 2.10862, size = 1041, normalized size = 28.92

$$\frac{1}{5120}\,b^{3}\cosh\left(x\right)^{10}+\frac{1}{5120}\,b^{3}\sinh\left(x\right)^{10}+\frac{1}{1024}\left(3\,ab^{2}-2\,b^{3}\right)\cosh\left(x\right)^{8}+\frac{1}{1024}\left(9\,b^{3}\cosh\left(x\right)^{2}+3\,ab^{2}-2\,b^{3}\right)\sinh\left(x\right)^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 - 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 - 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
- 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 - 2*b^3)*cosh(x)^4 + 32*a^3 - 96*a^2*b + 84*a*b^2 - 24*b^3 + 15*(
16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 - 1/512*(64*a^3 - 120*a^2
*b + 84*a*b^2 - 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 -
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^4 - 128*a^3 +
240*a^2*b - 168*a*b^2 + 42*b^3 + 24*(8*a^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

Sympy [B] time = 20.6006, size = 82, normalized size = 2.28

$$\frac{a^{3}\sinh^{4}\left(x\right)}{4}+\frac{3a^{2}b\sinh^{4}\left(x\right)\cosh^{2}\left(x\right)}{2}-\frac{3a^{2}b\sinh^{2}\left(x\right)\cosh^{4}\left(x\right)}{2}+\frac{a^{2}b\cosh^{6}\left(x\right)}{2}+\frac{3ab^{2}\sinh^{8}\left(x\right)}{8}+\frac{b^{3}\sinh^{10}\left(x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3*(a+b*sinh(x)**2)**3,x)
```

```
[Out] a**3*sinh(x)**4/4 + 3*a**2*b*sinh(x)**4*cosh(x)**2/2 - 3*a**2*b*sinh(x)**2*
cosh(x)**4/2 + a**2*b*cosh(x)**6/2 + 3*a*b**2*sinh(x)**8/8 + b**3*sinh(x)**
10/10
```

Giac [B] time = 1.23311, size = 302, normalized size = 8.39

$$\frac{1}{10240}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^5+\frac{3}{2048}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4-\frac{1}{1024}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4+\frac{1}{128}\,a^2b\left(e^{(2\,x)}+e^{(-2\,x)}\right)^3-\frac{3}{256}\,ab^2\left(e^{(2\,x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 -
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 - 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 - 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 - 1/128*b^3*(e^(2*x) + e^(-2*x))^2 - 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) - 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

3.1028 $\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Rubi [A] time = 0.0622082, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3198, 261}

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \text{Subst}\left(\int x\sqrt{a+bx^2}\,dx, x, \sinh(x)\right)$$
$$= \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.0120327, size = 19, normalized size = 1.

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Maple [A] time = 0.009, size = 16, normalized size = 0.8

$$\frac{1}{3\,b}\left(a+b\left(\sinh\left(x\right)\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/3*(a+b*sinh(x)^2)^(3/2)/b
```

Maxima [A] time = 1.06767, size = 20, normalized size = 1.05

$$\frac{\left(b\sinh\left(x\right)^{2}+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*sinh(x)^2 + a)^(3/2)/b
```

Fricas [B] time = 2.19973, size = 464, normalized size = 24.42

$$\frac{\sqrt{2}\left(b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a-b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+2\,a-b\right)\sinh(x)^2+4\left(b\right.\right.}{24\left(b\cosh(x)^3+3\,b\cosh(x)^2\sinh(x)+3\,b\cosh(x)\sinh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a -
b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*
a - b)*cosh(x))*sinh(x) + b)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(b*cosh(x)^3 + 3*b*cosh(x)^2*sinh
(x) + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3)
```

Sympy [A] time = 1.90936, size = 46, normalized size = 2.42

$$\begin{cases} \frac{a\sqrt{a+b\sinh^2(x)}}{3b}+\frac{\sqrt{a+b\sinh^2(x)}\sinh^2(x)}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\sinh^2(x)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*sinh(x)**2)/(3*b) + sqrt(a + b*sinh(x)**2)*sinh(x)*
*2/3, Ne(b, 0)), (sqrt(a)*sinh(x)**2/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(x)^2+a}\cosh(x)\sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a)*cosh(x)*sinh(x), x)
```

3.1029 $\int \mathbf{csch}(x)\sqrt{1+\log^2(\coth(x))}\mathbf{sech}(x)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}-\frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

```
[Out] -ArcSinh[Log[Coth[x]]]/2 - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

Rubi [A] time = 0.17014, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6696, 195, 215}

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}-\frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

```
[Out] -ArcSinh[Log[Coth[x]]]/2 - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

Rule 6696

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \mathrm{csch}(x)\sqrt{1+\log^2(\coth(x))}\,\mathrm{sech}(x)\,dx = -\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \log(\coth(x))\right)$$
$$= -\frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \log(\coth$$
$$= -\frac{1}{2}\sinh^{-1}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))}$$

Mathematica [A] time = 0.033826, size = 27, normalized size = 1.

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1} - \frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

```
[Out] -ArcSinh[Log[Coth[x]]]/2 - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

Maple [A] time = 0.074, size = 22, normalized size = 0.8

$$-\frac{\mathrm{Arcsinh}\left(\ln\left(\coth\left(x\right)\right)\right)}{2} - \frac{\ln\left(\coth\left(x\right)\right)}{2}\sqrt{1+\left(\ln\left(\coth\left(x\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)*sech(x)*(1+ln(coth(x))^2)^(1/2),x)
```

```
[Out] -1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\,\mathrm{csch}\left(x\right)\mathrm{sech}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

Fricas [B] time = 2.22986, size = 171, normalized size = 6.33

$$-\frac{1}{2}\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1}\log\left(\frac{\cosh(x)}{\sinh(x)}\right)+\frac{1}{2}\log\left(\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1}-\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(log(cosh(x)/sinh(x))^2 + 1)*log(cosh(x)/sinh(x)) + 1/2*log(sqrt(l
og(cosh(x)/sinh(x))^2 + 1) - log(cosh(x)/sinh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+ln(coth(x))**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\log(\coth(x))^2+1}\,\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

3.1030 $\int \frac{\coth(\sqrt{x})\mathbf{csch}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\text{csch}\left(\sqrt{x}\right)$$

```
[Out] -2*Csch[Sqrt[x]]
```

Rubi [A] time = 0.191162, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\text{csch}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csch[Sqrt[x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\operatorname{Subst}\left(\int \coth(x)\operatorname{csch}(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2i\operatorname{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}\left(\sqrt{x}\right)\right)\right)$$
$$= -2\operatorname{csch}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0168591, size = 8, normalized size = 1.

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csch[Sqrt[x]]
```

Maple [A] time = 0.013, size = 7, normalized size = 0.9

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*csch(x^(1/2))
```

Maxima [B] time = 1.07936, size = 23, normalized size = 2.88

$$\frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

Fricas [B] time = 2.08235, size = 146, normalized size = 18.25

$$-\frac{4\left(\cosh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^{2}+2\,\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x**(1/2))*csch(x**(1/2))/x**(1/2),x)
```

```
[Out] Integral(coth(sqrt(x))*csch(sqrt(x))/sqrt(x), x)
```

Giac [B] time = 1.13097, size = 23, normalized size = 2.88

$$\frac{4}{e^{\left(-\sqrt{x}\right)}-e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="giac")
```


```
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

3.1031 $\int \frac{\cosh(\sqrt{x})\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$\sinh^2\left(\sqrt{x}\right)$$

```
[Out] Sinh[Sqrt[x]]^2
```

Rubi [A] time = 0.0140994, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5370}

$$\sinh^2\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sinh[Sqrt[x]]^2
```

Rule 5370

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]
^(p_.), x_Symbol] :> Simp[Sinh[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; Free
Q[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0040892, size = 12, normalized size = 1.5

$$\frac{1}{2}\cosh\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Cosh[2*Sqrt[x]]/2
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\left(\cosh\left(\sqrt{x}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x)
```

```
[Out] cosh(x^(1/2))^2
```

Maxima [A] time = 1.17754, size = 8, normalized size = 1.

$$\cosh\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] cosh(sqrt(x))^2
```

Fricas [B] time = 1.95028, size = 58, normalized size = 7.25

$$\frac{1}{2}\cosh\left(\sqrt{x}\right)^2 + \frac{1}{2}\sinh\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(sqrt(x))^2 + 1/2*sinh(sqrt(x))^2
```

Sympy [A] time = 0.362589, size = 7, normalized size = 0.88

$$\cosh^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x**(1/2))*sinh(x**(1/2))/x**(1/2),x)
```

```
[Out] cosh(sqrt(x))**2
```

Giac [B] time = 1.11933, size = 23, normalized size = 2.88

$$\frac{1}{4}\, e^{\left(2\,\sqrt{x}\right)} + \frac{1}{4}\, e^{\left(-2\,\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*e^(2*sqrt(x)) + 1/4*e^(-2*sqrt(x))
```

3.1032 $\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

```
[Out] -2*Sech[Sqrt[x]]
```

Rubi [A] time = 0.188749, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Sech[Sqrt[x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\text{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \text{sech}(x)\tanh(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int 1\,dx, x, \text{sech}\left(\sqrt{x}\right)\right)\right)$$
$$= -2\text{sech}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0156439, size = 8, normalized size = 1.

$$-2\text{sech}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Sech[Sqrt[x]]
```

Maple [A] time = 0.012, size = 7, normalized size = 0.9

$$-2\,\text{sech}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*sech(x^(1/2))
```

Maxima [B] time = 1.04147, size = 20, normalized size = 2.5

$$-\frac{4}{e^{\left(-\sqrt{x}\right)} + e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))
```

Fricas [B] time = 1.76584, size = 146, normalized size = 18.25

$$-\frac{4\left(\cosh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^{2}+2\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 + 1)
```

Sympy [A] time = 0.431821, size = 8, normalized size = 1.

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x**(1/2))*tanh(x**(1/2))/x**(1/2),x)
```

```
[Out] -2*sech(sqrt(x))
```

Giac [B] time = 1.12011, size = 20, normalized size = 2.5

$$-\frac{4}{e^{\left(-\sqrt{x}\right)}+e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))
```

3.1033 $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(2x))}{4b}$$

```
[Out] ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[a + b*Si
nh[2*x]]/(4*b)
```

Rubi [A] time = 0.162048, antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1075, 12, 634, 618, 206, 628, 260}

$$\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}+\frac{\log(\cosh(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[Cosh[x]]
/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = -\text{Subst}\left(\int \frac{x^2}{(-1+x^2)(a+2bx-ax^2)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\text{Subst}\left(\int -\frac{2bx}{-1+x^2}\,dx, x, \tanh(x)\right)}{4b^2} + \frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, \tanh(x)\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{x}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{2b}$$
$$= \frac{\log(\cosh(x))}{2b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \tanh(x)\right.}{4b}$$
$$= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} + \text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\,\text{t}\right.$$
$$= \frac{\tanh^{-1}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}$$

Mathematica [A] time = 0.0837305, size = 59, normalized size = 1.13

$$\frac{1}{4}\left(\frac{\log(a+b\sinh(2x))}{b} - \frac{2\tan^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ((-2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b
*Sinh[2*x]]/b)/4
```

Maple [A] time = 0.053, size = 75, normalized size = 1.4

$$-\frac{\ln(1+\tanh(x))}{4b} + \frac{\ln\left(a(\tanh(x))^2 - 2b\tanh(x) - a\right)}{4b} - \frac{1}{2}\text{Artanh}\left(\frac{2a\tanh(x)-2b}{2}\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}} - \frac{\ln(\text{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(2*x)),x)
```

```
[Out] -1/4/b*ln(1+tanh(x))+1/4/b*ln(a*tanh(x)^2-2*b*tanh(x)-a)-1/2/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2))-1/4/b*ln(tanh(x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89932, size = 730, normalized size = 14.04

