Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.3-Hyperbolic-secant-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [201]. This is test number [179].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (201)	% 0. (0)
Mathematica	% 95.52 (192)	% 4.48 (9)
Maple	% 69.65 (140)	% 30.35 (61)
Maxima	% 44.78 (90)	% 55.22 (111)
Fricas	% 70.65 (142)	% 29.35 (59)
Sympy	% 3.98 (8)	% 96.02 (193)
Giac	% 56.22 (113)	% 43.78 (88)

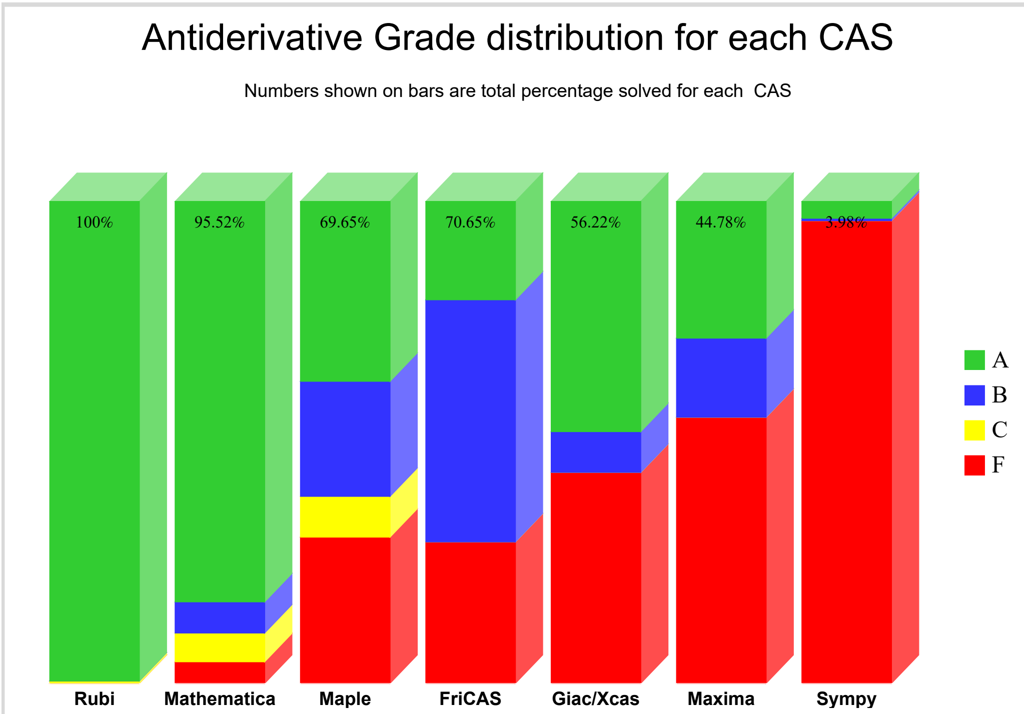
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

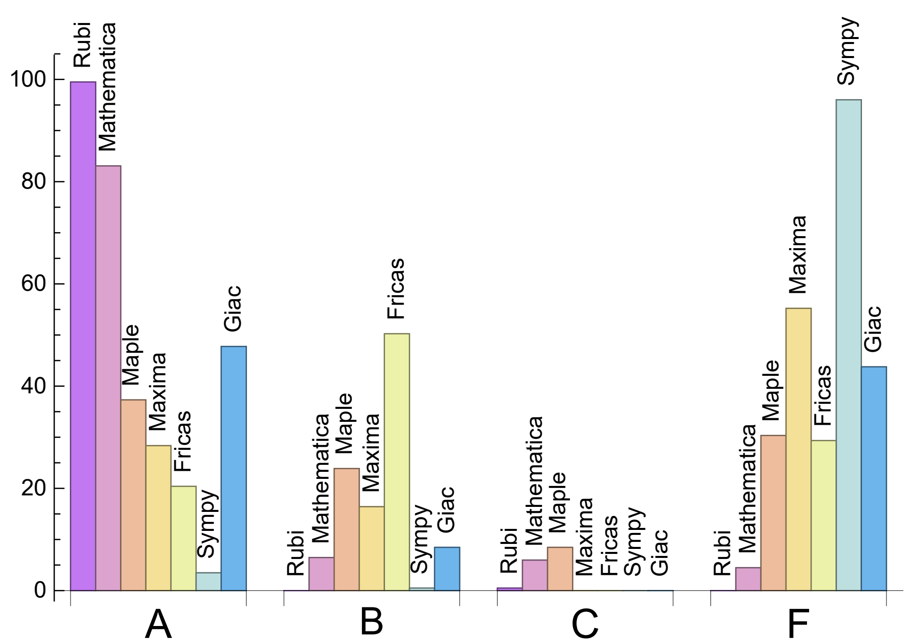
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.5	0.	0.5	0.
Mathematica	83.08	6.47	5.97	4.48
Maple	37.31	23.88	8.46	30.35
Maxima	28.36	16.42	0.	55.22
Fricas	20.4	50.25	0.	29.35
Sympy	3.48	0.5	0.	96.02
Giac	47.76	8.46	0.	43.78

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	93.02	1.01	66.	1.
Mathematica	0.7	84.67	1.09	58.	0.98
Maple	0.08	129.71	2.07	100.	1.5
Maxima	1.36	119.92	2.31	84.	2.11
Fricas	3.23	2099.11	24.5	784.	18.18
Sympy	13.63	41.88	1.37	39.	1.12
Giac	1.15	99.29	1.71	72.	1.61

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {186}

Mathematica {183, 184, 185}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

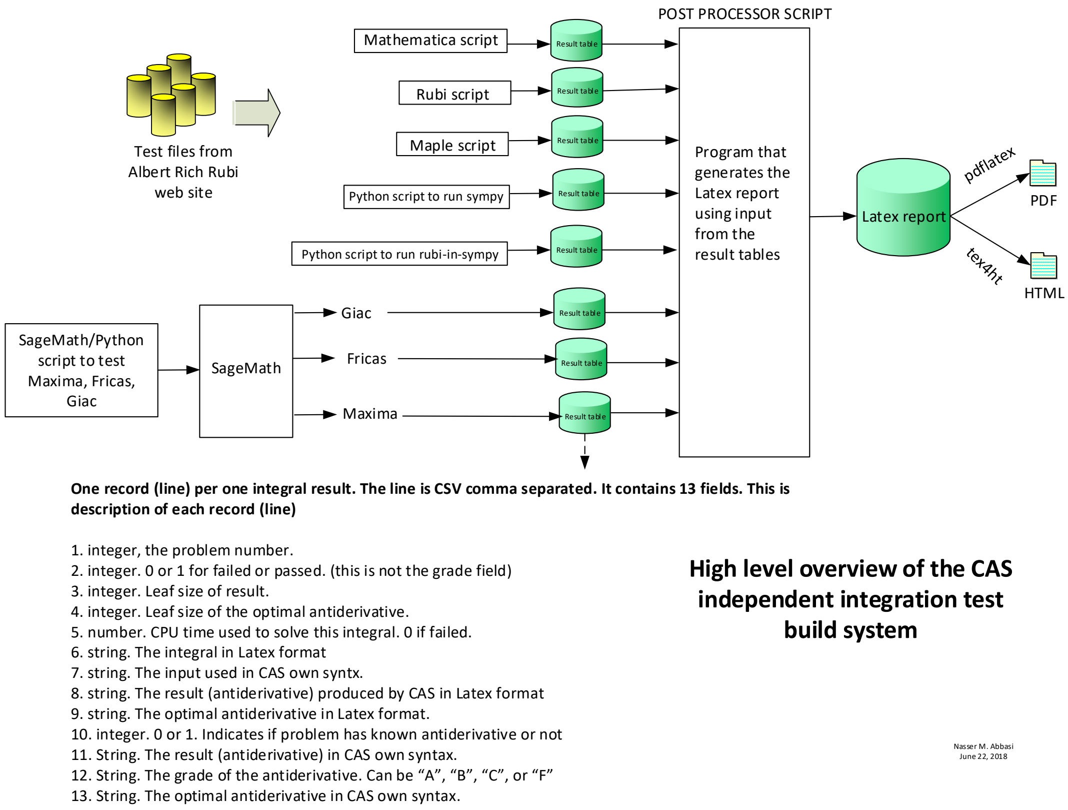
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.


POST PROCESSOR SCRIPT
Mathematica script
Rubi script
Maple script
Python script to run sympy
Python script to run rubi-in-sympy
Test files from Albert Rich Rubi web site
Result table
Program that generates the Latex report using input from the result tables
Latex report
pdflatex
tex4ht
PDF
HTML
SageMath/Python script to test Maxima, Fricas, Giac
SageMath
Giac
Fricas
Maxima
One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)
1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.
High level overview of the CAS independent integration test build system
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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { }

C grade: { 186 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 140, 142, 143, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 167, 169, 171, 173, 175, 178, 179, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { 27, 85, 86, 129, 132, 135, 136, 137, 145, 146, 185, 187, 188 }

C grade: { 158, 160, 162, 166, 168, 170, 172, 174, 176, 177, 180, 181 }

F grade: { 130, 131, 138, 139, 141, 147, 148, 149, 150}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 45, 46, 47, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 104, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 167, 169, 171, 173, 177, 178, 191, 192, 193, 194, 195, 197, 200 }

B grade: { 9, 11, 12, 13, 14, 19, 28, 29, 30, 31, 35, 36, 37, 38, 48, 49, 50, 51, 52, 53, 54, 60, 61, 62, 68, 69, 70, 71, 92, 93, 95, 96, 97, 103, 105, 106, 107, 113, 114, 115, 116, 117, 118, 164, 196, 198, 199, 201 }

C grade: { 24, 25, 26, 27, 32, 33, 34, 158, 160, 162, 166, 168, 170, 172, 174, 176, 186 }

F grade: { 15, 16, 17, 18, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 163, 165, 175, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190 }

2.1.4 Maxima

A grade: { 1, 2, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 49, 50, 51, 52, 54, 56, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 88, 89, 90, 107, 108, 109, 110, 111, 117, 119, 120, 122, 152, 153, 154, 155, 156, 157, 159, 171, 179, 191, 192

B grade: { 3, 4, 5, 6, 7, 8, 24, 25, 45, 46, 47, 53, 55, 57, 58, 59, 61, 63, 87, 103, 104, 105, 106, 112, 113, 115, 124, 151, 167, 186, 187, 188, 194 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 60, 62, 65, 67, 78, 79, 80, 81, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 114, 116, 118, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 193, 195, 196, 197, 198, 199, 200, 201 }

2.1.5 FriCAS

A grade: { 1, 27, 28, 29, 30, 31, 34, 52, 53, 54, 64, 70, 71, 72, 73, 76, 77, 90, 91, 99, 100, 107, 108, 110, 118, 119, 120, 154, 155, 156, 157, 161, 165, 167, 169, 173, 175, 177, 179, 181, 191 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 32, 33, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 142, 143, 144, 151, 152, 153, 159, 163, 171, 186, 187, 188, 189, 190, 192, 193, 194, 195 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 94, 129, 130, 131, 132, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 183, 184, 185, 196, 197, 198, 199, 200, 201 }

2.1.6 Sympy

A grade: { 28, 29, 30, 35, 36, 37, 119 }

B grade: { 108 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

2.1.7 Giac

A grade: { 1, 2, 4, 6, 7, 8, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 187, 188, 191, 192, 194, 195 }

B grade: { 3, 5, 26, 58, 59, 66, 79, 80, 83, 85, 86, 106, 113, 115, 117, 124, 193 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 81, 82, 84, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 196, 197, 198, 199, 200, 201 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	58	0	16
normalized size	1	1.	1.	1.09	1.36	5.27	0.	1.45
time (sec)	N/A	0.005	0.002	0.004	1.004	2.007	0.	1.097

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	24	111	0	24
normalized size	1	1.	1.	1.1	2.4	11.1	0.	2.4
time (sec)	N/A	0.01	0.003	0.005	0.991	2.071	0.	1.139

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	88	757	0	107
normalized size	1	1.	1.	0.88	2.59	22.26	0.	3.15
time (sec)	N/A	0.016	0.009	0.01	1.497	1.996	0.	1.132

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	122	448	0	42
normalized size	1	1.	1.	0.88	4.69	17.23	0.	1.62
time (sec)	N/A	0.012	0.006	0.012	1.018	1.963	0.	1.154

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	47	50	151	2263	0	140
normalized size	1	1.	0.85	0.91	2.75	41.15	0.	2.55
time (sec)	N/A	0.028	0.041	0.01	1.512	2.149	0.	1.144

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	277	954	0	57
normalized size	1	1.	1.	0.8	6.76	23.27	0.	1.39
time (sec)	N/A	0.015	0.011	0.013	1.018	2.092	0.	1.124

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	66	331	0	24
normalized size	1	1.	1.	0.89	3.47	17.42	0.	1.26
time (sec)	N/A	0.01	0.004	0.007	1.045	2.11	0.	1.134

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	27	185	859	0	41
normalized size	1	1.	1.	0.77	5.29	24.54	0.	1.17
time (sec)	N/A	0.014	0.004	0.009	1.028	2.129	0.	1.143

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	217	0	0	0	0
normalized size	1	1.	0.77	3.29	0.	0.	0.	0.
time (sec)	N/A	0.032	0.075	0.276	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	103	0	0	0	0
normalized size	1	1.	0.79	1.66	0.	0.	0.	0.
time (sec)	N/A	0.03	0.043	0.287	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	0	0	0
normalized size	1	1.	1.	3.38	0.	0.	0.	0.
time (sec)	N/A	0.02	0.029	0.241	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	0	0	0
normalized size	1	1.	1.	3.38	0.	0.	0.	0.
time (sec)	N/A	0.02	0.034	0.245	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	53	174	0	0	0	0
normalized size	1	1.	0.8	2.64	0.	0.	0.	0.
time (sec)	N/A	0.032	0.045	0.277	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	59	188	0	0	0	0
normalized size	1	1.	0.89	2.85	0.	0.	0.	0.
time (sec)	N/A	0.033	0.07	0.309	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	68	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.197	0.099	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	56	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.07	0.079	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.039	0.079	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.021	0.138	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	244	0	0	0	0
normalized size	1	1.	1.	5.81	0.	0.	0.	0.
time (sec)	N/A	0.022	0.03	0.141	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.063	0.083	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.081	0.082	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	70	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.127	0.082	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.062	0.191	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	81	230	211	4601	0	170
normalized size	1	1.	0.9	2.56	2.34	51.12	0.	1.89
time (sec)	N/A	0.03	0.101	0.177	1.525	2.334	0.	1.106

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	208	151	2263	0	140
normalized size	1	1.	0.85	3.2	2.32	34.82	0.	2.15
time (sec)	N/A	0.022	0.115	0.129	1.51	2.208	0.	1.14

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	183	88	757	0	107
normalized size	1	1.	1.15	4.58	2.2	18.92	0.	2.68
time (sec)	N/A	0.017	0.041	0.12	1.541	2.152	0.	1.122

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	29	130	15	58	0	16
normalized size	1	1.	2.64	11.82	1.36	5.27	0.	1.45
time (sec)	N/A	0.011	0.016	0.132	0.996	2.002	0.	1.141

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	97	35	23	29	31
normalized size	1	1.	1.	4.41	1.59	1.05	1.32	1.41
time (sec)	N/A	0.016	0.026	0.106	0.977	1.973	16.08	1.15

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	201	73	89	54	65
normalized size	1	1.	0.86	3.94	1.43	1.75	1.06	1.27
time (sec)	N/A	0.02	0.066	0.104	1.012	2.047	20.335	1.138

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	47	305	111	182	80	95
normalized size	1	1.	0.62	4.01	1.46	2.39	1.05	1.25
time (sec)	N/A	0.027	0.082	0.098	1.031	2.125	54.543	1.128

