Computer algebra independent integration tests

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/6.6.3-Hyperbolic-cosecant-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [175]. This is test number [183].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (175)	% 0. (0)
Mathematica	% 100. (175)	% 0. (0)
Maple	% 77.71 (136)	% 22.29 (39)
Maxima	% 52.57 (92)	% 47.43 (83)
Fricas	% 74.29 (130)	% 25.71 (45)
Sympy	% 0. (0)	% 100. (175)
Giac	% 60.57 (106)	% 39.43 (69)

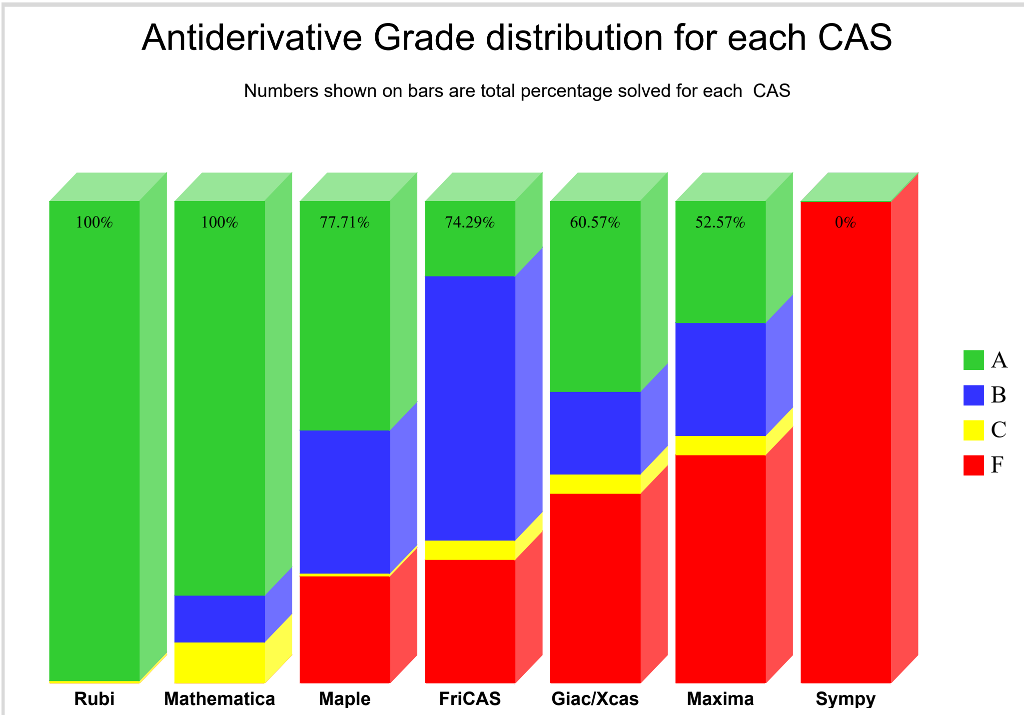
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

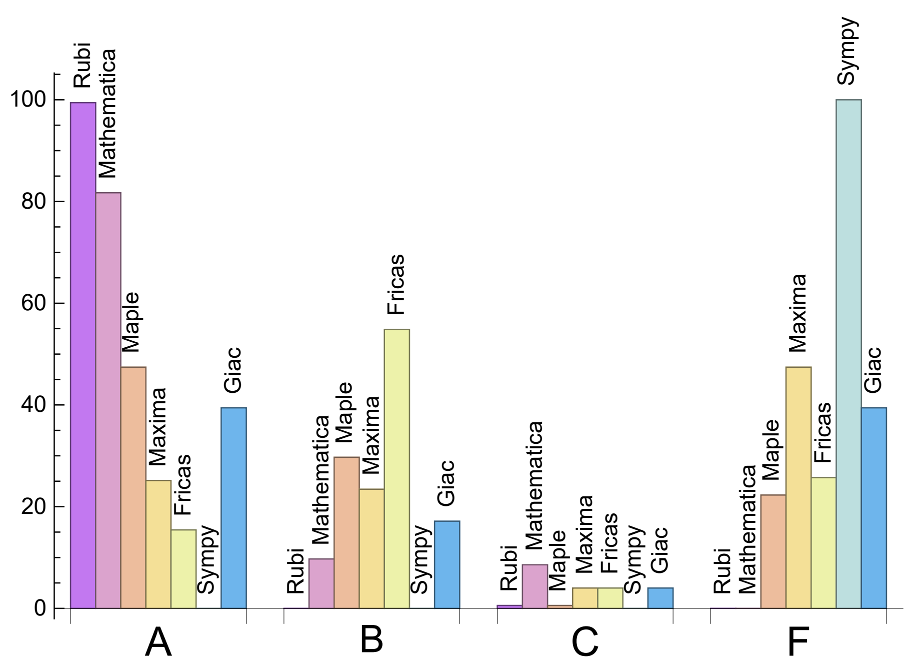
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.43	0.	0.57	0.
Mathematica	81.71	9.71	8.57	0.
Maple	47.43	29.71	0.57	22.29
Maxima	25.14	23.43	4.	47.43
Fricas	15.43	54.86	4.	25.71
Sympy	0.	0.	0.	100.
Giac	39.43	17.14	4.	39.43

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size
Rubi	0.08	68.71	1.01	62.
Mathematica	0.37	71.59	1.23	61.
Maple	0.06	143.17	2.26	99.
Maxima	1.3	128.91	2.52	82.5
Fricas	1.79	1252.73	15.9	439.5
Sympy	Round[Mean[], 0.01]	Round[Mean[], 0.01]	Round[Mean[], 0.01]	Round[Median[], 0.0
Giac	1.17	124.68	2.21	89.5

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {160}

Mathematica {157, 158, 159, 163}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

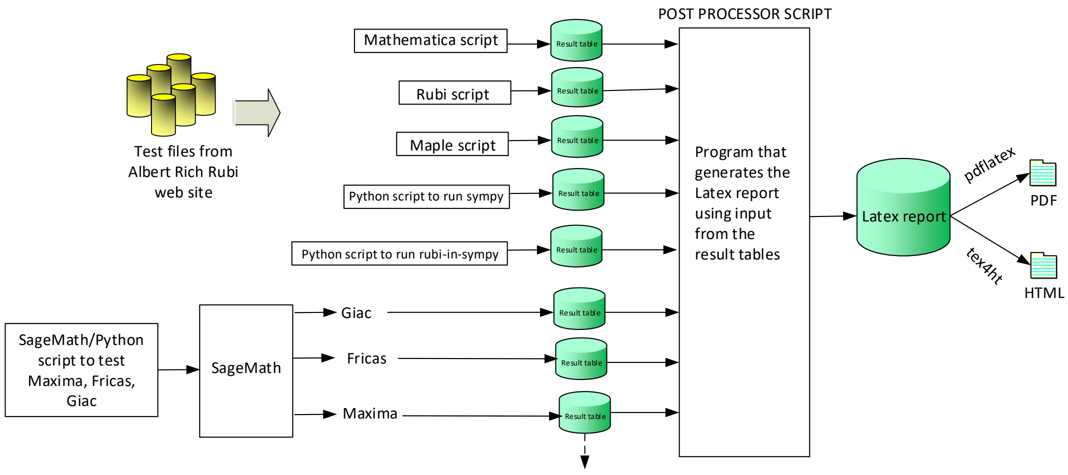
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { 160 }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 141, 143, 145, 147, 149, 152, 153, 156, 157, 158, 160, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { 1, 24, 55, 67, 68, 69, 70, 73, 89, 91, 103, 110, 112, 159, 161, 162, 165 }

C grade: { 113, 115, 117, 132, 134, 136, 140, 142, 144, 146, 148, 150, 151, 154, 155 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 42, 43, 44, 45, 49, 50, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 85, 87, 96, 97, 98, 100, 102, 104, 106, 107, 109, 114, 116, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 62, 63, 64, 65, 75, 76, 77, 78, 84, 86, 88, 89, 90, 91, 92, 93, 94, 95, 99, 101, 103, 105, 108, 110, 111, 112, 113, 115, 119, 120, 121, 122, 123, 124 }

C grade: { 160 }

F grade: { 13, 14, 15, 16, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 137, 139, 149, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164 }

2.1.4 Maxima

A grade: { 1, 2, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 71, 72, 73, 84, 87, 97, 105, 106, 107, 115, 117, 118, 126, 127, 128, 129, 130, 131, 133, 145, 162, 165, 166

B grade: { 3, 4, 5, 6, 42, 43, 44, 68, 69, 70, 85, 86, 88, 89, 90, 91, 92, 94, 96, 99, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 120, 122, 124, 125, 141, 153, 160, 161, 167, 168, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 93, 95, 98, 100, 114, 116, 119, 121, 123, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.1.5 FriCAS

A grade: { 49, 50, 62, 63, 66, 84, 97, 107, 108, 117, 118, 128, 129, 130, 131, 133, 135, 137, 139, 141, 143, 147, 149, 151, 153, 155, 162 }

B grade: { 1, 2, 3, 4, 5, 6, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 55, 56, 58, 59, 60, 61, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 145, 160, 161, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 54, 57, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

2.1.7 Giac

A grade: { 2, 4, 6, 29, 30, 31, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 93, 94, 95, 96, 97, 98, 100, 102, 103, 105, 107, 108, 114, 116, 117, 118, 119, 123, 125, 126, 127, 128, 129, 130, 131, 161, 162, 166, 168 }

B grade: { 1, 3, 5, 32, 68, 73, 85, 86, 87, 88, 90, 91, 92, 99, 101, 104, 106, 109, 110, 111, 112, 113, 115, 120, 121, 122, 124, 165, 167, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 170, 171, 172, 173, 174, 175 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	38	15	19	116	0	39
normalized size	1	1.	3.17	1.25	1.58	9.67	0.	3.25
time (sec)	N/A	0.006	0.018	0.003	1.005	1.553	0.	1.147

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	24	111	0	24
normalized size	1	1.	1.	1.09	2.18	10.09	0.	2.18
time (sec)	N/A	0.01	0.01	0.005	0.99	1.514	0.	1.139

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	27	113	1088	0	122
normalized size	1	1.	1.68	0.79	3.32	32.	0.	3.59
time (sec)	N/A	0.022	0.014	0.01	1.008	1.638	0.	1.147

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	35	23	122	448	0	42
normalized size	1	1.	1.35	0.88	4.69	17.23	0.	1.62
time (sec)	N/A	0.012	0.013	0.01	1.025	1.574	0.	1.126

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	95	41	180	3087	0	154
normalized size	1	1.	1.73	0.75	3.27	56.13	0.	2.8
time (sec)	N/A	0.043	0.016	0.011	1.03	1.722	0.	1.151

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	56	33	277	954	0	57
normalized size	1	1.	1.33	0.79	6.6	22.71	0.	1.36
time (sec)	N/A	0.015	0.016	0.012	1.011	1.496	0.	1.171

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	101	0	0	0	0
normalized size	1	1.	0.76	1.26	0.	0.	0.	0.
time (sec)	N/A	0.037	0.119	0.199	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	0	0	0
normalized size	1	1.	0.75	2.03	0.	0.	0.	0.
time (sec)	N/A	0.03	0.205	0.151	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	0	0	0
normalized size	1	1.	0.89	1.61	0.	0.	0.	0.
time (sec)	N/A	0.02	0.187	0.204	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	0	0	0
normalized size	1	1.	0.93	2.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.034	0.152	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	100	0	0	0	0
normalized size	1	1.	0.79	1.25	0.	0.	0.	0.
time (sec)	N/A	0.032	0.065	0.174	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	67	164	0	0	0	0
normalized size	1	1.	0.84	2.05	0.	0.	0.	0.
time (sec)	N/A	0.032	0.125	0.237	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.157	0.092	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.101	0.072	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	60	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.057	0.072	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.033	0.129	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	227	0	0	0	0
normalized size	1	1.	0.93	4.05	0.	0.	0.	0.
time (sec)	N/A	0.023	0.045	0.138	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	73	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.088	0.071	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	68	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.13	0.069	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.157	0.073	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	67	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.097	0.214	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	114	100	365	0	97
normalized size	1	1.	1.02	2.85	2.5	9.12	0.	2.42
time (sec)	N/A	0.017	0.119	0.053	1.697	1.605	0.	1.187

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	41	99	66	197	0	77
normalized size	1	1.	1.71	4.12	2.75	8.21	0.	3.21
time (sec)	N/A	0.012	0.059	0.043	1.691	1.526	0.	1.138

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	20	67	26	46	0	36
normalized size	1	1.	6.67	22.33	8.67	15.33	0.	12.
time (sec)	N/A	0.008	0.005	0.052	1.747	1.508	0.	1.154

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	15	39	0	34
normalized size	1	1.	1.	4.46	1.15	3.	0.	2.62
time (sec)	N/A	0.01	0.005	0.049	1.613	1.502	0.	1.153

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	118	31	80	0	68
normalized size	1	1.	0.82	3.58	0.94	2.42	0.	2.06
time (sec)	N/A	0.015	0.014	0.039	1.641	1.557	0.	1.153

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	178	47	135	0	86
normalized size	1	1.	0.67	3.63	0.96	2.76	0.	1.76
time (sec)	N/A	0.02	0.025	0.04	1.579	1.405	0.	1.179

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	39	238	63	184	0	103
normalized size	1	1.	0.6	3.66	0.97	2.83	0.	1.58
time (sec)	N/A	0.024	0.034	0.044	1.7	1.706	0.	1.195

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	41	123	124	3120	0	101
normalized size	1	1.	0.63	1.89	1.91	48.	0.	1.55
time (sec)	N/A	0.034	0.098	0.067	1.558	1.715	0.	1.14

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	103	81	1007	0	78
normalized size	1	1.	0.65	2.24	1.76	21.89	0.	1.7
time (sec)	N/A	0.025	0.064	0.053	1.638	1.627	0.	1.135

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	20	67	32	296	0	39
normalized size	1	1.	0.77	2.58	1.23	11.38	0.	1.5
time (sec)	N/A	0.018	0.005	0.072	1.582	1.901	0.	1.171

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	23	247	0	32
normalized size	1	1.	1.	4.46	1.77	19.	0.	2.46
time (sec)	N/A	0.014	0.006	0.065	1.732	1.886	0.	1.145

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	47	856	0	73
normalized size	1	1.	0.75	3.61	1.31	23.78	0.	2.03
time (sec)	N/A	0.022	0.023	0.049	1.7	1.842	0.	1.178

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	72	1885	0	90
normalized size	1	1.	0.65	3.56	1.31	34.27	0.	1.64
time (sec)	N/A	0.03	0.027	0.05	1.652	1.91	0.	1.152

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	96	3340	0	108
normalized size	1	1.	0.57	3.54	1.3	45.14	0.	1.46
time (sec)	N/A	0.04	0.052	0.052	1.715	2.003	0.	1.193

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	68	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.168	0.056	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.114	0.047	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	60	42	0	0	0	0	0
normalized size	1	1.07	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.028	0.066	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	43	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.073	0.066	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.075	0.048	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	71	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.12	0.048	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	59	72	837	9415	0	69
normalized size	1	1.	0.36	0.44	5.1	57.41	0.	0.42
time (sec)	N/A	0.042	0.049	0.076	1.661	2.336	0.	1.153

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	60	435	4632	0	53
normalized size	1	1.	0.4	0.51	3.69	39.25	0.	0.45
time (sec)	N/A	0.032	0.032	0.059	1.782	1.912	0.	1.168

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	46	162	1623	0	36
normalized size	1	1.	0.53	0.74	2.61	26.18	0.	0.58
time (sec)	N/A	0.023	0.018	0.053	1.703	1.657	0.	1.163

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	18	230	0	18
normalized size	1	1.	1.	1.81	1.12	14.38	0.	1.12
time (sec)	N/A	0.017	0.005	0.068	1.727	1.385	0.	1.167

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	89	30	771	0	35
normalized size	1	1.	0.67	2.47	0.83	21.42	0.	0.97
time (sec)	N/A	0.016	0.024	0.069	1.582	1.64	0.	1.202

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	62	3687	0	68
normalized size	1	1.	0.44	2.67	0.72	42.87	0.	0.79
time (sec)	N/A	0.033	0.038	0.053	1.579	1.779	0.	1.249

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	97	9443	0	103
normalized size	1	1.	0.42	2.74	0.73	71.54	0.	0.78
time (sec)	N/A	0.051	0.111	0.056	1.679	1.954	0.	1.176

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	63	47	80	0	42
normalized size	1	1.	1.69	1.97	1.47	2.5	0.	1.31
time (sec)	N/A	0.015	0.113	0.038	1.192	1.509	0.	1.152

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	63	47	80	0	42
normalized size	1	1.	1.69	1.97	1.47	2.5	0.	1.31
time (sec)	N/A	0.015	0.103	0.043	1.139	1.595	0.	1.171

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	136	0	0	1403	0	0
normalized size	1	1.	1.27	0.	0.	13.11	0.	0.
time (sec)	N/A	0.127	1.402	0.258	0.	2.419	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	100	0	0	1089	0	0
normalized size	1	1.	1.39	0.	0.	15.12	0.	0.
time (sec)	N/A	0.042	1.172	0.206	0.	2.405	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	814	0	0
normalized size	1	1.	2.	0.	0.	20.35	0.	0.
time (sec)	N/A	0.02	0.89	0.52	0.	2.396	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	118	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	1.081	0.324	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	327	0	0	2372	0	0
normalized size	1	1.	2.66	0.	0.	19.28	0.	0.
time (sec)	N/A	0.146	2.367	0.176	0.	3.065	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	815	0	0
normalized size	1	1.	2.	0.	0.	20.38	0.	0.
time (sec)	N/A	0.023	0.922	0.578	0.	2.39	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	117	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	1.071	0.446	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	581	0	0
normalized size	1	1.	2.	0.	0.	25.26	0.	0.
time (sec)	N/A	0.017	0.686	0.261	0.	2.12	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	582	0	0
normalized size	1	1.	2.	0.	0.	25.3	0.	0.
time (sec)	N/A	0.018	0.675	0.286	0.	2.112	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	590	0	0
normalized size	1	1.	2.	0.	0.	25.65	0.	0.
time (sec)	N/A	0.016	0.657	0.188	0.	2.16	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	589	0	0
normalized size	1	1.	2.	0.	0.	25.61	0.	0.
time (sec)	N/A	0.017	0.643	0.192	0.	2.157	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	182	96	244	0	89
normalized size	1	1.	1.09	3.14	1.66	4.21	0.	1.53
time (sec)	N/A	0.071	0.14	0.048	1.145	1.658	0.	1.18

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	56	137	80	194	0	68
normalized size	1	1.	1.22	2.98	1.74	4.22	0.	1.48
time (sec)	N/A	0.071	0.128	0.046	1.054	1.748	0.	1.196

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	46	96	63	151	0	54
normalized size	1	1.	1.28	2.67	1.75	4.19	0.	1.5
time (sec)	N/A	0.06	0.124	0.042	1.003	1.592	0.	1.173

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	35	51	42	108	0	35
normalized size	1	1.	1.75	2.55	2.1	5.4	0.	1.75
time (sec)	N/A	0.047	0.049	0.039	0.981	1.658	0.	1.216

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	27	12	16	20	0	11
normalized size	1	1.	1.93	0.86	1.14	1.43	0.	0.79
time (sec)	N/A	0.021	0.019	0.02	1.029	1.357	0.	1.151

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	19	39	89	0	30
normalized size	1	1.	2.18	1.12	2.29	5.24	0.	1.76
time (sec)	N/A	0.054	0.033	0.017	1.021	1.589	0.	1.18

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	70	35	74	216	0	62
normalized size	1	1.	2.69	1.35	2.85	8.31	0.	2.38
time (sec)	N/A	0.084	0.109	0.018	1.025	1.64	0.	1.21

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	81	53	109	381	0	68
normalized size	1	1.	2.19	1.43	2.95	10.3	0.	1.84
time (sec)	N/A	0.064	0.307	0.025	1.035	1.61	0.	1.164

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	508	92	316	3453	0	236
normalized size	1	1.	4.66	0.84	2.9	31.68	0.	2.17
time (sec)	N/A	0.128	6.23	0.039	1.026	1.733	0.	1.136

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	118	66	184	1844	0	174
normalized size	1	1.	1.57	0.88	2.45	24.59	0.	2.32
time (sec)	N/A	0.051	0.88	0.028	0.995	1.661	0.	1.185

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	61	37	59	603	0	90
normalized size	1	1.	1.79	1.09	1.74	17.74	0.	2.65
time (sec)	N/A	0.031	0.219	0.009	1.042	1.657	0.	1.175

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	43	20	26	131	0	49
normalized size	1	1.	2.53	1.18	1.53	7.71	0.	2.88
time (sec)	N/A	0.01	0.014	0.003	0.982	1.539	0.	1.16

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	87	0	473	0	115
normalized size	1	1.	1.19	1.61	0.	8.76	0.	2.13
time (sec)	N/A	0.059	0.104	0.015	0.	1.607	0.	1.177

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	142	238	0	1503	0	223
normalized size	1	1.	1.41	2.36	0.	14.88	0.	2.21
time (sec)	N/A	0.158	0.381	0.057	0.	1.929	0.	1.221

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	213	822	0	4658	0	402
normalized size	1	1.	1.31	5.04	0.	28.58	0.	2.47
time (sec)	N/A	0.316	0.929	0.076	0.	2.217	0.	1.305

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	262	0	1947	0	209
normalized size	1	1.	0.97	2.45	0.	18.2	0.	1.95
time (sec)	N/A	0.458	0.412	0.039	0.	2.056	0.	1.202

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	174	0	1142	0	155
normalized size	1	1.	1.02	2.17	0.	14.28	0.	1.94
time (sec)	N/A	0.288	0.136	0.034	0.	1.966	0.	1.155

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	0	640	0	116
normalized size	1	1.	1.07	1.61	0.	11.23	0.	2.04
time (sec)	N/A	0.107	0.105	0.033	0.	1.933	0.	1.177

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	35	0	321	0	76
normalized size	1	1.	1.22	0.95	0.	8.68	0.	2.05
time (sec)	N/A	0.066	0.024	0.015	0.	1.798	0.	1.163

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	0	456	0	111
normalized size	1	1.	1.16	0.98	0.	9.12	0.	2.22
time (sec)	N/A	0.115	0.049	0.014	0.	2.046	0.	1.184

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	73	0	926	0	132
normalized size	1	1.	1.2	1.24	0.	15.69	0.	2.24
time (sec)	N/A	0.16	0.278	0.016	0.	2.2	0.	1.209

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	124	108	0	2288	0	190
normalized size	1	1.	1.49	1.3	0.	27.57	0.	2.29
time (sec)	N/A	0.292	0.476	0.023	0.	2.81	0.	1.178

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	170	57	134	0	51
normalized size	1	1.	0.84	4.47	1.5	3.53	0.	1.34
time (sec)	N/A	0.128	0.033	0.039	1.04	1.858	0.	1.181

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	53	108	0	47
normalized size	1	1.	1.	0.79	2.79	5.68	0.	2.47
time (sec)	N/A	0.106	0.01	0.022	1.037	1.858	0.	1.141

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	84	41	89	0	35
normalized size	1	1.	1.	4.2	2.05	4.45	0.	1.75
time (sec)	N/A	0.092	0.029	0.033	1.044	1.893	0.	1.241

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	28	81	0	34
normalized size	1	1.	1.	1.25	1.75	5.06	0.	2.12
time (sec)	N/A	0.058	0.009	0.023	1.025	1.745	0.	1.131

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	20	43	55	155	0	72
normalized size	1	1.	0.71	1.54	1.96	5.54	0.	2.57
time (sec)	N/A	0.08	0.03	0.033	1.044	1.634	0.	1.143

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	64	49	109	93	0	36
normalized size	1	1.	3.37	2.58	5.74	4.89	0.	1.89
time (sec)	N/A	0.111	0.053	0.036	1.027	1.491	0.	1.2

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	32	89	124	431	0	127
normalized size	1	1.	0.8	2.22	3.1	10.78	0.	3.18
time (sec)	N/A	0.13	0.059	0.043	1.055	1.696	0.	1.162

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	96	93	347	212	0	74
normalized size	1	1.	3.31	3.21	11.97	7.31	0.	2.55
time (sec)	N/A	0.121	0.104	0.045	0.992	1.446	0.	1.166

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	97	600	327	3536	0	262
normalized size	1	1.	0.95	5.88	3.21	34.67	0.	2.57
time (sec)	N/A	0.198	0.277	0.041	1.01	1.752	0.	1.159

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	180	486	0	2414	0	298
normalized size	1	1.	1.44	3.89	0.	19.31	0.	2.38
time (sec)	N/A	0.378	1.146	0.04	0.	1.728	0.	1.173

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	274	171	1251	0	131
normalized size	1	1.	0.98	4.81	3.	21.95	0.	2.3
time (sec)	N/A	0.16	0.108	0.035	1.007	1.614	0.	1.158

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	80	172	0	859	0	163
normalized size	1	1.	1.04	2.23	0.	11.16	0.	2.12
time (sec)	N/A	0.206	0.228	0.03	0.	1.573	0.	1.168

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	65	246	0	53
normalized size	1	1.	0.95	1.55	3.25	12.3	0.	2.65
time (sec)	N/A	0.08	0.012	0.022	1.016	1.582	0.	1.163

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	84	89	177	0	120
normalized size	1	1.	0.56	1.31	1.39	2.77	0.	1.88
time (sec)	N/A	0.111	0.057	0.026	1.479	1.735	0.	1.149

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	67	81	0	693	0	115
normalized size	1	1.	1.12	1.35	0.	11.55	0.	1.92
time (sec)	N/A	0.142	0.177	0.029	0.	1.628	0.	1.151

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	78	275	217	1748	0	294
normalized size	1	1.	0.82	2.89	2.28	18.4	0.	3.09
time (sec)	N/A	0.22	0.149	0.036	1.512	1.739	0.	1.175

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	114	170	0	2822	0	235
normalized size	1	1.	1.1	1.63	0.	27.13	0.	2.26
time (sec)	N/A	0.27	0.611	0.04	0.	1.809	0.	1.22

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	138	1168	470	6677	0	505
normalized size	1	1.	0.93	7.84	3.15	44.81	0.	3.39
time (sec)	N/A	0.341	0.274	0.046	1.583	2.281	0.	1.178

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	75	155	194	991	0	162
normalized size	1	1.	0.69	1.42	1.78	9.09	0.	1.49
time (sec)	N/A	0.086	0.187	0.062	1.022	1.737	0.	1.163

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	126	99	130	394	0	84
normalized size	1	1.	2.42	1.9	2.5	7.58	0.	1.62
time (sec)	N/A	0.093	0.125	0.056	1.027	1.674	0.	1.212

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	109	130	568	0	132
normalized size	1	1.	0.79	1.42	1.69	7.38	0.	1.71
time (sec)	N/A	0.067	0.115	0.055	1.048	1.582	0.	1.138

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	71	67	57	149	0	51
normalized size	1	1.	1.97	1.86	1.58	4.14	0.	1.42
time (sec)	N/A	0.073	0.084	0.047	1.009	1.652	0.	1.175

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	65	61	207	0	74
normalized size	1	1.	0.87	1.44	1.36	4.6	0.	1.64
time (sec)	N/A	0.048	0.037	0.045	1.038	1.631	0.	1.151

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	20	32	0	18
normalized size	1	1.	1.	1.31	1.54	2.46	0.	1.38
time (sec)	N/A	0.023	0.007	0.023	1.01	1.683	0.	1.148

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	13	27	27	49	0	23
normalized size	1	1.	1.18	2.45	2.45	4.45	0.	2.09
time (sec)	N/A	0.038	0.033	0.029	1.003	1.619	0.	1.191

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	49	107	0	51
normalized size	1	1.	1.	1.	4.08	8.92	0.	4.25
time (sec)	N/A	0.039	0.01	0.017	1.035	1.779	0.	1.175

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	65	61	74	254	0	65
normalized size	1	1.	2.41	2.26	2.74	9.41	0.	2.41
time (sec)	N/A	0.056	0.036	0.044	1.03	1.622	0.	1.165

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	78	101	292	0	92
normalized size	1	1.	1.	2.6	3.37	9.73	0.	3.07
time (sec)	N/A	0.046	0.015	0.068	1.022	1.722	0.	1.155

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	129	95	130	478	0	104
normalized size	1	1.	3.	2.21	3.02	11.12	0.	2.42
time (sec)	N/A	0.075	0.041	0.092	1.034	1.798	0.	1.156

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	253	1323	517	9482	0	583
normalized size	1	1.	1.3	6.82	2.66	48.88	0.	3.01
time (sec)	N/A	0.255	0.493	0.063	1.608	2.645	0.	1.211

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	141	207	0	4128	0	290
normalized size	1	1.	0.77	1.13	0.	22.56	0.	1.58
time (sec)	N/A	0.384	0.696	0.053	0.	1.855	0.	1.203

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	191	324	232	2379	0	316
normalized size	1	1.	1.69	2.87	2.05	21.05	0.	2.8
time (sec)	N/A	0.163	0.19	0.049	1.587	1.96	0.	1.229

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	95	0	899	0	138
normalized size	1	1.	0.82	0.95	0.	8.99	0.	1.38
time (sec)	N/A	0.221	0.318	0.04	0.	1.658	0.	1.216

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	108	100	211	0	120
normalized size	1	1.	1.03	1.77	1.64	3.46	0.	1.97
time (sec)	N/A	0.099	0.059	0.036	1.519	1.55	0.	1.228

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	38	72	0	30
normalized size	1	1.	0.58	1.11	2.	3.79	0.	1.58
time (sec)	N/A	0.032	0.008	0.021	1.011	1.549	0.	1.107

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	65	110	0	424	0	120
normalized size	1	1.	1.14	1.93	0.	7.44	0.	2.11
time (sec)	N/A	0.178	0.081	0.027	0.	1.814	0.	1.162

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	37	106	111	543	0	108
normalized size	1	1.	1.16	3.31	3.47	16.97	0.	3.38
time (sec)	N/A	0.072	0.05	0.03	1.007	1.665	0.	1.193

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	151	207	0	2174	0	217
normalized size	1	1.	1.72	2.35	0.	24.7	0.	2.47
time (sec)	N/A	0.332	0.574	0.038	0.	2.26	0.	1.205

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	83	219	257	3216	0	230
normalized size	1	1.	1.19	3.13	3.67	45.94	0.	3.29
time (sec)	N/A	0.094	0.124	0.04	1.049	1.833	0.	1.151

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	269	360	0	7830	0	412
normalized size	1	1.	1.47	1.97	0.	42.79	0.	2.25
time (sec)	N/A	0.337	1.562	0.043	0.	3.258	0.	1.163

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	130	388	491	9682	0	398
normalized size	1	1.	1.09	3.26	4.13	81.36	0.	3.34
time (sec)	N/A	0.141	0.253	0.046	1.083	2.273	0.	1.16

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	84	91	521	1512	0	122
normalized size	1	1.	0.42	0.46	2.62	7.6	0.	0.61
time (sec)	N/A	0.303	0.077	0.203	1.564	1.615	0.	1.162

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	72	80	282	797	0	104
normalized size	1	1.	0.49	0.54	1.92	5.42	0.	0.71
time (sec)	N/A	0.171	0.064	0.164	1.555	1.564	0.	1.154

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	113	300	0	86
normalized size	1	1.	0.97	1.19	1.95	5.17	0.	1.48
time (sec)	N/A	0.118	0.043	0.16	1.571	1.58	0.	1.17

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	68	53	97	0	65
normalized size	1	1.	0.96	1.48	1.15	2.11	0.	1.41
time (sec)	N/A	0.091	0.036	0.183	1.547	1.596	0.	1.154

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	49	165	0	96
normalized size	1	1.	0.65	1.43	0.66	2.23	0.	1.3
time (sec)	N/A	0.115	0.05	0.175	1.613	1.493	0.	1.136

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	76	216	84	319	0	275
normalized size	1	1.	0.47	1.33	0.52	1.97	0.	1.7
time (sec)	N/A	0.153	0.063	0.171	1.555	1.549	0.	1.193

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	122	559	0	375
normalized size	1	1.	0.42	1.3	0.49	2.24	0.	1.5
time (sec)	N/A	0.2	0.103	0.174	1.569	1.585	0.	1.213

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	0	0	0
normalized size	1	1.	0.99	1.54	0.	0.	0.	0.
time (sec)	N/A	0.068	0.169	0.058	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	39	62	103	0	0
normalized size	1	1.	1.47	1.3	2.07	3.43	0.	0.
time (sec)	N/A	0.045	0.048	0.033	1.779	1.611	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	60	127	0	0	0	0
normalized size	1	1.	0.5	1.07	0.	0.	0.	0.
time (sec)	N/A	0.086	0.111	0.039	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	74	97	0	193	0	0
normalized size	1	1.	1.07	1.41	0.	2.8	0.	0.
time (sec)	N/A	0.058	0.136	0.05	0.	1.593	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	109	0	0	0	0
normalized size	1	1.	0.95	1.82	0.	0.	0.	0.
time (sec)	N/A	0.041	0.103	0.033	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	77	0	0	184	0	0
normalized size	1	1.	1.28	0.	0.	3.07	0.	0.
time (sec)	N/A	0.035	0.088	0.033	0.	1.535	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	90	0	0	0	0
normalized size	1	1.	0.93	1.96	0.	0.	0.	0.
time (sec)	N/A	0.032	0.068	0.168	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	0	0	96	0	0
normalized size	1	1.	1.32	0.	0.	2.34	0.	0.
time (sec)	N/A	0.046	0.111	0.033	0.	1.571	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	58	126	0	0	0	0
normalized size	1	1.	0.78	1.7	0.	0.	0.	0.
time (sec)	N/A	0.066	0.094	0.036	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	38	120	80	0	0
normalized size	1	1.	1.32	1.52	4.8	3.2	0.	0.
time (sec)	N/A	0.041	0.037	0.033	1.627	1.563	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	112	0	0	0	0
normalized size	1	1.	0.94	1.75	0.	0.	0.	0.
time (sec)	N/A	0.055	0.094	0.036	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	121	0	240	0	0
normalized size	1	1.	0.74	0.95	0.	1.88	0.	0.
time (sec)	N/A	0.08	0.199	0.038	0.	1.75	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	133	0	0	0	0
normalized size	1	1.	0.68	1.13	0.	0.	0.	0.
time (sec)	N/A	0.078	0.166	0.033	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	47	62	122	0	0
normalized size	1	1.	1.47	1.57	2.07	4.07	0.	0.
time (sec)	N/A	0.041	0.048	0.03	1.746	1.571	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	63	140	0	0	0	0
normalized size	1	1.	0.39	0.86	0.	0.	0.	0.
time (sec)	N/A	0.1	0.121	0.035	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	87	113	0	219	0	0
normalized size	1	1.	0.91	1.18	0.	2.28	0.	0.
time (sec)	N/A	0.067	0.175	0.035	0.	1.58	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	65	124	0	0	0	0
normalized size	1	1.	0.76	1.44	0.	0.	0.	0.
time (sec)	N/A	0.063	0.121	0.032	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	203	0	0
normalized size	1	1.	0.97	0.	0.	2.23	0.	0.
time (sec)	N/A	0.073	0.156	0.031	0.	1.55	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	60	152	0	0	0	0
normalized size	1	1.	0.46	1.17	0.	0.	0.	0.
time (sec)	N/A	0.073	0.108	0.037	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	63	130	0	225	0	0
normalized size	1	1.	0.66	1.35	0.	2.34	0.	0.
time (sec)	N/A	0.046	0.089	0.036	0.	1.703	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	163	0	0	0	0
normalized size	1	1.	0.81	2.43	0.	0.	0.	0.
time (sec)	N/A	0.037	0.094	0.168	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	0	117	59	0	0
normalized size	1	1.	1.22	0.	4.33	2.19	0.	0.
time (sec)	N/A	0.039	0.034	0.035	1.582	1.557	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.108	0.033	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	53	0	0	159	0	0
normalized size	1	1.	0.77	0.	0.	2.3	0.	0.
time (sec)	N/A	0.061	0.104	0.035	0.	1.633	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.157	0.093	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	126	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	4.243	1.119	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	101	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	5.297	1.283	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	200	0	0	0	0	0
normalized size	1	1.	2.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	8.479	0.107	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	42	137	30	509	128	602	0	0
normalized size	1	3.26	0.71	12.12	3.05	14.33	0.	0.
time (sec)	N/A	0.135	0.387	0.237	2.189	1.625	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	62	0	103	104	0	51
normalized size	1	1.	2.38	0.	3.96	4.	0.	1.96
time (sec)	N/A	0.039	0.117	0.049	1.029	1.548	0.	1.186

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	0	66	107	0	53
normalized size	1	1.	2.5	0.	2.54	4.12	0.	2.04
time (sec)	N/A	0.046	0.098	0.066	1.062	1.453	0.	1.172

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	115	0	0	1323	0	0
normalized size	1	1.	1.28	0.	0.	14.7	0.	0.
time (sec)	N/A	0.087	5.955	0.105	0.	1.819	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	1347	0	0
normalized size	1	1.	0.97	0.	0.	20.41	0.	0.
time (sec)	N/A	0.075	0.845	0.084	0.	1.702	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	54	23	30	219	0	190
normalized size	1	1.	2.7	1.15	1.5	10.95	0.	9.5
time (sec)	N/A	0.018	0.064	0.008	1.097	1.842	0.	1.211

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	39	219	0	38
normalized size	1	1.	1.	1.05	2.05	11.53	0.	2.
time (sec)	N/A	0.027	0.066	0.012	1.122	1.921	0.	1.145

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	81	51	203	2109	0	278
normalized size	1	1.	1.47	0.93	3.69	38.35	0.	5.05
time (sec)	N/A	0.045	0.059	0.019	1.155	1.845	0.	1.209

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	56	36	124	869	0	63
normalized size	1	1.	1.33	0.86	2.95	20.69	0.	1.5
time (sec)	N/A	0.033	0.065	0.01	1.132	1.936	0.	1.178

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	135	84	313	5852	0	329
normalized size	1	1.	1.52	0.94	3.52	65.75	0.	3.7
time (sec)	N/A	0.07	0.058	0.015	1.268	2.23	0.	1.233

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	84	144	0	0	0	0
normalized size	1	1.	0.76	1.3	0.	0.	0.	0.
time (sec)	N/A	0.068	0.207	0.036	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	0	0	0
normalized size	1	1.	0.75	1.98	0.	0.	0.	0.
time (sec)	N/A	0.063	0.112	0.188	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	0	0	0
normalized size	1	1.	0.92	1.67	0.	0.	0.	0.
time (sec)	N/A	0.043	0.092	0.118	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	0	0	0
normalized size	1	1.	0.94	2.03	0.	0.	0.	0.
time (sec)	N/A	0.044	0.076	0.027	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	86	143	0	0	0	0
normalized size	1	1.	0.77	1.29	0.	0.	0.	0.
time (sec)	N/A	0.06	0.128	0.027	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	227	0	0	0	0
normalized size	1	1.	0.86	2.05	0.	0.	0.	0.
time (sec)	N/A	0.061	0.176	0.197	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [114] had the largest ratio of [0.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	2	1	1.	8	0.125
5	A	3	2	1.	8	0.25
6	A	2	1	1.	8	0.125
7	A	3	3	1.	10	0.3
8	A	3	3	1.	10	0.3
9	A	2	2	1.	10	0.2
10	A	2	2	1.	10	0.2
11	A	3	3	1.	10	0.3
12	A	3	3	1.	10	0.3
13	A	4	3	1.	12	0.25
14	A	3	3	1.	12	0.25
15	A	3	3	1.	12	0.25
16	A	2	2	1.	12	0.167
17	A	2	2	1.	12	0.167
18	A	3	3	1.	12	0.25
19	A	3	3	1.	12	0.25
20	A	4	3	1.	12	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	2	2	1.	10	0.2
22	A	4	3	1.	10	0.3
23	A	3	3	1.	10	0.3
24	A	2	2	1.	10	0.2
25	A	2	2	1.	10	0.2
26	A	3	3	1.	10	0.3
27	A	4	3	1.	10	0.3
28	A	5	3	1.	10	0.3
29	A	5	4	1.	10	0.4
30	A	4	4	1.	10	0.4
31	A	3	3	1.	10	0.3
32	A	2	2	1.	10	0.2
33	A	3	3	1.	10	0.3
34	A	4	3	1.	10	0.3
35	A	5	3	1.	10	0.3
36	A	7	4	1.	10	0.4
37	A	5	4	1.	10	0.4
38	A	4	4	1.07	10	0.4
39	A	4	4	1.	10	0.4
40	A	5	4	1.	10	0.4
41	A	7	4	1.	10	0.4
42	A	3	2	1.	10	0.2
43	A	3	2	1.	10	0.2
44	A	3	2	1.	10	0.2
45	A	3	3	1.	10	0.3
46	A	3	3	1.	10	0.3
47	A	5	3	1.	10	0.3
48	A	7	3	1.	10	0.3
49	A	2	2	1.	15	0.133
50	A	2	2	1.	15	0.133
51	A	5	5	1.	17	0.294
52	A	4	4	1.	17	0.235
53	A	2	2	1.	17	0.118
54	A	5	4	1.	17	0.235
55	A	6	5	1.	17	0.294
56	A	2	2	1.	17	0.118
57	A	5	4	1.	17	0.235
58	A	2	2	1.	12	0.167
59	A	2	2	1.	12	0.167
60	A	2	2	1.	12	0.167
61	A	2	2	1.	12	0.167
62	A	7	5	1.	13	0.385
63	A	6	5	1.	13	0.385
64	A	5	5	1.	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
65	A	4	4	1.	11	0.364
66	A	1	1	1.	11	0.091
67	A	3	3	1.	13	0.231
68	A	4	4	1.	13	0.308
69	A	6	6	1.	13	0.462
70	A	6	5	1.	12	0.417
71	A	5	4	1.	12	0.333
72	A	4	4	1.	12	0.333
73	A	2	1	1.	10	0.1
74	A	4	4	1.	12	0.333
75	A	6	6	1.	12	0.5
76	A	7	7	1.	12	0.583
77	A	8	7	1.	13	0.538
78	A	7	7	1.	13	0.538
79	A	6	6	1.	11	0.546
80	A	4	4	1.	11	0.364
81	A	6	6	1.	13	0.462
82	A	7	7	1.	13	0.538
83	A	8	8	1.	13	0.615
84	A	7	7	1.	13	0.538
85	A	6	4	1.	13	0.308
86	A	5	5	1.	13	0.385
87	A	4	3	1.	11	0.273
88	A	6	6	1.	11	0.546
89	A	6	5	1.	13	0.385
90	A	7	7	1.	13	0.538
91	A	7	6	1.	13	0.462
92	A	5	4	1.	13	0.308
93	A	7	6	1.	13	0.462
94	A	5	4	1.	13	0.308
95	A	6	6	1.	13	0.462
96	A	5	4	1.	11	0.364
97	A	4	3	1.	11	0.273
98	A	6	6	1.	13	0.462
99	A	6	5	1.	13	0.385
100	A	7	6	1.	13	0.462
101	A	7	5	1.	13	0.385
102	A	3	2	1.	13	0.154
103	A	5	3	1.	13	0.231
104	A	3	2	1.	13	0.154
105	A	4	3	1.	13	0.231
106	A	3	2	1.	11	0.182
107	A	2	2	1.	11	0.182
108	A	3	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	3	2	1.	13	0.154
110	A	4	3	1.	13	0.231
111	A	3	2	1.	13	0.154
112	A	5	3	1.	13	0.231
113	A	11	7	1.	13	0.538
114	A	16	9	1.	13	0.692
115	A	8	6	1.	13	0.462
116	A	10	9	1.	13	0.692
117	A	6	5	1.	11	0.454
118	A	4	4	1.	11	0.364
119	A	8	8	1.	13	0.615
120	A	3	2	1.	13	0.154
121	A	7	7	1.	13	0.538
122	A	3	2	1.	13	0.154
123	A	16	9	1.	13	0.692
124	A	3	2	1.	13	0.154
125	A	6	5	1.	25	0.2
126	A	6	5	1.	25	0.2
127	A	4	4	1.	25	0.16
128	A	4	4	1.	25	0.16
129	A	5	4	1.	25	0.16
130	A	6	5	1.	25	0.2
131	A	6	5	1.	25	0.2
132	A	6	6	1.	15	0.4
133	A	3	3	1.	15	0.2
134	A	9	9	1.	15	0.6
135	A	6	6	1.	15	0.4
136	A	5	5	1.	13	0.385
137	A	6	6	1.	11	0.546
138	A	3	2	1.	15	0.133
139	A	5	5	1.	15	0.333
140	A	7	7	1.	15	0.467
141	A	3	3	1.	15	0.2
142	A	5	5	1.	15	0.333
143	A	8	7	1.	15	0.467
144	A	7	6	1.	15	0.4
145	A	3	3	1.	15	0.2
146	A	10	9	1.	15	0.6
147	A	7	6	1.	15	0.4
148	A	6	5	1.	15	0.333
149	A	7	6	1.	15	0.4
150	A	9	8	1.	13	0.615
151	A	7	7	1.	11	0.636
152	A	4	3	1.	15	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	3	3	1.	15	0.2
154	A	5	5	1.	15	0.333
155	A	6	6	1.	15	0.4
156	A	4	4	1.	11	0.364
157	A	4	4	1.	13	0.308
158	A	4	4	1.	13	0.308
159	A	4	4	1.	13	0.308
160	C	9	4	3.26	45	0.089
161	A	3	3	1.	15	0.2
162	A	4	4	1.	15	0.267
163	A	3	3	1.	20	0.15
164	A	3	3	1.	21	0.143
165	A	2	1	1.	15	0.067
166	A	3	2	1.	17	0.118
167	A	3	2	1.	17	0.118
168	A	3	1	1.	17	0.059
169	A	4	2	1.	17	0.118
170	A	4	3	1.	19	0.158
171	A	4	3	1.	19	0.158
172	A	3	2	1.	19	0.105
173	A	3	2	1.	19	0.105
174	A	4	3	1.	19	0.158
175	A	4	3	1.	19	0.158

Chapter 3

Listing of integrals

3.1 $\int \operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b)
```

Rubi [A] time = 0.0062549, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x],x]

[Out] -(ArcTanh[Cosh[a + b*x]]/b)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Mathematica [B] time = 0.0176559, size = 38, normalized size = 3.17

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x],x]
```

```
[Out] -(Log[Cosh[a/2 + (b*x)/2]]/b) + Log[Sinh[a/2 + (b*x)/2]]/b
```

Maple [A] time = 0.003, size = 15, normalized size = 1.3

$$\frac{1}{b}\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a),x)
```

```
[Out] 1/b*ln(tanh(1/2*b*x+1/2*a))
```

Maxima [A] time = 1.00528, size = 19, normalized size = 1.58

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*x + 1/2*a))/b
```

Fricas [B] time = 1.55341, size = 116, normalized size = 9.67

$$-\frac{\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*x + a) + sinh(b*x + a) + 1) - log(cosh(b*x + a) + sinh(b*x + a
) - 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x), x)
```

Giac [B] time = 1.14747, size = 39, normalized size = 3.25

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="giac")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(abs(e^(b*x + a) - 1))/b
```

3.2 $\int \operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\coth(a+bx)}{b}$$

```
[Out] -(Coth[a + b*x]/b)
```

Rubi [A] time = 0.009889, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767, 8}

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2,x]

[Out] -(Coth[a + b*x]/b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0099317, size = 11, normalized size = 1.

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2,x]

[Out] -(Coth[a + b*x]/b)
```

Maple [A] time = 0.005, size = 12, normalized size = 1.1

$$-\frac{\coth(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2,x)
```

```
[Out] -coth(b*x+a)/b
```

Maxima [A] time = 0.990234, size = 24, normalized size = 2.18

$$\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 1.5139, size = 111, normalized size = 10.09

$$-\frac{2}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2, x)
```

Giac [A] time = 1.1387, size = 24, normalized size = 2.18

$$-\frac{2}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) - 1))
```

3.3 $\int \operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[Out] ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rubi [A] time = 0.0221119, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(a+bx)\,dx&=-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}-\frac{1}{2}\int \operatorname{csch}(a+bx)\,dx\\&=\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0136658, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3,x]
```

```
[Out] -Csch[(a + b*x)/2]^2/(8*b) - Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

Maple [A] time = 0.01, size = 27, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a)))
```

Maxima [B] time = 1.00849, size = 113, normalized size = 3.32

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{2b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{2b}+\frac{e^{(-bx-a)}+e^{(-3bx-3a)}}{b\left(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(-b*x - a) + 1)/b - 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a) +
 e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 1.63838, size = 1088, normalized size = 32.

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3, x)
```

Giac [B] time = 1.1466, size = 122, normalized size = 3.59

$$\frac{\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right)}{4\,b} - \frac{\log\left(e^{(bx+a)} + e^{(-bx-a)} - 2\right)}{4\,b} - \frac{e^{(bx+a)} + e^{(-bx-a)}}{\left(\left(e^{(bx+a)} + e^{(-bx-a)}\right)^2 - 4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*log(e^(b*x + a) + e^(-b*x - a) + 2)/b - 1/4*log(e^(b*x + a) + e^(-b*x -
 a) - 2)/b - (e^(b*x + a) + e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2
- 4)*b)
```

3.4 $\int \operatorname{csch}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0118475, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4,x]
```

```
[Out] Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-i\coth(a+bx)\right)}{b}\\ &= \frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0128278, size = 35, normalized size = 1.35

$$\frac{2\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4,x]
```

```
[Out] (2*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b)
```

Maple [A] time = 0.01, size = 23, normalized size = 0.9

$$\frac{\coth(bx+a)}{b}\left(\frac{2}{3}-\frac{(\operatorname{csch}(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4,x)
```

```
[Out] 1/b*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
```

Maxima [B] time = 1.02507, size = 122, normalized size = 4.69

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}-\frac{4}{3\,b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1)) - 4/3/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1))
```

Fricas [B] time = 1.57385, size = 448, normalized size = 17.23

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}-3\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx+a\right)^{2}-3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 - 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 9*b*cosh(b*
x + a)^2 + 4*b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{4}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**4, x)
```

Giac [A] time = 1.12553, size = 42, normalized size = 1.62

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}-1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

3.5 $\int \operatorname{csch}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) + (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0430128, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) + (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\,dx &= -\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}-\frac{3}{4}\int \operatorname{csch}^3(a+bx)\,dx\\ &= \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3}{8}\int \operatorname{csch}(a+bx)\,dx\\ &= -\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0160215, size = 95, normalized size = 1.73

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5,x]
```

```
[Out] (3*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (3*Log[Tanh[(
a + b*x)/2]])/(8*b) + (3*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/
(64*b)
```

Maple [A] time = 0.011, size = 41, normalized size = 0.8

$$\frac{1}{b}\left(\left(-\frac{(\operatorname{csch}(bx+a))^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{Artanh}\left(e^{bx+a}\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5,x)
```

```
[Out] 1/b*((-1/4*csch(b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a
)))
```

Maxima [B] time = 1.03042, size = 180, normalized size = 3.27

$$-\frac{3\log\left(e^{(-bx-a)}+1\right)}{8b}+\frac{3\log\left(e^{(-bx-a)}-1\right)}{8b}-\frac{3e^{(-bx-a)}-11e^{(-3bx-3a)}-11e^{(-5bx-5a)}+3e^{(-7bx-7a)}}{4b\left(4e^{(-2bx-2a)}-6e^{(-4bx-4a)}+4e^{(-6bx-6a)}-e^{(-8bx-8a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b - 1/4*(3*e^(-b*x
 - a) - 11*e^(-3*b*x - 3*a) - 11*e^(-5*b*x - 5*a) + 3*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
 8*a) - 1))
```

Fricas [B] time = 1.72173, size = 3087, normalized size = 56.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)
^7 + 2*(63*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10*
(21*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(105*cosh(b*x +
 a)^4 - 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 22*cosh(b*x + a)^3 + 2*
(63*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)
^2 - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cos
h(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 3
0*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
+ a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) +
```

```
 1) + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^
8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*co
sh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 -
30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
 + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x +
a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a)
- 1) + 2*(21*cosh(b*x + a)^6 - 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 + 3)
*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*si
nh(b*x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*si
nh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b
*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x +
 a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*c
osh(b*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x +
 a)^2 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 -
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**5, x)
```

Giac [B] time = 1.15139, size = 154, normalized size = 2.8

$$-\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)}{16\,b}+\frac{3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}+\frac{3\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{3}-20\,e^{(bx+a)}-20\,e^{(-bx-a)}}{4\,\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}-4\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -3/16*log(e^(b*x + a) + e^(-b*x - a) + 2)/b + 3/16*log(e^(b*x + a) + e^(-b*
x - a) - 2)/b + 1/4*(3*(e^(b*x + a) + e^(-b*x - a))^3 - 20*e^(b*x + a) - 20
*e^(-b*x - a))/(((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2*b)
```

3.6 $\int \operatorname{csch}^6(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\coth^5(a+bx)}{5b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -(Coth[a + b*x]/b) + (2*Coth[a + b*x]^3)/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0153031, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$-\frac{\coth^5(a+bx)}{5b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^6,x]
```

```
[Out] -(Coth[a + b*x]/b) + (2*Coth[a + b*x]^3)/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(a+bx)\,dx &= -\frac{i\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0161284, size = 56, normalized size = 1.33

$$-\frac{8\coth(a+bx)}{15b}-\frac{\coth(a+bx)\operatorname{csch}^4(a+bx)}{5b}+\frac{4\coth(a+bx)\operatorname{csch}^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^6,x]
```

```
[Out] (-8*Coth[a + b*x])/(15*b) + (4*Coth[a + b*x]*Csch[a + b*x]^2)/(15*b) - (Cot
h[a + b*x]*Csch[a + b*x]^4)/(5*b)
```

Maple [A] time = 0.012, size = 33, normalized size = 0.8

$$\frac{\coth(bx+a)}{b}\left(-\frac{8}{15}-\frac{(\operatorname{csch}(bx+a))^4}{5}+\frac{4(\operatorname{csch}(bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^6,x)
```

```
[Out] 1/b*(-8/15-1/5*csch(b*x+a)^4+4/15*csch(b*x+a)^2)*coth(b*x+a)
```

Maxima [B] time = 1.01119, size = 277, normalized size = 6.6

$$-\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1\right)}+\frac{32\,e^{(-}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 32/3*e^(-4
*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15/(b*(5*e^(-2*b*x
 - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

Fricas [B] time = 1.49635, size = 954, normalized size = 22.71

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}-5\,b\cosh\left(bx+a\right)^{6}+\left(28\,b\cosh\left(bx+a\right)^{2}-5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 - 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 - 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 - 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 - 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 -
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 - 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 - 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{6}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**6,x)
```

```
[Out] Integral(csch(a + b*x)**6, x)
```

Giac [A] time = 1.17068, size = 57, normalized size = 1.36

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}-5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1
)^5)
```

3.7 $\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

```
[Out] (-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rubi [A] time = 0.0365541, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2641}

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx\\
&= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx\\
&= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b}
\end{aligned}$$

Mathematica [A] time = 0.118607, size = 61, normalized size = 0.76

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\coth(a+bx)+i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Coth[a + b*x] + I*EllipticF[((-2*I)*a + Pi - (2*I)
*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b)
```

Maple [A] time = 0.199, size = 101, normalized size = 1.3

$$-\frac{1}{3\cosh(bx+a)b}\left(i\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(5/2),x)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{csch}(bx+a)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

3.8 $\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] (-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0301108, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2639}

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx\\
&= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.204587, size = 57, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\cosh(a+bx)-\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b
*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b
```

Maple [A] time = 0.151, size = 154, normalized size = 2.

$$\frac{1}{\cosh(bx+a)\,b}\left(2\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},1/2\sqrt{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(3/2),x)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\operatorname{csch}(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

3.9 $\int \sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b}$$

```
[Out] ((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sin
h[a + b*x]])/b
```

Rubi [A] time = 0.0197578, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sin
h[a + b*x]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{csch}(a+bx)}\,dx &= \left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.187321, size = 48, normalized size = 0.89

$$\frac{2(i\sinh(a+bx))^{3/2}\operatorname{csch}^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Csch[a + b*x]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[
a + b*x])^(3/2))/b
```

Maple [A] time = 0.204, size = 87, normalized size = 1.6

$$\frac{i\sqrt{2}}{\cosh(bx+a)\,b}\sqrt{-i(i+\sinh(bx+a))}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\text{EllipticF}\left(\sqrt{-i(i+\sinh(bx+a))},\frac{\sqrt{2}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(1/2),x)
```

```
[Out] I*(-I*(I+sinh(b*x+a)))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(I+sinh(b*x+a)))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\operatorname{csch}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\operatorname{csch}(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\operatorname{csch}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(csch(a + b*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{csch}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(b*x + a)), x)
```

3.10 $\int \frac{1}{\sqrt{\mathbf{csch}(a+bx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I
*Sinh[a + b*x]])
```

Rubi [A] time = 0.0201483, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I
*Sinh[a + b*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx &= \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0339411, size = 50, normalized size = 0.93

$$\frac{2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csch[a + b*x]]*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a +
 b*x]])/b
```

Maple [A] time = 0.152, size = 108, normalized size = 2.

$$\frac{\sqrt{2}}{\cosh(bx+a)\,b}\sqrt{-i\,(i+\sinh(bx+a))}\sqrt{-i\,(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},1/2\,\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(1/2),x)
```

```
[Out] (-I*(I+sinh(b*x+a)))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{\operatorname{csch}(bx+a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(csch(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(csch(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

3.11 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

```
[Out] (2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]
]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rubi [A] time = 0.0324635, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]
]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx\\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx\\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0649159, size = 63, normalized size = 0.79

$$\frac{\sqrt{\operatorname{csch}(a+bx)}\left(\sinh(2(a+bx))-2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sq
rt[I*Sinh[a + b*x]] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [A] time = 0.174, size = 100, normalized size = 1.3

$$\frac{1}{\cosh(bx+a)\,b}\left(-\frac{i}{3}\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(3/2),x)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*sinh(b*x+a)*c
osh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}(bx + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

3.12 $\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}$$

```
[Out] (2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0324466, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx\\
&= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.125116, size = 67, normalized size = 0.84

$$\frac{2\left(\cosh(a+bx)-3\sqrt{i\sinh(a+bx)}\operatorname{csch}^2(a+bx)E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Cosh[a + b*x] - 3*Csch[a + b*x]^2*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)
/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(5*b*Csch[a + b*x]^(3/2))
```

Maple [A] time = 0.237, size = 164, normalized size = 2.1

$$\frac{1}{\cosh(bx+a)\,b}\left(-\frac{6\sqrt{2}}{5}\sqrt{1-i\sinh(bx+a)}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(5/2),x)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(-5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

3.13 $\int (b\operatorname{csch}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$\frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}$$

```
[Out] (6*b^3*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/(5*d) - (2*b*Cosh[c + d*x]*(b*C
sch[c + d*x])^(5/2))/(5*d) + (((6*I)/5)*b^4*EllipticE[(I*c - Pi/2 + I*d*x)/
2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0611193, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (6*b^3*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/(5*d) - (2*b*Cosh[c + d*x]*(b*C
sch[c + d*x])^(5/2))/(5*d) + (((6*I)/5)*b^4*EllipticE[(I*c - Pi/2 + I*d*x)/
2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\text{csch}(c+dx))^{7/2}\,dx = -\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}\left(3b^2\right)\int (b\text{csch}(c+dx))^{3/2}\,dx$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}\left(3b^4\right)\int \frac{}{\sqrt{b\text{csc}}}$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{\left(3b^4\right)\int \sqrt{i\sinh}}{5\sqrt{b\text{csch}(c+dx)}}$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} + \frac{6ib^4E\left(\frac{1}{2}\left(ic-\right.\right.}{5d\sqrt{b\text{csch}(c+dx)}}$$

Mathematica [A] time = 0.157263, size = 79, normalized size = 0.68

$$-\frac{2b^3\sqrt{b\text{csch}(c+dx)}\left(-3\cosh(c+dx)+\coth(c+dx)\text{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^3*Sqrt[b*Csch[c + d*x]]*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*
x] + 3*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/
(5*d)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (b\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(7/2),x)
```

```
[Out] int((b*csch(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\,\text{csch}\,(dx+c)}b^3\,\text{csch}\,(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b^3*csch(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{csch}\,(c+dx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

3.14 $\int (b\,\mathrm{csch}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2b\cosh(c+dx)(b\,\mathrm{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\,\mathrm{csch}(c+dx)}}{3d}$$

```
[Out] (-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b*
Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
d
```

Rubi [A] time = 0.038811, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2641}

$$-\frac{2b\cosh(c+dx)(b\,\mathrm{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\,\mathrm{csch}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b*
Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
d
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\,\mathrm{csch}(c+dx))^{5/2}\,dx &= -\frac{2b\cosh(c+dx)(b\,\mathrm{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}b^2\int\sqrt{b\,\mathrm{csch}(c+dx)}\,dx\\ &= -\frac{2b\cosh(c+dx)(b\,\mathrm{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}\left(b^2\sqrt{b\,\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int\frac{1}{\sqrt{i\sinh(c+}}\\ &= -\frac{2b\cosh(c+dx)(b\,\mathrm{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{b\,\mathrm{csch}(c+dx)}F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c}}{3d}\end{aligned}$$

Mathematica [A] time = 0.100849, size = 66, normalized size = 0.75

$$-\frac{2b^2\sqrt{b\operatorname{csch}(c+dx)}\left(\coth(c+dx)+i\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2ic-2idx+\pi),2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^2*Sqrt[b*Csch[c + d*x]]*(Coth[c + d*x] + I*EllipticF[((-2*I)*c + Pi -
 (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(3*d)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(5/2),x)
```

```
[Out] int((b*csch(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\operatorname{csch}(dx+c)}b^2\operatorname{csch}(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b^2*csch(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(c+dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{csch}(dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

3.15 $\int (b\,\mathrm{\mathbf{csch}}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\cosh(c+dx)\sqrt{b\mathrm{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\mathrm{csch}(c+dx)}}$$

```
[Out] (-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0376565, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$-\frac{2b\cosh(c+dx)\sqrt{b\mathrm{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\mathrm{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\mathrm{csch}(c+dx))^{3/2}\,dx &= -\frac{2b\cosh(c+dx)\sqrt{b\mathrm{csch}(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{b\mathrm{csch}(c+dx)}}\,dx\\
&= -\frac{2b\cosh(c+dx)\sqrt{b\mathrm{csch}(c+dx)}}{d}+\frac{b^2\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\
&= -\frac{2b\cosh(c+dx)\sqrt{b\mathrm{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0567943, size = 60, normalized size = 0.71

$$-\frac{2b\sqrt{b\operatorname{csch}(c+dx)}\left(\cosh(c+dx)-\sqrt{i\sinh(c+dx)}E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Csch[c + d*x]]*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*
I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(3/2),x)
```

```
[Out] int((b*csch(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{b\operatorname{csch}(dx+c)}b\operatorname{csch}(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b*csch(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

3.16 $\int \sqrt{b\,\mathrm{csch}(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\,\mathrm{csch}(c+dx)}}{d}$$

```
[Out] ((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rubi [A] time = 0.0218582, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\,\mathrm{csch}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\,\mathrm{csch}(c+dx)}\,dx &= \left(\sqrt{b\,\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{b\,\mathrm{csch}(c+dx)}F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0333764, size = 54, normalized size = 0.96

$$\frac{2i\sqrt{i\sinh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)\sqrt{b\,\mathrm{csch}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Si
nh[c + d*x]])/d
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int \sqrt{b\mathrm{csch}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(1/2),x)
```

```
[Out] int((b*csch(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{csch}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\,\mathrm{csch}\,(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{csch}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*csch(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{csch}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

3.17 $\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt
[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0226316, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt
[I*Sinh[c + d*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx &= \frac{\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0445771, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/(d*Sqrt[b*Csch[c + d*x]
]*Sqrt[I*Sinh[c + d*x]])
```

Maple [B] time = 0.138, size = 227, normalized size = 4.1

$$\frac{\sqrt{2}}{d}\frac{1}{\sqrt{\frac{b\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}\right)^{2}-1}}}-\frac{\sqrt{2}}{d\left(\left(\mathrm{e}^{dx+c}\right)^{2}-1\right)}\left(2\frac{b\left(\mathrm{e}^{dx+c}\right)^{2}-b}{b\sqrt{\mathrm{e}^{dx+c}\left(b\left(\mathrm{e}^{dx+c}\right)^{2}-b\right)}}-\sqrt{\mathrm{e}^{dx+c}+1}\sqrt{2-2\,\mathrm{e}^{dx+c}}\sqrt{-\mathrm{e}^{dx+c}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{dx}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)-1/d*(2*(b*exp(d*x+c)^2-b)
/b/(exp(d*x+c)*(b*exp(d*x+c)^2-b))^(1/2)-(exp(d*x+c)+1)^(1/2)*(2-2*exp(d*x+
c))^(1/2)*(-exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3-b*exp(d*x+c))^(1/2)*(-2*Ellip
ticE((exp(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(d*x+c)+1)^(1/2),1/2*2
^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)*(b*exp(d*x+c)*(exp(
d*x+c)^2-1))^(1/2)/(exp(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{csch}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\,\mathrm{csch}\left(dx+c\right)}}{b\,\mathrm{csch}\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))/(b*csch(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{csch}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*csch(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{csch}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

3.18 $\int \frac{1}{(b\,\mathbf{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\text{csch}(c+dx)}}{3b^2d}$$

```
[Out] (2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c
+ d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)
```

Rubi [A] time = 0.0385371, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c
+ d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\int \sqrt{b\text{csch}(c+dx)}\,dx}{3b^2}\\ &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\left(\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2}\\ &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}+\frac{2i\sqrt{b\text{csch}(c+dx)}F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}\end{aligned}$$

Mathematica [A] time = 0.088462, size = 73, normalized size = 0.81

$$\frac{\operatorname{csch}^2(c+dx)\left(\sinh(2(c+dx))-2i\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2ic-2idx+\pi),2\right)\right)}{3d(b\operatorname{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (Csch[c + d*x]^2*((-2*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I
*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(3*d*(b*Csch[c + d*x])^(3/2))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\operatorname{csch}(dx+c)}}{b^2\operatorname{csch}(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))/(b^2*csch(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{csch}\left(dx + c\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

3.19 $\int \frac{1}{(b\,\mathbf{csch}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{5bd(b\,\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\,\text{csch}(c+dx)}}$$

```
[Out] (2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(I
*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]
])
```

Rubi [A] time = 0.0382342, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh(c+dx)}{5bd(b\,\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\,\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(I
*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]
])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}}\,dx = \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} - \frac{3\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx}{5b^2}$$
$$= \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} - \frac{3\int \sqrt{i\sinh(c+dx)}\,dx}{5b^2\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$
$$= \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.13045, size = 68, normalized size = 0.76

$$\frac{\sinh(2(c+dx)) - \frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]]
 + Sinh[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Csch[c + d*x]])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(5/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\operatorname{csch}(dx+c)}}{b^3\operatorname{csch}(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))/(b^3*csch(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