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2+b^2+4\left(b^2\cosh(x)^3+ab\cosh(x)\right)\sinh(x)}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(x)^2+4\left(b\cosh(x)^3-\right.}\right.}{4\left(a^2b+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sinh(2*x)), x)
```

Giac [A] time = 1.21357, size = 124, normalized size = 2.38

$$-\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}}-\frac{x}{2\,b}+\frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*
a*e^(2*x) - b))/b
```

3.1034 $\int \frac{\cosh^2(x)}{a+b\sinh(2x)} \, dx$

Optimal. Leaf size=52

$$\frac{\log(a+b\sinh(2x))}{4b}-\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

```
[Out] -ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[a + b*S
inh[2*x]]/(4*b)
```

Rubi [A] time = 0.129991, antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {981, 634, 618, 206, 628, 12, 260}

$$-\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}+\frac{\log(\cosh(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] -ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[Cosh[x]
]/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 981

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)(a+2bx-ax^2)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{2bx}{1-x^2}\,dx, x, \tanh(x)\right)}{4b^2} - \frac{\text{Subst}\left(\int \frac{-4b^2+2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b} + \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\right.}{}$$
$$= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} - \text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\,\text{ta}\right.$$
$$= -\frac{\tanh^{-1}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}$$

Mathematica [A] time = 0.0607836, size = 59, normalized size = 1.13

$$\frac{1}{4}\left(\frac{2\tan^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\log(a+b\sinh(2x))}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ((2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b*
Sinh[2*x]]/b)/4
```

Maple [A] time = 0.055, size = 75, normalized size = 1.4

$$-\frac{\ln(1+\tanh(x))}{4b} + \frac{\ln\left(a(\tanh(x))^2-2b\tanh(x)-a\right)}{4b} + \frac{1}{2}\text{Artanh}\left(\frac{2a\tanh(x)-2b}{2}\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}} - \frac{\ln(\tan}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(2*x)),x)
```

```
[Out] -1/4/b*ln(1+tanh(x))+1/4/b*ln(a*tanh(x)^2-2*b*tanh(x)-a)+1/2/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2))-1/4/b*ln(tanh(x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9327, size = 730, normalized size = 14.04

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2+b^2+4\left(b^2\cosh(x)^3+ab\cosh(x)\right)\sinh(}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(x)^2+4\left(b\cosh(x)\right.}\right.}{4\left(a^2b+b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) - 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(2*x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sinh(2*x)), x)
```

Giac [A] time = 1.15244, size = 124, normalized size = 2.38

$$\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}}-\frac{x}{2\,b}+\frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*a
*e^(2*x) - b))/b
```

3.1035 $\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

```
[Out] x/(2*b) - (Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*Sqrt[
a - b]*b)
```

Rubi [A] time = 0.134041, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1130, 208}

$$\frac{x}{2b}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*Sqrt[
a - b]*b)
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = -\,\mathrm{Subst}\left(\int \frac{x^2}{-a-b+2ax^2+(-a+b)x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\left(-1+\frac{a}{b}\right)\mathrm{Subst}\left(\int \frac{1}{a-b+(-a+b)x^2}\,dx, x, \tanh(x)\right) - \frac{(a+b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \mathrm{ta}\right.}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}b}$$

Mathematica [A] time = 0.0774598, size = 48, normalized size = 0.92

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{(a-b)\tanh(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] (x + ((a + b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [B] time = 0.034, size = 92, normalized size = 1.8

$$\frac{\ln(1+\tanh(x))}{4b} - \frac{a}{2b}\mathrm{Artanh}\left((a-b)\tanh(x)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+b)(a-b)}} - \frac{1}{2}\mathrm{Artanh}\left((a-b)\tanh(x)\frac{1}{\sqrt{(a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(2*x)),x)
```

```
[Out] 1/4/b*ln(1+tanh(x))-1/2/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*
(a-b))^(1/2))*a-1/2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))
^(1/2))-1/4/b*ln(tanh(x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96352, size = 821, normalized size = 15.79

$$\left[\frac{\sqrt{\frac{a+b}{a-b}}\log\left(\frac{b^{2}\cosh(x)^{4}+4\,b^{2}\cosh(x)\sinh(x)^{3}+b^{2}\sinh(x)^{4}+2\,ab\cosh(x)^{2}+2\left(3\,b^{2}\cosh(x)^{2}+ab\right)\sinh(x)^{2}+2\,a^{2}-b^{2}+4\left(b^{2}\cosh(x)^{3}+ab\cosh(x)\right)\sinh(x)+2}{b\cosh(x)^{4}+4\,b\cosh(x)\sinh(x)^{3}+b\sinh(x)^{4}+2\,a\cosh(x)^{2}+2\left(3\,b\cosh(x)^{2}+a\right)\sinh(x)^{2}+4\left(b\,c\right.}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((a + b)/(a - b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b - b^2)*cosh(x)
^2 + 2*(a*b - b^2)*cosh(x)*sinh(x) + (a*b - b^2)*sinh(x)^2 + a^2 - a*b)*sqr
t((a + b)/(a - b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, -1/2*(sqrt(-(a + b)/(a - b))*arctan((b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a + b)/(a - b))/(a + b)) - x
)/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}(x)}{a+b\cosh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*cosh(2*x)), x)
```

Giac [A] time = 1.14763, size = 63, normalized size = 1.21

$$-\frac{(a+b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b}+\frac{x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

3.1036 $\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*b*Sqr
t[a + b])
```

Rubi [A] time = 0.106355, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 208}

$$\frac{x}{2b}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*b*Sqr
t[a + b])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \text{Subst}\left(\int \frac{1}{a+b-2ax^2+(a-b)x^4}\,dx,x,\tanh(x)\right)$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\tanh(x)\right)}{2b} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+(a-b)x^2}\,dx,x,\tanh(x)\right)}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A] time = 0.0607476, size = 50, normalized size = 0.96

$$\frac{\frac{(a-b)\tan^{-1}\left(\frac{(a-b)\tanh(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] (x + ((a - b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [B] time = 0.033, size = 92, normalized size = 1.8

$$\frac{\ln(1+\tanh(x))}{4b} - \frac{a}{2b}\text{Artanh}\left((a-b)\tanh(x)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+b)(a-b)}} + \frac{1}{2}\text{Artanh}\left((a-b)\tanh(x)\frac{1}{\sqrt{(a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(2*x)),x)
```

```
[Out] 1/4/b*ln(1+tanh(x))-1/2/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*
(a-b))^(1/2))*a+1/2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))
^(1/2))-1/4/b*ln(tanh(x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96277, size = 821, normalized size = 15.79

$$\left[\frac{\sqrt{\frac{a-b}{a+b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2-b^2+4\left(b^2\cosh(x)^3+ab\cosh(x)\right)\sinh(x)+2}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(x)^2+4\left(b\right.}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((a - b)/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b + b^2)*cosh(x)
^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a^2 + a*b)*sqr
t((a - b)/(a + b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, 1/2*(sqrt(-(a - b)/(a + b))*arctan(-(b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a - b)/(a + b))/(a - b)) + x
)/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*cosh(2*x)), x)
```

Giac [A] time = 1.14756, size = 66, normalized size = 1.27

$$-\frac{(a-b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b}+\frac{x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

3.1037 $\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] ArcTan[Sqrt[a*Sinh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rubi [A] time = 0.0385723, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
```

```
[Out] ArcTan[Sqrt[a*Sinh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{ax}(1+x)}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sinh^2(c+dx)}\right)}{ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0371999, size = 31, normalized size = 1.03

$$\frac{\sinh(c+dx)\tan^{-1}(\sinh(c+dx))}{d\sqrt{a\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
```

```
[Out] (ArcTan[Sinh[c + d*x]]*Sinh[c + d*x])/(d*Sqrt[a*Sinh[c + d*x]^2])
```

Maple [C] time = 0.114, size = 39, normalized size = 1.3

$$\frac{1}{d}\text{` int/indef0`}\left(\frac{\sinh(dx+c)}{(\cosh(dx+c))^2}\frac{1}{\sqrt{a(\sinh(dx+c))^2}}, \sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(d*x+c)/cosh(d*x+c)^2/(a*sinh(d*x+c)^2)^(1/2),sinh(d*x+c))
/d
```

Maxima [A] time = 1.63042, size = 24, normalized size = 0.8

$$\frac{2\,\arctan\left(e^{(-dx-c)}\right)}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-d*x - c))/(sqrt(a)*d)
```

Fricas [B] time = 1.88352, size = 842, normalized size = 28.07

$$\left[-\frac{\sqrt{-a}\log\left(-\frac{a\cosh(dx+c)^2+2\sqrt{ae^{(4dx+4c)}-2ae^{(2dx+2c)}+a}\left(\cosh(dx+c)e^{(dx+c)}+e^{(dx+c)}\sinh(dx+c)\right)\sqrt{-a}e^{(-dx-c)}-\left(ae^{(2dx+2c)}-a\right)\sinh(dx+c)^2-(a\cosh(dx\cdots}{\left(e^{(2dx+2c)}-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+\left(\cosh(dx+c)^2+1\right)e^{(2dx+2c)}+2\left(\cosh(dx+c)e^{(2dx+2c)}\cdots\right.}\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(-(a*cosh(d*x + c)^2 + 2*sqrt(a*e^(4*d*x + 4*c) - 2*a*e^(2*d*
x + 2*c) + a)*(cosh(d*x + c)*e^(d*x + c) + e^(d*x + c)*sinh(d*x + c))*sqrt(
-a)*e^(-d*x - c) - (a*e^(2*d*x + 2*c) - a)*sinh(d*x + c)^2 - (a*cosh(d*x +
c)^2 - a)*e^(2*d*x + 2*c) - 2*(a*cosh(d*x + c)*e^(2*d*x + 2*c) - a*cosh(d*x
 + c))*sinh(d*x + c) - a)/((e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 - cosh(d*x
 + c)^2 + (cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 2*(cosh(d*x + c)*e^(2*d*x
 + 2*c) - cosh(d*x + c))*sinh(d*x + c) - 1))/(a*d), 2*sqrt(a*e^(4*d*x + 4*c
) - 2*a*e^(2*d*x + 2*c) + a)*arctan(cosh(d*x + c) + sinh(d*x + c))/(a*d*e^(
2*d*x + 2*c) - a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh{(c + dx)}}{\sqrt{a \sinh^{2}{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)/sqrt(a*sinh(c + d*x)**2), x)
```

Giac [A] time = 1.2259, size = 49, normalized size = 1.63

$$\frac{2 \arctan\left(e^{(dx+c)}\right)}{\sqrt{a} d \operatorname{sgn}\left(e^{(3\,dx+3\,c)} - e^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(d*x + c))/(sqrt(a)*d*sgn(e^(3*d*x + 3*c) - e^(d*x + c)))
```

3.1038 $\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rubi [A] time = 0.0394633, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(c+dx)}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0661061, size = 49, normalized size = 1.58

$$\frac{\cosh(c+dx)\left(\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
```

```
[Out] (Cosh[c + d*x]*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]))/(d*Sqrt[
a*Cosh[c + d*x]^2])
```

Maple [A] time = 0.077, size = 31, normalized size = 1.

$$-\frac{\cosh(dx+c)\,\text{Artanh}(\cosh(dx+c))}{d}\frac{1}{\sqrt{a(\cosh(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/(a*cosh(d*x+c)^2)^(1/2)*cosh(d*x+c)*arctanh(cosh(d*x+c))/d
```

Maxima [A] time = 1.71249, size = 54, normalized size = 1.74

$$-\frac{\log\left(e^{(-dx-c)}+1\right)}{\sqrt{a}d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(e^(-d*x - c) + 1)/(sqrt(a)*d) + log(e^(-d*x - c) - 1)/(sqrt(a)*d)
```