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	57	409	135	302	0	124
normalized size	1	1.	0.56	4.05	1.34	2.99	0.	1.23
time (sec)	N/A	0.035	0.131	0.102	1.018	2.073	0.	1.12

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	127	97	3067	0	88
normalized size	1	1.	0.65	1.95	1.49	47.18	0.	1.35
time (sec)	N/A	0.034	0.034	0.08	1.686	2.238	0.	1.136

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	29	106	53	952	0	65
normalized size	1	1.	0.63	2.3	1.15	20.7	0.	1.41
time (sec)	N/A	0.024	0.019	0.066	1.832	2.263	0.	1.125

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	21	72	11	444	0	11
normalized size	1	1.	0.84	2.88	0.44	17.76	0.	0.44
time (sec)	N/A	0.016	0.006	0.082	1.771	2.234	0.	1.132

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	23	247	15	19
normalized size	1	1.	1.	4.46	1.77	19.	1.15	1.46
time (sec)	N/A	0.029	0.006	0.072	1.873	2.03	0.695	1.141

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	47	856	37	39
normalized size	1	1.	0.75	3.61	1.31	23.78	1.03	1.08
time (sec)	N/A	0.02	0.021	0.059	1.705	2.199	1.532	1.138

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	72	1885	60	55
normalized size	1	1.	0.65	3.56	1.31	34.27	1.09	1.
time (sec)	N/A	0.029	0.039	0.054	1.829	2.149	15.078	1.135

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	96	3340	0	72
normalized size	1	1.	0.57	3.54	1.3	45.14	0.	0.97
time (sec)	N/A	0.04	0.049	0.057	1.655	2.299	0.	1.114

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.096	0.059	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	47	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.036	0.054	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.019	0.073	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	38	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.04	0.079	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	47	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.089	0.058	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.091	0.053	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	72	837	9415	0	69
normalized size	1	1.	0.33	0.44	5.13	57.76	0.	0.42
time (sec)	N/A	0.045	0.172	0.089	1.657	3.054	0.	1.2

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	42	60	435	4632	0	53
normalized size	1	1.	0.36	0.51	3.72	39.59	0.	0.45
time (sec)	N/A	0.035	0.093	0.07	1.769	2.43	0.	1.172

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	46	162	1623	0	36
normalized size	1	1.	0.49	0.75	2.66	26.61	0.	0.59
time (sec)	N/A	0.023	0.054	0.06	1.723	2.04	0.	1.174

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	18	230	0	18
normalized size	1	1.	1.	1.93	1.2	15.33	0.	1.2
time (sec)	N/A	0.017	0.005	0.075	1.576	2.108	0.	1.176

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	89	41	771	0	38
normalized size	1	1.	0.64	2.47	1.14	21.42	0.	1.06
time (sec)	N/A	0.016	0.022	0.085	1.725	2.331	0.	1.132

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	88	3687	0	70
normalized size	1	1.	0.44	2.67	1.02	42.87	0.	0.81
time (sec)	N/A	0.036	0.036	0.058	1.689	2.647	0.	1.139

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	139	9443	0	103
normalized size	1	1.	0.42	2.74	1.05	71.54	0.	0.78
time (sec)	N/A	0.056	0.076	0.069	1.745	2.761	0.	1.124

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	28	130	73	113	0	57
normalized size	1	1.	0.64	2.95	1.66	2.57	0.	1.3
time (sec)	N/A	0.139	0.102	0.029	1.108	2.387	0.	1.09

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	67	62	100	0	50
normalized size	1	1.	1.	2.91	2.7	4.35	0.	2.17
time (sec)	N/A	0.122	0.045	0.024	1.163	2.302	0.	1.129

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	16	78	57	47	0	38
normalized size	1	1.	0.59	2.89	2.11	1.74	0.	1.41
time (sec)	N/A	0.101	0.06	0.023	1.083	2.484	0.	1.158

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	27	47	200	0	43
normalized size	1	1.	0.94	1.59	2.76	11.76	0.	2.53
time (sec)	N/A	0.073	0.018	0.017	1.127	2.408	0.	1.15

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	44	23	65	397	0	70
normalized size	1	1.	1.33	0.7	1.97	12.03	0.	2.12
time (sec)	N/A	0.098	0.049	0.02	1.112	2.416	0.	1.113

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	122	230	0	42
normalized size	1	1.	1.09	1.	5.3	10.	0.	1.83
time (sec)	N/A	0.139	0.038	0.022	1.143	2.292	0.	1.152

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	59	45	134	2060	0	122
normalized size	1	1.	1.28	0.98	2.91	44.78	0.	2.65
time (sec)	N/A	0.195	0.154	0.027	1.126	2.536	0.	1.194

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	39	394	695	0	80
normalized size	1	1.	1.15	1.15	11.59	20.44	0.	2.35
time (sec)	N/A	0.146	0.06	0.026	1.174	2.365	0.	1.182

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	219	488	0	4609	0	266
normalized size	1	1.	1.66	3.7	0.	34.92	0.	2.02
time (sec)	N/A	0.37	0.705	0.036	0.	2.742	0.	1.171

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	66	361	173	1251	0	117
normalized size	1	1.	1.08	5.92	2.84	20.51	0.	1.92
time (sec)	N/A	0.18	0.127	0.031	1.131	2.763	0.	1.172

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	213	0	1500	0	135
normalized size	1	1.	0.93	2.6	0.	18.29	0.	1.65
time (sec)	N/A	0.212	0.153	0.027	0.	2.759	0.	1.148

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	62	246	0	46
normalized size	1	1.	0.95	1.55	3.1	12.3	0.	2.3
time (sec)	N/A	0.088	0.01	0.02	1.134	2.574	0.	1.124

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	37	48	80	181	0	88
normalized size	1	1.	0.7	0.91	1.51	3.42	0.	1.66
time (sec)	N/A	0.118	0.073	0.022	1.107	2.475	0.	1.125

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	77	0	1156	0	86
normalized size	1	1.	1.14	1.17	0.	17.52	0.	1.3
time (sec)	N/A	0.133	0.252	0.023	0.	2.588	0.	1.134

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	86	82	200	2097	0	235
normalized size	1	1.	1.01	0.96	2.35	24.67	0.	2.76
time (sec)	N/A	0.238	0.33	0.029	1.112	2.743	0.	1.147

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	156	154	0	5416	0	201
normalized size	1	1.	1.41	1.39	0.	48.79	0.	1.81
time (sec)	N/A	0.304	0.577	0.032	0.	2.872	0.	1.136

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	139	108	491	0	116
normalized size	1	1.	0.94	2.07	1.61	7.33	0.	1.73
time (sec)	N/A	0.096	0.086	0.032	1.079	2.548	0.	1.181

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	111	89	346	0	95
normalized size	1	1.	0.98	2.06	1.65	6.41	0.	1.76
time (sec)	N/A	0.086	0.073	0.029	1.168	2.43	0.	1.148

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	45	87	76	242	0	69
normalized size	1	1.	1.1	2.12	1.85	5.9	0.	1.68
time (sec)	N/A	0.08	0.048	0.029	1.151	2.171	0.	1.123

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	32	59	55	151	0	47
normalized size	1	1.	1.23	2.27	2.12	5.81	0.	1.81
time (sec)	N/A	0.057	0.06	0.027	1.121	2.43	0.	1.166

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	16	43	0	15
normalized size	1	1.	0.91	0.82	1.45	3.91	0.	1.36
time (sec)	N/A	0.024	0.007	0.012	1.137	2.271	0.	1.148

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	31	117	0	27
normalized size	1	1.	1.1	1.05	1.55	5.85	0.	1.35
time (sec)	N/A	0.066	0.029	0.01	1.753	2.392	0.	1.119

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	45	39	61	467	0	49
normalized size	1	1.	1.73	1.5	2.35	17.96	0.	1.88
time (sec)	N/A	0.101	0.085	0.015	1.67	2.299	0.	1.114

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	51	61	99	1099	0	65
normalized size	1	1.	1.13	1.36	2.2	24.42	0.	1.44
time (sec)	N/A	0.085	0.084	0.017	1.695	2.436	0.	1.152

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	58	58	45	131	0	42
normalized size	1	1.	2.	2.	1.55	4.52	0.	1.45
time (sec)	N/A	0.015	0.139	0.036	1.144	2.385	0.	1.149

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	59	60	47	131	0	42
normalized size	1	1.	1.97	2.	1.57	4.37	0.	1.4
time (sec)	N/A	0.015	0.14	0.032	1.115	2.42	0.	1.169

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	0	0	2558	0	216
normalized size	1	1.	1.01	0.	0.	26.1	0.	2.2
time (sec)	N/A	0.12	0.328	0.178	0.	2.776	0.	1.363

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	0	0	1975	0	159
normalized size	1	1.	1.14	0.	0.	29.92	0.	2.41
time (sec)	N/A	0.042	0.195	0.131	0.	2.438	0.	1.228

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	60	0	0	1808	0	112
normalized size	1	1.	1.62	0.	0.	48.86	0.	3.03
time (sec)	N/A	0.019	0.089	0.217	0.	2.558	0.	1.178

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	118	0	0	2480	0	0
normalized size	1	1.	1.39	0.	0.	29.18	0.	0.
time (sec)	N/A	0.074	1.124	0.172	0.	2.877	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	3401	0	0
normalized size	1	1.	1.55	0.	0.	29.83	0.	0.
time (sec)	N/A	0.128	4.627	0.13	0.	2.915	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	70	0	0	1808	0	136
normalized size	1	1.	1.84	0.	0.	47.58	0.	3.58
time (sec)	N/A	0.023	2.266	0.227	0.	2.567	0.	1.185

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	2480	0	0
normalized size	1	1.	1.36	0.	0.	28.51	0.	0.
time (sec)	N/A	0.079	2.164	0.161	0.	2.811	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	39	0	0	833	0	70
normalized size	1	1.	2.05	0.	0.	43.84	0.	3.68
time (sec)	N/A	0.017	0.038	0.116	0.	2.496	0.	1.148

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	51	0	0	833	0	93
normalized size	1	1.	2.43	0.	0.	39.67	0.	4.43
time (sec)	N/A	0.019	0.526	0.111	0.	2.476	0.	1.121

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	78	121	285	2547	0	194
normalized size	1	1.	0.73	1.13	2.66	23.8	0.	1.81
time (sec)	N/A	0.124	0.237	0.033	1.817	2.488	0.	1.165

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	80	154	1319	0	131
normalized size	1	1.	0.75	1.1	2.11	18.07	0.	1.79
time (sec)	N/A	0.052	0.13	0.023	1.828	2.361	0.	1.133

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	42	55	428	0	65
normalized size	1	1.	0.97	1.27	1.67	12.97	0.	1.97
time (sec)	N/A	0.028	0.063	0.01	1.084	2.435	0.	1.166

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	74	0	23
normalized size	1	1.	1.	1.06	1.38	4.62	0.	1.44
time (sec)	N/A	0.009	0.002	0.003	1.219	2.312	0.	1.132

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	88	0	648	0	76
normalized size	1	1.	1.02	1.49	0.	10.98	0.	1.29
time (sec)	N/A	0.055	0.105	0.014	0.	2.519	0.	1.141

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	203	221	0	2708	0	185
normalized size	1	1.	1.86	2.03	0.	24.84	0.	1.7
time (sec)	N/A	0.159	0.402	0.046	0.	2.612	0.	1.124

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	205	660	0	8986	0	362
normalized size	1	1.	1.18	3.82	0.	51.94	0.	2.09
time (sec)	N/A	0.308	0.727	0.05	0.	3.172	0.	1.179

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.345	0.235	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	126	406	0	5667	0	246
normalized size	1	1.	0.86	2.78	0.	38.82	0.	1.68
time (sec)	N/A	0.657	0.275	0.037	0.	2.813	0.	1.147

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	264	0	3675	0	180
normalized size	1	1.	0.88	2.36	0.	32.81	0.	1.61
time (sec)	N/A	0.423	0.16	0.036	0.	2.695	0.	1.148

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	174	0	2067	0	124
normalized size	1	1.	0.92	2.05	0.	24.32	0.	1.46
time (sec)	N/A	0.265	0.124	0.031	0.	2.6	0.	1.133

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	1064	0	84
normalized size	1	1.	0.92	1.52	0.	17.16	0.	1.35
time (sec)	N/A	0.092	0.113	0.031	0.	2.63	0.	1.161

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	446	0	43
normalized size	1	1.	0.98	0.86	0.	10.62	0.	1.02
time (sec)	N/A	0.055	0.026	0.013	0.	2.337	0.	1.174

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	587	0	61
normalized size	1	1.	1.	0.94	0.	10.87	0.	1.13
time (sec)	N/A	0.1	0.05	0.014	0.	2.605	0.	1.158

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	1285	0	82
normalized size	1	1.	0.98	1.14	0.	20.08	0.	1.28
time (sec)	N/A	0.14	0.105	0.024	0.	2.704	0.	1.169

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	146	0	3321	0	120
normalized size	1	1.	0.94	1.68	0.	38.17	0.	1.38
time (sec)	N/A	0.242	0.2	0.021	0.	3.544	0.	1.137

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	117	126	2288	0	93
normalized size	1	1.	1.25	2.44	2.62	47.67	0.	1.94
time (sec)	N/A	0.097	0.118	0.059	1.702	2.609	0.	1.15

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	34	100	1403	0	82
normalized size	1	1.	1.06	0.94	2.78	38.97	0.	2.28
time (sec)	N/A	0.059	0.071	0.037	1.501	2.563	0.	1.153

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	41	75	69	711	0	57
normalized size	1	1.	1.32	2.42	2.23	22.94	0.	1.84
time (sec)	N/A	0.07	0.058	0.033	1.665	2.511	0.	1.143

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	10	54	45	288	0	47
normalized size	1	1.	0.71	3.86	3.21	20.57	0.	3.36
time (sec)	N/A	0.048	0.034	0.027	1.655	2.525	0.	1.224

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	35	22	50	0	19
normalized size	1	1.	1.07	2.5	1.57	3.57	0.	1.36
time (sec)	N/A	0.046	0.03	0.019	1.659	2.428	0.	1.15

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	19	24	53	19	23
normalized size	1	1.	1.33	2.11	2.67	5.89	2.11	2.56
time (sec)	N/A	0.027	0.007	0.016	1.132	2.419	0.205	1.152

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	47	70	497	0	76
normalized size	1	1.	1.1	1.18	1.75	12.42	0.	1.9
time (sec)	N/A	0.058	0.048	0.029	1.155	2.464	0.	1.167

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	56	63	149	0	54
normalized size	1	1.	0.87	1.47	1.66	3.92	0.	1.42
time (sec)	N/A	0.089	0.072	0.03	1.096	2.238	0.	1.173

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	69	146	2475	0	127
normalized size	1	1.	0.97	1.01	2.15	36.4	0.	1.87
time (sec)	N/A	0.086	0.147	0.036	1.145	2.475	0.	1.16

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	69	78	142	502	0	86
normalized size	1	1.	1.25	1.42	2.58	9.13	0.	1.56
time (sec)	N/A	0.12	0.102	0.037	1.198	2.443	0.	1.113

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	132	415	448	9682	0	360
normalized size	1	1.	1.09	3.43	3.7	80.02	0.	2.98
time (sec)	N/A	0.149	0.325	0.047	1.727	3.278	0.	1.161

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	185	575	0	12166	0	338
normalized size	1	1.	0.99	3.07	0.	65.06	0.	1.81
time (sec)	N/A	0.293	0.589	0.043	0.	5.44	0.	1.136

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	85	233	221	3216	0	205
normalized size	1	1.	1.18	3.24	3.07	44.67	0.	2.85
time (sec)	N/A	0.097	0.171	0.04	1.667	3.192	0.	1.151