3.20 $\int \frac{1}{(b\,\mathbf{csch}(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{10i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\,\text{csch}(c+dx)}}{21b^4d}-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\,\text{csch}(c+dx)}}+\frac{2\cosh(c+dx)}{7bd(b\,\text{csch}(c+dx))^{5/2}}$$

```
[Out] (2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (10*Cosh[c + d*x])/(21*
b^3*d*Sqrt[b*Csch[c + d*x]]) - (((10*I)/21)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^4*d)
```

Rubi [A] time = 0.0564636, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\,\text{csch}(c+dx)}}-\frac{10i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\,\text{csch}(c+dx)}}{21b^4d}+\frac{2\cosh(c+dx)}{7bd(b\,\text{csch}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (10*Cosh[c + d*x])/(21*
b^3*d*Sqrt[b*Csch[c + d*x]]) - (((10*I)/21)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^4*d)
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{5\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx}{7b^2}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} + \frac{5\int \sqrt{b\operatorname{csch}(c+dx)}\,dx}{21b^4}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} + \frac{\left(5\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh}}}{21b^4}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} - \frac{10i\sqrt{b\operatorname{csch}(c+dx)}F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{}}{21b^4d}$$

Mathematica [A] time = 0.156832, size = 80, normalized size = 0.68

$$\frac{\sqrt{b\operatorname{csch}(c+dx)}\left(40i\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2ic-2idx+\pi),2\right)-26\sinh(2(c+dx))+3\sinh(4(c+dx))\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Csch[c + d*x]]*((40*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*
Sqrt[I*Sinh[c + d*x]] - 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^
4*d)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(7/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \operatorname{csch}(dx+c)}}{b^4 \operatorname{csch}(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))/(b^4*csch(d*x + c)^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{csch}(c+dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{csch}(dx+c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

3.21 $\int (b\operatorname{csch}(c+dx))^n\,dx$

Optimal. Leaf size=74

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0346077, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^n,x]
```

```
[Out] (b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\operatorname{csch}(c+dx))^n\,dx &= (b\operatorname{csch}(c+dx))^n\left(\frac{\sinh(c+dx)}{b}\right)^n\int\left(\frac{\sinh(c+dx)}{b}\right)^{-n}dx\\ &= \frac{\cosh(c+dx)(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)\sinh(c+dx)}{d(1-n)\sqrt{\cosh^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0971341, size = 67, normalized size = 0.91

$$-\frac{\sinh(c+dx)\cosh(c+dx)\left(-\sinh^2(c+dx)\right)^{\frac{n-1}{2}}(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cosh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^n,x]
```

```
[Out] -((Cosh[c + d*x]*(b*Csch[c + d*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2,
 Cosh[c + d*x]^2]*Sinh[c + d*x]*(-Sinh[c + d*x]^2)^((-1 + n)/2))/d)
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^n,x)
```

```
[Out] int((b*csch(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\operatorname{csch}(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csch(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{csch}(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**n,x)
```

```
[Out] Integral((b*csch(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{csch}(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^n, x)
```

3.22 $\int \left(-\mathrm{csch}^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{3}{8}\sin^{-1}(\coth(x))+\frac{1}{4}\coth(x)\left(-\mathrm{csch}^2(x)\right)^{3/2}+\frac{3}{8}\coth(x)\sqrt{-\mathrm{csch}^2(x)}$$

```
[Out] (3*ArcSin[Coth[x]])/8 + (3*Coth[x]*Sqrt[-Csch[x]^2])/8 + (Coth[x]*(-Csch[x]
^2)^(3/2))/4
```

Rubi [A] time = 0.0170618, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 195, 216}

$$\frac{3}{8}\sin^{-1}(\coth(x))+\frac{1}{4}\coth(x)\left(-\mathrm{csch}^2(x)\right)^{3/2}+\frac{3}{8}\coth(x)\sqrt{-\mathrm{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Coth[x]])/8 + (3*Coth[x]*Sqrt[-Csch[x]^2])/8 + (Coth[x]*(-Csch[x]
^2)^(3/2))/4
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \left(-\mathrm{csch}^2(x)\right)^{5/2}\,dx &= \text{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx,x,\coth(x)\right)\\
&= \frac{1}{4}\coth(x)\left(-\mathrm{csch}^2(x)\right)^{3/2}+\frac{3}{4}\text{Subst}\left(\int \sqrt{1-x^2}\,dx,x,\coth(x)\right)\\
&= \frac{3}{8}\coth(x)\sqrt{-\mathrm{csch}^2(x)}+\frac{1}{4}\coth(x)\left(-\mathrm{csch}^2(x)\right)^{3/2}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,\coth(x)\right)\\
&= \frac{3}{8}\sin^{-1}(\coth(x))+\frac{3}{8}\coth(x)\sqrt{-\mathrm{csch}^2(x)}+\frac{1}{4}\coth(x)\left(-\mathrm{csch}^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.118668, size = 41, normalized size = 1.02

$$\frac{1}{64}\sinh(x)\left(-\text{csch}^2(x)\right)^{5/2}\left(6\left(\cosh(3x)+4\sinh^4(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)-22\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(5/2),x]
```

```
[Out] ((-Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]*
Sinh[x]^4)))/64
```

Maple [B] time = 0.053, size = 114, normalized size = 2.9

$$\frac{3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3}{4\left(e^{2x}-1\right)^3}\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}+\frac{3\,e^{-x}\left(e^{2x}-1\right)\ln\left(e^x-1\right)}{8}\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}-\frac{3\,e^{-x}\left(e^{2x}-1\right)\ln\left(e^x+1\right)}{8}\sqrt{-\frac{e}{\left(e^{2x}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(5/2),x)
```

```
[Out] 1/4/(exp(2*x)-1)^3*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp(4*x)
-11*exp(2*x)+3)+3/8*exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*l
n(exp(x)-1)-3/8*exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(ex
p(x)+1)
```

Maxima [C] time = 1.69723, size = 100, normalized size = 2.5

$$\frac{3i\,e^{(-x)}-11i\,e^{(-3x)}-11i\,e^{(-5x)}+3i\,e^{(-7x)}}{4\left(4\,e^{(-2x)}-6\,e^{(-4x)}+4\,e^{(-6x)}-e^{(-8x)}-1\right)}+\frac{3}{8}i\,\log\left(e^{(-x)}+1\right)-\frac{3}{8}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*I*e^(-x) - 11*I*e^(-3*x) - 11*I*e^(-5*x) + 3*I*e^(-7*x))/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3/8*I*log(e^(-x) + 1) - 3/8*I*
log(e^(-x) - 1)
```

Fricas [C] time = 1.60467, size = 365, normalized size = 9.12

$$\frac{\left(-3i\,e^{(8x)}+12i\,e^{(6x)}-18i\,e^{(4x)}+12i\,e^{(2x)}-3i\right)\log\left(e^x+1\right)+\left(3i\,e^{(8x)}-12i\,e^{(6x)}+18i\,e^{(4x)}-12i\,e^{(2x)}+3i\right)\log\left(e^x-1\right)+}{8\left(e^{(8x)}-4\,e^{(6x)}+6\,e^{(4x)}-4\,e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*((-3*I*e^(8*x) + 12*I*e^(6*x) - 18*I*e^(4*x) + 12*I*e^(2*x) - 3*I)*log(
e^x + 1) + (3*I*e^(8*x) - 12*I*e^(6*x) + 18*I*e^(4*x) - 12*I*e^(2*x) + 3*I)
```

```
*log(e^x - 1) + 6*I*e^(7*x) - 22*I*e^(5*x) - 22*I*e^(3*x) + 6*I*e^x)/(e^(8*
x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(5/2), x)
```

Giac [C] time = 1.18677, size = 97, normalized size = 2.42

$$-\frac{1}{16}\left(\frac{4i\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3i\,\log\left(e^{(-x)}+e^{x}+2\right)+3i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(4*I*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2 - 4)
^2 - 3*I*log(e^(-x) + e^x + 2) + 3*I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) +
e^x)
```

3.23 $\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=24

$$\frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

```
[Out] ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[-Csch[x]^2])/2
```

Rubi [A] time = 0.0115918, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 195, 216}

$$\frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[-Csch[x]^2])/2
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx &= \operatorname{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \coth(x)\right)\\ &= \frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \coth(x)\right)\\ &= \frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0586266, size = 41, normalized size = 1.71

$$\frac{1}{4}\operatorname{csch}\left(\frac{x}{2}\right)\sqrt{-\operatorname{csch}^2(x)}\operatorname{sech}\left(\frac{x}{2}\right)\left(\cosh(x)+\sinh^2(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(3/2),x]
```

```
[Out] (Csch[x/2]*Sqrt[-Csch[x]^2]*Sech[x/2]*(Cosh[x] + Log[Tanh[x/2]]*Sinh[x]^2))
/4
```

Maple [B] time = 0.043, size = 99, normalized size = 4.1

$$\frac{e^{2x}+1}{e^{2x}-1}\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}+\frac{e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)}{2}\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}-\frac{e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}+1\right)}{2}\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(3/2),x)
```

```
[Out] 1/(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(exp(2*x)+1)+1/2*exp(-x)*(e
xp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)-1/2*exp(-x)*(exp(2
*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)
```

Maxima [C] time = 1.69098, size = 66, normalized size = 2.75

$$\frac{i\,e^{(-x)}+i\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{1}{2}i\,\log\left(e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (I*e^(-x) + I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*I*log(e^(-x) + 1)
 - 1/2*I*log(e^(-x) - 1)
```

Fricas [C] time = 1.52588, size = 197, normalized size = 8.21

$$\frac{\left(-i\,e^{(4\,x)}+2i\,e^{(2\,x)}-i\right)\log\left(e^{x}+1\right)+\left(i\,e^{(4\,x)}-2i\,e^{(2\,x)}+i\right)\log\left(e^{x}-1\right)+2i\,e^{(3\,x)}+2i\,e^{x}}{2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*x) + 2*I*e^(2*x) - I)*log(e^x + 1) + (I*e^(4*x) - 2*I*e^(2*x)
 + I)*log(e^x - 1) + 2*I*e^(3*x) + 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\operatorname{csch}^{2}(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(3/2), x)
```

Giac [C] time = 1.13772, size = 77, normalized size = 3.21

$$-\frac{1}{4}\left(\frac{4i\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^{2}-4}-i\,\log\left(e^{(-x)}+e^{x}+2\right)+i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - I*log(e^(-x) + e^x + 2) +
 I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) + e^x)
```

3.24 $\int \sqrt{-\text{csch}^2(x)}\,dx$

Optimal. Leaf size=3

$$\sin^{-1}(\coth(x))$$

```
[Out] ArcSin[Coth[x]]
```

Rubi [A] time = 0.0081699, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4122, 216}

$$\sin^{-1}(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Csch[x]^2],x]
```

```
[Out] ArcSin[Coth[x]]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\text{csch}^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \coth(x)\right)\\ &= \sin^{-1}(\coth(x))\end{aligned}$$

Mathematica [B] time = 0.0048837, size = 20, normalized size = 6.67

$$\sinh(x)\sqrt{-\text{csch}^2(x)}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Csch[x]^2],x]
```

```
[Out] Sqrt[-Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

Maple [B] time = 0.052, size = 67, normalized size = 22.3

$$-e^{-x}\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(e^{x}+1\right)+e^{-x}\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(1/2),x)
```

```
[Out] -exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)+exp(-x)
*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

Maxima [C] time = 1.74671, size = 26, normalized size = 8.67

$$i\log\left(e^{(-x)}+1\right)-i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [C] time = 1.50787, size = 46, normalized size = 15.33

$$-i\log\left(e^{x}+1\right)+i\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*log(e^x + 1) + I*log(e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\operatorname{csch}^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-csch(x)**2), x)
```

Giac [C] time = 1.15374, size = 36, normalized size = 12.

$$\left(i\log\left(e^{x}+1\right)-i\log\left(|e^{x}-1|\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (I*log(e^x + 1) - I*log(abs(e^x - 1)))*sgn(-e^(3*x) + e^x)
```

3.25 $\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}} \, dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Rubi [A] time = 0.0097834, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4122, 191}

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}} \, dx &= \operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}} \, dx, x, \coth(x)\right)\\ &= \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0046467, size = 13, normalized size = 1.

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Maple [B] time = 0.049, size = 58, normalized size = 4.5

$$\frac{e^{2x}}{2e^{2x}-2}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}+\frac{1}{2e^{2x}-2}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(1/2),x)
```

```
[Out] 1/2/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(-exp(2*x)/(
exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)
```

Maxima [C] time = 1.6128, size = 15, normalized size = 1.15

$$\frac{1}{2}i\,e^{(-x)}+\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I*e^(-x) + 1/2*I*e^x
```

Fricas [C] time = 1.50158, size = 39, normalized size = 3.

$$\frac{1}{2}\left(-i\,e^{(2x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*e^(2*x) - I)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\operatorname{csch}^{2}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-csch(x)**2), x)
```

Giac [C] time = 1.15287, size = 34, normalized size = 2.62

$$-\frac{-i\,e^{(-x)} - i\,e^{x}}{2\,\mathrm{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(-I*e^(-x) - I*e^x)/sgn(-e^(3*x) + e^x)
```

3.26 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}+\frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}$$

```
[Out] Coth[x]/(3*(-Csch[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[-Csch[x]^2])
```

Rubi [A] time = 0.014503, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}+\frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(-Csch[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[-Csch[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0144007, size = 27, normalized size = 0.82

$$\frac{9\coth(x)-\cosh(3x)\text{csch}(x)}{12\sqrt{-\text{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-3/2),x]
```

```
[Out] (9*Coth[x] - Cosh[3*x]*Csch[x])/(12*Sqrt[-Csch[x]^2])
```

Maple [B] time = 0.039, size = 118, normalized size = 3.6

$$-\frac{e^{4x}}{24e^{2x}-24}\frac{1}{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{3e^{2x}}{8e^{2x}-8}\frac{1}{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{3}{8e^{2x}-8}\frac{1}{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}-\frac{e^{-2x}}{24e^{2x}-24}\frac{1}{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(3/2),x)
```

```
[Out] -1/24*exp(4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+3/8/(-exp(2*x)
/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+3/8/(-exp(2*x)/(exp(2*x)-1)^2)
^(1/2)/(exp(2*x)-1)-1/24*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^
(1/2)
```

Maxima [C] time = 1.64072, size = 31, normalized size = 0.94

$$-\frac{1}{24}ie^{(3x)}+\frac{3}{8}ie^{(-x)}-\frac{1}{24}ie^{(-3x)}+\frac{3}{8}ie^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*I*e^(3*x) + 3/8*I*e^(-x) - 1/24*I*e^(-3*x) + 3/8*I*e^x
```

Fricas [C] time = 1.55686, size = 80, normalized size = 2.42

$$\frac{1}{24}\left(ie^{(6x)}-9ie^{(4x)}-9ie^{(2x)}+i\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(I*e^(6*x) - 9*I*e^(4*x) - 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\operatorname{csch}^{2}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-3/2), x)
```

Giac [C] time = 1.1529, size = 68, normalized size = 2.06

$$\frac{i\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}}{24\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}-\frac{i\left(e^{(3\,x)}-9\,e^{x}\right)}{24\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*I*(9*e^(2*x) - 1)*e^(-3*x)/sgn(-e^(3*x) + e^x) - 1/24*I*(e^(3*x) - 9*e
^x)/sgn(-e^(3*x) + e^x)
```

3.27 $\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{8\coth(x)}{15\sqrt{-\mathrm{csch}^2(x)}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}$$

```
[Out] Coth[x]/(5*(-Csch[x]^2)^(5/2)) + (4*Coth[x])/(15*(-Csch[x]^2)^(3/2)) + (8*C
oth[x])/(15*Sqrt[-Csch[x]^2])
```

Rubi [A] time = 0.0196418, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{8\coth(x)}{15\sqrt{-\mathrm{csch}^2(x)}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(-Csch[x]^2)^(5/2)) + (4*Coth[x])/(15*(-Csch[x]^2)^(3/2)) + (8*C
oth[x])/(15*Sqrt[-Csch[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8}{15}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15\sqrt{-\operatorname{csch}^2(x)}}$$

Mathematica [A] time = 0.0249027, size = 33, normalized size = 0.67

$$\frac{(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\operatorname{csch}(x)}{240\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-5/2),x]

[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x])/(240*Sqrt[-Csch[x]^2])
```

Maple [B] time = 0.04, size = 178, normalized size = 3.6

$$\frac{e^{6x}}{160\,e^{2x}-160}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{5\,e^{4x}}{96\,e^{2x}-96}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{5\,e^{2x}}{16\,e^{2x}-16}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{5}{16\,e^{2x}-16}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{5\,e^{-2x}}{96\,e^{2x}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(5/2),x)

[Out] 1/160*exp(6*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96*exp(4*x)/
(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(-exp(2*x)/(exp(2*x)-1)^
2)^(1/2)/(exp(2*x)-1)*exp(2*x)+5/16/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2
*x)-1)-5/96*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+1/160*e
xp(-4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [C] time = 1.57892, size = 47, normalized size = 0.96

$$\frac{1}{160}i\,e^{(5x)} - \frac{5}{96}i\,e^{(3x)} + \frac{5}{16}i\,e^{(-x)} - \frac{5}{96}i\,e^{(-3x)} + \frac{1}{160}i\,e^{(-5x)} + \frac{5}{16}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*I*e^(5*x) - 5/96*I*e^(3*x) + 5/16*I*e^(-x) - 5/96*I*e^(-3*x) + 1/160*
I*e^(-5*x) + 5/16*I*e^x
```

Fricas [C] time = 1.40485, size = 135, normalized size = 2.76

$$\frac{1}{480}\left(-3i\,e^{(10\,x)}+25i\,e^{(8\,x)}-150i\,e^{(6\,x)}-150i\,e^{(4\,x)}+25i\,e^{(2\,x)}-3i\right)e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(10*x) + 25*I*e^(8*x) - 150*I*e^(6*x) - 150*I*e^(4*x) + 25*I*
e^(2*x) - 3*I)*e^(-5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-\operatorname{csch}^{2}(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-5/2), x)
```

Giac [C] time = 1.17908, size = 86, normalized size = 1.76

$$\frac{i\left(150\,e^{(4\,x)}-25\,e^{(2\,x)}+3\right)e^{(-5\,x)}}{480\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}+\frac{i\left(3\,e^{(5\,x)}-25\,e^{(3\,x)}+150\,e^{x}\right)}{480\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*I*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x)/sgn(-e^(3*x) + e^x) + 1/480
*I*(3*e^(5*x) - 25*e^(3*x) + 150*e^x)/sgn(-e^(3*x) + e^x)
```

3.28 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}} \, dx$

Optimal. Leaf size=65

$$\frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}+\frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}}$$

```
[Out] Coth[x]/(7*(-Csch[x]^2)^(7/2)) + (6*Coth[x])/(35*(-Csch[x]^2)^(5/2)) + (8*C
oth[x])/(35*(-Csch[x]^2)^(3/2)) + (16*Coth[x])/(35*Sqrt[-Csch[x]^2])
```

Rubi [A] time = 0.0241402, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}+\frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(-Csch[x]^2)^(7/2)) + (6*Coth[x])/(35*(-Csch[x]^2)^(5/2)) + (8*C
oth[x])/(35*(-Csch[x]^2)^(3/2)) + (16*Coth[x])/(35*Sqrt[-Csch[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{9/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{24}{35}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{16}{35}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \text{cot}\right.$$
$$= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}$$

Mathematica [A] time = 0.0341773, size = 39, normalized size = 0.6

$$\frac{(1225\cosh(x) - 245\cosh(3x) + 49\cosh(5x) - 5\cosh(7x))\text{csch}(x)}{2240\sqrt{-\text{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] ((1225*Cosh[x] - 245*Cosh[3*x] + 49*Cosh[5*x] - 5*Cosh[7*x])*Csch[x])/(2240
*Sqrt[-Csch[x]^2])
```

Maple [B] time = 0.044, size = 238, normalized size = 3.7

$$-\frac{e^{8x}}{896e^{2x}-896}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{7e^{6x}}{640e^{2x}-640}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{7e^{4x}}{128e^{2x}-128}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{35e^{2x}}{128e^{2x}-128}\frac{1}{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(7/2),x)
```

```
[Out] -1/896*exp(8*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(6*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/128*exp(4*x)/(exp(2*x)-1)
/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+35/128/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(
exp(2*x)-1)*exp(2*x)+35/128/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)-7
/128*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(-4*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-1/896*exp(-6*x)/(exp(2*x)-1
)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [C] time = 1.70011, size = 63, normalized size = 0.97

$$-\frac{1}{896}ie^{(7x)} + \frac{7}{640}ie^{(5x)} - \frac{7}{128}ie^{(3x)} + \frac{35}{128}ie^{(-x)} - \frac{7}{128}ie^{(-3x)} + \frac{7}{640}ie^{(-5x)} - \frac{1}{896}ie^{(-7x)} + \frac{35}{128}ie^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*I*e^(7*x) + 7/640*I*e^(5*x) - 7/128*I*e^(3*x) + 35/128*I*e^(-x) - 7/
128*I*e^(-3*x) + 7/640*I*e^(-5*x) - 1/896*I*e^(-7*x) + 35/128*I*e^x
```

Fricas [C] time = 1.70587, size = 184, normalized size = 2.83

$$\frac{1}{4480}\left(5i\,e^{(14\,x)} - 49i\,e^{(12\,x)} + 245i\,e^{(10\,x)} - 1225i\,e^{(8\,x)} - 1225i\,e^{(6\,x)} + 245i\,e^{(4\,x)} - 49i\,e^{(2\,x)} + 5i\right)e^{(-7\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*I*e^(14*x) - 49*I*e^(12*x) + 245*I*e^(10*x) - 1225*I*e^(8*x) - 12
25*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(7/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-7/2), x)
```

Giac [C] time = 1.19482, size = 103, normalized size = 1.58

$$\frac{i\left(1225\,e^{(6\,x)} - 245\,e^{(4\,x)} + 49\,e^{(2\,x)} - 5\right)e^{(-7\,x)}}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)} - \frac{i\left(5\,e^{(7\,x)} - 49\,e^{(5\,x)} + 245\,e^{(3\,x)} - 1225\,e^{x}\right)}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4480*I*(1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x)/sgn(-e^(3*x
) + e^x) - 1/4480*I*(5*e^(7*x) - 49*e^(5*x) + 245*e^(3*x) - 1225*e^x)/sgn(-
e^(3*x) + e^x)
```

3.29 $\int \left(a\operatorname{csch}^2(x)\right)^{5/2} dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^2\coth(x)\sqrt{a\operatorname{csch}^2(x)}-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{4}a\coth(x)\left(a\operatorname{csch}^2(x)\right)^{3/2}$$

```
[Out] (-3*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/8 + (3*a^2*Coth[x
]*Sqrt[a*Csch[x]^2])/8 - (a*Coth[x]*(a*Csch[x]^2)^(3/2))/4
```

Rubi [A] time = 0.0340968, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 206}

$$\frac{3}{8}a^2\coth(x)\sqrt{a\operatorname{csch}^2(x)}-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{4}a\coth(x)\left(a\operatorname{csch}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/8 + (3*a^2*Coth[x
]*Sqrt[a*Csch[x]^2])/8 - (a*Coth[x]*(a*Csch[x]^2)^(3/2))/4
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = -\left(a\,\text{Subst}\left(\int \left(-a+ax^2\right)^{3/2} dx, x, \coth(x)\right)\right)$$
$$= -\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{-a+ax^2}\,dx, x, \coth(x)\right)$$
$$= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}dx, x, \text{co}\right.$$
$$= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x, \frac{\text{co}}{\sqrt{a\text{cs}}}\right.$$
$$= -\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) + \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

Mathematica [A] time = 0.0984696, size = 41, normalized size = 0.63

$$\frac{1}{64}\sinh(x)\left(a\text{csch}^2(x)\right)^{5/2}\left(6\left(\cosh(3x)+4\sinh^4(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)-22\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] ((a*Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]
*Sinh[x]^4)))/64
```

Maple [B] time = 0.067, size = 123, normalized size = 1.9

$$\frac{a^2\left(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3\right)}{4\left(e^{2x}-1\right)^3}\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}} - \frac{3\,a^2e^{-x}\left(e^{2x}-1\right)\ln\left(e^x+1\right)}{8}\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}} + \frac{3\,a^2e^{-x}\left(e^{2x}-1\right)\ln\left(e^x-1\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(5/2),x)
```

```
[Out] 1/4*a^2/(exp(2*x)-1)^3*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp
(4*x)-11*exp(2*x)+3)-3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^
2)^(1/2)*ln(exp(x)+1)+3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)*ln(exp(x)-1)
```

Maxima [A] time = 1.55754, size = 124, normalized size = 1.91

$$\frac{3}{8}a^{\frac{5}{2}}\log\left(e^{(-x)}+1\right) - \frac{3}{8}a^{\frac{5}{2}}\log\left(e^{(-x)}-1\right) + \frac{3\,a^{\frac{5}{2}}e^{(-x)} - 11\,a^{\frac{5}{2}}e^{(-3\,x)} - 11\,a^{\frac{5}{2}}e^{(-5\,x)} + 3\,a^{\frac{5}{2}}e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^(5/2)*log(e^(-x) + 1) - 3/8*a^(5/2)*log(e^(-x) - 1) + 1/4*(3*a^(5/2)*
e^(-x) - 11*a^(5/2)*e^(-3*x) - 11*a^(5/2)*e^(-5*x) + 3*a^(5/2)*e^(-7*x))/(4
```

```
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] time = 1.71472, size = 3120, normalized size = 48.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(6*a^2*cosh(x)^7 - 6*(a^2*e^(2*x) - a^2)*sinh(x)^7 - 22*a^2*cosh(x)^5
- 42*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^6 + 2*(63*a^2*cosh(x)^2 -
11*a^2 - (63*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^5 - 22*a^2*cosh(x)^3
+ 10*(21*a^2*cosh(x)^3 - 11*a^2*cosh(x) - (21*a^2*cosh(x)^3 - 11*a^2*cosh(x
))*e^(2*x))*sinh(x)^4 + 2*(105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2 -
 (105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 + 6*a^
2*cosh(x) + 2*(63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) - (63*
a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 - 2*
(3*a^2*cosh(x)^7 - 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(
2*x) + 3*(a^2*cosh(x)^8 - (a^2*e^(2*x) - a^2)*sinh(x)^8 - 4*a^2*cosh(x)^6 -
 8*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 - a^2
 - (7*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*
cosh(x)^3 - 3*a^2*cosh(x) - (7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2*x))*sinh
(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2 - (35*a^2*cosh(x)^4
- 30*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2
*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) - (7*a^2*cosh(x)^5 - 10*a^2*c
osh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2 - (7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9
*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^8 - 4*a^2*cos
h(x)^6 + 6*a^2*cosh(x)^4 - 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^
7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^7 - 3*a^
2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x))*log((cosh(x)
 + sinh(x) - 1)/(cosh(x) + sinh(x) + 1)) + 2*(21*a^2*cosh(x)^6 - 55*a^2*cos
h(x)^4 - 33*a^2*cosh(x)^2 + 3*a^2 - (21*a^2*cosh(x)^6 - 55*a^2*cosh(x)^4 -
33*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1
))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x*s
inh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 - 30
*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh(x
)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (co
sh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \operatorname{csch}^{2}(x)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(5/2), x)
```

Giac [A] time = 1.13981, size = 101, normalized size = 1.55

$$\frac{1}{16}\,a^{\frac{5}{2}}\left(\frac{4\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3\,\log\left(e^{(-x)}+e^{x}+2\right)+3\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*a^(5/2)*(4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2
 - 4)^2 - 3*log(e^(-x) + e^x + 2) + 3*log(e^(-x) + e^x - 2))*sgn(e^(3*x) -
e^x)
```

3.30 $\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/2 - (a*Coth[x]*Sqrt[
a*Csch[x]^2])/2
```

Rubi [A] time = 0.0251034, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 206}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/2 - (a*Coth[x]*Sqrt[
a*Csch[x]^2])/2
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx &= -\left(a \operatorname{Subst}\left(\int \sqrt{-a+ax^2}\,dx, x, \coth(x)\right)\right)\\ &= -\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)} + \frac{1}{2}a^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\,dx, x, \coth(x)\right)\\ &= -\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)} + \frac{1}{2}a^2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)\\ &= \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0636664, size = 30, normalized size = 0.65

$$-\frac{1}{2}a\sinh(x)\sqrt{a\operatorname{csch}^2(x)}\left(\log\left(\tanh\left(\frac{x}{2}\right)\right)+\coth(x)\operatorname{csch}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] -(a*Sqrt[a*Csch[x]^2]*(Coth[x]*Csch[x] + Log[Tanh[x/2]])*Sinh[x])/2
```

Maple [B] time = 0.053, size = 103, normalized size = 2.2

$$-\frac{a\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}-\frac{ae^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)}{2}\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}+\frac{ae^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}+1\right)}{2}\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(3/2),x)
```

```
[Out] -a/(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(exp(2*x)+1)-1/2*a*exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)+1/2*a*exp(-x)
*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)
```

Maxima [A] time = 1.63817, size = 81, normalized size = 1.76

$$-\frac{1}{2}a^{\frac{3}{2}}\log\left(e^{(-x)}+1\right)+\frac{1}{2}a^{\frac{3}{2}}\log\left(e^{(-x)}-1\right)-\frac{a^{\frac{3}{2}}e^{(-x)}+a^{\frac{3}{2}}e^{(-3x)}}{2e^{(-2x)}-e^{(-4x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(e^(-x) + 1) + 1/2*a^(3/2)*log(e^(-x) - 1) - (a^(3/2)*e^(-x
) + a^(3/2)*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] time = 1.6269, size = 1007, normalized size = 21.89

$$\frac{\left(2\,a\cosh\left(x\right)^{3}-2\,\left(ae^{\left(2\,x\right)}-a\right)\sinh\left(x\right)^{3}-6\,\left(a\cosh\left(x\right)e^{\left(2\,x\right)}-a\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,a\cosh\left(x\right)-2\,\left(a\cosh\left(x\right)^{3}+a\cosh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cosh(x)^3 - 2*(a*e^(2*x) - a)*sinh(x)^3 - 6*(a*cosh(x)*e^(2*x) - a
*cosh(x))*sinh(x)^2 + 2*a*cosh(x) - 2*(a*cosh(x)^3 + a*cosh(x))*e^(2*x) - (
a*cosh(x)^4 - (a*e^(2*x) - a)*sinh(x)^4 - 4*(a*cosh(x)*e^(2*x) - a*cosh(x))
*sinh(x)^3 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)^2 - a)*e^(2*x)
 - a)*sinh(x)^2 - (a*cosh(x)^4 - 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^
3 - a*cosh(x) - (a*cosh(x)^3 - a*cosh(x))*e^(2*x))*sinh(x) + a)*log((cosh(x
) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(4*cosh
(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(
cosh(x)^3 - cosh(x))*e^x*sinh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\operatorname{csch}^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(3/2), x)
```

Giac [A] time = 1.13455, size = 78, normalized size = 1.7

$$-\frac{1}{4}\,a^{\frac{3}{2}}\left(\frac{4\,\left(e^{\left(-x\right)}+e^{x}\right)}{\left(e^{\left(-x\right)}+e^{x}\right)^{2}-4}-\log\left(e^{\left(-x\right)}+e^{x}+2\right)+\log\left(e^{\left(-x\right)}+e^{x}-2\right)\right)\operatorname{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*a^(3/2)*(4*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - log(e^(-x) + e^x +
2) + log(e^(-x) + e^x - 2))*sgn(e^(3*x) - e^x)
```

3.31 $\int \sqrt{a \operatorname{csch}^2(x)} \, dx$

Optimal. Leaf size=26

$$-\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \coth(x)}{\sqrt{a \operatorname{csch}^2(x)}}\right)$$

```
[Out] -(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])
```

Rubi [A] time = 0.018236, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 217, 206}

$$-\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \coth(x)}{\sqrt{a \operatorname{csch}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a \operatorname{csch}^2(x)} \, dx &= -\left(a \operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+a x^2}} \, dx, x, \coth(x)\right)\right)\\
&= -\left(a \operatorname{Subst}\left(\int \frac{1}{1-a x^2} \, dx, x, \frac{\coth(x)}{\sqrt{a \operatorname{csch}^2(x)}}\right)\right)\\
&= -\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \coth(x)}{\sqrt{a \operatorname{csch}^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0051363, size = 20, normalized size = 0.77

$$\sinh(x)\sqrt{a\text{csch}^2(x)}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^2],x]
```

```
[Out] Sqrt[a*Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

Maple [B] time = 0.072, size = 67, normalized size = 2.6

$$\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^2}}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}-1\right)-\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^2}}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(1/2),x)
```

```
[Out] (a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-(a*exp(
2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)
```

Maxima [A] time = 1.58171, size = 32, normalized size = 1.23

$$\sqrt{a}\log\left(e^{(-x)}+1\right)-\sqrt{a}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*log(e^(-x) + 1) - sqrt(a)*log(e^(-x) - 1)
```

Fricas [B] time = 1.90139, size = 296, normalized size = 11.38

$$\left[\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\left(e^{(2x)}-1\right)\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right), 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\left(e^{(2x)}-1\right)e^{x}}{a\cosh(x)e^{x}+ae^{x}\sinh(x)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*(e^(2*x) - 1)*log((cosh(x) + sinh(x) - 1
)/(cosh(x) + sinh(x) + 1)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(a/(e^(4*x) - 2*
e^(2*x) + 1))*(e^(2*x) - 1)*e^x/(a*cosh(x)*e^x + a*e^x*sinh(x)))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{csch}^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**2), x)
```

Giac [A] time = 1.17051, size = 39, normalized size = 1.5

$$-\sqrt{a}(\log(e^x + 1) - \log(|e^x - 1|))\operatorname{sgn}\left(e^{(3x)} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*(log(e^x + 1) - log(abs(e^x - 1)))*sgn(e^(3*x) - e^x)
```

3.32 $\int \frac{1}{\sqrt{a \operatorname{csch}^2(x)}} \, dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Rubi [A] time = 0.0135736, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4122, 191}

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}} \, dx &= -\left(a \operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}} \, dx, x, \coth(x)\right)\right)\\ &= \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0057898, size = 13, normalized size = 1.

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Maple [B] time = 0.065, size = 58, normalized size = 4.5

$$\frac{e^{2x}}{2e^{2x}-2}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}-1)^2}}}+\frac{1}{2e^{2x}-2}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}-1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(1/2),x)
```

```
[Out] 1/2/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(a*exp(2*x)
/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)
```

Maxima [A] time = 1.73158, size = 23, normalized size = 1.77

$$-\frac{e^{(-x)}}{2\sqrt{a}}-\frac{e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) - 1/2*e^x/sqrt(a)
```

Fricas [B] time = 1.88567, size = 247, normalized size = 19.

$$\frac{\left(\left(e^{(2x)}-1\right)\sinh(x)^2-\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}-\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}e^{x}}{2\left(a\cosh(x)e^{x}+ae^{x}\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) - 1)*sinh(x)^2 - cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) - cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**2), x)
```

Giac [B] time = 1.14493, size = 32, normalized size = 2.46

$$\frac{e^{(-x)} + e^{x}}{2\sqrt{a}\operatorname{sgn}\left(e^{(3x)} - e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/(sqrt(a)*sgn(e^(3*x) - e^x))
```

3.33 $\int \frac{1}{\left(a\,\mathbf{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] Coth[x]/(3*(a*Csch[x]^2)^(3/2)) - (2*Coth[x])/(3*a*Sqrt[a*Csch[x]^2])
```

Rubi [A] time = 0.0216984, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(a*Csch[x]^2)^(3/2)) - (2*Coth[x])/(3*a*Sqrt[a*Csch[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx &= -\left(a\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx,x,\coth(x)\right)\right)\\
&= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0232281, size = 27, normalized size = 0.75

$$\frac{(\cosh(3x) - 9\cosh(x))\text{csch}^3(x)}{12\left(a\text{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] ((-9*Cosh[x] + Cosh[3*x])*Csch[x]^3)/(12*(a*Csch[x]^2)^(3/2))
```

Maple [B] time = 0.049, size = 130, normalized size = 3.6

$$\frac{e^{4x}}{24a\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{3e^{2x}}{8a\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{3}{8a\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{e^{-2x}}{24a\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(3/2),x)
```

```
[Out] 1/24/a*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a*exp(2*
x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a/(exp(2*x)-1)/(a*exp
(2*x)/(exp(2*x)-1)^2)^(1/2)+1/24/a*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(
2*x)-1)^2)^(1/2)
```

Maxima [A] time = 1.70034, size = 47, normalized size = 1.31

$$-\frac{e^{(3x)}}{24a^{\frac{3}{2}}} + \frac{3e^{(-x)}}{8a^{\frac{3}{2}}} - \frac{e^{(-3x)}}{24a^{\frac{3}{2}}} + \frac{3e^{x}}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*e^(3*x)/a^(3/2) + 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^
x/a^(3/2)
```

Fricas [B] time = 1.84244, size = 856, normalized size = 23.78

$$\left(\left(e^{(2x)}-1\right)\sinh(x)^6 - \cosh(x)^6 + 6\left(\cosh(x)e^{(2x)} - \cosh(x)\right)\sinh(x)^5 - 3\left(5\cosh(x)^2 - \left(5\cosh(x)^2-3\right)e^{(2x)} - 3\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) - 1)*sinh(x)^6 - cosh(x)^6 + 6*(cosh(x)*e^(2*x) - cosh(x))*s
inh(x)^5 - 3*(5*cosh(x)^2 - (5*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^4 + 9*co
```

```
sh(x)^4 - 4*(5*cosh(x)^3 - (5*cosh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*s
inh(x)^3 - 3*(5*cosh(x)^4 - 18*cosh(x)^2 - (5*cosh(x)^4 - 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^6 - 9*cosh(x)^4 - 9*cosh(x
)^2 + 1)*e^(2*x) - 6*(cosh(x)^5 - 6*cosh(x)^3 - (cosh(x)^5 - 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-3/2), x)
```

Giac [A] time = 1.17792, size = 73, normalized size = 2.03

$$-\frac{\frac{\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}}{\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)}-\frac{e^{(3\,x)}-9\,e^{x}}{\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)}}{24\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) - 1)*e^(-3*x)/sgn(e^(3*x) - e^x) - (e^(3*x) - 9*e^x)/sgn(
e^(3*x) - e^x))/a^(3/2)
```

3.34 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}$$

```
[Out] Coth[x]/(5*(a*Csch[x]^2)^(5/2)) - (4*Coth[x])/(15*a*(a*Csch[x]^2)^(3/2)) +
(8*Coth[x])/(15*a^2*Sqrt[a*Csch[x]^2])
```

Rubi [A] time = 0.0303103, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(a*Csch[x]^2)^(5/2)) - (4*Coth[x])/(15*a*(a*Csch[x]^2)^(3/2)) +
(8*Coth[x])/(15*a^2*Sqrt[a*Csch[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = -\left(a\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{8\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)}{15a}$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [A] time = 0.027251, size = 36, normalized size = 0.65

$$\frac{\sinh(x)(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\sqrt{a\operatorname{csch}^2(x)}}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Sqrt[a*Csch[x]^2]*Sinh[x])/(240
*a^3)
```

Maple [B] time = 0.05, size = 196, normalized size = 3.6

$$\frac{e^{6x}}{160\,a^2\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{5\,e^{4x}}{96\,a^2\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{5\,e^{2x}}{16\,a^2\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{5}{16\,a^2\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{}{96\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(5/2),x)
```

```
[Out] 1/160/a^2*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2*
exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2*exp(2*x)/(
exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2/(exp(2*x)-1)/(a*exp(
2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp
(2*x)-1)^2)^(1/2)+1/160/a^2*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)
```

Maxima [A] time = 1.65226, size = 72, normalized size = 1.31

$$-\frac{e^{(5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} + \frac{5\,e^{(-3x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5x)}}{160\,a^{\frac{5}{2}}} - \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) + 5/96*
e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) - 5/16*e^x/a^(5/2)
```

Fricas [B] time = 1.91041, size = 1885, normalized size = 34.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) - 1)*sinh(x)^10 - 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) - co
sh(x))*sinh(x)^9 - 5*(27*cosh(x)^2 - (27*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x
)^8 + 25*cosh(x)^8 - 40*(9*cosh(x)^3 - (9*cosh(x)^3 - 5*cosh(x))*e^(2*x) -
5*cosh(x))*sinh(x)^7 - 10*(63*cosh(x)^4 - 70*cosh(x)^2 - (63*cosh(x)^4 - 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 - 150*cosh(x)^6 - 4*(189*cosh(x)^5
 - 350*cosh(x)^3 - (189*cosh(x)^5 - 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 - 10*(63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 -
 (63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^
4 - 150*cosh(x)^4 - 40*(9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 - (9*cosh
(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sin
h(x)^3 - 5*(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 -
(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 + 25*cosh(x)^2 + (3*cosh(x)^10 - 25*cosh(x)^8 + 150*cosh(x)^
6 + 150*cosh(x)^4 - 25*cosh(x)^2 + 3)*e^(2*x) - 10*(3*cosh(x)^9 - 20*cosh(x
)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - (3*cosh(x)^9 - 20*cosh(x)^7 + 90*cosh(x
)^5 + 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) - 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \operatorname{csch}^{2}(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-5/2), x)
```

Giac [A] time = 1.15165, size = 90, normalized size = 1.64

$$\frac{\frac{\left(150\, e^{(4\,x)}-25\, e^{(2\,x)}+3\right)e^{(-5\,x)}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}+\frac{3\, e^{(5\,x)}-25\, e^{(3\,x)}+150\, e^{x}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}}{480\, a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*((150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x)/sgn(e^(3*x) - e^x) + (3*e^(5
*x) - 25*e^(3*x) + 150*e^x)/sgn(e^(3*x) - e^x))/a^(5/2)
```

3.35 $\int \frac{1}{\left(a\,\mathrm{csch}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{16\coth(x)}{35a^3\sqrt{a\,\mathrm{csch}^2(x)}}+\frac{8\coth(x)}{35a^2\left(a\,\mathrm{csch}^2(x)\right)^{3/2}}-\frac{6\coth(x)}{35a\left(a\,\mathrm{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(a\,\mathrm{csch}^2(x)\right)^{7/2}}$$

```
[Out] Coth[x]/(7*(a*Csch[x]^2)^(7/2)) - (6*Coth[x])/(35*a*(a*Csch[x]^2)^(5/2)) +
(8*Coth[x])/(35*a^2*(a*Csch[x]^2)^(3/2)) - (16*Coth[x])/(35*a^3*Sqrt[a*Csch
[x]^2])
```

Rubi [A] time = 0.0401579, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$-\frac{16\coth(x)}{35a^3\sqrt{a\,\mathrm{csch}^2(x)}}+\frac{8\coth(x)}{35a^2\left(a\,\mathrm{csch}^2(x)\right)^{3/2}}-\frac{6\coth(x)}{35a\left(a\,\mathrm{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(a\,\mathrm{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(a*Csch[x]^2)^(7/2)) - (6*Coth[x])/(35*a*(a*Csch[x]^2)^(5/2)) +
(8*Coth[x])/(35*a^2*(a*Csch[x]^2)^(3/2)) - (16*Coth[x])/(35*a^3*Sqrt[a*Csch
[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\left(a\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{9/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{24\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)}{35a}$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \mathrm{c}\right.}{35a^2}$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [A] time = 0.052438, size = 42, normalized size = 0.57

$$\frac{\sinh(x)(-1225\cosh(x)+245\cosh(3x)-49\cosh(5x)+5\cosh(7x))\sqrt{a\operatorname{csch}^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] ((-1225*Cosh[x] + 245*Cosh[3*x] - 49*Cosh[5*x] + 5*Cosh[7*x])*Sqrt[a*Csch[x
]^2]*Sinh[x])/(2240*a^4)
```

Maple [B] time = 0.052, size = 262, normalized size = 3.5

$$\frac{e^{8x}}{896a^3\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{7e^{6x}}{640a^3\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{7e^{4x}}{128a^3\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{35e^{2x}}{128a^3\left(e^{2x}-1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}-1\right)^2}}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(7/2),x)
```

```
[Out] 1/896/a^3*exp(8*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/640/a^3
*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(4*x)
/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3*exp(2*x)/(exp(2*
x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3/(exp(2*x)-1)/(a*exp(2*x)
/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)
^(1/2)+1/896/a^3*exp(-6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [A] time = 1.71474, size = 96, normalized size = 1.3

$$-\frac{e^{(7x)}}{896a^{\frac{7}{2}}} + \frac{7e^{(5x)}}{640a^{\frac{7}{2}}} - \frac{7e^{(3x)}}{128a^{\frac{7}{2}}} + \frac{35e^{(-x)}}{128a^{\frac{7}{2}}} - \frac{7e^{(-3x)}}{128a^{\frac{7}{2}}} + \frac{7e^{(-5x)}}{640a^{\frac{7}{2}}} - \frac{e^{(-7x)}}{896a^{\frac{7}{2}}} + \frac{35e^{x}}{128a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) - 7/128*e^(3*x)/a^(7/2) + 35
/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) + 7/640*e^(-5*x)/a^(7/2) - 1/8
96*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Fricas [B] time = 2.00259, size = 3340, normalized size = 45.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) - 1)*sinh(x)^14 - 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) - c
osh(x))*sinh(x)^13 - 7*(65*cosh(x)^2 - (65*cosh(x)^2 - 7)*e^(2*x) - 7)*sinh
(x)^12 + 49*cosh(x)^12 - 28*(65*cosh(x)^3 - (65*cosh(x)^3 - 21*cosh(x))*e^(
2*x) - 21*cosh(x))*sinh(x)^11 - 7*(715*cosh(x)^4 - 462*cosh(x)^2 - (715*cos
h(x)^4 - 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 - 245*cosh(x)^10 - 70
*(143*cosh(x)^5 - 154*cosh(x)^3 - (143*cosh(x)^5 - 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 - 35*(429*cosh(x)^6 - 693*cosh(x)^4 + 3
15*cosh(x)^2 - (429*cosh(x)^6 - 693*cosh(x)^4 + 315*cosh(x)^2 - 35)*e^(2*x)
 - 35)*sinh(x)^8 + 1225*cosh(x)^8 - 8*(2145*cosh(x)^7 - 4851*cosh(x)^5 + 36
75*cosh(x)^3 - (2145*cosh(x)^7 - 4851*cosh(x)^5 + 3675*cosh(x)^3 - 1225*cos
h(x))*e^(2*x) - 1225*cosh(x))*sinh(x)^7 - 7*(2145*cosh(x)^8 - 6468*cosh(x)^
6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - (2145*cosh(x)^8 - 6468*cosh(x)^6 + 73
50*cosh(x)^4 - 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 + 1225*cosh(x
)^6 - 14*(715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3
- (715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 - 35*(143*cosh(x)^10 - 693*cosh(x)
^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 - (143*cosh(x)^10 - 69
3*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 + 7)*e^(2*x)
+ 7)*sinh(x)^4 - 245*cosh(x)^4 - 140*(13*cosh(x)^11 - 77*cosh(x)^9 + 210*co
sh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 - (13*cosh(x)^11 - 77*cosh(x)^9 + 2
10*cosh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 + 7*cosh(x))*e^(2*x) + 7*cosh(
x))*sinh(x)^3 - 7*(65*cosh(x)^12 - 462*cosh(x)^10 + 1575*cosh(x)^8 - 4900*c
osh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - (65*cosh(x)^12 - 462*cosh(x)^10
 + 1575*cosh(x)^8 - 4900*cosh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 + 49*cosh(x)^2 + (5*cosh(x)^14 - 49*cosh(x)^12 + 245*c
osh(x)^10 - 1225*cosh(x)^8 - 1225*cosh(x)^6 + 245*cosh(x)^4 - 49*cosh(x)^2
+ 5)*e^(2*x) - 14*(5*cosh(x)^13 - 42*cosh(x)^11 + 175*cosh(x)^9 - 700*cosh(
x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - (5*cosh(x)^13 - 42*cosh(x)^11 + 175*c
osh(x)^9 - 700*cosh(x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \operatorname{csch}^2(x)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(7/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-7/2), x)
```

Giac [A] time = 1.1931, size = 108, normalized size = 1.46

$$-\frac{\frac{\left(1225\,e^{(6\,x)}-245\,e^{(4\,x)}+49\,e^{(2\,x)}-5\right)e^{(-7\,x)}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}-\frac{5\,e^{(7\,x)}-49\,e^{(5\,x)}+245\,e^{(3\,x)}-1225\,e^{x}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}}{4480\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x)/sgn(e^(3*x)
 - e^x) - (5*e^(7*x) - 49*e^(5*x) + 245*e^(3*x) - 1225*e^x)/sgn(e^(3*x) - e
^x))/a^(7/2)
```

3.36 $\int \left(a\,\mathrm{csch}^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=135

$$-\frac{2}{13}a^2\coth(x)\mathrm{csch}^4(x)\sqrt{a\,\mathrm{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\mathrm{csch}^2(x)\sqrt{a\,\mathrm{csch}^3(x)}-\frac{154}{585}a^2\coth(x)\sqrt{a\,\mathrm{csch}^3(x)}+\frac{154}{195}a^2\sinh(x)$$

```
[Out] (-154*a^2*Coth[x]*Sqrt[a*Csch[x]^3])/585 + (22*a^2*Coth[x]*Csch[x]^2*Sqrt[a
*Csch[x]^3])/117 - (2*a^2*Coth[x]*Csch[x]^4*Sqrt[a*Csch[x]^3])/13 + (154*a^
2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x])/195 - (((154*I)/195)*a^2*Sqrt[a*Csch[x
]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rubi [A] time = 0.0698942, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$-\frac{2}{13}a^2\coth(x)\mathrm{csch}^4(x)\sqrt{a\,\mathrm{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\mathrm{csch}^2(x)\sqrt{a\,\mathrm{csch}^3(x)}-\frac{154}{585}a^2\coth(x)\sqrt{a\,\mathrm{csch}^3(x)}+\frac{154}{195}a^2\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Coth[x]*Sqrt[a*Csch[x]^3])/585 + (22*a^2*Coth[x]*Csch[x]^2*Sqrt[a
*Csch[x]^3])/117 - (2*a^2*Coth[x]*Csch[x]^4*Sqrt[a*Csch[x]^3])/13 + (154*a^
2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x])/195 - (((154*I)/195)*a^2*Sqrt[a*Csch[x
]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = -\frac{\left(a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{15/2}\,dx}{(i\text{csch}(x))^{3/2}}$$

$$= -\frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(11a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{11/2}\,dx}{13(i\text{csch}(x))^{3/2}}$$

$$= \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(77a^2\sqrt{a\text{csch}^3(x)}\right)}{117(i\text{csc}}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{}$$

Mathematica [A] time = 0.167947, size = 68, normalized size = 0.5

$$-\frac{2}{585}a^2\sinh(x)\sqrt{a\text{csch}^3(x)}\left(-231\cosh(x) + \coth(x)\text{csch}(x)\left(45\text{csch}^4(x) - 55\text{csch}^2(x) + 77\right) + 231\sqrt{i\sinh(x)}E\left(\frac{1}{4}(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a*Csch[x]^3]*(-231*Cosh[x] + Coth[x]*Csch[x]*(77 - 55*Csch[x]^
2 + 45*Csch[x]^4) + 231*EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sin
h[x])/585
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \left(a\,(\text{csch}\,(x))^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(5/2),x)
```

```
[Out] int((a*csch(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{csch}\,(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\,\mathrm{csch}\left(x\right)^{3}}a^{2}\,\mathrm{csch}\left(x\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)*a^2*csch(x)^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\mathrm{csch}^{3}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\mathrm{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

3.37 $\int \left(a\operatorname{csch}^3(x)\right)^{3/2} dx$

Optimal. Leaf size=81

$$\frac{10}{21} i a \sqrt{i \sinh(x)} \sinh(x) \text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right) \sqrt{a\operatorname{csch}^3(x)}+\frac{10}{21} a \cosh(x) \sqrt{a\operatorname{csch}^3(x)}-\frac{2}{7} a \coth(x) \operatorname{csch}(x) \sqrt{a\operatorname{csch}^3(x)}$$

```
[Out] (10*a*Cosh[x]*Sqrt[a*Csch[x]^3])/21 - (2*a*Coth[x]*Csch[x]*Sqrt[a*Csch[x]^3
])/7 + ((10*I)/21)*a*Sqrt[a*Csch[x]^3]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*
Sinh[x]]*Sinh[x]
```

Rubi [A] time = 0.0452946, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21} a \cosh(x) \sqrt{a\operatorname{csch}^3(x)}-\frac{2}{7} a \coth(x) \operatorname{csch}(x) \sqrt{a\operatorname{csch}^3(x)}+\frac{10}{21} i a \sqrt{i \sinh(x)} \sinh(x) F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right) \sqrt{a\operatorname{csch}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cosh[x]*Sqrt[a*Csch[x]^3])/21 - (2*a*Coth[x]*Csch[x]*Sqrt[a*Csch[x]^3
])/7 + ((10*I)/21)*a*Sqrt[a*Csch[x]^3]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*
Sinh[x]]*Sinh[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = \frac{\left(ia\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{9/2}\,dx}{(i\text{csch}(x))^{3/2}}$$
$$= -\frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{\left(5ia\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{5/2}\,dx}{7(i\text{csch}(x))^{3/2}}$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{\left(5ia\sqrt{a\text{csch}^3(x)}\right)\int \sqrt{i\text{csch}(x)}\,dx}{21(i\text{csch}(x))^{3/2}}$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{1}{21}\left(5a\sqrt{a\text{csch}^3(x)}\sqrt{i\sinh(x)}\sinh(\right.$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{10}{21}ia\sqrt{a\text{csch}^3(x)}F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\,\text{si}}$$

Mathematica [A] time = 0.113659, size = 56, normalized size = 0.69

$$-\frac{2}{21}a\sinh(x)\sqrt{a\text{csch}^3(x)}\left(\coth(x)\left(3\text{csch}^2(x)-5\right)-5i\sqrt{i\sinh(x)}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Csch[x]^3]*(Coth[x]*(-5 + 3*Csch[x]^2) - (5*I)*EllipticF[(Pi -
 (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/21
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \left(a\left(\text{csch}\left(x\right)\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(3/2),x)
```

```
[Out] int((a*csch(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{csch}\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\,\text{csch}\,(x)^3}a\,\text{csch}\,(x)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)*a*csch(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{csch}^3\,(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{csch}\,(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

3.38 $\int \sqrt{a\,\mathrm{csch}^3(x)}\,dx$

Optimal. Leaf size=56

$$-2\sinh(x)\cosh(x)\sqrt{a\,\mathrm{csch}^3(x)}-2i(i\sinh(x))^{3/2}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\,\mathrm{csch}^3(x)}$$

```
[Out] (-2*I)*Sqrt[a*Csch[x]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*(I*Sinh[x])^(3/2) - 2
*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x]
```

Rubi [A] time = 0.0338612, antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$-2\sinh(x)\cosh(x)\sqrt{a\,\mathrm{csch}^3(x)}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\,\mathrm{csch}^3(x)}}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x] + ((2*I)*Sqrt[a*Csch[x]^3]*EllipticE[P
i/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\mathrm{csch}^3(x)}\,dx = \frac{\sqrt{a\mathrm{csch}^3(x)}\int (i\mathrm{csch}(x))^{3/2}\,dx}{(i\mathrm{csch}(x))^{3/2}}$$
$$= -2\cosh(x)\sqrt{a\mathrm{csch}^3(x)}\sinh(x) - \frac{\sqrt{a\mathrm{csch}^3(x)}\int \frac{1}{\sqrt{i\mathrm{csch}(x)}}\,dx}{(i\mathrm{csch}(x))^{3/2}}$$
$$= -2\cosh(x)\sqrt{a\mathrm{csch}^3(x)}\sinh(x) + \frac{\left(\sqrt{a\mathrm{csch}^3(x)}\sinh^2(x)\right)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}$$
$$= -2\cosh(x)\sqrt{a\mathrm{csch}^3(x)}\sinh(x) + \frac{2i\sqrt{a\mathrm{csch}^3(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}}$$

Mathematica [A] time = 0.028467, size = 42, normalized size = 0.75

$$-2\sinh(x)\sqrt{a\mathrm{csch}^3(x)}\left(\cosh(x)-\sqrt{i\sinh(x)}E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Sqrt[a*Csch[x]^3]*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[
x]])*Sinh[x]
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \sqrt{a\,(\mathrm{csch}\,(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(1/2),x)
```

```
[Out] int((a*csch(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\mathrm{csch}\,(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\,\mathrm{csch}\,(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{csch}^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{csch}(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

3.39 $\int \frac{1}{\sqrt{a\mathbf{csch}^3(x)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\sqrt{i\sinh(x)}\text{csch}^2(x)\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{3\sqrt{a\text{csch}^3(x)}}$$

```
[Out] (2*Coth[x])/(3*Sqrt[a*Csch[x]^3]) - (((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (
I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[a*Csch[x]^3]
```

Rubi [A] time = 0.0341976, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\sqrt{i\sinh(x)}\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{3\sqrt{a\text{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^3],x]
```

```
[Out] (2*Coth[x])/(3*Sqrt[a*Csch[x]^3]) - (((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (
I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[a*Csch[x]^3]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{3/2}}\,dx}{\sqrt{a\operatorname{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} + \frac{(i\operatorname{csch}(x))^{3/2}\int \sqrt{i\operatorname{csch}(x)}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{2i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\operatorname{csch}^3(x)}}$$

Mathematica [A] time = 0.0731304, size = 43, normalized size = 0.69

$$\frac{2\left(\coth(x)+\frac{\operatorname{csch}(x)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}}\right)}{3\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^3],x]

[Out] (2*(Coth[x] + (Csch[x]*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(3
*Sqrt[a*Csch[x]^3])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\operatorname{csch}(x)\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(1/2),x)

[Out] int(1/(a*csch(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\operatorname{csch}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\,\text{csch}\,(x)^3}}{a\,\text{csch}\,(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)/(a*csch(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\text{csch}^3\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\text{csch}\,(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

3.40 $\int \frac{1}{\left(a\,\mathrm{csch}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{14\cosh(x)}{45a\sqrt{a\,\mathrm{csch}^3(x)}}+\frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\,\mathrm{csch}^3(x)}}+\frac{14i\,\mathrm{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\,\mathrm{csch}^3(x)}}$$

```
[Out] (-14*Cosh[x])/(45*a*Sqrt[a*Csch[x]^3]) + (((14*I)/15)*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2])/(a*Sqrt[a*Csch[x]^3]*Sqrt[I*Sinh[x]]) + (2*Cosh[x]*Sinh[x]
^2)/(9*a*Sqrt[a*Csch[x]^3])
```

Rubi [A] time = 0.0479715, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2639}

$$-\frac{14\cosh(x)}{45a\sqrt{a\,\mathrm{csch}^3(x)}}+\frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\,\mathrm{csch}^3(x)}}+\frac{14i\,\mathrm{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\,\mathrm{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-14*Cosh[x])/(45*a*Sqrt[a*Csch[x]^3]) + (((14*I)/15)*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2])/(a*Sqrt[a*Csch[x]^3]*Sqrt[I*Sinh[x]]) + (2*Cosh[x]*Sinh[x]
^2)/(9*a*Sqrt[a*Csch[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = -\frac{\left(i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{9/2}}\,dx}{a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{5/2}}\,dx}{9a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} + \frac{(7\operatorname{csch}(x))\int \sqrt{i\sinh(x)}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}$$

Mathematica [A] time = 0.0750177, size = 57, normalized size = 0.64

$$\frac{-33\cosh(x) + 5\cosh(3x) + 84\sqrt{i\sinh(x)}\operatorname{csch}^2(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)}{90a\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-33*Cosh[x] + 5*Cosh[3*x] + 84*Csch[x]^2*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]])/(90*a*Sqrt[a*Csch[x]^3])
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \left(a\left(\operatorname{csch}(x)\right)^3\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\,\text{csch}\,(x)^{3}}}{a^{2}\,\text{csch}\,(x)^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)/(a^2*csch(x)^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\text{csch}^{3}\,(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\text{csch}\,(x)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

3.41 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}} dx$

Optimal. Leaf size=135

$$\frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\sin}{385a^2}$$

```
[Out] (-26*Coth[x])/(77*a^2*Sqrt[a*Csch[x]^3]) + (((26*I)/77)*Csch[x]^2*EllipticF
[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/(a^2*Sqrt[a*Csch[x]^3]) + (78*Cosh[x]*
Sinh[x])/(385*a^2*Sqrt[a*Csch[x]^3]) - (26*Cosh[x]*Sinh[x]^3)/(165*a^2*Sqrt
[a*Csch[x]^3]) + (2*Cosh[x]*Sinh[x]^5)/(15*a^2*Sqrt[a*Csch[x]^3])
```

Rubi [A] time = 0.0707349, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\right.}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (-26*Coth[x])/(77*a^2*Sqrt[a*Csch[x]^3]) + (((26*I)/77)*Csch[x]^2*EllipticF
[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/(a^2*Sqrt[a*Csch[x]^3]) + (78*Cosh[x]*
Sinh[x])/(385*a^2*Sqrt[a*Csch[x]^3]) - (26*Cosh[x]*Sinh[x]^3)/(165*a^2*Sqrt
[a*Csch[x]^3]) + (2*Cosh[x]*Sinh[x]^5)/(15*a^2*Sqrt[a*Csch[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = -\frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{15/2}}\,dx}{a^2\sqrt{a\operatorname{csch}^3(x)}}$$

$$= \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(13(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{11/2}}\,dx}{15a^2\sqrt{a\operatorname{csch}^3(x)}}$$

$$= -\frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{7/2}}\,dx}{55a^2\sqrt{a\operatorname{csch}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{3/2}}}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(13(i\operatorname{csc}\right.}{7}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{\left(13\operatorname{csch}\right.}{}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{26i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sin}{165a^2\sqrt{a\operatorname{cscl}}}$$

Mathematica [A] time = 0.120328, size = 71, normalized size = 0.53

$$\frac{\sinh(x)\sqrt{a\operatorname{csch}^3(x)}\left(24960i\sqrt{i\sinh(x)}\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)-19122\sinh(2x)+4406\sinh(4x)-826\sinh(6x)+77\,\mathrm{s}\right.}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^3]*Sinh[x]*((24960*I)*EllipticF[(Pi - (2*I)*x)/4, 2]*Sqrt[I
*Sinh[x]] - 19122*Sinh[2*x] + 4406*Sinh[4*x] - 826*Sinh[6*x] + 77*Sinh[8*x]
))/(73920*a^3)
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \left(a\left(\operatorname{csch}(x)\right)^3\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{csch}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\,\mathrm{csch}\,(x)^3}}{a^3\,\mathrm{csch}\,(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)/(a^3*csch(x)^9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{csch}^3\,(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{csch}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

3.42 $\int \left(a\,\mathrm{csch}^4(x)\right)^{7/2}\,dx$

Optimal. Leaf size=164

$$-\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\,\mathrm{csch}^4(x)}-\frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{20}{7}a$$

```
[Out] 2*a^3*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4] - 3*a^3*Cosh[x]^2*Coth[x]^3*Sqrt[
a*Csch[x]^4] + (20*a^3*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (5*a^3*Co
sh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/3 + (6*a^3*Cosh[x]^2*Coth[x]^9*Sqrt[a*
Csch[x]^4])/11 - (a^3*Cosh[x]^2*Coth[x]^11*Sqrt[a*Csch[x]^4])/13 - a^3*Cosh
[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rubi [A] time = 0.0421675, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$-\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\,\mathrm{csch}^4(x)}-\frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{20}{7}a$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(7/2),x]
```


```
[Out] 2*a^3*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4] - 3*a^3*Cosh[x]^2*Coth[x]^3*Sqrt[
a*Csch[x]^4] + (20*a^3*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (5*a^3*Co
sh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/3 + (6*a^3*Cosh[x]^2*Coth[x]^9*Sqrt[a*
Csch[x]^4])/11 - (a^3*Cosh[x]^2*Coth[x]^11*Sqrt[a*Csch[x]^4])/13 - a^3*Cosh
[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```


Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\,\mathrm{csch}^4(x)\right)^{7/2}\,dx &= \left(a^3\sqrt{a\,\mathrm{csch}^4(x)}\sinh^2(x)\right)\int \mathrm{csch}^{14}(x)\,dx\\ &= -\left(\left(ia^3\sqrt{a\,\mathrm{csch}^4(x)}\sinh^2(x)\right)\mathrm{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)\,dx,x,-i\,\mathrm{co}\right.\right.\\ &= 2a^3\cosh^2(x)\coth(x)\sqrt{a\,\mathrm{csch}^4(x)}-3a^3\cosh^2(x)\coth^3(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{20}{7}a^3\cosh^2(x)\coth^5(x)\end{aligned}$$

Mathematica [A] time = 0.049322, size = 59, normalized size = 0.36

$$-\frac{a^3\sinh(x)\cosh(x)\left(231\,\mathrm{csch}^{12}(x)-252\,\mathrm{csch}^{10}(x)+280\,\mathrm{csch}^8(x)-320\,\mathrm{csch}^6(x)+384\,\mathrm{csch}^4(x)-512\,\mathrm{csch}^2(x)+1024\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] -(a^3*Cosh[x]*Sqrt[a*Csch[x]^4]*(1024 - 512*Csch[x]^2 + 384*Csch[x]^4 - 320
*Csch[x]^6 + 280*Csch[x]^8 - 252*Csch[x]^10 + 231*Csch[x]^12)*Sinh[x])/3003
```

Maple [A] time = 0.076, size = 72, normalized size = 0.4

$$-\frac{2048\,a^{3}\mathrm{e}^{-2x}\left(1716\,\mathrm{e}^{12x}-1287\,\mathrm{e}^{10x}+715\,\mathrm{e}^{8x}-286\,\mathrm{e}^{6x}+78\,\mathrm{e}^{4x}-13\,\mathrm{e}^{2x}+1\right)}{3003\left(\mathrm{e}^{2x}-1\right)^{11}}\sqrt{\frac{\mathrm{e}^{4x}a}{\left(\mathrm{e}^{2x}-1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(7/2),x)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(exp(2*x)-1)^11*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*
(1716*exp(12*x)-1287*exp(10*x)+715*exp(8*x)-286*exp(6*x)+78*exp(4*x)-13*exp
(2*x)+1)
```

Maxima [B] time = 1.66096, size = 837, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] -2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*
e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x
) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x)
 - 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x)
- 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^
(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(
-26*x) - 1) - 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(
-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1
287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x)
 + e^(-26*x) - 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) - 78*e^(-4*x) +
286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14
*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^
(-24*x) + e^(-26*x) - 1) - 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) - 78*e^(-4
*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*
e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) -
 13*e^(-24*x) + e^(-26*x) - 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) - 78
*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-2
2*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) - 78*
e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22
*x) - 13*e^(-24*x) + e^(-26*x) - 1)
```