Fricas [B] time = 1.85583, size = 455, normalized size = 14.68

$$\left[\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\log\left(\frac{\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{ade^{(2\,dx+2\,c)}+ad}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\sqrt{-a}}{a\cosh(dx+c)e^{(2\,dx+2\,c)}+a\cosh(dx+c)+\left(ae^{(2\,dx+2\,c)}+a\right)\sinh}\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*log((cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/(a*d*e^(2*d*x + 2*c) +
 a*d), 2*sqrt(-a)*arctan(sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*
sqrt(-a)/(a*cosh(d*x + c)*e^(2*d*x + 2*c) + a*cosh(d*x + c) + (a*e^(2*d*x +
 2*c) + a)*sinh(d*x + c)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(c+dx\right)}{\sqrt{a\cosh^{2}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(coth(c + d*x)/sqrt(a*cosh(c + d*x)**2), x)
```

Giac [A] time = 1.20626, size = 46, normalized size = 1.48

$$-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{\sqrt{a}}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{\sqrt{a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(e^(d*x + c) + 1)/sqrt(a) - log(abs(e^(d*x + c) - 1))/sqrt(a))/d
```

3.1039 $\int x \cosh(2x)\mathbf{sech}(x)\, dx$

Optimal. Leaf size=43

$$i\text{PolyLog}\left(2,-ie^x\right)-i\text{PolyLog}\left(2,ie^x\right)-2x\tan^{-1}\left(e^x\right)+2x\sinh(x)-2\cosh(x)$$

```
[Out] -2*x*ArcTan[E^x] - 2*Cosh[x] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2, I*E^x]
 + 2*x*Sinh[x]
```

Rubi [A] time = 0.0725748, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5473, 3296, 2638, 5449, 4180, 2279, 2391}

$$i\text{PolyLog}\left(2,-ie^x\right)-i\text{PolyLog}\left(2,ie^x\right)-2x\tan^{-1}\left(e^x\right)+2x\sinh(x)-2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x],x]
```


```
[Out] -2*x*ArcTan[E^x] - 2*Cosh[x] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2, I*E^x]
 + 2*x*Sinh[x]
```


Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cosh(2x)\operatorname{sech}(x)\,dx &= \int (x\cosh(x)+x\sinh(x)\tanh(x))\,dx\\
&= \int x\cosh(x)\,dx+\int x\sinh(x)\tanh(x)\,dx\\
&= x\sinh(x)+\int x\cosh(x)\,dx-\int x\operatorname{sech}(x)\,dx-\int \sinh(x)\,dx\\
&= -2x\tan^{-1}\left(e^x\right)-\cosh(x)+2x\sinh(x)+i\int \log\left(1-ie^x\right)\,dx-i\int \log\left(1+ie^x\right)\,dx-\int \sinh(\\
&= -2x\tan^{-1}\left(e^x\right)-2\cosh(x)+2x\sinh(x)+i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^x\right)-i\,\text{Subst}\left(\int \frac{\log}{}\right.\\
&= -2x\tan^{-1}\left(e^x\right)-2\cosh(x)+i\text{Li}_2\left(-ie^x\right)-i\text{Li}_2\left(ie^x\right)+2x\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0318339, size = 71, normalized size = 1.65

$$i\left(\text{PolyLog}\left(2,-ie^{-x}\right)-\text{PolyLog}\left(2,ie^{-x}\right)\right)+ix\left(\log\left(1-ie^{-x}\right)-\log\left(1+ie^{-x}\right)\right)+2x\sinh(x)-2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x],x]
```

```
[Out] -2*Cosh[x] + I*x*(Log[1 - I/E^x] - Log[1 + I/E^x]) + I*(PolyLog[2, (-I)/E^x
] - PolyLog[2, I/E^x]) + 2*x*Sinh[x]
```

Maple [A] time = 0.038, size = 68, normalized size = 1.6

$$2\left(-1/2+x/2\right)\mathrm{e}^{x}+2\left(-1/2-x/2\right)\mathrm{e}^{-x}+ix\ln\left(1+i\mathrm{e}^{x}\right)-ix\ln\left(1-i\mathrm{e}^{x}\right)+i\mathrm{dilog}\left(1+i\mathrm{e}^{x}\right)-i\mathrm{dilog}\left(1-i\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x),x)
```

```
[Out] 2*(-1/2+1/2*x)*exp(x)+2*(-1/2-1/2*x)*exp(-x)+I*x*ln(1+I*exp(x))-I*x*ln(1-I*
exp(x))+I*dilog(1+I*exp(x))-I*dilog(1-I*exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-(x+1)e^{(-x)}+(x-1)e^{x}-2\int\frac{xe^{x}}{e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + (x - 1)*e^x - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

Fricas [B] time = 1.835, size = 450, normalized size = 10.47

$$\frac{(x-1)\cosh(x)^{2}+2(x-1)\cosh(x)\sinh(x)+(x-1)\sinh(x)^{2}+(-i\cosh(x)-i\sinh(x))\mathrm{Li}_{2}(i\cosh(x)+i\sinh(x))}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x)^2 + 2*(x - 1)*cosh(x)*sinh(x) + (x - 1)*sinh(x)^2 + (-I*co
sh(x) - I*sinh(x))*dilog(I*cosh(x) + I*sinh(x)) + (I*cosh(x) + I*sinh(x))*d
ilog(-I*cosh(x) - I*sinh(x)) + (I*x*cosh(x) + I*x*sinh(x))*log(I*cosh(x) +
I*sinh(x) + 1) + (-I*x*cosh(x) - I*x*sinh(x))*log(-I*cosh(x) - I*sinh(x) +
1) - x - 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(2x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(2x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(2*x)*sech(x), x)
```

3.1040 $\int x \cosh(2x)\mathbf{sech}^2(x)\,dx$

Optimal. Leaf size=12

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Rubi [A] time = 0.0358555, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5473, 3720, 3475, 30}

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x]^2,x]
```

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cosh(2x)\text{sech}^2(x)\,dx &= \int \left(x + x\tanh^2(x)\right)\,dx\\ &= \frac{x^2}{2} + \int x\tanh^2(x)\,dx\\ &= \frac{x^2}{2} - x\tanh(x) + \int x\,dx + \int \tanh(x)\,dx\\ &= x^2 + \log(\cosh(x)) - x\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0203948, size = 12, normalized size = 1.

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^2,x]
```

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Maple [B] time = 0.032, size = 26, normalized size = 2.2

$$x^2 - 2\,x + 2\,\frac{x}{e^{2\,x}+1} + \ln\left(e^{2\,x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x)^2,x)
```

```
[Out] x^2-2*x+2*x/(exp(2*x)+1)+ln(exp(2*x)+1)
```

Maxima [B] time = 1.62947, size = 45, normalized size = 3.75

$$\frac{x^2+\left(x^2-2\,x\right)e^{(2\,x)}}{e^{(2\,x)}+1}+\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="maxima")
```

```
[Out] (x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

Fricas [B] time = 1.82887, size = 297, normalized size = 24.75

$$\frac{\left(x^2-2\,x\right)\cosh\left(x\right)^2+2\left(x^2-2\,x\right)\cosh\left(x\right)\sinh\left(x\right)+\left(x^2-2\,x\right)\sinh\left(x\right)^2+x^2+\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\right.}{\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x)*cosh(x)^2 + 2*(x^2 - 2*x)*cosh(x)*sinh(x) + (x^2 - 2*x)*sinh(x
)^2 + x^2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(
cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh\left(2x\right)\operatorname{sech}^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)**2,x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x)**2, x)
```

Giac [B] time = 1.14985, size = 63, normalized size = 5.25

$$\frac{x^2e^{(2x)} + x^2 - 2\,xe^{(2x)} + e^{(2x)}\log\left(e^{(2x)} + 1\right) + \log\left(e^{(2x)} + 1\right)}{e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="giac")
```

```
[Out] (x^2*e^(2*x) + x^2 - 2*x*e^(2*x) + e^(2*x)*log(e^(2*x) + 1) + log(e^(2*x) +
 1))/(e^(2*x) + 1)
```

3.1041 $\int x \cosh(2x)\mathbf{sech}^3(x)\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}i\text{PolyLog}\left(2,-ie^x\right)+\frac{3}{2}i\text{PolyLog}\left(2,ie^x\right)+3x\tan^{-1}\left(e^x\right)-\frac{\text{sech}(x)}{2}-\frac{1}{2}x\tanh(x)\text{sech}(x)$$

```
[Out] 3*x*ArcTan[E^x] - ((3*I)/2)*PolyLog[2, (-I)*E^x] + ((3*I)/2)*PolyLog[2, I*E
^x] - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

Rubi [A] time = 0.131812, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5473, 4180, 2279, 2391, 5455, 4185}

$$-\frac{3}{2}i\text{PolyLog}\left(2,-ie^x\right)+\frac{3}{2}i\text{PolyLog}\left(2,ie^x\right)+3x\tan^{-1}\left(e^x\right)-\frac{\text{sech}(x)}{2}-\frac{1}{2}x\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] 3*x*ArcTan[E^x] - ((3*I)/2)*PolyLog[2, (-I)*E^x] + ((3*I)/2)*PolyLog[2, I*E
^x] - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int \left(x\text{sech}(x) + x\text{sech}(x)\tanh^2(x)\right)\,dx$$

$$= \int x\text{sech}(x)\,dx + \int x\text{sech}(x)\tanh^2(x)\,dx$$

$$= 2x\tan^{-1}(e^x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie^x)\,dx + \int x\text{sech}(x)\,dx - \int x\text{sech}^3(x)\,dx$$

$$= 4x\tan^{-1}(e^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie^x)\,dx -$$

$$= 3x\tan^{-1}(e^x) - i\text{Li}_2(-ie^x) + i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\int \log(1-ie^x)\,d$$

$$= 3x\tan^{-1}(e^x) - 2i\text{Li}_2(-ie^x) + 2i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log}{}\right.$$

$$= 3x\tan^{-1}(e^x) - \frac{3}{2}i\text{Li}_2(-ie^x) + \frac{3}{2}i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x)$$

Mathematica [A] time = 0.0640587, size = 78, normalized size = 1.47

$$-\frac{1}{2}i\left(3\text{PolyLog}\left(2,-ie^{-x}\right)-3\text{PolyLog}\left(2,ie^{-x}\right)+3x\log\left(1-ie^{-x}\right)-3x\log\left(1+ie^{-x}\right)-i\,\mathbf{sech}(x)-ix\tanh(x)\mathbf{sech}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] (-I/2)*(3*x*Log[1 - I/E^x] - 3*x*Log[1 + I/E^x] + 3*PolyLog[2, (-I)/E^x] -
3*PolyLog[2, I/E^x] - I*Sech[x] - I*x*Sech[x]*Tanh[x])
```

Maple [A] time = 0.037, size = 75, normalized size = 1.4

$$-\frac{e^{x}\left(xe^{2x}+e^{2x}-x+1\right)}{\left(e^{2x}+1\right)^{2}}-\frac{3i}{2}x\ln\left(1+ie^{x}\right)+\frac{3i}{2}x\ln\left(1-ie^{x}\right)-\frac{3i}{2}\text{dilog}\left(1+ie^{x}\right)+\frac{3i}{2}\text{dilog}\left(1-ie^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x)^3,x)
```

```
[Out] -exp(x)*(x*exp(2*x)+exp(2*x)-x+1)/(exp(2*x)+1)^2-3/2*I*x*ln(1+I*exp(x))+3/2
*I*x*ln(1-I*exp(x))-3/2*I*dilog(1+I*exp(x))+3/2*I*dilog(1-I*exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(x+1)e^{(3x)}-(x-1)e^{x}}{e^{(4x)}+2e^{(2x)}+1}+12\int\frac{xe^{x}}{4\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="maxima")
```