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	113	248	0	3224	0	150
normalized size	1	1.	1.2	2.64	0.	34.3	0.	1.6
time (sec)	N/A	0.319	0.403	0.033	0.	3.971	0.	1.142

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	37	107	90	543	0	99
normalized size	1	1.	1.06	3.06	2.57	15.51	0.	2.83
time (sec)	N/A	0.075	0.084	0.03	1.565	2.572	0.	1.138

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	113	0	551	0	70
normalized size	1	1.	1.	1.82	0.	8.89	0.	1.13
time (sec)	N/A	0.171	0.081	0.026	0.	2.812	0.	1.172

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	35	72	41	26
normalized size	1	1.	0.58	1.11	1.84	3.79	2.16	1.37
time (sec)	N/A	0.032	0.017	0.014	1.132	2.437	0.586	1.134

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	44	78	90	220	0	90
normalized size	1	1.	0.67	1.18	1.36	3.33	0.	1.36
time (sec)	N/A	0.106	0.085	0.029	1.072	2.33	0.	1.117

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	81	104	0	1582	0	111
normalized size	1	1.	0.71	0.91	0.	13.88	0.	0.97
time (sec)	N/A	0.205	0.335	0.03	0.	2.36	0.	1.151

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	112	119	221	2885	0	261
normalized size	1	1.	0.99	1.05	1.96	25.53	0.	2.31
time (sec)	N/A	0.19	0.3	0.043	1.183	2.718	0.	1.141

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	166	179	0	8041	0	257
normalized size	1	1.	0.8	0.86	0.	38.85	0.	1.24
time (sec)	N/A	0.329	0.762	0.042	0.	2.621	0.	1.158

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	167	215	494	11934	0	513
normalized size	1	1.	0.94	1.21	2.78	67.04	0.	2.88
time (sec)	N/A	0.32	0.998	0.041	1.141	3.652	0.	1.189

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	160	0	0	10971	0	0
normalized size	1	1.	0.95	0.	0.	64.92	0.	0.
time (sec)	N/A	0.194	4.835	0.221	0.	9.438	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	108	0	0	4070	0	0
normalized size	1	1.	1.08	0.	0.	40.7	0.	0.
time (sec)	N/A	0.122	0.962	0.201	0.	9.05	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	90	43	0	1613	0	0
normalized size	1	1.	1.76	0.84	0.	31.63	0.	0.
time (sec)	N/A	0.054	0.141	0.03	0.	8.722	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	195	0	0	22756	0	0
normalized size	1	1.	1.84	0.	0.	214.68	0.	0.
time (sec)	N/A	0.175	2.22	0.181	0.	7.566	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	518	0	0	0	0	0
normalized size	1	1.	2.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.327	20.391	0.201	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	180.002	0.195	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	7.717	0.194	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	539	0	0	0	0	0
normalized size	1	1.	2.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	18.313	0.196	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	167	0	0	7063	0	0
normalized size	1	1.	1.13	0.	0.	47.72	0.	0.
time (sec)	N/A	0.165	4.432	0.22	0.	10.507	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	2434	0	0
normalized size	1	1.	1.41	0.	0.	30.81	0.	0.
time (sec)	N/A	0.108	0.624	0.305	0.	9.95	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	73	26	0	1492	0	0
normalized size	1	1.	2.35	0.84	0.	48.13	0.	0.
time (sec)	N/A	0.047	0.13	0.034	0.	9.529	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	230	0	0	23317	0	0
normalized size	1	1.	2.17	0.	0.	219.97	0.	0.
time (sec)	N/A	0.148	3.011	0.199	0.	12.038	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	902	0	0	0	0	0
normalized size	1	1.	3.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.297	7.281	0.404	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	610	610	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.791	180.001	0.342	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.258	180.001	0.199	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.626	0.	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.436	86.492	0.215	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	155	0	0	9172	0	0
normalized size	1	1.	1.05	0.	0.	61.97	0.	0.
time (sec)	N/A	0.191	3.077	0.174	0.	12.53	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	103	0	0	2801	0	0
normalized size	1	1.	1.17	0.	0.	31.83	0.	0.
time (sec)	N/A	0.141	0.65	0.167	0.	11.793	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	79	46	0	2456	0	0
normalized size	1	1.	1.46	0.85	0.	45.48	0.	0.
time (sec)	N/A	0.062	0.243	0.018	0.	10.067	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	904	0	0	0	0	0
normalized size	1	1.	6.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	7.319	0.154	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	996	0	0	0	0	0
normalized size	1	1.	3.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.427	7.523	0.167	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.366	180.004	0.171	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.42	180.003	0.148	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	347	347	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	81.578	0.142	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	665	665	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.983	104.747	0.166	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	84	91	521	1516	0	86
normalized size	1	1.	0.44	0.48	2.73	7.94	0.	0.45
time (sec)	N/A	0.282	0.088	0.228	1.132	3.883	0.	1.138

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	72	80	282	797	0	69
normalized size	1	1.	0.51	0.57	2.	5.65	0.	0.49
time (sec)	N/A	0.167	0.066	0.174	1.151	3.124	0.	1.126

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	44	69	113	302	0	51
normalized size	1	1.	0.79	1.23	2.02	5.39	0.	0.91
time (sec)	N/A	0.113	0.056	0.174	1.092	3.51	0.	1.122

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	66	28	97	0	27
normalized size	1	1.	0.95	1.5	0.64	2.2	0.	0.61
time (sec)	N/A	0.087	0.037	0.207	1.679	3.225	0.	1.138

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	39	163	0	45
normalized size	1	1.	0.65	1.43	0.53	2.2	0.	0.61
time (sec)	N/A	0.113	0.049	0.188	1.121	2.956	0.	1.127

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	78	216	100	323	0	111
normalized size	1	1.	0.48	1.33	0.62	1.99	0.	0.69
time (sec)	N/A	0.155	0.064	0.19	1.152	3.32	0.	1.127

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	151	562	0	149
normalized size	1	1.	0.42	1.3	0.6	2.25	0.	0.6
time (sec)	N/A	0.196	0.103	0.204	1.179	3.181	0.	1.122

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	77	130	0	0	0	0
normalized size	1	1.	0.71	1.2	0.	0.	0.	0.
time (sec)	N/A	0.087	0.17	0.075	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	39	41	103	0	0
normalized size	1	1.	1.57	1.39	1.46	3.68	0.	0.
time (sec)	N/A	0.042	0.047	0.035	1.811	3.024	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	65	134	0	0	0	0
normalized size	1	1.	0.32	0.66	0.	0.	0.	0.
time (sec)	N/A	0.13	0.112	0.041	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	77	97	0	194	0	0
normalized size	1	1.	1.15	1.45	0.	2.9	0.	0.
time (sec)	N/A	0.055	0.138	0.053	0.	3.01	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	58	114	0	0	0	0
normalized size	1	1.	0.67	1.31	0.	0.	0.	0.
time (sec)	N/A	0.06	0.096	0.035	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	0	213	0	0
normalized size	1	1.	1.31	0.	0.	3.61	0.	0.
time (sec)	N/A	0.033	0.089	0.033	0.	3.004	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	167	0	0	0	0
normalized size	1	1.	1.	4.64	0.	0.	0.	0.
time (sec)	N/A	0.029	0.055	0.417	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	0	124	0	0
normalized size	1	1.	1.38	0.	0.	3.1	0.	0.
time (sec)	N/A	0.045	0.118	0.04	0.	3.041	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	59	134	0	0	0	0
normalized size	1	1.	0.43	0.98	0.	0.	0.	0.
time (sec)	N/A	0.099	0.118	0.04	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	33	38	57	81	0	0
normalized size	1	1.	1.43	1.65	2.48	3.52	0.	0.
time (sec)	N/A	0.04	0.038	0.037	1.602	2.966	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	65	117	0	0	0	0
normalized size	1	1.	0.81	1.46	0.	0.	0.	0.
time (sec)	N/A	0.07	0.098	0.042	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	98	121	0	242	0	0
normalized size	1	1.	0.8	0.99	0.	1.98	0.	0.
time (sec)	N/A	0.075	0.185	0.039	0.	3.251	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	77	138	0	0	0	0
normalized size	1	1.	0.55	0.98	0.	0.	0.	0.
time (sec)	N/A	0.098	0.177	0.036	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	47	41	122	0	0
normalized size	1	1.	1.57	1.68	1.46	4.36	0.	0.
time (sec)	N/A	0.043	0.054	0.033	1.907	2.926	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	65	147	0	0	0	0
normalized size	1	1.	0.26	0.59	0.	0.	0.	0.
time (sec)	N/A	0.149	0.118	0.037	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	113	0	220	0	0
normalized size	1	1.	0.98	1.23	0.	2.39	0.	0.
time (sec)	N/A	0.066	0.167	0.036	0.	3.104	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	61	129	0	0	0	0
normalized size	1	1.	0.55	1.16	0.	0.	0.	0.
time (sec)	N/A	0.083	0.106	0.033	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	234	0	0
normalized size	1	1.	1.	0.	0.	2.66	0.	0.
time (sec)	N/A	0.067	0.168	0.033	0.	3.1	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	65	159	0	0	0	0
normalized size	1	1.	0.3	0.74	0.	0.	0.	0.
time (sec)	N/A	0.116	0.115	0.039	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	64	131	0	227	0	0
normalized size	1	1.	0.7	1.42	0.	2.47	0.	0.
time (sec)	N/A	0.039	0.086	0.037	0.	3.321	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	45	127	0	0	0	0
normalized size	1	1.	0.8	2.27	0.	0.	0.	0.
time (sec)	N/A	0.036	0.101	0.314	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	32	0	53	58	0	0
normalized size	1	1.	1.28	0.	2.12	2.32	0.	0.
time (sec)	N/A	0.04	0.035	0.036	1.51	3.135	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	65	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.108	0.03	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	0	0	194	0	0
normalized size	1	1.	0.77	0.	0.	2.94	0.	0.
time (sec)	N/A	0.057	0.105	0.036	0.	3.066	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.147	0.084	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	126	0	0	0	0	0
normalized size	1	1.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	5.41	1.099	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	101	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.886	1.303	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	192	0	0	0	0	0
normalized size	1	1.	2.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	13.402	0.105	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	40	139	29	509	130	603	0	0
normalized size	1	3.48	0.72	12.72	3.25	15.08	0.	0.
time (sec)	N/A	0.135	0.287	0.275	2.132	2.927	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	62	0	100	104	0	51
normalized size	1	1.	2.48	0.	4.	4.16	0.	2.04
time (sec)	N/A	0.039	0.116	0.051	1.104	2.876	0.	1.134

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	64	0	66	107	0	50
normalized size	1	1.	2.56	0.	2.64	4.28	0.	2.
time (sec)	N/A	0.046	0.096	0.066	1.06	2.851	0.	1.112

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	1322	0	0
normalized size	1	1.	0.64	0.	0.	14.85	0.	0.
time (sec)	N/A	0.089	0.715	0.165	0.	3.206	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	0	0	1343	0	0
normalized size	1	1.	0.95	0.	0.	20.66	0.	0.
time (sec)	N/A	0.076	0.817	0.119	0.	3.123	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	26	112	0	36
normalized size	1	1.	1.	1.05	1.37	5.89	0.	1.89
time (sec)	N/A	0.016	0.048	0.005	1.07	3.132	0.	1.129

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	38	219	0	38
normalized size	1	1.	1.	1.06	2.11	12.17	0.	2.11
time (sec)	N/A	0.028	0.059	0.013	1.202	3.036	0.	1.153

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	51	0	1486	0	155
normalized size	1	1.	1.	0.93	0.	27.02	0.	2.82
time (sec)	N/A	0.04	0.055	0.02	0.	3.192	0.	1.172

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	123	869	0	63
normalized size	1	1.	1.	0.86	2.93	20.69	0.	1.5
time (sec)	N/A	0.034	0.048	0.017	1.202	3.059	0.	1.17

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	75	84	0	4298	0	205
normalized size	1	1.	0.84	0.94	0.	48.29	0.	2.3
time (sec)	N/A	0.057	0.089	0.022	0.	3.311	0.	1.18

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	295	0	0	0	0
normalized size	1	1.	0.76	3.04	0.	0.	0.	0.
time (sec)	N/A	0.061	0.162	0.47	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	72	141	0	0	0	0
normalized size	1	1.	0.77	1.52	0.	0.	0.	0.
time (sec)	N/A	0.072	0.088	0.414	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	0	0	0
normalized size	1	1.	1.	3.16	0.	0.	0.	0.
time (sec)	N/A	0.069	0.066	0.268	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	0	0	0
normalized size	1	1.	1.	3.16	0.	0.	0.	0.
time (sec)	N/A	0.056	0.07	0.297	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	237	0	0	0	0
normalized size	1	1.	0.78	2.44	0.	0.	0.	0.
time (sec)	N/A	0.073	0.111	0.287	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	256	0	0	0	0
normalized size	1	1.	0.9	2.64	0.	0.	0.	0.
time (sec)	N/A	0.07	0.129	0.345	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [177] had the largest ratio of [0.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	2	1	1.	8	0.125
5	A	3	2	1.	8	0.25
6	A	2	1	1.	8	0.125
7	A	2	1	1.	6	0.167
8	A	2	1	1.	6	0.167
9	A	3	3	1.	10	0.3
10	A	3	3	1.	10	0.3
11	A	2	2	1.	10	0.2
12	A	2	2	1.	10	0.2
13	A	3	3	1.	10	0.3
14	A	3	3	1.	10	0.3
15	A	4	3	1.	12	0.25
16	A	3	3	1.	12	0.25
17	A	3	3	1.	12	0.25
18	A	2	2	1.	12	0.167
19	A	2	2	1.	12	0.167
20	A	3	3	1.	12	0.25