Fricas [B] time = 2.3358, size = 9415, normalized size = 57.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) -
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) - 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 - a^3 + (88*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(88*a^3*cosh(x)^2 - a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 - 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 - 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(11
88*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 - 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) - 2*(1188*a^3*cosh(x)^5 - 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3 + (5544*a^3*c
osh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(5544*a^
3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^
3*cosh(x))*e^(4*x) - 2*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 - 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
- 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) - 2*(32670*a^3*cosh(x)^8 - 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) - 2*(3630*a^3
*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 - 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3
 + (8712*a^3*cosh(x)^10 - 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(8712*a^3*cosh(x)^10 - 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 - a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) - 2*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x) + (792*a^3*cosh(x)^11 - 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a
^3*cosh(x))*e^(4*x) - 2*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 - 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 - 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 -
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 - 483*co
sh(x)^4 + 63*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 - 483*
cosh(x)^5 + 105*cosh(x)^3 - 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 - 12236*cosh(x)^6 + 3990*cosh(x)^4 - 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 - 15732*cosh(x)^7 + 7182*cosh(x)^5 - 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 - 66861*cosh
```

```
(x)^8 + 40698*cosh(x)^6 - 9690*cosh(x)^4 + 765*cosh(x)^2 - 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 - 81719*cosh(x)^9 + 63954*cosh(x)^7 - 21318*c
osh(x)^5 + 2805*cosh(x)^3 - 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 - 490314*cosh(x)^10 + 479655*cosh(x)^8 - 213180*cosh(x)^6 + 42075*c
osh(x)^4 - 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
- 4056234*cosh(x)^11 + 4849845*cosh(x)^9 - 2771340*cosh(x)^7 + 765765*cosh(
x)^5 - 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 - 676039*cosh(x)^12 + 969969*cosh(x)^10 - 692835*cosh(x)^8 + 255255*
cosh(x)^6 - 45045*cosh(x)^4 + 3003*cosh(x)^2 - 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 - 156009*cosh(x)^13 + 264537*cosh(x)^11 - 230945*cosh(x)
^9 + 109395*cosh(x)^7 - 27027*cosh(x)^5 + 3003*cosh(x)^3 - 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 - 178296*cosh(x)^14 + 352716*cosh(x)
^12 - 369512*cosh(x)^10 + 218790*cosh(x)^8 - 72072*cosh(x)^6 + 12012*cosh(x
)^4 - 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 - 59432
*cosh(x)^15 + 135660*cosh(x)^13 - 167960*cosh(x)^11 + 121550*cosh(x)^9 - 51
480*cosh(x)^7 + 12012*cosh(x)^5 - 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 - 66861*cosh(x)^16 + 174420*cosh(x)^14 - 2519
40*cosh(x)^12 + 218790*cosh(x)^10 - 115830*cosh(x)^8 + 36036*cosh(x)^6 - 59
40*cosh(x)^4 + 405*cosh(x)^2 - 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
- 3933*cosh(x)^17 + 11628*cosh(x)^15 - 19380*cosh(x)^13 + 19890*cosh(x)^11
- 12870*cosh(x)^9 + 5148*cosh(x)^7 - 1188*cosh(x)^5 + 135*cosh(x)^3 - 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 - 6118*cosh(x)^18 + 20349*cos
h(x)^16 - 38760*cosh(x)^14 + 46410*cosh(x)^12 - 36036*cosh(x)^10 + 18018*co
sh(x)^8 - 5544*cosh(x)^6 + 945*cosh(x)^4 - 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 - 966*cosh(x)^19 + 3591*cosh(x)^17 - 7752*cosh(x)
^15 + 10710*cosh(x)^13 - 9828*cosh(x)^11 + 6006*cosh(x)^9 - 2376*cosh(x)^7
+ 567*cosh(x)^5 - 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 - 5313*cosh(x)^20 + 21945*cosh(x)^18 - 53295*cosh(x)^16 + 84150*cos
h(x)^14 - 90090*cosh(x)^12 + 66066*cosh(x)^10 - 32670*cosh(x)^8 + 10395*cos
h(x)^6 - 1925*cosh(x)^4 + 165*cosh(x)^2 - 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 - 253*cosh(x)^21 + 1155*cosh(x)^19 - 3135*cosh(x)^17 + 5610*cosh(x
)^15 - 6930*cosh(x)^13 + 6006*cosh(x)^11 - 3630*cosh(x)^9 + 1485*cosh(x)^7
- 385*cosh(x)^5 + 55*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 - 276*cosh(x)^22 + 1386*cosh(x)^20 - 4180*cosh(x)^18 + 8415*cosh(x)^
16 - 11880*cosh(x)^14 + 12012*cosh(x)^12 - 8712*cosh(x)^10 + 4455*cosh(x)^8
 - 1540*cosh(x)^6 + 330*cosh(x)^4 - 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 - 12*cosh(x)^23 + 66*cosh(x)^21 - 220*cosh(x)^19 + 495*cosh(x
)^17 - 792*cosh(x)^15 + 924*cosh(x)^13 - 792*cosh(x)^11 + 495*cosh(x)^9 - 2
20*cosh(x)^7 + 66*cosh(x)^5 - 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 - 13*cosh(x)^24 + 78*cosh(x)^22 - 286*cosh(x)^20 + 715*cosh(x)^18
- 1287*cosh(x)^16 + 1716*cosh(x)^14 - 1716*cosh(x)^12 + 1287*cosh(x)^10 - 7
15*cosh(x)^8 + 286*cosh(x)^6 - 78*cosh(x)^4 + 13*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \operatorname{csch}^4(x)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(7/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(7/2), x)
```

Giac [A] time = 1.15254, size = 69, normalized size = 0.42

$$-\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}-1287\,e^{(10\,x)}+715\,e^{(8\,x)}-286\,e^{(6\,x)}+78\,e^{(4\,x)}-13\,e^{(2\,x)}+1\right)}{3003\,\left(e^{(2\,x)}-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) - 1287*e^(10*x) + 715*e^(8*x) - 286*e^(6*
x) + 78*e^(4*x) - 13*e^(2*x) + 1)/(e^(2*x) - 1)^13
```

3.43 $\int \left(a\,\mathrm{csch}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=118

$$-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\,\mathrm{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{4}{3}a^2$$

```
[Out] (4*a^2*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (6*a^2*Cosh[x]^2*Coth[x]^3*
Sqrt[a*Csch[x]^4])/5 + (4*a^2*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (a
^2*Cosh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/9 - a^2*Cosh[x]*Sqrt[a*Csch[x]^4]
*Sinh[x]
```

Rubi [A] time = 0.0322146, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\,\mathrm{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{4}{3}a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(5/2),x]
```


```
[Out] (4*a^2*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (6*a^2*Cosh[x]^2*Coth[x]^3*
Sqrt[a*Csch[x]^4])/5 + (4*a^2*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (a
^2*Cosh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/9 - a^2*Cosh[x]*Sqrt[a*Csch[x]^4]
*Sinh[x]
```


Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\,\mathrm{csch}^4(x)\right)^{5/2}\,dx &= \left(a^2\sqrt{a\,\mathrm{csch}^4(x)}\sinh^2(x)\right)\int \mathrm{csch}^{10}(x)\,dx\\ &= -\left(\left(ia^2\sqrt{a\,\mathrm{csch}^4(x)}\sinh^2(x)\right)\mathrm{Subst}\left(\int\left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-i\coth(x)\right)\right)\\ &= \frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\,\mathrm{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\,\mathrm{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(\end{aligned}$$

Mathematica [A] time = 0.0324982, size = 47, normalized size = 0.4

$$-\frac{1}{315}a^2\sinh(x)\cosh(x)\left(35\mathrm{csch}^8(x)-40\mathrm{csch}^6(x)+48\mathrm{csch}^4(x)-64\mathrm{csch}^2(x)+128\right)\sqrt{a\,\mathrm{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] -(a^2*Cosh[x]*Sqrt[a*Csch[x]^4]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csc
h[x]^6 + 35*Csch[x]^8)*Sinh[x])/315
```

Maple [A] time = 0.059, size = 60, normalized size = 0.5

$$-\frac{256\,a^{2}\mathrm{e}^{-2\,x}\left(126\,\mathrm{e}^{8\,x}-84\,\mathrm{e}^{6\,x}+36\,\mathrm{e}^{4\,x}-9\,\mathrm{e}^{2\,x}+1\right)}{315\,\left(\mathrm{e}^{2\,x}-1\right)^{7}}\sqrt{\frac{\mathrm{e}^{4\,x}a}{\left(\mathrm{e}^{2\,x}-1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(5/2),x)
```

```
[Out] -256/315*a^2*exp(-2*x)/(exp(2*x)-1)^7*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(12
6*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Maxima [B] time = 1.78231, size = 435, normalized size = 3.69

$$-\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\,\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x)}-1\right)}+\frac{}{35\,\left(9\,e^{(-\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-
8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) -
 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) +
 e^(-18*x) - 1) - 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e
^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^
(-16*x) + e^(-18*x) - 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1) + 256/315*a^(5/2)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1)
```

Fricas [B] time = 1.91199, size = 4632, normalized size = 39.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) - 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 - 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) - 2*a^2*cosh(x)*e^(2*x)
```

```
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 - a^2 + (42*a^2*cosh(x)^2
- a^2)*e^(4*x) - 2*(42*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 - a^2*cosh(x) + (14*a^2*cosh(x)^3 - a^2*cosh
(x))*e^(4*x) - 2*(14*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 - 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) - 2*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 - 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) - 2*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 -
a^2 + (392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^(4
*x) - 2*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(126*a^2*cosh(x)^8 - 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x) + (56*a^2*cosh(x)
^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(56*a^2*
cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 - 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 - 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 - 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 - 195*cosh(x)^4
+ 39*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 - 273*cosh(x)^5
 + 91*cosh(x)^3 - 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 - 4004
*cosh(x)^6 + 2002*cosh(x)^4 - 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 - 25740*cosh(x)^7 + 18018*cosh(x)^5 - 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 - 6435*cosh(x)^8 + 6006*cosh
(x)^6 - 2310*cosh(x)^4 + 315*cosh(x)^2 - 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 - 715*cosh(x)^9 + 858*cosh(x)^7 - 462*cosh(x)^5 + 105*cosh(x)^3 -
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 - 858*cosh(x)^10 + 1287*c
osh(x)^8 - 924*cosh(x)^6 + 315*cosh(x)^4 - 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 - 78*cosh(x)^11 + 143*cosh(x)^9 - 132*cosh(x)^7 +
63*cosh(x)^5 - 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 - 455*cosh(x)^12 + 1001*cosh(x)^10 - 1155*cosh(x)^8 + 735*cosh(x)^6 - 245
*cosh(x)^4 + 35*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 - 105*
cosh(x)^13 + 273*cosh(x)^11 - 385*cosh(x)^9 + 315*cosh(x)^7 - 147*cosh(x)^5
 + 35*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 - 120*cos
h(x)^14 + 364*cosh(x)^12 - 616*cosh(x)^10 + 630*cosh(x)^8 - 392*cosh(x)^6 +
 140*cosh(x)^4 - 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 - 8*c
osh(x)^15 + 28*cosh(x)^13 - 56*cosh(x)^11 + 70*cosh(x)^9 - 56*cosh(x)^7 + 2
8*cosh(x)^5 - 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 - 9*cosh
(x)^16 + 36*cosh(x)^14 - 84*cosh(x)^12 + 126*cosh(x)^10 - 126*cosh(x)^8 + 8
4*cosh(x)^6 - 36*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\operatorname{csch}^4(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(5/2), x)
```

Giac [A] time = 1.1685, size = 53, normalized size = 0.45

$$-\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(e
^(2*x) - 1)^9
```

3.44 $\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-a\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}$$

```
[Out] (2*a*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (a*Cosh[x]^2*Coth[x]^3*Sqrt[a
*Csch[x]^4])/5 - a*Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rubi [A] time = 0.0226303, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-a\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] (2*a*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (a*Cosh[x]^2*Coth[x]^3*Sqrt[a
*Csch[x]^4])/5 - a*Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx &= \left(a\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\int \operatorname{csch}^6(x)\,dx\\ &= -\left(\left(ia\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-i\coth(x)\right)\right)\\ &= \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}-a\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\end{aligned}$$

Mathematica [A] time = 0.0181743, size = 33, normalized size = 0.53

$$-\frac{1}{15}a\sinh(x)\cosh(x)\left(3\operatorname{csch}^4(x)-4\operatorname{csch}^2(x)+8\right)\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(3/2),x]

[Out] -(a*Cosh[x]*Sqrt[a*Csch[x]^4]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x])/15
```

Maple [A] time = 0.053, size = 46, normalized size = 0.7

$$-\frac{16\,a\mathrm{e}^{-2x}\left(10\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+1\right)}{15\,\left(\mathrm{e}^{2x}-1\right)^{3}}\sqrt{\frac{\mathrm{e}^{4x}a}{\left(\mathrm{e}^{2x}-1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(3/2),x)

[Out] -16/15*a*exp(-2*x)/(exp(2*x)-1)^3*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(10*exp
(4*x)-5*exp(2*x)+1)
```

Maxima [B] time = 1.70253, size = 162, normalized size = 2.61

$$-\frac{16\,a^{\frac{3}{2}}e^{(-2x)}}{3\left(5\,e^{(-2x)}-10\,e^{(-4x)}+10\,e^{(-6x)}-5\,e^{(-8x)}+e^{(-10x)}-1\right)}+\frac{32\,a^{\frac{3}{2}}e^{(-4x)}}{3\left(5\,e^{(-2x)}-10\,e^{(-4x)}+10\,e^{(-6x)}-5\,e^{(-8x)}+e^{(-10x)}-1\right)}+\frac{}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="maxima")

[Out] -16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^
(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) + 16/15*a^(3/2)/(5*e^(-2*x) - 10*e^(-4
*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1)
```

Fricas [B] time = 1.65655, size = 1623, normalized size = 26.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="fricas")

[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 - 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 - a)*e^(4*x) - 2*(12*a*cosh(x)^2 -
 a)*e^(2*x) - a)*sinh(x)^2 + (10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(4*x) -
 2*(10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 - a*cos
h(x) + (4*a*cosh(x)^3 - a*cosh(x))*e^(4*x) - 2*(4*a*cosh(x)^3 - a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 - 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 - 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 - 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 - 35*cos
h(x)^4 + 15*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 - 7*cosh(x)^
```

```
5 + 5*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 - 28*cosh(x)^
6 + 30*cosh(x)^4 - 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 - 4*
cosh(x)^7 + 6*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 - 5*cosh(x)^8 + 10*cosh(x)^6 - 10*cosh(x)^4 + 5*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\operatorname{csch}^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(3/2), x)
```

Giac [A] time = 1.16261, size = 36, normalized size = 0.58

$$-\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1\right)}{15\left(e^{(2\,x)} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

3.45 $\int \sqrt{a \operatorname{csch}^4(x)}\, dx$

Optimal. Leaf size=16

$$\sinh(x)(-\cosh(x))\sqrt{a \operatorname{csch}^4(x)}$$

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rubi [A] time = 0.0167914, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 3767, 8}

$$\sinh(x)(-\cosh(x))\sqrt{a \operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a \operatorname{csch}^4(x)}\, dx &= \left(\sqrt{a \operatorname{csch}^4(x)} \sinh^2(x)\right) \int \operatorname{csch}^2(x)\, dx \\
&= -\left(\left(i\sqrt{a \operatorname{csch}^4(x)} \sinh^2(x)\right) \operatorname{Subst}\left(\int 1\, dx, x, -i \coth(x)\right)\right) \\
&= -\cosh(x)\sqrt{a \operatorname{csch}^4(x)} \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.005449, size = 16, normalized size = 1.

$$\sinh(x)(-\cosh(x))\sqrt{a \operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Maple [A] time = 0.068, size = 29, normalized size = 1.8

$$-2\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^{4}}}e^{-2x}\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(1/2),x)
```

```
[Out] -2*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Maxima [A] time = 1.7271, size = 18, normalized size = 1.12

$$\frac{2\sqrt{a}}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) - 1)
```

Fricas [B] time = 1.38496, size = 230, normalized size = 14.38

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1}}\left(e^{(4x)}-2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^{2}+\left(\cosh(x)^{2}-1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*(e^(4*x) - 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 - 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\operatorname{csch}^{4}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**4), x)
```

Giac [A] time = 1.16743, size = 18, normalized size = 1.12

$$-\frac{2\sqrt{a}}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) - 1)
```

3.46 $\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[Out] Coth[x]/(2*Sqrt[a*Csch[x]^4]) - (x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rubi [A] time = 0.0161932, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] Coth[x]/(2*Sqrt[a*Csch[x]^4]) - (x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx&=\frac{\operatorname{csch}^2(x)\int \sinh^2(x)\,dx}{\sqrt{a\operatorname{csch}^4(x)}}\\&=\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{\operatorname{csch}^2(x)\int 1\,dx}{2\sqrt{a\operatorname{csch}^4(x)}}\\&=\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0242194, size = 24, normalized size = 0.67

$$\frac{\coth(x) - x\text{csch}^2(x)}{2\sqrt{a\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] (Coth[x] - x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Maple [B] time = 0.069, size = 89, normalized size = 2.5

$$-\frac{e^{2x}x}{2\left(e^{2x}-1\right)^{2}}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^{4}}}}+\frac{e^{4x}}{8\left(e^{2x}-1\right)^{2}}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^{4}}}}-\frac{1}{8\left(e^{2x}-1\right)^{2}}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(2*x)*x+1/8/(a*exp
(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(4*x)-1/8/(a*exp(4*x)/(exp(2*
x)-1)^4)^(1/2)/(exp(2*x)-1)^2
```

Maxima [A] time = 1.58208, size = 30, normalized size = 0.83

$$-\frac{\left(e^{(-4x)}-1\right)e^{(2x)}}{8\sqrt{a}}-\frac{x}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(e^(-4*x) - 1)*e^(2*x)/sqrt(a) - 1/2*x/sqrt(a)
```

Fricas [B] time = 1.63992, size = 771, normalized size = 21.42

$$\frac{\left(\left(e^{(4x)}-2e^{(2x)}+1\right)\sinh(x)^4+\cosh(x)^4+4\left(\cosh(x)e^{(4x)}-2\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^3-4x\cosh(x)^2+2\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) -
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 - 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 - 2*x)*e^(4*x) - 2*(3*cosh(x)^2 - 2*x)*e^(2*x) - 2*x)*sinh(x)^
2 + (cosh(x)^4 - 4*x*cosh(x)^2 - 1)*e^(4*x) - 2*(cosh(x)^4 - 4*x*cosh(x)^2
```

```
- 1)*e^(2*x) + 4*(cosh(x)^3 - 2*x*cosh(x) + (cosh(x)^3 - 2*x*cosh(x))*e^(4*
x) - 2*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*
e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \operatorname{csch}^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**4), x)
```

Giac [A] time = 1.20209, size = 35, normalized size = 0.97

$$\frac{\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-4\,x+e^{(2\,x)}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/sqrt(a)
```

3.47 $\int \frac{1}{\left(a\,\mathrm{csch}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{5x\mathrm{csch}^2(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\mathrm{csch}^4(x)}}-\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}}$$

```
[Out] (5*Coth[x])/(16*a*Sqrt[a*Csch[x]^4]) - (5*x*Csch[x]^2)/(16*a*Sqrt[a*Csch[x]
^4]) - (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^3)/(
6*a*Sqrt[a*Csch[x]^4])
```

Rubi [A] time = 0.0327562, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$-\frac{5x\mathrm{csch}^2(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\mathrm{csch}^4(x)}}-\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (5*Coth[x])/(16*a*Sqrt[a*Csch[x]^4]) - (5*x*Csch[x]^2)/(16*a*Sqrt[a*Csch[x]
^4]) - (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^3)/(
6*a*Sqrt[a*Csch[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\mathrm{csch}^4(x)\right)^{3/2}}\,dx = \frac{\mathrm{csch}^2(x)\int \sinh^6(x)\,dx}{a\sqrt{a\mathrm{csch}^4(x)}}$$
$$= \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\mathrm{csch}^4(x)}} - \frac{\left(5\mathrm{csch}^2(x)\right)\int \sinh^4(x)\,dx}{6a\sqrt{a\mathrm{csch}^4(x)}}$$
$$= -\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\mathrm{csch}^4(x)}} + \frac{\left(5\mathrm{csch}^2(x)\right)\int \sinh^2(x)\,dx}{8a\sqrt{a\mathrm{csch}^4(x)}}$$
$$= \frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\mathrm{csch}^4(x)}} - \frac{\left(5\mathrm{csch}^2(x)\right)\int 1\,dx}{16a\sqrt{a\mathrm{csch}^4(x)}}$$
$$= \frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}} - \frac{5x\mathrm{csch}^2(x)}{16a\sqrt{a\mathrm{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\mathrm{csch}^4(x)}}$$

Mathematica [A] time = 0.0384692, size = 38, normalized size = 0.44

$$\frac{(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))\mathrm{csch}^6(x)}{192\left(a\mathrm{csch}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (Csch[x]^6*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Csch[x
]^4)^(3/2))
```

Maple [B] time = 0.053, size = 230, normalized size = 2.7

$$-\frac{5\,e^{2x}x}{16\,a\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} + \frac{e^{8x}}{384\,a\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} - \frac{3\,e^{6x}}{128\,a\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} + \frac{15\,e^{4x}}{128\,a\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(3/2),x)
```

```
[Out] -5/16/a*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/384/a
*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-3/128/a*exp(6*x)
/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(2*
x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/128/a/(exp(2*x)-1)^2/(a*exp(4*
x)/(exp(2*x)-1)^4)^(1/2)+3/128/a*exp(-2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^
4)^(1/2)
```

Maxima [A] time = 1.57918, size = 62, normalized size = 0.72

$$-\frac{\left(9\,e^{(-2\,x)} - 45\,e^{(-4\,x)} + 45\,e^{(-8\,x)} - 9\,e^{(-10\,x)} + e^{(-12\,x)} - 1\right)e^{(6\,x)}}{384\,a^{\frac{3}{2}}} - \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*x) - 45*e^(-4*x) + 45*e^(-8*x) - 9*e^(-10*x) + e^(-12*x) -
1)*e^(6*x)/a^(3/2) - 5/16*x/a^(3/2)
```

Fricas [B] time = 1.77925, size = 3687, normalized size = 42.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 - 3)*e^(4*x) - 2*(22*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^10 - 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 - 9*cosh(x))*e^(4*x) - 2*(22*cosh(x
)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 - 9*cosh
(x)^2 + (11*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(4*x) - 2*(11*cosh(x)^4 - 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + (11*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(11*cosh(x)
^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 - 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 -
315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(4*x) - 2*(154*cosh(x)^6 - 315*cosh
(x)^4 + 210*cosh(x)^2 - 20*x)*e^(2*x) - 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
- 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x) + (22*cosh(x)^7 - 63*cosh(x)^5
 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(4*x) - 2*(22*cosh(x)^7 - 63*cosh(x)^5 +
70*cosh(x)^3 - 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 - 42*cos
h(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 + (11*cosh(x)^8 - 42*cosh(x)^6 + 70*
cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(4*x) - 2*(11*cosh(x)^8 - 42*cosh(x)^6 +
70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 + (11*cos
h(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) - 2*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 +
420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 - 135*cosh(
x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + 3)*e^(4*x) - 2*(22*
cosh(x)^10 - 135*cosh(x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2
 + 3)*e^(2*x) + 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^12 - 9*cosh(x)^10 + 4
5*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(4*x) - 2
*(cosh(x)^12 - 9*cosh(x)^10 + 45*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4
 + 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(x)^7
 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(
x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x))*e^(4*x) - 2*(2*cosh(x)^1
1 - 15*cosh(x)^9 + 60*cosh(x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x
))*e^(2*x) + 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x
) - 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{csch}^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-3/2), x)
```

Giac [A] time = 1.2494, size = 68, normalized size = 0.79

$$\frac{\left(110\,e^{(6\,x)} - 45\,e^{(4\,x)} + 9\,e^{(2\,x)} - 1\right)e^{(-6\,x)} - 120\,x + e^{(6\,x)} - 9\,e^{(4\,x)} + 45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))/a^(3/2)
```

3.48 $\int \frac{1}{\left(a\,\mathbf{csch}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{63x\,\text{csch}^2(x)}{256a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{63\coth(x)}{256a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\,\text{csch}^4(x)}}-\frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\,\text{csch}^4(x)}}-\frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\,\text{csch}^4(x)}}$$

```
[Out] (63*Coth[x])/(256*a^2*Sqrt[a*Csch[x]^4]) - (63*x*Csch[x]^2)/(256*a^2*Sqrt[a
*Csch[x]^4]) - (21*Cosh[x]*Sinh[x])/(128*a^2*Sqrt[a*Csch[x]^4]) + (21*Cosh[
x]*Sinh[x]^3)/(160*a^2*Sqrt[a*Csch[x]^4]) - (9*Cosh[x]*Sinh[x]^5)/(80*a^2*S
qrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^7)/(10*a^2*Sqrt[a*Csch[x]^4])
```

Rubi [A] time = 0.0511113, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$-\frac{63x\,\text{csch}^2(x)}{256a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{63\coth(x)}{256a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\,\text{csch}^4(x)}}-\frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\,\text{csch}^4(x)}}+\frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\,\text{csch}^4(x)}}-\frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\,\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (63*Coth[x])/(256*a^2*Sqrt[a*Csch[x]^4]) - (63*x*Csch[x]^2)/(256*a^2*Sqrt[a
*Csch[x]^4]) - (21*Cosh[x]*Sinh[x])/(128*a^2*Sqrt[a*Csch[x]^4]) + (21*Cosh[
x]*Sinh[x]^3)/(160*a^2*Sqrt[a*Csch[x]^4]) - (9*Cosh[x]*Sinh[x]^5)/(80*a^2*S
qrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^7)/(10*a^2*Sqrt[a*Csch[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \frac{\text{csch}^2(x)\int \sinh^{10}(x)\,dx}{a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(9\text{csch}^2(x)\right)\int \sinh^8(x)\,dx}{10a^2\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} + \frac{\left(63\text{csch}^2(x)\right)\int \sinh^6(x)\,dx}{80a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(21\text{csch}^2(x)\right)\int \sinh^4(x)\,dx}{32a^2\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} + \frac{(63\text{cs}}{1}$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(}{10a^2\sqrt{}}$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{63x\text{csch}^2(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh}{80a^2}$$

Mathematica [A] time = 0.110912, size = 55, normalized size = 0.42

$$\frac{\sinh^2(x)(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sinh(8x) + 2\sinh(10x))\sqrt{a\text{csch}^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^4]*Sinh[x]^2*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x] + 15
0*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] time = 0.056, size = 362, normalized size = 2.7

$$-\frac{63\,e^{2x}x}{256\,a^2\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} + \frac{e^{12x}}{10240\,a^2\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} - \frac{5\,e^{10x}}{4096\,a^2\left(e^{2x}-1\right)^2}\frac{1}{\sqrt{\frac{e^{4x}a}{\left(e^{2x}-1\right)^4}}} + \frac{15\,e^{8x}}{2048\,a^2\left(e^{2x}-1\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(5/2),x)
```

```
[Out] -63/256/a^2*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/1
0240/a^2*exp(12*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-5/4096/
a^2*exp(10*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/2048/a^2*
exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/512/a^2*exp(6*
x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+105/1024/a^2*exp(4*x)/(
exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-105/1024/a^2/(exp(2*x)-1)^2
/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/512/a^2*exp(-2*x)/(exp(2*x)-1)^2/(a*e
```

```
xp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4
*x)/(exp(2*x)-1)^4)^(1/2)+5/4096/a^2*exp(-6*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(
exp(2*x)-1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)
```

Maxima [A] time = 1.67863, size = 97, normalized size = 0.73

$$-\frac{\left(25\,e^{(-2\,x)} - 150\,e^{(-4\,x)} + 600\,e^{(-6\,x)} - 2100\,e^{(-8\,x)} + 2100\,e^{(-12\,x)} - 600\,e^{(-14\,x)} + 150\,e^{(-16\,x)} - 25\,e^{(-18\,x)} + 2\,e^{(-20\,x)} - 2\right)e^{(10}}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20480*(25*e^(-2*x) - 150*e^(-4*x) + 600*e^(-6*x) - 2100*e^(-8*x) + 2100*
e^(-12*x) - 600*e^(-14*x) + 150*e^(-16*x) - 25*e^(-18*x) + 2*e^(-20*x) - 2)
*e^(10*x)/a^(5/2) - 63/256*x/a^(5/2)
```

Fricas [B] time = 1.95369, size = 9443, normalized size = 71.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 - 5)*e^(4*x) - 2*(76*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^18 - 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 - 15*cosh(x))*e^(4*x) - 2*(7
6*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 - 255*cosh(x)^2 + (646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(4*x) - 2*(646
*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 - 425*cosh(x)^3 + (646*cosh(x)^5 - 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) - 2*(646*cosh(x)^5 - 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(4*x) - 2*(129
2*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^14
 - 600*cosh(x)^14 + 120*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4
 - 910*cosh(x)^2 + (4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4 - 910*
cosh(x)^2 + 35)*e^(4*x) - 2*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)
^4 - 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + (4199*co
sh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) - 2*(4199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 - 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x
)^6 - 300300*cosh(x)^4 + 69300*cosh(x)^2 - 2520*x)*e^(4*x) - 2*(184756*cosh
(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^4 + 69300*cos
h(x)^2 - 2520*x)*e^(2*x) - 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 - 6077
```

```
5*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 2520*x*
cosh(x) + (16796*cosh(x)^11 - 60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cos
h(x)^5 + 23100*cosh(x)^3 - 2520*x*cosh(x))*e^(4*x) - 2*(16796*cosh(x)^11 -
60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 - 36465*cosh(x)^10 +
64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 - 36465*cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 346
50*cosh(x)^4 - 7560*x*cosh(x)^2 - 70)*e^(4*x) - 2*(8398*cosh(x)^12 - 36465*
cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*cosh(x)^5 - 2
520*x*cosh(x)^3 + (646*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580
*cosh(x)^7 + 6930*cosh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*(6
46*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*co
sh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 - 7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8
+ 32340*cosh(x)^6 - 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 -
7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17
640*x*cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(4*x) - 2*(1292*cosh(x)^14 - 7735*c
osh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17640*x*
cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^6 + 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 +
 69300*cosh(x)^7 - 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 -
8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7 - 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(4*x) - 2*(1292*cosh(x)^1
5 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7
- 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(2*x) + 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*
cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cos
h(x)^2 + (323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*cosh(x
)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cosh(x)^2
 - 5)*e^(4*x) - 2*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 200
20*cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
- 170*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040
*x*cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 + (19*cosh(x)^17 - 170*cosh(x)
^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*cosh(x)^7
- 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) - 2*(19*cosh(x)^17 - 1
70*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*
cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
*sinh(x)^3 + 5*(76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14 - 10920*co
sh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^6 + 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14
 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^
6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(4*x) - 2*(76*cosh(x)^18 - 765*co
sh(x)^16 + 3600*cosh(x)^14 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(2*x) +
 5)*sinh(x)^2 + 25*cosh(x)^2 + (2*cosh(x)^20 - 25*cosh(x)^18 + 150*cosh(x)^
16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 5040*x*cosh(x)^10 - 2100*cosh(x)^8
+ 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(x)^2 - 2)*e^(4*x) - 2*(2*cosh(x)^
20 - 25*cosh(x)^18 + 150*cosh(x)^16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 + 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 840
*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 84
0*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*co
sh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(4*cosh(x)^19 - 45*cosh(x)^
17 + 240*cosh(x)^15 - 840*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 -
 1680*cosh(x)^7 + 360*cosh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cos
```

```
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-5/2), x)
```

Giac [A] time = 1.17581, size = 103, normalized size = 0.78

$$\frac{\left(5754\,e^{(10\,x)} - 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} - 150\,e^{(4\,x)} + 25\,e^{(2\,x)} - 2\right)e^{(-10\,x)} - 5040\,x + 2\,e^{(10\,x)} - 25\,e^{(8\,x)} + 150\,e^{(6\,x)} - 600\,e^{(4\,x)} +}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^(
2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 600*
e^(4*x) + 2100*e^(2*x))/a^(5/2)
```

3.49 $\int \frac{1}{a+ia\,\mathbf{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}$$

```
[Out] x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Rubi [A] time = 0.0148094, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\operatorname{csch}(a+bx)}\,dx &= -\frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.113411, size = 54, normalized size = 1.69

$$-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)-i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{x}{a}+\frac{1}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] - I*Sinh[(a +
b*x)/2]))
```

Maple [A] time = 0.038, size = 63, normalized size = 2.

$$\frac{1}{ab}\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-2\,\frac{1}{ab\,(\tanh\,(1/2\,bx+a/2)+i)}-\frac{1}{ab}\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(b*x+a)),x)
```

```
[Out] 1/b/a*ln(tanh(1/2*b*x+1/2*a)+1)-2/b/a/(tanh(1/2*b*x+1/2*a)+I)-1/b/a*ln(tanh
(1/2*b*x+1/2*a)-1)
```

Maxima [A] time = 1.19155, size = 47, normalized size = 1.47

$$\frac{bx+a}{ab}+\frac{2i}{\left(ae^{(-bx-a)}-i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)/(a*b) + 2*I/((a*e^(-b*x - a) - I*a)*b)
```

Fricas [A] time = 1.5088, size = 80, normalized size = 2.5

$$\frac{bxe^{(bx+a)}+i\,bx+2i}{abe^{(bx+a)}+i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*e^(b*x + a) + I*b*x + 2*I)/(a*b*e^(b*x + a) + I*a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{i\operatorname{csch}(a+bx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x)
```

```
[Out] Integral(1/(I*csch(a + b*x) + 1), x)/a
```

Giac [A] time = 1.15244, size = 42, normalized size = 1.31

$$\frac{bx+a}{ab}+\frac{2i}{ab\left(e^{(bx+a)}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*x + a)/(a*b) + 2*I/(a*b*(e^(b*x + a) + I))
```

3.50 $\int \frac{1}{a-ia\,\mathbf{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

```
[Out] x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Rubi [A] time = 0.0153436, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-ia\operatorname{csch}(a+bx)}\,dx &= -\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.103366, size = 54, normalized size = 1.69

$$-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)+i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{x}{a}+\frac{1}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] + I*Sinh[(a +
b*x)/2]))
```

Maple [A] time = 0.043, size = 63, normalized size = 2.

$$\frac{1}{ab}\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-2\,\frac{1}{ab\left(\tanh\left(1/2\,bx+a/2\right)-i\right)}-\frac{1}{ab}\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(b*x+a)),x)
```

```
[Out] 1/b/a*ln(tanh(1/2*b*x+1/2*a)+1)-2/b/a/(tanh(1/2*b*x+1/2*a)-I)-1/b/a*ln(tanh
(1/2*b*x+1/2*a)-1)
```

Maxima [A] time = 1.13904, size = 47, normalized size = 1.47

$$\frac{bx+a}{ab}-\frac{2i}{\left(ae^{(-bx-a)}+i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)/(a*b) - 2*I/((a*e^(-b*x - a) + I*a)*b)
```

Fricas [A] time = 1.59477, size = 80, normalized size = 2.5

$$\frac{bxe^{(bx+a)}-i\,bx-2i}{abe^{(bx+a)}-i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*e^(b*x + a) - I*b*x - 2*I)/(a*b*e^(b*x + a) - I*a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{i\operatorname{csch}(a+bx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x)
```

```
[Out] -Integral(1/(I*csch(a + b*x) - 1), x)/a
```

Giac [A] time = 1.17111, size = 42, normalized size = 1.31

$$\frac{bx+a}{ab} - \frac{2i}{ab\left(e^{(bx+a)} - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*x + a)/(a*b) - 2*I/(a*b*(e^(b*x + a) - I))
```

3.51 $\int (a + ia\operatorname{csch}(c + dx))^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (14*a^3*Coth[c + d*x])/(3*d*Sqrt[a + I*a*Csch[c + d*x]]) + (2*a^2*Coth[c
+ d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d)
```

Rubi [A] time = 0.12708, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (14*a^3*Coth[c + d*x])/(3*d*Sqrt[a + I*a*Csch[c + d*x]]) + (2*a^2*Coth[c
+ d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d)
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\operatorname{csch}(c+dx))^{5/2}\,dx &= \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{1}{3}(2a)\int \sqrt{a+ia\operatorname{csch}(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}ia\operatorname{csch}(c+\right.\\ &= \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{1}{3}\left(7ia^2\right)\int \operatorname{csch}(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\,dx\\ &= \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}-\frac{\left(2ia^3\right)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,d\right.}{d}\\ &= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(}{3d}\end{aligned}$$

Mathematica [A] time = 1.40158, size = 136, normalized size = 1.27

$$\frac{2a^2\sqrt{a+ia\operatorname{csch}(c+dx)}\left(\coth(c+dx)+\frac{14\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)}+\frac{3(-1)^{3/4}\coth(c+dx)\tanh^{-1}\left((-1)^{3/4}\sqrt{\operatorname{csch}(c+dx)+i}\right)}{(\operatorname{csch}(c+dx)-i)\sqrt{\operatorname{csch}(c+dx)+i}}-7i\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[a + I*a*Csch[c + d*x]]*(-7*I + Coth[c + d*x] + (3*(-1)^(3/4)*Ar
cTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x])/((-I + Csch[c + d*
x])*Sqrt[I + Csch[c + d*x]]) + (14*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2])))/(3*d)
```

Maple [F] time = 0.258, size = 0, normalized size = 0.

$$\int (a+ia\operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\operatorname{csch}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.41947, size = 1403, normalized size = 13.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(a^5/d^2)*(3*d*e^(3*d*x + 3*c) + 3*I*d*e^(2*d*x + 2*c) - 3*d*e^(d*x
 + c) - 3*I*d)*log((2*sqrt(a^5/d^2)*(-(I - 4)*d*e^(3*d*x + 3*c) - (4*I + 1)
*d) + ((2*I - 8)*a^2*e^(3*d*x + 3*c) + (8*I + 2)*a^2*e^(2*d*x + 2*c) - (2*I
 - 8)*a^2*e^(d*x + c) - (8*I + 2)*a^2)*sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d
*x + c) - a)/(e^(2*d*x + 2*c) - 1)))/((20*I + 48)*a^2*e^(2*d*x + 2*c) + (48
*I - 20)*a^2*e^(d*x + c))) - 2*sqrt(a^5/d^2)*(3*d*e^(3*d*x + 3*c) + 3*I*d*e
^(2*d*x + 2*c) - 3*d*e^(d*x + c) - 3*I*d)*log((2*sqrt(a^5/d^2)*((I - 4)*d*e
^(3*d*x + 3*c) + (4*I + 1)*d) + ((2*I - 8)*a^2*e^(3*d*x + 3*c) + (8*I + 2)*
a^2*e^(2*d*x + 2*c) - (2*I - 8)*a^2*e^(d*x + c) - (8*I + 2)*a^2)*sqrt((a*e^
(2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1)))/((20*I + 48)
*a^2*e^(2*d*x + 2*c) + (48*I - 20)*a^2*e^(d*x + c))) - (64*a^2*e^(3*d*x + 3
*c) - 48*I*a^2*e^(2*d*x + 2*c) - 48*a^2*e^(d*x + c) + 64*I*a^2)*sqrt((a*e^(
2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1)))/(12*d*e^(3*d*
x + 3*c) + 12*I*d*e^(2*d*x + 2*c) - 12*d*e^(d*x + c) - 12*I*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\operatorname{csch}(c+dx)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\operatorname{csch}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

3.52 $\int (a + ia\mathbf{csch}(c + dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])
```

Rubi [A] time = 0.0415648, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(3/2),x]
```


```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])
```


Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx = \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} + (2a)\int \frac{\frac{a}{2}+\frac{1}{2}ia\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}dx$$
$$= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} + a\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx$$
$$= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} - \frac{(2ia^2)\operatorname{Subst}\left(\int \frac{1}{a+x^2}dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$
$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}$$

Mathematica [A] time = 1.17214, size = 100, normalized size = 1.39

$$-\frac{2ia\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\left(\sqrt{\operatorname{csch}(c+dx)+i}-\sqrt[4]{-1}\tanh^{-1}\left((-1)^{3/4}\sqrt{\operatorname{csch}(c+dx)+i}\right)\right)}{d(\operatorname{csch}(c+dx)-i)\sqrt{\operatorname{csch}(c+dx)+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]]*(-((-1)^(1/4)*ArcTanh[(
-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]) + Sqrt[I + Csch[c + d*x]]))/(d*(-I + Cs
ch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int (a+ia\operatorname{csch}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\operatorname{csch}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.40457, size = 1089, normalized size = 15.12

$$-\frac{2\left(de^{(dx+c)}+i\,d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{2\left(-(i-4)\,de^{(3\,dx+3\,c)}-(4i+1)\,d\right)\sqrt{\frac{a^3}{d^2}}+\left((2i-8)\,ae^{(3\,dx+3\,c)}+(8i+2)\,ae^{(2\,dx+2\,c)}-(2i-8)\,ae^{(dx+c)}-(8i+2)\,a\right)\sqrt{\frac{ae^{(2\,dx+2\,c)}+2i\,ae^{(dx+c)}-}{e^{(2\,dx+2\,c)}-1}}}{(20i+48)\,ae^{(2\,dx+2\,c)}+(48i-20)\,ae^{(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*(d*e^(d*x + c) + I*d)*sqrt(a^3/d^2)*log((2*(-(I - 4)*d*e^(3*d*x + 3*c)
- (4*I + 1)*d)*sqrt(a^3/d^2) + ((2*I - 8)*a*e^(3*d*x + 3*c) + (8*I + 2)*a*e
^(2*d*x + 2*c) - (2*I - 8)*a*e^(d*x + c) - (8*I + 2)*a)*sqrt((a*e^(2*d*x +
2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1)))/((20*I + 48)*a*e^(2*d
*x + 2*c) + (48*I - 20)*a*e^(d*x + c))) - 2*(d*e^(d*x + c) + I*d)*sqrt(a^3/
d^2)*log((2*((I - 4)*d*e^(3*d*x + 3*c) + (4*I + 1)*d)*sqrt(a^3/d^2) + ((2*I
 - 8)*a*e^(3*d*x + 3*c) + (8*I + 2)*a*e^(2*d*x + 2*c) - (2*I - 8)*a*e^(d*x
+ c) - (8*I + 2)*a)*sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*
d*x + 2*c) - 1)))/((20*I + 48)*a*e^(2*d*x + 2*c) + (48*I - 20)*a*e^(d*x + c
))) - (8*a*e^(d*x + c) - 8*I*a)*sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c)
 - a)/(e^(2*d*x + 2*c) - 1)))/(4*d*e^(d*x + c) + 4*I*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\operatorname{csch}(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\operatorname{csch}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

3.53 $\int \sqrt{a+ia\,\mathbf{csch}(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Rubi [A] time = 0.0197913, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\,\mathrm{csch}(c+dx)}\,dx &= -\frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.890078, size = 80, normalized size = 2.

$$\frac{2(-1)^{3/4}\coth(c+dx)\sqrt{a+ia\,\mathrm{csch}(c+dx)}\tanh^{-1}\left((-1)^{3/4}\sqrt{\mathrm{csch}(c+dx)+i}\right)}{d(\mathrm{csch}(c+dx)-i)\sqrt{\mathrm{csch}(c+dx)+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x]*Sqr
t[a + I*a*Csch[c + d*x]])/(d*(-I + Csch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \sqrt{a + ia\operatorname{csch}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\operatorname{csch}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Fricas [B] time = 2.39565, size = 814, normalized size = 20.35

$$-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(-(i-4)\,de^{(3\,dx+3\,c)}-(4i+1)\,d\right)\sqrt{\frac{a}{d^2}}+\sqrt{\frac{ae^{(2\,dx+2\,c)}+2i\,ae^{(dx+c)}-a}{e^{(2\,dx+2\,c)}-1}}\left((2i-8)\,e^{(3\,dx+3\,c)}+(8i+2)\,e^{(2\,dx+2\,c)}-(2i\right.}{(20i+48)\,e^{(2\,dx+2\,c)}+(48i-20)\,e^{(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a/d^2)*log((2*(-(I - 4)*d*e^(3*d*x + 3*c) - (4*I + 1)*d)*sqrt(a/d
^2) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1
))*((2*I - 8)*e^(3*d*x + 3*c) + (8*I + 2)*e^(2*d*x + 2*c) - (2*I - 8)*e^(d*
x + c) - 8*I - 2))/((20*I + 48)*e^(2*d*x + 2*c) + (48*I - 20)*e^(d*x + c)))
 + 1/2*sqrt(a/d^2)*log((2*((I - 4)*d*e^(3*d*x + 3*c) + (4*I + 1)*d)*sqrt(a/
d^2) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) -
1))*((2*I - 8)*e^(3*d*x + 3*c) + (8*I + 2)*e^(2*d*x + 2*c) - (2*I - 8)*e^(d
*x + c) - 8*I - 2))/((20*I + 48)*e^(2*d*x + 2*c) + (48*I - 20)*e^(d*x + c))
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia\operatorname{csch}(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*csch(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\,\mathrm{csch}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

3.54 $\int \frac{1}{\sqrt{a+ia\,\mathbf{csch}(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0873695, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = -\left(i\int \frac{\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx\right)+\frac{\int \sqrt{a+ia\text{csch}(c+dx)}\,dx}{a}$$
$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 1.08103, size = 118, normalized size = 1.3

$$\frac{\sqrt{a}\coth(c+dx)\left(2\tan^{-1}\left(\frac{\sqrt{ia(\text{csch}(c+dx)+i)}}{\sqrt{a}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{ia(\text{csch}(c+dx)+i)}}{\sqrt{2}\sqrt{a}}\right)\right)}{d\sqrt{ia(\text{csch}(c+dx)+i)}\sqrt{a+ia\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*ArcTan[
Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d*Sqrt[I*
a*(I + Csch[c + d*x])]*Sqrt[a + I*a*Csch[c + d*x]])
```

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+ia\text{csch}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia\operatorname{csch}(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*csch(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\operatorname{csch}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

3.55 $\int \frac{1}{(a+ia\,\mathbf{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(a^(3/2)*d
) - (5*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x])^(3/2
))
```

Rubi [A] time = 0.145928, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(a^(3/2)*d
) - (5*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x])^(3/2
))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\text{csch}(c+dx))^{3/2}}dx = -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}ia\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}dx}{2a^2}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} + \frac{\int \sqrt{a+ia\text{csch}(c+dx)}\,dx}{a^2} - \frac{(5i)\int \frac{\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}dx}{4a}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} - \frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{ad} + \frac{(5i)\,\text{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

Mathematica [B] time = 2.36687, size = 327, normalized size = 2.66

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sqrt{ia(\text{csch}(c+dx)+i)}\tan^{-1}\left(\frac{\sqrt{ia(\text{csch}(c+dx)+i)}}{\sqrt{a}}\right)+i\text{csch}(c+dx)\left(-8\sqrt{ia(\text{csch}(c\right.\right.}{4a^{3/2}d(\text{csch}(c+dx)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*Sqrt[a] - 8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I
+ Csch[c + d*x])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]
*Sqrt[a])]*Sqrt[I*a*(I + Csch[c + d*x])] + I*Csch[c + d*x]*(2*Sqrt[a] - 8*A
rcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I + Csch[c + d*x])]
+ 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])]*Sqrt[I*
a*(I + Csch[c + d*x])]))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))/(4*a^(3
/2)*d*(I + Csch[c + d*x])*Sqrt[a + I*a*Csch[c + d*x]]*(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2]))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int (a+ia\text{csch}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\,\mathrm{csch}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 3.06548, size = 2372, normalized size = 19.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*(5*a^2*d*e^(3*d*x + 3*c) + 15*I*a^2*d*e^(2*d*x + 2*c) - 15*a^2*d
*e^(d*x + c) - 5*I*a^2*d)*sqrt(1/(a^3*d^2))*log((sqrt(1/2)*(4*I*a^2*d*e^(2*
d*x + 2*c) + 4*I*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a
*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1))*(-4*I*e^(2*d*x + 2*c) + 4*I))/((10
*I + 5)*e^(2*d*x + 2*c) + (10*I - 20)*e^(d*x + c) - 10*I - 5)) - sqrt(1/2)*
(5*a^2*d*e^(3*d*x + 3*c) + 15*I*a^2*d*e^(2*d*x + 2*c) - 15*a^2*d*e^(d*x + c
) - 5*I*a^2*d)*sqrt(1/(a^3*d^2))*log((sqrt(1/2)*(-4*I*a^2*d*e^(2*d*x + 2*c)
 - 4*I*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x +
c) - a)/(e^(2*d*x + 2*c) - 1))*(-4*I*e^(2*d*x + 2*c) + 4*I))/((10*I + 5)*e^
(2*d*x + 2*c) + (10*I - 20)*e^(d*x + c) - 10*I - 5)) - (2*a^2*d*e^(3*d*x +
3*c) + 6*I*a^2*d*e^(2*d*x + 2*c) - 6*a^2*d*e^(d*x + c) - 2*I*a^2*d)*sqrt(1/
(a^3*d^2))*log(((-(I - 4)*a^2*d*e^(3*d*x + 3*c) - (4*I + 1)*a^2*d)*sqrt(1/(
a^3*d^2)) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*
c) - 1))*((I - 4)*e^(3*d*x + 3*c) + (4*I + 1)*e^(2*d*x + 2*c) - (I - 4)*e^(
d*x + c) - 4*I - 1))/((10*I + 24)*e^(2*d*x + 2*c) + (24*I - 10)*e^(d*x + c)
)) + (2*a^2*d*e^(3*d*x + 3*c) + 6*I*a^2*d*e^(2*d*x + 2*c) - 6*a^2*d*e^(d*x
+ c) - 2*I*a^2*d)*sqrt(1/(a^3*d^2))*log((((I - 4)*a^2*d*e^(3*d*x + 3*c) + (
4*I + 1)*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x
+ c) - a)/(e^(2*d*x + 2*c) - 1))*((I - 4)*e^(3*d*x + 3*c) + (4*I + 1)*e^(2*
d*x + 2*c) - (I - 4)*e^(d*x + c) - 4*I - 1))/((10*I + 24)*e^(2*d*x + 2*c) +
 (24*I - 10)*e^(d*x + c))) - sqrt((a*e^(2*d*x + 2*c) + 2*I*a*e^(d*x + c) -
a)/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - 2*e^(d
*x + c) + 2*I))/(4*a^2*d*e^(3*d*x + 3*c) + 12*I*a^2*d*e^(2*d*x + 2*c) - 12*
a^2*d*e^(d*x + c) - 4*I*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia\,\mathrm{csch}\,(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\,\mathrm{csch}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

3.56 $\int \sqrt{a - ia\,\mathbf{csch}(c + dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Rubi [A] time = 0.0226346, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a - ia\text{csch}(c + dx)}\,dx &= \frac{(2ia)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.922423, size = 80, normalized size = 2.

$$-\frac{2(-1)^{3/4}\coth(c+dx)\sqrt{a-ia\text{csch}(c+dx)}\tan^{-1}\left((-1)^{3/4}\sqrt{\text{csch}(c+dx)-i}\right)}{d\sqrt{\text{csch}(c+dx)-i}(\text{csch}(c+dx)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[-I + Csch[c + d*x]]]*Coth[c + d*x]*Sq
rt[a - I*a*Csch[c + d*x]])/(d*Sqrt[-I + Csch[c + d*x]]*(I + Csch[c + d*x]))
```

Maple [F] time = 0.578, size = 0, normalized size = 0.

$$\int \sqrt{a - ia\text{csch}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a-I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-i\,a\,\text{csch}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Fricas [B] time = 2.38993, size = 815, normalized size = 20.38

$$\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(-(4i-1)\,de^{(3\,dx+3\,c)} + (i+4)\,d\right)\sqrt{\frac{a}{d^2}} + \sqrt{\frac{ae^{(2\,dx+2\,c)} - 2i\,ae^{(dx+c)} - a}{e^{(2\,dx+2\,c)} - 1}}\left(-(8i-2)\,e^{(3\,dx+3\,c)} + (2i+8)\,e^{(2\,dx+2\,c)} + (8i\right.}{(48i+20)\,e^{(2\,dx+2\,c)} - (20i-48)\,e^{(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log((2*(-(4*I - 1)*d*e^(3*d*x + 3*c) + (I + 4)*d)*sqrt(a/d^
2) + sqrt((a*e^(2*d*x + 2*c) - 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) - 1)
)*(-(8*I - 2)*e^(3*d*x + 3*c) + (2*I + 8)*e^(2*d*x + 2*c) + (8*I - 2)*e^(d*
x + c) - 2*I - 8))/((48*I + 20)*e^(2*d*x + 2*c) - (20*I - 48)*e^(d*x + c)))
 - 1/2*sqrt(a/d^2)*log((2*((4*I - 1)*d*e^(3*d*x + 3*c) - (I + 4)*d)*sqrt(a/
d^2) + sqrt((a*e^(2*d*x + 2*c) - 2*I*a*e^(d*x + c) - a)/(e^(2*d*x + 2*c) -
1))*(-(8*I - 2)*e^(3*d*x + 3*c) + (2*I + 8)*e^(2*d*x + 2*c) + (8*I - 2)*e^(
d*x + c) - 2*I - 8))/((48*I + 20)*e^(2*d*x + 2*c) - (20*I - 48)*e^(d*x + c)
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ia\,\text{csch}\,(c + dx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-I*a*csch(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-i\,a\,\mathrm{csch}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

3.57 $\int \frac{1}{\sqrt{a-ia\,\mathbf{csch}(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0843303, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ia\text{csch}(c+dx)}}dx = i\int \frac{\text{csch}(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}dx + \frac{\int \sqrt{a-ia\text{csch}(c+dx)}dx}{a}$$
$$= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d} - \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a+x^2}dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 1.07126, size = 117, normalized size = 1.29

$$\frac{\sqrt{a}\coth(c+dx)\left(2\tan^{-1}\left(\frac{\sqrt{-ia(\text{csch}(c+dx)-i)}}{\sqrt{a}}\right) - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{-ia(\text{csch}(c+dx)-i)}}{\sqrt{2}\sqrt{a}}\right)\right)}{d\sqrt{a(-1-i\text{csch}(c+dx))}\sqrt{a-ia\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*Arc
Tan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d
*Sqrt[a*(-1 - I*Csch[c + d*x])]*Sqrt[a - I*a*Csch[c + d*x]])
```

Maple [F] time = 0.446, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-ia\text{csch}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-ia\operatorname{csch}(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-I*a*csch(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-i\,a\operatorname{csch}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

3.58 $\int \sqrt{3+3i\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\operatorname{csch}(x)}}\right)$$

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Rubi [A] time = 0.017324, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\operatorname{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3i\operatorname{csch}(x)}\,dx &= -\left(6i\operatorname{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{3+3i\operatorname{csch}(x)}}\right)\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\operatorname{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.685641, size = 46, normalized size = 2.

$$\frac{2\sqrt{3}\coth(x)\tan^{-1}\left(\sqrt{-1+i\operatorname{csch}(x)}\right)}{\sqrt{-1+i\operatorname{csch}(x)}\sqrt{1+i\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqrt
[1 + I*Csch[x]])
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \sqrt{3+3\,i\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((3+3*I*csch(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3i\,\mathrm{csch}\,(x)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

Fricas [B] time = 2.12005, size = 581, normalized size = 25.26

$$-\frac{1}{2}\,\sqrt{3}\log\left(\frac{\sqrt{\frac{3\,e^{(2\,x)}+6i\,e^{x}-3}{e^{(2\,x)}-1}}\left((i-4)\,e^{(3\,x)}+(4i+1)\,e^{(2\,x)}-(i-4)\,e^{x}-4i-1\right)-(i-4)\,\sqrt{3}e^{(3\,x)}-(4i+1)\,\sqrt{3}}{(10i+24)\,e^{(2\,x)}+(24i-10)\,e^{x}}\right)+\frac{1}{2}\,\sqrt{3}\log\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*log((sqrt((3*e^(2*x) + 6*I*e^x - 3)/(e^(2*x) - 1))*((I - 4)*e^
(3*x) + (4*I + 1)*e^(2*x) - (I - 4)*e^x - 4*I - 1) - (I - 4)*sqrt(3)*e^(3*x
) - (4*I + 1)*sqrt(3))/((10*I + 24)*e^(2*x) + (24*I - 10)*e^x)) + 1/2*sqrt(
3)*log((sqrt((3*e^(2*x) + 6*I*e^x - 3)/(e^(2*x) - 1))*((I - 4)*e^(3*x) + (4
*I + 1)*e^(2*x) - (I - 4)*e^x - 4*I - 1) + (I - 4)*sqrt(3)*e^(3*x) + (4*I +
 1)*sqrt(3))/((10*I + 24)*e^(2*x) + (24*I - 10)*e^x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{i\,\mathrm{csch}\,(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) + 1), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3i\,\operatorname{csch}\left(x\right) + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

3.59 $\int \sqrt{3-3i\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\operatorname{csch}(x)}}\right)$$

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]
```

Rubi [A] time = 0.0184587, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\operatorname{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-3i\operatorname{csch}(x)}\,dx &= 6i\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{3-3i\operatorname{csch}(x)}}\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\operatorname{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.674897, size = 46, normalized size = 2.

$$\frac{2\sqrt{3}\coth(x)\tan^{-1}\left(\sqrt{-1-i\operatorname{csch}(x)}\right)}{\sqrt{-1-i\operatorname{csch}(x)}\sqrt{1-i\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqrt
[1 - I*Csch[x]])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int \sqrt{3 - 3\,i\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((3-3*I*csch(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3i\,\mathrm{csch}\,(x) + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

Fricas [B] time = 2.11173, size = 582, normalized size = 25.3

$$\frac{1}{2}\,\sqrt{3}\log\left(\frac{\sqrt{\frac{3\,e^{(2\,x)}-6i\,e^{x}-3}{e^{(2\,x)}-1}}\left(-\left(4i-1\right)\,e^{(3\,x)}+\left(i+4\right)\,e^{(2\,x)}+\left(4i-1\right)\,e^{x}-i-4\right)-\left(4i-1\right)\,\sqrt{3}e^{(3\,x)}+\left(i+4\right)\,\sqrt{3}}{\left(24i+10\right)\,e^{(2\,x)}-\left(10i-24\right)\,e^{x}}\right)-\frac{1}{2}\,\sqrt{3}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((sqrt((3*e^(2*x) - 6*I*e^x - 3)/(e^(2*x) - 1))*(-(4*I - 1)*
e^(3*x) + (I + 4)*e^(2*x) + (4*I - 1)*e^x - I - 4) - (4*I - 1)*sqrt(3)*e^(3
*x) + (I + 4)*sqrt(3))/((24*I + 10)*e^(2*x) - (10*I - 24)*e^x)) - 1/2*sqrt(
3)*log((sqrt((3*e^(2*x) - 6*I*e^x - 3)/(e^(2*x) - 1))*(-(4*I - 1)*e^(3*x) +
 (I + 4)*e^(2*x) + (4*I - 1)*e^x - I - 4) + (4*I - 1)*sqrt(3)*e^(3*x) - (I
+ 4)*sqrt(3))/((24*I + 10)*e^(2*x) - (10*I - 24)*e^x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{-i\,\mathrm{csch}\,(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3i\,\mathrm{csch}\,(x) + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

3.60 $\int \sqrt{-3+3i\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Rubi [A] time = 0.0159079, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 207}

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx &= -\left(6i\operatorname{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{-3+3i\operatorname{csch}(x)}}\right)\right)\\ &= -2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.657496, size = 46, normalized size = 2.

$$-\frac{2\sqrt{3}\coth(x)\tanh^{-1}\left(\sqrt{1+i\operatorname{csch}(x)}\right)}{\sqrt{-1+i\operatorname{csch}(x)}\sqrt{1+i\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqr
t[1 + I*Csch[x]])
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \sqrt{-3+3\,i\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3+3*I*csch(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3i\,\operatorname{csch}\left(x\right)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

Fricas [B] time = 2.16028, size = 590, normalized size = 25.65

$$\frac{1}{2}i\sqrt{3}\log\left(\frac{\sqrt{-\frac{3\,e^{(2\,x)}-6i\,e^{x}-3}{e^{(2\,x)}-1}}\left(-(4i-1)\,e^{(3\,x)}+(i+4)\,e^{(2\,x)}+(4i-1)\,e^{x}-i-4\right)+(i+4)\,\sqrt{3}e^{(3\,x)}+(4i-1)\,\sqrt{3}}{(24i+10)\,e^{(2\,x)}-(10i-24)\,e^{x}}\right)-\frac{1}{2}i\sqrt{3}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(3)*log((sqrt(-(3*e^(2*x) - 6*I*e^x - 3)/(e^(2*x) - 1))*(-(4*I -
1)*e^(3*x) + (I + 4)*e^(2*x) + (4*I - 1)*e^x - I - 4) + (I + 4)*sqrt(3)*e^(
3*x) + (4*I - 1)*sqrt(3))/((24*I + 10)*e^(2*x) - (10*I - 24)*e^x)) - 1/2*I*
sqrt(3)*log((sqrt(-(3*e^(2*x) - 6*I*e^x - 3)/(e^(2*x) - 1))*(-(4*I - 1)*e^(
3*x) + (I + 4)*e^(2*x) + (4*I - 1)*e^x - I - 4) - (I + 4)*sqrt(3)*e^(3*x) -
 (4*I - 1)*sqrt(3))/((24*I + 10)*e^(2*x) - (10*I - 24)*e^x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{i\operatorname{csch}\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3i\,\mathrm{csch}\,(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

3.61 $\int \sqrt{-3-3i\,\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\,\mathrm{csch}(x)}}\right)$$

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]
```

Rubi [A] time = 0.0169177, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 207}

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\,\mathrm{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3-3i\,\mathrm{csch}(x)}\,dx &= 6i\,\mathrm{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{-3-3i\,\mathrm{csch}(x)}}\right)\\ &= -2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\,\mathrm{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.643309, size = 46, normalized size = 2.

$$-\frac{2\sqrt{3}\coth(x)\tanh^{-1}\left(\sqrt{1-i\,\mathrm{csch}(x)}\right)}{\sqrt{-1-i\,\mathrm{csch}(x)}\sqrt{1-i\,\mathrm{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqr
t[1 - I*Csch[x]])
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \sqrt{-3-3\,i\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3-3*I*csch(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3i\,\mathrm{csch}\,(x)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

Fricas [B] time = 2.15661, size = 589, normalized size = 25.61

$$-\frac{1}{2}i\sqrt{3}\log\left(\frac{\sqrt{-\frac{3\,e^{(2\,x)}+6i\,e^{x}-3}{e^{(2\,x)}-1}}\left((i-4)\,e^{(3\,x)}+(4i+1)\,e^{(2\,x)}-(i-4)\,e^{x}-4i-1\right)+(4i+1)\,\sqrt{3}e^{(3\,x)}-(i-4)\,\sqrt{3}}{(10i+24)\,e^{(2\,x)}+(24i-10)\,e^{x}}\right)+\frac{1}{2}i\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(3)*log((sqrt(-(3*e^(2*x) + 6*I*e^x - 3)/(e^(2*x) - 1))*((I - 4)
*e^(3*x) + (4*I + 1)*e^(2*x) - (I - 4)*e^x - 4*I - 1) + (4*I + 1)*sqrt(3)*e
^(3*x) - (I - 4)*sqrt(3))/((10*I + 24)*e^(2*x) + (24*I - 10)*e^x)) + 1/2*I*
sqrt(3)*log((sqrt(-(3*e^(2*x) + 6*I*e^x - 3)/(e^(2*x) - 1))*((I - 4)*e^(3*x
) + (4*I + 1)*e^(2*x) - (I - 4)*e^x - 4*I - 1) - (4*I + 1)*sqrt(3)*e^(3*x)
+ (I - 4)*sqrt(3))/((10*I + 24)*e^(2*x) + (24*I - 10)*e^x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{-i\,\mathrm{csch}\,(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3i \operatorname{csch}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

3.62 $\int \frac{\sinh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{15ix}{8}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{5}{4}i\sinh^3(x)\cosh(x)+\frac{15}{8}i\sinh(x)\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\text{csch}(x)+i}$$

```
[Out] ((-15*I)/8)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 + ((15*I)/8)*Cosh[x]*Sinh[x] -
((5*I)/4)*Cosh[x]*Sinh[x]^3 - (Cosh[x]*Sinh[x]^3)/(I + Csch[x])
```

Rubi [A] time = 0.0705064, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2633}

$$-\frac{15ix}{8}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{5}{4}i\sinh^3(x)\cosh(x)+\frac{15}{8}i\sinh(x)\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\text{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-15*I)/8)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 + ((15*I)/8)*Cosh[x]*Sinh[x] -
((5*I)/4)*Cosh[x]*Sinh[x]^3 - (Cosh[x]*Sinh[x]^3)/(I + Csch[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \int (-5i+4\operatorname{csch}(x))\sinh^4(x)\,dx \\ &= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - 5i\int \sinh^4(x)\,dx + 4\int \sinh^3(x)\,dx \\ &= -\frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \frac{15}{4}i\int \sinh^2(x)\,dx - 4\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) \\ &= -4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - \frac{15}{8}i\int 1 \\ &= -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [A] time = 0.140282, size = 63, normalized size = 1.09

$$\frac{1}{96}\left(-180ix + 48i\sinh(2x) - 3i\sinh(4x) - 168\cosh(x) + 8\cosh(3x) + \frac{192\sinh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right) - i\cosh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-180*I)*x - 168*Cosh[x] + 8*Cosh[3*x] + (192*Sinh[x/2])/((-I)*Cosh[x/2] +
 Sinh[x/2]) + (48*I)*Sinh[2*x] - (3*I)*Sinh[4*x])/96
```

Maple [B] time = 0.048, size = 182, normalized size = 3.1

$$-\frac{15i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{15i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + \frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{5i}{8}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+csch(x)),x)
```

```
[Out] -15/8*I*ln(tanh(1/2*x)+1)+15/8*I*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^3+5/
8*I/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)+1)^2+1/4*I/(tanh(1/2*x)+1)^4-3/2/(ta
nh(1/2*x)+1)+2*I/(tanh(1/2*x)-I)+7/8*I/(tanh(1/2*x)+1)-1/2*I/(tanh(1/2*x)-1
)^3-5/8*I/(tanh(1/2*x)+1)^2+3/2/(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)+1)^3-1/2
/(tanh(1/2*x)-1)^2+7/8*I/(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)-1)^3-1/4*I/(tanh(
1/2*x)-1)^4
```