```
[Out] -((x + 1)*e^(3*x) - (x - 1)*e^x)/(e^(4*x) + 2*e^(2*x) + 1) + 12*integrate(1
/4*x*e^x/(e^(2*x) + 1), x)
```

Fricas [B] time = 1.93736, size = 1442, normalized size = 27.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*cosh(x)^3 + 6*(x + 1)*cosh(x)*sinh(x)^2 + 2*(x + 1)*sinh(x)
^3 - 2*(x - 1)*cosh(x) - (3*I*cosh(x)^4 + 12*I*cosh(x)*sinh(x)^3 + 3*I*sinh
(x)^4 + (18*I*cosh(x)^2 + 6*I)*sinh(x)^2 + 6*I*cosh(x)^2 + (12*I*cosh(x)^3
+ 12*I*cosh(x))*sinh(x) + 3*I)*dilog(I*cosh(x) + I*sinh(x)) - (-3*I*cosh(x)
^4 - 12*I*cosh(x)*sinh(x)^3 - 3*I*sinh(x)^4 + (-18*I*cosh(x)^2 - 6*I)*sinh(
x)^2 - 6*I*cosh(x)^2 + (-12*I*cosh(x)^3 - 12*I*cosh(x))*sinh(x) - 3*I)*dilo
g(-I*cosh(x) - I*sinh(x)) - (-3*I*x*cosh(x)^4 - 12*I*x*cosh(x)*sinh(x)^3 -
3*I*x*sinh(x)^4 - 6*I*x*cosh(x)^2 + (-18*I*x*cosh(x)^2 - 6*I*x)*sinh(x)^2 +
 (-12*I*x*cosh(x)^3 - 12*I*x*cosh(x))*sinh(x) - 3*I*x)*log(I*cosh(x) + I*si
nh(x) + 1) - (3*I*x*cosh(x)^4 + 12*I*x*cosh(x)*sinh(x)^3 + 3*I*x*sinh(x)^4
+ 6*I*x*cosh(x)^2 + (18*I*x*cosh(x)^2 + 6*I*x)*sinh(x)^2 + (12*I*x*cosh(x)^
3 + 12*I*x*cosh(x))*sinh(x) + 3*I*x)*log(-I*cosh(x) - I*sinh(x) + 1) + 2*(3
*(x + 1)*cosh(x)^2 - x + 1)*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(
x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(2x)\operatorname{sech}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)**3,x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cosh(2x)\operatorname{sech}(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(2*x)*sech(x)^3, x)
```

3.1042 $\int \sqrt{\operatorname{csch}(x)}(x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\,dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\operatorname{csch}(x)}} - \frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[Out] (2*x)/Sqrt[Csch[x]] - (4*Sech[x])/Csch[x]^(3/2)
```

Rubi [A] time = 0.166912, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6742, 5445, 3771, 2639, 2626}

$$\frac{2x}{\sqrt{\operatorname{csch}(x)}} - \frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
```

```
[Out] (2*x)/Sqrt[Csch[x]] - (4*Sech[x])/Csch[x]^(3/2)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\text{csch}(x)}(x\cosh(x)-4\text{sech}(x)\tanh(x))\,dx &= \int \left(x\cosh(x)\sqrt{\text{csch}(x)}-\frac{4\text{sech}^2(x)}{\sqrt{\text{csch}(x)}}\right)dx\\ &= -\left(4\int \frac{\text{sech}^2(x)}{\sqrt{\text{csch}(x)}}\,dx\right)+\int x\cosh(x)\sqrt{\text{csch}(x)}\,dx\\ &= \frac{2x}{\sqrt{\text{csch}(x)}}-\frac{4\text{sech}(x)}{\text{csch}^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 1.10116, size = 17, normalized size = 0.85

$$\frac{2(x\text{csch}(x)-2\text{sech}(x))}{\text{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
```

```
[Out] (2*(x*Csch[x] - 2*Sech[x]))/Csch[x]^(3/2)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \sqrt{\text{csch}\,(x)}\,(x\cosh\,(x)-4\,\text{sech}\,(x)\tanh\,(x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

```
[Out] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x\cosh(x) - 4\,\mathrm{sech}(x)\tanh(x))\sqrt{\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="maxima"
)
```

```
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x\cosh(x) - 4\,\mathrm{sech}(x)\tanh(x))\sqrt{\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

3.1043 $\int \sinh(x)(\cosh(x)+\sinh(x))\,dx$

Optimal. Leaf size=22

$$-\frac{x}{2}+\frac{\sinh^2(x)}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] -x/2 + (Cosh[x]*Sinh[x])/2 + Sinh[x]^2/2
```

Rubi [A] time = 0.0342234, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3089, 2564, 30, 2635, 8}

$$-\frac{x}{2}+\frac{\sinh^2(x)}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

```
[Out] -x/2 + (Cosh[x]*Sinh[x])/2 + Sinh[x]^2/2
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(x)(\cosh(x)+\sinh(x))\,dx &= -\left(i\int\left(i\cosh(x)\sinh(x)+i\sinh^2(x)\right)\,dx\right)\\ &= \int\cosh(x)\sinh(x)\,dx+\int\sinh^2(x)\,dx\\ &= \frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-\text{Subst}(\int x\,dx,x,i\sinh(x))\\ &= -\frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0041325, size = 22, normalized size = 1.

$$-\frac{x}{2}+\frac{1}{4}\sinh(2x)+\frac{\cosh^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

```
[Out] -x/2 + Cosh[x]^2/2 + Sinh[2*x]/4
```

Maple [A] time = 0.007, size = 17, normalized size = 0.8

$$\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}+\frac{(\cosh(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*(cosh(x)+sinh(x)),x)
```

[Out] 1/2*cosh(x)*sinh(x)-1/2*x+1/2*cosh(x)^2

Maxima [A] time = 1.01952, size = 30, normalized size = 1.36

$$\frac{1}{2}\cosh(x)^2 - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(x)^2 - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 2.03138, size = 89, normalized size = 4.05

$$-\frac{(2x-1)\cosh(x) - (2x+1)\sinh(x)}{4(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.21644, size = 31, normalized size = 1.41

$$\frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} + \frac{\cosh^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x)
```

```
[Out] x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2 + cosh(x)**2/2
```

Giac [A] time = 1.14004, size = 14, normalized size = 0.64

$$-\frac{1}{2}x + \frac{1}{4}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*x + 1/4*e^(2*x)
```

3.1044 $\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=69

$$\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

```
[Out] Log[1 - Tanh[x/2]]/4 + (3*Log[1 + Tanh[x/2]])/4 + 1/(2*(1 - Tanh[x/2])) - 1
/(2*(1 + Tanh[x/2])^2) + (1 + Tanh[x/2])^(-1)
```

Rubi [A] time = 0.200526, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4397, 12, 894}

$$\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] Log[1 - Tanh[x/2]]/4 + (3*Log[1 + Tanh[x/2]])/4 + 1/(2*(1 - Tanh[x/2])) - 1
/(2*(1 + Tanh[x/2])^2) + (1 + Tanh[x/2])^(-1)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx &= \int \frac{\cosh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{2(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}+\frac{1}{4(-1+x)}+\frac{1}{(1+x)^3}-\frac{1}{(1+x)^2}+\frac{3}{4(1+x)}\right)dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^2}+\end{aligned}$$

Mathematica [A] time = 0.0344364, size = 37, normalized size = 0.54

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{\cosh(x)}{2}-\frac{1}{8}\cosh(2x)-\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/4 + Cosh[x]/2 - Cosh[2*x]/8 - Log[Cosh[x/2]] + Sinh[2*x]/8
```

Maple [A] time = 0.039, size = 48, normalized size = 0.7

$$-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3}{4}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{1}{4}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] -1/2/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)+3/4*ln(tanh(1/2*x)+1)-1/2/(tanh(1/
2*x)-1)+1/4*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05191, size = 39, normalized size = 0.57

$$-\frac{1}{4}\,x+\frac{1}{4}\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{4}\,e^{x}-\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*x + 1/4*e^(-x) - 1/8*e^(-2*x) + 1/4*e^x - log(e^(-x) + 1)
```

Fricas [A] time = 2.03597, size = 346, normalized size = 5.01

$$\frac{6\,x\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)^{3}+6\,\left(x+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,\sinh\left(x\right)^{3}-8\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\log}{8\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 1.41054, size = 382, normalized size = 5.54

$$-\frac{x\tanh^{3}\left(\frac{x}{2}\right)}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh^{2}\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}-\frac{x\tanh^{2}\left(\frac{x}{2}\right)}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh^{2}\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}+\frac{x\tanh\left(\frac{x}{2}\right.}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh^{2}\left(\frac{x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] -x*tanh(x/2)**3/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - x*tan
h(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x*tanh(x/2)
/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x/(4*tanh(x/2)**3 +
```

```
4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(x/2)**3/(4*ta
nh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(
x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*log(tanh(x/
2) + 1)*tanh(x/2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*l
og(tanh(x/2) + 1)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 6*t
anh(x/2)**3/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*tanh(x/
2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 2/(4*tanh(x/2)*
*3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4)
```

Giac [A] time = 1.12233, size = 36, normalized size = 0.52

$$\frac{1}{8}\,(2\,e^{x}-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

3.1045 $\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx$

Optimal. Leaf size=129

$$-\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{192b^2}-\frac{7\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{1152b^2}-\frac{35\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4608b^2}-\frac{35\sinh\left(a+\right.}{3}$$

```
[Out] (-35*x^3)/(3072*b) + (x^3*Cosh[a + b*x^3]^8)/(24*b) - (35*Cosh[a + b*x^3]*S
inh[a + b*x^3])/(3072*b^2) - (35*Cosh[a + b*x^3]^3*Sinh[a + b*x^3])/(4608*b
^2) - (7*Cosh[a + b*x^3]^5*Sinh[a + b*x^3])/(1152*b^2) - (Cosh[a + b*x^3]^7
*Sinh[a + b*x^3])/(192*b^2)
```

Rubi [A] time = 0.140261, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5373, 5321, 2635, 8}

$$-\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{192b^2}-\frac{7\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{1152b^2}-\frac{35\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4608b^2}-\frac{35\sinh\left(a+\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

```
[Out] (-35*x^3)/(3072*b) + (x^3*Cosh[a + b*x^3]^8)/(24*b) - (35*Cosh[a + b*x^3]*S
inh[a + b*x^3])/(3072*b^2) - (35*Cosh[a + b*x^3]^3*Sinh[a + b*x^3])/(4608*b
^2) - (7*Cosh[a + b*x^3]^5*Sinh[a + b*x^3])/(1152*b^2) - (Cosh[a + b*x^3]^7
*Sinh[a + b*x^3])/(192*b^2)
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx = \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\int x^2\cosh^8\left(a+bx^3\right)\,dx}{8b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\text{Subst}\left(\int \cosh^8(a+bx)\,dx, x, x^3\right)}{24b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{192b^2} - \frac{7\,\text{Subst}\left(\int \cosh^6(a+\right.}{192b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{7\cosh^5\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{1152b^2} - \frac{\cosh^7\left(a+bx^3\right)\sin}{192b^2}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh^3\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{4608b^2} - \frac{7\cosh^5\left(a+bx^3\right)}{1152b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{3072b^2} - \frac{35\cosh^3\left(a+bx^3\right)}{4608b}$$
$$= -\frac{35x^3}{3072b} + \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{3072b^2} - \frac{35\cosh^3}{}$$

Mathematica [A] time = 0.498366, size = 120, normalized size = 0.93

$$\frac{-672\sinh\left(2\left(a+bx^3\right)\right)-168\sinh\left(4\left(a+bx^3\right)\right)-32\sinh\left(6\left(a+bx^3\right)\right)-3\sinh\left(8\left(a+bx^3\right)\right)+1344bx^3\cosh\left(2\left(a+\right.\right.}{73728b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