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	3	1.	12	0.25
22	A	4	3	1.	12	0.25
23	A	2	2	1.	10	0.2
24	A	5	3	1.	12	0.25
25	A	4	3	1.	12	0.25
26	A	3	3	1.	12	0.25
27	A	2	2	1.	12	0.167
28	A	2	2	1.	12	0.167
29	A	3	3	1.	12	0.25
30	A	4	3	1.	12	0.25
31	A	5	3	1.	12	0.25
32	A	5	4	1.	10	0.4
33	A	4	4	1.	10	0.4
34	A	3	3	1.	10	0.3
35	A	2	2	1.	10	0.2
36	A	3	3	1.	10	0.3
37	A	4	3	1.	10	0.3
38	A	5	3	1.	10	0.3
39	A	7	4	1.	10	0.4
40	A	5	4	1.	10	0.4
41	A	4	4	1.	10	0.4
42	A	4	4	1.	10	0.4
43	A	5	4	1.	10	0.4
44	A	7	4	1.	10	0.4
45	A	3	2	1.	10	0.2
46	A	3	2	1.	10	0.2
47	A	3	2	1.	10	0.2
48	A	3	3	1.	10	0.3
49	A	3	3	1.	10	0.3
50	A	5	3	1.	10	0.3
51	A	7	3	1.	10	0.3
52	A	7	7	1.	13	0.538
53	A	6	5	1.	13	0.385
54	A	5	5	1.	13	0.385
55	A	5	4	1.	11	0.364
56	A	6	6	1.	11	0.546
57	A	6	5	1.	13	0.385
58	A	7	7	1.	13	0.538
59	A	7	6	1.	13	0.462
60	A	6	5	1.	13	0.385
61	A	5	4	1.	13	0.308
62	A	5	5	1.	13	0.385
63	A	5	4	1.	11	0.364
64	A	4	3	1.	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
65	A	5	5	1.	13	0.385
66	A	6	5	1.	13	0.385
67	A	6	5	1.	13	0.385
68	A	7	5	1.	13	0.385
69	A	6	5	1.	13	0.385
70	A	5	5	1.	13	0.385
71	A	4	4	1.	11	0.364
72	A	1	1	1.	11	0.091
73	A	3	3	1.	13	0.231
74	A	4	4	1.	13	0.308
75	A	6	6	1.	13	0.462
76	A	2	2	1.	12	0.167
77	A	2	2	1.	13	0.154
78	A	5	5	1.	14	0.357
79	A	4	4	1.	14	0.286
80	A	2	2	1.	14	0.143
81	A	5	4	1.	14	0.286
82	A	6	5	1.	14	0.357
83	A	2	2	1.	15	0.133
84	A	5	4	1.	15	0.267
85	A	2	2	1.	10	0.2
86	A	2	2	1.	10	0.2
87	A	6	5	1.	12	0.417
88	A	5	4	1.	12	0.333
89	A	4	4	1.	12	0.333
90	A	2	1	1.	10	0.1
91	A	3	3	1.	12	0.25
92	A	5	5	1.	12	0.417
93	A	6	6	1.	12	0.5
94	A	1	1	1.	14	0.071
95	A	8	6	1.	13	0.462
96	A	7	6	1.	13	0.462
97	A	6	6	1.	13	0.462
98	A	5	5	1.	11	0.454
99	A	3	3	1.	11	0.273
100	A	5	5	1.	13	0.385
101	A	6	6	1.	13	0.462
102	A	7	7	1.	13	0.538
103	A	5	3	1.	13	0.231
104	A	3	2	1.	13	0.154
105	A	4	3	1.	13	0.231
106	A	3	2	1.	13	0.154
107	A	3	2	1.	13	0.154
108	A	2	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	3	2	1.	11	0.182
110	A	4	3	1.	13	0.231
111	A	3	2	1.	13	0.154
112	A	5	3	1.	13	0.231
113	A	3	2	1.	13	0.154
114	A	15	8	1.	13	0.615
115	A	3	2	1.	13	0.154
116	A	6	6	1.	13	0.462
117	A	3	2	1.	13	0.154
118	A	7	7	1.	13	0.538
119	A	4	4	1.	11	0.364
120	A	3	2	1.	11	0.182
121	A	9	8	1.	13	0.615
122	A	3	2	1.	13	0.154
123	A	15	8	1.	13	0.615
124	A	3	2	1.	13	0.154
125	A	5	4	1.	23	0.174
126	A	5	4	1.	23	0.174
127	A	4	4	1.	21	0.19
128	A	7	5	1.	21	0.238
129	A	13	9	1.	23	0.391
130	A	7	7	1.	23	0.304
131	A	1	1	1.	14	0.071
132	A	5	4	1.	23	0.174
133	A	5	4	1.	23	0.174
134	A	5	4	1.	23	0.174
135	A	3	3	1.	21	0.143
136	A	7	5	1.	21	0.238
137	A	11	6	1.	23	0.261
138	A	11	8	1.	23	0.348
139	A	6	6	1.	23	0.261
140	A	1	1	1.	14	0.071
141	A	9	8	1.	23	0.348
142	A	5	4	1.	23	0.174
143	A	5	4	1.	23	0.174
144	A	4	4	1.	21	0.19
145	A	7	4	1.	21	0.19
146	A	11	5	1.	23	0.217
147	A	17	11	1.	23	0.478
148	A	7	7	1.	23	0.304
149	A	6	6	1.	14	0.429
150	A	14	11	1.	23	0.478
151	A	6	5	1.	25	0.2
152	A	6	5	1.	25	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	4	4	1.	25	0.16
154	A	4	4	1.	25	0.16
155	A	5	4	1.	25	0.16
156	A	6	5	1.	25	0.2
157	A	6	5	1.	25	0.2
158	A	6	6	1.	15	0.4
159	A	3	3	1.	15	0.2
160	A	8	8	1.	15	0.533
161	A	6	6	1.	15	0.4
162	A	5	5	1.	13	0.385
163	A	6	6	1.	11	0.546
164	A	3	2	1.	15	0.133
165	A	5	5	1.	15	0.333
166	A	6	6	1.	15	0.4
167	A	3	3	1.	15	0.2
168	A	5	5	1.	15	0.333
169	A	8	7	1.	15	0.467
170	A	7	6	1.	15	0.4
171	A	3	3	1.	15	0.2
172	A	9	8	1.	15	0.533
173	A	7	6	1.	15	0.4
174	A	6	5	1.	15	0.333
175	A	7	6	1.	15	0.4
176	A	8	7	1.	13	0.538
177	A	7	7	1.	11	0.636
178	A	4	3	1.	15	0.2
179	A	3	3	1.	15	0.2
180	A	5	5	1.	15	0.333
181	A	6	6	1.	15	0.4
182	A	4	4	1.	11	0.364
183	A	4	4	1.	13	0.308
184	A	4	4	1.	13	0.308
185	A	4	4	1.	13	0.308
186	C	9	4	3.48	44	0.091
187	A	3	3	1.	15	0.2
188	A	4	4	1.	15	0.267
189	A	3	3	1.	20	0.15
190	A	3	3	1.	21	0.143
191	A	2	1	1.	15	0.067
192	A	3	2	1.	17	0.118
193	A	3	2	1.	17	0.118
194	A	3	1	1.	17	0.059
195	A	4	2	1.	17	0.118
196	A	4	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	4	3	1.	19	0.158
198	A	3	2	1.	19	0.105
199	A	3	2	1.	19	0.105
200	A	4	3	1.	19	0.158
201	A	4	3	1.	19	0.158

Chapter 3

Listing of integrals

3.1 $\int \operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0050338, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(a+bx)\,dx=\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.0020975, size = 11, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a),x)
```

```
[Out] arctan(sinh(b*x+a))/b
```

Maxima [A] time = 1.0037, size = 15, normalized size = 1.36

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*x + a))/b
```

Fricas [A] time = 2.00671, size = 58, normalized size = 5.27

$$\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x)
```

```
[Out] Integral(sech(a + b*x), x)
```

Giac [A] time = 1.09724, size = 16, normalized size = 1.45

$$\frac{2 \arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b
```

3.2 $\int \operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tanh(a+bx)}{b}$$

```
[Out] Tanh[a + b*x]/b
```

Rubi [A] time = 0.0100053, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767, 8}

$$\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]/b
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\,dx &= \frac{i\,\mathrm{Subst}\left(\int 1\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0028304, size = 10, normalized size = 1.

$$\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]/b
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\frac{\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2,x)
```

```
[Out] tanh(b*x+a)/b
```

Maxima [A] time = 0.991004, size = 24, normalized size = 2.4

$$\frac{2}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] time = 2.07124, size = 111, normalized size = 11.1

$$-\frac{2}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2,x)
```

```
[Out] Integral(sech(a + b*x)**2, x)
```

Giac [A] time = 1.13855, size = 24, normalized size = 2.4

$$-\frac{2}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) + 1))
```

3.3 $\int \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0156794, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(a+bx)\,dx &= \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0088326, size = 34, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] time = 0.01, size = 30, normalized size = 0.9

$$\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2b} + \frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3,x)
```

```
[Out] 1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x+a))/b
```

Maxima [B] time = 1.49696, size = 88, normalized size = 2.59

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 1.99584, size = 757, normalized size = 22.26

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \right.}{b\cosh(bx+a)^4 + 4\,b\cosh(bx+a)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3,x)
```

```
[Out] Integral(sech(a + b*x)**3, x)
```

Giac [B] time = 1.13166, size = 107, normalized size = 3.15

$$\frac{\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2 bx+2 a)} - 1\right)e^{(-bx-a)}\right)}{4 b} + \frac{e^{(bx+a)} - e^{(-bx-a)}}{\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + (e^(b*x + a
) - e^(-b*x - a))/(((e^(b*x + a) - e^(-b*x - a))^2 + 4)*b)
```

3.4 $\int \operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.011633, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4,x]
```

```
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0064148, size = 26, normalized size = 1.

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4,x]
```

```
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.012, size = 23, normalized size = 0.9

$$\frac{\tanh(bx+a)}{b}\left(\frac{2}{3}+\frac{(\operatorname{sech}(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4,x)
```

```
[Out] 1/b*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)
```

Maxima [B] time = 1.01786, size = 122, normalized size = 4.69

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}+\frac{4}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1)) + 4/3/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1))
```

Fricas [B] time = 1.96322, size = 448, normalized size = 17.23

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}+3\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx+a\right)^{2}+3\,l\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 + 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 4*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 + 9*b*cosh(b*
x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4,x)
```

```
[Out] Integral(sech(a + b*x)**4, x)
```

Giac [A] time = 1.15398, size = 42, normalized size = 1.62

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

3.5 $\int \text{sech}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rubi [A] time = 0.027885, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^5(a+bx)\,dx &= \frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{4}\int \text{sech}^3(a+bx)\,dx\\ &= \frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{8}\int \text{sech}(a+bx)\,dx\\ &= \frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0405043, size = 47, normalized size = 0.85

$$\frac{3\tan^{-1}(\sinh(a+bx))+2\tanh(a+bx)\text{sech}^3(a+bx)+3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]] + 3*Sech[a + b*x]*Tanh[a + b*x] + 2*Sech[a + b*x]^
3*Tanh[a + b*x])/(8*b)
```

Maple [A] time = 0.01, size = 50, normalized size = 0.9

$$\frac{(\operatorname{sech}(bx+a))^3 \tanh(bx+a)}{4b} + \frac{3\operatorname{sech}(bx+a)\tanh(bx+a)}{8b} + \frac{3\arctan\left(e^{bx+a}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^5,x)
```

```
[Out] 1/4*sech(b*x+a)^3*tanh(b*x+a)/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+3/4*arctan(ex
p(b*x+a))/b
```

Maxima [B] time = 1.51204, size = 151, normalized size = 2.75

$$-\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b} + \frac{3e^{(-bx-a)} + 11e^{(-3bx-3a)} - 11e^{(-5bx-5a)} - 3e^{(-7bx-7a)}}{4b\left(4e^{(-2bx-2a)} + 6e^{(-4bx-4a)} + 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] time = 2.14876, size = 2263, normalized size = 41.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
```

```
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**5,x)
```

```
[Out] Integral(sech(a + b*x)**5, x)
```

Giac [B] time = 1.1439, size = 140, normalized size = 2.55

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)\right)}{16\,b}+\frac{3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20\,e^{(bx+a)}-20\,e^{(-bx-a)}}{4\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + 1/4*(3*(e^
(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-b*x - a))/(((e^(b*x +
 a) - e^(-b*x - a))^2 + 4)^2*b)
```

3.6 $\int \operatorname{sech}^6(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0147582, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0105993, size = 41, normalized size = 1.

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Maple [A] time = 0.013, size = 33, normalized size = 0.8

$$\frac{\tanh(bx+a)}{b}\left(\frac{8}{15}+\frac{(\operatorname{sech}(bx+a))^4}{5}+\frac{4\,(\operatorname{sech}(bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^6,x)
```

```
[Out] 1/b*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b*x+a)^2)*tanh(b*x+a)
```

Maxima [B] time = 1.01809, size = 277, normalized size = 6.76

$$\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)}+\frac{32\,e^{(-4}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 32/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15/(b*(5*e^(-2*b*x
- 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e
^(-10*b*x - 10*a) + 1))
```

Fricas [B] time = 2.09174, size = 954, normalized size = 23.27

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}+5\,b\cosh\left(bx+a\right)^{6}+\left(28\,b\cosh\left(bx+a\right)^{2}+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 + 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 + 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 + 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 + 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 + 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 +
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 + 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{6}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**6,x)
```

```
[Out] Integral(sech(a + b*x)**6, x)
```

Giac [A] time = 1.12395, size = 57, normalized size = 1.39

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1
)^5)
```

3.7 $\int \operatorname{sech}^4(7x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rubi [A] time = 0.0096991, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767}

$$\frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(7x)\,dx &= \frac{1}{7}i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(7x)\right)\\ &= \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)\end{aligned}$$

Mathematica [A] time = 0.0036122, size = 19, normalized size = 1.

$$\frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$\frac{\tanh(7x)}{7}\left(\frac{2}{3} + \frac{(\operatorname{sech}(7x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(7*x)^4,x)
```

```
[Out] 1/7*(2/3+1/3*sech(7*x)^2)*tanh(7*x)
```

Maxima [B] time = 1.04548, size = 66, normalized size = 3.47

$$\frac{4\,e^{(-14\,x)}}{7\left(3\,e^{(-14\,x)}+3\,e^{(-28\,x)}+e^{(-42\,x)}+1\right)}+\frac{4}{21\left(3\,e^{(-14\,x)}+3\,e^{(-28\,x)}+e^{(-42\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="maxima")
```

```
[Out] 4/7*e^(-14*x)/(3*e^(-14*x) + 3*e^(-28*x) + e^(-42*x) + 1) + 4/21/(3*e^(-14*
x) + 3*e^(-28*x) + e^(-42*x) + 1)
```

Fricas [B] time = 2.10968, size = 331, normalized size = 17.42

$$-\frac{8\,(2\,\cosh\left(7\,x\right)+\sinh(7}{21\left(\cosh\left(7\,x\right)^{5}+5\,\cosh\left(7\,x\right)\sinh\left(7\,x\right)^{4}+\sinh\left(7\,x\right)^{5}+\left(10\,\cosh\left(7\,x\right)^{2}+3\right)\sinh\left(7\,x\right)^{3}+3\,\cosh\left(7\,x\right)^{3}+\left(10\,\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="fricas")
```

```
[Out] -8/21*(2*cosh(7*x) + sinh(7*x))/(cosh(7*x)^5 + 5*cosh(7*x)*sinh(7*x)^4 + si
nh(7*x)^5 + (10*cosh(7*x)^2 + 3)*sinh(7*x)^3 + 3*cosh(7*x)^3 + (10*cosh(7*x
)^3 + 9*cosh(7*x))*sinh(7*x)^2 + (5*cosh(7*x)^4 + 9*cosh(7*x)^2 + 2)*sinh(7
*x) + 4*cosh(7*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{4}(7x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)**4,x)
```

```
[Out] Integral(sech(7*x)**4, x)
```

Giac [A] time = 1.13372, size = 24, normalized size = 1.26

$$-\frac{4\left(3\,e^{(14\,x)}+1\right)}{21\left(e^{(14\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="giac")
```

```
[Out] -4/21*(3*e^(14*x) + 1)/(e^(14*x) + 1)^3
```

3.8 $\int \mathbf{sech}^6(\pi x)\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rubi [A] time = 0.013558, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767}

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(\pi x)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(\pi x)\right)}{\pi}\\ &= \frac{\tanh(\pi x)}{\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh^5(\pi x)}{5\pi}\end{aligned}$$

Mathematica [A] time = 0.0040945, size = 35, normalized size = 1.

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Maple [A] time = 0.009, size = 27, normalized size = 0.8

$$\frac{\tanh(\pi x)}{\pi}\left(\frac{8}{15}+\frac{(\operatorname{sech}(\pi x))^4}{5}+\frac{4\,(\operatorname{sech}(\pi x))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(Pi*x)^6,x)
```

```
[Out] 1/Pi*(8/15+1/5*sech(Pi*x)^4+4/15*sech(Pi*x)^2)*tanh(Pi*x)
```