Maxima [A] time = 1.14486, size = 96, normalized size = 1.66

$$-\frac{15}{8}ix - \frac{-5ie^{(-x)} + 40e^{(-2x)} + 120ie^{(-3x)} + 552e^{(-4x)} - 3}{16\left(12ie^{(-4x)} + 12e^{(-5x)}\right)} - \frac{7}{8}e^{(-x)} - \frac{1}{4}ie^{(-2x)} + \frac{1}{24}e^{(-3x)} + \frac{1}{64}ie^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -15/8*I*x - 1/16*(-5*I*e^(-x) + 40*e^(-2*x) + 120*I*e^(-3*x) + 552*e^(-4*x)
 - 3)/(12*I*e^(-4*x) + 12*e^(-5*x)) - 7/8*e^(-x) - 1/4*I*e^(-2*x) + 1/24*e^
```

```
(-3*x) + 1/64*I*e^(-4*x)
```

Fricas [A] time = 1.65756, size = 244, normalized size = 4.21

$$\frac{(-360 i x + 168 i) e^{(5 x)} - 24 (15 x + 23) e^{(4 x)} - 3 i e^{(9 x)} + 5 e^{(8 x)} + 40 i e^{(7 x)} - 120 e^{(6 x)} + 120 i e^{(3 x)} - 40 e^{(2 x)} - 5 i e^{x} + 3}{192 \left(e^{(5 x)} - i e^{(4 x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*((-360*I*x + 168*I)*e^(5*x) - 24*(15*x + 23)*e^(4*x) - 3*I*e^(9*x) +
5*e^(8*x) + 40*I*e^(7*x) - 120*e^(6*x) + 120*I*e^(3*x) - 40*e^(2*x) - 5*I*e
^x + 3)/(e^(5*x) - I*e^(4*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18036, size = 89, normalized size = 1.53

$$-\frac{\left(552 e^{(4 x)} - 120 i e^{(3 x)} + 40 e^{(2 x)} + 5 i e^{x} - 3\right) e^{(-4 x)}}{192 (e^{x} - i)} - \frac{1}{64} i e^{(4 x)} + \frac{1}{24} e^{(3 x)} + \frac{1}{4} i e^{(2 x)} - \frac{7}{8} e^{x} - \frac{15}{8} i \log(i e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/192*(552*e^(4*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e^x - 3)*e^(-4*x)/(e
^x - I) - 1/64*I*e^(4*x) + 1/24*e^(3*x) + 1/4*I*e^(2*x) - 7/8*e^x - 15/8*I*
log(I*e^x)
```

3.63 $\int \frac{\sinh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{3x}{2}-\frac{4}{3}i\cosh^3(x)+4i\cosh(x)+\frac{3}{2}\sinh(x)\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[Out] (-3*x)/2 + (4*I)*Cosh[x] - ((4*I)/3)*Cosh[x]^3 + (3*Cosh[x]*Sinh[x])/2 - (C
osh[x]*Sinh[x]^2)/(I + Csch[x])
```

Rubi [A] time = 0.0711676, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{3x}{2}-\frac{4}{3}i\cosh^3(x)+4i\cosh(x)+\frac{3}{2}\sinh(x)\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] (-3*x)/2 + (4*I)*Cosh[x] - ((4*I)/3)*Cosh[x]^3 + (3*Cosh[x]*Sinh[x])/2 - (C
osh[x]*Sinh[x]^2)/(I + Csch[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + \int (-4i+3\operatorname{csch}(x))\sinh^3(x)\,dx$$
$$= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} - 4i\int \sinh^3(x)\,dx + 3\int \sinh^2(x)\,dx$$
$$= \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + 4i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) - \frac{3\int 1\,dx}{2}$$
$$= -\frac{3x}{2} + 4i\cosh(x) - \frac{4}{3}i\cosh^3(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] time = 0.127716, size = 56, normalized size = 1.22

$$\frac{1}{12}\left(21i\cosh(x) - i\cosh(3x) + 3\left(-6x + \sinh(2x) + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((21*I)*Cosh[x] - I*Cosh[3*x] + 3*(-6*x + (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh
[x/2]) + Sinh[2*x]))/12
```

Maple [B] time = 0.046, size = 137, normalized size = 3.

$$-\frac{i}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{3i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+csch(x)),x)
```

```
[Out] -1/3*I/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)+3/2*I/(tanh(1/2*x)+1)-1/2/(tan
h(1/2*x)+1)^2+1/2*I/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+2/(tanh(1/2*x)-
I)+1/3*I/(tanh(1/2*x)-1)^3+1/2/(tanh(1/2*x)-1)^2+1/2*I/(tanh(1/2*x)-1)^2+1/
2/(tanh(1/2*x)-1)-3/2*I/(tanh(1/2*x)-1)+3/2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05446, size = 80, normalized size = 1.74

$$-\frac{3}{2}x + \frac{2i\,e^{(-x)} - 18\,e^{(-2x)} + 69i\,e^{(-3x)} + 1}{8\left(3i\,e^{(-3x)} + 3\,e^{(-4x)}\right)} + \frac{7}{8}i\,e^{(-x)} - \frac{1}{8}\,e^{(-2x)} - \frac{1}{24}i\,e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x + 1/8*(2*I*e^(-x) - 18*e^(-2*x) + 69*I*e^(-3*x) + 1)/(3*I*e^(-3*x) +
 3*e^(-4*x)) + 7/8*I*e^(-x) - 1/8*e^(-2*x) - 1/24*I*e^(-3*x)
```

Fricas [A] time = 1.74752, size = 194, normalized size = 4.22

$$-\frac{3\,(12\,x-7)e^{(4\,x)}-(36i\,x+69i)e^{(3\,x)}+i\,e^{(7\,x)}-2\,e^{(6\,x)}-18i\,e^{(5\,x)}-18\,e^{(2\,x)}-2i\,e^{x}+1}{24\,\left(e^{(4\,x)}-i\,e^{(3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(12*x - 7)*e^(4*x) - (36*I*x + 69*I)*e^(3*x) + I*e^(7*x) - 2*e^(6*
x) - 18*I*e^(5*x) - 18*e^(2*x) - 2*I*e^x + 1)/(e^(4*x) - I*e^(3*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19581, size = 68, normalized size = 1.48

$$-\frac{3}{2}\,x+\frac{i\left(69\,e^{(3\,x)}-18i\,e^{(2\,x)}+2\,e^{x}+i\right)e^{(-3\,x)}}{24\,(e^{x}-i)}-\frac{1}{24}i\,e^{(3\,x)}+\frac{1}{8}\,e^{(2\,x)}+\frac{7}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/24*I*(69*e^(3*x) - 18*I*e^(2*x) + 2*e^x + I)*e^(-3*x)/(e^x - I)
- 1/24*I*e^(3*x) + 1/8*e^(2*x) + 7/8*I*e^x
```

3.64 $\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=36

$$\frac{3ix}{2}+2\cosh(x)-\frac{3}{2}i\sinh(x)\cosh(x)-\frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[Out] ((3*I)/2)*x + 2*Cosh[x] - ((3*I)/2)*Cosh[x]*Sinh[x] - (Cosh[x]*Sinh[x])/(I
+ Csch[x])
```

Rubi [A] time = 0.0598771, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2638}

$$\frac{3ix}{2}+2\cosh(x)-\frac{3}{2}i\sinh(x)\cosh(x)-\frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] ((3*I)/2)*x + 2*Cosh[x] - ((3*I)/2)*Cosh[x]*Sinh[x] - (Cosh[x]*Sinh[x])/(I
+ Csch[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \int (-3i+2\operatorname{csch}(x))\sinh^2(x)\,dx$$
$$= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} - 3i\int \sinh^2(x)\,dx + 2\int \sinh(x)\,dx$$
$$= 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \frac{3}{2}i\int 1\,dx$$
$$= \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] time = 0.124247, size = 46, normalized size = 1.28

$$\cosh(x) + \frac{1}{4}i\left(6x - \sinh(2x) - \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] Cosh[x] + (I/4)*(6*x - (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2]) - Sinh[2*x])
```

Maple [B] time = 0.042, size = 96, normalized size = 2.7

$$\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{3i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - 2i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} - \frac{3i}{2}\ln\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+csch(x)),x)
```

```
[Out] 1/2*I/(tanh(1/2*x)+1)^2+3/2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)-1/2*I/(ta
nh(1/2*x)+1)-2*I/(tanh(1/2*x)-I)-3/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)
-1)^2-1/(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.00302, size = 63, normalized size = 1.75

$$\frac{3}{2}i\,x + \frac{3i\,e^{(-x)} + 20\,e^{(-2\,x)} + 1}{4\left(2i\,e^{(-2\,x)} + 2\,e^{(-3\,x)}\right)} + \frac{1}{2}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x + 1/4*(3*I*e^(-x) + 20*e^(-2*x) + 1)/(2*I*e^(-2*x) + 2*e^(-3*x)) +
1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Fricas [B] time = 1.59214, size = 151, normalized size = 4.19

$$\frac{(12i\,x - 4i)e^{(3\,x)} + 4\,(3\,x + 5)e^{(2\,x)} - i\,e^{(5\,x)} + 3\,e^{(4\,x)} - 3i\,e^{x} + 1}{8\left(e^{(3\,x)} - i\,e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((12*I*x - 4*I)*e^(3*x) + 4*(3*x + 5)*e^(2*x) - I*e^(5*x) + 3*e^(4*x) -
 3*I*e^x + 1)/(e^(3*x) - I*e^(2*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17289, size = 54, normalized size = 1.5

$$\frac{3}{2}i\,x + \frac{\left(-20i\,e^{(2\,x)} - 3\,e^{x} - i\right)e^{(-2\,x)}}{8\,(-i\,e^{x} - 1)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x + 1/8*(-20*I*e^(2*x) - 3*e^x - I)*e^(-2*x)/(-I*e^x - 1) - 1/8*I*e^(
2*x) + 1/2*e^x
```

3.65 $\int \frac{\sinh(x)}{i+\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=20

$$x - 2i\cosh(x) - \frac{\cosh(x)}{\mathrm{csch}(x) + i}$$

```
[Out] x - (2*I)*Cosh[x] - Cosh[x]/(I + Csch[x])
```

Rubi [A] time = 0.0472941, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3819, 3787, 2638, 8}

$$x - 2i\cosh(x) - \frac{\cosh(x)}{\mathrm{csch}(x) + i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Csch[x]),x]
```

```
[Out] x - (2*I)*Cosh[x] - Cosh[x]/(I + Csch[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{i+\mathrm{csch}(x)}\,dx &= -\frac{\cosh(x)}{i+\mathrm{csch}(x)} + \int (-2i+\mathrm{csch}(x))\sinh(x)\,dx \\
&= -\frac{\cosh(x)}{i+\mathrm{csch}(x)} - 2i\int \sinh(x)\,dx + \int 1\,dx \\
&= x - 2i\cosh(x) - \frac{\cosh(x)}{i+\mathrm{csch}(x)}
\end{aligned}$$

Mathematica [A] time = 0.0491476, size = 35, normalized size = 1.75

$$x - i\cosh(x) - \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Csch[x]),x]
```

```
[Out] x - I*Cosh[x] - (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [B] time = 0.039, size = 51, normalized size = 2.6

$$-i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\left(\tanh(x/2)-i\right)^{-1} + i\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+csch(x)),x)
```

```
[Out] -I/(tanh(1/2*x)+1)+ln(tanh(1/2*x)+1)-2/(tanh(1/2*x)-I)+I/(tanh(1/2*x)-1)-ln
(tanh(1/2*x)-1)
```

Maxima [A] time = 0.98078, size = 42, normalized size = 2.1

$$x - \frac{5i\,e^{(-x)} - 1}{2\left(i\,e^{(-x)} + e^{(-2\,x)}\right)} - \frac{1}{2}i\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] x - 1/2*(5*I*e^(-x) - 1)/(I*e^(-x) + e^(-2*x)) - 1/2*I*e^(-x)
```

Fricas [B] time = 1.65848, size = 108, normalized size = 5.4

$$\frac{(2\,x-1)e^{(2\,x)} + (-2i\,x - 5i)e^{x} - i\,e^{(3\,x)} - 1}{2\,e^{(2\,x)} - 2i\,e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((2*x - 1)*e^(2*x) + (-2*I*x - 5*I)*e^x - I*e^(3*x) - 1)/(2*e^(2*x) - 2*I*e
^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)/(csch(x) + I), x)
```

Giac [A] time = 1.21594, size = 35, normalized size = 1.75

$$x + \frac{(5 e^{x} - i)e^{(-x)}}{2 (i e^{x} + 1)} - \frac{1}{2} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] x + 1/2*(5*e^x - I)*e^(-x)/(I*e^x + 1) - 1/2*I*e^x
```

3.66 $\int \frac{\mathbf{csch}(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

```
[Out] (I*Coth[x])/(I + Csch[x])
```

Rubi [A] time = 0.0205694, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3794}

$$\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Csch[x]),x]
```

```
[Out] (I*Coth[x])/(I + Csch[x])
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx=\frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] time = 0.0187179, size = 27, normalized size = 1.93

$$\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Csch[x]),x]
```

```
[Out] (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] time = 0.02, size = 12, normalized size = 0.9

$$2\left(\tanh(x/2)-i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+csch(x)),x)
```

```
[Out] 2/(tanh(1/2*x)-I)
```

Maxima [A] time = 1.02864, size = 16, normalized size = 1.14

$$-\frac{2}{i\,e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2/(I*e^(-x) - 1)
```

Fricas [A] time = 1.35672, size = 20, normalized size = 1.43

$$\frac{2i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 2*I/(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)/(csch(x) + I), x)
```

Giac [A] time = 1.15148, size = 11, normalized size = 0.79

$$\frac{2i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*I/(e^x - I)
```

3.67 $\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=17

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\operatorname{csch}(x)+i}$$

```
[Out] -ArcTanh[Cosh[x]] + Coth[x]/(I + Csch[x])
```

Rubi [A] time = 0.0539062, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3789, 3770, 3794}

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Coth[x]/(I + Csch[x])
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx &= -\left(i\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx\right)+\int \operatorname{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] time = 0.0325784, size = 37, normalized size = 2.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] Log[Tanh[x/2]] - ((2*I)*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] time = 0.017, size = 19, normalized size = 1.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right) - 2\,i\left(\tanh\left(\frac{x}{2}\right) - i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+csch(x)),x)
```

```
[Out] ln(tanh(1/2*x))-2*I/(tanh(1/2*x)-I)
```

Maxima [A] time = 1.02085, size = 39, normalized size = 2.29

$$\frac{4}{2\,e^{(-x)} + 2i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 4/(2*e^(-x) + 2*I) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] time = 1.58911, size = 89, normalized size = 5.24

$$-\frac{(e^x - i)\log(e^x + 1) - (e^x - i)\log(e^x - 1) - 2}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -((e^x - I)*log(e^x + 1) - (e^x - I)*log(e^x - 1) - 2)/(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(csch(x) + I), x)
```

Giac [A] time = 1.18024, size = 30, normalized size = 1.76

$$\frac{2}{e^x - i} - \log\left(e^x + 1\right) + \log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2/(e^x - I) - log(e^x + 1) + log(abs(e^x - 1))
```

3.68 $\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=26

$$-\coth(x)+i\tanh^{-1}(\cosh(x))-\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

```
[Out] I*ArcTanh[Cosh[x]] - Coth[x] - (I*Coth[x])/(I + Csch[x])
```

Rubi [A] time = 0.0836923, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3790, 3789, 3770, 3794}

$$-\coth(x)+i\tanh^{-1}(\cosh(x))-\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] - Coth[x] - (I*Coth[x])/(I + Csch[x])
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx &= -\coth(x)-i\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx\\
&= -\coth(x)-i\int \operatorname{csch}(x)\,dx-\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx\\
&= i\tanh^{-1}(\cosh(x))-\coth(x)-\frac{i\coth(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [B] time = 0.108823, size = 70, normalized size = 2.69

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)-\frac{1}{2}\coth\left(\frac{x}{2}\right)-i\log\left(\sinh\left(\frac{x}{2}\right)\right)+i\log\left(\cosh\left(\frac{x}{2}\right)\right)-\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] -Coth[x/2]/2 + I*Log[Cosh[x/2]] - I*Log[Sinh[x/2]] - (2*Sinh[x/2])/(Cosh[x/
2] + I*Sinh[x/2]) - Tanh[x/2]/2
```

Maple [A] time = 0.018, size = 35, normalized size = 1.4

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)-2\left(\tanh\left(x/2\right)-i\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+csch(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)-2/(tanh(1/2*x)-I)-1/2/tanh(1/2*x)-I*ln(tanh(1/2*x))
```

Maxima [B] time = 1.02521, size = 74, normalized size = 2.85

$$-\frac{8\left(e^{(-x)}-i\,e^{(-2\,x)}+2i\right)}{4\,e^{(-x)}-4i\,e^{(-2\,x)}-4\,e^{(-3\,x)}+4i}+i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -8*(e^(-x) - I*e^(-2*x) + 2*I)/(4*e^(-x) - 4*I*e^(-2*x) - 4*e^(-3*x) + 4*I)
 + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] time = 1.64018, size = 216, normalized size = 8.31

$$\frac{\left(i\,e^{(3\,x)}+e^{(2\,x)}-i\,e^{x}-1\right)\log\left(e^{x}+1\right)+\left(-i\,e^{(3\,x)}-e^{(2\,x)}+i\,e^{x}+1\right)\log\left(e^{x}-1\right)-2i\,e^{(2\,x)}-2\,e^{x}+4i}{e^{(3\,x)}-i\,e^{(2\,x)}-e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*x) + e^(2*x) - I*e^x - 1)*log(e^x + 1) + (-I*e^(3*x) - e^(2*x) + I
*e^x + 1)*log(e^x - 1) - 2*I*e^(2*x) - 2*e^x + 4*I)/(e^(3*x) - I*e^(2*x) -
e^x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(csch(x) + I), x)
```

Giac [B] time = 1.21046, size = 62, normalized size = 2.38

$$\frac{2\left(e^{(2x)} - i e^{x} - 2\right)}{i e^{(3x)} + e^{(2x)} - i e^{x} - 1} + i \log\left(e^{x} + 1\right) - i \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) - I*e^x - 2)/(I*e^(3*x) + e^(2*x) - I*e^x - 1) + I*log(e^x + 1)
- I*log(abs(e^x - 1))
```

3.69 $\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=37

$$2i\coth(x) + \frac{3}{2}\tanh^{-1}(\cosh(x)) + \frac{\coth(x)\operatorname{csch}^2(x)}{\operatorname{csch}(x)+i} - \frac{3}{2}\coth(x)\operatorname{csch}(x)$$

```
[Out] (3*ArcTanh[Cosh[x]])/2 + (2*I)*Coth[x] - (3*Coth[x]*Csch[x])/2 + (Coth[x]*C
sch[x]^2)/(I + Csch[x])
```

Rubi [A] time = 0.064237, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$2i\coth(x) + \frac{3}{2}\tanh^{-1}(\cosh(x)) + \frac{\coth(x)\operatorname{csch}^2(x)}{\operatorname{csch}(x)+i} - \frac{3}{2}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 + (2*I)*Coth[x] - (3*Coth[x]*Csch[x])/2 + (Coth[x]*C
sch[x]^2)/(I + Csch[x])
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \int (2i-3\operatorname{csch}(x))\operatorname{csch}^2(x)\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - 2i\int \operatorname{csch}^2(x)\,dx + 3\int \operatorname{csch}^3(x)\,dx\\ &= -\frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \frac{3}{2}\int \operatorname{csch}(x)\,dx - 2\,\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\\ &= \frac{3}{2}\tanh^{-1}(\cosh(x)) + 2i\coth(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] time = 0.307058, size = 81, normalized size = 2.19

$$\frac{1}{8}\left(4i\tanh\left(\frac{x}{2}\right) + 4i\coth\left(\frac{x}{2}\right) - \operatorname{csch}^2\left(\frac{x}{2}\right) - \operatorname{sech}^2\left(\frac{x}{2}\right) - 12\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{16\sinh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right) - i\cosh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] ((4*I)*Coth[x/2] - Csch[x/2]^2 - 12*Log[Tanh[x/2]] - Sech[x/2]^2 + (16*Sinh
[x/2])/((-I)*Cosh[x/2] + Sinh[x/2]) + (4*I)*Tanh[x/2])/8
```

Maple [A] time = 0.025, size = 53, normalized size = 1.4

$$\frac{i}{2}\tanh\left(\frac{x}{2}\right) + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 2i\left(\tanh\left(\frac{x}{2}\right) - i\right)^{-1} - \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \frac{3}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+csch(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2+2*I/(tanh(1/2*x)-I)-1/8/tanh(1/2*x)^2+1
/2*I/tanh(1/2*x)-3/2*ln(tanh(1/2*x))
```

Maxima [B] time = 1.03503, size = 109, normalized size = 2.95

$$-\frac{16\left(-i e^{(-x)} - 5 e^{(-2x)} + 3i e^{(-3x)} + 3 e^{(-4x)} + 4\right)}{16 e^{(-x)} - 32i e^{(-2x)} - 32 e^{(-3x)} + 16i e^{(-4x)} + 16 e^{(-5x)} + 16i} + \frac{3}{2}\log\left(e^{(-x)} + 1\right) - \frac{3}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -16*(-I*e^(-x) - 5*e^(-2*x) + 3*I*e^(-3*x) + 3*e^(-4*x) + 4)/(16*e^(-x) - 3
2*I*e^(-2*x) - 32*e^(-3*x) + 16*I*e^(-4*x) + 16*e^(-5*x) + 16*I) + 3/2*log(
e^(-x) + 1) - 3/2*log(e^(-x) - 1)
```

Fricas [B] time = 1.60965, size = 381, normalized size = 10.3

$$\frac{\left(3\,e^{(5\,x)} - 3i\,e^{(4\,x)} - 6\,e^{(3\,x)} + 6i\,e^{(2\,x)} + 3\,e^{x} - 3i\right)\log\left(e^{x} + 1\right) - \left(3\,e^{(5\,x)} - 3i\,e^{(4\,x)} - 6\,e^{(3\,x)} + 6i\,e^{(2\,x)} + 3\,e^{x} - 3i\right)\log\left(e^{x} - \right.}{2\,e^{(5\,x)} - 2i\,e^{(4\,x)} - 4\,e^{(3\,x)} + 4i\,e^{(2\,x)} + 2\,e^{x} - 2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((3*e^(5*x) - 3*I*e^(4*x) - 6*e^(3*x) + 6*I*e^(2*x) + 3*e^x - 3*I)*log(e^x
+ 1) - (3*e^(5*x) - 3*I*e^(4*x) - 6*e^(3*x) + 6*I*e^(2*x) + 3*e^x - 3*I)*lo
g(e^x - 1) - 6*e^(4*x) + 6*I*e^(3*x) + 10*e^(2*x) - 2*I*e^x - 8)/(2*e^(5*x)
 - 2*I*e^(4*x) - 4*e^(3*x) + 4*I*e^(2*x) + 2*e^x - 2*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(csch(x) + I), x)
```

Giac [A] time = 1.16423, size = 68, normalized size = 1.84

$$-\frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(e^{(2\,x)} - 1\right)^{2}} - \frac{2i}{i\,e^{x} + 1} + \frac{3}{2}\log\left(e^{x} + 1\right) - \frac{3}{2}\log\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 2*I/(I*e^x + 1) + 3/
2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

3.70 $\int (a+b\,\mathrm{csch}(c+dx))^4\,dx$

Optimal. Leaf size=109

$$-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}+a^4x-\frac{4ab^3\coth(c+dx)\mathrm{csch}(c+dx)}{3d}-\frac{b^2\coth$$

```
[Out] a^4*x - (2*a*b*(2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^2*(17*a^2 - 2*b
^2)*Coth[c + d*x])/(3*d) - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/(3*d) - (b
^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.127954, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}+a^4x-\frac{4ab^3\coth(c+dx)\mathrm{csch}(c+dx)}{3d}-\frac{b^2\coth$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^4,x]
```


```
[Out] a^4*x - (2*a*b*(2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^2*(17*a^2 - 2*b
^2)*Coth[c + d*x])/(3*d) - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/(3*d) - (b
^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/(3*d)
```


Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx = -\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\operatorname{csch}(c+dx))\left(3a^3+b\left(9a^2-2b^2\right)\operatorname{cs}\right.$$
$$= -\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+\frac{1}{6}\int \left(6a^4+1\right.$$
$$= a^4x-\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+\frac{1}{3}\left(b^2\left(17\right.\right.$$
$$= a^4x-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-\frac{b^2\coth(}{}$$
$$= a^4x-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{4ab^3\coth(}{}$$

Mathematica [B] time = 6.22972, size = 508, normalized size = 4.66

$$\frac{\sinh^4(c+dx)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)\left(b^4\cosh\left(\frac{1}{2}(c+dx)\right)-9a^2b^2\cosh\left(\frac{1}{2}(c+dx)\right)\right)(a+b\operatorname{csch}(c+dx))^4}{3d(a\sinh(c+dx)+b)^4}+\frac{\sinh^4(c+dx)\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] (a^4*(c + d*x)*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c +
d*x])^4) + ((-9*a^2*b^2*Cosh[(c + d*x)/2] + b^4*Cosh[(c + d*x)/2])*Csch[(c
+ d*x)/2]*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x
])^4) - (a*b^3*Csch[(c + d*x)/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)
/(2*d*(b + a*Sinh[c + d*x])^4) - (b^4*Coth[(c + d*x)/2]*Csch[(c + d*x)/2]^2
*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(24*d*(b + a*Sinh[c + d*x])^4) +
(2*a*b*(2*a^2 - b^2)*(a + b*Csch[c + d*x])^4*Log[Tanh[(c + d*x)/2]]*Sinh[c
+ d*x]^4)/(d*(b + a*Sinh[c + d*x])^4) - (a*b^3*(a + b*Csch[c + d*x])^4*Sech
[(c + d*x)/2]^2*Sinh[c + d*x]^4)/(2*d*(b + a*Sinh[c + d*x])^4) + ((a + b*Cs
ch[c + d*x])^4*Sech[(c + d*x)/2]*(-9*a^2*b^2*Sinh[(c + d*x)/2] + b^4*Sinh[(
c + d*x)/2])*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x])^4) + (b^4*(a + b*C
sch[c + d*x])^4*Sech[(c + d*x)/2]^2*Sinh[c + d*x]^4*Tanh[(c + d*x)/2])/(24*
d*(b + a*Sinh[c + d*x])^4)
```

Maple [A] time = 0.039, size = 92, normalized size = 0.8

$$\frac{1}{d}\left(a^4(dx+c)-8a^3b\operatorname{Artanh}\left(e^{dx+c}\right)-6a^2b^2\coth(dx+c)+4ab^3\left(-1/2\operatorname{csch}(dx+c)\coth(dx+c)+\operatorname{Artanh}\left(e^{dx+c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)-8*a^3*b*arctanh(exp(d*x+c))-6*a^2*b^2*coth(d*x+c)+4*a*b^3*
(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^4*(2/3-1/3*csch(d*x+c)
^2)*coth(d*x+c))
```

Maxima [B] time = 1.02551, size = 316, normalized size = 2.9

$$a^4x + 2\,ab^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right) + \frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*a*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1))) + 4*a^3*b*log(tanh(1/2*d*x + 1/2*c))/d + 12*a^2*
b^2/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.73342, size = 3453, normalized size = 31.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 - 12*a*b^3*cosh(
d*x + c)^5 - 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) - 2*a*b^3)*sinh(d*x + c
)^5 + 12*a*b^3*cosh(d*x + c) - 9*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 - 3*a^4*d*x - 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 + 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 -
10*a*b^3*cosh(d*x + c)^2 - 3*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 - 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x + 24*a^2*b^2 - 4*b^
4 - 18*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((2*a^3*b
 - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)
^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^
4 - 3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^4 - 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b
- a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)
^2 + 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b
- a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x +
 c)^5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x +
c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 6*((2*a^3*b - a
*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 -
3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 +
 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x + c)^
5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(3*a^4*d*x*cosh(d
*x + c)^5 - 10*a*b^3*cosh(d*x + c)^4 + 2*a*b^3 - 6*(a^4*d*x + 4*a^2*b^2)*co
sh(d*x + c)^3 + (3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c
)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
```

```
 c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{csch}(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**4, x)
```

Giac [A] time = 1.13597, size = 236, normalized size = 2.17

$$\frac{(dx+c)a^4}{d} - \frac{2\left(2\,a^3b - ab^3\right)\log\left(e^{(dx+c)}+1\right)}{d} + \frac{2\left(2\,a^3b - ab^3\right)\log\left(\left|e^{(dx+c)}-1\right|\right)}{d} - \frac{4\left(3\,ab^3e^{(5\,dx+5\,c)} + 9\,a^2b^2e^{(4\,dx+4\,c)} -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^4/d - 2*(2*a^3*b - a*b^3)*log(e^(d*x + c) + 1)/d + 2*(2*a^3*b -
 a*b^3)*log(abs(e^(d*x + c) - 1))/d - 4/3*(3*a*b^3*e^(5*d*x + 5*c) + 9*a^2*
b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) -
3*a*b^3*e^(d*x + c) + 9*a^2*b^2 - b^4)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.71 $\int (a+b\,\mathbf{csch}(c+dx))^3\,dx$

Optimal. Leaf size=75

$$-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}+a^3x-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\,\text{csch}(c+dx))}{2d}$$

```
[Out] a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/(2*d) - (5*a*b^2*Coth[c +
d*x])/(2*d) - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Rubi [A] time = 0.051388, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}+a^3x-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\,\text{csch}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/(2*d) - (5*a*b^2*Coth[c +
d*x])/(2*d) - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\operatorname{csch}(c+dx))^3\,dx = -\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} + \frac{1}{2}\int \left(2a^3 + b\left(6a^2-b^2\right)\operatorname{csch}(c+dx) + 5ab^2\operatorname{csc}\right.$$

$$= a^3x - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} + \frac{1}{2}\left(5ab^2\right)\int \operatorname{csch}^2(c+dx)\,dx + \frac{1}{2}\left(b\left(6a^2-\right.\right.$$

$$= a^3x - \frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} - \frac{\left(5iab^2\right)}{}$$

$$= a^3x - \frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{5ab^2\coth(c+dx)}{2d} - \frac{b^2\coth(c+dx)(a+bcs}{2d}$$

Mathematica [A] time = 0.880053, size = 118, normalized size = 1.57

$$-\frac{-24a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 8a^3c - 8a^3dx + 12ab^2\tanh\left(\frac{1}{2}(c+dx)\right) + 12ab^2\coth\left(\frac{1}{2}(c+dx)\right) + b^3\operatorname{csch}^2\left(\frac{1}{2}(c+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] -(-8*a^3*c - 8*a^3*d*x + 12*a*b^2*Coth[(c + d*x)/2] + b^3*Csch[(c + d*x)/2]
^2 - 24*a^2*b*Log[Tanh[(c + d*x)/2]] + 4*b^3*Log[Tanh[(c + d*x)/2]] + b^3*S
ech[(c + d*x)/2]^2 + 12*a*b^2*Tanh[(c + d*x)/2])/(8*d)
```

Maple [A] time = 0.028, size = 66, normalized size = 0.9

$$\frac{1}{d}\left(a^3(dx+c) - 6a^2b\operatorname{Artanh}\left(e^{dx+c}\right) - 3ab^2\coth(dx+c) + b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{Artanh}\left(e^{dx+c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))-3*a*b^2*coth(d*x+c)+b^3*(-1/2*
csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c))))
```

Maxima [A] time = 0.994928, size = 184, normalized size = 2.45

$$a^3x + \frac{1}{2}b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right) + \frac{3\,a^2b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/2*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 3*a^2*b*log(tanh(1/2*d*x + 1/2*c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) -
 1))
```

Fricas [B] time = 1.66081, size = 1844, normalized size = 24.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*d*x*cosh(d*x + c)^4 + 2*a^3*d*x*sinh(d*x + c)^4 - 2*b^3*cosh(d*x
 + c)^3 + 2*a^3*d*x - 2*b^3*cosh(d*x + c) + 2*(4*a^3*d*x*cosh(d*x + c) - b^
3)*sinh(d*x + c)^3 + 12*a*b^2 - 4*(a^3*d*x + 3*a*b^2)*cosh(d*x + c)^2 + 2*(
6*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x - 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh
(d*x + c)^2 - ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2
*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x + c)*si
nh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2*(6*a^2*
b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*
(4*a^3*d*x*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c)^2 - b^3 - 4*(a^3*d*x + 3*a
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh
(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{csch}\,(c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**3, x)
```

Giac [A] time = 1.18527, size = 174, normalized size = 2.32

$$\frac{(dx+c)a^3}{d} - \frac{\left(6\,a^2b - b^3\right)\log\left(e^{(dx+c)} + 1\right)}{2\,d} + \frac{\left(6\,a^2b - b^3\right)\log\left(\left|e^{(dx+c)} - 1\right|\right)}{2\,d} - \frac{b^3e^{(3\,dx+3\,c)} + 6\,ab^2e^{(2\,dx+2\,c)} + b^3e^{(dx+c)} - 6\,al}{d\left(e^{(2\,dx+2\,c)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^3/d - 1/2*(6*a^2*b - b^3)*log(e^(d*x + c) + 1)/d + 1/2*(6*a^2*b
 - b^3)*log(abs(e^(d*x + c) - 1))/d - (b^3*e^(3*d*x + 3*c) + 6*a*b^2*e^(2*d
*x + 2*c) + b^3*e^(d*x + c) - 6*a*b^2)/(d*(e^(2*d*x + 2*c) - 1)^2)
```

3.72 $\int (a+b\,\mathrm{csch}(c+dx))^2\,dx$

Optimal. Leaf size=34

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[Out] a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Rubi [A] time = 0.0311796, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\mathrm{csch}(c+dx))^2\,dx&=a^2x+(2ab)\int \mathrm{csch}(c+dx)\,dx+b^2\int \mathrm{csch}^2(c+dx)\,dx\\&=a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{\left(ib^2\right)\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(c+dx)\right)}{d}\\&=a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.218753, size = 61, normalized size = 1.79

$$-\frac{-2a\left(ac+adx+2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+b^2\tanh\left(\frac{1}{2}(c+dx)\right)+b^2\coth\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] -(b^2*Coth[(c + d*x)/2] - 2*a*(a*c + a*d*x + 2*b*Log[Tanh[(c + d*x)/2]]) +
b^2*Tanh[(c + d*x)/2])/(2*d)
```

Maple [A] time = 0.009, size = 37, normalized size = 1.1

$$\frac{a^2(dx+c) - 4ab\mathrm{Artanh}\left(e^{dx+c}\right) - b^2\coth(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)-4*a*b*arctanh(exp(d*x+c))-b^2*coth(d*x+c))
```

Maxima [A] time = 1.04168, size = 59, normalized size = 1.74

$$a^2x + \frac{2ab\log\left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d} + \frac{2b^2}{d\left(e^{(-2dx-2c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(tanh(1/2*d*x + 1/2*c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) - 1
))
```

Fricas [B] time = 1.65717, size = 603, normalized size = 17.74

$$\frac{a^2dx\cosh(dx+c)^2 + 2a^2dx\cosh(dx+c)\sinh(dx+c) + a^2dx\sinh(dx+c)^2 - a^2dx - 2b^2 - 2\left(ab\cosh(dx+c)^2 + 2ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 - a^2*d*x - 2*b^2 - 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) +
 a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(d*cosh
(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{csch}(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**2, x)
```

Giac [A] time = 1.17489, size = 90, normalized size = 2.65

$$\frac{(dx+c)a^2}{d} - \frac{2\,ab\log\left(e^{(dx+c)}+1\right)}{d} + \frac{2\,ab\log\left(\left|e^{(dx+c)}-1\right|\right)}{d} - \frac{2\,b^2}{d\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^2/d - 2*a*b*log(e^(d*x + c) + 1)/d + 2*a*b*log(abs(e^(d*x + c)
- 1))/d - 2*b^2/(d*(e^(2*d*x + 2*c) - 1))
```

3.73 $\int (a+b\operatorname{csch}(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Rubi [A] time = 0.0101213, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3770}

$$ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Csch[c + d*x],x]

[Out] a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\operatorname{csch}(c+dx))\,dx &= ax+b\int \operatorname{csch}(c+dx)\,dx\\ &= ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.0139126, size = 43, normalized size = 2.53

$$ax+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Csch[c + d*x],x]

[Out] a*x - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A] time = 0.003, size = 20, normalized size = 1.2

$$ax+\frac{b}{d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*csch(d*x+c),x)
```

```
[Out] a*x+b/d*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.982444, size = 26, normalized size = 1.53

$$ax + \frac{b \log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 1.53912, size = 131, normalized size = 7.71

$$\frac{adx - b \log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) + 1\right) + b \log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{csch}\left(c + dx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x)
```

```
[Out] Integral(a + b*csch(c + d*x), x)
```

Giac [B] time = 1.15953, size = 49, normalized size = 2.88

$$ax - b\left(\frac{\log\left(e^{(dx+c)} + 1\right)}{d} - \frac{\log\left(\left|e^{(dx+c)} - 1\right|\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - b*(log(e^(d*x + c) + 1)/d - log(abs(e^(d*x + c) - 1))/d)
```

3.74 $\int \frac{1}{a+b\,\text{csch}(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x}{a}$$

```
[Out] x/a + (2*b*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2
+ b^2]*d)
```

Rubi [A] time = 0.0594489, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3783, 2660, 618, 204}

$$\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] x/a + (2*b*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2
+ b^2]*d)
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\text{csch}(c+dx)}\,dx = \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a}$$
$$= \frac{x}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$
$$= \frac{x}{a} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$
$$= \frac{x}{a} + \frac{2b\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

Mathematica [A] time = 0.103844, size = 64, normalized size = 1.19

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\sqrt{-a^2-b^2}}+\frac{c}{d}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] (c/d + x - (2*b*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/a
```

Maple [A] time = 0.015, size = 87, normalized size = 1.6

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\,\frac{b}{da\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,b\tanh\left(1/2\,dx+c/2\right)-2\,a}{\sqrt{a^2+b^2}}\right)-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*ta
nh(1/2*d*x+1/2*c)-2*a)/(a^2+b^2)^(1/2))-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60734, size = 473, normalized size = 8.76

$$\frac{\left(a^2+b^2\right)dx+\sqrt{a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,b^2+2\left(a^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\,\sqrt{a^2+b^2}(a\cosh(dx+c)+}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx+c)-a}\right.}{\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b))/
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x
+ c) + b)*sinh(d*x + c) - a)))/((a^3 + a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*csch(c + d*x)), x)
```

Giac [A] time = 1.17666, size = 115, normalized size = 2.13

$$-\frac{b\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="giac")
```

```
[Out] -b*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x + c) +
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + (d*x + c)/(a*d)
```

3.75 $\int \frac{1}{(a+b\mathbf{csch}(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2 + (2*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2
]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b
*Csch[c + d*x]))
```

Rubi [A] time = 0.157509, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 3919, 3831, 2660, 618, 204}

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (2*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2
]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b
*Csch[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = -\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))} - \frac{\int \frac{-a^2-b^2+ab\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))} - \frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))} - \frac{\left(2a^2+b^2\right)\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))} + \frac{\left(2i\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(ic$$

$$\frac{}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))} - \frac{\left(4i\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+\right.}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{x}{a^2} + \frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}$$

Mathematica [A] time = 0.3813, size = 142, normalized size = 1.41

$$\frac{\operatorname{csch}(c+dx)(a\sinh(c+dx)+b)\left(-\frac{ab^2\coth(c+dx)}{a^2+b^2} + \frac{2b\left(2a^2+b^2\right)(a+b\operatorname{csch}(c+dx))\tan^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + (c+dx)(a+b\operatorname{csch}(c+dx\right.}{a^2d(a+b\operatorname{csch}(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] (Csch[c + d*x]*(-((a*b^2*Coth[c + d*x])/(a^2 + b^2)) + (c + d*x)*(a + b*Csc
h[c + d*x]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2
 - b^2]]*(a + b*Csch[c + d*x]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh[c + d*x]))/
```

```
(a^2*d*(a + b*Csch[c + d*x])^2)
```

Maple [B] time = 0.057, size = 238, normalized size = 2.4

$$\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\,\frac{b\tanh(1/2\,dx+c/2)}{d\left((\tanh(1/2\,dx+c/2))^2b-2\,a\tanh(1/2\,dx+c/2)-b\right)\left(a^2+b^2\right)}+2\,\frac{}{da\left((\tanh(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d*b/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1
/2*d*x+1/2*c)-b)/(a^2+b^2)*tanh(1/2*d*x+1/2*c)+2/d/a*b^2/(tanh(1/2*d*x+1/2*
c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)/(a^2+b^2)-4/d*b/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*b*tanh(1/2*d*x+1/2*c)-2*a)/(a^2+b^2)^(1/2))-2/d/a^2*b^3/(a^2+b^2)^(3/
2)*arctanh(1/2*(2*b*tanh(1/2*d*x+1/2*c)-2*a)/(a^2+b^2)^(1/2))-1/d/a^2*ln(ta
nh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9289, size = 1503, normalized size = 14.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^
5 + 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x
+ (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x + c)^2 - (2*a^3*b + a*b^3)*
sinh(d*x + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x + c) - 2*(2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*co
sh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2
*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x +
c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cos
h(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(a^2*b^3 + b^5
 + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) - 2*(a^2*b^3 + b^5 + (a^5 +
 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*sinh
(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^
2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x
 + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-2), x)
```

Giac [A] time = 1.22093, size = 223, normalized size = 2.21

$$-\frac{\left(2\,a^2b+b^3\right)\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4d+a^2b^2d\right)\sqrt{a^2+b^2}}+\frac{2\left(b^3e^{(dx+c)}-ab^2\right)}{\left(a^4d+a^2b^2d\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}-a\right)}+\frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b + b^3)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4*d + a^2*b^2*d)*sqrt(a^2 + b^
2)) + 2*(b^3*e^(d*x + c) - a*b^2)/((a^4*d + a^2*b^2*d)*(a*e^(2*d*x + 2*c) +
 2*b*e^(d*x + c) - a)) + (d*x + c)/(a^2*d)
```

3.76 $\int \frac{1}{(a+b\,\mathbf{csch}(c+dx))^3}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{5/2}}-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2d\left(a^2+b^2\right)^2(a+b\,\mathrm{csch}(c+dx))}-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\,\mathrm{csch}(c+$$

```
[Out] x/a^3 + (b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(5/2)*d) - (b^2*Coth[c + d*x])/(2*a*(a^2 +
 b^2)*d*(a + b*Csch[c + d*x])^2) - (b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(2*a
^2*(a^2 + b^2)^2*d*(a + b*Csch[c + d*x]))
```

Rubi [A] time = 0.316353, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3785, 4060, 3919, 3831, 2660, 618, 204}

$$\frac{b\left(5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{5/2}}-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2d\left(a^2+b^2\right)^2(a+b\,\mathrm{csch}(c+dx))}-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\,\mathrm{csch}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + (b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(5/2)*d) - (b^2*Coth[c + d*x])/(2*a*(a^2 +
 b^2)*d*(a + b*Csch[c + d*x])^2) - (b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(2*a
^2*(a^2 + b^2)^2*d*(a + b*Csch[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\,\mathrm{csch}(c+dx))^3}\,dx = -\frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{\int \frac{-2\left(a^2+b^2\right)+2ab\,\mathrm{csch}(c+dx)-b^2\mathrm{csch}^2(c+dx)}{(a+b\,\mathrm{csch}(c+dx))^2}\,dx}{2a\left(a^2+b^2\right)}$$

$$= -\frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\,\mathrm{csch}(c+dx))} + \frac{\int \frac{2\left(a^2+b^2\right)^2-ab\cdots}{a+b\cdots}}{2a^2\cdots}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\,\mathrm{csch}(c+dx))} - \frac{\left(b\left(6a^4+\cdots\right.\right.}{\cdots}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\,\mathrm{csch}(c+dx))} - \frac{\left(6a^4+5a\cdots\right.}{\cdots}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\,\mathrm{csch}(c+dx))} + \frac{\left(i\left(6a^4+\cdots\right.\right.}{\cdots}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\,\mathrm{csch}(c+dx))} - \frac{\left(2i\left(6a^4+\cdots\right.\right.}{\cdots}$$

$$= \frac{x}{a^3} + \frac{b\left(6a^4+5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{5/2}d} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\,\mathrm{csch}(c+dx))\cdots}$$

Mathematica [A] time = 0.929081, size = 213, normalized size = 1.31

$$\frac{\operatorname{csch}^2(c+dx)(a\sinh(c+dx)+b)\left(\frac{ab^3\coth(c+dx)}{a^2+b^2}-\frac{3ab^2\left(2a^2+b^2\right)\coth(c+dx)(a\sinh(c+dx)+b)}{\left(a^2+b^2\right)^2}-\frac{2b\left(5a^2b^2+6a^4+2b^4\right)\operatorname{csch}(c+dx)(a\sinh(c+}{\left(-a^2-b^2\right)^5}\right.}{2a^3d(a+b\operatorname{csch}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] (Csch[c + d*x]^2*(b + a*Sinh[c + d*x])*((a*b^3*Coth[c + d*x])/(a^2 + b^2) -
 (3*a*b^2*(2*a^2 + b^2)*Coth[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2)^2
+ 2*(c + d*x)*Csch[c + d*x]*(b + a*Sinh[c + d*x])^2 - (2*b*(6*a^4 + 5*a^2*b
^2 + 2*b^4)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]*Csch[c + d*x
]*(b + a*Sinh[c + d*x])^2)/(-a^2 - b^2)^(5/2)))/(2*a^3*d*(a + b*Csch[c + d*
x])^3)
```

Maple [B] time = 0.076, size = 822, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)+4/d*a*b^2/(tanh(1/2*d*x+1/2*c)^2*b-2*a*ta
nh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3+1/d/a*b^4/
(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*t
anh(1/2*d*x+1/2*c)^3-10/d*a^2*b/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1
/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2+1/d*b^3/(tanh(1/2*d*x+
1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/
2*c)^2+2/d/a^2*b^5/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a
^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2-16/d*a*b^2/(tanh(1/2*d*x+1/2*c)^2*b
-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)-7/d/a
*b^4/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b
^4)*tanh(1/2*d*x+1/2*c)-5/d*b^3/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1
/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)-2/d/a^2*b^5/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tan
h(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)-6/d*a*b/(a^4+2*a^2*b^2+b^4)/(a^2+
b^2)^(1/2)*arctanh(1/2*(2*b*tanh(1/2*d*x+1/2*c)-2*a)/(a^2+b^2)^(1/2))-5/d/a
*b^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tanh(1/2*d*x+1/2*
c)-2*a)/(a^2+b^2)^(1/2))-2/d/a^3*b^5/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*tanh(1/2*d*x+1/2*c)-2*a)/(a^2+b^2)^(1/2))-1/d/a^3*ln(tanh(1/
2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21722, size = 4658, normalized size = 28.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^6*b^2 + 18*a^4*b^4 + 6*a^2*b^6 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 +
 a^2*b^6)*d*x*cosh(d*x + c)^4 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d
*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5
*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 + 11*a^3*b^5
+ 4*a*b^7 + 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c) + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*sinh(d*x + c)^3 + 2*(a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6
- 6*b^8 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x +
 c)^2 - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8 - 6*(a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 -
 5*a^2*b^6 - 2*b^8)*d*x - 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b +
3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a^6
*b + 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c
)^4 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 + 5*
a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6 + (
6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6*
b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7)*cosh(d*x + c)^2 - 2*(6*a^6*b - 7*a^4*b^3
 - 8*a^2*b^5 - 4*b^7 - 3*(6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2
- 6*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(6
*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) - 4*(6*a^5*b^2 + 5*a^3*b^4 +
2*a*b^6 - (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 - 3*(6*a^5*b^2
+ 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b - 7*a^4*b^3 - 8*a^2*b^5 -
 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*cosh(d*x + c
)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh
(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sin
h(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
 + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(17*a^5*b^3 + 25*a^3*b^5
 + 8*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c) -
 2*(17*a^5*b^3 + 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6)*d*x*cosh(d*x + c)^3 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x
- 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8
 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c)^4 +
(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b + 3*
a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c)^3 - 2*(a^11 + a^9*b^2 - 3*a^
7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 + 3*a^9*b^2 + 3
*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c) + (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4
*b^7)*d)*sinh(d*x + c)^3 - 4*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c) + 2*(3*(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c
)^2 + 6*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c) - (a^11
+ a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 +
 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d + 4*((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^
5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c)^2 - (a^11 + a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cos
h(d*x + c) - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{csch}(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-3), x)
```

Giac [A] time = 1.30529, size = 402, normalized size = 2.47

$$-\frac{\left(6\,a^4b + 5\,a^2b^3 + 2\,b^5\right)\log\left(\frac{\left|2\,ae^{(dx+c)} + 2\,b - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)} + 2\,b + 2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^7d + 2\,a^5b^2d + a^3b^4d\right)\sqrt{a^2+b^2}} + \frac{7\,a^3b^3e^{(3\,dx+3\,c)} + 4\,ab^5e^{(3\,dx+3\,c)} - 6\,a^4b^2e^{(2\,dx+2\,c)} + 9\,a^2b^4e^{(2\,dx+2}}{\left(a^7d + 2\,a^5b^2d + a^3b^4d\right)(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a^4*b + 5*a^2*b^3 + 2*b^5)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^7*d + 2*a^5*
b^2*d + a^3*b^4*d)*sqrt(a^2 + b^2)) + (7*a^3*b^3*e^(3*d*x + 3*c) + 4*a*b^5*
e^(3*d*x + 3*c) - 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x + 2*c) + 6
*b^6*e^(2*d*x + 2*c) - 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x + c) + 6*a^4
*b^2 + 3*a^2*b^4)/((a^7*d + 2*a^5*b^2*d + a^3*b^4*d)*(a*e^(2*d*x + 2*c) + 2
*b*e^(d*x + c) - a)^2) + (d*x + c)/(a^3*d)
```

3.77 $\int \frac{\sinh^3(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=107

$$\frac{bx\left(a^2-2b^2\right)}{2a^4}-\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh^2(x)\cosh(x)}{3a}$$

```
[Out] (b*(a^2 - 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - ((2*a^2 - 3*b^2)*Cosh[x])/(3*a^3) - (b*Cosh[x
]*Sinh[x])/(2*a^2) + (Cosh[x]*Sinh[x]^2)/(3*a)
```

Rubi [A] time = 0.4582, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$\frac{bx\left(a^2-2b^2\right)}{2a^4}-\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh^2(x)\cosh(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (b*(a^2 - 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - ((2*a^2 - 3*b^2)*Cosh[x])/(3*a^3) - (b*Cosh[x
]*Sinh[x])/(2*a^2) + (Cosh[x]*Sinh[x]^2)/(3*a)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{i\int \frac{\left(-3ib-2ia\operatorname{csch}(x)-2ib\operatorname{csch}^2(x)\right)\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\int \frac{\left(-2\left(2a^2-3b^2\right)-ab\operatorname{csch}(x)+3b^2\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{6a^2}$$

$$= -\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{i\int \frac{-3ib\left(a^2-2b^2\right)-3iab^2\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{6a^3}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}}{a^4}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}}{a^4}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\left(2b^3\right)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{\left(4b^3\right)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \cosh($$

Mathematica [A] time = 0.412106, size = 104, normalized size = 0.97

$$\frac{\left(12ab^2-9a^3\right)\cosh(x)+3b\left(\frac{8b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+2a^2x-a^2\sinh(2x)-4b^2x\right)+a^3\cosh(3x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] + a^3*Cosh[3*x] + 3*b*(2*a^2*x - 4*b^2*x + (8*
b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - a^2*Sinh
[2*x]))/(12*a^4)
```

Maple [B] time = 0.039, size = 262, normalized size = 2.5

$$\frac{1}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*csch(x)),x)
```

```
[Out] 1/3/a/(tanh(1/2*x)+1)^3-1/2/a/(tanh(1/2*x)+1)^2+1/2/a^2/(tanh(1/2*x)+1)^2*b
-1/2/a/(tanh(1/2*x)+1)-1/2/a^2/(tanh(1/2*x)+1)*b+1/a^3/(tanh(1/2*x)+1)*b^2+
1/2*b/a^2*ln(tanh(1/2*x)+1)-b^3/a^4*ln(tanh(1/2*x)+1)+2*b^4/a^4/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/3/a/(tanh(1/2*x)-
1)^3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*b+1/2/a/(tanh(1/2*x)
-1)-1/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2-1/2*b/a^2*ln(tanh(1
/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0559, size = 1947, normalized size = 18.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^5 + a^3*b^2)*cosh(x)^6 + (a^5 + a^3*b^2)*sinh(x)^6 - 3*(a^4*b + a^
2*b^3)*cosh(x)^5 - 3*(a^4*b + a^2*b^3 - 2*(a^5 + a^3*b^2)*cosh(x))*sinh(x)^
5 + a^5 + a^3*b^2 + 12*(a^4*b - a^2*b^3 - 2*b^5)*x*cosh(x)^3 - 3*(3*a^5 - a
^3*b^2 - 4*a*b^4)*cosh(x)^4 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b
^2)*cosh(x)^2 + 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 + a^3*b
^2)*cosh(x)^3 - 15*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b - a^2*b^3 - 2*b^5
)*x - 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 - a^3*b^2
 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b^2)*co
sh(x)^4 + 10*(a^4*b + a^2*b^3)*cosh(x)^3 - 12*(a^4*b - a^2*b^3 - 2*b^5)*x*c
osh(x) + 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 24*(b^4*cosh(
x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2
*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sin
h(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sin
h(x) - a)) + 3*(a^4*b + a^2*b^3)*cosh(x) + 3*(2*(a^5 + a^3*b^2)*cosh(x)^5 +
 a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 12*(a^4*b - a^2*b^3 - 2*
b^5)*x*cosh(x)^2 - 4*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 - a^3
*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x)^3 + 3*(a^6 + a^4
*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 + a^4*
b^2)*sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{a + b\operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*csch(x)), x)
```

Giac [A] time = 1.20164, size = 209, normalized size = 1.95

$$\frac{b^4\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} + \frac{a^2e^{(3\,x)} - 3\,abe^{(2\,x)} - 9\,a^2e^x + 12\,b^2e^x}{24\,a^3} + \frac{\left(a^2b - 2\,b^3\right)x}{2\,a^4} + \frac{\left(3\,a^2be^x + a^3 - 3\left(3\,a^3 - 4\,ab^2\right)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/24*(a^2*e^(3*x) - 3*a*b*e^(2*x) - 9*a
^2*e^x + 12*b^2*e^x)/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4 + 1/24*(3*a^2*b*e^x +
a^3 - 3*(3*a^3 - 4*a*b^2)*e^(2*x))*e^(-3*x)/a^4
```

3.78 $\int \frac{\sinh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=80

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{b\cosh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*b^3*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^
2]])/(a^3*Sqrt[a^2 + b^2]) - (b*Cosh[x])/a^2 + (Cosh[x]*Sinh[x])/(2*a)
```

Rubi [A] time = 0.288274, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{b\cosh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*b^3*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^
2]])/(a^3*Sqrt[a^2 + b^2]) - (b*Cosh[x])/a^2 + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2a} - \frac{i\int \frac{\left(-2ib-ia\operatorname{csch}(x)-ib\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{2a}\\
&= -\frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{-a^2+2b^2-ab\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{2a^2}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right.\right.}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{2b^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.135727, size = 82, normalized size = 1.02

$$\frac{-\frac{8b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 2a^2x + a^2\sinh(2x) - 4ab\cosh(x) + 4b^2x}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] time = 0.034, size = 174, normalized size = 2.2

$$-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*csch(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)-1/a^2/(tanh(1/2*x)+1)*b-1/2/
a*ln(tanh(1/2*x)+1)+1/a^3*ln(tanh(1/2*x)+1)*b^2-2*b^3/a^3/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+1/2/a/(tanh(1/2*x)-1)^2+1
/2/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b+1/2/a*ln(tanh(1/2*x)-1)-1/a^3*
ln(tanh(1/2*x)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9658, size = 1142, normalized size = 14.28

$$\frac{\left(a^4+a^2b^2\right)\cosh(x)^4+\left(a^4+a^2b^2\right)\sinh(x)^4-a^4-a^2b^2-4\left(a^4-a^2b^2-2\,b^4\right)x\cosh(x)^2-4\left(a^3b+ab^3\right)\cosh(x)^3-4\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^4 + a^2*b^2)*cosh(x)^4 + (a^4 + a^2*b^2)*sinh(x)^4 - a^4 - a^2*b^2
- 4*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 + a^2*b^2)*c
osh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/
(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x)^3
+ 2*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^5 + a^3*b^2)*cosh(x)*sinh(x) + (a^5 +
a^3*b^2)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.15516, size = 155, normalized size = 1.94

$$-\frac{b^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}+\frac{ae^{(2\,x)}-4\,be^x}{8\,a^2}-\frac{\left(a^2-2\,b^2\right)x}{2\,a^3}-\frac{\left(4\,abe^x+a^2\right)e^{(-2\,x)}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a
^2 - 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x + a^2)*e^(-2*x)/a^3
```

3.79 $\int \frac{\sinh(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{bx}{a^2}+\frac{\cosh(x)}{a}$$

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2*Sqrt
[a^2 + b^2]) + Cosh[x]/a
```

Rubi [A] time = 0.106813, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3853, 12, 3783, 2660, 618, 206}

$$-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{bx}{a^2}+\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2*Sqrt
[a^2 + b^2]) + Cosh[x]/a
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\cosh(x)}{a} - \frac{\int \frac{b}{a+b\operatorname{csch}(x)}\,dx}{a}\\
&= \frac{\cosh(x)}{a} - \frac{b\int \frac{1}{a+b\operatorname{csch}(x)}\,dx}{a}\\
&= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} + \frac{b\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} + \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} - \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2} - \frac{2b^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} + \frac{\cosh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.104826, size = 61, normalized size = 1.07

$$\frac{b\left(\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-x\right)+a\cosh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] (b*(-x + (2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + a*Cosh[x])/a^2
```

Maple [A] time = 0.033, size = 92, normalized size = 1.6

$$\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + 2\frac{b^2}{a^2\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right) - \frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*csch(x)),x)
```

```
[Out] 1/a/(tanh(1/2*x)+1)-b/a^2*ln(tanh(1/2*x)+1)+2/a^2*b^2/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/a/(tanh(1/2*x)-1)+b/a^2*ln(
tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93253, size = 640, normalized size = 11.23

$$\frac{a^3 + ab^2 - 2\left(a^2b + b^3\right)x\cosh\left(x\right) + \left(a^3 + ab^2\right)\cosh\left(x\right)^2 + \left(a^3 + ab^2\right)\sinh\left(x\right)^2 + 2\left(b^2\cosh\left(x\right) + b^2\sinh\left(x\right)\right)\sqrt{a^2 + b^2}\,\mathrm{lo}}{2\left(\left(a^4 + a^2b^2\right)\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + a*b^2 - 2*(a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*cosh(x)^2 + (a
^3 + a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 + b^2)*log((
a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x
)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 2*((a^2
*b + b^3)*x - (a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + a^2*b^2)*cosh(x) + (a
^4 + a^2*b^2)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.1766, size = 116, normalized size = 2.04

$$\frac{b^2\log\left(\frac{\left|2\,ae^x + 2\,b - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,ae^x + 2\,b + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

3.80 $\int \frac{\mathbf{csch}(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] (-2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Rubi [A] time = 0.0663334, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3831, 2660, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b}$$
$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{4\,\mathrm{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.024058, size = 45, normalized size = 1.22

$$\frac{2\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
```

Maple [A] time = 0.015, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*csch(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79808, size = 321, normalized size = 8.68

$$\frac{\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,\left(a^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*c
osh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))/sqrt
(a^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.16349, size = 76, normalized size = 2.05

$$\frac{\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 +
 b^2)))/sqrt(a^2 + b^2)
```

3.81 $\int \frac{\mathbf{csch}^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=50

$$\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] -(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b
*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.114566, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3789, 3770, 3831, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b
*Sqrt[a^2 + b^2])
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{b}-\frac{a\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{a\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2a\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.0488853, size = 58, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/b
```

Maple [A] time = 0.014, size = 49, normalized size = 1.

$$-2\,\frac{a}{b\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\,\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)+\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*csch(x)),x)
```

```
[Out] -2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+1
/b*ln(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04562, size = 456, normalized size = 9.12

$$\frac{\sqrt{a^2+b^2}a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)-\left(a^2+b^2\right)\log(}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a
*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)
*sinh(x) - a)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^2*b + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.18426, size = 111, normalized size = 2.22

$$-\frac{a\log\left(\frac{\left|2\,ae^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b}-\frac{\log\left(e^x+1\right)}{b}+\frac{\log\left(\left|e^x-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) - log(e^x + 1)/b + log(abs(e^x - 1))/b
```

3.82 $\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}$$

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (2*a^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(b^2*Sqrt[a^2 + b^2]) - Coth[x]/b
```

Rubi [A] time = 0.159803, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3790, 3789, 3770, 3831, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (2*a^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(b^2*Sqrt[a^2 + b^2]) - Coth[x]/b
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\coth(x)}{b}-\frac{a\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx}{b}\\
&= -\frac{\coth(x)}{b}-\frac{a\int \operatorname{csch}(x)\,dx}{b^2}+\frac{a^2\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^2}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}+\frac{a^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}-\frac{\left(4a^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.278098, size = 71, normalized size = 1.2

$$
\frac{\frac{4a^2\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-2a\log\left(\tanh\left(\frac{x}{2}\right)\right)-2b\coth(x)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((4*a^2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 2*b*
Coth[x] - 2*a*Log[Tanh[x/2]])/(2*b^2)
```

Maple [A] time = 0.016, size = 73, normalized size = 1.2

$$
-\frac{1}{2b}\tanh\left(\frac{x}{2}\right)+2\frac{a^2}{b^2\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)-\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/2/b*tanh(1/2*x)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2
*a)/(a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19974, size = 926, normalized size = 15.69

$$2\,a^2b + 2\,b^3 - \left(a^2\cosh\left(x\right)^2 + 2\,a^2\cosh\left(x\right)\sinh\left(x\right) + a^2\sinh\left(x\right)^2 - a^2\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\cosh(x) + a^2 + 2\,b}{a\cosh(x)^2 + a\sinh(x)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^2*b + 2*b^3 - (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 -
 a^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x)
+ a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
 b)*sinh(x) - a)) + (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2
)*cosh(x)*sinh(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2)*cosh(x)*sinh(x) - (
a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 + b^4 - (a^2*b
^2 + b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^2*b^2 + b^4)*s
inh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*csch(x)), x)
```

Giac [A] time = 1.20938, size = 132, normalized size = 2.24

$$\frac{a^2 \log\left(\frac{\left|2 ae^x+2 b-2 \sqrt{a^2+b^2}\right|}{\left|2 ae^x+2 b+2 \sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{a \log\left(e^x+1\right)}{b^2}-\frac{a \log\left(\left|e^x-1\right|\right)}{b^2}-\frac{2}{b\left(e^{(2 x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1)
)/b^2 - 2/(b*(e^(2*x) - 1))
```

3.83 $\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=83

$$\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[Out] -((2*a^2 - b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*a^3*ArcTanh[(a - b*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]) + (a*Coth[x])/b^2 - (Coth[x]*Csch
[x])/(2*b)
```

Rubi [A] time = 0.291833, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3851, 4082, 3998, 3770, 3831, 2660, 618, 206}

$$\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -((2*a^2 - b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*a^3*ArcTanh[(a - b*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]) + (a*Coth[x])/b^2 - (Coth[x]*Csch
[x])/(2*b)
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\int \frac{\operatorname{csch}(x)\left(a+b\operatorname{csch}(x)+2a\operatorname{csch}^2(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b}\\
&= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{i\int \frac{\operatorname{csch}(x)\left(iab-i\left(2a^2-b^2\right)\operatorname{csch}(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b^2}\\
&= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^3} + \frac{\left(2a^2-b^2\right)\int \operatorname{csch}(x)\,dx}{2b^3}\\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^4}\\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \text{t}\right.}{b^4}\\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{\left(4a^3\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x\right.}{b^4}\\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.475925, size = 124, normalized size = 1.49

$$-\frac{\frac{16a^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-8a^2\log\left(\tanh\left(\frac{x}{2}\right)\right)-4ab\tanh\left(\frac{x}{2}\right)-4ab\coth\left(\frac{x}{2}\right)+b^2\text{csch}^2\left(\frac{x}{2}\right)+b^2\text{sech}^2\left(\frac{x}{2}\right)+4b^2\log\left(\tanh\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -((16*a^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*
a*b*Coth[x/2] + b^2*Csch[x/2]^2 - 8*a^2*Log[Tanh[x/2]] + 4*b^2*Log[Tanh[x/2
]] + b^2*Sech[x/2]^2 - 4*a*b*Tanh[x/2])/(8*b^3)
```

Maple [A] time = 0.023, size = 108, normalized size = 1.3

$$\frac{1}{8b}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{a}{2b^2}\tanh\left(\frac{x}{2}\right)-2\frac{a^3}{b^3\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\frac{2\tanh(x/2)b-2a}{\sqrt{a^2+b^2}}\right)-\frac{1}{8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a^2}{b^3}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*csch(x)),x)
```

```
[Out] 1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/2*x)^2+1/b^3*ln(ta
nh(1/2*x))*a^2-1/2/b*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.8103, size = 2288, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x)^3 + 2*(a^2*b^2 + b^4)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^2*b^2
+ b^4)*cosh(x))*sinh(x)^2 - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^
3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
```

```
h(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 2*(a^2*b^2 + b^4)*c
osh(x) + ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cos
h(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4
- 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4
 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)
^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) -
 ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cosh(x)*sin
h(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4 - 2*(2*a
^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4 + a^2*b
^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)^3 - (2*
a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*
b^2 + b^4 + 3*(a^2*b^2 + b^4)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^2*b^3 + b^5 + (a^2*b^3 + b^5)*cosh(x)^4 + 4*(a^2*b^3 + b^5)*cosh(x)*s
inh(x)^3 + (a^2*b^3 + b^5)*sinh(x)^4 - 2*(a^2*b^3 + b^5)*cosh(x)^2 - 2*(a^2
*b^3 + b^5 - 3*(a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 + b^5)*co
sh(x)^3 - (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*csch(x)), x)
```

Giac [A] time = 1.1781, size = 190, normalized size = 2.29

$$-\frac{a^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3}-\frac{\left(2\,a^2-b^2\right)\log\left(e^x+1\right)}{2\,b^3}+\frac{\left(2\,a^2-b^2\right)\log\left(\left|e^x-1\right|\right)}{2\,b^3}-\frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 1/2*(2*a^2 - b^2)*log(e^x + 1)/b^3 + 1
/2*(2*a^2 - b^2)*log(abs(e^x - 1))/b^3 - (b*e^(3*x) - 2*a*e^(2*x) + b*e^x +
 2*a)/(b^2*(e^(2*x) - 1)^2)
```

3.84 $\int \frac{\cosh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=38

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}-\frac{1}{4}i\sinh(x)\cosh^3(x)+\frac{1}{8}i\sinh(x)\cosh(x)$$