```
[Out] (1344*b*x^3*Cosh[2*(a + b*x^3)] + 672*b*x^3*Cosh[4*(a + b*x^3)] + 192*b*x^3
*Cosh[6*(a + b*x^3)] + 24*b*x^3*Cosh[8*(a + b*x^3)] - 672*Sinh[2*(a + b*x^3
)] - 168*Sinh[4*(a + b*x^3)] - 32*Sinh[6*(a + b*x^3)] - 3*Sinh[8*(a + b*x^3
)])/(73728*b^2)
```

Maple [A] time = 0.069, size = 194, normalized size = 1.5

$$\frac{\left(8\,bx^3-1\right)\mathrm{e}^{8\,bx^3+8\,a}}{49152\,b^2}+\frac{\left(6\,bx^3-1\right)\mathrm{e}^{6\,bx^3+6\,a}}{4608\,b^2}+\frac{\left(28\,bx^3-7\right)\mathrm{e}^{4\,bx^3+4\,a}}{6144\,b^2}+\frac{\left(14\,bx^3-7\right)\mathrm{e}^{2\,bx^3+2\,a}}{1536\,b^2}+\frac{\left(14\,bx^3+7\right)\mathrm{e}^{-2\,bx^3-2\,a}}{1536\,b^2}+\frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x)
```

```
[Out] 1/49152*(8*b*x^3-1)/b^2*exp(8*b*x^3+8*a)+1/4608*(6*b*x^3-1)/b^2*exp(6*b*x^3
+6*a)+7/6144*(4*b*x^3-1)/b^2*exp(4*b*x^3+4*a)+7/1536*(2*b*x^3-1)/b^2*exp(2*
b*x^3+2*a)+7/1536*(2*b*x^3+1)/b^2*exp(-2*b*x^3-2*a)+7/6144*(4*b*x^3+1)/b^2*
exp(-4*b*x^3-4*a)+1/4608*(6*b*x^3+1)/b^2*exp(-6*b*x^3-6*a)+1/49152*(8*b*x^3
+1)/b^2*exp(-8*b*x^3-8*a)
```

Maxima [A] time = 1.10528, size = 288, normalized size = 2.23

$$\frac{\left(8\,bx^3e^{(8\,a)}-e^{(8\,a)}\right)e^{\left(8\,bx^3\right)}}{49152\,b^2}+\frac{\left(6\,bx^3e^{(6\,a)}-e^{(6\,a)}\right)e^{\left(6\,bx^3\right)}}{4608\,b^2}+\frac{7\left(4\,bx^3e^{(4\,a)}-e^{(4\,a)}\right)e^{\left(4\,bx^3\right)}}{6144\,b^2}+\frac{7\left(2\,bx^3e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^3\right)}}{1536\,b^2}+\frac{7\,(2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/49152*(8*b*x^3*e^(8*a) - e^(8*a))*e^(8*b*x^3)/b^2 + 1/4608*(6*b*x^3*e^(6*
a) - e^(6*a))*e^(6*b*x^3)/b^2 + 7/6144*(4*b*x^3*e^(4*a) - e^(4*a))*e^(4*b*x
^3)/b^2 + 7/1536*(2*b*x^3*e^(2*a) - e^(2*a))*e^(2*b*x^3)/b^2 + 7/1536*(2*b*
x^3 + 1)*e^(-2*b*x^3 - 2*a)/b^2 + 7/6144*(4*b*x^3 + 1)*e^(-4*b*x^3 - 4*a)/b
^2 + 1/4608*(6*b*x^3 + 1)*e^(-6*b*x^3 - 6*a)/b^2 + 1/49152*(8*b*x^3 + 1)*e^
(-8*b*x^3 - 8*a)/b^2
```

Fricas [B] time = 2.24002, size = 976, normalized size = 7.57

$$\frac{3\,bx^3\cosh\left(bx^3+a\right)^8+3\,bx^3\sinh\left(bx^3+a\right)^8+24\,bx^3\cosh\left(bx^3+a\right)^6+84\,bx^3\cosh\left(bx^3+a\right)^4-3\,\cosh\left(bx^3+a\right)\sinh\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9216*(3*b*x^3*cosh(b*x^3 + a)^8 + 3*b*x^3*sinh(b*x^3 + a)^8 + 24*b*x^3*co
sh(b*x^3 + a)^6 + 84*b*x^3*cosh(b*x^3 + a)^4 - 3*cosh(b*x^3 + a)*sinh(b*x^3
 + a)^7 + 12*(7*b*x^3*cosh(b*x^3 + a)^2 + 2*b*x^3)*sinh(b*x^3 + a)^6 + 168*
b*x^3*cosh(b*x^3 + a)^2 - 3*(7*cosh(b*x^3 + a)^3 + 8*cosh(b*x^3 + a))*sinh(
b*x^3 + a)^5 + 6*(35*b*x^3*cosh(b*x^3 + a)^4 + 60*b*x^3*cosh(b*x^3 + a)^2 +
 14*b*x^3)*sinh(b*x^3 + a)^4 - (21*cosh(b*x^3 + a)^5 + 80*cosh(b*x^3 + a)^3
 + 84*cosh(b*x^3 + a))*sinh(b*x^3 + a)^3 + 12*(7*b*x^3*cosh(b*x^3 + a)^6 +
30*b*x^3*cosh(b*x^3 + a)^4 + 42*b*x^3*cosh(b*x^3 + a)^2 + 14*b*x^3)*sinh(b*
x^3 + a)^2 - 3*(cosh(b*x^3 + a)^7 + 8*cosh(b*x^3 + a)^5 + 28*cosh(b*x^3 + a
)^3 + 56*cosh(b*x^3 + a))*sinh(b*x^3 + a))/b^2
```

Sympy [A] time = 137.366, size = 241, normalized size = 1.87

$$\begin{cases} -\frac{35x^3\sinh^8\left(a+bx^3\right)}{3072b}+\frac{35x^3\sinh^6\left(a+bx^3\right)\cosh^2\left(a+bx^3\right)}{768b}-\frac{35x^3\sinh^4\left(a+bx^3\right)\cosh^4\left(a+bx^3\right)}{512b}+\frac{35x^3\sinh^2\left(a+bx^3\right)\cosh^6\left(a+bx^3\right)}{768b}+\frac{31x^3\cosh^8\left(a\cdots\right)}{1024b}\cdots \\ \frac{x^6\sinh\left(a\right)\cosh^7\left(a\right)}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(b*x**3+a)**7*sinh(b*x**3+a),x)
```

```
[Out] Piecewise((-35*x**3*sinh(a + b*x**3)**8/(3072*b) + 35*x**3*sinh(a + b*x**3)
**6*cosh(a + b*x**3)**2/(768*b) - 35*x**3*sinh(a + b*x**3)**4*cosh(a + b*x*
*3)**4/(512*b) + 35*x**3*sinh(a + b*x**3)**2*cosh(a + b*x**3)**6/(768*b) +
31*x**3*cosh(a + b*x**3)**8/(1024*b) + 35*sinh(a + b*x**3)**7*cosh(a + b*x*
*3)/(3072*b**2) - 385*sinh(a + b*x**3)**5*cosh(a + b*x**3)**3/(9216*b**2) +
 511*sinh(a + b*x**3)**3*cosh(a + b*x**3)**5/(9216*b**2) - 31*sinh(a + b*x*
*3)*cosh(a + b*x**3)**7/(1024*b**2), Ne(b, 0)), (x**6*sinh(a)*cosh(a)**7/6,
 True))
```

Giac [B] time = 1.16338, size = 516, normalized size = 4.

$$\frac{24\left(bx^3+a\right)e^{\left(8\,bx^3+8\,a\right)} - 24\,ae^{\left(8\,bx^3+8\,a\right)} + 192\left(bx^3+a\right)e^{\left(6\,bx^3+6\,a\right)} - 192\,ae^{\left(6\,bx^3+6\,a\right)} + 672\left(bx^3+a\right)e^{\left(4\,bx^3+4\,a\right)} - 672\,ae^{\left(4\,bx^3\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/147456*(24*(b*x^3 + a)*e^(8*b*x^3 + 8*a) - 24*a*e^(8*b*x^3 + 8*a) + 192*(
b*x^3 + a)*e^(6*b*x^3 + 6*a) - 192*a*e^(6*b*x^3 + 6*a) + 672*(b*x^3 + a)*e^
(4*b*x^3 + 4*a) - 672*a*e^(4*b*x^3 + 4*a) + 1344*(b*x^3 + a)*e^(2*b*x^3 + 2
*a) - 1344*a*e^(2*b*x^3 + 2*a) + 1344*(b*x^3 + a)*e^(-2*b*x^3 - 2*a) - 1344
*a*e^(-2*b*x^3 - 2*a) + 672*(b*x^3 + a)*e^(-4*b*x^3 - 4*a) - 672*a*e^(-4*b*
x^3 - 4*a) + 192*(b*x^3 + a)*e^(-6*b*x^3 - 6*a) - 192*a*e^(-6*b*x^3 - 6*a)
+ 24*(b*x^3 + a)*e^(-8*b*x^3 - 8*a) - 24*a*e^(-8*b*x^3 - 8*a) - 3*e^(8*b*x^
3 + 8*a) - 32*e^(6*b*x^3 + 6*a) - 168*e^(4*b*x^3 + 4*a) - 672*e^(2*b*x^3 +
2*a) + 672*e^(-2*b*x^3 - 2*a) + 168*e^(-4*b*x^3 - 4*a) + 32*e^(-6*b*x^3 - 6
*a) + 3*e^(-8*b*x^3 - 8*a))/b^2
```

3.1046 $\int \frac{\cosh^2(x)}{1+e^x} dx$

Optimal. Leaf size=39

$$\frac{3x}{4} - \frac{e^{-2x}}{8} + \frac{e^{-x}}{4} + \frac{e^x}{4} - \log\left(e^x + 1\right)$$

```
[Out] -1/(8*E^(2*x)) + 1/(4*E^x) + E^x/4 + (3*x)/4 - Log[1 + E^x]
```

Rubi [A] time = 0.0488001, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 894}

$$\frac{3x}{4} - \frac{e^{-2x}}{8} + \frac{e^{-x}}{4} + \frac{e^x}{4} - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + E^x),x]
```

```
[Out] -1/(8*E^(2*x)) + 1/(4*E^x) + E^x/4 + (3*x)/4 - Log[1 + E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{4x^3(1+x)}\,dx, x, e^x\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3(1+x)}\,dx, x, e^x\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{3}{x}-\frac{4}{1+x}\right)dx, x, e^x\right)\\ &= -\frac{1}{8}e^{-2x}+\frac{e^{-x}}{4}+\frac{e^x}{4}+\frac{3x}{4}-\log\left(1+e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0322403, size = 33, normalized size = 0.85

$$\frac{1}{4}\left(3x-\frac{e^{-2x}}{2}+e^{-x}+e^x-4\log\left(e^x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + E^x),x]
```

```
[Out] (-1/(2*E^(2*x)) + E^(-x) + E^x + 3*x - 4*Log[1 + E^x])/4
```

Maple [A] time = 0.011, size = 48, normalized size = 1.2

$$-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3}{4}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{1}{4}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(exp(x)+1),x)
```

```
[Out] -1/2/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)+3/4*ln(tanh(1/2*x)+1)-1/2/(tanh(1/
2*x)-1)+1/4*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.03582, size = 36, normalized size = 0.92

$$\frac{1}{8}\,(2\,e^{x}-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

Fricas [B] time = 2.09387, size = 346, normalized size = 8.87

$$\frac{6\,x\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)^{3}+6\,(x+\cosh\left(x\right))\sinh\left(x\right)^{2}+2\,\sinh\left(x\right)^{3}-8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\mathrm{lo}}{8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh^{2}\left(x\right)}{e^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+exp(x)),x)
```

```
[Out] Integral(cosh(x)**2/(exp(x) + 1), x)
```

Giac [A] time = 1.13807, size = 36, normalized size = 0.92

$$\frac{1}{8}\left(2\,e^{x}-1\right)e^{\left(-2\,x\right)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="giac")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

3.1047 $\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx$

Optimal. Leaf size=25

$$\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

```
[Out] (-4*(1 + Sech[x])^(5/2))/5 + (2*(1 + Sech[x])^(7/2))/7
```