Maxima [B] time = 1.02779, size = 185, normalized size = 5.29

$$\frac{16\,e^{(-2\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1\right)} + \frac{32\,e^{(-4\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*pi*x)/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e
^(-8*pi*x) + e^(-10*pi*x) + 1)) + 32/3*e^(-4*pi*x)/(pi*(5*e^(-2*pi*x) + 10*
e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 16/15/(
pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-1
0*pi*x) + 1))
```

Fricas [B] time = 2.12901, size = 859, normalized size = 24.54

$$-\frac{}{15\left(5\,\pi + \pi\cosh\left(\pi x\right)^{8} + 8\,\pi\cosh\left(\pi x\right)\sinh\left(\pi x\right)^{7} + \pi\sinh\left(\pi x\right)^{8} + 5\,\pi\cosh\left(\pi x\right)^{6} + \left(5\,\pi + 28\,\pi\cosh\left(\pi x\right)^{2}\right)\sinh\left(\pi x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(pi*x)^2 + 18*cosh(pi*x)*sinh(pi*x) + 11*sinh(pi*x)^2 + 5)/(
5*pi + pi*cosh(pi*x)^8 + 8*pi*cosh(pi*x)*sinh(pi*x)^7 + pi*sinh(pi*x)^8 + 5
*pi*cosh(pi*x)^6 + (5*pi + 28*pi*cosh(pi*x)^2)*sinh(pi*x)^6 + 2*(28*pi*cosh
(pi*x)^3 + 15*pi*cosh(pi*x))*sinh(pi*x)^5 + 10*pi*cosh(pi*x)^4 + 5*(2*pi +
14*pi*cosh(pi*x)^4 + 15*pi*cosh(pi*x)^2)*sinh(pi*x)^4 + 4*(14*pi*cosh(pi*x)
^5 + 25*pi*cosh(pi*x)^3 + 10*pi*cosh(pi*x))*sinh(pi*x)^3 + 11*pi*cosh(pi*x)
^2 + (11*pi + 28*pi*cosh(pi*x)^6 + 75*pi*cosh(pi*x)^4 + 60*pi*cosh(pi*x)^2)
*sinh(pi*x)^2 + 2*(4*pi*cosh(pi*x)^7 + 15*pi*cosh(pi*x)^5 + 20*pi*cosh(pi*x
)^3 + 9*pi*cosh(pi*x))*sinh(pi*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{6}\left(\pi x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)**6,x)
```

```
[Out] Integral(sech(pi*x)**6, x)
```

Giac [A] time = 1.14291, size = 41, normalized size = 1.17

$$-\frac{16\left(10\,e^{(4\,\pi x)}+5\,e^{(2\,\pi x)}+1\right)}{15\,\pi\left(e^{(2\,\pi x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*pi*x) + 5*e^(2*pi*x) + 1)/(pi*(e^(2*pi*x) + 1)^5)
```

3.9 $\int \mathbf{sech}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Rubi [A] time = 0.031515, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2641}

$$\frac{2\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}+\frac{1}{3}\int \sqrt{\text{sech}(a+bx)}\,dx\\ &= \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}+\frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{3b}+\frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0745525, size = 51, normalized size = 0.77

$$\frac{2\text{sech}^{\frac{3}{2}}(a+bx)\left(\sinh(a+bx)-i\cosh^{\frac{3}{2}}(a+bx)\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*((-I)*Cosh[a + b*x]^(3/2)*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[a + b*x]))/(3*b)
```

Maple [B] time = 0.276, size = 217, normalized size = 3.3

$$\frac{2}{3b}\left(2\sqrt{-(\sinh(1/2\,bx+a/2))^2}\sqrt{-2\,(\sinh(1/2\,bx+a/2))^2-1}\text{EllipticF}\left(\cosh(1/2\,bx+a/2),\sqrt{2}\right)(\sinh(1/2\,bx+a/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1
/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/
2*a),2^(1/2))+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{sech}(bx+a)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(sech(b*x + a)^(5/2), x)
```

3.10 $\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] ((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]
])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0300481, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2639}

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]
])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}-\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx\\ &= \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}-\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx\\ &= \frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0428349, size = 49, normalized size = 0.79

$$\frac{2\sqrt{\mathrm{sech}(a+bx)}\left(\sinh(a+bx)+i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sech[a + b*x]]*(I*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]
 + Sinh[a + b*x]))/b
```

Maple [A] time = 0.287, size = 103, normalized size = 1.7

$$2\,\frac{\mathrm{EllipticE}\left(\cosh(1/2\,bx+a/2),\sqrt{2}\right)\sqrt{-(\sinh(1/2\,bx+a/2))^2}\sqrt{-2\,(\sinh(1/2\,bx+a/2))^2-1}+2\,\cosh(1/2\,bx+a/2)\,(\mathrm{si}}{\sinh(1/2\,bx+a/2)\sqrt{2\,(\cosh(1/2\,bx+a/2))^2-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(3/2),x)
```

```
[Out] 2*(EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-
2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^
2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{sech}\,(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{sech}\,(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

3.11 $\int \sqrt{\text{sech}(a+bx)}\,dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rubi [A] time = 0.0199072, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\text{sech}(a+bx)}\,dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0292461, size = 40, normalized size = 1.

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Maple [B] time = 0.241, size = 135, normalized size = 3.4

$$2\,{\frac{\sqrt{\left(2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{Ellipt}}{\sqrt{2\,\left(\sinh\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sinh\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cosh\left(1/2\,bx+a/2\right)\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{sech}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\operatorname{sech}\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{sech}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{sech}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(b*x + a)), x)
```

3.12 $\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rubi [A] time = 0.0198356, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2639}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0342797, size = 40, normalized size = 1.

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/(b*Sqrt[Cosh[a + b*x]]*Sqrt[Sech[a +
 b*x]])
```

Maple [B] time = 0.245, size = 135, normalized size = 3.4

$$-2\,\frac{\sqrt{\left(2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{Elliptic}}{\sqrt{2\,\left(\sinh\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sinh\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(1/2),x)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sech(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

3.13 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Rubi [A] time = 0.0316396, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx &= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}\, dx\\ &= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}} dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0447442, size = 53, normalized size = 0.8

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(\sinh(2(a+bx))-2i\sqrt{\cosh(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [B] time = 0.277, size = 174, normalized size = 2.6

$$\frac{2}{3b}\sqrt{\left(2\left(\cosh(1/2\,bx+a/2)\right)^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\left(\cosh(1/2\,bx+a/2)\right)^5-6\left(\cosh(1/2\,bx+a/2)\right)^3+\sqrt{-\left(\sinh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(3/2),x)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

3.14 $\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)} \, dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{6i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Rubi [A] time = 0.0329784, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{6i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)} \, dx &= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx \\
&= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{1}{5}\left(3\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)} \, dx \\
&= -\frac{6i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b} + \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0697306, size = 59, normalized size = 0.89

$$\frac{\sqrt{\text{sech}(a+bx)}\left(\sinh(a+bx)+\sinh(3(a+bx))-12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*((-12*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x)
, 2] + Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(10*b)
```

Maple [B] time = 0.309, size = 188, normalized size = 2.9

$$\frac{2}{5b}\sqrt{\left(2\left(\cosh(1/2\,bx+a/2)\right)^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(8\left(\cosh(1/2\,bx+a/2)\right)^7-16\left(\cosh(1/2\,bx+a/2)\right)^5+10\left(\cosh(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(5/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\text{sech}\,(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\text{sech}\,(bx+a)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(-5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(5/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

3.15 $\int (b\,\mathbf{sech}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=102

$$\frac{6b^3\sinh(c+dx)\sqrt{b\,\mathrm{sech}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}+\frac{2b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{5/2}}{5d}$$

```
[Out] (((6*I)/5)*b^4*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b
*Sech[c + d*x]]) + (6*b^3*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/(5*d) + (2*b
*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d)
```

Rubi [A] time = 0.0602533, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{6b^3\sinh(c+dx)\sqrt{b\,\mathrm{sech}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}+\frac{2b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (((6*I)/5)*b^4*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b
*Sech[c + d*x]]) + (6*b^3*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/(5*d) + (2*b
*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\text{sech}(c+dx))^{7/2}\,dx &= \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\text{sech}(c+dx))^{3/2}\,dx \\ &= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{1}{5}\left(3b^4\right)\int \frac{}{\sqrt{b\text{sech}}} \\ &= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{\left(3b^4\right)\int \sqrt{\cosh}}{5\sqrt{\cosh(c+dx)}\sqrt{}} \\ &= \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5}}{5d}\end{aligned}$$

Mathematica [A] time = 0.196679, size = 68, normalized size = 0.67

$$\frac{b^2(b\text{sech}(c+dx))^{3/2}\left(3\sinh(2(c+dx)) + 2\tanh(c+dx) + 6i\cosh^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (b^2*(b*Sech[c + d*x])^(3/2)*((6*I)*Cosh[c + d*x]^(3/2)*EllipticE[(I/2)*(c
+ d*x), 2] + 3*Sinh[2*(c + d*x)] + 2*Tanh[c + d*x]))/(5*d)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (b\text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(7/2),x)
```

```
[Out] int((b*sech(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}b^3\,\text{sech}\,(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))*b^3*sech(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

3.16 $\int (b\,\mathbf{sech}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=74

$$\frac{2b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{3/2}}{3d}-\frac{2ib^2\sqrt{\cosh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{3d}$$

```
[Out] (((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Se
ch[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Rubi [A] time = 0.0381697, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2641}

$$\frac{2b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{3/2}}{3d}-\frac{2ib^2\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Se
ch[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\,\mathrm{sech}(c+dx))^{5/2}\,dx &= \frac{2b(b\,\mathrm{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}+\frac{1}{3}b^2\int\sqrt{b\,\mathrm{sech}(c+dx)}\,dx\\ &= \frac{2b(b\,\mathrm{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}+\frac{1}{3}\left(b^2\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}\right)\int\frac{1}{\sqrt{\cosh(c+dx)}}\\ &= -\frac{2ib^2\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{3d}+\frac{2b(b\,\mathrm{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0695537, size = 56, normalized size = 0.76

$$\frac{2b^2\sqrt{b\text{sech}(c+dx)}\left(\tanh(c+dx)-i\sqrt{\cosh(c+dx)}\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sech[c + d*x]]*((-I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c +
 d*x), 2] + Tanh[c + d*x]))/(3*d)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (b\text{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(5/2),x)
```

```
[Out] int((b*sech(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}b^2\,\text{sech}\,(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))*b^2*sech(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{sech}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

3.17 $\int (b\,\mathbf{sech}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b\sinh(c+dx)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}$$

```
[Out] ((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sec
h[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0383941, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{2b\sinh(c+dx)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sec
h[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\,\mathrm{sech}(c+dx))^{3/2}\,dx&=\frac{2b\sqrt{b\,\mathrm{sech}(c+dx)}\sinh(c+dx)}{d}-b^2\int \frac{1}{\sqrt{b\,\mathrm{sech}(c+dx)}}\,dx\\&=\frac{2b\sqrt{b\,\mathrm{sech}(c+dx)}\sinh(c+dx)}{d}-\frac{b^2\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}\\&=\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}+\frac{2b\sqrt{b\,\mathrm{sech}(c+dx)}\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0393364, size = 52, normalized size = 0.74

$$\frac{2b\sqrt{b\text{sech}(c+dx)}\left(\sinh(c+dx)+i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sech[c + d*x]]*(I*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x)
, 2] + Sinh[c + d*x]))/d
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (b\text{sech}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(3/2),x)
```

```
[Out] int((b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}b\,\text{sech}\,(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))*b*sech(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{sech}(dx + c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

3.18 $\int \sqrt{b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=42

$$-\frac{2i\sqrt{\cosh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}$$

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Rubi [A] time = 0.0215461, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\,\mathrm{sech}(c+dx)}\,dx &= \left(\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0214048, size = 42, normalized size = 1.

$$-\frac{2i\sqrt{\cosh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \sqrt{b\text{sech}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(1/2),x)
```

```
[Out] int((b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

3.19 $\int \frac{1}{\sqrt{b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Rubi [A] time = 0.0220029, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\operatorname{sech}(c+dx)}}\,dx &= \frac{\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0295799, size = 42, normalized size = 1.

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Maple [B] time = 0.141, size = 244, normalized size = 5.8

$$\frac{\sqrt{2}}{d}\frac{1}{\sqrt{\frac{b e^{dx+c}}{\left(e^{dx+c}\right)^2+1}}}+\frac{\sqrt{2}}{d\left(\left(e^{dx+c}\right)^2+1\right)}\left(-2\frac{b\left(e^{dx+c}\right)^2+b}{b\sqrt{e^{dx+c}\left(b\left(e^{dx+c}\right)^2+b\right)}}+i\sqrt{2}\sqrt{-i\left(e^{dx+c}+i\right)}\sqrt{i\left(e^{dx+c}-i\right)}\sqrt{ie^{dx+c}}\left(-2\,i\mathrm{Elli}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2+1))^(1/2)+1/d*(-2*(b*exp(d*x+c)^2+b
)/b/(exp(d*x+c)*(b*exp(d*x+c)^2+b))^(1/2)+I*(-I*(exp(d*x+c)+I))^(1/2)*2^(1/
2)*(I*(exp(d*x+c)-I))^(1/2)*(I*exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3+b*exp(d*x+
c))^(1/2)*(-2*I*EllipticE((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))+I*Elliptic
F((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c
)^2+1))^(1/2)*(b*exp(d*x+c)*(exp(d*x+c)^2+1))^(1/2)/(exp(d*x+c)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\,\mathrm{sech}\,(dx+c)}}{b\,\mathrm{sech}\,(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b*sech(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

3.20 $\int \frac{1}{(b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c
 + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Rubi [A] time = 0.0388664, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c
 + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx &= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\int \sqrt{b\text{sech}(c+dx)}\,dx}{3b^2}\\ &= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx}{3b^2}\\ &= -\frac{2i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.063421, size = 63, normalized size = 0.83

$$\frac{\operatorname{sech}^2(c+dx)\left(\sinh(2(c+dx))-2i\sqrt{\cosh(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\right)}{3d(b\operatorname{sech}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Sech[c + d*x]^2*((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]
+ Sinh[2*(c + d*x)]))/(3*d*(b*Sech[c + d*x])^(3/2))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int (b\operatorname{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\operatorname{sech}(dx+c)}}{b^2\operatorname{sech}(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^2*sech(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

3.21 $\int \frac{1}{(b\,\mathbf{sech}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt[
b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Rubi [A] time = 0.0391326, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt[
b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx &= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx}{5b^2}\\ &= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \sqrt{\cosh(c+dx)}\,dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}\\ &= -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0808668, size = 64, normalized size = 0.84

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(\sinh(c+dx)+\sinh(3(c+dx))-12i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{10b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-12*I)*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*
x), 2] + Sinh[c + d*x] + Sinh[3*(c + d*x)]))/(10*b^3*d)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (b\text{sech}\,(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(5/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b^3\,\text{sech}\,(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^3*sech(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{sech}\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

3.22 $\int \frac{1}{(b\,\mathbf{sech}(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{10i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\,\text{sech}(c+dx)}}{21b^4d}+\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\,\text{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{7bd(b\,\text{sech}(c+dx))^{5/2}}$$