```
[Out] (I/8)*x + Cosh[x]^3/3 + (I/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Sinh[x]
```

Rubi [A] time = 0.128245, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2839, 2565, 30, 2568, 2635, 8}

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}-\frac{1}{4}i\sinh(x)\cosh^3(x)+\frac{1}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]^3/3 + (I/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Sinh[x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^4(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \cosh^2(x)\sinh^2(x)\,dx\right)+\int \cosh^2(x)\sinh(x)\,dx\\
&= -\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{4}i\int \cosh^2(x)\,dx+\operatorname{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{8}i\int 1\,dx\\
&= \frac{ix}{8}+\frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0328512, size = 32, normalized size = 0.84

$$\frac{ix}{8}-\frac{1}{32}i\sinh(4x)+\frac{\cosh(x)}{4}+\frac{1}{12}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]/4 + Cosh[3*x]/12 - (I/32)*Sinh[4*x]
```

Maple [B] time = 0.039, size = 170, normalized size = 4.5

$$\frac{i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-3}+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3i}{8}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+csch(x)),x)
```

```
[Out] 1/8*I*ln(tanh(1/2*x)+1)-1/8*I*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^3-1/2*I
/(tanh(1/2*x)-1)^3+1/2/(tanh(1/2*x)+1)+3/8*I/(tanh(1/2*x)+1)^2-1/2/(tanh(1/
2*x)+1)^2-3/8*I/(tanh(1/2*x)-1)^2-1/2*I/(tanh(1/2*x)+1)^3+1/4*I/(tanh(1/2*x
)+1)^4-1/3/(tanh(1/2*x)-1)^3-1/8*I/(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)-1)^2-1/
8*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)-1)-1/4*I/(tanh(1/2*x)-1)^4
```

Maxima [A] time = 1.03984, size = 57, normalized size = 1.5

$$\frac{1}{192}\left(8e^{(-x)}+24e^{(-3x)}-3i\right)e^{(4x)}+\frac{1}{8}ix+\frac{1}{8}e^{(-x)}+\frac{1}{24}e^{(-3x)}+\frac{1}{64}ie^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/192*(8*e^(-x) + 24*e^(-3*x) - 3*I)*e^(4*x) + 1/8*I*x + 1/8*e^(-x) + 1/24*
e^(-3*x) + 1/64*I*e^(-4*x)
```

Fricas [A] time = 1.85841, size = 134, normalized size = 3.53

$$\frac{1}{192}\left(24 i\, x e^{(4\,x)} - 3 i\, e^{(8\,x)} + 8\, e^{(7\,x)} + 24\, e^{(5\,x)} + 24\, e^{(3\,x)} + 8\, e^{x} + 3 i\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(24*I*x*e^(4*x) - 3*I*e^(8*x) + 8*e^(7*x) + 24*e^(5*x) + 24*e^(3*x) +
 8*e^x + 3*I)*e^(-4*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1809, size = 51, normalized size = 1.34

$$\frac{1}{192}\left(24\, e^{(3\,x)} + 8\, e^{x} + 3 i\right)e^{(-4\,x)} + \frac{1}{8} i\, x - \frac{1}{64} i\, e^{(4\,x)} + \frac{1}{24}\, e^{(3\,x)} + \frac{1}{8}\, e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(24*e^(3*x) + 8*e^x + 3*I)*e^(-4*x) + 1/8*I*x - 1/64*I*e^(4*x) + 1/24
*e^(3*x) + 1/8*e^x
```

3.85 $\int \frac{\cosh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3

Rubi [A] time = 0.105568, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2835, 2564, 30}

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(I + Csch[x]),x]

[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^3(x)\sinh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \cosh(x)\sinh^2(x)\,dx\right)+\int \cosh(x)\sinh(x)\,dx\\ &= -\operatorname{Subst}\left(\int x\,dx,x,i\sinh(x)\right)+\operatorname{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\\ &= \frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)\end{aligned}$$

Mathematica [A] time = 0.0097886, size = 19, normalized size = 1.

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Csch[x]),x]
```

```
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Maple [A] time = 0.022, size = 15, normalized size = 0.8

$$\frac{1}{2\left(\operatorname{csch}(x)\right)^2}-\frac{\frac{i}{3}}{\left(\operatorname{csch}(x)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+csch(x)),x)
```

```
[Out] 1/2/csch(x)^2-1/3*I/csch(x)^3
```

Maxima [B] time = 1.03729, size = 53, normalized size = 2.79

$$\frac{1}{24}\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-i\right)e^{(3\,x)}-\frac{1}{8}i\,e^{(-x)}+\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{24}i\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 3*I*e^(-2*x) - I)*e^(3*x) - 1/8*I*e^(-x) + 1/8*e^(-2*x) +
1/24*I*e^(-3*x)
```

Fricas [B] time = 1.85812, size = 108, normalized size = 5.68

$$\frac{1}{24}\left(-i\,e^{(6\,x)}+3\,e^{(5\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+3\,e^{x}+i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(6*x) + 3*e^(5*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + 3*e^x + I)*e^(-3
*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14124, size = 47, normalized size = 2.47

$$-\frac{1}{24}\left(3i\,e^{(2\,x)} - 3\,e^{x} - i\right)e^{(-3\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(3*I*e^(2*x) - 3*e^x - I)*e^(-3*x) - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 1
/8*I*e^x
```

3.86 $\int \frac{\cosh^2(x)}{i+\mathbf{csch}(x)} \, dx$

Optimal. Leaf size=20

$$\frac{ix}{2} + \cosh(x) - \frac{1}{2} i \sinh(x) \cosh(x)$$

```
[Out] (I/2)*x + Cosh[x] - (I/2)*Cosh[x]*Sinh[x]
```

Rubi [A] time = 0.0923256, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2638, 2635, 8}

$$\frac{ix}{2} + \cosh(x) - \frac{1}{2} i \sinh(x) \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/2)*Cosh[x]*Sinh[x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \sinh^2(x)\,dx\right)+\int \sinh(x)\,dx\\ &= \cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)+\frac{1}{2}i\int 1\,dx\\ &= \frac{ix}{2}+\cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0294516, size = 20, normalized size = 1.

$$\frac{ix}{2}-\frac{1}{4}i\sinh(2x)+\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/4)*Sinh[2*x]
```

Maple [B] time = 0.033, size = 84, normalized size = 4.2

$$\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{i}{2}\left(\mathrm{tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+csch(x)),x)
```

```
[Out] 1/2*I/(tanh(1/2*x)+1)^2+1/2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)-1/2*I/(ta
nh(1/2*x)+1)-1/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)^2-1/(tanh(1/2*x)
-1)-1/2*I/(tanh(1/2*x)-1)
```

Maxima [B] time = 1.04373, size = 41, normalized size = 2.05

$$\frac{1}{8}\left(4e^{(-x)}-i\right)e^{(2x)}+\frac{1}{2}ix+\frac{1}{2}e^{(-x)}+\frac{1}{8}ie^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*e^(-x) - I)*e^(2*x) + 1/2*I*x + 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Fricas [B] time = 1.89306, size = 89, normalized size = 4.45

$$\frac{1}{8}\left(4ixe^{(2x)}-ie^{(4x)}+4e^{(3x)}+4e^{x}+i\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*I*x*e^(2*x) - I*e^(4*x) + 4*e^(3*x) + 4*e^x + I)*e^(-2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24136, size = 35, normalized size = 1.75

$$\frac{1}{8}\,(4\,e^{x}+i)e^{(-2\,x)}+\frac{1}{2}\,i\,x-\frac{1}{8}\,i\,e^{(2\,x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(4*e^x + I)*e^(-2*x) + 1/2*I*x - 1/8*I*e^(2*x) + 1/2*e^x
```

3.87 $\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=16

$$\log(-\sinh(x)+i)-i\sinh(x)$$

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Rubi [A] time = 0.0584481, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2833, 43}

$$\log(-\sinh(x)+i)-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= i\,\text{Subst}\left(\int \frac{x}{i+x}\,dx, x, -\sinh(x)\right)\\
&= i\,\text{Subst}\left(\int \left(1-\frac{i}{i+x}\right)dx, x, -\sinh(x)\right)\\
&= \log(i-\sinh(x))-i\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0093764, size = 16, normalized size = 1.

$$\log(-\sinh(x)+i)-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Maple [A] time = 0.023, size = 20, normalized size = 1.3

$$-\ln\left(\operatorname{csch}\left(x\right)\right)-\frac{i}{\operatorname{csch}\left(x\right)}+\ln\left(i+\operatorname{csch}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+csch(x)),x)
```

```
[Out] -ln(csch(x))-I/csch(x)+ln(I+csch(x))
```

Maxima [A] time = 1.02466, size = 28, normalized size = 1.75

$$x+\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}+2\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^(-x) + I)
```

Fricas [B] time = 1.74491, size = 81, normalized size = 5.06

$$-\frac{1}{2}\left(2\,xe^{x}-4\,e^{x}\log\left(e^{x}-i\right)+i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - 4*e^x*log(e^x - I) + I*e^(2*x) - I)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)/(csch(x) + I), x)
```

Giac [B] time = 1.1312, size = 34, normalized size = 2.12

$$\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}-\log\left(-i\,e^{x}\right)+2\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/2*I*e^(-x) - 1/2*I*e^x - log(-I*e^x) + 2*log(e^x - I)
```

3.88 $\int \frac{\text{sech}(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\text{sech}^2(x)-\frac{1}{2}i\tan^{-1}(\sinh(x))+\frac{1}{2}i\tanh(x)\text{sech}(x)$$

```
[Out] (-I/2)*ArcTan[Sinh[x]] - Sech[x]^2/2 + (I/2)*Sech[x]*Tanh[x]
```

Rubi [A] time = 0.0801613, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\text{sech}^2(x)-\frac{1}{2}i\tan^{-1}(\sinh(x))+\frac{1}{2}i\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-I/2)*ArcTan[Sinh[x]] - Sech[x]^2/2 + (I/2)*Sech[x]*Tanh[x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{i-\sinh(x)}\,dx \\
&= -\left(i\int \operatorname{sech}(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^2(x)\tanh(x)\,dx \\
&= \frac{1}{2}i\operatorname{sech}(x)\tanh(x)-\frac{1}{2}i\int \operatorname{sech}(x)\,dx-\operatorname{Subst}\left(\int x\,dx,x,\operatorname{sech}(x)\right) \\
&= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^2(x)}{2}+\frac{1}{2}i\operatorname{sech}(x)\tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.0295167, size = 20, normalized size = 0.71

$$-\frac{1}{2}i\left(\tan^{-1}(\sinh(x))+\frac{1}{-\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-I/2)*(ArcTan[Sinh[x]] + (I - Sinh[x])^(-1))
```

Maple [B] time = 0.033, size = 43, normalized size = 1.5

$$-i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+csch(x)),x)
```

```
[Out] -I/(tanh(1/2*x)-I)+1/(tanh(1/2*x)-I)^2-1/2*ln(tanh(1/2*x)-I)+1/2*ln(tanh(1/
2*x)+I)
```

Maxima [B] time = 1.04381, size = 55, normalized size = 1.96

$$-\frac{2i\,e^{(-x)}}{4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\,\log\left(e^{(-x)}+i\right)+\frac{1}{2}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*I*e^(-x)/(4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Fricas [B] time = 1.63444, size = 155, normalized size = 5.54

$$\frac{\left(e^{(2x)} - 2i\,e^x - 1\right)\log\left(e^x + i\right) - \left(e^{(2x)} - 2i\,e^x - 1\right)\log\left(e^x - i\right) + 2i\,e^x}{2\,e^{(2x)} - 4i\,e^x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) - 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) - 2*I*e^x - 1)*log(e^x - I
) + 2*I*e^x)/(2*e^(2*x) - 4*I*e^x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)/(csch(x) + I), x)
```

Giac [B] time = 1.1428, size = 72, normalized size = 2.57

$$\frac{e^{(-x)} - e^x - 2i}{4\left(e^{(-x)} - e^x + 2i\right)} + \frac{1}{4}\log\left(-i\,e^{(-x)} + i\,e^x - 2\right) - \frac{1}{4}\log\left(-e^{(-x)} + e^x - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) - e^x - 2*I)/(e^(-x) - e^x + 2*I) + 1/4*log(-I*e^(-x) + I*e^x -
 2) - 1/4*log(-e^(-x) + e^x - 2*I)
```

3.89 $\int \frac{\mathrm{sech}^2(x)}{i+\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{3}\mathrm{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

```
[Out] -Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rubi [A] time = 0.111021, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2606, 30, 2607}

$$-\frac{1}{3}\mathrm{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] -Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}(x)\tanh(x)}{i-\sinh(x)}\,dx$$
$$= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right) + \int \operatorname{sech}^3(x)\tanh(x)\,dx$$
$$= -\operatorname{Subst}\left(\int x^2\,dx, x, \operatorname{sech}(x)\right) + \operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right)$$
$$= -\frac{1}{3}\operatorname{sech}^3(x) - \frac{1}{3}i\tanh^3(x)$$

Mathematica [B] time = 0.0531439, size = 64, normalized size = 3.37

$$\frac{-2i\sinh(x)+\cosh(x)+\cosh(2x)+i\sinh(x)\cosh(x)-3}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] (-3 + Cosh[x] + Cosh[2*x] - (2*I)*Sinh[x] + I*Cosh[x]*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] time = 0.036, size = 49, normalized size = 2.6

$$\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} - \frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3} - \left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2} - \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+csch(x)),x)
```

```
[Out] 1/2*I/(tanh(1/2*x)-I)-2/3*I/(tanh(1/2*x)-I)^3-1/(tanh(1/2*x)-I)^2-1/2*I/(ta
nh(1/2*x)+I)
```

Maxima [B] time = 1.02708, size = 109, normalized size = 5.74

$$\frac{8\,e^{(-x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6} - \frac{12i\,e^{(-2\,x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6} + \frac{4i}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*e^(-x)/(12*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) - 12*I*e^(-2*x)/(12
*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) + 4*I/(12*I*e^(-x) + 12*I*e^(-3
*x) + 6*e^(-4*x) - 6)
```

Fricas [B] time = 1.49085, size = 93, normalized size = 4.89

$$\frac{6i\,e^{(2\,x)}+4\,e^{x}-2i}{3\,e^{(4\,x)}-6i\,e^{(3\,x)}-6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (6*I*e^(2*x) + 4*e^x - 2*I)/(3*e^(4*x) - 6*I*e^(3*x) - 6*I*e^x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(csch(x) + I), x)
```

Giac [A] time = 1.20023, size = 36, normalized size = 1.89

$$-\frac{i}{2\,(i\,e^{x}-1)}+\frac{3\,e^{(2\,x)}-1}{6\,(e^{x}-i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*I/(I*e^x - 1) + 1/6*(3*e^(2*x) - 1)/(e^x - I)^3
```

3.90 $\int \frac{\mathrm{sech}^3(x)}{i+\mathrm{csch}(x)} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{4}\mathrm{sech}^4(x) - \frac{1}{8}i\tan^{-1}(\sinh(x)) + \frac{1}{4}i\tanh(x)\mathrm{sech}^3(x) - \frac{1}{8}i\tanh(x)\mathrm{sech}(x)$$

```
[Out] (-I/8)*ArcTan[Sinh[x]] - Sech[x]^4/4 - (I/8)*Sech[x]*Tanh[x] + (I/4)*Sech[x
]^3*Tanh[x]
```

Rubi [A] time = 0.130221, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{1}{4}\mathrm{sech}^4(x) - \frac{1}{8}i\tan^{-1}(\sinh(x)) + \frac{1}{4}i\tanh(x)\mathrm{sech}^3(x) - \frac{1}{8}i\tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] (-I/8)*ArcTan[Sinh[x]] - Sech[x]^4/4 - (I/8)*Sech[x]*Tanh[x] + (I/4)*Sech[x
]^3*Tanh[x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \operatorname{sech}^3(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^4(x)\tanh(x)\,dx\\
&= \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{4}i\int \operatorname{sech}^3(x)\,dx-\operatorname{Subst}\left(\int x^3\,dx,x,\operatorname{sech}(x)\right)\\
&= -\frac{1}{4}\operatorname{sech}^4(x)-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{8}i\int \operatorname{sech}(x)\,dx\\
&= -\frac{1}{8}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^4(x)}{4}-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0586016, size = 32, normalized size = 0.8

$$\frac{1}{8}\left(-\frac{i}{\sinh(x)+i}+\frac{1}{(\sinh(x)-i)^2}-i\tan^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-I)*ArcTan[Sinh[x]] + (-I + Sinh[x])^(-2) - I/(I + Sinh[x]))/8
```

Maple [B] time = 0.043, size = 89, normalized size = 2.2

$$i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-\frac{1}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{i}{4}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+csch(x)),x)
```

```
[Out] I/(tanh(1/2*x)-I)^3-1/2*I/(tanh(1/2*x)-I)-1/2/(tanh(1/2*x)-I)^4+1/(tanh(1/2
*x)-I)^2-1/8*ln(tanh(1/2*x)-I)+1/4*I/(tanh(1/2*x)+I)+1/4/(tanh(1/2*x)+I)^2+
1/8*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.05542, size = 124, normalized size = 3.1

$$\frac{8\left(i\,e^{(-x)}+2\,e^{(-2\,x)}-10i\,e^{(-3\,x)}-2\,e^{(-4\,x)}+i\,e^{(-5\,x)}\right)}{64i\,e^{(-x)}-32\,e^{(-2\,x)}+128i\,e^{(-3\,x)}+32\,e^{(-4\,x)}+64i\,e^{(-5\,x)}+32\,e^{(-6\,x)}-32}-\frac{1}{8}\,\log\left(e^{(-x)}+i\right)+\frac{1}{8}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*(I*e^(-x) + 2*e^(-2*x) - 10*I*e^(-3*x) - 2*e^(-4*x) + I*e^(-5*x))/(64*I*e
^(-x) - 32*e^(-2*x) + 128*I*e^(-3*x) + 32*e^(-4*x) + 64*I*e^(-5*x) + 32*e^(
-6*x) - 32) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

Fricas [B] time = 1.69641, size = 431, normalized size = 10.78

$$\frac{\left(e^{(6\,x)}-2i\,e^{(5\,x)}+e^{(4\,x)}-4i\,e^{(3\,x)}-e^{(2\,x)}-2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(6\,x)}-2i\,e^{(5\,x)}+e^{(4\,x)}-4i\,e^{(3\,x)}-e^{(2\,x)}-2i\,e^{x}-1\right)\log\left(e^{x}\right.}{8\,e^{(6\,x)}-16i\,e^{(5\,x)}+8\,e^{(4\,x)}-32i\,e^{(3\,x)}-8\,e^{(2\,x)}-16i\,e^{x}-8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I*e^x - 1)*lo
g(e^x + I) - (e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I
*e^x - 1)*log(e^x - I) - 2*I*e^(5*x) - 4*e^(4*x) + 20*I*e^(3*x) + 4*e^(2*x)
 - 2*I*e^x)/(8*e^(6*x) - 16*I*e^(5*x) + 8*e^(4*x) - 32*I*e^(3*x) - 8*e^(2*x
) - 16*I*e^x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^3(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(csch(x) + I), x)
```

Giac [B] time = 1.16183, size = 127, normalized size = 3.18

$$-\frac{-i\,e^{(-x)}+i\,e^{x}-6}{16\left(-i\,e^{(-x)}+i\,e^{x}-2\right)}+\frac{3\left(e^{(-x)}-e^{x}\right)^2+12i\,e^{(-x)}-12i\,e^{x}+4}{32\left(e^{(-x)}-e^{x}+2i\right)^2}+\frac{1}{16}\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{16}\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/16*(-I*e^(-x) + I*e^x - 6)/(-I*e^(-x) + I*e^x - 2) + 1/32*(3*(e^(-x) - e
^x)^2 + 12*I*e^(-x) - 12*I*e^x + 4)/(e^(-x) - e^x + 2*I)^2 + 1/16*log(-e^(-
x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

3.91 $\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=29

$$\frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\operatorname{sech}^5(x)$$

```
[Out] -Sech[x]^5/5 - (I/3)*Tanh[x]^3 + (I/5)*Tanh[x]^5
```

Rubi [A] time = 0.121423, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$\frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\operatorname{sech}^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] -Sech[x]^5/5 - (I/3)*Tanh[x]^3 + (I/5)*Tanh[x]^5
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{i-\sinh(x)}\,dx \\
&= -\left(i\int \operatorname{sech}^4(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^5(x)\tanh(x)\,dx \\
&= -\operatorname{Subst}\left(\int x^4\,dx, x, \operatorname{sech}(x)\right)+\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\tanh(x)\right) \\
&= -\frac{1}{5}\operatorname{sech}^5(x)+\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\tanh(x)\right) \\
&= -\frac{1}{5}\operatorname{sech}^5(x)-\frac{1}{3}i\tanh^3(x)+\frac{1}{5}i\tanh^5(x)
\end{aligned}$$

Mathematica [B] time = 0.104372, size = 96, normalized size = 3.31

$$\frac{-96i\sinh(x)+18i\sinh(2x)-32i\sinh(3x)+9i\sinh(4x)+54\cosh(x)+32\cosh(2x)+18\cosh(3x)+16\cosh(4x)-240}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] (-240 + 54*Cosh[x] + 32*Cosh[2*x] + 18*Cosh[3*x] + 16*Cosh[4*x] - (96*I)*Si
nh[x] + (18*I)*Sinh[2*x] - (32*I)*Sinh[3*x] + (9*I)*Sinh[4*x])/(960*(Cosh[x
/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [B] time = 0.045, size = 93, normalized size = 3.2

$$-\frac{4i}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}+\frac{3i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{2i}{5}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-5}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}-\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{i}{6}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+csch(x)),x)
```

```
[Out] -4/3*I/(tanh(1/2*x)-I)^3+3/8*I/(tanh(1/2*x)-I)+2/5*I/(tanh(1/2*x)-I)^5+1/(t
anh(1/2*x)-I)^4-1/(tanh(1/2*x)-I)^2+1/6*I/(tanh(1/2*x)+I)^3-3/8*I/(tanh(1/2
*x)+I)-1/4/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 0.992296, size = 347, normalized size = 11.97

$$\frac{32\,e^{(-x)}}{120i\,e^{(-x)}-120\,e^{(-2\,x)}+360i\,e^{(-3\,x)}+360i\,e^{(-5\,x)}+120\,e^{(-6\,x)}+120i\,e^{(-7\,x)}+60\,e^{(-8\,x)}-60}+\frac{}{120i\,e^{(-x)}-120\,e^{(-2\,x)}+360i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 32*e^(-x)/(120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) +
120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60) + 32*I*e^(-2*x)/(120*I*e^
(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x) + 120*
I*e^(-7*x) + 60*e^(-8*x) - 60) + 96*e^(-3*x)/(120*I*e^(-x) - 120*e^(-2*x) +
 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8
*x) - 60) - 240*I*e^(-4*x)/(120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) +
360*I*e^(-5*x) + 120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60) + 16*I/(
120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x
) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60)
```

Fricas [B] time = 1.44565, size = 212, normalized size = 7.31

$$\frac{60i\, e^{(4\,x)} + 24\, e^{(3\,x)} - 8i\, e^{(2\,x)} + 8\, e^{x} - 4i}{15\, e^{(8\,x)} - 30i\, e^{(7\,x)} + 30\, e^{(6\,x)} - 90i\, e^{(5\,x)} - 90i\, e^{(3\,x)} - 30\, e^{(2\,x)} - 30i\, e^{x} - 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (60*I*e^(4*x) + 24*e^(3*x) - 8*I*e^(2*x) + 8*e^x - 4*I)/(15*e^(8*x) - 30*I*
e^(7*x) + 30*e^(6*x) - 90*I*e^(5*x) - 90*I*e^(3*x) - 30*e^(2*x) - 30*I*e^x
- 15)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{4}(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(csch(x) + I), x)
```

Giac [B] time = 1.16577, size = 74, normalized size = 2.55

$$-\frac{-3i\, e^{(2\,x)} + 12\, e^{x} + 5i}{24\,(i\, e^{x} - 1)^{3}} + \frac{15\, e^{(4\,x)} - 60i\, e^{(3\,x)} - 10\, e^{(2\,x)} + 20i\, e^{x} + 7}{120\,(e^{x} - i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*e^(2*x) + 12*e^x + 5*I)/(I*e^x - 1)^3 + 1/120*(15*e^(4*x) - 60*
I*e^(3*x) - 10*e^(2*x) + 20*I*e^x + 7)/(e^x - I)^5
```

3.92 $\int \frac{\cosh^5(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=102

$$\frac{(2a^2+b^2)\sinh^3(x)}{3a^3}-\frac{b(2a^2+b^2)\sinh^2(x)}{2a^4}+\frac{(a^2+b^2)^2\sinh(x)}{a^5}-\frac{b(a^2+b^2)^2\log(a\sinh(x)+b)}{a^6}-\frac{b\sinh^4(x)}{4a^2}+\frac{\sinh^5(x)}{5a}$$

```
[Out] -((b*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/a^6) + ((a^2 + b^2)^2*Sinh[x])/a^5 -
 (b*(2*a^2 + b^2)*Sinh[x]^2)/(2*a^4) + ((2*a^2 + b^2)*Sinh[x]^3)/(3*a^3) -
(b*Sinh[x]^4)/(4*a^2) + Sinh[x]^5/(5*a)
```

Rubi [A] time = 0.198158, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$\frac{(2a^2+b^2)\sinh^3(x)}{3a^3}-\frac{b(2a^2+b^2)\sinh^2(x)}{2a^4}+\frac{(a^2+b^2)^2\sinh(x)}{a^5}-\frac{b(a^2+b^2)^2\log(a\sinh(x)+b)}{a^6}-\frac{b\sinh^4(x)}{4a^2}+\frac{\sinh^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/a^6) + ((a^2 + b^2)^2*Sinh[x])/a^5 -
 (b*(2*a^2 + b^2)*Sinh[x]^2)/(2*a^4) + ((2*a^2 + b^2)*Sinh[x]^3)/(3*a^3) -
(b*Sinh[x]^4)/(4*a^2) + Sinh[x]^5/(5*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh^5(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^5}$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^6}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(\left(a^2+b^2\right)^2 - \frac{b\left(a^2+b^2\right)^2}{b-ix} + ib\left(2a^2+b^2\right)x - \left(2a^2+b^2\right)x^2 - ibx^3 + x^4\right)dx, x, ia\sinh(\right.}{a^6}$$
$$= -\frac{b\left(a^2+b^2\right)^2\log(b+a\sinh(x))}{a^6} + \frac{\left(a^2+b^2\right)^2\sinh(x)}{a^5} - \frac{b\left(2a^2+b^2\right)\sinh^2(x)}{2a^4} + \frac{\left(2a^2+b^2\right)\sin}{3a^3}$$

Mathematica [A] time = 0.277388, size = 97, normalized size = 0.95

$$\frac{20a^3\left(2a^2+b^2\right)\sinh^3(x) - 30a^2b\left(2a^2+b^2\right)\sinh^2(x) + 60a\left(a^2+b^2\right)^2\sinh(x) - 60b\left(a^2+b^2\right)^2\log(a\sinh(x)+b) - 15}{60a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^2 + b^2)^2*Log[b + a*Sinh[x]] + 60*a*(a^2 + b^2)^2*Sinh[x] - 30*a
^2*b*(2*a^2 + b^2)*Sinh[x]^2 + 20*a^3*(2*a^2 + b^2)*Sinh[x]^3 - 15*a^4*b*Si
nh[x]^4 + 12*a^5*Sinh[x]^5)/(60*a^6)
```

Maple [B] time = 0.041, size = 600, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*csch(x)),x)
```

```
[Out] -1/5/a/(tanh(1/2*x)+1)^5-1/5/a/(tanh(1/2*x)-1)^5+1/2/a/(tanh(1/2*x)+1)^4-11
/12/a/(tanh(1/2*x)+1)^3+7/8/a/(tanh(1/2*x)+1)^2-1/a/(tanh(1/2*x)+1)-1/2/a/(
tanh(1/2*x)-1)^4-11/12/a/(tanh(1/2*x)-1)^3-7/8/a/(tanh(1/2*x)-1)^2-1/a/(tan
h(1/2*x)-1)+b^5/a^6*ln(tanh(1/2*x)-1)-1/2/a^4/(tanh(1/2*x)+1)^2*b^3-1/a^5/(
tanh(1/2*x)+1)*b^4+b^5/a^6*ln(tanh(1/2*x)+1)-1/4/a^2/(tanh(1/2*x)+1)^4*b-1/
3/a^3/(tanh(1/2*x)+1)^3*b^2-1/a^2*b*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-2
/a^4*b^3*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/a^6*b^5*ln(tanh(1/2*x)^2*b
-2*a*tanh(1/2*x)-b)-1/4/a^2/(tanh(1/2*x)-1)^4*b-1/3/a^3/(tanh(1/2*x)-1)^3*b
^2-1/2/a^4/(tanh(1/2*x)-1)^2*b^3-1/a^5/(tanh(1/2*x)-1)*b^4+b/a^2*ln(tanh(1/
2*x)+1)+2*b^3/a^4*ln(tanh(1/2*x)+1)+b/a^2*ln(tanh(1/2*x)-1)+2*b^3/a^4*ln(ta
nh(1/2*x)-1)-1/2/a^3/(tanh(1/2*x)-1)^2*b^2-7/8/a^2/(tanh(1/2*x)-1)*b-2/a^3/
(tanh(1/2*x)-1)*b^2-1/2/a^4/(tanh(1/2*x)-1)*b^3+7/8/a^2/(tanh(1/2*x)+1)*b-2
/a^3/(tanh(1/2*x)+1)*b^2+1/2/a^4/(tanh(1/2*x)+1)*b^3-1/2/a^2/(tanh(1/2*x)-1
)^3*b-9/8/a^2/(tanh(1/2*x)-1)^2*b+1/2/a^2/(tanh(1/2*x)+1)^3*b-9/8/a^2/(tanh
(1/2*x)+1)^2*b+1/2/a^3/(tanh(1/2*x)+1)^2*b^2
```

Maxima [B] time = 1.01022, size = 327, normalized size = 3.21

$$-\frac{\left(15\,a^3be^{(-x)} - 6\,a^4 - 10\left(5\,a^4 + 4\,a^2b^2\right)e^{(-2\,x)} + 60\left(3\,a^3b + 2\,ab^3\right)e^{(-3\,x)} - 60\left(5\,a^4 + 14\,a^2b^2 + 8\,b^4\right)e^{(-4\,x)}\right)e^{(5\,x)}}{960\,a^5} - \frac{15\,a^3be^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a^3*b*e^(-x) - 6*a^4 - 10*(5*a^4 + 4*a^2*b^2)*e^(-2*x) + 60*(3*a
^3*b + 2*a*b^3)*e^(-3*x) - 60*(5*a^4 + 14*a^2*b^2 + 8*b^4)*e^(-4*x))*e^(5*x
)/a^5 - 1/960*(15*a^3*b*e^(-4*x) + 6*a^4*e^(-5*x) + 60*(5*a^4 + 14*a^2*b^2
+ 8*b^4)*e^(-x) + 60*(3*a^3*b + 2*a*b^3)*e^(-2*x) + 10*(5*a^4 + 4*a^2*b^2)*
e^(-3*x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*x/a^6 - (a^4*b + 2*a^2*b^3 + b^5)
*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^6
```

Fricas [B] time = 1.75196, size = 3536, normalized size = 34.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*a^5*cosh(x)^10 + 6*a^5*sinh(x)^10 - 15*a^4*b*cosh(x)^9 + 15*(4*a^5
*cosh(x) - a^4*b)*sinh(x)^9 + 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^8 + 5*(54*a^5*
cosh(x)^2 - 27*a^4*b*cosh(x) + 10*a^5 + 8*a^3*b^2)*sinh(x)^8 - 60*(3*a^4*b
+ 2*a^2*b^3)*cosh(x)^7 + 20*(36*a^5*cosh(x)^3 - 27*a^4*b*cosh(x)^2 - 9*a^4*
b - 6*a^2*b^3 + 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x)^7 + 960*(a^4*b + 2*a
^2*b^3 + b^5)*x*cosh(x)^5 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^6 + 2
0*(63*a^5*cosh(x)^4 - 63*a^4*b*cosh(x)^3 + 15*a^5 + 42*a^3*b^2 + 24*a*b^4 +
 14*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 - 21*(3*a^4*b + 2*a^2*b^3)*cosh(x))*sinh(
x)^6 - 15*a^4*b*cosh(x) + 2*(756*a^5*cosh(x)^5 - 945*a^4*b*cosh(x)^4 + 280*
(5*a^5 + 4*a^3*b^2)*cosh(x)^3 - 630*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 + 480*(
a^4*b + 2*a^2*b^3 + b^5)*x + 180*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x))*si
nh(x)^5 - 6*a^5 - 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 + 10*(126*a^5
*cosh(x)^6 - 189*a^4*b*cosh(x)^5 - 30*a^5 - 84*a^3*b^2 - 48*a*b^4 + 70*(5*a
^5 + 4*a^3*b^2)*cosh(x)^4 - 210*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 480*(a^4*
b + 2*a^2*b^3 + b^5)*x*cosh(x) + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^
2)*sinh(x)^4 - 60*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 20*(36*a^5*cosh(x)^7 -
63*a^4*b*cosh(x)^6 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^5 - 9*a^4*b - 6*a^2*b^3
 - 105*(3*a^4*b + 2*a^2*b^3)*cosh(x)^4 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x*co
sh(x)^2 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 12*(5*a^5 + 14*a^3*
b^2 + 8*a*b^4)*cosh(x))*sinh(x)^3 - 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 + 10*(
27*a^5*cosh(x)^8 - 54*a^4*b*cosh(x)^7 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^6 -
126*(3*a^4*b + 2*a^2*b^3)*cosh(x)^5 - 5*a^5 - 4*a^3*b^2 + 960*(a^4*b + 2*a^
2*b^3 + b^5)*x*cosh(x)^3 + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 - 36
*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^2 - 18*(3*a^4*b + 2*a^2*b^3)*cosh(x
))*sinh(x)^2 - 960*((a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4*sinh(x) + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^3*sinh(
x)^2 + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)*sinh(x)^4 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^5)*log(2*(
a*sinh(x) + b)/(cosh(x) - sinh(x))) + 5*(12*a^5*cosh(x)^9 - 27*a^4*b*cosh(x
)^8 + 16*(5*a^5 + 4*a^3*b^2)*cosh(x)^7 - 84*(3*a^4*b + 2*a^2*b^3)*cosh(x)^6
 + 960*(a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^4 + 72*(5*a^5 + 14*a^3*b^2 + 8*a
```

```
*b^4)*cosh(x)^5 - 3*a^4*b - 48*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 3
6*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 - 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x))
/(a^6*cosh(x)^5 + 5*a^6*cosh(x)^4*sinh(x) + 10*a^6*cosh(x)^3*sinh(x)^2 + 10
*a^6*cosh(x)^2*sinh(x)^3 + 5*a^6*cosh(x)*sinh(x)^4 + a^6*sinh(x)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15935, size = 262, normalized size = 2.57

$$-\frac{6\,a^4\left(e^{(-x)}-e^x\right)^5+15\,a^3b\left(e^{(-x)}-e^x\right)^4+80\,a^4\left(e^{(-x)}-e^x\right)^3+40\,a^2b^2\left(e^{(-x)}-e^x\right)^3+240\,a^3b\left(e^{(-x)}-e^x\right)^2+120\,ab^3(e^{(-x)}}{960\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/960*(6*a^4*(e^(-x) - e^x)^5 + 15*a^3*b*(e^(-x) - e^x)^4 + 80*a^4*(e^(-x)
 - e^x)^3 + 40*a^2*b^2*(e^(-x) - e^x)^3 + 240*a^3*b*(e^(-x) - e^x)^2 + 120*
a*b^3*(e^(-x) - e^x)^2 + 480*a^4*(e^(-x) - e^x) + 960*a^2*b^2*(e^(-x) - e^x
) + 480*b^4*(e^(-x) - e^x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(-a*(e^(
-x) - e^x) + 2*b))/a^6
```

3.93 $\int \frac{\cosh^4(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=125

$$\frac{x\left(12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}-\frac{\cosh^3(\ldots}{\ldots}$$

```
[Out] ((3*a^4 + 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b*(a^2 + b^2)^(3/2)*ArcTanh[(
a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - (Cosh[x]^3*(4*b - 3*a*Sinh[x]))/(1
2*a^2) - (Cosh[x]*(8*b*(a^2 + b^2) - a*(3*a^2 + 4*b^2)*Sinh[x]))/(8*a^4)
```

Rubi [A] time = 0.377696, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2865, 2735, 2660, 618, 204}

$$\frac{x\left(12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}-\frac{\cosh^3(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*a^4 + 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b*(a^2 + b^2)^(3/2)*ArcTanh[(
a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - (Cosh[x]^3*(4*b - 3*a*Sinh[x]))/(1
2*a^2) - (Cosh[x]*(8*b*(a^2 + b^2) - a*(3*a^2 + 4*b^2)*Sinh[x]))/(8*a^4)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^4(x)\sinh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}+\frac{\int \frac{\cosh^2(x)\left(-iab+i\left(3a^2+4b^2\right)\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{4a^2}\\
&= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}+\frac{\int \frac{-iab\left(5a^2+4b^2\right)+\ldots}{ib\ldots}}{\ldots}\\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sin\ldots\right.}{8a^4}\\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sin\ldots\right.}{8a^4}\\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sin\ldots\right.}{8a^4}\\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\ldots
\end{aligned}$$

Mathematica [A] time = 1.14641, size = 180, normalized size = 1.44

$$\frac{-24ab\left(5a^2+4b^2\right)\cosh(x)+3\left(48a^2b^2x+8a^2\left(a^2+b^2\right)\sinh(2x)+64a^2b\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+64b^3\sqrt{-a^2\ldots}\right.}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (-24*a*b*(5*a^2 + 4*b^2)*Cosh[x] - 8*a^3*b*Cosh[3*x] + 3*(12*a^4*x + 48*a^2
*b^2*x + 32*b^4*x + 64*a^2*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt
[-a^2 - b^2]] + 64*b^3*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2
- b^2]] + 8*a^2*(a^2 + b^2)*Sinh[2*x] + a^4*Sinh[4*x]))/(96*a^5)
```

Maple [B] time = 0.04, size = 486, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*csch(x)),x)
```

```
[Out] -2*b^5/a^5/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2
))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))
-1/4/a/(tanh(1/2*x)+1)^4+1/2/a/(tanh(1/2*x)+1)^3-7/8/a/(tanh(1/2*x)+1)^2+5/
8/a/(tanh(1/2*x)+1)+3/8/a*ln(tanh(1/2*x)+1)+1/4/a/(tanh(1/2*x)-1)^4+1/2/a/(
tanh(1/2*x)-1)^3+7/8/a/(tanh(1/2*x)-1)^2+5/8/a/(tanh(1/2*x)-1)-3/8/a*ln(tan
h(1/2*x)-1)-4*b^3/a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^
2+b^2)^(1/2))+1/2/a^3/(tanh(1/2*x)-1)^2*b^2-3/2/a^3*ln(tanh(1/2*x)-1)*b^2-1
/a^5*ln(tanh(1/2*x)-1)*b^4+3/2/a^2/(tanh(1/2*x)-1)*b+1/2/a^3/(tanh(1/2*x)-1
)*b^2+1/a^4/(tanh(1/2*x)-1)*b^3-3/2/a^2/(tanh(1/2*x)+1)*b+1/2/a^3/(tanh(1/2
*x)+1)*b^2-1/a^4/(tanh(1/2*x)+1)*b^3+1/3/a^2/(tanh(1/2*x)-1)^3*b+1/2/a^2/(t
anh(1/2*x)-1)^2*b+3/2/a^3*ln(tanh(1/2*x)+1)*b^2+1/a^5*ln(tanh(1/2*x)+1)*b^4
-1/3/a^2/(tanh(1/2*x)+1)^3*b+1/2/a^2/(tanh(1/2*x)+1)^2*b-1/2/a^3/(tanh(1/2*
x)+1)^2*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72759, size = 2414, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cos
h(x) - a^3*b)*sinh(x)^7 + 24*(a^4 + a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^
2 - 14*a^3*b*cosh(x) + 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 + 12*a^2*b^
2 + 8*b^4)*x*cosh(x)^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(
x)^3 - 7*a^3*b*cosh(x)^2 - 5*a^3*b - 4*a*b^3 + 6*(a^4 + a^2*b^2)*cosh(x))*s
inh(x)^5 - 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 + 1
80*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x - 60*(5*a^
3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^
3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b - 12*a*b^3 + 60*(a^
4 + a^2*b^2)*cosh(x)^3 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x) - 30*(5*
a^3*b + 4*a*b^3)*cosh(x)^2)*sinh(x)^3 - 24*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(
7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 + 30*(a^4 + a^2*b^2)*cosh(x)^4 - 2*a^4
 - 2*a^2*b^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 - 20*(5*a^3*b +
4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a^2*b
 + b^3)*cosh(x)^4 + 4*(a^2*b + b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b + b^3)*cos
h(x)^2*sinh(x)^2 + 4*(a^2*b + b^3)*cosh(x)*sinh(x)^3 + (a^2*b + b^3)*sinh(x
)^4)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a
^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) +
```

```
 a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
b)*sinh(x) - a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 + 18*(a^4 + a^2*b
^2)*cosh(x)^5 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 - 15*(5*a^3*b +
 4*a*b^3)*cosh(x)^4 - a^3*b - 9*(5*a^3*b + 4*a*b^3)*cosh(x)^2 - 6*(a^4 + a^
2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*c
osh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17293, size = 298, normalized size = 2.38

$$\frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-120\,a^2be^x-96\,b^3e^x}{192\,a^4}+\frac{\left(3\,a^4+12\,a^2b^2+8\,b^4\right)x}{8\,a^5}-\frac{\left(8\,a^3be^x+3\,a^4+24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
- 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x/a^5
- 1/192*(8*a^3*b*e^x + 3*a^4 + 24*(5*a^3*b + 4*a*b^3)*e^(3*x) + 24*(a^4 + a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(2*a*e^x +
 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2
 + b^2)*a^5)
```

3.94 $\int \frac{\cosh^3(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$\frac{(a^2+b^2)\sinh(x)}{a^3}-\frac{b\left(a^2+b^2\right)\log(a\sinh(x)+b)}{a^4}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

```
[Out] -((b*(a^2 + b^2)*Log[b + a*Sinh[x]])/a^4) + ((a^2 + b^2)*Sinh[x])/a^3 - (b*
Sinh[x]^2)/(2*a^2) + Sinh[x]^3/(3*a)
```

Rubi [A] time = 0.15953, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$\frac{(a^2+b^2)\sinh(x)}{a^3}-\frac{b\left(a^2+b^2\right)\log(a\sinh(x)+b)}{a^4}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)*Log[b + a*Sinh[x]])/a^4) + ((a^2 + b^2)*Sinh[x])/a^3 - (b*
Sinh[x]^2)/(2*a^2) + Sinh[x]^3/(3*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh^3(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^3}$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x(a^2-x^2)}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^4}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)-\frac{b(a^2+b^2)}{b-ix}+ibx-x^2\right)dx, x, ia\sinh(x)\right)}{a^4}$$
$$= -\frac{b\left(a^2+b^2\right)\log(b+a\sinh(x))}{a^4}+\frac{\left(a^2+b^2\right)\sinh(x)}{a^3}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

Mathematica [A] time = 0.107951, size = 56, normalized size = 0.98

$$\frac{6a\left(a^2+b^2\right)\sinh(x)-6b\left(a^2+b^2\right)\log(a\sinh(x)+b)-3a^2b\sinh^2(x)+2a^3\sinh^3(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-6*b*(a^2 + b^2)*Log[b + a*Sinh[x]] + 6*a*(a^2 + b^2)*Sinh[x] - 3*a^2*b*Si
nh[x]^2 + 2*a^3*Sinh[x]^3)/(6*a^4)
```

Maple [B] time = 0.035, size = 274, normalized size = 4.8

$$-\frac{1}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)+1)^3+1/2/a/(tanh(1/2*x)+1)^2-1/2/a^2/(tanh(1/2*x)+1)^2*
b-1/a/(tanh(1/2*x)+1)+1/2/a^2/(tanh(1/2*x)+1)*b-1/a^3/(tanh(1/2*x)+1)*b^2+b
/a^2*ln(tanh(1/2*x)+1)+b^3/a^4*ln(tanh(1/2*x)+1)-1/a^2*b*ln(tanh(1/2*x)^2*b
-2*a*tanh(1/2*x)-b)-1/a^4*b^3*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/3/a/(
tanh(1/2*x)-1)^3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*b-1/a/(t
anh(1/2*x)-1)-1/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2+b/a^2*ln(
tanh(1/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.00699, size = 171, normalized size = 3.

$$-\frac{\left(3\,abe^{(-x)}-a^2-3\left(3\,a^2+4\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}-\frac{3\,abe^{(-2\,x)}+a^2e^{(-3\,x)}+3\left(3\,a^2+4\,b^2\right)e^{(-x)}}{24\,a^3}-\frac{\left(a^2b+b^3\right)x}{a^4}-\frac{\left(a^2b+b^3\right)\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 - 3*(3*a^2 + 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) + a^2*e^(-3*x) + 3*(3*a^2 + 4*b^2)*e^(-x))/a^3 - (a^2*b + b
^3)*x/a^4 - (a^2*b + b^3)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^4
```

Fricas [B] time = 1.61395, size = 1251, normalized size = 21.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 + 24*(a^2*b + b^3)*x*cosh(x)^3 + 3*(3*a^3 + 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) + 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 + 12*(a^2*b +
b^3)*x + 6*(3*a^3 + 4*a*b^2)*cosh(x))*sinh(x)^3 - a^3 - 3*(3*a^3 + 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 - 4*a*b^2 + 24
*(a^2*b + b^3)*x*cosh(x) + 6*(3*a^3 + 4*a*b^2)*cosh(x)^2)*sinh(x)^2 - 24*((
a^2*b + b^3)*cosh(x)^3 + 3*(a^2*b + b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b + b^3
)*cosh(x)*sinh(x)^2 + (a^2*b + b^3)*sinh(x)^3)*log(2*(a*sinh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 + 24*(a^2*b + b^3)*
x*cosh(x)^2 + 4*(3*a^3 + 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 + 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*csch(x)), x)
```

Giac [A] time = 1.15823, size = 131, normalized size = 2.3

$$-\frac{a^2\left(e^{(-x)} - e^x\right)^3 + 3\,ab\left(e^{(-x)} - e^x\right)^2 + 12\,a^2\left(e^{(-x)} - e^x\right) + 12\,b^2\left(e^{(-x)} - e^x\right)}{24\,a^3} - \frac{\left(a^2b + b^3\right)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(a^2*(e^(-x) - e^x)^3 + 3*a*b*(e^(-x) - e^x)^2 + 12*a^2*(e^(-x) - e^x
) + 12*b^2*(e^(-x) - e^x))/a^3 - (a^2*b + b^3)*log(abs(-a*(e^(-x) - e^x) +
2*b))/a^4
```

3.95 $\int \frac{\cosh^2(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=77

$$\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/
Sqrt[a^2 + b^2]])/a^3 - (Cosh[x]*(2*b - a*Sinh[x]))/(2*a^2)
```

Rubi [A] time = 0.206218, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2865, 2735, 2660, 618, 204}

$$\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/
Sqrt[a^2 + b^2]])/a^3 - (Cosh[x]*(2*b - a*Sinh[x]))/(2*a^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\,\text{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{\int \frac{-iab+i\left(a^2+2b^2\right)\sinh(x)}{ib+ia\sinh(x)}\,dx}{2a^2}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{\left(ib\left(a^2+b^2\right)\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{\left(2ib\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{\left(4ib\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,2ia-2ib\tan
\end{aligned}
$$

$$
= \frac{\left(a^2+2b^2\right)x}{2a^3}+\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}
$$

Mathematica [A] time = 0.227557, size = 80, normalized size = 1.04

$$
\frac{8b\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+2a^2x+a^2\sinh(2x)-4ab\cosh(x)+4b^2x}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + 8*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^
2 - b^2]] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] time = 0.03, size = 172, normalized size = 2.2

$$
-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*csch(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)-1/a^2/(tanh(1/2*x)+1)*b+1/2/
a*ln(tanh(1/2*x)+1)+1/a^3*ln(tanh(1/2*x)+1)*b^2-2*b*(a^2+b^2)^(1/2)/a^3*arc
tanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+1/2/a/(tanh(1/2*x)-1)^2+1/2
/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b-1/2/a*ln(tanh(1/2*x)-1)-1/a^3*ln
(tanh(1/2*x)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57327, size = 859, normalized size = 11.16

$$\frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 + 4\left(a^2 + 2\,b^2\right)x\cosh(x)^2 + 4\left(a^2\cosh(x) - ab\right)\sinh(x)^3 - 4\,ab\cosh(x) + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 + 4*(a^2 + 2*b^2)*x*co
sh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*a^2*cosh(x
)^2 - 6*a*b*cosh(x) + 2*(a^2 + 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*c
osh(x)*sinh(x) + b*sinh(x)^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh
(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*
cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a*b*
cosh(x)^2 + 2*(a^2 + 2*b^2)*x*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.1684, size = 163, normalized size = 2.12

$$\frac{ae^{(2x)} - 4be^{x}}{8a^{2}} + \frac{\left(a^{2} + 2b^{2}\right)x}{2a^{3}} - \frac{\left(4abe^{x} + a^{2}\right)e^{(-2x)}}{8a^{3}} - \frac{\left(a^{2}b + b^{3}\right)\log\left(\frac{\left|2ae^{x}+2b-2\sqrt{a^{2}+b^{2}}\right|}{\left|2ae^{x}+2b+2\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2} + b^{2}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x +
a^2)*e^(-2*x)/a^3 - (a^2*b + b^3)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2)
)/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

3.96 $\int \frac{\cosh(x)}{a+b\,\textbf{csch}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\sinh(x)}{a}-\frac{b\log(a\sinh(x)+b)}{a^2}$$

```
[Out] -((b*Log[b + a*Sinh[x]])/a^2) + Sinh[x]/a
```

Rubi [A] time = 0.0800283, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\sinh(x)}{a}-\frac{b\log(a\sinh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*Log[b + a*Sinh[x]])/a^2) + Sinh[x]/a
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\cosh(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\operatorname{Subst}\left(\int \frac{x}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a}$$
$$= -\frac{i\operatorname{Subst}\left(\int \frac{x}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{i\operatorname{Subst}\left(\int \left(1-\frac{b}{b-ix}\right)\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

Mathematica [A] time = 0.0124498, size = 19, normalized size = 0.95

$$\frac{a\sinh(x)-b\log(a\sinh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

Maple [A] time = 0.022, size = 31, normalized size = 1.6

$$\frac{1}{a\operatorname{csch}(x)}+\frac{b\ln(\operatorname{csch}(x))}{a^2}-\frac{b\ln(a+b\operatorname{csch}(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*csch(x)),x)
```

```
[Out] 1/a/csch(x)+1/a^2*b*ln(csch(x))-1/a^2*b*ln(a+b*csch(x))
```

Maxima [B] time = 1.01637, size = 65, normalized size = 3.25

$$-\frac{bx}{a^2}-\frac{e^{(-x)}}{2a}+\frac{e^x}{2a}-\frac{b\log\left(-2be^{(-x)}+ae^{(-2x)}-a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 - 1/2*e^(-x)/a + 1/2*e^x/a - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
^2
```

Fricas [B] time = 1.58189, size = 246, normalized size = 12.3

$$\frac{2bx\cosh(x)+a\cosh(x)^2+a\sinh(x)^2-2(b\cosh(x)+b\sinh(x))\log\left(\frac{2(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)+2(bx+a\cosh(x))\sinh(x)-a}{2\left(a^2\cosh(x)+a^2\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) -
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{a + b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.16308, size = 53, normalized size = 2.65

$$-\frac{e^{(-x)} - e^{x}}{2\,a} - \frac{b \log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a - b*log(abs(-a*(e^(-x) - e^x) + 2*b))/a^2
```

3.97 $\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}+\frac{\log(-\sinh(x)+i)}{2(b+ia)}-\frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

```
[Out] Log[I - Sinh[x]]/(2*(I*a + b)) - Log[I + Sinh[x]]/(2*(I*a - b)) - (b*Log[b
+ a*Sinh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.110849, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2721, 801}

$$-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}+\frac{\log(-\sinh(x)+i)}{2(b+ia)}-\frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]]/(2*(I*a + b)) - Log[I + Sinh[x]]/(2*(I*a - b)) - (b*Log[b
+ a*Sinh[x]])/(a^2 + b^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\left(i\,\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\left(i\,\text{Subst}\left(\int \left(\frac{1}{2(a+ib)(a-x)}-\frac{b}{\left(a^2+b^2\right)(b-ix)}+\frac{1}{2(a-ib)(a+x)}\right)dx, x, ia\sinh(x)\right)\right)\\
&= \frac{\log(i-\sinh(x))}{2(ia+b)}-\frac{\log(i+\sinh(x))}{2(ia-b)}-\frac{b\log(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.0574008, size = 36, normalized size = 0.56

$$\frac{-b\log(a\sinh(x)+b)+2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] (2*a*ArcTan[Tanh[x/2]] + b*Log[Cosh[x]] - b*Log[b + a*Sinh[x]])/(a^2 + b^2)
```

Maple [A] time = 0.026, size = 84, normalized size = 1.3

$$-2\,\frac{b\ln\left((\tanh(x/2))^2 b-2\,a\tanh(x/2)-b\right)}{2\,a^2+2\,b^2}+2\,\frac{b\ln\left((\tanh(x/2))^2+1\right)}{2\,a^2+2\,b^2}+4\,\frac{a\arctan(\tanh(x/2))}{2\,a^2+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*csch(x)),x)
```

```
[Out] -2*b/(2*a^2+2*b^2)*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+2/(2*a^2+2*b^2)*b*
ln(tanh(1/2*x)^2+1)+4/(2*a^2+2*b^2)*a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.47917, size = 89, normalized size = 1.39

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^2
+ b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A] time = 1.73521, size = 177, normalized size = 2.77

$$\frac{2\,a\arctan(\cosh(x)+\sinh(x))-b\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)+b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) - b*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x)
)) + b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.14872, size = 120, normalized size = 1.88

$$-\frac{ab\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^3 + ab^2} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)a}{2\left(a^2 + b^2\right)} + \frac{b\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)}{2\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) + 1/2*b*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

3.98 $\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

```
[Out] (2*a*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (Sec
h[x]*(b - a*Sinh[x]))/(a^2 + b^2)
```

Rubi [A] time = 0.14187, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2866, 12, 2660, 618, 204}

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (Sec
h[x]*(b - a*Sinh[x]))/(a^2 + b^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{i\int \frac{ab}{ib+ia\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(iab)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(2iab)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}+\frac{(4iab)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.176511, size = 67, normalized size = 1.12

$$\frac{a\left(\tanh(x)-\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)-b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((-2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] + Tanh[x]))/(a^2 + b^2)
```

Maple [A] time = 0.029, size = 81, normalized size = 1.4

$$-4\,\frac{ab}{\left(2\,a^2+2\,b^2\right)\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\,\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)-2\,\frac{-a\tanh(x/2)+b}{\left(a^2+b^2\right)\left((\tanh(x/2))^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*csch(x)),x)
```

```
[Out] -4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2
+b^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*x)+b)/(tanh(1/2*x)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62838, size = 693, normalized size = 11.55

$$-\frac{2\,a^3+2\,ab^2-\left(ab\cosh\left(x\right)^2+2\,ab\cosh\left(x\right)\sinh\left(x\right)+ab\sinh\left(x\right)^2+ab\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)\cdots}{a\cosh(x)^2+a\,\mathrm{s}\cdots}\right.\cdots}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\left(a^4+2\,a^2b^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
+ a*b)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) - a)) + 2*(a^2*b + b^3)*cosh(x) + 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.15072, size = 115, normalized size = 1.92

$$-\frac{ab\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,\left(be^x+a\right)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x + a)/((a^2 + b^2)*(e^(2*x) + 1))
```

3.99 $\int \frac{\mathbf{sech}^3(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2 b \log(a\sinh(x)+b)}{(a^2+b^2)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2(a^2+b^2)}-\frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2}+\frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

```
[Out] ((-I/4)*a*Log[I - Sinh[x]])/(a - I*b)^2 + ((I/4)*a*Log[I + Sinh[x]])/(a + I
*b)^2 - (a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^2 - (Sech[x]^2*(b - a*Sinh[x
]))/(2*(a^2 + b^2))
```

Rubi [A] time = 0.220397, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^2 b \log(a\sinh(x)+b)}{(a^2+b^2)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2(a^2+b^2)}-\frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2}+\frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-I/4)*a*Log[I - Sinh[x]])/(a - I*b)^2 + ((I/4)*a*Log[I + Sinh[x]])/(a + I
*b)^2 - (a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^2 - (Sech[x]^2*(b - a*Sinh[x
]))/(2*(a^2 + b^2))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\left(\left(ia^3\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\left(\left(ia^2\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \frac{-ia^2b+a^2x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \left(\frac{a(a-ib)}{2(a+ib)(a-x)} - \frac{2a^2b}{\left(a^2+b^2\right)(b-ix)} + \frac{a(a+ib)}{2(a-ib)(a+x)}\right)dx, x, ia\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= -\frac{ia\log(i-\sinh(x))}{4(a-ib)^2} + \frac{ia\log(i+\sinh(x))}{4(a+ib)^2} - \frac{a^2b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.148813, size = 78, normalized size = 0.82

$$
\frac{-b\left(a^2+b^2\right)\operatorname{sech}^2(x) + a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}(x) + 2a\left(\left(a^2-b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + ab(\log(\cosh(x)) - \log(a\sinh(x)\right.}{2\left(a^2+b^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (2*a*((a^2 - b^2)*ArcTan[Tanh[x/2]] + a*b*(Log[Cosh[x]] - Log[b + a*Sinh[x]
])) - b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^
2)^2)
```

Maple [B] time = 0.036, size = 275, normalized size = 2.9

$$
-\frac{a^2b}{\left(a^2+b^2\right)^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 b - 2a\tanh(x/2) - b\right) - \frac{a^3}{\left(a^2+b^2\right)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - \frac{ab^2}{\left(a^2+b^2\right)^2}\left(\tanh\left(\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*csch(x)),x)
```

```
[Out] -a^2*b/(a^2+b^2)^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/(a^2+b^2)^2/(tan
h(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^3*a*b^2+2/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^2*b+2/(a^2+b
^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*b^3+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)
^2*tanh(1/2*x)*a^3+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a*b^2+1/(a
^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a^2*b+1/(a^2+b^2)^2*arctan(tanh(1/2*x))*a^3-1
/(a^2+b^2)^2*arctan(tanh(1/2*x))*a*b^2
```

Maxima [B] time = 1.51223, size = 217, normalized size = 2.28

$$-\frac{a^2 b \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{a^2 b \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{\left(a^3 - ab^2\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{ae^{(-x)} - 2\,be^{(-2\,x)} - ae^{(-}}{a^2 + b^2 + 2\left(a^2 + b^2\right)e^{(-2\,x)} + (a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^4 + 2*a^2*b^2 + b^4) + a^2*b*lo
g(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 - a*b^2)*arctan(e^(-x))/(a^4
 + 2*a^2*b^2 + b^4) + (a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Fricas [B] time = 1.73911, size = 1748, normalized size = 18.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 - 2*(a^2*b + b^3)*cosh(x
)^2 - (2*a^2*b + 2*b^3 - 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 - a*b^2
)*cosh(x)^4 + 4*(a^3 - a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - a*b^2)*sinh(x)^4 +
 a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))
*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(x) - (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3
 + a*b^2)*cosh(x)^2 + 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^
2*b^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)
*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)
*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*csch(x)), x)
```

Giac [B] time = 1.1754, size = 294, normalized size = 3.09

$$-\frac{a^{3}b\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^{5}+2\,a^{3}b^{2}+ab^{4}}+\frac{a^{2}b\log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^{3}-ab^{2}\right)}{4\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-\frac{a^{2}b\left(e^{(-x)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) + 1/2*a^
2*b*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*(a^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2
*b*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) + 2*a*b^2*(e^(-x) - e^x) + 8*a^2
*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

3.100 $\int \frac{\text{sech}^4(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=104

$$\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\text{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

```
[Out] (2*a^3*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (S
ech[x]^3*(b - a*Sinh[x]))/(3*(a^2 + b^2)) - (Sech[x]*(3*a^2*b - a*(2*a^2 -
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rubi [A] time = 0.270334, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2866, 12, 2660, 618, 204}

$$\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\text{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (2*a^3*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (S
ech[x]^3*(b - a*Sinh[x]))/(3*(a^2 + b^2)) - (Sech[x]*(3*a^2*b - a*(2*a^2 -
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}+\frac{\int \frac{\operatorname{sech}^2(x)\left(-iab+2ia^2\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)}\\
&= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}+\frac{\int -\frac{3ia^3b}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)^2}\\
&= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}-\frac{\left(ia^3b\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}\\
&= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}-\frac{\left(2ia^3b\right)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,d\right.}{\left(a^2+b^2\right)^2}\\
&= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}+\frac{\left(4ia^3b\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\right.}{\left(a^2+b\right.}\\
&= \frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.61086, size = 114, normalized size = 1.1

$$-\frac{\left(ab^2-2a^3\right)\tanh(x)+b\left(a^2+b^2\right)\operatorname{sech}^3(x)+\frac{6a^3b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}^2(x)+3a^2b\operatorname{sech}(x)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -((6*a^3*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3
*a^2*b*Sech[x] + b*(a^2 + b^2)*Sech[x]^3 + (-2*a^3 + a*b^2)*Tanh[x] - a*(a^
2 + b^2)*Sech[x]^2*Tanh[x])/(3*(a^2 + b^2)^2)
```

Maple [A] time = 0.04, size = 170, normalized size = 1.6

$$-4\,\frac{a^3b}{\left(2\,a^4+4\,a^2b^2+2\,b^4\right)\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,\tanh\left(x/2\right)b-2\,a}{\sqrt{a^2+b^2}}\right)-2\,\frac{-a^3\left(\tanh\left(x/2\right)\right)^5+\left(2\,a^2b+b^3\right)\left(\tanh\left(x/2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*csch(x)),x)
```

```
[Out] -4*a^3*b/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)
*b-2*a)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5+(2*a^2*b
+b^3)*tanh(1/2*x)^4+(-2/3*a^3+4/3*a*b^2)*tanh(1/2*x)^3+2*a^2*b*tanh(1/2*x)^
2-a^3*tanh(1/2*x)+4/3*a^2*b+1/3*b^3)/(tanh(1/2*x)^2+1)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80851, size = 2822, normalized size = 27.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 4*a^5
 + 2*a^3*b^2 - 2*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4
 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5)
*cosh(x)^3 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5 + 15*(a^4*b + a^2*b^3)*cosh(x)^
2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 +
 12*(a^5 + a^3*b^2 + 5*(a^4*b + a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 + a*b^4)*co
sh(x)^2 + (5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(
x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 + 3*a^3*b*cosh(x)^4 + 3*
a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^4 + a^3*b + 4*(5*a^
3*b*cosh(x)^3 + 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 + 6*a^3*b
*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 + 2*a^3*b*cosh(x)^3 + a^
3*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 +
2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) - a)) + 6*(a^4*b + a^2*b^3)*cosh(x) + 6*(a^4*b
+ a^2*b^3 + 5*(a^4*b + a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 +
 2*(5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 + a^3*b^2)*cosh(x))*sin
h(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
inh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b
```

```
^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sin
h(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*csch(x)), x)
```

Giac [A] time = 1.21967, size = 235, normalized size = 2.26

$$-\frac{a^3 b\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}+10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x+2\,a^3-ab^2\right)}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqr
t(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(
5*x) - 3*a*b^2*e^(4*x) + 10*a^2*b*e^(3*x) + 4*b^3*e^(3*x) + 6*a^3*e^(2*x) +
 3*a^2*b*e^x + 2*a^3 - a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

3.101 $\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=149

$$-\frac{a^4b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^3}-\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{a(b+3ia)\log(-\sinh(x$$

```
[Out] -(a*((3*I)*a + b)*Log[I - Sinh[x]])/(16*(a - I*b)^3) + (a*(3*a + I*b)*Log[I
 + Sinh[x]])/(16*(I*a - b)^3) - (a^4*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^3 -
(Sech[x]^4*(b - a*Sinh[x]))/(4*(a^2 + b^2)) - (Sech[x]^2*(4*a^2*b - a*(3*a^
2 - b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rubi [A] time = 0.340972, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^4b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^3}-\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{a(b+3ia)\log(-\sinh(x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Csch[x]),x]
```


```
[Out] -(a*((3*I)*a + b)*Log[I - Sinh[x]])/(16*(a - I*b)^3) + (a*(3*a + I*b)*Log[I
 + Sinh[x]])/(16*(I*a - b)^3) - (a^4*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^3 -
(Sech[x]^4*(b - a*Sinh[x]))/(4*(a^2 + b^2)) - (Sech[x]^2*(4*a^2*b - a*(3*a^
2 - b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}^4(x)\tanh(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\left(\left(ia^5\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x)\right)\right)$$

$$= -\left(\left(ia^4\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x)\right)\right)$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\left(ia^2\right)\operatorname{Subst}\left(\int \frac{-ia^2b+3a^2x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)}{4\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} - \frac{i\operatorname{Subst}\left(\int \frac{-ia^2b\left(5a^2+b^2\right)+a^2\left(3a^2\right.}{(ib+x)\left(a^2-x^2\right)}\right.}{8\left(a^2+b^2\right.}$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} - \frac{i\operatorname{Subst}\left(\int\left(\frac{a(a-ib)^2(3a+ib)}{2(a+ib)(a-x)} - \right.\right.}{}$$

$$= -\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3} + \frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3} - \frac{a^4b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^3} - \frac{\operatorname{sech}^4(x)(b-}{4\left(a^2+\right.}$$

Mathematica [A] time = 0.274322, size = 138, normalized size = 0.93

$$\frac{-4a^2b\left(a^2+b^2\right)\operatorname{sech}^2(x)-2b\left(a^2+b^2\right)^2\operatorname{sech}^4(x)+2a\left(-6a^2b^2+3a^4-b^4\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+2a\left(a^2+b^2\right)^2\tanh(x)\operatorname{sec}}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (2*a*(3*a^4 - 6*a^2*b^2 - b^4)*ArcTan[Tanh[x/2]] + 8*a^4*b*(Log[Cosh[x]] -
Log[b + a*Sinh[x]]) - 4*a^2*b*(a^2 + b^2)*Sech[x]^2 - 2*b*(a^2 + b^2)^2*Sec
h[x]^4 + a*(3*a^4 + 2*a^2*b^2 - b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*Se
ch[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