Rubi [A] time = 0.100077, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4373, 1570, 1469, 627, 43}

$$\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

```
[Out] (-4*(1 + Sech[x])^(5/2))/5 + (2*(1 + Sech[x])^(7/2))/7
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\left(1-x^2\right)}{x^4}\,dx, x, \cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cosh(x)\right)\\ &= \operatorname{Subst}\left(\int \sqrt{1+x}\left(-1+x^2\right)dx, x, \operatorname{sech}(x)\right)\\ &= \operatorname{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \operatorname{sech}(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \operatorname{sech}(x)\right)\\ &= -\frac{4}{5}(1+\operatorname{sech}(x))^{5/2}+\frac{2}{7}(1+\operatorname{sech}(x))^{7/2}\end{aligned}$$

Mathematica [A] time = 0.200845, size = 30, normalized size = 1.2

$$-\frac{8}{35}\cosh^4\left(\frac{x}{2}\right)(9\cosh(x)-5)\operatorname{sech}^3(x)\sqrt{\operatorname{sech}(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

```
[Out] (-8*Cosh[x/2]^4*(-5 + 9*Cosh[x])*Sech[x]^3*Sqrt[1 + Sech[x]])/35
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\,(\tanh(x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x)
```

```
[Out] int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{sech}(x)+1}\,\operatorname{sech}(x)\tanh(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(x) + 1)*sech(x)*tanh(x)^3, x)
```

Fricas [B] time = 2.07246, size = 1465, normalized size = 58.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] -2/35*(9*cosh(x)^6 + 54*cosh(x)*sinh(x)^5 + 9*sinh(x)^6 + 27*(5*cosh(x)^2 +
 1)*sinh(x)^4 + 27*cosh(x)^4 + 36*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 27*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 27*cosh(x)^2 + 54*(cosh(x)^5 +
2*cosh(x)^3 + cosh(x))*sinh(x) + (9*cosh(x)^7 + 7*(9*cosh(x) + 5)*sinh(x)^6
 + 9*sinh(x)^7 + 35*cosh(x)^6 + 7*(27*cosh(x)^2 + 30*cosh(x) + 7)*sinh(x)^5
 + 49*cosh(x)^5 + 35*(9*cosh(x)^3 + 15*cosh(x)^2 + 7*cosh(x) + 1)*sinh(x)^4
 + 35*cosh(x)^4 + 35*(9*cosh(x)^4 + 20*cosh(x)^3 + 14*cosh(x)^2 + 4*cosh(x)
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 7*(27*cosh(x)^5 + 75*cosh(x)^4 + 70*cosh(x
```

```
)^3 + 30*cosh(x)^2 + 15*cosh(x) + 7)*sinh(x)^2 + 49*cosh(x)^2 + 7*(9*cosh(x
)^6 + 30*cosh(x)^5 + 35*cosh(x)^4 + 20*cosh(x)^3 + 15*cosh(x)^2 + 14*cosh(x
) + 5)*sinh(x) + 35*cosh(x) + 9)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 + 1) + 9)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{sech}(x)+1}\tanh^3(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))**(1/2)*tanh(x)**3,x)
```

```
[Out] Integral(sqrt(sech(x) + 1)*tanh(x)**3*sech(x), x)
```

Giac [B] time = 1.1923, size = 62, normalized size = 2.48

$$-\frac{2\,(((((((9\,e^{x}+35)e^{x}+49)e^{x}+35)e^{x}+35)e^{x}+49)e^{x}+35)e^{x}+9)}{35\,\left(e^{(2\,x)}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] -2/35*(((((((9*e^x + 35)*e^x + 49)*e^x + 35)*e^x + 35)*e^x + 49)*e^x + 35)*
e^x + 9)/(e^(2*x) + 1)^(7/2)
```

3.1048 $\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{2}{7}(\operatorname{csch}(x)+1)^{7/2}+\frac{4}{5}(\operatorname{csch}(x)+1)^{5/2}-\frac{4}{3}(\operatorname{csch}(x)+1)^{3/2}$$

```
[Out] (-4*(1 + Csch[x])^(3/2))/3 + (4*(1 + Csch[x])^(5/2))/5 - (2*(1 + Csch[x])^(
7/2))/7
```

Rubi [A] time = 0.105672, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4372, 1570, 1469, 697}

$$-\frac{2}{7}(\operatorname{csch}(x)+1)^{7/2}+\frac{4}{5}(\operatorname{csch}(x)+1)^{5/2}-\frac{4}{3}(\operatorname{csch}(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

```
[Out] (-4*(1 + Csch[x])^(3/2))/3 + (4*(1 + Csch[x])^(5/2))/5 - (2*(1 + Csch[x])^(
7/2))/7
```

Rule 4372

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
```

```
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\left(1+x^2\right)}{x^4}\,dx, x, \sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \frac{\left(1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sinh(x)\right)\\
&= -\operatorname{Subst}\left(\int \sqrt{1+x}\left(1+x^2\right)dx, x, \operatorname{csch}(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(2\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \operatorname{csch}(x)\right)\\
&= -\frac{4}{3}(1+\operatorname{csch}(x))^{3/2}+\frac{4}{5}(1+\operatorname{csch}(x))^{5/2}-\frac{2}{7}(1+\operatorname{csch}(x))^{7/2}
\end{aligned}
$$

Mathematica [A] time = 0.0585229, size = 34, normalized size = 0.92

$$
-\frac{1}{210}\operatorname{csch}^3(x)\sqrt{\operatorname{csch}(x)+1}(-117\sinh(x)+43\sinh(3x)+62\cosh(2x)-2)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

```
[Out] -(Csch[x]^3*Sqrt[1 + Csch[x]]*(-2 + 62*Cosh[2*x] - 117*Sinh[x] + 43*Sinh[3*
x]))/210
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$
\int (\coth(x))^3 \operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x)
```

```
[Out] int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x)
```

Maxima [B] time = 1.44684, size = 525, normalized size = 14.19

$$\frac{124\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}-\frac{78\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-2x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}-\frac{}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 124/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) - 78/35*sqrt(-2*e^(-x) + e^
(-2*x) - 1)*e^(-2*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(
-4*x) + e^(-6*x) - 1)) - 8/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-3*x)/(sqr
t(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) +
78/35*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-4*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 124/105*sqrt(-2*e^(-x) +
e^(-2*x) - 1)*e^(-5*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e
^(-4*x) + e^(-6*x) - 1)) - 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-6*x)/(
sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
 + 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)
*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
```

Fricas [B] time = 2.09575, size = 953, normalized size = 25.76

$$-\frac{2\left(43\cosh(x)^{6}+2(129\cosh(x)+31)\sinh(x)^{5}+43\sinh(x)^{6}+62\cosh(x)^{5}+\left(645\cosh(x)^{2}+310\cosh(x)-11\right.}{105\left(\cosh(x)^{6}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(43*cosh(x)^6 + 2*(129*cosh(x) + 31)*sinh(x)^5 + 43*sinh(x)^6 + 62*c
osh(x)^5 + (645*cosh(x)^2 + 310*cosh(x) - 117)*sinh(x)^4 - 117*cosh(x)^4 +
4*(215*cosh(x)^3 + 155*cosh(x)^2 - 117*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3
 + (645*cosh(x)^4 + 620*cosh(x)^3 - 702*cosh(x)^2 - 12*cosh(x) + 117)*sinh(
x)^2 + 117*cosh(x)^2 + 2*(129*cosh(x)^5 + 155*cosh(x)^4 - 234*cosh(x)^3 - 6
*cosh(x)^2 + 117*cosh(x) + 31)*sinh(x) + 62*cosh(x) - 43)*sqrt((sinh(x) + 1
)/sinh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 -
1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*c
osh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh
(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*csch(x)*(1+csch(x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{csch}(x)+1} \coth(x)^3 \operatorname{csch}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(x) + 1)*coth(x)^3*csch(x), x)
```

3.1049 $\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx$

Optimal. Leaf size=4

$$\cosh^x(x)$$

```
[Out] Cosh[x]^x
```

Rubi [A] time = 0.139696, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2553}

$$\cosh^x(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

```
[Out] Cosh[x]^x
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2553

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx &= \int \left(\cosh^x(x)\log(\cosh(x)) + x\cosh^{-1+x}(x)\sinh(x)\right)\,dx\\ &= \int \cosh^x(x)\log(\cosh(x))\,dx + \int x\cosh^{-1+x}(x)\sinh(x)\,dx\\ &= \cosh^x(x)\end{aligned}$$

Mathematica [A] time = 0.0707959, size = 4, normalized size = 1.

$$\cosh^x(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

```
[Out] Cosh[x]^x
```

Maple [A] time = 0.02, size = 5, normalized size = 1.3

$$(\cosh(x))^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^x*(ln(cosh(x))+x*tanh(x)),x)
```

```
[Out] cosh(x)^x
```

Maxima [B] time = 2.23277, size = 28, normalized size = 7.

$$e^{\left(-x^2 - x\log(2) + x\log\left(e^{(2x)} + 1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x^2 - x*log(2) + x*log(e^(2*x) + 1))
```

Fricas [B] time = 2.07264, size = 61, normalized size = 15.25

$$\cosh(x\log(\cosh(x))) + \sinh(x\log(\cosh(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="fricas")
```

```
[Out] cosh(x*log(cosh(x))) + sinh(x*log(cosh(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**x*(ln(cosh(x))+x*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x\tanh(x) + \log(\cosh(x)))\cosh(x)^x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate((x*tanh(x) + log(cosh(x)))*cosh(x)^x, x)
```

3.1050 $\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$

Optimal. Leaf size=27

$$\frac{F^{a+bx}\left(e^{c+dx}\right)^n}{b\log(F)+dn}$$

```
[Out] ((E^(c + d*x))^n*F^(a + b*x))/(d*n + b*Log[F])
```

Rubi [A] time = 0.0910117, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5648, 2281, 2287, 2194}

$$\frac{F^{a+bx}\left(e^{c+dx}\right)^n}{b\log(F)+dn}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]

[Out] ((E^(c + d*x))^n*F^(a + b*x))/(d*n + b*Log[F])
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx &= \int \left(e^{c+dx}\right)^n F^{a+bx}\,dx\\ &= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{n(c+dx)}F^{a+bx}\,dx\\ &= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{cn+a\log(F)+x(dn+b\log(F))}\,dx\\ &= \frac{\left(e^{c+dx}\right)^n F^{a+bx}}{dn+b\log(F)}\end{aligned}$$

Mathematica [A] time = 0.0861154, size = 33, normalized size = 1.22

$$\frac{F^{a+bx}(\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)+dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

```
[Out] (F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n)/(d*n + b*Log[F])
```

Maple [A] time = 0.012, size = 34, normalized size = 1.3

$$\frac{F^{bx+a}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)^n}{dn+b\ln\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x)
```

```
[Out] 1/(d*n+b*ln(F))*F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n
```

Maxima [A] time = 1.05323, size = 38, normalized size = 1.41

$$\frac{F^a e^{(dnx+bx\log(F)+cn)}}{dn+b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] F^a*e^(d*n*x + b*x*log(F) + c*n)/(d*n + b*log(F))
```

Fricas [B] time = 2.07509, size = 200, normalized size = 7.41

$$\frac{(\cosh(dnx+cn)+\sinh(dnx+cn))\cosh((bx+a)\log(F))+(\cosh(dnx+cn)+\sinh(dnx+cn))\sinh((bx+a)\log(F))}{dn+b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((cosh(d*n*x + c*n) + sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*n
*x + c*n) + sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n + b*log(F))
```

Sympy [A] time = 8.38276, size = 94, normalized size = 3.48

$$\begin{cases} \frac{F^a F^{bx}(\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)+dn} & \text{for } b \neq -\frac{dn}{\log(F)} \\ F^a x(\sinh(c+dx)+\cosh(c+dx))^n e^{-dnx} + \frac{F^a(\sinh(c+dx)+\cosh(c+dx))^n e^{-dnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**a*F**(b*x)*(sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) + d*n
), Ne(b, -d*n/log(F))), (F**a*x*(sinh(c + d*x) + cosh(c + d*x))**n*exp(-d*n
*x) + F**a*(sinh(c + d*x) + cosh(c + d*x))**n*exp(-d*n*x)/(d*n), True))
```

Giac [C] time = 1.2019, size = 370, normalized size = 13.7

$$2\left(\frac{2\left(dn+b\log\left(|F|\right)\right)\cos\left(-\frac{1}{2}\pi bx\text{sgn}\left(F\right)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\text{sgn}\left(F\right)+\frac{1}{2}\pi a\right)}{\left(\pi b\text{sgn}\left(F\right)-\pi b\right)^{2}+4\left(dn+b\log\left(|F|\right)\right)^{2}}-\frac{\left(\pi b\text{sgn}\left(F\right)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\text{sgn}\left(F\right)+\frac{1}{2}\pi\right.}{\left(\pi b\text{sgn}\left(F\right)-\pi b\right)^{2}+4\left(dn+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] 2*(2*(d*n + b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*s
gn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2) - (p
i*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2))*e^(c*n + (d
*n + b*log(abs(F)))*x + a*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(F)
 - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b +
 2*d*n + 2*b*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/
2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b + 2*d*n + 2*b*log(ab
s(F))))*e^(c*n + (d*n + b*log(abs(F)))*x + a*log(abs(F)))
```