```
[Out] (((-10*I)/21)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech
[c + d*x]])/(b^4*d) + (2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (
10*Sinh[c + d*x])/(21*b^3*d*Sqrt[b*Sech[c + d*x]])
```

Rubi [A] time = 0.0581648, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\,\text{sech}(c+dx)}}-\frac{10i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)|2\right)\sqrt{b\,\text{sech}(c+dx)}}{21b^4d}+\frac{2\sinh(c+dx)}{7bd(b\,\text{sech}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (((-10*I)/21)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech
[c + d*x]])/(b^4*d) + (2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (
10*Sinh[c + d*x])/(21*b^3*d*Sqrt[b*Sech[c + d*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx}{7b^2}$$

$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{5\int\sqrt{b\text{sech}(c+dx)}\,dx}{21b^4}$$

$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{\left(5\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(\ldots}}}{21b^4}$$

$$= -\frac{10i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c\ldots}{21b^3d\sqrt{b\text{sec}\ldots}}$$

Mathematica [A] time = 0.126529, size = 70, normalized size = 0.67

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(-40i\sqrt{\cosh(c+dx)}\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+26\sinh(2(c+dx))+3\sinh(4(c+dx))\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-40*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*
x), 2] + 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^4*d)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (b\text{sech}\,(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(7/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b^4\,\text{sech}\,(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^4*sech(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

3.23 $\int (b\,\mathbf{sech}(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b\sinh(c+dx)(b\,\mathbf{sech}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech[
c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rubi [A] time = 0.0365971, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{b\sinh(c+dx)(b\,\mathbf{sech}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech[
c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\,\mathbf{sech}(c+dx))^n\,dx &= \left(\frac{\cosh(c+dx)}{b}\right)^n (b\,\mathbf{sech}(c+dx))^n \int \left(\frac{\cosh(c+dx)}{b}\right)^{-n} dx\\ &= -\frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)(b\,\mathbf{sech}(c+dx))^n\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0622459, size = 60, normalized size = 0.8

$$-\frac{\sqrt{\tanh^2(c+dx)}\coth(c+dx)(b\,\mathbf{sech}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\mathbf{sech}^2(c+dx)\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((Coth[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sech[c + d*x]^2]*(b
*Sech[c + d*x])^n*Sqrt[Tanh[c + d*x]^2])/(d*n))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (b\text{sech}\,(dx+c))^n\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^n,x)
```

```
[Out] int((b*sech(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(dx+c))^n\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{sech}\,(dx+c))^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sech(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{sech}\,(c+dx))^n\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**n,x)
```

```
[Out] Integral((b*sech(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{sech}(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

3.24 $\int \mathbf{sech}^2(a+bx)^{7/2}\,dx$

Optimal. Leaf size=90

$$\frac{5\sin^{-1}(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\mathrm{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\mathrm{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\mathrm{sech}^2(a}}{16b}$$

```
[Out] (5*ArcSin[Tanh[a + b*x]])/(16*b) + (5*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/
(16*b) + (5*(Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(24*b) + ((Sech[a + b*x]
^2)^(5/2)*Tanh[a + b*x])/(6*b)
```

Rubi [A] time = 0.0297275, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 195, 216}

$$\frac{5\sin^{-1}(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\mathrm{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\mathrm{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\mathrm{sech}^2(a}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (5*ArcSin[Tanh[a + b*x]])/(16*b) + (5*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/
(16*b) + (5*(Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(24*b) + ((Sech[a + b*x]
^2)^(5/2)*Tanh[a + b*x])/(6*b)
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^{5/2}dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\,\text{Subst}\left(\int \left(1-x^2\right)^{3/2}dx, x, \tanh(a+bx)\right)}{6b}$$
$$= \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\,\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \text{t}\right.}{8b}$$
$$= \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+}{6b}$$
$$= \frac{5\sin^{-1}(\tanh(a+bx))}{16b} + \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\text{s}}{}$$

Mathematica [A] time = 0.100585, size = 81, normalized size = 0.9

$$\frac{\cosh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}\left(15\tan^{-1}(\sinh(a+bx)) + 8\tanh(a+bx)\operatorname{sech}^5(a+bx) + 10\tanh(a+bx)\operatorname{sech}^3(a+bx) + \right.}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2]*(15*ArcTan[Sinh[a + b*x]] + 15*Sech[a
+ b*x]*Tanh[a + b*x] + 10*Sech[a + b*x]^3*Tanh[a + b*x] + 8*Sech[a + b*x]^5
*Tanh[a + b*x]))/(48*b)
```

Maple [C] time = 0.177, size = 230, normalized size = 2.6

$$\frac{15\,e^{10\,bx+10\,a}+85\,e^{8\,bx+8\,a}+198\,e^{6\,bx+6\,a}-198\,e^{4\,bx+4\,a}-85\,e^{2\,bx+2\,a}-15}{24\left(1+e^{2\,bx+2\,a}\right)^5 b}\sqrt{\frac{e^{2\,bx+2\,a}}{\left(1+e^{2\,bx+2\,a}\right)^2}} + \frac{\frac{5i}{16}\left(1+e^{2\,bx+2\,a}\right)\ln\left(e^{bx}+ie\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(7/2),x)
```

```
[Out] 1/24/(1+exp(2*b*x+2*a))^5*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(15
*exp(10*b*x+10*a)+85*exp(8*b*x+8*a)+198*exp(6*b*x+6*a)-198*exp(4*b*x+4*a)-8
5*exp(2*b*x+2*a)-15)/b+5/16*I*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*ex
p(2*b*x+2*a))^(1/2)/b*ln(exp(b*x)+I*exp(-a))*exp(-b*x-a)-5/16*I*(1+exp(2*b*
x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/b*ln(exp(b*x)-I*exp(-
a))*exp(-b*x-a)
```

Maxima [B] time = 1.52539, size = 211, normalized size = 2.34

$$-\frac{5\arctan\left(e^{(-bx-a)}\right)}{8\,b} + \frac{15\,e^{(-bx-a)}+85\,e^{(-3\,bx-3\,a)}+198\,e^{(-5\,bx-5\,a)}-198\,e^{(-7\,bx-7\,a)}-85\,e^{(-9\,bx-9\,a)}-15\,e^{(-11\,bx-11\,a)}}{24\,b\left(6\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}+20\,e^{(-6\,bx-6\,a)}+15\,e^{(-8\,bx-8\,a)}+6\,e^{(-10\,bx-10\,a)}+e^{(-12\,bx-12\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -5/8*arctan(e^(-b*x - a))/b + 1/24*(15*e^(-b*x - a) + 85*e^(-3*b*x - 3*a) +
 198*e^(-5*b*x - 5*a) - 198*e^(-7*b*x - 7*a) - 85*e^(-9*b*x - 9*a) - 15*e^(
-11*b*x - 11*a))/(b*(6*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) + 20*e^(-6*b*
x - 6*a) + 15*e^(-8*b*x - 8*a) + 6*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a)
+ 1))
```

Fricas [B] time = 2.3338, size = 4601, normalized size = 51.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/24*(15*cosh(b*x + a)^11 + 165*cosh(b*x + a)*sinh(b*x + a)^10 + 15*sinh(b*
x + a)^11 + 5*(165*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^9 + 85*cosh(b*x + a)
^9 + 45*(55*cosh(b*x + a)^3 + 17*cosh(b*x + a))*sinh(b*x + a)^8 + 18*(275*c
osh(b*x + a)^4 + 170*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^7 + 198*cosh(b*x +
 a)^7 + 42*(165*cosh(b*x + a)^5 + 170*cosh(b*x + a)^3 + 33*cosh(b*x + a))*s
inh(b*x + a)^6 + 18*(385*cosh(b*x + a)^6 + 595*cosh(b*x + a)^4 + 231*cosh(b
*x + a)^2 - 11)*sinh(b*x + a)^5 - 198*cosh(b*x + a)^5 + 90*(55*cosh(b*x + a
)^7 + 119*cosh(b*x + a)^5 + 77*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x
 + a)^4 + 5*(495*cosh(b*x + a)^8 + 1428*cosh(b*x + a)^6 + 1386*cosh(b*x + a
)^4 - 396*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^3 - 85*cosh(b*x + a)^3 + 3*(2
75*cosh(b*x + a)^9 + 1020*cosh(b*x + a)^7 + 1386*cosh(b*x + a)^5 - 660*cosh
(b*x + a)^3 - 85*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(b*x + a)^12 + 12
*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 6*(11*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^10 + 6*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + 3*cos
h(b*x + a))*sinh(b*x + a)^9 + 15*(33*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^8 + 15*cosh(b*x + a)^8 + 24*(33*cosh(b*x + a)^5 + 30*cosh
(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 3
15*cosh(b*x + a)^4 + 105*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^6 + 20*cosh(b*x
 + a)^6 + 24*(33*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 + 35*cosh(b*x + a)^3
+ 5*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(33*cosh(b*x + a)^8 + 84*cosh(b*x +
 a)^6 + 70*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*c
osh(b*x + a)^4 + 20*(11*cosh(b*x + a)^9 + 36*cosh(b*x + a)^7 + 42*cosh(b*x
+ a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(11*cosh
(b*x + a)^10 + 45*cosh(b*x + a)^8 + 70*cosh(b*x + a)^6 + 50*cosh(b*x + a)^4
 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 + 12*(cosh(b
*x + a)^11 + 5*cosh(b*x + a)^9 + 10*cosh(b*x + a)^7 + 10*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a)
+ sinh(b*x + a)) + 3*(55*cosh(b*x + a)^10 + 255*cosh(b*x + a)^8 + 462*cosh(
b*x + a)^6 - 330*cosh(b*x + a)^4 - 85*cosh(b*x + a)^2 - 5)*sinh(b*x + a) -
15*cosh(b*x + a))/(b*cosh(b*x + a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11
 + b*sinh(b*x + a)^12 + 6*b*cosh(b*x + a)^10 + 6*(11*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^10 + 20*(11*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 15*b*cosh(b*x + a)^8 + 15*(33*b*cosh(b*x + a)^4 + 18*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^8 + 24*(33*b*cosh(b*x + a)^5 + 30*b*cosh(b*x + a)^3
 + 5*b*cosh(b*x + a))*sinh(b*x + a)^7 + 20*b*cosh(b*x + a)^6 + 4*(231*b*cos
h(b*x + a)^6 + 315*b*cosh(b*x + a)^4 + 105*b*cosh(b*x + a)^2 + 5*b)*sinh(b*
x + a)^6 + 24*(33*b*cosh(b*x + a)^7 + 63*b*cosh(b*x + a)^5 + 35*b*cosh(b*x
+ a)^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*b*cosh(b*x + a)^4 + 15*(33
*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 70*b*cosh(b*x + a)^4 + 20*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^4 + 20*(11*b*cosh(b*x + a)^9 + 36*b*cosh(b*
x + a)^7 + 42*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))
```

```
*sinh(b*x + a)^3 + 6*b*cosh(b*x + a)^2 + 6*(11*b*cosh(b*x + a)^10 + 45*b*co
sh(b*x + a)^8 + 70*b*cosh(b*x + a)^6 + 50*b*cosh(b*x + a)^4 + 15*b*cosh(b*x
 + a)^2 + b)*sinh(b*x + a)^2 + 12*(b*cosh(b*x + a)^11 + 5*b*cosh(b*x + a)^9
 + 10*b*cosh(b*x + a)^7 + 10*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 + b*co
sh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.106, size = 170, normalized size = 1.89

$$\frac{5\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)\right)}{32b}+\frac{15\left(e^{(bx+a)}-e^{(-bx-a)}\right)^5+160\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+528e^{(bx+a)}-528e^{(-bx-a)}}{24\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 5/32*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + 1/24*(15*(
e^(b*x + a) - e^(-b*x - a))^5 + 160*(e^(b*x + a) - e^(-b*x - a))^3 + 528*e^
(b*x + a) - 528*e^(-b*x - a))/(((e^(b*x + a) - e^(-b*x - a))^2 + 4)^3*b)
```

3.25 $\int \operatorname{sech}^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3\sin^{-1}(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{8b}$$

```
[Out] (3*ArcSin[Tanh[a + b*x]])/(8*b) + (3*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(
8*b) + ((Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(4*b)
```

Rubi [A] time = 0.0220126, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 195, 216}

$$\frac{3\sin^{-1}(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Tanh[a + b*x]])/(8*b) + (3*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(
8*b) + ((Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(4*b)
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{\operatorname{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\operatorname{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{4b}$$
$$= \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan$$
$$= \frac{3\sin^{-1}(\tanh(a+bx))}{8b} + \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b}$$

Mathematica [A] time = 0.114824, size = 55, normalized size = 0.85

$$\frac{\operatorname{sech}^2(a+bx)^{3/2}\left(3\sinh(2(a+bx))+4\tanh(a+bx)+6\cosh^3(a+bx)\tan^{-1}(\sinh(a+bx))\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] ((Sech[a + b*x]^2)^(3/2)*(6*ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]^3 + 3*Sinh[
2*(a + b*x)] + 4*Tanh[a + b*x]))/(16*b)
```

Maple [C] time = 0.129, size = 208, normalized size = 3.2

$$\frac{3\,e^{6\,bx+6\,a}+11\,e^{4\,bx+4\,a}-11\,e^{2\,bx+2\,a}-3}{4\,\left(1+e^{2\,bx+2\,a}\right)^3 b}\sqrt{\frac{e^{2\,bx+2\,a}}{\left(1+e^{2\,bx+2\,a}\right)^2}}+\frac{\frac{3i}{8}\left(1+e^{2\,bx+2\,a}\right)\ln\left(e^{bx}+ie^{-a}\right)e^{-bx-a}}{b}\sqrt{\frac{e^{2\,bx+2\,a}}{\left(1+e^{2\,bx+2\,a}\right)^2}}-\frac{3i}{8}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(5/2),x)
```

```
[Out] 1/4/(1+exp(2*b*x+2*a))^3*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(3*e
xp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b+3/8*I*(1+exp(2*b*x+2
*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/b*ln(exp(b*x)+I*exp(-a))
*exp(-b*x-a)-3/8*I*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a
))^(1/2)/b*ln(exp(b*x)-I*exp(-a))*exp(-b*x-a)
```

Maxima [B] time = 1.5103, size = 151, normalized size = 2.32

$$-\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{3\,e^{(-bx-a)}+11\,e^{(-3\,bx-3\,a)}-11\,e^{(-5\,bx-5\,a)}-3\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
```

```
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] time = 2.20845, size = 2263, normalized size = 34.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14046, size = 140, normalized size = 2.15

$$\frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)\right)}{16\,b} + \frac{3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 + 20\,e^{(bx+a)} - 20\,e^{(-bx-a)}}{4\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + 1/4*(3*(e^
(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-b*x - a))/(((e^(b*x +
 a) - e^(-b*x - a))^2 + 4)^2*b)
```

3.26 $\int \text{sech}^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{\sin^{-1}(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\text{sech}^2(a+bx)}}{2b}$$

```
[Out] ArcSin[Tanh[a + b*x]]/(2*b) + (Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0168868, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 195, 216}

$$\frac{\sin^{-1}(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\text{sech}^2(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/(2*b) + (Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(2*b)
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^2(a+bx)^{3/2}\,dx&=\frac{\text{Subst}\left(\int\sqrt{1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}\\&=\frac{\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{2b}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,\tanh(a+bx)\right)}{2b}\\&=\frac{\sin^{-1}(\tanh(a+bx))}{2b}+\frac{\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0407185, size = 46, normalized size = 1.15

$$\frac{\operatorname{sech}(a+bx)\left(\tan^{-1}(\sinh(a+bx))+\tanh(a+bx)\operatorname{sech}(a+bx)\right)}{2b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] (Sech[a + b*x]*(ArcTan[Sinh[a + b*x]] + Sech[a + b*x]*Tanh[a + b*x]))/(2*b*
Sqrt[Sech[a + b*x]^2])
```

Maple [C] time = 0.12, size = 183, normalized size = 4.6

$$\frac{e^{2bx+2a}-1}{\left(1+e^{2bx+2a}\right)b}\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}+\frac{\frac{i}{2}\left(1+e^{2bx+2a}\right)\ln\left(e^{bx}+ie^{-a}\right)e^{-bx-a}}{b}\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}-\frac{\frac{i}{2}\left(1+e^{2bx+2a}\right)\ln\left(e^{bx}-ie^{-}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(3/2),x)
```

```
[Out] 1/(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b
*x+2*a)-1)/b+1/2*I*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a
))^(1/2)/b*ln(exp(b*x)+I*exp(-a))*exp(-b*x-a)-1/2*I*(1+exp(2*b*x+2*a))*(1/(
1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/b*ln(exp(b*x)-I*exp(-a))*exp(-b*x
-a)
```

Maxima [A] time = 1.54095, size = 88, normalized size = 2.2

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 2.15229, size = 757, normalized size = 18.92

$$\frac{\cosh(bx+a)^3+3\,\cosh(bx+a)\sinh(bx+a)^2+\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\,\cosh(bx+a)\sinh(bx+a)^3+\sin\right.}{b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.12229, size = 107, normalized size = 2.68

$$\frac{\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b} + \frac{e^{(bx+a)} - e^{(-bx-a)}}{\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b + (e^(b*x + a
) - e^(-b*x - a))/(((e^(b*x + a) - e^(-b*x - a))^2 + 4)*b)
```