Maple [B] time = 0.046, size = 1168, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*csch(x)),x)
```

```
[Out] -3/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^3*b^
2+6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^2*b^3
+3/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^3*b^2-
1/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a*b^4+4
/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^4*b+4/(a
^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^2*b^3-5/2/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^3*b^2-7/4/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a*b^4+4/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^4*b+1/4/(a^4
+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a*b^4+6/(a^4+2*a^
2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^2*b^3+5/2/(a^4+2*a
^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^3*b^2+7/4/(a^4+2*
a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a*b^4+4/(a^4+2*a^2
*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^4*b+3/4/(a^4+2*a^2*
b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^5+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/
(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*b^5-a^4*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*l
n(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arct
an(tanh(1/2*x))*a*b^4+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2+1)*a
^4*b+3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^
5-3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^5+2
/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*b^5-5/4/(a
^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^5-3/2/(a^4+
2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^3*b^2+5/4/(a^4+2*a^2*b^2+b^4
)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^5
```

Maxima [B] time = 1.58297, size = 470, normalized size = 3.15

$$-\frac{a^{4} b \log\left(-2\,b e^{(-x)} + a e^{(-2\,x)} - a\right)}{a^{6} + 3\,a^{4} b^{2} + 3\,a^{2} b^{4} + b^{6}} + \frac{a^{4} b \log\left(e^{(-2\,x)} + 1\right)}{a^{6} + 3\,a^{4} b^{2} + 3\,a^{2} b^{4} + b^{6}} - \frac{\left(3\,a^{5} - 6\,a^{3} b^{2} - a b^{4}\right) \arctan\left(e^{(-x)}\right)}{4\left(a^{6} + 3\,a^{4} b^{2} + 3\,a^{2} b^{4} + b^{6}\right)} - \frac{8\,a^{2} b e^{(-2\,x)} + 8}{4\left(a^{4} + 2\,a^{2} b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + a^4*b*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5 - 6*a^3*b^2 - a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 1/4*(8*a^2*b*e^(-2*x) + 8*a^2*b*e^(-6*x) - (3*a^3 - a*b^2)*e^(-x) - (11*a^
3 + 7*a*b^2)*e^(-3*x) + 16*(2*a^2*b + b^3)*e^(-4*x) + (11*a^3 + 7*a*b^2)*e^
(-5*x) + (3*a^3 - a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b
^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] time = 2.28082, size = 6677, normalized size = 44.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^7 + (3*a^5 + 2*a^3*b^2 - a*b^4)*si
nh(x)^7 - 8*(a^4*b + a^2*b^3)*cosh(x)^6 - (8*a^4*b + 8*a^2*b^3 - 7*(3*a^5 +
 2*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*co
sh(x)^5 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4 + 21*(3*a^5 + 2*a^3*b^2 - a*b^4)*c
osh(x)^2 - 48*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^5 - 16*(2*a^4*b + 3*a^2*b^
3 + b^5)*cosh(x)^4 - (32*a^4*b + 48*a^2*b^3 + 16*b^5 - 35*(3*a^5 + 2*a^3*b^
2 - a*b^4)*cosh(x)^3 + 120*(a^4*b + a^2*b^3)*cosh(x)^2 - 5*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*cosh(x
)^3 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4 - 35*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(
x)^4 + 160*(a^4*b + a^2*b^3)*cosh(x)^3 - 10*(11*a^5 + 18*a^3*b^2 + 7*a*b^4)
*cosh(x)^2 + 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 - 8*(a^4*b +
 a^2*b^3)*cosh(x)^2 + (21*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^5 - 8*a^4*b -
 8*a^2*b^3 - 120*(a^4*b + a^2*b^3)*cosh(x)^4 + 10*(11*a^5 + 18*a^3*b^2 + 7*
a*b^4)*cosh(x)^3 - 96*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 3*(11*a^5 + 1
8*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 - 6*a^3*b^2 - a*b^4)*cosh
(x)^8 + 8*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 - 6*a^3*b^
2 - a*b^4)*sinh(x)^8 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 4*(3*a^5 -
 6*a^3*b^2 - a*b^4 + 7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^6 + 8
*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^5 + 3*a^5 - 6*a^3*b^2 - a*b^4 + 6*(3*a^5 - 6*a^3*b^2 - a*b^
4)*cosh(x)^4 + 2*(9*a^5 - 18*a^3*b^2 - 3*a*b^4 + 35*(3*a^5 - 6*a^3*b^2 - a*
b^4)*cosh(x)^4 + 30*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7
*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 10*(3*a^5 - 6*a^3*b^2 - a*b^4)*cos
h(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 - 6*a^
3*b^2 - a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 3*a
^5 - 6*a^3*b^2 - a*b^4 + 15*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^4 + 9*(3*a^
5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 6*a^3*b^2 - a*b^4
)*cosh(x)^7 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(3*a^5 - 6*a^3*b^
2 - a*b^4)*cosh(x)^3 + (3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan
(cosh(x) + sinh(x)) - (3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x) - 4*(a^4*b*cosh(x
)^8 + 8*a^4*b*cosh(x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a
^4*b*cosh(x)^4 + 4*a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^
6 + 8*(7*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b
*cosh(x)^4 + 30*a^4*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5
 + 10*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 +
 15*a^4*b*cosh(x)^4 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(
x)^7 + 3*a^4*b*cosh(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(
2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 4*(a^4*b*cosh(x)^8 + 8*a^4*b*cosh(
x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a^4*b*cosh(x)^4 + 4*
a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^6 + 8*(7*a^4*b*cosh
(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b*cosh(x)^4 + 30*a^4
*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5 + 10*a^4*b*cosh(x)
^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 + 15*a^4*b*cosh(x)^4
 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(x)^7 + 3*a^4*b*cosh
(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
- sinh(x))) + (7*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^6 - 48*(a^4*b + a^2*b^
3)*cosh(x)^5 - 3*a^5 - 2*a^3*b^2 + a*b^4 + 5*(11*a^5 + 18*a^3*b^2 + 7*a*b^4
)*cosh(x)^4 - 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x)^2 - 16*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^8 + 4*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 + 6*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2 + 9*a^2*b
^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 30*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)
```

```
^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^
2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 + 3*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*csch(x)), x)
```

Giac [B] time = 1.1783, size = 505, normalized size = 3.39

$$-\frac{a^5 b \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^7 + 3\,a^5 b^2 + 3\,a^3 b^4 + a b^6} + \frac{a^4 b \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^5 - 6\,a^3 b^2 -\right.}{16\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^5*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6) + 1/2*a^4*b*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 - 6*a^3*b^2 - a*
b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b*(e^(-x) - e^x)^4 +
3*a^5*(e^(-x) - e^x)^3 + 2*a^3*b^2*(e^(-x) - e^x)^3 - a*b^4*(e^(-x) - e^x)^
3 + 32*a^4*b*(e^(-x) - e^x)^2 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
 - e^x) + 24*a^3*b^2*(e^(-x) - e^x) + 4*a*b^4*(e^(-x) - e^x) + 96*a^4*b + 6
4*a^2*b^3 + 16*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2
+ 4)^2)
```

3.102 $\int \frac{\tanh^5(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=109

$$-\frac{i}{4(1-i\sinh(x))}-\frac{15i}{16(1+i\sinh(x))}+\frac{i}{32(1-i\sinh(x))^2}+\frac{9i}{32(1+i\sinh(x))^2}-\frac{i}{24(1+i\sinh(x))^3}-\frac{21}{32}i\log(-\sinh(x)$$

```
[Out] ((-21*I)/32)*Log[I - Sinh[x]] - ((11*I)/32)*Log[I + Sinh[x]] + (I/32)/(1 -
I*Sinh[x])^2 - (I/4)/(1 - I*Sinh[x]) - (I/24)/(1 + I*Sinh[x])^3 + ((9*I)/32
)/(1 + I*Sinh[x])^2 - ((15*I)/16)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0860622, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$-\frac{i}{4(1-i\sinh(x))}-\frac{15i}{16(1+i\sinh(x))}+\frac{i}{32(1-i\sinh(x))^2}+\frac{9i}{32(1+i\sinh(x))^2}-\frac{i}{24(1+i\sinh(x))^3}-\frac{21}{32}i\log(-\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-21*I)/32)*Log[I - Sinh[x]] - ((11*I)/32)*Log[I + Sinh[x]] + (I/32)/(1 -
I*Sinh[x])^2 - (I/4)/(1 - I*Sinh[x]) - (I/24)/(1 + I*Sinh[x])^3 + ((9*I)/32
)/(1 + I*Sinh[x])^2 - ((15*I)/16)/(1 + I*Sinh[x])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{x^6}{(i-ix)^3(i+ix)^4}\,dx, x, i\sinh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-\frac{i}{16(-1+x)^3}-\frac{i}{4(-1+x)^2}-\frac{11i}{32(-1+x)}+\frac{i}{8(1+x)^4}-\frac{9i}{16(1+x)^3}+\frac{15i}{16(1+x)^2}-\frac{21}{32(1\right.\right.\\ &= -\frac{21}{32}i\log(i-\sinh(x))-\frac{11}{32}i\log(i+\sinh(x))+\frac{i}{32(1-i\sinh(x))^2}-\frac{i}{4(1-i\sinh(x))}-\frac{i}{24(1+i\sinh(\end{aligned}$$

Mathematica [A] time = 0.187191, size = 75, normalized size = 0.69

$$\frac{1}{96}\left(-\frac{2\left(33\sinh^4(x)+39i\sinh^3(x)+79\sinh^2(x)+29i\sinh(x)+44\right)}{(\sinh(x)-i)^3(\sinh(x)+i)^2}-63i\log(-\sinh(x)+i)-33i\log(\sinh(x)+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-63*I)*Log[I - Sinh[x]] - (33*I)*Log[I + Sinh[x]] - (2*(44 + (29*I)*Sinh[
x] + 79*Sinh[x]^2 + (39*I)*Sinh[x]^3 + 33*Sinh[x]^4))/((-I + Sinh[x])^3*(I
+ Sinh[x])^2))/96
```

Maple [A] time = 0.062, size = 155, normalized size = 1.4

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{21\,i}{16}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{3\,i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{i}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-6}-\frac{3\,i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}+\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)+1)-21/16*I*ln(tanh(1/2*x)-I)+3/8*I/(tanh(1/2*x)-I)^2+1/3*I
/(tanh(1/2*x)-I)^6-3/8*I/(tanh(1/2*x)-I)^4+1/(tanh(1/2*x)-I)^5+11/12/(tanh(
1/2*x)-I)^3+1/(tanh(1/2*x)-I)+I*ln(tanh(1/2*x)-1)-11/16*I*ln(tanh(1/2*x)+I)
+1/8*I/(tanh(1/2*x)+I)^4+1/4*I/(tanh(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)^3-3/8/
(tanh(1/2*x)+I)
```

Maxima [B] time = 1.02221, size = 194, normalized size = 1.78

$$-i\,x+\frac{33\,e^{(-x)}+78i\,e^{(-2\,x)}+184\,e^{(-3\,x)}-2i\,e^{(-4\,x)}+270\,e^{(-5\,x)}+2i\,e^{(-6\,x)}+184\,e^{(-7\,x)}-78i\,e^{(-8\,x)}+33\,e^{(-9\,x)}}{48i\,e^{(-x)}-72\,e^{(-2\,x)}+192i\,e^{(-3\,x)}-48\,e^{(-4\,x)}+288i\,e^{(-5\,x)}+48\,e^{(-6\,x)}+192i\,e^{(-7\,x)}+72\,e^{(-8\,x)}+48i\,e^{(-9\,x)}+24\,e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (33*e^(-x) + 78*I*e^(-2*x) + 184*e^(-3*x) - 2*I*e^(-4*x) + 270*e^(-5
*x) + 2*I*e^(-6*x) + 184*e^(-7*x) - 78*I*e^(-8*x) + 33*e^(-9*x))/(48*I*e^(-
x) - 72*e^(-2*x) + 192*I*e^(-3*x) - 48*e^(-4*x) + 288*I*e^(-5*x) + 48*e^(-6
*x) + 192*I*e^(-7*x) + 72*e^(-8*x) + 48*I*e^(-9*x) + 24*e^(-10*x) - 24) - 1
1/16*I*log(e^(-x) - I) - 21/16*I*log(I*e^(-x) - 1)
```

Fricas [B] time = 1.73703, size = 991, normalized size = 9.09

$$\frac{48i\,xe^{(10\,x)}+6\,(16\,x-11)e^{(9\,x)}+(144i\,x-156i)e^{(8\,x)}+16\,(24\,x-23)e^{(7\,x)}+(96i\,x+4i)e^{(6\,x)}+36\,(16\,x-15)e^{(5\,x)}+(-9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (48*I*x*e^(10*x) + 6*(16*x - 11)*e^(9*x) + (144*I*x - 156*I)*e^(8*x) + 16*(
24*x - 23)*e^(7*x) + (96*I*x + 4*I)*e^(6*x) + 36*(16*x - 15)*e^(5*x) + (-96
*I*x - 4*I)*e^(4*x) + 16*(24*x - 23)*e^(3*x) + (-144*I*x + 156*I)*e^(2*x) +
 6*(16*x - 11)*e^x + (-33*I*e^(10*x) - 66*e^(9*x) - 99*I*e^(8*x) - 264*e^(7
*x) - 66*I*e^(6*x) - 396*e^(5*x) + 66*I*e^(4*x) - 264*e^(3*x) + 99*I*e^(2*x
```

```
) - 66*e^x + 33*I)*log(e^x + I) + (-63*I*e^(10*x) - 126*e^(9*x) - 189*I*e^(
8*x) - 504*e^(7*x) - 126*I*e^(6*x) - 756*e^(5*x) + 126*I*e^(4*x) - 504*e^(3
*x) + 189*I*e^(2*x) - 126*e^x + 63*I)*log(e^x - I) - 48*I*x)/(48*e^(10*x) -
 96*I*e^(9*x) + 144*e^(8*x) - 384*I*e^(7*x) + 96*e^(6*x) - 576*I*e^(5*x) -
96*e^(4*x) - 384*I*e^(3*x) - 144*e^(2*x) - 96*I*e^x - 48)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(csch(x) + I), x)
```

Giac [A] time = 1.16329, size = 162, normalized size = 1.49

$$-\frac{33i\left(e^{(-x)} - e^{x}\right)^2 + 100\,e^{(-x)} - 100\,e^{x} - 76i}{64\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)^2} - \frac{-231i\left(e^{(-x)} - e^{x}\right)^3 + 1026\left(e^{(-x)} - e^{x}\right)^2 + 1548i\,e^{(-x)} - 1548i\,e^{x} - 776}{192\left(e^{(-x)} - e^{x} + 2i\right)^3} - \frac{11}{32}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/64*(33*I*(e^(-x) - e^x)^2 + 100*e^(-x) - 100*e^x - 76*I)/(-I*e^(-x) + I*
e^x - 2)^2 - 1/192*(-231*I*(e^(-x) - e^x)^3 + 1026*(e^(-x) - e^x)^2 + 1548*
I*e^(-x) - 1548*I*e^x - 776)/(e^(-x) - e^x + 2*I)^3 - 11/32*I*log(-e^(-x) +
 e^x + 2*I) - 21/32*I*log(-e^(-x) + e^x - 2*I)
```

3.103 $\int \frac{\tanh^4(x)}{i+\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=52

$$-ix+\frac{1}{5}\tanh^5(x)(-\mathrm{csch}(x)+i)+\frac{1}{15}\tanh^3(x)(-4\mathrm{csch}(x)+5i)+\frac{1}{15}\tanh(x)(-8\mathrm{csch}(x)+15i)$$

```
[Out] (-I)*x + ((15*I - 8*Csch[x])*Tanh[x])/15 + ((5*I - 4*Csch[x])*Tanh[x]^3)/15
 + ((I - Csch[x])*Tanh[x]^5)/5
```

Rubi [A] time = 0.0934978, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$-ix+\frac{1}{5}\tanh^5(x)(-\mathrm{csch}(x)+i)+\frac{1}{15}\tanh^3(x)(-4\mathrm{csch}(x)+5i)+\frac{1}{15}\tanh(x)(-8\mathrm{csch}(x)+15i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((15*I - 8*Csch[x])*Tanh[x])/15 + ((5*I - 4*Csch[x])*Tanh[x]^3)/15
 + ((I - Csch[x])*Tanh[x]^5)/5
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{i+\mathrm{csch}(x)}\,dx &= \int (-i+\mathrm{csch}(x))\tanh^6(x)\,dx\\
&= \frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)-\frac{1}{5}\int (5i-4\mathrm{csch}(x))\tanh^4(x)\,dx\\
&= \frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)+\frac{1}{15}\int (-15i+8\mathrm{csch}(x))\tanh^2(x)\,dx\\
&= \frac{1}{15}(15i-8\mathrm{csch}(x))\tanh(x)+\frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)-\frac{1}{15}\int 15i\\
&= -ix+\frac{1}{15}(15i-8\mathrm{csch}(x))\tanh(x)+\frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)
\end{aligned}$$

Mathematica [B] time = 0.125434, size = 126, normalized size = 2.42

$$\frac{64i\sinh(x)+240x\sinh(2x)+178i\sinh(2x)+128i\sinh(3x)+120x\sinh(4x)+89i\sinh(4x)+6(89-120ix)\cosh(x)-1}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-200 + 6*(89 - (120*I)*x)*Cosh[x] - 128*Cosh[2*x] + 178*Cosh[3*x] - (240*I
)*x*Cosh[3*x] - 184*Cosh[4*x] + (64*I)*Sinh[x] + (178*I)*Sinh[2*x] + 240*x*
Sinh[2*x] + (128*I)*Sinh[3*x] + (89*I)*Sinh[4*x] + 120*x*Sinh[4*x])/(960*(C
osh[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [B] time = 0.056, size = 99, normalized size = 1.9

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{11\,i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{2\,i}{5}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-5}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+i\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+csch(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+11/8*I/(tanh(1/2*x)-I)+2/5*I/(tanh(1/2*x)-I)^5+1/(tanh
(1/2*x)-I)^4+1/(tanh(1/2*x)-I)^2+I*ln(tanh(1/2*x)-1)+5/8*I/(tanh(1/2*x)+I)+
1/6*I/(tanh(1/2*x)+I)^3-1/4/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 1.02734, size = 130, normalized size = 2.5

$$-i\,x-\frac{62\,e^{(-x)}+62i\,e^{(-2\,x)}+146\,e^{(-3\,x)}+50i\,e^{(-4\,x)}+130\,e^{(-5\,x)}-30i\,e^{(-6\,x)}+30\,e^{(-7\,x)}+46i}{30i\,e^{(-x)}-30\,e^{(-2\,x)}+90i\,e^{(-3\,x)}+90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}+30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - (62*e^(-x) + 62*I*e^(-2*x) + 146*e^(-3*x) + 50*I*e^(-4*x) + 130*e^(-
5*x) - 30*I*e^(-6*x) + 30*e^(-7*x) + 46*I)/(30*I*e^(-x) - 30*e^(-2*x) + 90*
I*e^(-3*x) + 90*I*e^(-5*x) + 30*e^(-6*x) + 30*I*e^(-7*x) + 15*e^(-8*x) - 15
)
```

Fricas [B] time = 1.67382, size = 394, normalized size = 7.58

$$\frac{-15i\,xe^{(8\,x)}-30\,(x+1)e^{(7\,x)}+(-30i\,x-30i)e^{(6\,x)}-10\,(9\,x+13)e^{(5\,x)}-2\,(45\,x+73)e^{(3\,x)}+(30i\,x+62i)e^{(2\,x)}-2\,(15\,x+3}{15\,e^{(8\,x)}-30i\,e^{(7\,x)}+30\,e^{(6\,x)}-90i\,e^{(5\,x)}-90i\,e^{(3\,x)}-30\,e^{(2\,x)}-30i\,e^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (-15*I*x*e^(8*x) - 30*(x + 1)*e^(7*x) + (-30*I*x - 30*I)*e^(6*x) - 10*(9*x
+ 13)*e^(5*x) - 2*(45*x + 73)*e^(3*x) + (30*I*x + 62*I)*e^(2*x) - 2*(15*x +
```

```
31)*e^x + 15*I*x + 50*I*e^(4*x) + 46*I)/(15*e^(8*x) - 30*I*e^(7*x) + 30*e^
(6*x) - 90*I*e^(5*x) - 90*I*e^(3*x) - 30*e^(2*x) - 30*I*e^x - 15)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(csch(x) + I), x)
```

Giac [A] time = 1.21189, size = 84, normalized size = 1.62

$$-\frac{21 i e^{(2x)}-36 e^x-19 i}{24 (i e^x-1)^3}-\frac{115 e^{(4x)}-380 i e^{(3x)}-530 e^{(2x)}+340 i e^x+91}{40 (e^x-i)^5}-i \log(i e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(21*I*e^(2*x) - 36*e^x - 19*I)/(I*e^x - 1)^3 - 1/40*(115*e^(4*x) - 38
0*I*e^(3*x) - 530*e^(2*x) + 340*I*e^x + 91)/(e^x - I)^5 - I*log(I*e^x)
```

3.104 $\int \frac{\tanh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{i}{8(1-i\sinh(x))}-\frac{3i}{4(1+i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{11}{16}i\log(-\sinh(x)+i)-\frac{5}{16}i\log(\sinh(x)+i)$$

```
[Out] ((-11*I)/16)*Log[I - Sinh[x]] - ((5*I)/16)*Log[I + Sinh[x]] - (I/8)/(1 - I*
Sinh[x]) + (I/8)/(1 + I*Sinh[x])^2 - ((3*I)/4)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0672505, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$-\frac{i}{8(1-i\sinh(x))}-\frac{3i}{4(1+i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{11}{16}i\log(-\sinh(x)+i)-\frac{5}{16}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)/16)*Log[I - Sinh[x]] - ((5*I)/16)*Log[I + Sinh[x]] - (I/8)/(1 - I*
Sinh[x]) + (I/8)/(1 + I*Sinh[x])^2 - ((3*I)/4)/(1 + I*Sinh[x])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{x^4}{(i-ix)^2(i+ix)^3}\,dx,x,i\sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(-\frac{i}{8(-1+x)^2}-\frac{5i}{16(-1+x)}-\frac{i}{4(1+x)^3}+\frac{3i}{4(1+x)^2}-\frac{11i}{16(1+x)}\right)dx,x,i\sinh(x)\right)\\
&= -\frac{11}{16}i\log(i-\sinh(x))-\frac{5}{16}i\log(i+\sinh(x))-\frac{i}{8(1-i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{3i}{4(1+i\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.115431, size = 61, normalized size = 0.79

$$\frac{1}{16}\left(-\frac{2\left(5\sinh^2(x)+3i\sinh(x)+6\right)}{(\sinh(x)-i)^2(\sinh(x)+i)}-11i\log(-\sinh(x)+i)-5i\log(\sinh(x)+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)*Log[I - Sinh[x]] - (5*I)*Log[I + Sinh[x]] - (2*(6 + (3*I)*Sinh[x]
+ 5*Sinh[x]^2))/((-I + Sinh[x])^2*(I + Sinh[x])))/16
```

Maple [A] time = 0.055, size = 109, normalized size = 1.4

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{11\,i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}+\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)+1)-11/8*I*ln(tanh(1/2*x)-I)+1/2*I/(tanh(1/2*x)-I)^4+1/2*I/
(tanh(1/2*x)-I)^2+1/(tanh(1/2*x)-I)^3+1/(tanh(1/2*x)-I)+I*ln(tanh(1/2*x)-1)
-5/8*I*ln(tanh(1/2*x)+I)+1/4*I/(tanh(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.04781, size = 130, normalized size = 1.69

$$-i\,x+\frac{5\,e^{(-x)}+6i\,e^{(-2\,x)}+14\,e^{(-3\,x)}-6i\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{8i\,e^{(-x)}-4\,e^{(-2\,x)}+16i\,e^{(-3\,x)}+4\,e^{(-4\,x)}+8i\,e^{(-5\,x)}+4\,e^{(-6\,x)}-4}-\frac{5}{8}i\log\left(e^{(-x)}-i\right)-\frac{11}{8}i\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (5*e^(-x) + 6*I*e^(-2*x) + 14*e^(-3*x) - 6*I*e^(-4*x) + 5*e^(-5*x))/
(8*I*e^(-x) - 4*e^(-2*x) + 16*I*e^(-3*x) + 4*e^(-4*x) + 8*I*e^(-5*x) + 4*e^
(-6*x) - 4) - 5/8*I*log(e^(-x) - I) - 11/8*I*log(I*e^(-x) - 1)
```

Fricas [B] time = 1.58237, size = 568, normalized size = 7.38

$$\frac{8i\,xe^{(6\,x)}+2\,(8\,x-5)e^{(5\,x)}+(8i\,x-12i)e^{(4\,x)}+4\,(8\,x-7)e^{(3\,x)}+(-8i\,x+12i)e^{(2\,x)}+2\,(8\,x-5)e^{x}+\left(-5i\,e^{(6\,x)}-10\,e^{(5\,x)}\right.}{8\,e^{(6\,x)}-16i\,e^{(5\,x)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (8*I*x*e^(6*x) + 2*(8*x - 5)*e^(5*x) + (8*I*x - 12*I)*e^(4*x) + 4*(8*x - 7)
*e^(3*x) + (-8*I*x + 12*I)*e^(2*x) + 2*(8*x - 5)*e^x + (-5*I*e^(6*x) - 10*e
^(5*x) - 5*I*e^(4*x) - 20*e^(3*x) + 5*I*e^(2*x) - 10*e^x + 5*I)*log(e^x + I
) + (-11*I*e^(6*x) - 22*e^(5*x) - 11*I*e^(4*x) - 44*e^(3*x) + 11*I*e^(2*x)
- 22*e^x + 11*I)*log(e^x - I) - 8*I*x)/(8*e^(6*x) - 16*I*e^(5*x) + 8*e^(4*x
) - 32*I*e^(3*x) - 8*e^(2*x) - 16*I*e^x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(csch(x) + I), x)
```

Giac [B] time = 1.13793, size = 132, normalized size = 1.71

$$\frac{5\,e^{(-x)}-5\,e^{x}-6i}{16\left(-i\,e^{(-x)}+i\,e^{x}-2\right)}+\frac{33i\left(e^{(-x)}-e^{x}\right)^{2}-84\,e^{(-x)}+84\,e^{x}-52i}{32\left(e^{(-x)}-e^{x}+2i\right)^{2}}-\frac{5}{16}i\,\log\left(i\,e^{(-x)}-i\,e^{x}+2\right)-\frac{11}{16}i\,\log\left(i\,e^{(-x)}-i\,e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/16*(5*e^(-x) - 5*e^x - 6*I)/(-I*e^(-x) + I*e^x - 2) + 1/32*(33*I*(e^(-x)
- e^x)^2 - 84*e^(-x) + 84*e^x - 52*I)/(e^(-x) - e^x + 2*I)^2 - 5/16*I*log(I
*e^(-x) - I*e^x + 2) - 11/16*I*log(I*e^(-x) - I*e^x - 2)
```

3.105 $\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=36

$$-ix+\frac{1}{3}\tanh^3(x)(-\operatorname{csch}(x)+i)+\frac{1}{3}\tanh(x)(-2\operatorname{csch}(x)+3i)$$

```
[Out] (-I)*x + ((3*I - 2*Csch[x])*Tanh[x])/3 + ((I - Csch[x])*Tanh[x]^3)/3
```

Rubi [A] time = 0.0732533, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$-ix+\frac{1}{3}\tanh^3(x)(-\operatorname{csch}(x)+i)+\frac{1}{3}\tanh(x)(-2\operatorname{csch}(x)+3i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((3*I - 2*Csch[x])*Tanh[x])/3 + ((I - Csch[x])*Tanh[x]^3)/3
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx &= \int (-i+\operatorname{csch}(x))\tanh^4(x)\,dx\\
&= \frac{1}{3}(i-\operatorname{csch}(x))\tanh^3(x)-\frac{1}{3}\int (3i-2\operatorname{csch}(x))\tanh^2(x)\,dx\\
&= \frac{1}{3}(3i-2\operatorname{csch}(x))\tanh(x)+\frac{1}{3}(i-\operatorname{csch}(x))\tanh^3(x)+\frac{1}{3}\int -3i\,dx\\
&= -ix+\frac{1}{3}(3i-2\operatorname{csch}(x))\tanh(x)+\frac{1}{3}(i-\operatorname{csch}(x))\tanh^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0836784, size = 71, normalized size = 1.97

$$\frac{2i\sinh(x)-4\cosh(2x)+(6x+5i)(\sinh(x)-i)\cosh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-4*Cosh[2*x] + (2*I)*Sinh[x] + (5*I + 6*x)*Cosh[x]*(-I + Sinh[x]))/(6*(Cos
h[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] time = 0.047, size = 67, normalized size = 1.9

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{3i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{i}{2}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+csch(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+3/2*I/(tanh(1/2*x)-I)+2/3*I/(tanh(1/2*x)-I)^3+1/(tanh(
1/2*x)-I)^2+I*ln(tanh(1/2*x)-1)+1/2*I/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.00852, size = 57, normalized size = 1.58

$$-ix-\frac{10\,e^{(-x)}+6\,e^{(-3\,x)}+8i}{6i\,e^{(-x)}+6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - (10*e^(-x) + 6*e^(-3*x) + 8*I)/(6*I*e^(-x) + 6*I*e^(-3*x) + 3*e^(-4*
x) - 3)
```

Fricas [B] time = 1.6519, size = 149, normalized size = 4.14

$$\frac{-3i\,xe^{(4\,x)}-6\,(x+1)e^{(3\,x)}-2\,(3\,x+5)e^{x}+3i\,x+8i}{3\,e^{(4\,x)}-6i\,e^{(3\,x)}-6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (-3*I*x*e^(4*x) - 6*(x + 1)*e^(3*x) - 2*(3*x + 5)*e^x + 3*I*x + 8*I)/(3*e^(
4*x) - 6*I*e^(3*x) - 6*I*e^x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(csch(x) + I), x)
```

Giac [A] time = 1.17459, size = 51, normalized size = 1.42

$$\frac{i}{2\,(i\,e^{x}-1)}-\frac{15\,e^{(2\,x)}-24i\,e^{x}-13}{6\,(e^{x}-i)^{3}}-i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/2*I/(I*e^x - 1) - 1/6*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - I*log(I*
e^x)
```

3.106 $\int \frac{\tanh(x)}{i+\textbf{csch}(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{i}{2(1+i\sinh(x))}-\frac{3}{4}i\log(-\sinh(x)+i)-\frac{1}{4}i\log(\sinh(x)+i)$$

```
[Out] ((-3*I)/4)*Log[I - Sinh[x]] - (I/4)*Log[I + Sinh[x]] - (I/2)/(1 + I*Sinh[x]
)
```

Rubi [A] time = 0.0475287, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 88}

$$-\frac{i}{2(1+i\sinh(x))}-\frac{3}{4}i\log(-\sinh(x)+i)-\frac{1}{4}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)/4)*Log[I - Sinh[x]] - (I/4)*Log[I + Sinh[x]] - (I/2)/(1 + I*Sinh[x]
)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(i-ix)(i+ix)^2}\,dx, x, i\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{4(-1+x)}+\frac{i}{2(1+x)^2}-\frac{3i}{4(1+x)}\right)dx, x, i\sinh(x)\right)\\ &= -\frac{3}{4}i\log(i-\sinh(x))-\frac{1}{4}i\log(i+\sinh(x))-\frac{i}{2(1+i\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0374018, size = 39, normalized size = 0.87

$$\frac{1}{4}\left(-\frac{2}{\sinh(x)-i}-3i\log(-\sinh(x)+i)-i\log(\sinh(x)+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)*Log[I - Sinh[x]] - I*Log[I + Sinh[x]] - 2/(-I + Sinh[x]))/4
```

Maple [A] time = 0.045, size = 65, normalized size = 1.4

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{3i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{i}{2}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)+1)-3/2*I*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)-I)^2+1/(tanh(1/2
*x)-I)+I*ln(tanh(1/2*x)-1)-1/2*I*ln(tanh(1/2*x)+I)
```

Maxima [A] time = 1.03818, size = 61, normalized size = 1.36

$$-ix+\frac{e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}-\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{3}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) - 1/2*I*log(I*e^(-x) + 1) - 3/2*I
*log(I*e^(-x) - 1)
```

Fricas [B] time = 1.6308, size = 207, normalized size = 4.6

$$\frac{2i\,xe^{(2\,x)}+2\,(2\,x-1)e^{x}+\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}+i\right)+\left(-3i\,e^{(2\,x)}-6\,e^{x}+3i\right)\log\left(e^{x}-i\right)-2i\,x}{2\,e^{(2\,x)}-4i\,e^{x}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (2*I*x*e^(2*x) + 2*(2*x - 1)*e^x + (-I*e^(2*x) - 2*e^x + I)*log(e^x + I) +
(-3*I*e^(2*x) - 6*e^x + 3*I)*log(e^x - I) - 2*I*x)/(2*e^(2*x) - 4*I*e^x - 2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)/(csch(x) + I), x)
```

Giac [B] time = 1.1512, size = 74, normalized size = 1.64

$$\frac{3i\,e^{(-x)}-3i\,e^{x}-2}{4\left(e^{(-x)}-e^{x}+2i\right)}-\frac{1}{4}i\,\log\left(-i\,e^{(-x)}+i\,e^{x}-2\right)-\frac{3}{4}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/4*I*log(-I*e^(-x) +
 I*e^x - 2) - 3/4*I*log(-e^(-x) + e^x - 2*I)
```

3.107 $\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=13

$$-i\log(-\sinh(x)+i)$$

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rubi [A] time = 0.0227739, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 31}

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{i+ix}\,dx, x, i\sinh(x)\right)\\ &= -i\log(i-\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0072043, size = 13, normalized size = 1.

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Maple [A] time = 0.023, size = 17, normalized size = 1.3

$$i\ln(\operatorname{csch}(x)) - i\ln(i + \operatorname{csch}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+csch(x)),x)
```

```
[Out] I*ln(csch(x))-I*ln(I+csch(x))
```

Maxima [A] time = 1.0097, size = 20, normalized size = 1.54

$$-ix - 2i\log\left(ie^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

Fricas [A] time = 1.68254, size = 32, normalized size = 2.46

$$ix - 2i\log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)/(csch(x) + I), x)
```

Giac [A] time = 1.14821, size = 18, normalized size = 1.38

$$ix - 2i\log(ie^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] I*x - 2*I*log(I*e^x + 1)
```

3.108 $\int \frac{\coth^2(x)}{i+\mathbf{csch}(x)} \, dx$

Optimal. Leaf size=11

$$-\tanh^{-1}(\cosh(x)) - ix$$

```
[Out] (-I)*x - ArcTanh[Cosh[x]]
```

Rubi [A] time = 0.0383253, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3888, 3770}

$$-\tanh^{-1}(\cosh(x)) - ix$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(x)}{i+\text{csch}(x)} \, dx &= \int (-i+\text{csch}(x)) \, dx \\ &= -ix + \int \text{csch}(x) \, dx \\ &= -ix - \tanh^{-1}(\cosh(x)) \end{aligned}$$

Mathematica [A] time = 0.0329229, size = 13, normalized size = 1.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right) - ix$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + Log[Tanh[x/2]]
```

Maple [B] time = 0.029, size = 27, normalized size = 2.5

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+csch(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+ln(tanh(1/2*x))+I*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.00286, size = 27, normalized size = 2.45

$$-ix-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [A] time = 1.61851, size = 49, normalized size = 4.45

$$-ix-\log\left(e^{x}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -I*x - log(e^x + 1) + log(e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^2(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**2/(csch(x) + I), x)
```

Giac [A] time = 1.19109, size = 23, normalized size = 2.09

$$-ix-\log\left(e^{x}+1\right)+\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x - log(e^x + 1) + log(abs(e^x - 1))
```

3.109 $\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=12

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

```
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Rubi [A] time = 0.0386526, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 43}

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Csch[x]),x]

[Out] -Csch[x] - I*Log[Sinh[x]]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{i-ix}{x^2}\,dx, x, i\sinh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(\frac{i}{x^2}-\frac{i}{x}\right)dx, x, i\sinh(x)\right)\\ &= -\operatorname{csch}(x) - i\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.010364, size = 12, normalized size = 1.

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Csch[x]),x]
```

```
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Maple [A] time = 0.017, size = 12, normalized size = 1.

$$-\operatorname{csch}(x) + i\ln(\operatorname{csch}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+csch(x)),x)
```

```
[Out] -csch(x)+I*ln(csch(x))
```

Maxima [B] time = 1.03452, size = 49, normalized size = 4.08

$$-i\,x + \frac{2\,e^{(-x)}}{e^{(-2\,x)} - 1} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 2*e^(-x)/(e^(-2*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] time = 1.77861, size = 107, normalized size = 8.92

$$\frac{i\,xe^{(2\,x)} + \left(-i\,e^{(2\,x)} + i\right)\log\left(e^{(2\,x)} - 1\right) - i\,x - 2\,e^{x}}{e^{(2\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) + (-I*e^(2*x) + I)*log(e^(2*x) - 1) - I*x - 2*e^x)/(e^(2*x) -
1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(csch(x) + I), x)
```

Giac [B] time = 1.17523, size = 51, normalized size = 4.25

$$\frac{i e^{(-x)} - i e^{x} + 2}{e^{(-x)} - e^{x}} - i \log\left(\left|-e^{(-x)} + e^{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] (I*e^(-x) - I*e^x + 2)/(e^(-x) - e^x) - I*log(abs(-e^(-x) + e^x))
```

3.110 $\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=27

$$-ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(-\operatorname{csch}(x) + 2i)$$

```
[Out] (-I)*x - ArcTanh[Cosh[x]]/2 + (Coth[x]*(2*I - Csch[x]))/2
```

Rubi [A] time = 0.056028, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$-ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(-\operatorname{csch}(x) + 2i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]/2 + (Coth[x]*(2*I - Csch[x]))/2
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} dx &= \int \coth^2(x)(-i+\operatorname{csch}(x)) dx \\
&= \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int (-2i+\operatorname{csch}(x)) dx \\
&= -ix + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int \operatorname{csch}(x) dx \\
&= -ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x))
\end{aligned}$$

Mathematica [B] time = 0.0360378, size = 65, normalized size = 2.41

$$-ix+\frac{1}{2}i\tanh\left(\frac{x}{2}\right)+\frac{1}{2}i\coth\left(\frac{x}{2}\right)-\frac{1}{8}\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\text{sech}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + (I/2)*Coth[x/2] - Csch[x/2]^2/8 + Log[Tanh[x/2]]/2 - Sech[x/2]^2/8
 + (I/2)*Tanh[x/2]
```

Maple [B] time = 0.044, size = 61, normalized size = 2.3

$$\frac{i}{2}\tanh\left(\frac{x}{2}\right)+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\ln\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+csch(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-I*ln(tanh(1/2*x)+1)-1/8/tanh(1/2*x)^2+1
/2*I/tanh(1/2*x)+1/2*ln(tanh(1/2*x))+I*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.02958, size = 74, normalized size = 2.74

$$-ix+\frac{e^{(-x)}+2ie^{(-2x)}+e^{(-3x)}-2i}{2e^{(-2x)}-e^{(-4x)}-1}-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (e^(-x) + 2*I*e^(-2*x) + e^(-3*x) - 2*I)/(2*e^(-2*x) - e^(-4*x) - 1)
 - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] time = 1.6225, size = 254, normalized size = 9.41

$$\frac{-2ixe^{(4x)}+(4ix+4i)e^{(2x)}-\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^x+1\right)+\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^x-1\right)-2ix-2e^{(3x)}-2e^x-4}{2\left(e^{(4x)}-2e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^(4*x) + (4*I*x + 4*I)*e^(2*x) - (e^(4*x) - 2*e^(2*x) + 1)*log
(e^x + 1) + (e^(4*x) - 2*e^(2*x) + 1)*log(e^x - 1) - 2*I*x - 2*e^(3*x) - 2*
e^x - 4*I)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(csch(x) + I), x)
```

Giac [B] time = 1.16535, size = 65, normalized size = 2.41

$$\frac{e^{(3x)} - 2i\,e^{(2x)} + e^x + 2i}{\left(i\,e^{(2x)} - i\right)^2} - i\,\log\left(-i\,e^x\right) - \frac{1}{2}\,\log\left(e^x + 1\right) + \frac{1}{2}\,\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] (e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(I*e^(2*x) - I)^2 - I*log(-I*e^x) - 1/2
*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.111 $\int \frac{\coth^5(x)}{i+\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{3}\mathrm{csch}^3(x)+\frac{1}{2}i\,\mathrm{csch}^2(x)-\mathrm{csch}(x)-i\log(\sinh(x))$$

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rubi [A] time = 0.0459022, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 75}

$$-\frac{1}{3}\mathrm{csch}^3(x)+\frac{1}{2}i\,\mathrm{csch}^2(x)-\mathrm{csch}(x)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Csch[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{i+\mathrm{csch}(x)}\,dx &= \mathrm{Subst}\left(\int \frac{(i-ix)^2(i+ix)}{x^4}\,dx,x,i\sinh(x)\right)\\ &= \mathrm{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{i}{x^3}+\frac{i}{x^2}-\frac{i}{x}\right)dx,x,i\sinh(x)\right)\\ &= -\mathrm{csch}(x)+\frac{1}{2}i\,\mathrm{csch}^2(x)-\frac{\mathrm{csch}^3(x)}{3}-i\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0145455, size = 30, normalized size = 1.

$$-\frac{1}{3}\mathrm{csch}^3(x)+\frac{1}{2}i\,\mathrm{csch}^2(x)-\mathrm{csch}(x)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Csch[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Maple [B] time = 0.068, size = 78, normalized size = 2.6

$$\frac{3}{8}\tanh\left(\frac{x}{2}\right)+\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}+\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{i}{8}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+csch(x)),x)
```

```
[Out] 3/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3+1/8*I*tanh(1/2*x)^2+I*ln(tanh(1/2*x)+1)-
1/24/tanh(1/2*x)^3-I*ln(tanh(1/2*x))+1/8*I/tanh(1/2*x)^2-3/8/tanh(1/2*x)+I*
ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.02183, size = 101, normalized size = 3.37

$$-ix+\frac{6e^{(-x)}-6ie^{(-2x)}-4e^{(-3x)}+6ie^{(-4x)}+6e^{(-5x)}}{3\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}-i\log\left(e^{(-x)}+1\right)-i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/3*(6*e^(-x) - 6*I*e^(-2*x) - 4*e^(-3*x) + 6*I*e^(-4*x) + 6*e^(-5*x
))/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(
-x) - 1)
```

Fricas [B] time = 1.72243, size = 292, normalized size = 9.73

$$\frac{3ixe^{(6x)}+(-9ix+6i)e^{(4x)}+(9ix-6i)e^{(2x)}+\left(-3ie^{(6x)}+9ie^{(4x)}-9ie^{(2x)}+3i\right)\log\left(e^{(2x)}-1\right)-3ix-6e^{(5x)}+4e^{(3x)}-}{3\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*x*e^(6*x) + (-9*I*x + 6*I)*e^(4*x) + (9*I*x - 6*I)*e^(2*x) + (-3*I
*e^(6*x) + 9*I*e^(4*x) - 9*I*e^(2*x) + 3*I)*log(e^(2*x) - 1) - 3*I*x - 6*e^
(5*x) + 4*e^(3*x) - 6*e^x)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^{5}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(csch(x) + I), x)
```

Giac [B] time = 1.15493, size = 92, normalized size = 3.07

$$-\frac{11\left(e^{(-x)}-e^{x}\right)^{3}-12i\left(e^{(-x)}-e^{x}\right)^{2}+12\,e^{(-x)}-12\,e^{x}-16i}{6\left(-i\,e^{(-x)}+i\,e^{x}\right)^{3}}-i\,\log\left(-i\,e^{(-x)}+i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/6*(11*(e^(-x) - e^x)^3 - 12*I*(e^(-x) - e^x)^2 + 12*e^(-x) - 12*e^x - 16
*I)/(-I*e^(-x) + I*e^x)^3 - I*log(-I*e^(-x) + I*e^x)
```

3.112 $\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=43

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(-3\operatorname{csch}(x)+4i)+\frac{1}{8}\coth(x)(-3\operatorname{csch}(x)+8i)$$

```
[Out] (-I)*x - (3*ArcTanh[Cosh[x]])/8 + (Coth[x]^3*(4*I - 3*Csch[x]))/12 + (Coth[
x]*(8*I - 3*Csch[x]))/8
```

Rubi [A] time = 0.0749438, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(-3\operatorname{csch}(x)+4i)+\frac{1}{8}\coth(x)(-3\operatorname{csch}(x)+8i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x - (3*ArcTanh[Cosh[x]])/8 + (Coth[x]^3*(4*I - 3*Csch[x]))/12 + (Coth[
x]*(8*I - 3*Csch[x]))/8
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx &= \int \coth^4(x)(-i+\operatorname{csch}(x))\,dx\\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x))+\frac{1}{4}\int \coth^2(x)(-4i+3\operatorname{csch}(x))\,dx\\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x))+\frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))+\frac{1}{8}\int (-8i+3\operatorname{csch}(x))\,dx\\
&= -ix+\frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x))+\frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))+\frac{3}{8}\int \operatorname{csch}(x)\,dx\\
&= -ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x))+\frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))
\end{aligned}$$

Mathematica [B] time = 0.0405122, size = 129, normalized size = 3.

$$-ix+\frac{2}{3}i\tanh\left(\frac{x}{2}\right)+\frac{2}{3}i\coth\left(\frac{x}{2}\right)-\frac{1}{64}\mathrm{csch}^4\left(\frac{x}{2}\right)-\frac{5}{32}\mathrm{csch}^2\left(\frac{x}{2}\right)+\frac{1}{64}\mathrm{sech}^4\left(\frac{x}{2}\right)-\frac{5}{32}\mathrm{sech}^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((2*I)/3)*Coth[x/2] - (5*Csch[x/2]^2)/32 + (I/24)*Coth[x/2]*Csch[x
/2]^2 - Csch[x/2]^4/64 + (3*Log[Tanh[x/2]])/8 - (5*Sech[x/2]^2)/32 + Sech[x
/2]^4/64 + ((2*I)/3)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

Maple [B] time = 0.092, size = 95, normalized size = 2.2

$$\frac{5i}{8}\tanh\left(\frac{x}{2}\right)+\frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{i}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^{-4}+\frac{5i}{8}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+csch(x)),x)
```

```
[Out] 5/8*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2
-I*ln(tanh(1/2*x)+1)-1/64/tanh(1/2*x)^4+5/8*I/tanh(1/2*x)+1/24*I/tanh(1/2*x
)^3-1/8/tanh(1/2*x)^2+3/8*ln(tanh(1/2*x))+I*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.03415, size = 130, normalized size = 3.02

$$-ix+\frac{15e^{(-x)}+80ie^{(-2x)}+9e^{(-3x)}-96ie^{(-4x)}+9e^{(-5x)}+48ie^{(-6x)}+15e^{(-7x)}-32i}{12\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}-\frac{3}{8}\log\left(e^{(-x)}+1\right)+\frac{3}{8}\log\left(e^{(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/12*(15*e^(-x) + 80*I*e^(-2*x) + 9*e^(-3*x) - 96*I*e^(-4*x) + 9*e^(
-5*x) + 48*I*e^(-6*x) + 15*e^(-7*x) - 32*I)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^
(-6*x) - e^(-8*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Fricas [B] time = 1.79844, size = 478, normalized size = 11.12

$$\frac{-24ixe^{(8x)}+(96ix+96i)e^{(6x)}+(-144ix-192i)e^{(4x)}+(96ix+160i)e^{(2x)}-9\left(e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1\right)\mathrm{lo}}{24\left(e^{(8x)}-4e^{(6x)}+6e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-24*I*x*e^(8*x) + (96*I*x + 96*I)*e^(6*x) + (-144*I*x - 192*I)*e^(4*x
) + (96*I*x + 160*I)*e^(2*x) - 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*
x) + 1)*log(e^x + 1) + 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)*
```

```
log(e^x - 1) - 24*I*x - 30*e^(7*x) - 18*e^(5*x) - 18*e^(3*x) - 30*e^x - 64*
I)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+csch(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15593, size = 104, normalized size = 2.42

$$-\frac{15\,e^{(7\,x)} - 48i\,e^{(6\,x)} + 9\,e^{(5\,x)} + 96i\,e^{(4\,x)} + 9\,e^{(3\,x)} - 80i\,e^{(2\,x)} + 15\,e^{x} + 32i}{12\,\left(i\,e^{(2\,x)} - i\right)^{4}} - i\,\log\left(-i\,e^{x}\right) - \frac{3}{8}\,\log\left(e^{x} + 1\right) + \frac{3}{8}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/12*(15*e^(7*x) - 48*I*e^(6*x) + 9*e^(5*x) + 96*I*e^(4*x) + 9*e^(3*x) - 8
0*I*e^(2*x) + 15*e^x + 32*I)/(I*e^(2*x) - I)^4 - I*log(-I*e^x) - 3/8*log(e^
x + 1) + 3/8*log(abs(e^x - 1))
```

3.113 $\int \frac{\tanh^5(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=194

$$-\frac{a\left(3a^2b^2+a^4+3b^4\right)\log(\tanh(x))}{\left(a^2+b^2\right)^3}-\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)}+\frac{b^6\log(a+b\,\mathrm{csc}}{a\left(a^2+b^2\right)^3}$$

```
[Out] -((b^5*ArcTan[Sinh[x]])/(a^2 + b^2)^3) - (b^3*ArcTan[Sinh[x]])/(2*(a^2 + b^
2)^2) - (3*b*ArcTan[Sinh[x]])/(8*(a^2 + b^2)) + (b^6*Log[a + b*Csch[x]])/(a
*(a^2 + b^2)^3) + Log[Sinh[x]]/a - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Tanh[x]
])/(a^2 + b^2)^3 + (3*b*Sech[x]*Tanh[x])/(8*(a^2 + b^2)) - ((a*(a^2 + 2*b^2
) - b^3*Csch[x])*Tanh[x]^2)/(2*(a^2 + b^2)^2) - ((a - b*Csch[x])*Tanh[x]^4)
/(4*(a^2 + b^2))
```

Rubi [A] time = 0.254877, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3885, 894, 639, 199, 203, 635, 260}

$$-\frac{a\left(3a^2b^2+a^4+3b^4\right)\log(\tanh(x))}{\left(a^2+b^2\right)^3}-\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)}+\frac{b^6\log(a+b\,\mathrm{csc}}{a\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b^5*ArcTan[Sinh[x]])/(a^2 + b^2)^3) - (b^3*ArcTan[Sinh[x]])/(2*(a^2 + b^
2)^2) - (3*b*ArcTan[Sinh[x]])/(8*(a^2 + b^2)) + (b^6*Log[a + b*Csch[x]])/(a
*(a^2 + b^2)^3) + Log[Sinh[x]]/a - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Tanh[x]
])/(a^2 + b^2)^3 + (3*b*Sech[x]*Tanh[x])/(8*(a^2 + b^2)) - ((a*(a^2 + 2*b^2
) - b^3*Csch[x])*Tanh[x]^2)/(2*(a^2 + b^2)^2) - ((a - b*Csch[x])*Tanh[x]^4)
/(4*(a^2 + b^2))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = b^6\operatorname{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^3}\,dx,x,b\operatorname{csch}(x)\right)$$

$$= b^6\operatorname{Subst}\left(\int\left(-\frac{1}{ab^6x}+\frac{1}{a\left(a^2+b^2\right)^3(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^3}+\frac{b^4+a\left(a^2+2b^2\right)x}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)^2}+\right.\right.$$

$$= \frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}+\frac{\operatorname{Subst}\left(\int\frac{b^6+a\left(a^4+3a^2b^2+3b^4\right)x}{b^2+x^2}\,dx,x,b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^3}+\frac{b^2\operatorname{Subst}\left(\right.}{}$$

$$= \frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{\left(a\left(a^2+2b^2\right)-b^3\operatorname{csch}(x)\right)\tanh^2(x)}{2\left(a^2+b^2\right)^2}-\frac{(a-b\operatorname{csch}(x))\tanh}{4\left(a^2+b^2\right)}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{a\left(a^4+3a^2b^2+\right.}{\left(a^2\right.}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(}{a}$$

Mathematica [C] time = 0.492813, size = 253, normalized size = 1.3

$$\frac{-2a^2\left(a^2+b^2\right)^2\operatorname{sech}^4(x)+4a^2\left(5a^2b^2+2a^4+3b^4\right)\operatorname{sech}^2(x)+4a\left(3a^3b^2+3ia^2b^3+ia^4b+a^5+3ab^4+3ib^5\right)\log(-\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*ArcTan[Sinh[x]] + 4*a*(a^5 + I*a^4*b + 3*
a^3*b^2 + (3*I)*a^2*b^3 + 3*a*b^4 + (3*I)*b^5)*Log[I - Sinh[x]] + 4*a*(a^5
- I*a^4*b + 3*a^3*b^2 - (3*I)*a^2*b^3 + 3*a*b^4 - (3*I)*b^5)*Log[I + Sinh[x
]] + 8*b^6*Log[b + a*Sinh[x]] + 4*a^2*(2*a^4 + 5*a^2*b^2 + 3*b^4)*Sech[x]^2
 - 2*a^2*(a^2 + b^2)^2*Sech[x]^4 + a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*Sech[x]
*Tanh[x] - 2*a*b*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(8*a*(a^2 + b^2)^3)
```

Maple [B] time = 0.063, size = 1323, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*csch(x)),x)
```

```
[Out] -5/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^2*b^
3-20/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^3*b^
2-6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^3*b^2
-6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^3*b^2-
4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a*b^4-13/
2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^2*b^3+3
/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^4*b-12/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a*b^4+13/2/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^2*b^3+11/4
/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^4*b-11/4
/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^4*b-3/4/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^4*b+5/2/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^2*b^3-4/(a^4
+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a*b^4-1/a*ln(ta
nh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-15/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arcta
n(tanh(1/2*x))*b^5+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2+1)*a^5+
15/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*b^5-2/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^5+7/4/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*b^5+3/(a^4+2*a^2
*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2+1)*a^3*b^2+3/(a^4+2*a^2*b^2+b^4)/(a^2+
b^2)*ln(tanh(1/2*x)^2+1)*a*b^4-7/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*
x)^2+1)^4*tanh(1/2*x)^7*b^5+b^6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/a*ln(tanh(1/2
*x)^2*b-2*a*tanh(1/2*x)-b)-5/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/
2*x))*a^2*b^3-15/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1
/2*x)^5*b^5-8/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)
^4*a^5-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^
5-3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^4*b
```

Maxima [B] time = 1.60776, size = 517, normalized size = 2.66

$$\frac{b^6 \log\left(-2\, be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6} + \frac{\left(3\,a^4b + 10\,a^2b^3 + 15\,b^5\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{\left(a^5 + 3\,a^3b^2 + 3\,ab^4\right)\log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^6*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 + 1/4*(3*a^4*b + 10*a^2*b^3 + 15*b^5)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*
```

```
a^2*b^4 + b^6) + (a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + 1/4*((5*a^2*b + 9*b^3)*e^(-x) + 8*(2*a^3 + 3*a*b^
2)*e^(-2*x) - (3*a^2*b - b^3)*e^(-3*x) + 16*(a^3 + 2*a*b^2)*e^(-4*x) + (3*a
^2*b - b^3)*e^(-5*x) + 8*(2*a^3 + 3*a*b^2)*e^(-6*x) - (5*a^2*b + 9*b^3)*e^(
-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4
 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 +
2*a^2*b^2 + b^4)*e^(-8*x)) + x/a
```

Fricas [B] time = 2.64532, size = 9482, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^8 + 4*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*sinh(x)^8 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)
^7 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*x*cosh(x))*sinh(x)^7 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^6 - (16*a^6 + 40*a^4*b^2 + 24*a^2*b^4 -
112*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 16*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*x + 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^6
 + (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^5 + (3*a^5*b + 2*a^3*b^3 - a*b^5 +
 224*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 21*(5*a^5*b + 14*a^3
*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^5 - 8*(2*a^6 + 6*a^4*b^2 + 4
*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 - (16*a^6 + 4
8*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
4 + 35*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 120*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 - 24*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))
*sinh(x)^4 - (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^3 + (224*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 3*a^5*b - 2*a^3*b^3 + a*b^5 - 35*(5*a^5*
b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 - 160*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 10*(3*a^5*b + 2*a^3*b^
3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*
b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + (112*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 - 16*a^6 - 40*a^4*b^2 - 24*a^2*b^4 - 2
1*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 - 120*(2*a^6 + 5*a^4*b^2 + 3*a
^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 10*(3*a^5*b +
 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + 16*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*x - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))*sinh(x)^2 + 4*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + ((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^8 + 8*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*sinh(x)^8 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^6 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 7*(3*a^5*b + 10*a^3*b^3 + 15*a*b
^5)*cosh(x)^2)*sinh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 8*(7*(3*a^5*b
+ 10*a^3*b^3 + 15*a*b^5)*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*co
sh(x))*sinh(x)^5 + 6*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4 + 2*(9*a^5
*b + 30*a^3*b^3 + 45*a*b^5 + 35*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4
 + 30*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5*
b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 10*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)
*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x)^3 + 4*(3*
a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2 + 4*(7*(3*a^5*b + 10*a^3*b^3 + 15*
```

```
a*b^5)*cosh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 15*(3*a^5*b + 10*a^3*b
^3 + 15*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*s
inh(x)^2 + 8*((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^7 + 3*(3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^3 + (3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) +
sinh(x)) + (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x) - 4*(b^6*cosh(x)^8 + 8*
b^6*cosh(x)*sinh(x)^7 + b^6*sinh(x)^8 + 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 +
 4*b^6*cosh(x)^2 + 4*(7*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cos
h(x)^3 + 3*b^6*cosh(x))*sinh(x)^5 + 2*(35*b^6*cosh(x)^4 + 30*b^6*cosh(x)^2
+ 3*b^6)*sinh(x)^4 + 8*(7*b^6*cosh(x)^5 + 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))
*sinh(x)^3 + 4*(7*b^6*cosh(x)^6 + 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 + b^6)
*sinh(x)^2 + 8*(b^6*cosh(x)^7 + 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 + b^6*cos
h(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) - 4*((a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)*sinh(
x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 7*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^2)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 8*(7*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*s
inh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2
 + 9*a^2*b^4 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 30*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)
*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)
^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
7 - 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^6 + 5*a^5*b + 14*a^3*b^3 + 9
*a*b^5 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*x)*cosh(x)^5 + 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^4 - 32*(2*a^
6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x
)^3 - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 16*(2*a^6 + 5*a^4*b^2 + 3
*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/((a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^8 + 8*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cosh(x)*sinh(x)^7 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sinh(x
)^8 + a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cosh(x)^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 3*a^5*b^2 + 3
*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(
x))*sinh(x)^5 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(3*a^
7 + 9*a^5*b^2 + 9*a^3*b^4 + 3*a*b^6 + 35*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x)^4 + 30*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)
^4 + 8*(7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 + 10*(a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x))*sinh(x)^3 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 +
 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cosh(x)^6 + 15*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 9*(
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^7 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x)^5 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + (a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{a + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*csch(x)), x)
```

Giac [B] time = 1.21098, size = 583, normalized size = 3.01

$$\frac{b^6 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2b\right|\right)}{a^7 + 3a^5b^2 + 3a^3b^4 + ab^6} - \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)\left(3a^4b + 10a^2b^3 + 15b^5\right)}{16\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)} + \frac{\left(a^5 + 3a^3b^2 + 3ab^4\right)\log\left(\right.}{2\left(a^6 + 3a^4b^2 + 3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 - 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^4*b + 10*a^2*b^3 + 1
5*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^5 + 3*a^3*b^2 + 3*a*b^4
)*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5*(e^(-x) - e^x)^4 + 9*a^3*b^2*(e^(-x) - e^x)^4 + 9*a*b^4*(e^(-x) - e^x)^4
+ 5*a^4*b*(e^(-x) - e^x)^3 + 14*a^2*b^3*(e^(-x) - e^x)^3 + 9*b^5*(e^(-x) -
e^x)^3 + 8*a^5*(e^(-x) - e^x)^2 + 32*a^3*b^2*(e^(-x) - e^x)^2 + 48*a*b^4*(e
^(-x) - e^x)^2 + 12*a^4*b*(e^(-x) - e^x) + 40*a^2*b^3*(e^(-x) - e^x) + 28*b
^5*(e^(-x) - e^x) + 16*a^3*b^2 + 64*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*((e^(-x) - e^x)^2 + 4)^2)
```

3.114 $\int \frac{\tanh^4(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{b^4x}{a\left(a^2+b^2\right)^2}+\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\,\text{sech}^3(x)}{3\left(a^2+b^2\right)}$$

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (b^4*x)/(a*(a^2 + b^2)^2) + (a*x)/(a^2 + b^2) + (
2*b^5*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5/2)) + (
b^3*Sech[x])/(a^2 + b^2)^2 + (b*Sech[x])/(a^2 + b^2) - (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a*b^2*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x])/(a^2 + b^2) - (a*Ta
nh[x]^3)/(3*(a^2 + b^2))
```

Rubi [A] time = 0.384148, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2660, 618, 204}

$$\frac{b^4x}{a\left(a^2+b^2\right)^2}+\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\,\text{sech}^3(x)}{3\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (b^4*x)/(a*(a^2 + b^2)^2) + (a*x)/(a^2 + b^2) + (
2*b^5*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5/2)) + (
b^3*Sech[x])/(a^2 + b^2)^2 + (b*Sech[x])/(a^2 + b^2) - (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a*b^2*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x])/(a^2 + b^2) - (a*Ta
nh[x]^3)/(3*(a^2 + b^2))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\sinh(x)\tanh^4(x)}{ib+ia\sinh(x)}\,dx$$

$$= \frac{a\int \tanh^4(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh^3(x)\,dx}{a^2+b^2} + \frac{\left(ib^2\right)\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(ab^2\right)\int \tanh^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{b^3\int \text{sech}(x)\tanh(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(ib^4\right)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\int \text{t}}{}$$

$$= \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(ab^2\right)\int 1\,dx}{\left(a^2+b^2\right)^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a}{}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a}{}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}} + \frac{b^3\text{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{\text{bse}}{3(}$$

Mathematica [A] time = 0.695776, size = 141, normalized size = 0.77

$$\frac{1}{3}\left(-\frac{a\left(4a^2+7b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{b\text{sech}^3(x)}{a^2+b^2} + \frac{3b\left(a^2+2b^2\right)\text{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{3\left(x - \frac{2b^5\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{5/2}}\right)}{a} + \frac{a\tanh(x)\text{sech}^2(x)}{a^2+b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*(x - (2*b^5*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(5
/2)))/a + (3*b*(a^2 + 2*b^2)*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x]^3)/(a^2 +
b^2) - (a*(4*a^2 + 7*b^2)*Tanh[x])/(a^2 + b^2)^2 + (a*Sech[x]^2*Tanh[x])/(a
^2 + b^2))/3
```

Maple [A] time = 0.053, size = 207, normalized size = 1.1

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\frac{b^5}{a\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right) - \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*csch(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-2/a*b^5/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/a*ln(tanh(1/2*x)-1)+2/(a^4+2*a^
2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+b^3*tanh(1/2*x)^4+(-10/3*a^3-16/3*
a*b^2)*tanh(1/2*x)^3+(2*a^2*b+4*b^3)*tanh(1/2*x)^2+(-a^3-2*a*b^2)*tanh(1/2*
x)+2/3*a^2*b+5/3*b^3)/(tanh(1/2*x)^2+1)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85507, size = 4128, normalized size = 22.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 + 3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*x*sinh(x)^6 + 8*a^6 + 22*a^4*b^2 + 14*a^2*b^4 + 6*(a^5*
b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5 + 3*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x))*sinh(x)^5 + 3*(4*a^6 + 10*a^4*b
^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 3*(4*
a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cos
h(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 10*(a^5*b + 3*a^3*b^3 +
2*a*b^5)*cosh(x))*sinh(x)^4 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 + 4
*(a^5*b + 5*a^3*b^3 + 4*a*b^5 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*co
sh(x)^3 + 15*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 + 10*a^4*b^
2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3
 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*x)*cosh(x)^2 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*x*cosh(x)^4 + 20*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 +
 6*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
x)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 4*(a^5*b + 5*a^3*b
^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 + 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)
^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cos
h(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3
*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2
*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 +
 a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x
) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 3*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*x + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 + a^5*b + 3*a^3*b^3 + 2*a*b^5 + 5*(
a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 + 2*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 2*(a^5*b + 5*a^3*b^3
 + 4*a*b^5)*cosh(x)^2 + (4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^6 + 6*(a^7 + 3*a^5*b^2
```

```
+ 3*a^3*b^4 + a*b^6)*cosh(x)*sinh(x)^5 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*sinh(x)^6 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 3*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x)^2)*sinh(x)^4 + 4*(5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3
+ 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^3 + 3*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 6*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^5 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 +
 (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*csch(x)), x)
```

Giac [A] time = 1.20349, size = 290, normalized size = 1.58

$$-\frac{b^5\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5+2\,a^3b^2+ab^4\right)\sqrt{a^2+b^2}}+\frac{x}{a}+\frac{2\left(3\,a^2be^{(5\,x)}+6\,b^3e^{(5\,x)}+6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}+2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}\cdots\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) + x/a + 2/3*(3*a^2
*b*e^(5*x) + 6*b^3*e^(5*x) + 6*a^3*e^(4*x) + 9*a*b^2*e^(4*x) + 2*a^2*b*e^(3
*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*a^2*b*e^x + 6*b^
3*e^x + 4*a^3 + 7*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

3.115 $\int \frac{\tanh^3(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=113

$$-\frac{a\left(a^2+2b^2\right)\log(\tanh(x))}{\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}-\frac{\tanh^2(x)(a-b\text{csch}(x))}{2\left(a^2+b^2\right)}+$$

```
[Out] -((b^3*ArcTan[Sinh[x]])/(a^2 + b^2)^2) - (b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)
) + (b^4*Log[a + b*Csch[x]])/(a*(a^2 + b^2)^2) + Log[Sinh[x]]/a - (a*(a^2 +
 2*b^2)*Log[Tanh[x]])/(a^2 + b^2)^2 - ((a - b*Csch[x])*Tanh[x]^2)/(2*(a^2 +
 b^2))
```

Rubi [A] time = 0.162987, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3885, 894, 639, 203, 635, 260}

$$-\frac{a\left(a^2+2b^2\right)\log(\tanh(x))}{\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}-\frac{\tanh^2(x)(a-b\text{csch}(x))}{2\left(a^2+b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[x]])/(a^2 + b^2)^2) - (b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)
) + (b^4*Log[a + b*Csch[x]])/(a*(a^2 + b^2)^2) + Log[Sinh[x]]/a - (a*(a^2 +
 2*b^2)*Log[Tanh[x]])/(a^2 + b^2)^2 - ((a - b*Csch[x])*Tanh[x]^2)/(2*(a^2 +
 b^2))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx = -\left(b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\text{csch}(x)\right)\right)$$

$$= -\left(b^4\,\text{Subst}\left(\int\left(\frac{1}{ab^4x}-\frac{1}{a\left(a^2+b^2\right)^2(a+x)}+\frac{-b^2-ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2}+\frac{-b^4-a\left(a^2+2b^2\right)x}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right.\right.\right.$$

$$= \frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{\text{Subst}\left(\int\frac{-b^4-a\left(a^2+2b^2\right)x}{b^2+x^2}\,dx,x,b\text{csch}(x)\right)}{\left(a^2+b^2\right)^2}-\frac{b^2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{(a-b\text{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)}+\frac{b^4\,\text{Subst}\left(\int\frac{1}{b^2+x^2}\,dx,x,b\text{csc}\right.}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{a\left(a^2+2b^2\right)\text{l}}{\left(a^2+\right.}$$

Mathematica [C] time = 0.189544, size = 191, normalized size = 1.69

$$\frac{a^2\left(a^2+b^2\right)\text{sech}^2(x)+2a^2b^2\log(-\sinh(x)+i)+2a^2b^2\log(\sinh(x)+i)+ab\left(a^2+b^2\right)\tan^{-1}(\sinh(x))+ab\left(a^2+b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^4*Log[I - Sinh[x]] + I*a^3*b*Log[I - S
inh[x]] + 2*a^2*b^2*Log[I - Sinh[x]] + (2*I)*a*b^3*Log[I - Sinh[x]] + a^4*L
og[I + Sinh[x]] - I*a^3*b*Log[I + Sinh[x]] + 2*a^2*b^2*Log[I + Sinh[x]] - (
2*I)*a*b^3*Log[I + Sinh[x]] + 2*b^4*Log[b + a*Sinh[x]] + a^2*(a^2 + b^2)*Se
ch[x]^2 + a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*a*(a^2 + b^2)^2)
```

Maple [B] time = 0.049, size = 324, normalized size = 2.9

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^4}{a\left(a^2+b^2\right)^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b-2a\tanh(x/2)-b\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{a^2b}{\left(a^2+b^2\right)^2}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)+b^4/a/(a^2+b^2)^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)
-b)-1/a*ln(tanh(1/2*x)-1)-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a
^2*b-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^3-2/(a^2+b^2)^2/(tan
h(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^3-2/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^2*a*b^2+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^2*b+1/(a^2+b^2
)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^3+1/(a^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a
^3+2/(a^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a*b^2-3/(a^2+b^2)^2*arctan(tanh(1/2*x)
)*b^3-1/(a^2+b^2)^2*arctan(tanh(1/2*x))*a^2*b
```

Maxima [A] time = 1.58664, size = 232, normalized size = 2.05

$$\frac{b^4\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^5+2\,a^3b^2+ab^4}+\frac{\left(a^2b+3\,b^3\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{be^{(-x)}+2\,ae^{(-2\,x)}}{a^2+b^2+2\,\left(a^2+b^2\right)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^5 + 2*a^3*b^2 + a*b^4) + (a^2*b +
3*b^3)*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 2*a*b^2)*log(e^(-2*x
) + 1)/(a^4 + 2*a^2*b^2 + b^4) + (b*e^(-x) + 2*a*e^(-2*x) - b*e^(-3*x))/(a^
2 + b^2 + 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x)) + x/a
```

Fricas [B] time = 1.95952, size = 2379, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*x*sinh(x)^4
 - (a^3*b + a*b^3)*cosh(x)^3 - (a^3*b + a*b^3 - 4*(a^4 + 2*a^2*b^2 + b^4)*x
*cosh(x))*sinh(x)^3 - 2*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x)
^2 - (2*a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 2*(a^4 +
2*a^2*b^2 + b^4)*x + 3*(a^3*b + a*b^3)*cosh(x))*sinh(x)^2 + (a^4 + 2*a^2*b^
2 + b^4)*x + ((a^3*b + 3*a*b^3)*cosh(x)^4 + 4*(a^3*b + 3*a*b^3)*cosh(x)*sin
h(x)^3 + (a^3*b + 3*a*b^3)*sinh(x)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3
)*cosh(x)^2 + 2*(a^3*b + 3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2)*sinh(x)^2
 + 4*((a^3*b + 3*a*b^3)*cosh(x)^3 + (a^3*b + 3*a*b^3)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) + (a^3*b + a*b^3)*cosh(x) - (b^4*cosh(x)^4 + 4*b^4*c
osh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*log(2*(a*sin
h(x) + b)/(cosh(x) - sinh(x))) - ((a^4 + 2*a^2*b^2)*cosh(x)^4 + 4*(a^4 + 2*
a^2*b^2)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2)*sinh(x)^4 + a^4 + 2*a^2*b^2
+ 2*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*
cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*a^2*b^2)*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (4*(a^4 + 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 + a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 - 4*(a^4 +
a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2
```

```
+ a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^3 + (a^5 + 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{a + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*csch(x)), x)
```

Giac [B] time = 1.22909, size = 316, normalized size = 2.8

$$\frac{b^4 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^5 + 2\,a^3b^2 + ab^4} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^2b + 3\,b^3\right)}{4\left(a^4 + 2\,a^2b^2 + b^4\right)} + \frac{\left(a^3 + 2\,ab^2\right)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2b^2 + b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) - 1/4*(pi +
 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4
) + 1/2*(a^3 + 2*a*b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) -
 1/2*(a^3*(e^(-x) - e^x)^2 + 2*a*b^2*(e^(-x) - e^x)^2 + 2*a^2*b*(e^(-x) - e
^x) + 2*b^3*(e^(-x) - e^x) + 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e
^x)^2 + 4))
```

3.116 $\int \frac{\tanh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=100

$$\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\text{sech}(x)}{a^2+b^2}$$

```
[Out] (a*x)/(a^2 + b^2) + (b^2*x)/(a*(a^2 + b^2)) + (2*b^3*ArcTanh[(a - b*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)) + (b*Sech[x])/(a^2 + b^2) - (a*
Tanh[x])/(a^2 + b^2)
```

Rubi [A] time = 0.220667, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\text{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^2*x)/(a*(a^2 + b^2)) + (2*b^3*ArcTanh[(a - b*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)) + (b*Sech[x])/(a^2 + b^2) - (a*
Tanh[x])/(a^2 + b^2)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+b\,\text{csch}(x)}\,dx = i\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx$$

$$= \frac{a\int \tanh^2(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh(x)\,dx}{a^2+b^2} + \frac{\left(ib^2\right)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$

$$= \frac{b^2x}{a\left(a^2+b^2\right)} - \frac{a\tanh(x)}{a^2+b^2} + \frac{a\int 1\,dx}{a^2+b^2} + \frac{b\,\text{Subst}\left(\int 1\,dx, x, \text{sech}(x)\right)}{a^2+b^2} - \frac{\left(ib^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{b\,\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{\left(2ib^3\right)\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2+b^2\right)}$$

$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{b\,\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} + \frac{\left(4ib^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, 2ia-2ib\,\text{ta}\ldots\right)}{a\left(a^2+b^2\right)}$$

$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}} + \frac{b\,\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

Mathematica [A] time = 0.318356, size = 82, normalized size = 0.82

$$-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\text{sech}(x)}{a^2+b^2}+\frac{\frac{2b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (x + (2*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2))
/a + (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Maple [A] time = 0.04, size = 95, normalized size = 1.