3.1051 $\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{F^{a+bx}\left(e^{-c-dx}\right)^n}{dn-b\log(F)}$$

```
[Out] -(((E^(-c - d*x))^n*F^(a + b*x))/(d*n - b*Log[F]))
```

Rubi [A] time = 0.0946486, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5648, 2281, 2287, 2194}

$$-\frac{F^{a+bx}\left(e^{-c-dx}\right)^n}{dn-b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

```
[Out] -(((E^(-c - d*x))^n*F^(a + b*x))/(d*n - b*Log[F]))
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx &= \int \left(e^{-c-dx}\right)^n F^{a+bx}\,dx\\ &= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{n(-c-dx)}F^{a+bx}\,dx\\ &= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{-cn+a\log(F)-x(dn-b\log(F))}\,dx\\ &= -\frac{\left(e^{-c-dx}\right)^n F^{a+bx}}{dn-b\log(F)}\end{aligned}$$

Mathematica [A] time = 0.0720742, size = 37, normalized size = 1.16

$$-\frac{F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n}{dn-b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

```
[Out] -((F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n)/(d*n - b*Log[F]))
```

Maple [A] time = 0.006, size = 37, normalized size = 1.2

$$\frac{F^{bx+a}\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^n}{b\ln\left(F\right)-dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x)
```

```
[Out] 1/(b*ln(F)-d*n)*F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n
```

Maxima [A] time = 1.15712, size = 49, normalized size = 1.53

$$-\frac{F^a e^{(-dnx+bx\log(F))}}{dne^{(cn)} - be^{(cn)}\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -F^a*e^(-d*n*x + b*x*log(F))/(d*n*e^(c*n) - b*e^(c*n)*log(F))
```

Fricas [B] time = 2.11387, size = 201, normalized size = 6.28

$$-\frac{(\cosh(dnx+cn) - \sinh(dnx+cn))\cosh((bx+a)\log(F)) + (\cosh(dnx+cn) - \sinh(dnx+cn))\sinh((bx+a)\log(F))}{dn - b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -((cosh(d*n*x + c*n) - sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*
n*x + c*n) - sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n - b*log(F))
```

Sympy [A] time = 8.52737, size = 92, normalized size = 2.88

$$\begin{cases} \frac{F^a F^{bx}(-\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)-dn} & \text{for } b \neq \frac{dn}{\log(F)} \\ F^a x(-\sinh(c+dx)+\cosh(c+dx))^n e^{dnx} - \frac{F^a(-\sinh(c+dx)+\cosh(c+dx))^n e^{dnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**a*F**(b*x)*(-sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) - d*
n), Ne(b, d*n/log(F))), (F**a*x*(-sinh(c + d*x) + cosh(c + d*x))**n*exp(d*n
*x) - F**a*(-sinh(c + d*x) + cosh(c + d*x))**n*exp(d*n*x)/(d*n), True))
```

<u>Giac</u> [C] time = 1.16676, size = 381, normalized size = 11.91

$$-2\left(\frac{2\left(dn-b\log\left(|F|\right)\right)\cos\left(-\frac{1}{2}\pi bx\mathrm{sgn}\left(F\right)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\mathrm{sgn}\left(F\right)+\frac{1}{2}\pi a\right)}{\left(\pi b\mathrm{sgn}\left(F\right)-\pi b\right)^{2}+4\left(dn-b\log\left(|F|\right)\right)^{2}}+\frac{\left(\pi b\mathrm{sgn}\left(F\right)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\mathrm{sgn}\left(F\right)+\frac{1}{2}\right.}{\left(\pi b\mathrm{sgn}\left(F\right)-\pi b\right)^{2}+4\left(dn-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -2*(2*(d*n - b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*
sgn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2) + (
pi*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2))*e^(-c*n -
(d*n - b*log(abs(F)))*x + a*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(
F) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b
 - 2*d*n + 2*b*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x -
1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b - 2*d*n + 2*b*log(
abs(F))))*e^(-c*n - (d*n - b*log(abs(F)))*x + a*log(abs(F)))
```

3.1052 $\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b)) + ArcTan[1 + Sqrt[2]*Tanh[
a + b*x]]/(Sqrt[2]*b)
```

Rubi [A] time = 0.17189, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b*x]^4)
,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b)) + ArcTan[1 + Sqrt[2]*Tanh[
a + b*x]]/(Sqrt[2]*b)
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\tanh(a+b$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\tan$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}+\frac{\tan^{-1}\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

Mathematica [A] time = 0.0357443, size = 25, normalized size = 0.49

$$\frac{\tan^{-1}\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b
*x]^4),x]
```

```
[Out] ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

Maple [C] time = 0.097, size = 138, normalized size = 2.7

$$\frac{\frac{i}{4}\sqrt{2}}{b}\ln\left(-2i\sqrt{2}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4-2i\sqrt{2}\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\tanh(1/2\,bx+a/2)\right)^2+1\right)-\frac{\frac{i}{4}\sqrt{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x)
```

```
[Out] 1/4*I/b*2^(1/2)*ln(-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4
-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)-1/4*I/b*2^(1/2)
*ln(2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4+2*I*2^(1/2)*tan
h(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\int \frac{\left(e^{(-bx-a)}+e^{(-5\,bx-5\,a)}\right)e^{(-bx-a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx+2\int \frac{e^{(6\,bx+6\,a)}}{e^{(8\,bx+8\,a)}+6\,e^{(4\,bx+4\,a)}+1}\,dx+2\int \frac{e^{(-6\,bx-6\,a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="maxima")
```

```
[Out] 2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x - 4
*a) + e^(-8*b*x - 8*a) + 1), x) + 2*integrate(e^(6*b*x + 6*a)/(e^(8*b*x + 8
*a) + 6*e^(4*b*x + 4*a) + 1), x) + 2*integrate(e^(-6*b*x - 6*a)/(6*e^(-4*b*
x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

Fricas [B] time = 2.06874, size = 559, normalized size = 10.96

$$-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a)}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right)+\sqrt{2}\,\mathrm{arct}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a
)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 -
7*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*co
sh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x +
a)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a)
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**4-sinh(b*x+a)**4)/(cosh(b*x+a)**4+sinh(b*x+a)**4),x
)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="giac")
```

```
[Out] sage0*x
```

3.1053 $\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{3b(\tanh(a+bx)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}$$

```
[Out] (-4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) - 1/(3*b*(1 + Tanh
[a + b*x]))
```

Rubi [A] time = 0.327863, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 618, 204}

$$-\frac{1}{3b(\tanh(a+bx)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b*x]^3)
,x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) - 1/(3*b*(1 + Tanh
[a + b*x]))
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3(1-x+x^2)}\right)dx, x, \tanh(a+bx)\right)}{b}\\ &= -\frac{1}{3b(1+\tanh(a+bx))}+\frac{2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(a+bx)\right)}{3b}\\ &= -\frac{1}{3b(1+\tanh(a+bx))}-\frac{4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(a+bx)\right)}{3b}\\ &= -\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}-\frac{1}{3b(1+\tanh(a+bx))}\end{aligned}$$

Mathematica [B] time = 1.33687, size = 115, normalized size = 2.45

$$\frac{(\sinh(a+bx)-\cosh(a+bx))\left(\cosh(a+bx)\left(8\sqrt{3}\tan^{-1}\left(\frac{\text{sech}(bx)(\cosh(2a+bx)-2\sinh(2a+bx))}{\sqrt{3}}\right)+3\right)+\sinh(a+bx)\left(8\sqrt{3}\tan\right.\right.}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b
*x]^3),x]
```

```
[Out] ((-Cosh[a + b*x] + Sinh[a + b*x])*((3 + 8*Sqrt[3]*ArcTan[(Sech[b*x]*(Cosh[2
*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Cosh[a + b*x] + (-3 + 8*Sqrt[3]*A
rcTan[(Sech[b*x]*(Cosh[2*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Sinh[a +
b*x]))/(18*b)
```

Maple [C] time = 0.142, size = 120, normalized size = 2.6

$$-\frac{2}{3b}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{-2}+\frac{2}{3b}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{-1}+\frac{\frac{2i}{9}\sqrt{3}}{b}\ln\left(\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x)
```

```
[Out] -2/3/b/(tanh(1/2*b*x+1/2*a)+1)^2+2/3/b/(tanh(1/2*b*x+1/2*a)+1)+2/9*I/b*3^(1
/2)*ln(tanh(1/2*b*x+1/2*a)^2+(-I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)+1)-2/9*I/b*
3^(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)+1)
```

Maxima [B] time = 1.57628, size = 126, normalized size = 2.68

$$\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9b}-\frac{e^{(-2bx-2a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="maxima")
```

```
[Out] 4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*s
qrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^(
1/4)*sqrt(2))))/b - 1/6*e^(-2*b*x - 2*a)/b
```

Fricas [B] time = 2.17371, size = 370, normalized size = 7.87

$$-\frac{8\left(\sqrt{3}\cosh\left(bx+a\right)^{2}+2\sqrt{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\sinh\left(bx+a\right)^{2}\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)}{3\left(\cosh(bx+a)-\sinh(bx+a)\right)}\right)+3}{18\left(b\cosh\left(bx+a\right)^{2}+2b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="fricas")
```

```
[Out] -1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh
(b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*c
osh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**3-sinh(b*x+a)**3)/(cosh(b*x+a)**3+sinh(b*x+a)**3),x
)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.17791, size = 51, normalized size = 1.09

$$\frac{4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2\,bx+2\,a)}\right)}{9\,b} - \frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="giac")
```

```
[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a))/b - 1/6*e^(-2*b*x - 2*a)/b
```

3.1054 $\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

```
[Out] ArcTan[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0616019, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {4380, 203}

$$\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b*x]^2)
,x]
```

```
[Out] ArcTan[Tanh[a + b*x]]/b
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \int \frac{1}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\tan^{-1}(\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.004128, size = 17, normalized size = 1.55

$$\frac{\tan^{-1}(\sinh(2a+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b
*x]^2),x]
```

```
[Out] ArcTan[Sinh[2*a + 2*b*x]]/(2*b)
```

Maple [B] time = 0.063, size = 148, normalized size = 13.5

$$-2\,\frac{\sqrt{2}}{b\left(2+2\sqrt{2}\right)}\arctan\left(2\,\frac{\tanh\left(1/2\,bx+a/2\right)}{2+2\sqrt{2}}\right)-2\,\frac{1}{b\left(2+2\sqrt{2}\right)}\arctan\left(2\,\frac{\tanh\left(1/2\,bx+a/2\right)}{2+2\sqrt{2}}\right)+2\,\frac{\sqrt{2}}{b\left(-2+2\sqrt{2}\right)}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x)
```

```
[Out] -2/b*2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))-2/b/
(2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))+2/b*2^(1/2)/(-2+2
*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))-2/b/(-2+2*2^(1/2))*a
rctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))
```

Maxima [B] time = 1.75337, size = 66, normalized size = 6.

$$\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-bx-a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="maxima")
```

```
[Out] (arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(-b*x - a))))/b
```