3.27 $\int \sqrt{\text{sech}^2(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(\tanh(a+bx))}{b}$$

```
[Out] ArcSin[Tanh[a + b*x]]/b
```

Rubi [A] time = 0.0109753, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4122, 216}

$$\frac{\sin^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/b
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\text{sech}^2(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\sin^{-1}(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.01561, size = 29, normalized size = 2.64

$$\frac{\cosh(a+bx)\sqrt{\text{sech}^2(a+bx)}\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] (ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2])/b
```

Maple [C] time = 0.132, size = 130, normalized size = 11.8

$$\frac{i\left(1+e^{2bx+2a}\right)\ln\left(e^{bx}+ie^{-a}\right)e^{-bx-a}}{b}\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}-\frac{i\left(1+e^{2bx+2a}\right)\ln\left(e^{bx}-ie^{-a}\right)e^{-bx-a}}{b}\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(1/2),x)
```

```
[Out] I*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))/b*ln(exp
(b*x)+I*exp(-a))*exp(-b*x-a)-I*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2
)*(1+exp(2*b*x+2*a))/b*ln(exp(b*x)-I*exp(-a))*exp(-b*x-a)
```

Maxima [A] time = 0.995687, size = 15, normalized size = 1.36

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*x + a))/b
```

Fricas [A] time = 2.00202, size = 58, normalized size = 5.27

$$\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\operatorname{sech}^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(a + b*x)**2), x)
```

<u>Giac</u> [A] time = 1.14072, size = 16, normalized size = 1.45

$$\frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b
```

3.28 $\int \frac{1}{\sqrt{\mathbf{sech}^2(a+bx)}}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] time = 0.0164002, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4122, 191}

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0255771, size = 22, normalized size = 1.

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] time = 0.106, size = 97, normalized size = 4.4

$$\frac{e^{2bx+2a}}{\left(2+2e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}-\frac{1}{\left(2+2e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(
2*b*x+2*a)-1/2/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2
*a))/b
```

Maxima [A] time = 0.976915, size = 35, normalized size = 1.59

$$\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Fricas [A] time = 1.97329, size = 23, normalized size = 1.05

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sinh(b*x + a)/b
```

Sympy [A] time = 16.0804, size = 29, normalized size = 1.32

$$\begin{cases}\frac{\tanh\left(a+bx\right)}{b\sqrt{\operatorname{sech}^{2}\left(a+bx\right)}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{\operatorname{sech}^{2}\left(a\right)}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise((tanh(a + b*x)/(b*sqrt(sech(a + b*x)**2)), Ne(b, 0)), (x/sqrt(sec
h(a)**2), True))
```

Giac [A] time = 1.15026, size = 31, normalized size = 1.41

$$\frac{e^{(bx+a)} - e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) - e^(-b*x - a))/b
```

3.29 $\int \frac{1}{\mathbf{sech}^2(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=51

$$\frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

```
[Out] Tanh[a + b*x]/(3*b*(Sech[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*b*Sqrt[S
ech[a + b*x]^2])
```

Rubi [A] time = 0.0197899, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 192, 191}

$$\frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] Tanh[a + b*x]/(3*b*(Sech[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*b*Sqrt[S
ech[a + b*x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{2\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{3b}\\
&= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0657508, size = 44, normalized size = 0.86

$$\frac{\tanh^3(a+bx)+3\tanh(a+bx)\text{sech}^2(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] (3*Sech[a + b*x]^2*Tanh[a + b*x] + Tanh[a + b*x]^3)/(3*b*(Sech[a + b*x]^2)^
(3/2))
```

Maple [B] time = 0.104, size = 201, normalized size = 3.9

$$\frac{e^{4bx+4a}}{\left(24+24e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{3e^{2bx+2a}}{\left(8+8e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{3}{\left(8+8e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{e^{-2bx-2a}}{\left(24+24e^{2bx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(3/2),x)
```

```
[Out] 1/24/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp
(4*b*x+4*a)+3/8/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+
2*a))/b*exp(2*b*x+2*a)-3/8/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1
+exp(2*b*x+2*a))/b-1/24/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+ex
p(2*b*x+2*a))/b*exp(-2*b*x-2*a)
```

Maxima [A] time = 1.01207, size = 73, normalized size = 1.43

$$\frac{e^{(3bx+3a)}}{24b}+\frac{3e^{(bx+a)}}{8b}-\frac{3e^{(-bx-a)}}{8b}-\frac{e^{(-3bx-3a)}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A] time = 2.04689, size = 89, normalized size = 1.75

$$\frac{\sinh(bx+a)^3+3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A] time = 20.3354, size = 54, normalized size = 1.06

$$\begin{cases} -\frac{2\tanh^{3}(a+bx)}{3b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{3}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^{2}(a)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Piecewise((-2*tanh(a + b*x)**3/(3*b*(sech(a + b*x)**2)**(3/2)) + tanh(a + b
*x)/(b*(sech(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(sech(a)**2)**(3/2), True)
)
```

Giac [A] time = 1.13752, size = 65, normalized size = 1.27

$$-\frac{\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)} - e^{(3\,bx+3\,a)} - 9\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))/b
```

3.30 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}}$$

```
[Out] Tanh[a + b*x]/(5*b*(Sech[a + b*x]^2)^(5/2)) + (4*Tanh[a + b*x])/(15*b*(Sech
[a + b*x]^2)^(3/2)) + (8*Tanh[a + b*x])/(15*b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] time = 0.0266368, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 192, 191}

$$\frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] Tanh[a + b*x]/(5*b*(Sech[a + b*x]^2)^(5/2)) + (4*Tanh[a + b*x])/(15*b*(Sech
[a + b*x]^2)^(3/2)) + (8*Tanh[a + b*x])/(15*b*Sqrt[Sech[a + b*x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\text{sech}^2(a+bx)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{5b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{15b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}$$

Mathematica [A] time = 0.0817697, size = 47, normalized size = 0.62

$$\frac{\left(3\sinh^4(a+bx)+10\sinh^2(a+bx)+15\right)\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] ((15 + 10*Sinh[a + b*x]^2 + 3*Sinh[a + b*x]^4)*Tanh[a + b*x])/(15*b*Sqrt[Se
ch[a + b*x]^2])
```

Maple [B] time = 0.098, size = 305, normalized size = 4.

$$\frac{e^{6bx+6a}}{\left(160+160\,e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{4bx+4a}}{\left(96+96\,e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{2bx+2a}}{\left(16+16\,e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{5}{(16+16\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(5/2),x)
```

```
[Out] 1/160/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*ex
p(6*b*x+6*a)+5/96/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*
x+2*a))/b*exp(4*b*x+4*a)+5/16/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)
/(1+exp(2*b*x+2*a))/b*exp(2*b*x+2*a)-5/16/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x
+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b-5/96/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*
a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(-2*b*x-2*a)-1/160/(1/(1+exp(2*b*x+2*a))^
2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(-4*b*x-4*a)
```

Maxima [A] time = 1.03074, size = 111, normalized size = 1.46

$$\frac{e^{(5bx+5a)}}{160\,b}+\frac{5\,e^{(3bx+3a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}-\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3bx-3a)}}{96\,b}-\frac{e^{(-5bx-5a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Fricas [A] time = 2.12512, size = 182, normalized size = 2.39

$$\frac{3\sinh(bx+a)^5 + 5\left(6\cosh(bx+a)^2 + 5\right)\sinh(bx+a)^3 + 15\left(\cosh(bx+a)^4 + 5\cosh(bx+a)^2 + 10\right)\sinh(bx+a)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A] time = 54.5431, size = 80, normalized size = 1.05

$$\begin{cases} \frac{8\tanh^5(a+bx)}{15b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(a+bx)}{3b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^2(a)\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Piecewise((8*tanh(a + b*x)**5/(15*b*(sech(a + b*x)**2)**(5/2)) - 4*tanh(a +
 b*x)**3/(3*b*(sech(a + b*x)**2)**(5/2)) + tanh(a + b*x)/(b*(sech(a + b*x)*
*2)**(5/2)), Ne(b, 0)), (x/(sech(a)**2)**(5/2), True))
```

Giac [A] time = 1.12807, size = 95, normalized size = 1.25

$$-\frac{\left(150\,e^{(4\,bx+4\,a)} + 25\,e^{(2\,bx+2\,a)} + 3\right)e^{(-5\,bx-5\,a)} - 3\,e^{(5\,bx+5\,a)} - 25\,e^{(3\,bx+3\,a)} - 150\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) - 3
*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

3.31 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=101

$$\frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}}+\frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}}$$

```
[Out] Tanh[a + b*x]/(7*b*(Sech[a + b*x]^2)^(7/2)) + (6*Tanh[a + b*x])/(35*b*(Sech
[a + b*x]^2)^(5/2)) + (8*Tanh[a + b*x])/(35*b*(Sech[a + b*x]^2)^(3/2)) + (1
6*Tanh[a + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] time = 0.0345122, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 192, 191}

$$\frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}}+\frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] Tanh[a + b*x]/(7*b*(Sech[a + b*x]^2)^(7/2)) + (6*Tanh[a + b*x])/(35*b*(Sech
[a + b*x]^2)^(5/2)) + (8*Tanh[a + b*x])/(35*b*(Sech[a + b*x]^2)^(3/2)) + (1
6*Tanh[a + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{9/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{7b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{24\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{35b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{35b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Mathematica [A] time = 0.130937, size = 57, normalized size = 0.56

$$\frac{\left(5\sinh^6(a+bx)+21\sinh^4(a+bx)+35\sinh^2(a+bx)+35\right)\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] ((35 + 35*Sinh[a + b*x]^2 + 21*Sinh[a + b*x]^4 + 5*Sinh[a + b*x]^6)*Tanh[a
+ b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] time = 0.102, size = 409, normalized size = 4.1

$$\frac{e^{8bx+8a}}{\left(896+896e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}} + \frac{7e^{6bx+6a}}{\left(640+640e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}} + \frac{7e^{4bx+4a}}{\left(128+128e^{2bx+2a}\right)b}\frac{1}{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(7/2),x)
```

```
[Out] 1/896/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*ex
p(8*b*x+8*a)+7/640/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b
*x+2*a))/b*exp(6*b*x+6*a)+7/128/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/
2)/(1+exp(2*b*x+2*a))/b*exp(4*b*x+4*a)+35/128/(1/(1+exp(2*b*x+2*a))^2*exp(2
*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(2*b*x+2*a)-35/128/(1/(1+exp(2*b*x
+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b-7/128/(1/(1+exp(2*b*x+2
*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(-2*b*x-2*a)-7/640/(1/
(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))/b*exp(-4*b*x-
4*a)-1/896/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)/(1+exp(2*b*x+2*a))
/b*exp(-6*b*x-6*a)
```

Maxima [A] time = 1.01803, size = 135, normalized size = 1.34

$$\frac{\left(49\,e^{(-2\,bx-2\,a)}+245\,e^{(-4\,bx-4\,a)}+1225\,e^{(-6\,bx-6\,a)}+5\right)e^{(7\,bx+7\,a)}}{4480\,b}-\frac{1225\,e^{(-bx-a)}+245\,e^{(-3\,bx-3\,a)}+49\,e^{(-5\,bx-5\,a)}+5\,e^{(-7\,bx-7}}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4480*(49*e^(-2*b*x - 2*a) + 245*e^(-4*b*x - 4*a) + 1225*e^(-6*b*x - 6*a)
+ 5)*e^(7*b*x + 7*a)/b - 1/4480*(1225*e^(-b*x - a) + 245*e^(-3*b*x - 3*a) +
 49*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/b
```

Fricas [A] time = 2.07259, size = 302, normalized size = 2.99

$$\frac{5\,\sinh\left(bx+a\right)^{7}+7\left(15\,\cosh\left(bx+a\right)^{2}+7\right)\sinh\left(bx+a\right)^{5}+35\left(5\,\cosh\left(bx+a\right)^{4}+14\,\cosh\left(bx+a\right)^{2}+7\right)\sinh\left(bx+a\right)}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*sinh(b*x + a)^7 + 7*(15*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^5 + 35
*(5*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 35*(cosh(b*
x + a)^6 + 7*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 35)*sinh(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12029, size = 124, normalized size = 1.23

$$-\frac{\left(1225\,e^{(6\,bx+6\,a)}+245\,e^{(4\,bx+4\,a)}+49\,e^{(2\,bx+2\,a)}+5\right)e^{(-7\,bx-7\,a)}-5\,e^{(7\,bx+7\,a)}-49\,e^{(5\,bx+5\,a)}-245\,e^{(3\,bx+3\,a)}-1225\,e^{(bx+a)}}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*b*x + 6*a) + 245*e^(4*b*x + 4*a) + 49*e^(2*b*x + 2*a) +
 5)*e^(-7*b*x - 7*a) - 5*e^(7*b*x + 7*a) - 49*e^(5*b*x + 5*a) - 245*e^(3*b*
x + 3*a) - 1225*e^(b*x + a))/b
```

3.32 $\int \left(a\,\mathrm{sech}^2(x)\right)^{5/2} dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^2\tanh(x)\sqrt{a\,\mathrm{sech}^2(x)}+\frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{4}a\tanh(x)\left(a\,\mathrm{sech}^2(x)\right)^{3/2}$$

```
[Out] (3*a^(5/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/8 + (3*a^2*Sqrt[a*S
ech[x]^2]*Tanh[x])/8 + (a*(a*Sech[x]^2)^(3/2)*Tanh[x])/4
```

Rubi [A] time = 0.0336254, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 203}

$$\frac{3}{8}a^2\tanh(x)\sqrt{a\,\mathrm{sech}^2(x)}+\frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{4}a\tanh(x)\left(a\,\mathrm{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/8 + (3*a^2*Sqrt[a*S
ech[x]^2]*Tanh[x])/8 + (a*(a*Sech[x]^2)^(3/2)*Tanh[x])/4
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a\operatorname{sech}^2(x)\right)^{5/2} dx = a\,\text{Subst}\left(\int \left(a-ax^2\right)^{3/2} dx, x, \tanh(x)\right)$$
$$= \frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a-ax^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}dx, x, \tanh\right.$$
$$= \frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}dx, x, \frac{\tanh}{\sqrt{a\operatorname{sec}}}\right.$$
$$= \frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x)$$

Mathematica [A] time = 0.0341739, size = 42, normalized size = 0.65

$$\frac{1}{8}\cosh(x)\left(a\operatorname{sech}^2(x)\right)^{5/2}\left(2\sinh(x) + 3\sinh(x)\cosh^2(x) + 6\cosh^4(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(a*Sech[x]^2)^(5/2)*(6*ArcTan[Tanh[x/2]]*Cosh[x]^4 + 2*Sinh[x] + 3
*Cosh[x]^2*Sinh[x]))/8
```

Maple [C] time = 0.08, size = 127, normalized size = 2.

$$\frac{a^2\left(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3\right)}{4\left(e^{2x}+1\right)^3}\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}} + \frac{3\,i}{8}a^2e^{-x}\left(e^{2x}+1\right)\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}\ln\left(e^x+i\right) - \frac{3\,i}{8}a^2e^{-x}\left(e^{2x}+1\right)\sqrt{\frac{ae^{2x}}{\left(e^{2x}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(5/2),x)
```