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{b^3}{a\left(a^2+b^2\right)^{3/2}}\text{Artanh}\left(1/2\frac{2\tanh(x/2)b-2a}{\sqrt{a^2+b^2}}\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{-a\tanh(x/2)+}{\left(a^2+b^2\right)\left((\tanh(x/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*csch(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tanh(1/2*x)*b-
2*a)/(a^2+b^2)^(1/2))-1/a*ln(tanh(1/2*x)-1)+2/(a^2+b^2)*(-a*tanh(1/2*x)+b)/
(tanh(1/2*x)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65838, size = 899, normalized size = 8.99

$$\frac{2a^4+2a^2b^2+\left(a^4+2a^2b^2+b^4\right)x\cosh(x)^2+\left(a^4+2a^2b^2+b^4\right)x\sinh(x)^2+\left(b^3\cosh(x)^2+2b^3\cosh(x)\sinh(x)+b^3\right.}{a^5+2a^3b^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^4 + 2*a^2*b^2 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 + (a^4 + 2*a^2*b^2
 + b^4)*x*sinh(x)^2 + (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^
2 + b^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
```

```
 + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) - a)) + (a^4 + 2*a^2*b^2 + b^4)*x + 2*(a^3*b + a*b^3)*cosh(x
) + 2*(a^3*b + a*b^3 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 + 2
*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*cosh(x)*sinh(x) + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.21591, size = 138, normalized size = 1.38

$$-\frac{b^3 \log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^3+ab^2\right)\sqrt{a^2+b^2}} + \frac{x}{a} + \frac{2\,(be^x+a)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + x/a + 2*(b*e^x + a)/((a^2 +
b^2)*(e^(2*x) + 1))
```

3.117 $\int \frac{\tanh(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{a\log(\tanh(x))}{a^2+b^2}-\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\mathrm{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}$$

```
[Out] -((b*ArcTan[Sinh[x]])/(a^2 + b^2)) + (b^2*Log[a + b*Csch[x]])/(a*(a^2 + b^2
)) + Log[Sinh[x]]/a - (a*Log[Tanh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.0994827, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3885, 894, 635, 203, 260}

$$-\frac{a\log(\tanh(x))}{a^2+b^2}-\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\mathrm{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/(a^2 + b^2)) + (b^2*Log[a + b*Csch[x]])/(a*(a^2 + b^2
)) + Log[Sinh[x]]/a - (a*Log[Tanh[x]])/(a^2 + b^2)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = b^2\operatorname{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\operatorname{csch}(x)\right)$$
$$= b^2\operatorname{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a\left(a^2+b^2\right)(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\operatorname{csch}(x)\right)$$
$$= \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{\operatorname{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2}$$
$$= \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{a\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,d\right.}{a^2+b}$$
$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}$$

Mathematica [C] time = 0.0587164, size = 63, normalized size = 1.03

$$\frac{2b^2\log(a\sinh(x)+b)+a(a+ib)\log(-\sinh(x)+i)+a(a-ib)\log(\sinh(x)+i)}{2a\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] (a*(a + I*b)*Log[I - Sinh[x]] + a*(a - I*b)*Log[I + Sinh[x]] + 2*b^2*Log[b
+ a*Sinh[x]])/(2*a*(a^2 + b^2))
```

Maple [A] time = 0.036, size = 108, normalized size = 1.8

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{a\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b-2a\tanh(x/2)-b\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+4\frac{a\ln\left((\tanh(x/\right.}{4a^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)+b^2/a/(a^2+b^2)*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b
)-1/a*ln(tanh(1/2*x)-1)+4/(4*a^2+4*b^2)*a*ln(tanh(1/2*x)^2+1)-8/(4*a^2+4*b^
2)*b*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.51938, size = 100, normalized size = 1.64

$$\frac{b^2\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^3+ab^2}+\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^3 + a*b^2) + 2*b*arctan(e^(-x))/(a
^2 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2) + x/a
```

Fricas [A] time = 1.54987, size = 211, normalized size = 3.46

$$-\frac{2\,ab\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b^{2}\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)-a^{2}\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\left(a^{2}+b^{2}\right)x}{a^{3}+ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(x) + sinh(x)) - b^2*log(2*(a*sinh(x) + b)/(cosh(x) - si
nh(x))) - a^2*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^2 + b^2)*x)/(a^3 + a*
b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.22835, size = 120, normalized size = 1.97

$$\frac{b^{2}\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^{3}+ab^{2}}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\left(a^{2}+b^{2}\right)}+\frac{a\log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) - 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

3.118 $\int \frac{\coth(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

```
[Out] Log[a + b*Csch[x]]/a + Log[Sinh[x]]/a
```

Rubi [A] time = 0.0316641, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3885, 36, 29, 31}

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[a + b*Csch[x]]/a + Log[Sinh[x]]/a
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\operatorname{csch}(x)\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, b\operatorname{csch}(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\operatorname{csch}(x)\right)}{a}\\ &= \frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0079351, size = 11, normalized size = 0.58

$$\frac{\log(a\sinh(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[b + a*Sinh[x]]/a
```

Maple [A] time = 0.021, size = 21, normalized size = 1.1

$$-\frac{\ln(\operatorname{csch}(x))}{a}+\frac{\ln(a+b\operatorname{csch}(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(csch(x))+ln(a+b*csch(x))/a
```

Maxima [A] time = 1.01121, size = 38, normalized size = 2.

$$\frac{x}{a}+\frac{\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
```

Fricas [A] time = 1.54861, size = 72, normalized size = 3.79

$$-\frac{x-\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*csch(x)), x)
```

Giac [A] time = 1.10651, size = 30, normalized size = 1.58

$$\frac{\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(-a*(e^(-x) - e^x) + 2*b))/a
```

3.119 $\int \frac{\coth^2(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}+\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] x/a - ArcTanh[Cosh[x]]/b + (2*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a*b)
```

Rubi [A] time = 0.17816, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3894, 4051, 3770, 3919, 3831, 2660, 618, 206}

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}+\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] x/a - ArcTanh[Cosh[x]]/b + (2*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a*b)
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx &= -\int \frac{-1-\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx \\
&= \frac{i\int \frac{-ib+ia\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b} + \frac{\int \operatorname{csch}(x)\,dx}{b} \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \frac{\left(a^2+b^2\right)\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{ab} \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \left(2\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} + \left(4\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} + \frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{ab}
\end{aligned}
$$

Mathematica [A] time = 0.0813885, size = 65, normalized size = 1.14

$$
\frac{2\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)+bx}{ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (b*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Lo
g[Tanh[x/2]])/(a*b)
```

Maple [B] time = 0.027, size = 110, normalized size = 1.9

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{a}{b\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)-2\frac{b}{a\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,\tanh(x/2)\,b-2\,a}{\sqrt{a^2+b^2}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*csch(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*
a)/(a^2+b^2)^(1/2))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)
/(a^2+b^2)^(1/2))+1/b*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81417, size = 424, normalized size = 7.44

$$\frac{bx-a\log\left(\cosh(x)+\sinh(x)+1\right)+a\log\left(\cosh(x)+\sinh(x)-1\right)+\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2}{a\cosh(x)^2+a\sinh(x)^2+}\right.}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(cosh(x) + sinh(x) + 1) + a*log(cosh(x) + sinh(x) - 1) + sqrt(a
^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2
+ 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x)
+ b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x)
- a)))/(a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*csch(x)), x)
```

Giac [A] time = 1.16172, size = 120, normalized size = 2.11

$$\frac{x}{a} - \frac{\log\left(e^{x} + 1\right)}{b} + \frac{\log\left(\left|e^{x} - 1\right|\right)}{b} - \frac{\sqrt{a^{2} + b^{2}} \log\left(\frac{\left|2\, ae^{x} + 2\, b - 2\, \sqrt{a^{2} + b^{2}}\right|}{\left|2\, ae^{x} + 2\, b + 2\, \sqrt{a^{2} + b^{2}}\right|}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - log(e^x + 1)/b + log(abs(e^x - 1))/b - sqrt(a^2 + b^2)*log(abs(2*a*e^
x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(a*b)
```

3.120 $\int \frac{\coth^3(x)}{a+b\textbf{csch}(x)}\,dx$

Optimal. Leaf size=32

$$\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\text{csch}(x))+\frac{\log(\sinh(x))}{a}-\frac{\text{csch}(x)}{b}$$

```
[Out] -(Csch[x]/b) + (a^(-1) + a/b^2)*Log[a + b*Csch[x]] + Log[Sinh[x]]/a
```

Rubi [A] time = 0.0721172, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\text{csch}(x))+\frac{\log(\sinh(x))}{a}-\frac{\text{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -(Csch[x]/b) + (a^(-1) + a/b^2)*Log[a + b*Csch[x]] + Log[Sinh[x]]/a
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx,x,b\text{csch}(x)\right)}{b^2}\\ &= \frac{\text{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)dx,x,b\text{csch}(x)\right)}{b^2}\\ &= -\frac{\text{csch}(x)}{b}+\left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\text{csch}(x))+\frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0502377, size = 37, normalized size = 1.16

$$\frac{\left(a^2+b^2\right)\log(a\sinh(x)+b)+a^2(-\log(\sinh(x)))-ab\text{csch}(x)}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-(a*b*Csch[x]) - a^2*Log[Sinh[x]] + (a^2 + b^2)*Log[b + a*Sinh[x]])/(a*b^2
)
```

Maple [B] time = 0.03, size = 106, normalized size = 3.3

$$\frac{1}{2b}\tanh\left(\frac{x}{2}\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{a}{b^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 b-2a\tanh(x/2)-b\right)+\frac{1}{a}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 b-2a\tanh(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*csch(x)),x)
```

```
[Out] 1/2/b*tanh(1/2*x)-1/a*ln(tanh(1/2*x)+1)+a/b^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1
/2*x)-b)+1/a*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/2/b/tanh(1/2*x)-a/b^2*
ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.00749, size = 111, normalized size = 3.47

$$\frac{x}{a}+\frac{2e^{(-x)}}{be^{(-2x)}-b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{\left(a^2+b^2\right)\log\left(-2be^{(-x)}+ae^{(-2x)}-a\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) - b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)
/b^2 + (a^2 + b^2)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a*b^2)
```

Fricas [B] time = 1.66492, size = 543, normalized size = 16.97

$$-\frac{b^2x\cosh(x)^2+b^2x\sinh(x)^2-b^2x+2ab\cosh(x)-\left(\left(a^2+b^2\right)\cosh(x)^2+2\left(a^2+b^2\right)\cosh(x)\sinh(x)+\left(a^2+b^2\right)}{ab^2\cosh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 - b^2*x + 2*a*b*cosh(x) - ((a^2 + b^2)*
cosh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2 - a^2 - b
^2)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) + a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 - a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*csch(x)), x)
```

Giac [B] time = 1.19312, size = 108, normalized size = 3.38

$$-\frac{a\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{b^2}+\frac{\left(a^2+b^2\right)\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{ab^2}+\frac{a\left(e^{(-x)}-e^{x}\right)+2\,b}{b^2\left(e^{(-x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(-e^(-x) + e^x))/b^2 + (a^2 + b^2)*log(abs(-a*(e^(-x) - e^x) + 2*
b))/(a*b^2) + (a*(e^(-x) - e^x) + 2*b)/(b^2*(e^(-x) - e^x))
```

3.121 $\int \frac{\coth^4(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{\coth(x)\mathbf{csch}(x)}{2b}$$

```
[Out] x/a - ((2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*(a^2 + b^2)^(3/2)*Arc
Tanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^3) + (a*Coth[x])/b^2 - (Coth[
x]*Csch[x])/(2*b)
```

Rubi [A] time = 0.332179, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3898, 2893, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{\coth(x)\mathbf{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] x/a - ((2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*(a^2 + b^2)^(3/2)*Arc
Tanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^3) + (a*Coth[x])/b^2 - (Coth[
x]*Csch[x])/(2*b)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
```

```
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh(x)\coth^3(x)}{ib+ia\sinh(x)}\,dx \\
&= \frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}-\frac{i\int \frac{\operatorname{csch}(x)\left(-2a^2-3b^2+ab\sinh(x)-2b^2\sinh^2(x)\right)}{ib+ia\sinh(x)}\,dx}{2b^2} \\
&= \frac{x}{a}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}-\frac{\left(i\left(a^2+b^2\right)^2\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^3}+\frac{\left(2a^2+3b^2\right)\int \operatorname{csch}(x)\,dx}{2b^3} \\
&= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}-\frac{\left(2i\left(a^2+b^2\right)^2\right)\operatorname{Subst}\left(\int \frac{1}{ib+2ia\ldots}\right.}{ab^3} \\
&= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}+\frac{\left(4i\left(a^2+b^2\right)^2\right)\operatorname{Subst}\left(\int \frac{}{-4(a^2\ldots}\right.}{a\ldots} \\
&= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csc}\ldots}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.574497, size = 151, normalized size = 1.72

$$
\frac{\operatorname{csch}(x)(a\sinh(x)+b)\left(4a\left(2a^2+3b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-16\left(-a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+4a^2b\tanh\left(\frac{x}{2}\right)+4a^2b\coth\ldots\right.}{8ab^3(a+b\operatorname{csch}(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(8*b^3*x - 16*(-a^2 - b^2)^(3/2)*ArcTan[(a - b*Tan
h[x/2])/Sqrt[-a^2 - b^2]] + 4*a^2*b*Coth[x/2] - a*b^2*Csch[x/2]^2 + 4*a*(2*
a^2 + 3*b^2)*Log[Tanh[x/2]] - a*b^2*Sech[x/2]^2 + 4*a^2*b*Tanh[x/2]))/(8*a*
b^3*(a + b*Csch[x]))
```

Maple [B] time = 0.038, size = 207, normalized size = 2.4

$$\frac{1}{8\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{a}{2\,b^{2}}\tanh\left(\frac{x}{2}\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,\frac{a^{3}}{b^{3}\sqrt{a^{2}+b^{2}}}\mathrm{Artanh}\left(1/2\,\frac{2\,\tanh\left(x/2\right)b-2\,a}{\sqrt{a^{2}+b^{2}}}\right)-4\,\frac{a}{b\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*csch(x)),x)
```

```
[Out] 1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)+1/a*ln(tanh(1/2*x)+1)-2*a^3/b^3/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-4*a/b/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-2/a*b/(a^2+
b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/
2*x)^2+1/b^3*ln(tanh(1/2*x))*a^2+3/2/b*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x
)-1/a*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26011, size = 2174, normalized size = 24.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x*cosh(x)^4 + 2*b^3*x*sinh(x)^4 - 2*a*b^2*cosh(x)^3 + 2*b^3*x -
2*a*b^2*cosh(x) + 2*(4*b^3*x*cosh(x) - a*b^2)*sinh(x)^3 - 4*a^2*b - 4*(b^3*
x - a^2*b)*cosh(x)^2 + 2*(6*b^3*x*cosh(x)^2 - 2*b^3*x - 3*a*b^2*cosh(x) + 2
*a^2*b)*sinh(x)^2 + 2*((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*co
sh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^
2 + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cos
h(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - ((2*a^3 + 3*a*b^2)*cosh(x)^4 + 4*(
2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)^4 + 2*a^3 +
3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 - 3*(2*a^3 + 3
*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3 - (2*a^3 + 3*
```

```
a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((2*a^3 + 3*a*b^2)*co
sh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)
^4 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 -
 3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3
- (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*b^3
*x*cosh(x)^3 - 3*a*b^2*cosh(x)^2 - a*b^2 - 4*(b^3*x - a^2*b)*cosh(x))*sinh(
x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 - 2*a*b^
3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 - a*b^3)*sinh(x)^2 + 4*(a*b^3*co
sh(x)^3 - a*b^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*csch(x)), x)
```

Giac [B] time = 1.20455, size = 217, normalized size = 2.47

$$\frac{x}{a} - \frac{(2a^2 + 3b^2)\log(e^x + 1)}{2b^3} + \frac{(2a^2 + 3b^2)\log(|e^x - 1|)}{2b^3} - \frac{(a^4 + 2a^2b^2 + b^4)\log\left(\frac{|2ae^x + 2b - 2\sqrt{a^2+b^2}|}{|2ae^x + 2b + 2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2 + b^2}ab^3} - \frac{be^{(3x)} - 2ae^{(2x)}}{b^2(e^{(2x)} -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2*(2*a^2 + 3*b^2)*log(e^x + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(abs(e^
x - 1))/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3) - (b*
e^(3*x) - 2*a*e^(2*x) + b*e^x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

3.122 $\int \frac{\coth^5(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=70

$$-\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3}+\frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{a\operatorname{csch}^2(x)}{2b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^3(x)}{3b}$$

```
[Out] -(((a^2 + 2*b^2)*Csch[x])/b^3) + (a*Csch[x]^2)/(2*b^2) - Csch[x]^3/(3*b) +
((a^2 + b^2)^2*Log[a + b*Csch[x]])/(a*b^4) + Log[Sinh[x]]/a
```

Rubi [A] time = 0.094092, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3}+\frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{a\operatorname{csch}^2(x)}{2b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -(((a^2 + 2*b^2)*Csch[x])/b^3) + (a*Csch[x]^2)/(2*b^2) - Csch[x]^3/(3*b) +
((a^2 + b^2)^2*Log[a + b*Csch[x]])/(a*b^4) + Log[Sinh[x]]/a
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(-b^2-x^2)^2}{x(a+x)}\,dx,x,b\operatorname{csch}(x)\right)}{b^4}\\
&= -\frac{\operatorname{Subst}\left(\int \left(a^2\left(1+\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2+b^2)^2}{a(a+x)}\right)dx,x,b\operatorname{csch}(x)\right)}{b^4}\\
&= -\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3}+\frac{a\operatorname{csch}^2(x)}{2b^2}-\frac{\operatorname{csch}^3(x)}{3b}+\frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{\log(\sinh(x))}{a}
\end{aligned}$$

Mathematica [A] time = 0.123764, size = 83, normalized size = 1.19

$$\frac{3a^2b^2\operatorname{csch}^2(x)-6ab\left(a^2+2b^2\right)\operatorname{csch}(x)-6a^2\left(a^2+2b^2\right)\log(\sinh(x))+6\left(a^2+b^2\right)^2\log(a\sinh(x)+b)-2ab^3\operatorname{csch}^3(x)}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-6*a*b*(a^2 + 2*b^2)*Csch[x] + 3*a^2*b^2*Csch[x]^2 - 2*a*b^3*Csch[x]^3 - 6
*a^2*(a^2 + 2*b^2)*Log[Sinh[x]] + 6*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/(6*a*
b^4)
```

Maple [B] time = 0.04, size = 219, normalized size = 3.1

$$\frac{1}{24b}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{a}{8b^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{a^2}{2b^3}\tanh\left(\frac{x}{2}\right)+\frac{7}{8b}\tanh\left(\frac{x}{2}\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{a^3}{b^4}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*csch(x)),x)
```

```
[Out] 1/24/b*tanh(1/2*x)^3+1/8/b^2*tanh(1/2*x)^2*a+1/2/b^3*a^2*tanh(1/2*x)+7/8/b*
tanh(1/2*x)-1/a*ln(tanh(1/2*x)+1)+a^3/b^4*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x
)-b)+2*a/b^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+1/a*ln(tanh(1/2*x)^2*b-2
*a*tanh(1/2*x)-b)-1/24/b/tanh(1/2*x)^3-1/2/b^3/tanh(1/2*x)*a^2-7/8/b/tanh(1
/2*x)+1/8*a/b^2/tanh(1/2*x)^2-1/b^4*a^3*ln(tanh(1/2*x))-2*a/b^2*ln(tanh(1/2
*x))-1/a*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.04943, size = 257, normalized size = 3.67

$$-\frac{2\left(3\,abe^{(-2\,x)}-3\,abe^{(-4\,x)}-3\left(a^2+2\,b^2\right)e^{(-x)}+2\left(3\,a^2+4\,b^2\right)e^{(-3\,x)}-3\left(a^2+2\,b^2\right)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)}-3\,b^3e^{(-4\,x)}+b^3e^{(-6\,x)}-b^3\right)}+\frac{x}{a}-\frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-x)}+\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) - 3*(a^2 + 2*b^2)*e^(-x) + 2*(3*a^2 +
 4*b^2)*e^(-3*x) - 3*(a^2 + 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*
x) + b^3*e^(-6*x) - b^3) + x/a - (a^3 + 2*a*b^2)*log(e^(-x) + 1)/b^4 - (a^3
 + 2*a*b^2)*log(e^(-x) - 1)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*b*e^(-x) +
 a*e^(-2*x) - a)/(a*b^4)
```

Fricas [B] time = 1.83275, size = 3216, normalized size = 45.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b + 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b + 2*a*b^3)*sinh(x)^5 - 3*b^4*x - 3*(3*b^4*x +
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 - 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b + 2*a*b^3)*cosh(x))*sinh(x)^4 - 4*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 - 3*a^3*b - 4*a*b^3 + 15*(a^3*b + 2*a*b^3)*cosh(x)^2 - 3*
(3*b^4*x + 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x + 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x + 2*a^2*b^2 + 20*(a^3*b + 2*a*b^3)*cosh
(x)^3 - 6*(3*b^4*x + 2*a^2*b^2)*cosh(x)^2 - 4*(3*a^3*b + 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b + 2*a*b^3)*cosh(x) - 3*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2 + b^4)
*sinh(x)^6 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 + b^4
 - 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 - a^4 - 2*a^2*b^2 - b^4 +
 4*(5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 + 2*a^2*b^2 + b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*sinh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 + 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 + 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2)*sinh(x)^6 - 3*(a^4 + 2*a^2*b^
2)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 - 5*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 - a^4 - 2*a^2*b^2 + 4*(5*(a^4 + 2*a^2*b^2)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^
2)*cosh(x)^4 + a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 + 2*a^2*b^2)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b + 2*a*b^3)*cosh(x)^4 + a^3*b + 2*a*b^3 - 2*(3*b^4*x +
2*a^2*b^2)*cosh(x)^3 - 2*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (3*b^4*x + 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 - 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 - a*b^4 + 3*(5*a*b^4*cosh
(x)^2 - a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 - 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 - 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 - 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*csch(x)), x)
```

Giac [B] time = 1.15052, size = 230, normalized size = 3.29

$$-\frac{\left(a^3 + 2\,ab^2\right)\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^4} + \frac{\left(a^4 + 2\,a^2b^2 + b^4\right)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} - e^x\right)^3 + 22\,ab^2\left(e^{(-x)} - e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(-e^(-x) + e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(
abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) - e^x)^3 + 22*a
*b^2*(e^(-x) - e^x)^3 + 12*a^2*b*(e^(-x) - e^x)^2 + 24*b^3*(e^(-x) - e^x)^2
 + 12*a*b^2*(e^(-x) - e^x) + 16*b^3)/(b^4*(e^(-x) - e^x)^3)
```

3.123 $\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}+\frac{a\left(a^2+3b^2\right)\coth(x)}{b^4}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(3a^2b^2+a^4+3b^4\right)\tanh^{-}}{b^5}$$

```
[Out] x/a - (3*ArcTanh[Cosh[x]])/(8*b) + ((a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3)
 - ((a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 + (2*(a^2 + b^2)^(5/2)*
ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5) - (a*Coth[x])/b^2 + (a*
(a^2 + 3*b^2)*Coth[x])/b^4 + (a*Coth[x]^3)/(3*b^2) + (3*Coth[x]*Csch[x])/(8
*b) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*b^3) - (Coth[x]*Csch[x]^3)/(4*b)
```

Rubi [A] time = 0.336513, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2897, 3770, 3767, 8, 3768, 2660, 618, 204}

$$\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}+\frac{a\left(a^2+3b^2\right)\coth(x)}{b^4}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(3a^2b^2+a^4+3b^4\right)\tanh^{-}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cosh[x]])/(8*b) + ((a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3)
 - ((a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 + (2*(a^2 + b^2)^(5/2)*
ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5) - (a*Coth[x])/b^2 + (a*
(a^2 + 3*b^2)*Coth[x])/b^4 + (a*Coth[x]^3)/(3*b^2) + (3*Coth[x]*Csch[x])/(8
*b) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*b^3) - (Coth[x]*Csch[x]^3)/(4*b)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\cosh(x)\coth^5(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\int \left(-\frac{1}{a} - \frac{\left(a^4+3a^2b^2+3b^4\right)\operatorname{csch}(x)}{b^5} - \frac{a\left(-a^2-3b^2\right)\operatorname{csch}^2(x)}{b^4} - \frac{\left(a^2+3b^2\right)\operatorname{csch}^3(x)}{b^3} + \frac{a\operatorname{csch}^4(\ldots}{b^2}\right.$$

$$= \frac{x}{a} - \frac{a\int \operatorname{csch}^4(x)\,dx}{b^2} + \frac{\int \operatorname{csch}^5(x)\,dx}{b} - \frac{\left(i\left(a^2+b^2\right)^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^5} - \frac{\left(a\left(a^2+3b^2\right)\right)\int \operatorname{csch}^2(x)\ldots}{b^4}$$

$$= \frac{x}{a} - \frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x))}{b^5} - \frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2b^3} - \frac{\coth(x)\operatorname{csch}^3(x)}{4b} - \frac{(ia)\,\mathrm{S}\ldots}{\ldots}$$

$$= \frac{x}{a} + \frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} - \frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x))}{b^5} - \frac{a\coth(x)}{b^2} + \frac{a\left(a^2+3b^2\ldots}{b^4\ldots}$$

$$= \frac{x}{a} - \frac{3\tanh^{-1}(\cosh(x))}{8b} + \frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} - \frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x))}{b^5} + \frac{2}{\ldots}$$

Mathematica [A] time = 1.56235, size = 269, normalized size = 1.47

$$\operatorname{csch}(x)(a\sinh(x)+b)\left(32a^2b\left(3a^2+7b^2\right)\tanh\left(\tfrac{x}{2}\right)+32a^2b\left(3a^2+7b^2\right)\coth\left(\tfrac{x}{2}\right)-6ab^2\left(4a^2+9b^2\right)\operatorname{csch}^2\left(\tfrac{x}{2}\right)-6ab^2\left(4\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(192*b^5*x + 384*(-a^2 - b^2)^(5/2)*ArcTan[(a - b*
Tanh[x/2])/Sqrt[-a^2 - b^2]] + 32*a^2*b*(3*a^2 + 7*b^2)*Coth[x/2] - 6*a*b^2
*(4*a^2 + 9*b^2)*Csch[x/2]^2 - 3*a*b^4*Csch[x/2]^4 + 24*a*(8*a^4 + 20*a^2*b
^2 + 15*b^4)*Log[Tanh[x/2]] - 6*a*b^2*(4*a^2 + 9*b^2)*Sech[x/2]^2 + 3*a*b^4
*Sech[x/2]^4 - 64*a^2*b^3*Csch[x]^3*Sinh[x/2]^4 + 4*a^2*b^3*Csch[x/2]^4*Sin
h[x] + 32*a^2*b*(3*a^2 + 7*b^2)*Tanh[x/2]))/(192*a*b^5*(a + b*Csch[x]))
```

Maple [B] time = 0.043, size = 360, normalized size = 2.

$$\frac{1}{64\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^{4}+\frac{a}{24\,b^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}+\frac{a^{2}}{8\,b^{3}}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{1}{4\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{a^{3}}{2\,b^{4}}\tanh\left(\frac{x}{2}\right)+\frac{9\,a}{8\,b^{2}}\tanh\left(\frac{x}{2}\right)+\frac{1}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(a+b*csch(x)),x)
```

```
[Out] 1/64/b*tanh(1/2*x)^4+1/24/b^2*a*tanh(1/2*x)^3+1/8/b^3*a^2*tanh(1/2*x)^2+1/4
/b*tanh(1/2*x)^2+1/2/b^4*a^3*tanh(1/2*x)+9/8/b^2*a*tanh(1/2*x)+1/a*ln(tanh(
1/2*x)+1)-2*a^5/b^5/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+
b^2)^(1/2))-6*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^
2+b^2)^(1/2))-6*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+
b^2)^(1/2))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^
2)^(1/2))-1/64/b/tanh(1/2*x)^4-1/8/b^3/tanh(1/2*x)^2*a^2-1/4/b/tanh(1/2*x)^
2+1/b^5*ln(tanh(1/2*x))*a^4+5/2/b^3*ln(tanh(1/2*x))*a^2+15/8/b*ln(tanh(1/2*
x))+1/24*a/b^2/tanh(1/2*x)^3+1/2*a^3/b^4/tanh(1/2*x)+9/8*a/b^2/tanh(1/2*x)-
1/a*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.25833, size = 7830, normalized size = 42.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^5*x*cosh(x)^8 + 24*b^5*x*sinh(x)^8 - 6*(4*a^3*b^2 + 9*a*b^4)*cos
h(x)^7 + 6*(32*b^5*x*cosh(x) - 4*a^3*b^2 - 9*a*b^4)*sinh(x)^7 - 48*(2*b^5*x
```

- a^4*b - 3*a^2*b^3)*cosh(x)^6 + 6*(112*b^5*x*cosh(x)^2 - 16*b^5*x + 8*a^4
*b + 24*a^2*b^3 - 7*(4*a^3*b^2 + 9*a*b^4)*cosh(x))*sinh(x)^6 + 24*b^5*x + 6
*(4*a^3*b^2 + a*b^4)*cosh(x)^5 + 6*(224*b^5*x*cosh(x)^3 + 4*a^3*b^2 + a*b^4
- 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^2 - 48*(2*b^5*x - a^4*b - 3*a^2*b^3)*co
sh(x))*sinh(x)^5 - 48*a^4*b - 112*a^2*b^3 + 48*(3*b^5*x - 3*a^4*b - 7*a^2*b
^3)*cosh(x)^4 + 6*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b - 56*a^2*b^3 -
35*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^3 - 120*(2*b^5*x - a^4*b - 3*a^2*b^3)*cos
h(x)^2 + 5*(4*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 6*(4*a^3*b^2 + a*b^4)*c
osh(x)^3 + 6*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 + a*b^4 - 35*(4*a^3*b^2 + 9*a
*b^4)*cosh(x)^4 - 160*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^3 + 10*(4*a^3*b
^2 + a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x))*sinh(x)
^3 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cosh(x)^2 + 2*(336*b^5*x*cosh(x)^6
- 48*b^5*x - 63*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^5 + 72*a^4*b + 152*a^2*b^3 -
360*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^4 + 30*(4*a^3*b^2 + a*b^4)*cosh(
x)^3 + 144*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 + a*b^4
)*cosh(x))*sinh(x)^2 + 24*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 + 2*a
^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^8 - 4*(a^
4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 4*(a^4 + 2*a^2*b^2 + b^4 - 7*(a^4 + 2*a^2*
b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 -
3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 + 2*a^2*b^2 + b^4)*co
sh(x)^4 + 2*(35*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 + 6*a^2*b^2 + 3*b
^4 - 30*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^
4 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 10*(a^4 + 2*a^2*b^2 + b^4)*cos
h(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 4*(a^4 + 2*a^2*b^2
+ b^4)*cosh(x)^2 + 4*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 15*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^4 - a^4 - 2*a^2*b^2 - b^4 + 9*(a^4 + 2*a^2*b^2 + b^4)*c
osh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^7 - 3*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 + 2*a^2*
b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(
x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt
(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*c
osh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 6*(4*a^3*b^2 + 9*a*b^4)*cosh(x)
- 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 - 4*(
8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
- 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 2
0*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)
)*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 + 15*a*
b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a^3*b^2
+ 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sin
h(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^
2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8
- 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a
^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*c
osh(x))*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 +
15*a*b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2
)*sinh(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 +

```
20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) + 2*(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^6 -
144*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^5 - 12*a^3*b^2 - 27*a*b^4 + 15*(4
*a^3*b^2 + a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^3
+ 9*(4*a^3*b^2 + a*b^4)*cosh(x)^2 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cos
h(x))*sinh(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)
^8 - 4*a*b^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 - 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5
*cosh(x)^2 - a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 - 3*a*b^5*cosh
(x))*sinh(x)^5 + 2*(35*a*b^5*cosh(x)^4 - 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh
(x)^4 + 8*(7*a*b^5*cosh(x)^5 - 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x
)^3 + 4*(7*a*b^5*cosh(x)^6 - 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 - a*b^5
)*sinh(x)^2 + 8*(a*b^5*cosh(x)^7 - 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 -
a*b^5*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**6/(a + b*csch(x)), x)
```

Giac [A] time = 1.16333, size = 412, normalized size = 2.25

$$\frac{x}{a}-\frac{\left(8\,a^4+20\,a^2b^2+15\,b^4\right)\log\left(e^x+1\right)}{8\,b^5}+\frac{\left(8\,a^4+20\,a^2b^2+15\,b^4\right)\log\left(|e^x-1|\right)}{8\,b^5}-\frac{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\log\left(\frac{|2\,a}{|2\,a}\right.}{\sqrt{a^2+b^2}ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^x + 1)/b^5 + 1/8*(8*a^4 + 20*
a^2*b^2 + 15*b^4)*log(abs(e^x - 1))/b^5 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5) - 1/12*(12*a^2*b*e^(7*x) + 27*b^3*e^(7*x
) - 24*a^3*e^(6*x) - 72*a*b^2*e^(6*x) - 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) +
72*a^3*e^(4*x) + 168*a*b^2*e^(4*x) - 12*a^2*b*e^(3*x) - 3*b^3*e^(3*x) - 72*
a^3*e^(2*x) - 152*a*b^2*e^(2*x) + 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 + 56*a
*b^2)/(b^4*(e^(2*x) - 1)^4)
```

3.124 $\int \frac{\coth^7(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(a^2+3b^2\right)\text{csch}^3(x)}{3b^3}+\frac{a\left(a^2+3b^2\right)\text{csch}^2(x)}{2b^4}-\frac{\left(3a^2b^2+a^4+3b^4\right)\text{csch}(x)}{b^5}+\frac{\left(a^2+b^2\right)^3\log(a+b\,\text{csch}(x))}{ab^6}+\frac{a\,\text{csch}^4(x)}{4b^2}$$

```
[Out] -(((a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x])/b^5) + (a*(a^2 + 3*b^2)*Csch[x]^2)/(2
*b^4) - ((a^2 + 3*b^2)*Csch[x]^3)/(3*b^3) + (a*Csch[x]^4)/(4*b^2) - Csch[x]
^5/(5*b) + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/(a*b^6) + Log[Sinh[x]]/a
```

Rubi [A] time = 0.140522, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{\left(a^2+3b^2\right)\text{csch}^3(x)}{3b^3}+\frac{a\left(a^2+3b^2\right)\text{csch}^2(x)}{2b^4}-\frac{\left(3a^2b^2+a^4+3b^4\right)\text{csch}(x)}{b^5}+\frac{\left(a^2+b^2\right)^3\log(a+b\,\text{csch}(x))}{ab^6}+\frac{a\,\text{csch}^4(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] -(((a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x])/b^5) + (a*(a^2 + 3*b^2)*Csch[x]^2)/(2
*b^4) - ((a^2 + 3*b^2)*Csch[x]^3)/(3*b^3) + (a*Csch[x]^4)/(4*b^2) - Csch[x]
^5/(5*b) + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/(a*b^6) + Log[Sinh[x]]/a
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\coth^7(x)}{a+b\,\text{csch}(x)}\,dx=\frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^3}{x(a+x)}\,dx,x,b\,\text{csch}(x)\right)}{b^6}$$

$$=\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(a^2+b^2\right)}{a^4}\right)-\frac{b^6}{ax}+a\left(a^2+3b^2\right)x-\left(a^2+3b^2\right)x^2+ax^3-x^4+\frac{\left(a^2+b^2\right)^3}{a(a+x)}\right)dx,x,b\,\text{c}\right.}{b^6}$$

$$=-\frac{\left(a^4+3a^2b^2+3b^4\right)\text{csch}(x)}{b^5}+\frac{a\left(a^2+3b^2\right)\text{csch}^2(x)}{2b^4}-\frac{\left(a^2+3b^2\right)\text{csch}^3(x)}{3b^3}+\frac{a\,\text{csch}^4(x)}{4b^2}-\frac{\text{csch}^5}{5b}$$

Mathematica [A] time = 0.252734, size = 130, normalized size = 1.09

$$\frac{-20b^3\left(a^2+3b^2\right)\operatorname{csch}^3(x)+30ab^2\left(a^2+3b^2\right)\operatorname{csch}^2(x)-60b\left(3a^2b^2+a^4+3b^4\right)\operatorname{csch}(x)-60a\left(3a^2b^2+a^4+3b^4\right)\log}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x] + 30*a*b^2*(a^2 + 3*b^2)*Csch[x]^2
 - 20*b^3*(a^2 + 3*b^2)*Csch[x]^3 + 15*a*b^4*Csch[x]^4 - 12*b^5*Csch[x]^5 -
 60*a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + (60*(a^2 + b^2)^3*Log[b + a*
Sinh[x]])/a)/(60*b^6)
```

Maple [B] time = 0.046, size = 388, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/(a+b*csch(x)),x)
```

```
[Out] -3*a/b^2*ln(tanh(1/2*x))+5/16/b^2*tanh(1/2*x)^2*a+19/16/b*tanh(1/2*x)-19/16
/b/tanh(1/2*x)-1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*
x)^2*b-2*a*tanh(1/2*x)-b)-1/b^6*a^5*ln(tanh(1/2*x))+1/64*a/b^2/tanh(1/2*x)^
4-1/24/b^3/tanh(1/2*x)^3*a^2-1/2/b^5/tanh(1/2*x)*a^4+11/8/b^3*a^2*tanh(1/2*
x)+3*a^3/b^4*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-11/8/b^3/tanh(1/2*x)*a^2
+5/16*a/b^2/tanh(1/2*x)^2-3/b^4*a^3*ln(tanh(1/2*x))+1/8*a^3/b^4/tanh(1/2*x)
^2+1/64/b^2*a*tanh(1/2*x)^4+1/24/b^3*a^2*tanh(1/2*x)^3+1/8/b^4*tanh(1/2*x)^
2*a^3+1/2/b^5*a^4*tanh(1/2*x)+1/b^6*a^5*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-
b)+3/32/b*tanh(1/2*x)^3-3/32/b/tanh(1/2*x)^3+3*a/b^2*ln(tanh(1/2*x)^2*b-2*a
*tanh(1/2*x)-b)+1/160/b*tanh(1/2*x)^5-1/160/b/tanh(1/2*x)^5
```

Maxima [B] time = 1.08311, size = 491, normalized size = 4.13

$$\frac{2\left(15\left(a^4+3a^2b^2+3b^4\right)e^{(-x)}-15\left(a^3b+3ab^3\right)e^{(-2x)}-20\left(3a^4+8a^2b^2+6b^4\right)e^{(-3x)}+15\left(3a^3b+7ab^3\right)e^{(-4x)}+2\left(\right.}{15\left(5b^5e^{(-2x)}-10b^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b + 3*a*b^3)*e^(-2*x) -
 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-3*x) + 15*(3*a^3*b + 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 + 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b + 7*a*b^3)*e^(-6
*x) - 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-7*x) + 15*(a^3*b + 3*a*b^3)*e^(-8*
x) + 15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) - 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) - 5*b^5*e^(-8*x) + b^5*e^(-10*x) - b^5) + x/a - (a^5 +
 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + 1)/b^6 - (a^5 + 3*a^3*b^2 + 3*a*b^4)*log
(e^(-x) - 1)/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*b*e^(-x) + a*
e^(-2*x) - a)/(a*b^6)
```

Fricas [B] time = 2.2728, size = 9682, normalized size = 81.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 + 30*(a^5*b + 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) + a^5*b + 3*a^3*b^3 + 3*a*b^5)*sin
h(x)^9 - 15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 - 5*b^6*x - 2*a^4*b^2 - 6*a^2*b^4 + 18*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 + 27*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 - 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x))*sinh(x)^7
 - 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 + 21*a^2*b^4 + 252*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 - 42*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 + 4*(45*a^5*b + 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 + 45*a^5*b + 115*a^3*b^3 +
99*a*b^5 + 945*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 210*(5*b^6*x + 2*a
^4*b^2 + 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^5 - 30*(5*b^6*x
 + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 - 15*b^6*x - 9
*a^4*b^2 - 21*a^2*b^4 + 378*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 - 105*(
5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b + 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^2 + 2*(45*a^
5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b + 8*a^3*b^3
 + 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 + 63*(a^5*b + 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 - 21*(5*b^6*x + 2*a^4*b^2 +
 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 + (45*a^5*b + 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 - 3*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 + 2
16*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x - 84*(5*b^6*x + 2*a^4
*b^2 + 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 + 18*a^2*b^4 - 168*(3*a^5*b + 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4
 + 8*(45*a^5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 - 36*(5*b^6*x + 3*a^4*b^2
 + 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 + 30*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x) - 15*((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*
sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^10 - 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 -
9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 10
*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 -
 a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 + 4*(63*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 15*(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*cosh(x)^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 - 7*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 5*(9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
```

```
4 + b^6)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cos
h(x)^4 - 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*cosh(x)^7 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 - 4*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 15*
((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 - 5*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 - 9*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*(63*(a^6 + 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 - a^6
 - 3*a^4*b^2 - 3*a^2*b^4 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 - 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 -
 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 + 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 + 27*(a^5*b + 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 - 12*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 + 3*a^5*b + 9*a^3*b^3 + 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^5 + 2*(45*a^5*b + 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 - 12*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 - 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 - 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 - a*b^6*cosh(x))*sinh(x)^7 - a*b^6 + 10*(21*a*b^6*cosh(x)^4 - 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 - 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 - 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 - 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 - a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 - 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 - 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 - 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
- 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^7(x)}{a + b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**7/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**7/(a + b*csch(x)), x)
```

Giac [B] time = 1.15962, size = 398, normalized size = 3.34

$$-\frac{\left(a^{5}+3\,a^{3}b^{2}+3\,ab^{4}\right)\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{b^{6}}+\frac{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{ab^{6}}+\frac{137\,a^{5}\left(e^{(-x)}-e^{x}\right)^{5}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(abs(-e^(-x) + e^x))/b^6 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^6) + 1/60*(137*a
^5*(e^(-x) - e^x)^5 + 411*a^3*b^2*(e^(-x) - e^x)^5 + 411*a*b^4*(e^(-x) - e^
x)^5 + 120*a^4*b*(e^(-x) - e^x)^4 + 360*a^2*b^3*(e^(-x) - e^x)^4 + 360*b^5*
(e^(-x) - e^x)^4 + 120*a^3*b^2*(e^(-x) - e^x)^3 + 360*a*b^4*(e^(-x) - e^x)^
3 + 160*a^2*b^3*(e^(-x) - e^x)^2 + 480*b^5*(e^(-x) - e^x)^2 + 240*a*b^4*(e^
(-x) - e^x) + 384*b^5)/(b^6*(e^(-x) - e^x)^5)
```

3.125 $\int e^{c(a+bx)}\mathrm{csch}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=199

$$\frac{64\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{48\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{192\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{5bc\left(1-e^{2c(a+bx)}\right)^5}-$$

```
[Out] (-32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b
*x)))^6) + (192*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^
(2*c*(a + b*x)))^5) - (48*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c
*(1 - E^(2*c*(a + b*x)))^4) + (64*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3)
```

Rubi [A] time = 0.302527, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{64\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{48\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{192\sinh(ac+bcx)\sqrt{\mathrm{csch}^2(ac+bcx)}}{5bc\left(1-e^{2c(a+bx)}\right)^5}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```


```
[Out] (-32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b
*x)))^6) + (192*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^
(2*c*(a + b*x)))^5) - (48*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c
*(1 - E^(2*c*(a + b*x)))^4) + (64*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx &= \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int\frac{128x^7}{\left(-1+x^2\right)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(128\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int\frac{x^7}{\left(-1+x^2\right)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int\frac{x^3}{(-1+x)^7}\,dx,x,e^{2c(a+bx)}\right)}{bc}\\
&= \frac{\left(64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int\left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right)\right.}{bc}\\
&= -\frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6}+\frac{192\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5}-\frac{48\sqrt{c}}{}
\end{aligned}$$

Mathematica [A] time = 0.0768268, size = 84, normalized size = 0.42

$$-\frac{16\left(6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}-1\right)\sinh(c(a+bx))\sqrt{\operatorname{csch}^2(c(a+bx))}}{15bc\left(e^{2c(a+bx)}-1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sqrt[Csch[c*(a + b*x)]^2]*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b
*x)))^6)
```

Maple [A] time = 0.203, size = 91, normalized size = 0.5

$$-\frac{\left(320\,e^{6\,c(bx+a)}-240\,e^{4\,c(bx+a)}+96\,e^{2\,c(bx+a)}-16\right)e^{-c(bx+a)}}{15\,\left(e^{2\,c(bx+a)}-1\right)^5 cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] -16/15/(exp(2*c*(b*x+a))-1)^5*(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(
1/2)*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)/c/b*exp
(-c*(b*x+a))
```

Maxima [B] time = 1.56405, size = 521, normalized size = 2.62

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{}{bc\left(e^{(12\,bcx+12\,ac}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.61539, size = 1512, normalized size = 7.6

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}-6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{7}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(7/2)*exp(b*c*x), x)
```

Giac [A] time = 1.16192, size = 122, normalized size = 0.61

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}-15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}-1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{6}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))
```

3.126 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=147

$$-\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

```
[Out] (-4*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)
))^4) + (32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3) - (8*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 -
 E^(2*c*(a + b*x)))^2)
```

Rubi [A] time = 0.171003, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)
))^4) + (32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3) - (8*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 -
 E^(2*c*(a + b*x)))^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{32x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2c(a}\right.}{bc}$$

$$= -\frac{4\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{8\sqrt{\operatorname{csch}}}{}$$

Mathematica [A] time = 0.064242, size = 72, normalized size = 0.49

$$-\frac{4\left(-4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\sinh(c(a+bx))\sqrt{\operatorname{csch}^2(c(a+bx))}}{3bc\left(e^{2c(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sqrt[Csch[c*(a + b*x)]^
2]*Sinh[c*(a + b*x)])/(3*b*c*(-1 + E^(2*c*(a + b*x)))^4)
```

Maple [A] time = 0.164, size = 80, normalized size = 0.5

$$-\frac{\left(24\,e^{4\,c(bx+a)}-16\,e^{2\,c(bx+a)}+4\right)e^{-c(bx+a)}}{3\left(e^{2\,c(bx+a)}-1\right)^3 cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -4/3/(exp(2*c*(b*x+a))-1)^3*(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/
2)*(6*exp(4*c*(b*x+a))-4*exp(2*c*(b*x+a))+1)/c/b*exp(-c*(b*x+a))
```

Maxima [A] time = 1.55461, size = 282, normalized size = 1.92

$$-\frac{8\, e^{(4\, bcx+4\, ac)}}{bc\left(e^{(8\, bcx+8\, ac)} - 4\, e^{(6\, bcx+6\, ac)} + 6\, e^{(4\, bcx+4\, ac)} - 4\, e^{(2\, bcx+2\, ac)} + 1\right)} + \frac{16\, e^{(2\, bcx+2\, ac)}}{3\, bc\left(e^{(8\, bcx+8\, ac)} - 4\, e^{(6\, bcx+6\, ac)} + 6\, e^{(4\, bcx+4\, ac)} - 4\, e^{(2\, b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.56371, size = 797, normalized size = 5.42

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6} + 6\, bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5} + bc\sinh\left(bcx+ac\right)^{6} - 4\, bc\cosh\left(bcx+ac\right)^{4} + \left(15\, bc\, c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{5}{2}} e^{bcx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(5/2)*exp(b*c*x), x)
```

Giac [A] time = 1.15435, size = 104, normalized size = 0.71

$$-\frac{4\left(6\, e^{(4\, bcx+4\, ac)} - 4\, e^{(2\, bcx+2\, ac)} + 1\right)}{3\, bc\left(e^{(2\, bcx+2\, ac)} - 1\right)^{4}\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) - 1)^4*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

3.127 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[Out] (-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1
- E^(2*c*(a + b*x)))^2)
```

Rubi [A] time = 0.117562, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 264}

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1
- E^(2*c*(a + b*x)))^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(8\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= -\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Mathematica [A] time = 0.0427245, size = 56, normalized size = 0.97

$$-\frac{2e^{4c(a+bx)}\sinh^3(c(a+bx))\operatorname{csch}^2(c(a+bx))^{3/2}}{bc\left(e^{2c(a+bx)}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*(Csch[c*(a + b*x)]^2)^(3/2)*Sinh[c*(a + b*x)]^3)/(b*c
*(-1 + E^(2*c*(a + b*x)))^2)
```

Maple [A] time = 0.16, size = 69, normalized size = 1.2

$$-2\,\frac{\left(2\,\mathrm{e}^{2\,c(bx+a)}-1\right)\mathrm{e}^{-c(bx+a)}}{\left(\mathrm{e}^{2\,c(bx+a)}-1\right)cb}\sqrt{\frac{\mathrm{e}^{2\,c(bx+a)}}{\left(\mathrm{e}^{2\,c(bx+a)}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] -2/(exp(2*c*(b*x+a))-1)*(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*(
2*exp(2*c*(b*x+a))-1)/c/b*exp(-c*(b*x+a))
```

Maxima [A] time = 1.57095, size = 113, normalized size = 1.95

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.57986, size = 300, normalized size = 5.17

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\left(bc\cosh\left(bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(3/2)*exp(b*c*x), x)
```

Giac [A] time = 1.16995, size = 86, normalized size = 1.48

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)}{bc\left(e^{(2\,bcx+2\,ac)}-1\right)^2\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(b*c*
x + a*c) - e^(-b*c*x - a*c)))
```

3.128 $\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

```
[Out] (Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b
*c)
```

Rubi [A] time = 0.090595, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b
*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(2\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

Mathematica [A] time = 0.0358906, size = 44, normalized size = 0.96

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))\sqrt{\operatorname{csch}^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[c*(a + b*x)]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b
*c)
```

Maple [A] time = 0.183, size = 68, normalized size = 1.5

$$\frac{\left(e^{2\,c(bx+a)}-1\right)\ln\left(e^{2\,bcx}-e^{-2\,ac}\right)e^{-c(bx+a)}}{cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] (1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*(exp(2*c*(b*x+a))-1)/c/b*
ln(exp(2*b*c*x)-exp(-2*a*c))*exp(-c*(b*x+a))
```

Maxima [A] time = 1.54704, size = 53, normalized size = 1.15

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.59559, size = 97, normalized size = 2.11

$$\frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\sqrt{\operatorname{csch}^2(ac+bcx)}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(csch(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Giac [A] time = 1.15407, size = 65, normalized size = 1.41

$$\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(e^(2*b*c*x + 2*a*c) - 1))/(b*c*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*
c)))
```

3.129 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(4*b*c*Sqrt[Csch[a*c + b*c*x]^2]) - (
x*Csch[a*c + b*c*x])/(2*Sqrt[Csch[a*c + b*c*x]^2])
```

Rubi [A] time = 0.114765, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(4*b*c*Sqrt[Csch[a*c + b*c*x]^2]) - (
x*Csch[a*c + b*c*x])/(2*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\,\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\,\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\,\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)dx,x,e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}\end{aligned}$$

Mathematica [A] time = 0.050228, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))}{4bc\sqrt{\operatorname{csch}^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)])/(4*b*c*Sqrt[Csch[c*(a + b
*x)]^2])
```

Maple [A] time = 0.175, size = 106, normalized size = 1.4

$$-\frac{xe^{c(bx+a)}}{2\,e^{2\,c(bx+a)}-2}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}}+\frac{e^{3\,c(bx+a)}}{\left(4\,e^{2\,c(bx+a)}-4\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] -1/2/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)
*x*exp(c*(b*x+a))+1/4/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(ex
p(2*c*(b*x+a))-1)/c/b*exp(3*c*(b*x+a))
```

Maxima [A] time = 1.61341, size = 49, normalized size = 0.66

$$-\frac{bcx+ac}{2\,bc}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A] time = 1.49332, size = 165, normalized size = 2.23

$$-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\operatorname{csch}^{2}\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(csch(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.13629, size = 96, normalized size = 1.3

$$-\frac{1}{2}\,x\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\frac{e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

3.130 $\int \frac{e^{c(a+bx)}}{\mathrm{csch}^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\mathrm{csch}(ac+bcx)}{32bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{3x\mathrm{csch}(ac+bcx)}{8\sqrt{\mathrm{csch}^2(ac+bcx)}}$$

```
[Out] Csch[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (3
*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(16*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(3*x*Csch[a*c + b*c*x])/(8*Sqrt[Csch[a*c + b*c*x]^2])
```

Rubi [A] time = 0.153345, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\mathrm{csch}(ac+bcx)}{32bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{3x\mathrm{csch}(ac+bcx)}{8\sqrt{\mathrm{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (3
*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(16*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(3*x*Csch[a*c + b*c*x])/(8*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{3x\operatorname{csch}(}{8\sqrt{\operatorname{csch}^2}}$$

Mathematica [A] time = 0.0633141, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)} - 3e^{2c(a+bx)} + \frac{1}{2}e^{4c(a+bx)} + 6bcx\right)\operatorname{csch}^3(c(a+bx))}{16bc\operatorname{csch}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3)/(16*b*c*(Csch[c*(a + b*x)]^2)^(3/2))
```

Maple [A] time = 0.171, size = 216, normalized size = 1.3

$$\frac{3\,x e^{c(bx+a)}}{8\,e^{2\,c(bx+a)}-8}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}} + \frac{e^{5\,c(bx+a)}}{\left(32\,e^{2\,c(bx+a)}-32\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}} - \frac{3\,e^{3\,c(bx+a)}}{\left(16\,e^{2\,c(bx+a)}-16\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(e^{2\,c(bx+a)}-1\right)^2}}} + \frac{e^{-c}}{\left(16\,e^{2\,c(bx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 3/8/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*
x*exp(c*(b*x+a))+1/32/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(ex
p(2*c*(b*x+a))-1)/c/b*exp(5*c*(b*x+a))-3/16/(1/(exp(2*c*(b*x+a))-1)^2*exp(2
*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)/c/b*exp(3*c*(b*x+a))+1/16/(1/(exp(2
*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)/c/b*exp(-c*(b
*x+a))
```

Maxima [A] time = 1.55534, size = 84, normalized size = 0.52

$$\frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.54929, size = 319, normalized size = 1.97

$$\frac{3\,\cosh\left(bcx+ac\right)^{3}+9\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-\sinh\left(bcx+ac\right)^{3}+6\,(2\,bcx-1)\cosh\left(bcx+ac\right)-3\,\left(4\,bcx+\mathrm{co}\right.}{32\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A] time = 1.19345, size = 275, normalized size = 1.7

$$\frac{\left(12\,bcxe^{(-ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2\,bcx-3\,ac)}+\right.}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2
*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*
c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x - 3*a*c) + (e^(4*b*c*x + 9*a*c)*sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 7*a*c)*sgn(e^(b*c*x + a*c
) - e^(-b*c*x - a*c)))*e^(-6*a*c))*e^(a*c)/(b*c)
```

3.131 $\int \frac{e^{c(a+bx)}}{\mathbf{csch}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\mathrm{csch}(ac+bcx)}{128bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\mathrm{csch}(ac+bcx)}{64bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\mathrm{csch}(ac+bcx)}{32bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\mathrm{csch}(ac+bcx)}{128bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\mathrm{cs}}{192bc\sqrt{\mathrm{csc}}}$$

```
[Out] Csch[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (
5*Csch[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) +
 (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2])
 - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(128*b*c*Sqrt[Csch[a*c + b*c*x]^
2]) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x])/(192*b*c*Sqrt[Csch[a*c + b*c*x]
^2]) - (5*x*Csch[a*c + b*c*x])/(16*Sqrt[Csch[a*c + b*c*x]^2])
```

Rubi [A] time = 0.199711, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-4c(a+bx)}\mathrm{csch}(ac+bcx)}{128bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\mathrm{csch}(ac+bcx)}{64bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\mathrm{csch}(ac+bcx)}{32bc\sqrt{\mathrm{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\mathrm{csch}(ac+bcx)}{128bc\sqrt{\mathrm{csch}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\mathrm{cs}}{192bc\sqrt{\mathrm{csc}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] Csch[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (
5*Csch[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) +
 (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2])
 - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(128*b*c*Sqrt[Csch[a*c + b*c*x]^
2]) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x])/(192*b*c*Sqrt[Csch[a*c + b*c*x]
^2]) - (5*x*Csch[a*c + b*c*x])/(16*Sqrt[Csch[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{4c(a+b}}{128bc\sqrt{}}$$

Mathematica [A] time = 0.102871, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)} - 5e^{-2c(a+bx)} + 10e^{2c(a+bx)} - \frac{5}{2}e^{4c(a+bx)} + \frac{1}{3}e^{6c(a+bx)} - 20bcx\right)\operatorname{csch}^5(c(a+bx))}{64bc\operatorname{csch}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
)/(64*b*c*(Csch[c*(a + b*x)]^2)^(5/2))
```

Maple [A] time = 0.174, size = 326, normalized size = 1.3

$$-\frac{5\,x\mathrm{e}^{c(bx+a)}}{16\,\mathrm{e}^{2\,c(bx+a)}-16}\frac{1}{\sqrt{\frac{\mathrm{e}^{2\,c(bx+a)}}{\left(\mathrm{e}^{2\,c(bx+a)}-1\right)^2}}} + \frac{\mathrm{e}^{7\,c(bx+a)}}{\left(192\,\mathrm{e}^{2\,c(bx+a)}-192\right)cb}\frac{1}{\sqrt{\frac{\mathrm{e}^{2\,c(bx+a)}}{\left(\mathrm{e}^{2\,c(bx+a)}-1\right)^2}}} - \frac{5\,\mathrm{e}^{5\,c(bx+a)}}{\left(128\,\mathrm{e}^{2\,c(bx+a)}-128\right)cb}\frac{1}{\sqrt{\frac{\mathrm{e}^{2\,c(bx+a)}}{\left(\mathrm{e}^{2\,c(bx+a)}-1\right)^2}}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -5/16/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1
)*x*exp(c*(b*x+a))+1/192/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/
(exp(2*c*(b*x+a))-1)/c/b*exp(7*c*(b*x+a))-5/128/(1/(exp(2*c*(b*x+a))-1)^2*e
xp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)/c/b*exp(5*c*(b*x+a))+5/32/(1/(e
xp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)/c/b*exp(3
*c*(b*x+a))-5/64/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c
*(b*x+a))-1)/c/b*exp(-c*(b*x+a))+1/128/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b
*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)/c/b*exp(-3*c*(b*x+a))
```

Maxima [A] time = 1.56872, size = 122, normalized size = 0.49

$$\frac{\left(2\, e^{(10\, bcx+10\, ac)} - 15\, e^{(8\, bcx+8\, ac)} + 60\, e^{(6\, bcx+6\, ac)} - 30\, e^{(2\, bcx+2\, ac)} + 3\right) e^{(-4\, bcx-4\, ac)}}{384\, bc} - \frac{5\, (bcx + ac)}{16\, bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Fricas [A] time = 1.58458, size = 559, normalized size = 2.24

$$\frac{5\, \cosh\left(bcx + ac\right)^{5} + 25\, \cosh\left(bcx + ac\right) \sinh\left(bcx + ac\right)^{4} - \sinh\left(bcx + ac\right)^{5} - 5\left(2\, \cosh\left(bcx + ac\right)^{2} - 3\right) \sinh\left(bcx + ac\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int \frac{e^{bcx}}{\left(\operatorname{csch}^{2}\left(ac + bcx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(5/2), x)
```