Fricas [B] time = 2.01314, size = 104, normalized size = 9.45

$$-\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="fricas")
```

```
[Out] -arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**2-sinh(b*x+a)**2)/(cosh(b*x+a)**2+sinh(b*x+a)**2),x
)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.16033, size = 19, normalized size = 1.73

$$\frac{\arctan\left(e^{(2\,bx+2\,a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="giac")
```

```
[Out] arctan(e^(2*b*x + 2*a))/b
```

3.1055 $\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

```
[Out] -1/(2*b*(Cosh[a + b*x] + Sinh[a + b*x])^2)
```

Rubi [A] time = 0.0503993, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {4385}

$$-\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x]
```

```
[Out] -1/(2*b*(Cosh[a + b*x] + Sinh[a + b*x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2b(\cosh(a+bx)+\sinh(a+bx))^2}$$

Mathematica [B] time = 0.0240751, size = 65, normalized size = 2.95

$$-\frac{\sinh(2a)\sinh(2bx)}{2b}-\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\sinh(2a)\cosh(2bx)}{2b}+\frac{\cosh(2a)\sinh(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x
]
```

```
[Out] -(Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Cosh[2*b*x]*Sinh[2*a])/(2*b) + (Cosh[2*a]
*Sinh[2*b*x])/(2*b) - (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

Maple [A] time = 0., size = 36, normalized size = 1.6

$$\frac{-\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{2\,b\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
```

```
[Out] 1/2*(-cosh(b*x+a)+sinh(b*x+a))/b/(cosh(b*x+a)+sinh(b*x+a))
```

Maxima [A] time = 1.0248, size = 19, normalized size = 0.86

$$-\frac{e^{\left(-2\,bx-2\,a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

Fricas [A] time = 2.00972, size = 108, normalized size = 4.91

$$-\frac{1}{2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"fricas")
```

```
[Out] -1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

Sympy [A] time = 0.649011, size = 39, normalized size = 1.77

$$\begin{cases} -\frac{\cosh(a+bx)}{b\sinh(a+bx)+b\cosh(a+bx)} & \text{for } b \neq 0 \\ \frac{x(-\sinh(a)+\cosh(a))}{\sinh(a)+\cosh(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
```

```
[Out] Piecewise((-cosh(a + b*x)/(b*sinh(a + b*x) + b*cosh(a + b*x)), Ne(b, 0)), (
x*(-sinh(a) + cosh(a))/(sinh(a) + cosh(a)), True))
```

Giac [A] time = 1.13724, size = 19, normalized size = 0.86

$$-\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"giac")
```

```
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

3.1056 $\int \frac{-\mathbf{csch}(a+bx)+\mathbf{sech}(a+bx)}{\mathbf{csch}(a+bx)+\mathbf{sech}(a+bx)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{b(\tanh(a+bx)+1)}$$

```
[Out] 1/(b*(1 + Tanh[a + b*x]))
```

Rubi [A] time = 0.206698, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {32}

$$\frac{1}{b(\tanh(a+bx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),x]
```

```
[Out] 1/(b*(1 + Tanh[a + b*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-\text{csch}(a+bx)+\text{sech}(a+bx)}{\text{csch}(a+bx)+\text{sech}(a+bx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{1}{b(1+\tanh(a+bx))}\end{aligned}$$

Mathematica [B] time = 0.0248082, size = 65, normalized size = 4.64

$$\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}-\frac{\sinh(2a)\cosh(2bx)}{2b}-\frac{\cosh(2a)\sinh(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),
x]
```

```
[Out] (Cosh[2*a]*Cosh[2*b*x])/(2*b) - (Cosh[2*b*x]*Sinh[2*a])/(2*b) - (Cosh[2*a]*
Sinh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

Maple [B] time = 0.129, size = 36, normalized size = 2.6

$$\frac{1}{b}\left(2\,(\tanh(1/2\,bx+a/2)+1)^{-2}-2\,(\tanh(1/2\,bx+a/2)+1)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

```
[Out] 1/b*(2/(tanh(1/2*b*x+1/2*a)+1)^2-2/(tanh(1/2*b*x+1/2*a)+1))
```

Maxima [A] time = 1.10967, size = 19, normalized size = 1.36

$$\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="maxima")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

Fricas [B] time = 2.0868, size = 107, normalized size = 7.64

$$\frac{1}{2\left(b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="fricas")
```

```
[Out] 1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{csch}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx-\int -\frac{\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

```
[Out] -Integral(csch(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x) - Integral(-sec
h(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x)
```

Giac [A] time = 1.13312, size = 19, normalized size = 1.36

$$\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="giac")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

3.1057 $\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx$

Optimal. Leaf size=12

$$-\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

```
[Out] -(ArcTan[Tanh[a + b*x]]/b)
```

Rubi [A] time = 0.263468, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {204}

$$-\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + b*x]^2
),x]
```

```
[Out] -(ArcTan[Tanh[a + b*x]]/b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= -\frac{\tan^{-1}(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0076216, size = 17, normalized size = 1.42

$$-\frac{\tan^{-1}(\sinh(2a+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a +
b*x]^2),x]
```

```
[Out] -ArcTan[Sinh[2*a + 2*b*x]]/(2*b)
```

Maple [B] time = 0.101, size = 148, normalized size = 12.3

$$2\frac{\sqrt{2}}{b\left(2+2\sqrt{2}\right)}\arctan\left(2\frac{\tanh\left(1/2\,bx+a/2\right)}{2+2\sqrt{2}}\right)+2\frac{1}{b\left(2+2\sqrt{2}\right)}\arctan\left(2\frac{\tanh\left(1/2\,bx+a/2\right)}{2+2\sqrt{2}}\right)-2\frac{\sqrt{2}}{b\left(-2+2\sqrt{2}\right)}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x)
```

```
[Out] 2/b*2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))+2/b/(
2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))-2/b*2^(1/2)/(-2+2*
2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))+2/b/(-2+2*2^(1/2))*ar
ctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))
```

Maxima [B] time = 1.58104, size = 68, normalized size = 5.67

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-bx-a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="maxima")
```

```
[Out] -(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*e^(-b*x - a))))/b
```

Fricas [B] time = 1.94676, size = 103, normalized size = 8.58

$$\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="fricas")
```

```
[Out] arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx-\int -\frac{\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**2+sech(b*x+a)**2)/(csch(b*x+a)**2+sech(b*x+a)**2),
x)
```

```
[Out] -Integral(csch(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x) - Inte
gral(-sech(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x)
```

Giac [A] time = 1.16565, size = 20, normalized size = 1.67

$$-\frac{\arctan\left(e^{(2\,bx+2\,a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="giac")
```

```
[Out] -arctan(e^(2*b*x + 2*a))/b
```

3.1058 $\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)} \, dx$

Optimal. Leaf size=47

$$\frac{1}{3b(\tanh(a+bx)+1)}+\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}$$

```
[Out] (4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) + 1/(3*b*(1 + Tanh[
a + b*x]))
```

Rubi [A] time = 0.40999, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 618, 204}

$$\frac{1}{3b(\tanh(a+bx)+1)}+\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a + b*x]^3
),x]
```

```
[Out] (4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) + 1/(3*b*(1 + Tanh[
a + b*x]))
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1-x-x^2}{1+x+x^3+x^4}\,dx,x,\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)^2}-\frac{2}{3\left(1-x+x^2\right)}\right)dx,x,\tanh(a+bx)\right)}{b}\\ &= \frac{1}{3b(1+\tanh(a+bx))}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\tanh(a+bx)\right)}{3b}\\ &= \frac{1}{3b(1+\tanh(a+bx))}+\frac{4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\tanh(a+bx)\right)}{3b}\\ &= \frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}+\frac{1}{3b(1+\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.318515, size = 52, normalized size = 1.11

$$\frac{-3\sinh(2(a+bx))+3\cosh(2(a+bx))-8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(a+bx)-1}{\sqrt{3}}\right)}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a +
b*x]^3),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[a + b*x])/Sqrt[3]] + 3*Cosh[2*(a + b*x)] -
3*Sinh[2*(a + b*x)])/(18*b)
```

Maple [C] time = 0.247, size = 120, normalized size = 2.6

$$\frac{2}{3\,b}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{-2}-\frac{2}{3\,b}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{-1}+\frac{\frac{2i}{9}\sqrt{3}}{b}\ln\left(\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2+\left(i\sqrt{3}-1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x)
```

```
[Out] 2/3/b/(tanh(1/2*b*x+1/2*a)+1)^2-2/3/b/(tanh(1/2*b*x+1/2*a)+1)+2/9*I/b*3^(1/
2)*ln(tanh(1/2*b*x+1/2*a)^2+(I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)+1)-2/9*I/b*3^
(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(-I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)+1)
```

Maxima [B] time = 1.77548, size = 126, normalized size = 2.68

$$-\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9\,b}+\frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="maxima")
```

```
[Out] -4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*
sqrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^
(1/4)*sqrt(2))))/b + 1/6*e^(-2*b*x - 2*a)/b
```

Fricas [B] time = 2.04038, size = 369, normalized size = 7.85

$$\frac{8\left(\sqrt{3}\cosh\left(bx+a\right)^{2}+2\sqrt{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\sinh\left(bx+a\right)^{2}\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)}{3\,(\cosh(bx+a)-\sinh(bx+a))}\right)+3}{18\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="fricas")
```

```
[Out] 1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*co
sh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{csch}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx-\int -\frac{\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**3+sech(b*x+a)**3)/(csch(b*x+a)**3+sech(b*x+a)**3),
x)
```

```
[Out] -Integral(csch(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x) - Inte
gral(-sech(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x)
```

Giac [A] time = 1.1638, size = 51, normalized size = 1.09

$$-\frac{4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2bx+2a)}\right)}{9b}+\frac{e^{(-2bx-2a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="giac")
```

```
[Out] -4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a))/b + 1/6*e^(-2*b*x - 2*a)/b
```

3.1059 $\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b}$$

```
[Out] ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b) - ArcTan[1 + Sqrt[2]*Tanh[a +
 b*x]]/(Sqrt[2]*b)
```

Rubi [A] time = 1.41791, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a + b*x]^4
),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b) - ArcTan[1 + Sqrt[2]*Tanh[a +
 b*x]]/(Sqrt[2]*b)
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{-1-x^2}{1+x^4}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \tanh(a+\right.}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\,\mathrm{ta}\right.}{\sqrt{2}b}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\tan^{-1}\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

Mathematica [A] time = 0.0258839, size = 26, normalized size = 0.51

$$-\frac{\tan^{-1}\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a +
b*x]^4),x]
```

```
[Out] -(ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

Maple [C] time = 0.134, size = 138, normalized size = 2.7

$$\frac{\frac{i}{4}\sqrt{2}}{b}\ln\left(2i\sqrt{2}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4+2i\sqrt{2}\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\tanh(1/2\,bx+a/2)\right)^2+1\right)-\frac{\frac{i}{4}\sqrt{2}}{b}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x)
```

```
[Out] 1/4*I/b*2^(1/2)*ln(2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4+
2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)-1/4*I/b*2^(1/2)*
ln(-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4-2*I*2^(1/2)*tan
h(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\left(e^{(-bx-a)}+e^{(-5\,bx-5\,a)}\right)e^{(-bx-a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx-2\int\frac{\left(e^{(-4\,bx-4\,a)}+1\right)e^{(-2\,bx-2\,a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="maxima")
```

```
[Out] -2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x -
4*a) + e^(-8*b*x - 8*a) + 1), x) - 2*integrate((e^(-4*b*x - 4*a) + 1)*e^(-2
*b*x - 2*a)/(6*e^(-4*b*x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

Fricas [B] time = 1.70971, size = 558, normalized size = 10.94

$$\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a)}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right)+\sqrt{2}\,\mathrm{arct}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)
*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 - 7
*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*cos
h(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a
)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))
/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx-\int -\frac{\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**4+sech(b*x+a)**4)/(csch(b*x+a)**4+sech(b*x+a)**4),
x)
```

```
[Out] -Integral(csch(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x) - Inte
gral(-sech(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="giac")
```

```
[Out] sage0*x
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```