Giac [A] time = 1.2132, size = 375, normalized size = 1.5

$$-\frac{\left(120\,bcxe^{(-ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\left(30\,e^{(4\,bcx+4\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-b}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*
e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))*e^(-4*b*c*x - 5*a*c) - (2*e^(6*b*c*x + 20*a*c)*sgn(e^(b*c
*x + a*c) - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) + 60*e^(2*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)))*e^(-15*a*c))*e^(a*c)/(b*c)
```

3.132 $\int \frac{x^5}{\sqrt{\mathbf{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=81

$$\frac{2\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}-\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}}$$

```
[Out] (-2*x^2)/(21*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^6/(7*Sqrt[Csch[2*Log[c*x]]]) +
 (2*EllipticF[ArcCsc[c*x], -1])/(21*c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2
*Log[c*x]]])
```

Rubi [A] time = 0.0679058, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 335, 277, 325, 221}

$$-\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (-2*x^2)/(21*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^6/(7*Sqrt[Csch[2*Log[c*x]]]) +
 (2*EllipticF[ArcCsc[c*x], -1])/(21*c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2
*Log[c*x]]])
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^6}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^6\,dx, x, cx\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [C] time = 0.169403, size = 80, normalized size = 0.99

$$\frac{x^2\left({}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)-\left(1-c^4x^4\right)^{3/2}\right)}{7c^4\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*(-(1 - c^4*x^4)^(3/2) + Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4]))/(
7*c^4*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.058, size = 125, normalized size = 1.5

$$\frac{x^2\left(3c^4x^4-2\right)\sqrt{2}}{42c^4}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{\sqrt{2}x}{21c^4\left(c^4x^4-1\right)}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/42*x^2*(3*c^4*x^4-2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/21/c^4/(-c
^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^
2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sqrt(csch(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(csch(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

3.133 $\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=30

$$\frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] ((c^4 - x^(-4))*x^5)/(6*c^4*Sqrt[Csch[2*Log[c*x]]])
```

Rubi [A] time = 0.0452772, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5552, 5550, 264}

$$\frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] ((c^4 - x^(-4))*x^5)/(6*c^4*Sqrt[Csch[2*Log[c*x]]])
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^5}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^5\,dx, x, cx\right)}{c^6\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.0475923, size = 44, normalized size = 1.47

$$\frac{\left(c^4x^4-1\right)^2\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] ((-1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A] time = 0.033, size = 39, normalized size = 1.3

$$\frac{\sqrt{2}x\left(c^4x^4-1\right)}{12\,c^4}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^4*x^4-1)/c^4
```

Maxima [A] time = 1.77885, size = 62, normalized size = 2.07

$$\frac{\left(\sqrt{2}c^4x^4-\sqrt{2}\right)\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{cx-1}}{12\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*c^4*x^4 - sqrt(2))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x -
 1)/c^5
```

Fricas [A] time = 1.61067, size = 103, normalized size = 3.43

$$\frac{\sqrt{2}\left(c^8x^8-2\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*(c^8*x^8 - 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(csch(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(csch(2*log(c*x))), x)
```

3.134 $\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=119

$$\frac{2\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -2/(5*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^4/(5*Sqrt[Csch[2*Log[c*x]]]) - (2*Ell
ipticE[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x
]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[C
sch[2*Log[c*x]]])
```

Rubi [A] time = 0.0863124, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5552, 5550, 335, 277, 325, 307, 221, 1181, 424}

$$\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] -2/(5*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^4/(5*Sqrt[Csch[2*Log[c*x]]]) - (2*Ell
ipticE[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x
]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[C
sch[2*Log[c*x]]])
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^4\,dx, x, cx\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\,\text{Subst}\Big(}{5c^5\sqrt{1-\frac{1}{c^4x}}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\,\text{Subst}\Big(}{5c^5\sqrt{1-\frac{1}{c^4x}}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\big(c}{5c^5\sqrt{1-\frac{1}{c^4x}}}$$

Mathematica [C] time = 0.110618, size = 60, normalized size = 0.5

$$\frac{x^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{3\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^4*Hypergeometric2F1[-1/2, 3/4, 7/4, c^4*x^4])/(3*Sqrt[2 - 2*c^4*x^4]*Sqr
t[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.039, size = 127, normalized size = 1.1

$$\frac{x^4\sqrt{2}}{10}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\sqrt{2}x}{\left(5\,c^4x^4-5\right)c^2}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/5/(-c^2)^(1/2)*(c^2*x^2+1)^(
1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^2*(EllipticF(x*(-c^2)^(1/2),I)-Ellipt
```

```
icE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(csch(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(csch(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

3.135 $\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=69

$$\frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] x^3/(4*Sqrt[Csch[2*Log[c*x]]]) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rubi [A] time = 0.0580091, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 266, 47, 63, 206}

$$\frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Csch[2*Log[c*x]]]) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^3}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^3\,dx, x, cx\right)}{c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}
$$

Mathematica [A] time = 0.136255, size = 74, normalized size = 1.07

$$
\frac{x\left(c^2x^2\sqrt{1-c^4x^4}+\sin^{-1}\left(c^2x^2\right)\right)}{4c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 - c^4*x^4] + ArcSin[c^2*x^2]))/(4*c^2*Sqrt[2 - 2*c^4*x^4
]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.05, size = 97, normalized size = 1.4

$$
\frac{x^3\sqrt{2}}{8}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{\sqrt{2}x}{8}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}\frac{1}{\sqrt{c^4x^4-1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)/(c^4*x^4-1
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Fricas [A] time = 1.59258, size = 193, normalized size = 2.8

$$\frac{2\sqrt{2}\left(c^5x^5 - cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + \sqrt{2}\log\left(2\,c^4x^4 - 2\left(c^5x^5 - cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + sqrt(2)*log(2
*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) - 1))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(csch(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

3.136 $\int \frac{x}{\sqrt{\mathbf{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{2\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}+\frac{x^2}{3\sqrt{\text{csch}(2\log(cx))}}$$

```
[Out] x^2/(3*Sqrt[Csch[2*Log[c*x]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(3*c^3*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rubi [A] time = 0.0405378, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5552, 5550, 335, 277, 221}

$$\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}+\frac{x^2}{3\sqrt{\text{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Csch[2*Log[c*x]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(3*c^3*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^2}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^2\,dx, x, cx\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [C] time = 0.103116, size = 57, normalized size = 0.95

$$\frac{x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4])/(Sqrt[2 - 2*c^4*x^4]*Sqrt[
(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.033, size = 109, normalized size = 1.8

$$\frac{x^2\sqrt{2}}{6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{\sqrt{2}x}{3\,c^4x^4-3}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/3/(-c^2)^(1/2)*(c^2*x^2+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c
^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(csch(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(csch(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

3.137 $\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{\csc^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] x/(2*Sqrt[Csch[2*Log[c*x]]]) + ArcCsc[c^2*x^2]/(2*c^2*Sqrt[1 - 1/(c^4*x^4)]
*x*Sqrt[Csch[2*Log[c*x]]])
```

Rubi [A] time = 0.0353633, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5546, 5544, 335, 275, 277, 216}

$$\frac{\csc^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Csch[2*Log[c*x]]]) + ArcCsc[c^2*x^2]/(2*c^2*Sqrt[1 - 1/(c^4*x^4)]
*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5544

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Csch[d*(a
 + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c} \\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x\,dx, x, cx\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}
$$

Mathematica [A] time = 0.0876883, size = 77, normalized size = 1.28

$$
\frac{x\left(2\sqrt{c^4x^4-1}-2\tan^{-1}\left(\sqrt{c^4x^4-1}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\sqrt{c^4x^4-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(2*Sqrt[-1 + c^4*x^4] - 2*ArcTan[Sqrt[-1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(
c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{\operatorname{csch}\left(2\ \ln\left(cx\right)\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] int(1/csch(2*ln(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csch(2*log(c*x))), x)
```

Fricas [A] time = 1.53477, size = 184, normalized size = 3.07

$$-\frac{\sqrt{2}cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)-\sqrt{2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) -
 sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.138 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=46

$$i\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}\operatorname{EllipticF}\left(\frac{\pi}{4}-i\log(cx),2\right)$$

```
[Out] I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log[
c*x]]]
```

Rubi [A] time = 0.0321486, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3771, 2641}

$$i\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log[
c*x]]]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx &= \operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(2x)}\,dx,x,\log(cx)\right)\\
&= \left(\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(2x)}}\,dx,x,\log(cx)\right)\\
&= i\sqrt{\operatorname{csch}(2\log(cx))}F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.0680655, size = 43, normalized size = 0.93

$$(i\sinh(2\log(cx)))^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\operatorname{EllipticF}\left(\frac{\pi}{4}-i\log(cx),2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] Csch[2*Log[c*x]]^(3/2)*EllipticF[Pi/4 - I*Log[c*x], 2]*(I*Sinh[2*Log[c*x]])
^(3/2)
```

Maple [A] time = 0.168, size = 90, normalized size = 2.

$$\frac{\frac{i}{2}\sqrt{2}}{\cosh\left(2\ln\left(cx\right)\right)}\sqrt{-i\left(i+\sinh\left(2\ln\left(cx\right)\right)\right)}\sqrt{-i\left(-\sinh\left(2\ln\left(cx\right)\right)+i\right)}\sqrt{i\sinh\left(2\ln\left(cx\right)\right)}\mathrm{EllipticF}\left(\sqrt{-i\left(i+\sinh\left(2\ln\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x,x)
```

```
[Out] 1/2*I*(-I*(I+sinh(2*ln(c*x))))^(1/2)*2^(1/2)*(-I*(-sinh(2*ln(c*x))+I))^(1/2
)*(I*sinh(2*ln(c*x)))^(1/2)*EllipticF((-I*(I+sinh(2*ln(c*x))))^(1/2),1/2*2^
(1/2))/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(2*log(c*x)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x, x)
```

3.139 $\int \frac{\sqrt{\mathbf{csch}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] -(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcCsc[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])/2
```

Rubi [A] time = 0.0460243, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 275, 216}

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] -(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcCsc[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = c\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^2}\,dx, x, cx\right)$$
$$= \left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^3}\,dx, x, cx\right)$$
$$= -\left(\left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\left(\frac{1}{2}\left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)$$
$$= -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Mathematica [A] time = 0.110806, size = 54, normalized size = 1.32

$$\frac{\sqrt{c^4x^4-1}\sqrt{\frac{c^2x^2}{2c^4x^4-2}}\tan^{-1}\left(\sqrt{c^4x^4-1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] (Sqrt[-1 + c^4*x^4]*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)]*ArcTan[Sqrt[-1 + c^4*x
^4]])/x
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{\operatorname{csch}(2\ln(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^2, x)
```

Fricas [A] time = 1.57141, size = 96, normalized size = 2.34

$$\frac{1}{2}\sqrt{2}c\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*c*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.140 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=74

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\text{EllipticF}\left(\csc^{-1}(cx),-1\right)-c^3x\sqrt{1-\frac{1}{c^4x^4}}E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] -(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticE[ArcCsc[c*x],
 -1]) + c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc
[c*x], -1]
```

Rubi [A] time = 0.0656654, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5552, 5550, 335, 307, 221, 1181, 424}

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}-c^3x\sqrt{1-\frac{1}{c^4x^4}}E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticE[ArcCsc[c*x],
 -1]) + c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc
[c*x], -1]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx &= c^2\,\mathrm{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^3}\,dx,x,cx\right)\\ &= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^4}\,dx,x,cx\right)\\ &= -\left(\left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx,x,\frac{1}{cx}\right)\right)\\ &= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx,x,\frac{1}{cx}\right)-\left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\right.\\ &= c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}F\left(\csc^{-1}(cx)\middle|-1\right)-\left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Sub}\\ &= -c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}E\left(\csc^{-1}(cx)\middle|-1\right)+c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}F\left(\mathrm{cs}\right.\end{aligned}$$

Mathematica [C] time = 0.0936235, size = 58, normalized size = 0.78

$$-\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};c^4x^4\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/
4, 1/2, 3/4, c^4*x^4])/x^2)
```

Maple [A] time = 0.036, size = 126, normalized size = 1.7

$$\frac{\left(c^4x^4-1\right)\sqrt{2}}{x^2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}-\frac{c^2\sqrt{2}}{x}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{-c^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}\frac{1}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^3,x)
```

```
[Out] (c^4*x^4-1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)-c^2/(-c^2)^(1/2)*(c^2*x
^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(EllipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c
^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(2*log(c*x)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

3.141 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] ((c^4 - x^(-4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Rubi [A] time = 0.0409001, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5552, 5550, 261}

$$\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] ((c^4 - x^(-4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx &= c^3\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^4}\,dx,x,cx\right)\\ &= \left(c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^5}\,dx,x,cx\right)\\ &= \frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\sqrt{\operatorname{csch}(2\log(cx))}\end{aligned}$$

Mathematica [A] time = 0.0372616, size = 33, normalized size = 1.32

$$\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] c^2/(2*x*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])
```

Maple [A] time = 0.033, size = 38, normalized size = 1.5

$$\frac{\sqrt{2}\left(c^4x^4-1\right)}{2\,x^3}\sqrt{\frac{c^2x^2}{c^4x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^4,x)
```

```
[Out] 1/2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x^3*(c^4*x^4-1)
```

Maxima [B] time = 1.62694, size = 120, normalized size = 4.8

$$\frac{1}{2}\,c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(sqrt(2)/(sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2) + 1
)) - sqrt(2)/(c^4*x^4*sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2)
 + 1)))
```

Fricas [A] time = 1.56276, size = 80, normalized size = 3.2

$$\frac{\sqrt{2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}(2\ \log(cx))}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^4, x)
```

3.142 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

```
[Out] ((c^4 - x^(-4))*Sqrt[Csch[2*Log[c*x]]])/3 - (c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sq
rt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc[c*x], -1])/3
```

Rubi [A] time = 0.0546849, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 321, 221}

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] ((c^4 - x^(-4))*Sqrt[Csch[2*Log[c*x]]])/3 - (c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sq
rt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc[c*x], -1])/3
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\text{csch}(2\log(cx))}}{x^5}\,dx &= c^4\,\text{Subst}\left(\int \frac{\sqrt{\text{csch}(2\log(x))}}{x^5}\,dx, x, cx\right)\\
&= \left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)\\
&= -\left(\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\text{csch}(2\log(cx))}-\frac{1}{3}\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx\right.\\
&= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\text{csch}(2\log(cx))}-\frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}F\left(\csc^{-1}(cx)\middle|-1\right)
\end{aligned}
$$

Mathematica [C] time = 0.0937916, size = 60, normalized size = 0.94

$$
-\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};c^4x^4\right)}{3x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] -(Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/4
, 1/2, 1/4, c^4*x^4])/(3*x^4)
```

Maple [A] time = 0.036, size = 112, normalized size = 1.8

$$
\frac{\left(c^4x^4-1\right)\sqrt{2}}{3\,x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}+\frac{c^4\sqrt{2}}{3\,x}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\,\text{EllipticF}\left(x\sqrt{-c^2},i\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}\frac{1}{\sqrt{-c^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^5,x)
```

```
[Out] 1/3*(c^4*x^4-1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)+1/3*c^4/(-c^2)^(1/2
)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*
(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{\text{csch}\left(2\,\log\left(cx\right)\right)}}{x^5}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(2*log(c*x)))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

3.143 $\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=128

$$-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\cdots}{12\operatorname{csch}\cdots}$$

```
[Out] x/(32*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - x^5/(16*(c^4 - x^(-4))*C
sch[2*Log[c*x]]^(3/2)) + x^9/(12*Csch[2*Log[c*x]]^(3/2)) + ArcTanh[Sqrt[1 -
 1/(c^4*x^4)]]/(32*c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0799591, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5552, 5550, 266, 47, 51, 63, 206}

$$-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\cdots}{12\operatorname{csch}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - x^5/(16*(c^4 - x^(-4))*C
sch[2*Log[c*x]]^(3/2)) + x^9/(12*Csch[2*Log[c*x]]^(3/2)) + ArcTanh[Sqrt[1 -
 1/(c^4*x^4)]]/(32*c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(c\ldots}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\ldots}{64\ldots}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\ldots}{32\ldots}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\ldots}{32\ldots}$$

Mathematica [A] time = 0.199421, size = 95, normalized size = 0.74

$$\frac{c^3x^3\sqrt{1-c^4x^4}\left(8c^8x^8-14c^4x^4+3\right)-3cx\sin^{-1}\left(c^2x^2\right)}{192c^9\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(3 - 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSin[c^2*
x^2])/(192*c^9*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.038, size = 121, normalized size = 1.

$$\frac{x^3\left(8c^8x^8-14c^4x^4+3\right)\sqrt{2}}{384c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}} + \frac{\sqrt{2}x}{128c^6}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{c^4x^4-1}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/384*x^3*(8*c^8*x^8-14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 1.75048, size = 240, normalized size = 1.88

$$\frac{2\sqrt{2}\left(8c^{13}x^{13}-22c^9x^9+17c^5x^5-3cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+3\sqrt{2}\log\left(2c^4x^4+2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{768c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 - 22*c^9*x^9 + 17*c^5*x^5 - 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 1))/c^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**8/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

3.144 $\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=118

$$-\frac{4\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 4/(77*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - (6*x^4)/(77*(c^4 - x^(-4
))*Csch[2*Log[c*x]]^(3/2)) + x^8/(11*Csch[2*Log[c*x]]^(3/2)) - (4*EllipticF
[ArcCsc[c*x], -1])/(77*c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3
/2))
```

Rubi [A] time = 0.077669, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 335, 277, 325, 221}

$$-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - (6*x^4)/(77*(c^4 - x^(-4
))*Csch[2*Log[c*x]]^(3/2)) + x^8/(11*Csch[2*Log[c*x]]^(3/2)) - (4*EllipticF
[ArcCsc[c*x], -1])/(77*c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3
/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\,\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(c$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{}{77}$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{}{77}$$

Mathematica [C] time = 0.165896, size = 80, normalized size = 0.68

$$\frac{x^2\left(\left(1-c^4x^4\right)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)\right)}{22c^6\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x^2*((1 - c^4*x^4)^(5/2) - Hypergeometric2F1[-3/2, 1/4, 5/4, c^4*x^4]))/(2
2*c^6*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.033, size = 133, normalized size = 1.1

$$\frac{x^2\left(7\,c^8x^8-13\,c^4x^4+4\right)\sqrt{2}}{308\,c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{2}x}{77\,c^6\left(c^4x^4-1\right)}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\mathrm{EllipticF}\left(x\sqrt{-c^2},i\right)\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/308*x^2*(7*c^8*x^8-13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/77/c^6/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*Elli
pticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/csch(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**7/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

3.145 $\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=30

$$\frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] ((c^4 - x^(-4))*x^7)/(10*c^4*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0408179, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5552, 5550, 264}

$$\frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 - x^(-4))*x^7)/(10*c^4*Csch[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] time = 0.0481718, size = 44, normalized size = 1.47

$$\frac{\left(c^4x^4-1\right)^3\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((-1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A] time = 0.03, size = 47, normalized size = 1.6

$$\frac{\sqrt{2}x\left(c^8x^8-2\,c^4x^4+1\right)}{40\,c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^8*x^8-2*c^4*x^4+1)
```

Maxima [A] time = 1.74628, size = 62, normalized size = 2.07

$$\frac{\left(\sqrt{2}c^4x^4-\sqrt{2}\right)\left(c^2x^2+1\right)^{\frac{3}{2}}(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 - sqrt(2))*(c^2*x^2 + 1)^(3/2)*(c*x + 1)^(3/2)*(c*x -
 1)^(3/2)/c^7
```

Fricas [B] time = 1.5711, size = 122, normalized size = 4.07

$$\frac{\sqrt{2}\left(c^{12}x^{12}-3\,c^8x^8+3\,c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(2)*(c^12*x^12 - 3*c^8*x^8 + 3*c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4
- 1))/(c^8*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**6/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/csch(2*log(c*x))^(3/2), x)
```

3.146 $\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=162

$$-\frac{4\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{15c^9x^3}$$

```
[Out] 4/(15*c^4*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) - (2*x^2)/(15*(c^4 - x
^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^6/(9*Csch[2*Log[c*x]]^(3/2)) + (4*Ellipt
icE[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2)) - (4*EllipticF[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0999622, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5552, 5550, 335, 277, 325, 307, 221, 1181, 424}

$$-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{15c^9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(15*c^4*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) - (2*x^2)/(15*(c^4 - x
^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^6/(9*Csch[2*Log[c*x]]^(3/2)) + (4*Ellipt
icE[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2)) - (4*EllipticF[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \cdots$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \cdots$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \cdots$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \cdots$$

Mathematica [C] time = 0.120538, size = 63, normalized size = 0.39

$$-\frac{x^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{6c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -(x^4*Hypergeometric2F1[-3/2, 3/4, 7/4, c^4*x^4])/(6*c^2*Sqrt[2 - 2*c^4*x^4
]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.035, size = 140, normalized size = 0.9

$$\frac{x^4\left(5c^4x^4-11\right)\sqrt{2}}{180\,c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}} + \frac{\sqrt{2}x}{\left(15\,c^4x^4-15\right)c^4}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right) \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/180*x^4*(5*c^4*x^4-11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/15/(-c^2
)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^4*(EllipticF(x*(
-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/csch(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**5/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="giac")

[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

3.147 $\int \frac{x^4}{\mathbf{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] (-3*x)/(16*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^5/(8*Csch[2*Log[c*x]]
^(3/2)) + (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(16*c^8*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0673089, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 266, 47, 63, 206}

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (-3*x)/(16*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^5/(8*Csch[2*Log[c*x]]
^(3/2)) + (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(16*c^8*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx,x,cx\right)}{c^5}\\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^7\,dx,x,cx\right)}{c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^{3/2}}{x^3}\,dx,x,\frac{1}{c^4x^4}\right)}{4c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,\frac{1}{c^4x^4}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,\frac{1}{c^4x^4}\right)}{32c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx)}\\
&= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx)}\\
&= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx)}
\end{aligned}
$$

Mathematica [A] time = 0.175055, size = 87, normalized size = 0.91

$$
\frac{c^3x^3\sqrt{1-c^4x^4}\left(2c^4x^4-5\right)-3cx\sin^{-1}\left(c^2x^2\right)}{32c^5\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(-5 + 2*c^4*x^4) - 3*c*x*ArcSin[c^2*x^2])/(32*c^
5*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.035, size = 113, normalized size = 1.2

$$\frac{x^{3}\left(2c^{4}x^{4}-5\right)\sqrt{2}}{64\,c^{2}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}+\frac{3\sqrt{2}x}{64\,c^{2}}\ln\left(c^{4}x^{2}\frac{1}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}-1}\right)\frac{1}{\sqrt{c^{4}}}\frac{1}{\sqrt{c^{4}x^{4}-1}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/64*x^3*(2*c^4*x^4-5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4-1)^(1
/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 1.57994, size = 219, normalized size = 2.28

$$\frac{2\sqrt{2}\left(2c^{9}x^{9}-7c^{5}x^{5}+5cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}+3\sqrt{2}\log\left(2c^{4}x^{4}+2\left(c^{5}x^{5}-cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}-1\right)}{128\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 - 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)
) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1))
 - 1))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

3.148 $\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=86

$$-\frac{4\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -2/(7*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^4/(7*Csch[2*Log[c*x]]^(3/2
)) - (4*EllipticF[ArcCsc[c*x], -1])/(7*c^7*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0633788, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 277, 221}

$$-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)|-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -2/(7*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^4/(7*Csch[2*Log[c*x]]^(3/2
)) - (4*EllipticF[ArcCsc[c*x], -1])/(7*c^7*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4}\\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

Mathematica [C] time = 0.12084, size = 65, normalized size = 0.76

$$\frac{\sqrt{1-c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{2\sqrt{2}c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 - c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/2, 1
/4, 5/4, c^4*x^4])/(2*Sqrt[2]*c^4)
```

Maple [A] time = 0.032, size = 124, normalized size = 1.4

$$\frac{x^2\left(c^4x^4-3\right)\sqrt{2}}{28\,c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{2}x}{\left(7\,c^4x^4-7\right)c^2}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/28*x^2*(c^4*x^4-3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/7/(-c^2)^(1/
2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2
),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^3}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/csch(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="giac")

[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

3.149 $\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \, dx$

Optimal. Leaf size=91

$$-\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -1/(2*(c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + x^3/(6*Csch[2*Log[c*x]]^(3
/2)) - ArcCsc[c^2*x^2]/(2*c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2))
```

Rubi [A] time = 0.072699, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 335, 275, 277, 216}

$$-\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/(2*(c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + x^3/(6*Csch[2*Log[c*x]]^(3
/2)) - ArcCsc[c^2*x^2]/(2*c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3} \\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right) x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right) x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.156129, size = 88, normalized size = 0.97

$$\frac{x\left(\sqrt{c^4x^4-1}\left(c^4x^4-4\right)+3\tan^{-1}\left(\sqrt{c^4x^4-1}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\sqrt{c^4x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*((-4 + c^4*x^4)*Sqrt[-1 + c^4*x^4] + 3*ArcTan[Sqrt[-1 + c^4*x^4]]))/(12*
Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{csch}(2\ln(cx)))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 1.54962, size = 203, normalized size = 2.23

$$\frac{3\sqrt{2}cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\left(c^8x^8-5c^4x^4+4\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{24c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
 + sqrt(2)*(c^8*x^8 - 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

3.150 $\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=130

$$\frac{12\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -6/(5*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + x^2/(5*Csch[2*Log[c*x]]^
(3/2)) - (12*EllipticE[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2)) + (12*EllipticF[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c
^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0732644, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5552, 5550, 335, 277, 307, 221, 1181, 424}

$$-\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{12F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -6/(5*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + x^2/(5*Csch[2*Log[c*x]]^
(3/2)) - (12*EllipticE[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2)) + (12*EllipticF[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c
^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} -$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} -$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} +$$

Mathematica [C] time = 0.107892, size = 60, normalized size = 0.46

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};c^4x^4\right)}{2c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/4, 3/4, c^4*x^4]/(2*c^2*Sqrt[2 - 2*c^4*x^4]*Sqrt
[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.037, size = 152, normalized size = 1.2

$$\frac{\left(c^8x^8+4\,c^4x^4-5\right)\sqrt{2}}{\left(20\,c^4x^4-20\right)c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{3\sqrt{2}x}{5\,c^4x^4-5}\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\frac{1}{\sqrt{-c^2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/20*(c^8*x^8+4*c^4*x^4-5)/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/5/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*(Elli
pticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^
4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/csch(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x/csch(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

3.151 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$\frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/(4*(c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2)) + x/(4*Csch[2*Log[c*x]]^(3/
2)) - (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(4*c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0464677, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5546, 5544, 266, 47, 50, 63, 206}

$$\frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] 3/(4*(c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2)) + x/(4*Csch[2*Log[c*x]]^(3/
2)) - (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(4*c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5544

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Csch[d*(a
 + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}\\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [C] time = 0.0889616, size = 63, normalized size = 0.66

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};c^4x^4\right)}{4c^2x\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/2, 1/2, c^4*x^4]/(4*c^2*x*Sqrt[2 - 2*c^4*x^4]*Sq
rt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] time = 0.036, size = 130, normalized size = 1.4

$$\frac{\left(c^8x^8+c^4x^4-2\right)\sqrt{2}}{16\,x\left(c^4x^4-1\right)c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{3\,c^2\sqrt{2}x}{16}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{c^4x^4-1}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/16*(c^8*x^8+c^4*x^4-2)/x/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(-3/2), x)
```

Fricas [A] time = 1.70339, size = 225, normalized size = 2.34

$$\frac{3\,\sqrt{2}c^3x^3\log\left(2\,c^4x^4-2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)+2\,\sqrt{2}\left(c^8x^8+c^4x^4-2\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{32\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4
*x^4 - 1)) - 1) + 2*sqrt(2)*(c^8*x^8 + c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4 -
```

```
1)))/(c^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(csch(2*log(c*x))**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.152 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=67

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x]
, 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rubi [A] time = 0.0372023, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3768, 3771, 2639}

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] -(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x]
, 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx &= \operatorname{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(2x)\,dx,x,\log(cx)\right)\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2x)}}\,dx,x,\log(cx)\right)\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(2x)}\,dx,x,\log(cx)\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.0935348, size = 54, normalized size = 0.81

$$\sqrt{\text{csch}(2\log(cx))}\left(-\cosh(2\log(cx))+\sqrt{i\sinh(2\log(cx))}E\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] Sqrt[Csch[2*Log[c*x]]]*(-Cosh[2*Log[c*x]] + EllipticE[Pi/4 - I*Log[c*x], 2]
*Sqrt[I*Sinh[2*Log[c*x]]])
```

Maple [A] time = 0.168, size = 163, normalized size = 2.4

$$\frac{1}{2\cosh(2\ln(cx))}\left(2\sqrt{1-i\sinh(2\ln(cx))}\sqrt{2}\sqrt{1+i\sinh(2\ln(cx))}\sqrt{i\sinh(2\ln(cx))}\text{EllipticE}\left(\sqrt{1-i\sinh(2\ln(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x,x)
```

```
[Out] 1/2*(2*(1-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(1+I*sinh(2*ln(c*x)))^(1/2)*(I*s
inh(2*ln(c*x)))^(1/2)*EllipticE((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-(1
-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(1+I*sinh(2*ln(c*x)))^(1/2)*(I*sinh(2*ln(
c*x)))^(1/2)*EllipticF((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-2*cosh(2*ln
(c*x))^2)/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{csch}(2\log(cx))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(2\log(cx))^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(2*log(c*x))^(3/2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x, x)
```

3.153 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -((c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2))/2
```

Rubi [A] time = 0.039279, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5552, 5550, 261}

$$-\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -((c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx &= c\,\mathrm{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)\\
&= \left(c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [A] time = 0.033557, size = 33, normalized size = 1.22

$$-\sqrt{2}c^2x\sqrt{\frac{c^2x^2}{c^4x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -(Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\operatorname{csch}\left(2\ln\left(cx\right)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

Maxima [B] time = 1.58188, size = 117, normalized size = 4.33

$$-c\left(\frac{\sqrt{2}}{\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}}{c^4x^4\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -c*(sqrt(2)/((1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2) + 1)^(3/
2)) - sqrt(2)/(c^4*x^4*(1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2
) + 1)^(3/2)))
```

Fricas [A] time = 1.55671, size = 59, normalized size = 2.19

$$-\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.154 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\text{EllipticF}\left(\csc^{-1}(cx),-1\right)-\frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -((c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2))/2 + (c^5*(1 - 1/(c^4*x^4))^(3/
2)*x^3*Csch[2*Log[c*x]]^(3/2)*EllipticF[ArcCsc[c*x], -1])/2
```

Rubi [A] time = 0.0539875, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 288, 221}

$$\frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}F\left(\csc^{-1}(cx)\middle|-1\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```


```
[Out] -((c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2))/2 + (c^5*(1 - 1/(c^4*x^4))^(3/
2)*x^3*Csch[2*Log[c*x]]^(3/2)*EllipticF[ArcCsc[c*x], -1])/2
```


Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = c^2\,\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)$$
$$= \left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right)$$
$$= -\left(\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.$$
$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\csc^{-1}(cx)\middle|-1\right)$$

Mathematica [C] time = 0.107646, size = 66, normalized size = 0.96

$$-\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\left(\sqrt{1-c^4x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] -(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*(1 + Sqrt[1 - c^4*x^4]*Hyperge
ometric2F1[1/4, 1/2, 5/4, c^4*x^4]))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\operatorname{csch}(2\ln(cx))\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(csch(2*log(c*x))^(3/2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

3.155 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2))/2 + (c^6*(1 - 1/(c^4*x^4))^(3/2)
*x^3*ArcCsc[c^2*x^2]*Csch[2*Log[c*x]]^(3/2))/2
```

Rubi [A] time = 0.0606238, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5552, 5550, 335, 275, 288, 216}

$$\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2))/2 + (c^6*(1 - 1/(c^4*x^4))^(3/2)
*x^3*ArcCsc[c^2*x^2]*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx &= c^3\,\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)\\
&= \left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^7}\,dx, x, cx\right)\\
&= -\left(\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\right.\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))
\end{aligned}
$$

Mathematica [C] time = 0.104476, size = 53, normalized size = 0.77

$$
-\frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-c^4x^4\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/
2, 1 - c^4*x^4])/x)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$
\int \frac{1}{x^4}\left(\operatorname{csch}(2\ln(cx))\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^4, x)
```

Fricas [A] time = 1.63338, size = 159, normalized size = 2.3

$$-\frac{\sqrt{2}c^3x\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*c^3*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) + s
qrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int \operatorname{csch}(a + b\log(cx^n))\,dx$

Optimal. Leaf size=62

$$-\frac{2e^a x (cx^n)^b\, {}_2F_1\left(1, \frac{b+\frac{1}{n}}{2b}; \frac{1}{2}\left(3+\frac{1}{bn}\right); e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Rubi [A] time = 0.0547365, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5546, 5548, 263, 364}

$$-\frac{2e^a x (cx^n)^b\, {}_2F_1\left(1, \frac{b+\frac{1}{n}}{2b}; \frac{1}{2}\left(3+\frac{1}{bn}\right); e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{csch}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1-e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{-e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}$$
$$= -\frac{2e^a x(cx^n)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

Mathematica [A] time = 1.15678, size = 62, normalized size = 1.

$$-\frac{2e^a x(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2(a+b\log(cx^n))}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
E^(2*(a + b*Log[c*x^n]))])/(1 + b*n)
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \operatorname{csch}(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n)),x)
```

```
[Out] int(csch(a+b*ln(c*x^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(b\log(cx^n)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{csch}(b\log(cx^n)+a),x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(a + b\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(csch(a + b*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}(b\log(cx^n) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

3.157 $\int \operatorname{csch}^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Rubi [A] time = 0.0653747, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^2(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^2}dx, x, cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^2}dx, x, cx^n\right)}{n}$$
$$= \frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

Mathematica [A] time = 4.24291, size = 126, normalized size = 1.85

$$\frac{x\left(-\frac{e^{2a}(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)}{2bn+1} - {}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right) - \coth(a+b\log(cx^n))\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-Coth[a + b*Log[c*x^n]] - (E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1
 + 1/(2*b*n), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))])/(1 + 2*b*n) - Hyper
geometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))]))/(b*n)
```

Maple [F] time = 1.119, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(a+b\ln(cx^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2x}{bc^{2b}ne^{(2b\log(x^n)+2a)} - bn} - 4\int \frac{1}{4\left(bc^bne^{(b\log(x^n)+a)} + bn\right)}dx + 4\int \frac{1}{4\left(bc^bne^{(b\log(x^n)+a)} - bn\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 4*integrate(1/4/(b*c^b*n*
e^(b*log(x^n) + a) + b*n), x) + 4*integrate(1/4/(b*c^b*n*e^(b*log(x^n) + a)
 - b*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^2, x)
```

3.158 $\int \operatorname{csch}^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$-\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

```
[Out] (-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Rubi [A] time = 0.0694034, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$-\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^3}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^3}\,dx,x,cx^n\right)}{n}$$

$$= -\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A] time = 5.29714, size = 101, normalized size = 1.46

$$\frac{8e^a x(bn-1)(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2(a+b\log(cx^n))}\right)-4x(bn\coth(a+b\log(cx^n))+1)\operatorname{csch}(a+b\log(c\ldots}{8b^2n^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-4*x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]] + 8*E^a*(-1 +
 b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, E^
(2*(a + b*Log[c*x^n]))])/(8*b^2*n^2)
```

Maple [F] time = 1.283, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(a+b\ln(cx^n)))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8\left(b^2n^2-1\right)\int \frac{1}{16\left(b^2c^bn^2e^{(b\log(x^n)+a)}+b^2n^2\right)}\,dx-8\left(b^2n^2-1\right)\int \frac{1}{16\left(b^2c^bn^2e^{(b\log(x^n)+a)}-b^2n^2\right)}\,dx-\frac{\left(bc^{3b}n+c^{3b}\right)\ldots}{b^2c^{4b}n^2e^{(4b\log\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) + b^2*n^2),
 x) - 8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) - b^2*
n^2), x) - ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c
^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b*log(x^n) + 4*a) - 2*b^2*c
```

```
^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{csch}\left(b\log\left(cx^n\right)+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{csch}^3\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{csch}\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^3, x)
```

3.159 $\int \mathrm{csch}^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=68

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Rubi [A] time = 0.0688398, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^4(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$= \frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

Mathematica [B] time = 8.4795, size = 200, normalized size = 2.94

$$\frac{x\left(4\left(4b^2n^2-1\right){}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)+\operatorname{csch}^3(a+b\log(cx^n))\left(\left(1-12b^2n^2\right)\cosh(a+b\log(cx^n))+\left(4b^2n^2\right.\right.\right.}{24l}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(4*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n)
, 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + 4*(-1 + 4*b^2*n^2)*Hypergeomet
ric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + Csch[a + b*
Log[c*x^n]]^3*((1 - 12*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (-1 + 4*b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 4*b*n*Sinh[a + b*Log[c*x^n]])))/(24*b^3*n^3)
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(a+b\ln(cx^n)))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\left(4b^2n^2-1\right)\int \frac{1}{96\left(b^3c^bn^3e^{(b\log(x^n)+a)}+b^3n^3\right)}\,dx-16\left(4b^2n^2-1\right)\int \frac{1}{96\left(b^3c^bn^3e^{(b\log(x^n)+a)}-b^3n^3\right)}\,dx-\frac{\left(2bc^{4b}n+c^4\right.}{3\left(b^3c^{6b}n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) + b^3*n^3
), x) - 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) -
```

```
 b^3*n^3), x) - 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) + 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) - 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) - b^3*n^
3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{4}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^4, x)
```

3.160 $\int \left(-\left(1-b^2n^2\right) \mathbf{csch}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\mathbf{csch}^3\left(a+b\right.\right.$

Optimal. Leaf size=42

$$-x\text{csch}\left(a+b\log\left(cx^n\right)\right)-bnx\coth\left(a+b\log\left(cx^n\right)\right)\text{csch}\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -(x*Csch[a + b*Log[c*x^n]]) - b*n*x*Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c
*x^n]]
```

Rubi [C] time = 0.135159, antiderivative size = 137, normalized size of antiderivative = 3.26, number of steps used = 9, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {5546, 5548, 263, 364}

$$2e^a x(1-bn)\left(cx^n\right)^b {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)-\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b} {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

Warning: Unable to verify antiderivative.

```
[In] Int[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x^
n]]^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)] - (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*Hyp
ergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, E^(2*a)*(c*x^n)^(2
*b)])/(1 + 3*b*n)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(-\left(1-b^2n^2\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)\right)dx = \left(2b^2n^2\right)\int \operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)dx+\left(-\right.$$

$$= \left(2b^2nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(\right.$$

$$= \left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-}}{\left(1-e^{-2}\right.}\right.$$

$$= \left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3}}{\left(-e^{-2a}+\right.}\right.$$

$$= 2e^a(1-bn)x\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\right.\right.$$

Mathematica [A] time = 0.387069, size = 30, normalized size = 0.71

$$-x\left(bn\coth\left(a+b\log\left(cx^n\right)\right)+1\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Lo
g[c*x^n]]^3,x]
```

```
[Out] -(x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]])
```

Maple [C] time = 0.237, size = 509, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(-b^2*n^2+1)*csch(a+b*ln(c*x^n))+2*b^2*n^2*csch(a+b*ln(c*x^n))^3,x)
```

```
[Out] -2*c^b*(x^n)^b*x/((c^b)^2*((x^n)^b)^2*exp(2*a)*exp(-I*b*Pi*csgn(I*c*x^n)^3)
*exp(I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^
n))*exp(-I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))-1)^2*((c^b)^2*((x^n)^b
)^2*b*n*exp(3*a)*exp(-3/2*I*b*Pi*csgn(I*c*x^n)^3)*exp(3/2*I*b*Pi*csgn(I*c*x
^n)^2*csgn(I*c))*exp(3/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(-3/2*I*b*P
i*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))+exp(a)*exp(-1/2*I*b*Pi*csgn(I*c*x^n)
^3)*exp(1/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(1/2*I*b*Pi*csgn(I*c*x^n)^
2*csgn(I*x^n))*exp(-1/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*b*n+(c^
b)^2*((x^n)^b)^2*exp(3*a)*exp(-3/2*I*b*Pi*csgn(I*c*x^n)^3)*exp(3/2*I*b*Pi*c
sgn(I*c*x^n)^2*csgn(I*c))*exp(3/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(-
3/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))-exp(a)*exp(-1/2*I*b*Pi*csgn
(I*c*x^n)^3)*exp(1/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(1/2*I*b*Pi*csgn(
I*c*x^n)^2*csgn(I*x^n))*exp(-1/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)
))
```

Maxima [B] time = 2.18901, size = 128, normalized size = 3.05

$$-\frac{2\left(\left(bc^{3b}n+c^{3b}\right)xe^{(3b\log(x^n)+3a)}+\left(bc^{b}n-c^{b}\right)xe^{(b\log(x^n)+a)}\right)}{c^{4b}e^{(4b\log(x^n)+4a)}-2c^{2b}e^{(2b\log(x^n)+2a)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="maxima")
```

```
[Out] -2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c^b)*x*e^
(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) - 2*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)
```

Fricas [B] time = 1.62483, size = 602, normalized size = 14.33

$$-\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\left(bn+1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)\right.\right.}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="fricas")
```

```
[Out] -2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b
*n*log(x) + b*log(c) + a)^2 + (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 +
sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 -
1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(2b^{2}n^{2}\operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)+b^{2}n^{2}-1\right)\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b**2*n**2+1)*csch(a+b*ln(c*x**n))+2*b**2*n**2*csch(a+b*ln(c*x*
*n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*csch(a + b*log(c*x**n))**2 + b**2*n**2 - 1)*csch(a +
b*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 2\,b^{2}n^{2}\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{3}+\left(b^{2}n^{2}-1\right)\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="giac")
```

```
[Out] integrate(2*b^2*n^2*csch(b*log(c*x^n) + a)^3 + (b^2*n^2 - 1)*csch(b*log(c*x
^n) + a), x)
```

3.161 $\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

```
[Out] (-2*c^6)/(E^a*(c^4 - 1/(E^(2*a)*x^2))^2)
```

Rubi [A] time = 0.0386493, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5546, 5548, 261}

$$-\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]

[Out] (-2*c^6)/(E^a*(c^4 - 1/(E^(2*a)*x^2))^2)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx &= \frac{2\,\text{Subst}\left(\int x\operatorname{csch}^3(a+2\log(x))\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= \frac{\left(16e^{-3a}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3x^5}\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= -\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}
\end{aligned}$$

Mathematica [B] time = 0.116666, size = 62, normalized size = 2.38

$$\frac{2(\cosh(a)-\sinh(a))\left(\sinh^2(a)+\cosh^2(a)-2\sinh(a)\cosh(a)-2c^4x^2\right)}{c^2\left(\sinh(a)\left(c^4x^2+1\right)+\cosh(a)\left(c^4x^2-1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*(Cosh[a] - Sinh[a])*(-2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((-1 + c^4*x^2)*Cosh[a] + (1 + c^4*x^2)*Sinh[a])^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int\left(\operatorname{csch}\left(a+2\ln\left(c\sqrt{x}\right)\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

Maxima [B] time = 1.02855, size = 103, normalized size = 3.96

$$-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}-\frac{1}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a) - 1/(c^8*
x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Fricas [B] time = 1.54766, size = 104, normalized size = 4.

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)}{c^{10}x^4e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)/(c^10*x^4*e^(5*a) - 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c*x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c*sqrt(x)))**3, x)
```

Giac [A] time = 1.18587, size = 51, normalized size = 1.96

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}-1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)*e^(-a)/((c^4*x^2*e^(2*a) - 1)^2*c^2)
```

3.162 $\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal. Leaf size=26

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)
```

Rubi [A] time = 0.0460046, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5546, 5548, 263, 261}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx = -\left((2c^2)\operatorname{Subst}\left(\int \frac{\operatorname{csch}^3(a+2\log(x))}{x^3}dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left((16c^2e^{-3a})\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3 x^9}dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left((16c^2e^{-3a})\operatorname{Subst}\left(\int \frac{x^3}{\left(-e^{-2a}+x^4\right)^3}dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= \frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

Mathematica [B] time = 0.0982161, size = 65, normalized size = 2.5

$$-\frac{2c^6(\sinh(2a)+\cosh(2a))\left(\sinh(a)\left(c^4+2x^2\right)+\cosh(a)\left(c^4-2x^2\right)\right)}{\left(\cosh(a)\left(x^2-c^4\right)-\sinh(a)\left(c^4+x^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 - 2*x^2)*Cosh[a] + (c^4 + 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((-c^4 + x^2)*Cosh[a] - (c^4 + x^2)*Sinh[a])^2
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

Maxima [A] time = 1.06159, size = 66, normalized size = 2.54

$$-\frac{2\left(c^{10}e^{(5a)}-2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}-2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Fricas [A] time = 1.45316, size = 107, normalized size = 4.12

$$-\frac{2\left(c^{10}e^{(5a)}-2c^{6}x^{2}e^{(3a)}\right)}{c^{8}e^{(4a)}-2c^{4}x^{2}e^{(2a)}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c/sqrt(x)))**3, x)
```

Giac [A] time = 1.17209, size = 53, normalized size = 2.04

$$-\frac{2\left(c^{10}e^{(5a)}-2c^{6}x^{2}e^{(3a)}\right)}{\left(c^{4}e^{(2a)}-x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) - x^2)^2
```

3.163 $\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=90

$$-\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] -(E^(2*a)*(2 - p)*x*(1 - (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Csch[a - Log[c*x^
n]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rubi [A] time = 0.0872534, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5546, 5550, 261}

$$-\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -(E^(2*a)*(2 - p)*x*(1 - (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Csch[a - Log[c*x^
n]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}}\right.}{n}$$

$$= -\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 5.95475, size = 115, normalized size = 1.28

$$\frac{2^{p-1}(p-2)x\left(\frac{e^a(cx^n)^{\frac{1}{n(p-2)}}}{e^{2a}(cx^n)^{\frac{2}{n(p-2)}}-1}\right)^p\left(e^{2a}(cx^n)^{\frac{2}{n(p-2)}}\left(\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(p-2)}}\right)^p-1\right)+1\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^(2*a)*(c*x^n
)^(2/(n*(-2 + p)))))^p*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 - 1/(
E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(a+\frac{\ln(cx^n)}{n(-2+p)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [B] time = 1.81932, size = 1323, normalized size = 14.7

$$-\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\right)}{\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2+2\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2-\cdots}\right.\right.}{(p-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p
- 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log
(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p -
2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh
((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(p*log(2*
(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*
n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(
c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a
*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n)))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n
)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{p}\left(a+\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csch(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(a+\frac{\log\left(cx^{n}\right)}{n(p-2)}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

3.164 $\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right) dx$

Optimal. Leaf size=66

$$\frac{(2-p)x\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] ((2 - p)*x*(1 - 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rubi [A] time = 0.0752071, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5546, 5550, 264}

$$\frac{(2-p)x\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 - 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-\right.\right.}{n}$$

$$= \frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 0.845, size = 64, normalized size = 0.97

$$\frac{e^{-2a}(p-2)x\left(e^{2a}-(cx^n)^{\frac{2}{n(p-2)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{2n-np}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(E^(2*a) - (c*x^n)^(2/(n*(-2 + p))))*Csch[a + Log[c*x^n]/(2*n -
 n*p)]^p)/(2*E^(2*a)*(-1 + p))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(a-\frac{\ln(cx^n)}{n\left(-2+p\right)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\operatorname{csch}\left(-a+\frac{\log(cx^n)}{n\left(p-2\right)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate((-csch(-a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Fricas [B] time = 1.70206, size = 1347, normalized size = 20.41

$$-\frac{(p-2)x\cosh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\right)}{\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)^{2}+2\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(\cdots}{np-2n}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*
n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n -
n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(
n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)
))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(
p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c)
)/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sin
h(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)))*sinh(-(a*n*p -
 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/((p - 1)*cosh(-(a*n*p - 2*a*n - n
*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) -
log(c))/(n*p - 2*n)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^{p}\left(a-\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csch(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(a-\frac{\log\left(cx^{n}\right)}{n(p-2)}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csch(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

3.165 $\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{bn}$$

```
[Out] -(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rubi [A] time = 0.0175591, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3770}

$$-\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [B] time = 0.0638428, size = 54, normalized size = 2.7

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{1}{2}b\log(cx^n)\right)\right)}{bn}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{1}{2}b\log(cx^n)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(Log[Cosh[a/2 + (b*Log[c*x^n])/2]]/(b*n)) + Log[Sinh[a/2 + (b*Log[c*x^n])/
2]]/(b*n)
```

Maple [A] time = 0.008, size = 23, normalized size = 1.2

$$\frac{1}{bn}\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))/x,x)
```

```
[Out] 1/n/b*ln(tanh(1/2*a+1/2*b*ln(c*x^n)))
```

Maxima [A] time = 1.09665, size = 30, normalized size = 1.5

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,b\log\left(cx^{n}\right)+\frac{1}{2}\,a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*log(c*x^n) + 1/2*a))/(b*n)
```

Fricas [B] time = 1.84175, size = 219, normalized size = 10.95

$$-\frac{\log\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)-\log\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1
) - log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) -
 1))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))/x, x)
```

Giac [B] time = 1.21073, size = 190, normalized size = 9.5

$$-\frac{1}{2}\,c^{b}\left(\frac{c^{b}e^{(-a)}\log\left(2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1\right)}{bc^{2\,b}n}-\frac{c^{b}e^{(-a)}\log\left(-2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}\left(c\right.\right.\right.}{bc^{2\,b}n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] -1/2*c^b*(c^b*e^(-a)*log(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b
)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1)/(b*c^(2*b)*n) - c^b*e^(-a)*log(
-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)
^(2*b)*e^(2*a) + 1)/(b*c^(2*b)*n))*e^a
```

3.166 $\int \frac{\mathbf{csch}^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rubi [A] time = 0.0271739, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3767, 8}

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{csch}^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \text{csch}^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.0663661, size = 19, normalized size = 1.

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Maple [A] time = 0.012, size = 20, normalized size = 1.1

$$-\frac{\coth\left(a+b\ln\left(cx^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] -coth(a+b*ln(c*x^n))/b/n
```

Maxima [A] time = 1.12217, size = 39, normalized size = 2.05

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n)
```

Fricas [B] time = 1.92079, size = 219, normalized size = 11.53

$$-\frac{2}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 - b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2/x, x)
```

Giac [A] time = 1.14547, size = 38, normalized size = 2.

$$-\frac{2}{\left(c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}-1\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)*b*n)
```

3.167 $\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}$$

```
[Out] ArcTanh[Cosh[a + b*Log[c*x^n]]]/(2*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a +
b*Log[c*x^n]])/(2*b*n)
```

Rubi [A] time = 0.0448704, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Cosh[a + b*Log[c*x^n]]]/(2*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a +
b*Log[c*x^n]])/(2*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}-\frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx,x,\log(cx^n)\right)}{2n}\\ &= \frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A] time = 0.0591374, size = 81, normalized size = 1.47

$$-\frac{\log\left(\tanh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -Csch[(a + b*Log[c*x^n])/2]^2/(8*b*n) - Log[Tanh[(a + b*Log[c*x^n])/2]]/(2*
b*n) - Sech[(a + b*Log[c*x^n])/2]^2/(8*b*n)
```

Maple [A] time = 0.019, size = 51, normalized size = 0.9

$$-\frac{\operatorname{csch}(a+b\ln(cx^n))\coth(a+b\ln(cx^n))}{2\,bn}+\frac{\operatorname{Artanh}\left(e^{a+b\ln(cx^n)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] -1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))/b/n+1/b/n*arctanh(exp(a+b*ln(c
*x^n)))
```

Maxima [B] time = 1.15472, size = 203, normalized size = 3.69

$$-\frac{c^{3b}e^{(3b\log(x^n)+3a)}+c^be^{(b\log(x^n)+a)}}{bc^{4b}ne^{(4b\log(x^n)+4a)}-2\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn}+\frac{\log\left(\frac{\left(c^be^{\left(b\log(x^n)+a\right)}+1\right)e^{(-a)}}{c^b}\right)}{2\,bn}-\frac{\log\left(\frac{\left(c^be^{\left(b\log(x^n)+a\right)}-1\right)e^{(-a)}}{c^b}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -(c^(3*b)*e^(3*b*log(x^n) + 3*a) + c^b*e^(b*log(x^n) + a))/(b*c^(4*b)*n*e^(
4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/2*log
((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) - 1/2*log((c^b*e^(b*log(x^n
) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Fricas [B] time = 1.84464, size = 2109, normalized size = 38.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sinh(b*n*log(x) + b*log(c) + a)^3
- (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*
cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 -
2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3
- cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log
(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + (
cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cos
h(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*c
```

```
osh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(co
sh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(3
*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + 2
*cosh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 +
4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b
*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b
*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3/x, x)
```

Giac [B] time = 1.20888, size = 278, normalized size = 5.05

$$\frac{1}{4}\,c^{3\,b}\left(\frac{c^b e^{(-3\,a)}\log\left(2\,x^{bn}|c|^b\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^a+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1\right)}{bc^{4\,b}n}-\frac{c^b e^{(-3\,a)}\log\left(-2\,x^{bn}|c|^b\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}\left(c\right)\right.\right.}{bc^{4\,b}n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/4*c^(3*b)*(c^b*e^(-3*a)*log(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2
*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1)/(b*c^(4*b)*n) - c^b*e^(-3*
a)*log(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(2*b*n)
*abs(c)^(2*b)*e^(2*a) + 1)/(b*c^(4*b)*n) - 4*(c^(2*b)*x^(3*b*n)*e^(2*a) + x
^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*c^(2*b)*n))*e^(3*a)
```

3.168 $\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}$$

```
[Out] Coth[a + b*Log[c*x^n]]/(b*n) - Coth[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] time = 0.0329257, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3767}

$$\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Coth[a + b*Log[c*x^n]]/(b*n) - Coth[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^4(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{i\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}\end{aligned}$$

Mathematica [A] time = 0.0648677, size = 56, normalized size = 1.33

$$\frac{2\coth(a+b\log(cx^n))}{3bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^2(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (2*Coth[a + b*Log[c*x^n]])/(3*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log
[c*x^n]]^2)/(3*b*n)
```

Maple [A] time = 0.01, size = 36, normalized size = 0.9

$$\frac{\coth\left(a+b\ln\left(cx^{n}\right)\right)}{bn}\left(\frac{2}{3}-\frac{\left(\operatorname{csch}\left(a+b\ln\left(cx^{n}\right)\right)\right)^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/n/b*(2/3-1/3*csch(a+b*ln(c*x^n))^2)*coth(a+b*ln(c*x^n))
```

Maxima [B] time = 1.13217, size = 124, normalized size = 2.95

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}-3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) - b*n)
```

Fricas [B] time = 1.93634, size = 869, normalized size = 20.69

$$-\frac{}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+bn\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*n*log(x) + b*log(c) + a) + 2*sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 - 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 2*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 - 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 4*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4/x, x)
```

Giac [A] time = 1.17783, size = 63, normalized size = 1.5

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{3}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) - 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*b
*n)
```

3.169 $\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}$$

```
[Out] (-3*ArcTanh[Cosh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Coth[a + b*Log[c*x^n]]*Cs
ch[a + b*Log[c*x^n]])/(8*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^
n]]^3)/(4*b*n)
```

Rubi [A] time = 0.0702584, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$-\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^5/x,x]
```


```
[Out] (-3*ArcTanh[Cosh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Coth[a + b*Log[c*x^n]]*Cs
ch[a + b*Log[c*x^n]])/(8*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^
n]]^3)/(4*b*n)
```


Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^5(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}-\frac{3\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx,x,\log(cx^n)\right)}{4n}\\ &= \frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}\\ &= -\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}-\frac{\coth}{}\end{aligned}$$

Mathematica [A] time = 0.0581078, size = 135, normalized size = 1.52

$$\frac{3\log\left(\tanh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{8bn}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+b\log(cx^n))\right)}{64bn}+\frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{32bn}-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+b\log(cx^n)\right)}{64bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*Csch[(a + b*Log[c*x^n])/2]^2)/(32*b*n) - Csch[(a + b*Log[c*x^n])/2]^4/(6
4*b*n) + (3*Log[Tanh[(a + b*Log[c*x^n])/2]])/(8*b*n) + (3*Sech[(a + b*Log[c
*x^n])/2]^2)/(32*b*n) + Sech[(a + b*Log[c*x^n])/2]^4/(64*b*n)
```

Maple [A] time = 0.015, size = 84, normalized size = 0.9

$$-\frac{\coth\left(a+b\ln\left(cx^{n}\right)\right)\left(\operatorname{csch}\left(a+b\ln\left(cx^{n}\right)\right)\right)^{3}}{4\,bn}+\frac{3\operatorname{csch}\left(a+b\ln\left(cx^{n}\right)\right)\coth\left(a+b\ln\left(cx^{n}\right)\right)}{8\,bn}-\frac{3\operatorname{Artanh}\left(e^{a+b\ln\left(cx^{n}\right)}\right)}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] -1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n+3/8*coth(a+b*ln(c*x^n))*
csch(a+b*ln(c*x^n))/b/n-3/4/b/n*arctanh(exp(a+b*ln(c*x^n)))
```

Maxima [B] time = 1.26848, size = 313, normalized size = 3.52

$$\frac{3\,c^{7\,b}e^{\left(7\,b\log\left(x^{n}\right)+7\,a\right)}-11\,c^{5\,b}e^{\left(5\,b\log\left(x^{n}\right)+5\,a\right)}-11\,c^{3\,b}e^{\left(3\,b\log\left(x^{n}\right)+3\,a\right)}+3\,c^{b}e^{\left(b\log\left(x^{n}\right)+a\right)}}{4\left(bc^{8\,b}ne^{\left(8\,b\log\left(x^{n}\right)+8\,a\right)}-4\,bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}+6\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-4\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}-\frac{3\log\left(\left(c^{b}e^{\left(b\log(x\right.}\right.\right.}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) - 11*c^(5*b)*e^(5*b*log(x^n) + 5*a) -
 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) + 3*c^b*e^(b*log(x^n) + a))/(b*c^(8*b)*n
*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b
)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) -
3/8*log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + 3/8*log((c^b*e^(b*
log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Fricas [B] time = 2.23032, size = 5852, normalized size = 65.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*n*log(x) + b*log(c) + a)^7 + 42*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*sinh(b*n*log(x) + b*log(c) + a)^7
+ 2*(63*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c)
+ a)^5 - 22*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*(21*cosh(b*n*log(x) + b*
log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log
```

```
(c) + a)^4 + 2*(105*cosh(b*n*log(x) + b*log(c) + a)^4 - 110*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 22*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^5 - 110*cosh
(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log
(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*
n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cos
h(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*c
osh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6
*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5
 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^
6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*log(x) + b*log(c)
 + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
+ a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log(x) + b*log(c) +
 a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + 3*(cosh(b*n*log(x) + b*log(c)
+ a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^
7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(
c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) +
 b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x)
 + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x)
+ b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x
) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log
(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*l
og(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*lo
g(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(21*cosh(b*n
*log(x) + b*log(c) + a)^6 - 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(
b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(
b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*
cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sin
h(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 +
4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c
) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(
x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(
x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*b*n
*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)
^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b
*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*
log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*cosh(b*n*log(x
) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^5(a + b\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**5/x, x)
```

Giac [B] time = 1.23348, size = 329, normalized size = 3.7

$$-\frac{1}{16}\,c^{5\,b}\left(\frac{3\,c^{b}e^{(-5\,a)}\log\left(2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1\right)}{bc^{6\,b}n}-\frac{3\,c^{b}e^{(-5\,a)}\log\left(-2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] -1/16*c^(5*b)*(3*c^b*e^(-5*a)*log(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) +
 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1)/(b*c^(6*b)*n) - 3*c^b*
e^(-5*a)*log(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(
2*b*n)*abs(c)^(2*b)*e^(2*a) + 1)/(b*c^(6*b)*n) - 4*(3*c^(6*b)*x^(7*b*n)*e^(
6*a) - 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x^(3*b*n)*e^(2*a) + 3*x^(b
*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*c^(4*b)*n))*e^(5*a)
```

3.170 $\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$-\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\right.\right.}{3bn}$$

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.068457, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2641}

$$-\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}F\left(\frac{1}{2}\left(ia+ib\log(cx^n\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\,\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}\,dx, x, \log\right.}{3n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\,\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\left(\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a\right.}{}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\,\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{\text{csch}(a+b\log(cx^n))}F\left(\frac{1}{2}\left(ia-\right.\right.}{}$$

Mathematica [A] time = 0.207335, size = 84, normalized size = 0.76

$$-\frac{2\sqrt{\text{csch}(a+b\log(cx^n))}\left(\coth(a+b\log(cx^n)) + i\sqrt{i\sinh(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi), 2\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Coth[a + b*Log[c*x^n]] + I*EllipticF[((-2
*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(3*
b*n)
```

Maple [A] time = 0.036, size = 144, normalized size = 1.3

$$-\frac{1}{3n\cosh(a+b\ln(cx^n))\,b}\left(i\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{csch}(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(5/2)/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.171 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/(b*n) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c*x^n]
]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.062964, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2639}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/(b*n) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c*x^n]
]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx,x,\log(cx^n)\right)}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx,x,\log(cx^n)\right)}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx,x,\operatorname{lo}\right.}{n\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+}}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\right.}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a-}}$$

Mathematica [A] time = 0.112479, size = 80, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\cosh(a+b\log(cx^n))-\sqrt{i\sinh(a+b\log(cx^n))}E\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi)\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I
)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A] time = 0.188, size = 212, normalized size = 2.

$$\frac{1}{n\cosh(a+b\ln(cx^n))\,b}\left(2\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\operatorname{EllipticE}\left(\sqrt{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.172 $\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right),2\right)}{bn}$$

```
[Out] ((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n]
)/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0426943, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n]
)/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\left(\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0919914, size = 66, normalized size = 0.92

$$\frac{2\left(i\sinh\left(a+b\log\left(cx^n\right)\right)\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\text{EllipticF}\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right),2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Csch[a + b*Log[c*x^n]]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^
n])/4, 2]*(I*Sinh[a + b*Log[c*x^n]])^(3/2))/(b*n)
```

Maple [A] time = 0.118, size = 120, normalized size = 1.7

$$\frac{i\sqrt{2}}{n\cosh\left(a+b\ln\left(cx^{n}\right)\right)b}\sqrt{-i\left(i+\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)}\sqrt{-i\left(-\sinh\left(a+b\ln\left(cx^{n}\right)\right)+i\right)}\sqrt{i\sinh\left(a+b\ln\left(cx^{n}\right)\right)}\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] I/n*(-I*(I+sinh(a+b*ln(c*x^n))))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(I+sinh(a+b*ln(c*x^n)))
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.173 $\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b
*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0444987, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b
*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx,x,\log\left(cx^n\right)\right)}{n\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}\\
&= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}$$

Mathematica [A] time = 0.0755923, size = 68, normalized size = 0.94

$$\frac{2\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i\left(a+b\log\left(cx^n\right)\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] time = 0.027, size = 146, normalized size = 2.

$$\frac{\sqrt{2}}{n\cosh\left(a+b\ln\left(cx^{n}\right)\right)b}\sqrt{-i\left(i+\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)}\sqrt{-i\left(-\sinh\left(a+b\ln\left(cx^{n}\right)\right)+i\right)}\sqrt{i\sinh\left(a+b\ln\left(cx^{n}\right)\right)}\left(2\,\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] 1/n*(-I*(I+sinh(a+b*ln(c*x^n))))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(csch(a + b*log(c*x**n)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

3.174 $\int \frac{1}{x\,\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right),2\right)}{3bn}$$

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2
]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0601368, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```


```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2
]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```


Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} - \frac{\left(\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}\right)\text{Sub}}{3n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} + \frac{2i\sqrt{\text{csch}(a+b\log(cx^n))}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\right|}{3bn}$$

Mathematica [A] time = 0.128335, size = 86, normalized size = 0.77

$$\frac{\sqrt{\text{csch}(a+b\log(cx^n))}\left(\sinh(2(a+b\log(cx^n))) - 2i\sqrt{i\sinh(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi),\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Csch[a + b*Log[c*x^n]]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Lo
g[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]] + Sinh[2*(a + b*Log[c*x^n])]
))/(3*b*n)
```

Maple [A] time = 0.027, size = 143, normalized size = 1.3

$$\frac{1}{n\cosh(a+b\ln(cx^n))b}\left(-\frac{i}{3}\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{csch}(b\log(cx^n)+a)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")

[Out] integral(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csch(a+b*ln(c*x**n))**(3/2),x)

[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="giac")

[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)

3.175 $\int \frac{1}{x\mathbf{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh(a+b\log(cx^n))}{5bn\mathrm{csch}^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\mathrm{csch}(a+b\log(cx^n))}}$$

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(5*b*n*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c
*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0606222, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh(a+b\log(cx^n))}{5bn\mathrm{csch}^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\mathrm{csch}(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(5*b*n*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c
*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx, x, \log(cx^n)\right)}{5n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{3\,\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{5n\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)}{5bn\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

Mathematica [A] time = 0.176031, size = 95, normalized size = 0.86

$$\frac{2\left(\cosh(a+b\log(cx^n)) - 3\sqrt{i\sinh(a+b\log(cx^n))}\text{csch}^2(a+b\log(cx^n))\,E\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi)\middle|2\right)\right)}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Cosh[a + b*Log[c*x^n]] - 3*Csch[a + b*Log[c*x^n]]^2*EllipticE[((-2*I)*a
 + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(5*b*n*C
sch[a + b*Log[c*x^n]]^(3/2))
```

Maple [A] time = 0.197, size = 227, normalized size = 2.1

$$\frac{1}{n\cosh(a+b\ln(cx^n))\,b}\left(-\frac{6\sqrt{2}}{5}\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{EllipticE}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{csch}(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
            print("both result and optimal complex");
         fi;
         #both result and optimal complex
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      else #result contains complex but optimal is not
         if debug then
            print("result contains complex but optimal is not");
         fi;
         return "C";
      end if
   else # result do not contain complex
        # this assumes optimal do not as well
        if debug then
            print("result do not contain complex, this assumes optimal do
   not as well");
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
           if debug then
              print("leaf_count_result<=2*leaf_count_optimal");
           fi;
           return "A";
        else
           if debug then
              print("leaf_count_result>2*leaf_count_optimal");
           fi;
           return "B";
        end if
   end if
 else #ExpnType(result) > ExpnType(optimal)
     if debug then
        print("ExpnType(result) > ExpnType(optimal)");
     fi;
     return "C";
 end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```