```
[Out] 1/4*a^2/(exp(2*x)+1)^3*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*(3*exp(6*x)+11*exp
(4*x)-11*exp(2*x)-3)+3/8*I*a^2*exp(-x)*(exp(2*x)+1)*(a*exp(2*x)/(exp(2*x)+1
)^2)^(1/2)*ln(exp(x)+I)-3/8*I*a^2*exp(-x)*(exp(2*x)+1)*(a*exp(2*x)/(exp(2*x
)+1)^2)^(1/2)*ln(exp(x)-I)
```

Maxima [A] time = 1.68599, size = 97, normalized size = 1.49

$$\frac{3}{4}a^{\frac{5}{2}}\arctan\left(e^x\right) + \frac{3\,a^{\frac{5}{2}}e^{(7x)} + 11\,a^{\frac{5}{2}}e^{(5x)} - 11\,a^{\frac{5}{2}}e^{(3x)} - 3\,a^{\frac{5}{2}}e^x}{4\left(e^{(8x)} + 4\,e^{(6x)} + 6\,e^{(4x)} + 4\,e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*a^(5/2)*arctan(e^x) + 1/4*(3*a^(5/2)*e^(7*x) + 11*a^(5/2)*e^(5*x) - 11*
a^(5/2)*e^(3*x) - 3*a^(5/2)*e^x)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*
```

```
x) + 1)
```

Fricas [B] time = 2.23797, size = 3067, normalized size = 47.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cosh(x)^7 + 3*(a^2*e^(2*x) + a^2)*sinh(x)^7 + 11*a^2*cosh(x)^5 +
 21*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^6 + (63*a^2*cosh(x)^2 + 11*
a^2 + (63*a^2*cosh(x)^2 + 11*a^2)*e^(2*x))*sinh(x)^5 - 11*a^2*cosh(x)^3 + 5
*(21*a^2*cosh(x)^3 + 11*a^2*cosh(x) + (21*a^2*cosh(x)^3 + 11*a^2*cosh(x))*e
^(2*x))*sinh(x)^4 + (105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2 + (105*
a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 - 3*a^2*cosh
(x) + (63*a^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) + (63*a^2*cosh
(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 + 3*(a^2*cos
h(x)^8 + (a^2*e^(2*x) + a^2)*sinh(x)^8 + 4*a^2*cosh(x)^6 + 8*(a^2*cosh(x)*e
^(2*x) + a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 + a^2 + (7*a^2*cosh(x)
^2 + a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2
*cosh(x) + (7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 2*(35*a^2
*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2 + (35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^
2 + 3*a^2)*e^(2*x))*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 + 10*a
^2*cosh(x)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2
*cosh(x)^2 + a^2 + (7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^8 + 4*a^2*cosh(x)^6 + 6*a^2*co
sh(x)^4 + 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)
^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 + 3*a
^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (
3*a^2*cosh(x)^7 + 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2
*x) + (21*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2 + (21
*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2)*e^(2*x))*sinh
(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*s
inh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cos
h(x)^4 + 9*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*co
sh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4
*cosh(x)^2 + 1)*e^x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1358, size = 88, normalized size = 1.35

$$\frac{1}{16}\left(3\pi - \frac{4\left(3\left(e^{(-x)} - e^{x}\right)^{3} + 20\,e^{(-x)} - 20\,e^{x}\right)}{\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2}} + 6\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)a^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi - 4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 6*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a^(5/2)
```

3.33 $\int \left(a\,\mathrm{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\,\mathrm{sech}^2(x)}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/2 + (a*Sqrt[a*Sech[x]
^2]*Tanh[x])/2
```

Rubi [A] time = 0.0244867, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 203}

$$\frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\,\mathrm{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/2 + (a*Sqrt[a*Sech[x]
^2]*Tanh[x])/2
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a\,\text{sech}^2(x)\right)^{3/2} dx = a\,\text{Subst}\left(\int \sqrt{a-ax^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}\tanh(x) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}\tanh(x) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}\right)$$
$$= \frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}\right) + \frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}\tanh(x)$$

Mathematica [A] time = 0.0192026, size = 29, normalized size = 0.63

$$\frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}\left(\tanh(x) + 2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))/2
```

Maple [C] time = 0.066, size = 106, normalized size = 2.3

$$\frac{a\left(e^{2x}-1\right)}{e^{2x}+1}\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}} + \frac{i}{2}ae^{-x}\left(e^{2x}+1\right)\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}\ln\left(e^x+i\right) - \frac{i}{2}ae^{-x}\left(e^{2x}+1\right)\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}\ln\left(e^x-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(3/2),x)
```

```
[Out] a/(exp(2*x)+1)*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*(exp(2*x)-1)+1/2*I*a*exp(-
x)*(exp(2*x)+1)*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*ln(exp(x)+I)-1/2*I*a*exp(
-x)*(exp(2*x)+1)*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*ln(exp(x)-I)
```

Maxima [A] time = 1.83217, size = 53, normalized size = 1.15

$$a^{\frac{3}{2}}\arctan\left(e^x\right) + \frac{a^{\frac{3}{2}}e^{(3x)} - a^{\frac{3}{2}}e^x}{e^{(4x)} + 2e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a^(3/2)*arctan(e^x) + (a^(3/2)*e^(3*x) - a^(3/2)*e^x)/(e^(4*x) + 2*e^(2*x)
+ 1)
```

Fricas [B] time = 2.26341, size = 952, normalized size = 20.7

$$\frac{\left(a\cosh\left(x\right)^{3} + \left(ae^{\left(2\,x\right)} + a\right)\sinh\left(x\right)^{3} + 3\,\left(a\cosh\left(x\right)e^{\left(2\,x\right)} + a\cosh\left(x\right)\right)\sinh\left(x\right)^{2} + \left(a\cosh\left(x\right)^{4} + \left(ae^{\left(2\,x\right)} + a\right)\sinh\left(x\right)^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*cosh(x)^3 + (a*e^(2*x) + a)*sinh(x)^3 + 3*(a*cosh(x)*e^(2*x) + a*cosh(x)
)*sinh(x)^2 + (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(2*
x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2*x)
+ 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh(x) +
 a)*arctan(cosh(x) + sinh(x)) - a*cosh(x) + (a*cosh(x)^3 - a*cosh(x))*e^(2*
x) + (3*a*cosh(x)^2 + (3*a*cosh(x)^2 - a)*e^(2*x) - a)*sinh(x))*sqrt(a/(e^(
4*x) + 2*e^(2*x) + 1))*e^x/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*
cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\operatorname{sech}^{2}\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**2)**(3/2), x)
```

Giac [A] time = 1.12518, size = 65, normalized size = 1.41

$$\frac{1}{4}\left(\pi - \frac{4\left(e^{\left(-x\right)} - e^{x}\right)}{\left(e^{\left(-x\right)} - e^{x}\right)^{2} + 4} + 2\arctan\left(\frac{1}{2}\left(e^{\left(2\,x\right)} - 1\right)e^{\left(-x\right)}\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*(e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 2*arctan(1/2*(e^(2*x) -
 1)*e^(-x)))*a^(3/2)
```

3.34 $\int \sqrt{a\text{sech}^2(x)}\,dx$

Optimal. Leaf size=25

$$\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

```
[Out] Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]]
```

Rubi [A] time = 0.0157782, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 217, 203}

$$\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\text{sech}^2(x)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)\\ &= a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\\ &= \sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.0059364, size = 21, normalized size = 0.84

$$2\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^2],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[a*Sech[x]^2]
```

Maple [C] time = 0.082, size = 72, normalized size = 2.9

$$i\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^2}}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}+1\right)\ln\left(\mathrm{e}^{x}+i\right)-i\sqrt{\frac{a\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}+1\right)^2}}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}+1\right)\ln\left(\mathrm{e}^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(1/2),x)
```

```
[Out] I*(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(exp(2*x)+1)*ln(exp(x)+I)-I*(a*
exp(2*x)/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(exp(2*x)+1)*ln(exp(x)-I)
```

Maxima [A] time = 1.77084, size = 11, normalized size = 0.44

$$2\sqrt{a}\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*arctan(e^x)
```

Fricas [A] time = 2.23395, size = 444, normalized size = 17.76

$$\left[\sqrt{-a}\log\left(\frac{2\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)+ae^{x}\sinh\left(x\right)^2+2\left(\cosh\left(x\right)e^{(2\,x)}+\left(e^{(2\,x)}+1\right)\sinh\left(x\right)+\cosh\left(x\right)\right)\sqrt{-a}\sqrt{\frac{a}{e^{(4\,x)}+2\,e^{(2\,x)}}}}{2\,\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^2+\left(\cosh\left(x\right)^2+1\right)e^{x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cosh(x)*e^x*sinh(x) + a*e^x*sinh(x)^2 + 2*(cosh(x)*e^(2*
x) + (e^(2*x) + 1)*sinh(x) + cosh(x))*sqrt(-a)*sqrt(a/(e^(4*x) + 2*e^(2*x)
+ 1))*e^x + (a*cosh(x)^2 - a)*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 +
 (cosh(x)^2 + 1)*e^x)), 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*a
rctan(cosh(x) + sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{sech}^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**2), x)
```

Giac [A] time = 1.13218, size = 11, normalized size = 0.44

$$2\sqrt{a}\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*arctan(e^x)
```

3.35 $\int \frac{1}{\sqrt{a\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

[Out] Tanh[x]/Sqrt[a*Sech[x]^2]

Rubi [A] time = 0.0285025, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4122, 191}

$$\frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*Sech[x]^2],x]

[Out] Tanh[x]/Sqrt[a*Sech[x]^2]

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{sech}^2(x)}}\,dx &= a\operatorname{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0064179, size = 13, normalized size = 1.

$$\frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*Sech[x]^2],x]

```
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Maple [B] time = 0.072, size = 58, normalized size = 4.5

$$\frac{e^{2x}}{2e^{2x}+2}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}}-\frac{1}{2e^{2x}+2}\frac{1}{\sqrt{\frac{ae^{2x}}{(e^{2x}+1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(1/2),x)
```

```
[Out] 1/2/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(2*x)-1/2/(a*exp(2*x)
/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)
```

Maxima [A] time = 1.87257, size = 23, normalized size = 1.77

$$-\frac{e^{(-x)}}{2\sqrt{a}}+\frac{e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) + 1/2*e^x/sqrt(a)
```

Fricas [B] time = 2.0295, size = 247, normalized size = 19.

$$\frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}e^{x}}{2\left(a\cosh(x)e^{x}+ae^{x}\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 - 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [A] time = 0.695431, size = 15, normalized size = 1.15

$$\frac{\tanh(x)}{\sqrt{a}\sqrt{\operatorname{sech}^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(1/2),x)
```

```
[Out] tanh(x)/(sqrt(a)*sqrt(sech(x)**2))
```

Giac [A] time = 1.14099, size = 19, normalized size = 1.46

$$-\frac{e^{(-x)} - e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/sqrt(a)
```

3.36 $\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=36

$$\frac{2\tanh(x)}{3a\sqrt{a\,\text{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\,\text{sech}^2(x)\right)^{3/2}}$$

```
[Out] Tanh[x]/(3*(a*Sech[x]^2)^(3/2)) + (2*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rubi [A] time = 0.0201417, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{2\tanh(x)}{3a\sqrt{a\,\text{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\,\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] Tanh[x]/(3*(a*Sech[x]^2)^(3/2)) + (2*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{3/2}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{3\left(a\,\text{sech}^2(x)\right)^{3/2}}+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{3\left(a\,\text{sech}^2(x)\right)^{3/2}}+\frac{2\tanh(x)}{3a\sqrt{a\,\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0211334, size = 27, normalized size = 0.75

$$\frac{(9\sinh(x)+\sinh(3x))\operatorname{sech}^3(x)}{12\left(a\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] (Sech[x]^3*(9*Sinh[x] + Sinh[3*x]))/(12*(a*Sech[x]^2)^(3/2))
```

Maple [B] time = 0.059, size = 130, normalized size = 3.6

$$\frac{e^{4x}}{24a\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}}+\frac{3e^{2x}}{8a\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}}-\frac{3}{8a\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}}-\frac{e^{-2x}}{24a\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(3/2),x)
```

```
[Out] 1/24/a*exp(4*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+3/8/a*exp(2*
x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)-3/8/a/(exp(2*x)+1)/(a*exp
(2*x)/(exp(2*x)+1)^2)^(1/2)-1/24/a*exp(-2*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(
2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.70525, size = 47, normalized size = 1.31

$$\frac{e^{(3x)}}{24a^{\frac{3}{2}}}-\frac{3e^{(-x)}}{8a^{\frac{3}{2}}}-\frac{e^{(-3x)}}{24a^{\frac{3}{2}}}+\frac{3e^{x}}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*x)/a^(3/2) - 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^x
/a^(3/2)
```

Fricas [B] time = 2.19873, size = 856, normalized size = 23.78

$$\left(\left(e^{(2x)}+1\right)\sinh(x)^6+\cosh(x)^6+6\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^5+3\left(5\cosh(x)^2+\left(5\cosh(x)^2+3\right)e^{(2x)}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(cosh(x)*e^(2*x) + cosh(x))*s
inh(x)^5 + 3*(5*cosh(x)^2 + (5*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^4 + 9*co
```

```
sh(x)^4 + 4*(5*cosh(x)^3 + (5*cosh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*s
inh(x)^3 + 3*(5*cosh(x)^4 + 18*cosh(x)^2 + (5*cosh(x)^4 + 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^6 + 9*cosh(x)^4 - 9*cosh(x
)^2 - 1)*e^(2*x) + 6*(cosh(x)^5 + 6*cosh(x)^3 + (cosh(x)^5 + 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [A] time = 1.53155, size = 37, normalized size = 1.03

$$-\frac{2\tanh^{3}(x)}{3a^{\frac{3}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{a^{\frac{3}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(3/2),x)
```

```
[Out] -2*tanh(x)**3/(3*a**(3/2)*(sech(x)**2)**(3/2)) + tanh(x)/(a**(3/2)*(sech(x)
**2)**(3/2))
```

Giac [A] time = 1.13785, size = 39, normalized size = 1.08

$$-\frac{\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}}{24\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a^(3/2)
```

3.37 $\int \frac{1}{\left(a\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{8\tanh(x)}{15a^2\sqrt{a\operatorname{sech}^2(x)}}+\frac{4\tanh(x)}{15a\left(a\operatorname{sech}^2(x)\right)^{3/2}}+\frac{\tanh(x)}{5\left(a\operatorname{sech}^2(x)\right)^{5/2}}$$

```
[Out] Tanh[x]/(5*(a*Sech[x]^2)^(5/2)) + (4*Tanh[x])/(15*a*(a*Sech[x]^2)^(3/2)) +
(8*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rubi [A] time = 0.0291431, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{8\tanh(x)}{15a^2\sqrt{a\operatorname{sech}^2(x)}}+\frac{4\tanh(x)}{15a\left(a\operatorname{sech}^2(x)\right)^{3/2}}+\frac{\tanh(x)}{5\left(a\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] Tanh[x]/(5*(a*Sech[x]^2)^(5/2)) + (4*Tanh[x])/(15*a*(a*Sech[x]^2)^(3/2)) +
(8*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{7/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{15a}$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] time = 0.0388786, size = 36, normalized size = 0.65

$$\frac{(150\sinh(x) + 25\sinh(3x) + 3\sinh(5x))\cosh(x)\sqrt{a\text{sech}^2(x)}}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/(240
*a^3)
```

Maple [B] time = 0.054, size = 196, normalized size = 3.6

$$\frac{e^{6x}}{160\,a^2\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{a e^{2x}}{\left(e^{2x}+1\right)^2}}} + \frac{5\,e^{4x}}{96\,a^2\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{a e^{2x}}{\left(e^{2x}+1\right)^2}}} + \frac{5\,e^{2x}}{16\,a^2\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{a e^{2x}}{\left(e^{2x}+1\right)^2}}} - \frac{5}{16\,a^2\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{a e^{2x}}{\left(e^{2x}+1\right)^2}}} - \frac{5}{96\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(5/2),x)
```

```
[Out] 1/160/a^2*exp(6*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+5/96/a^2*
exp(4*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+5/16/a^2*exp(2*x)/(
exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)-5/16/a^2/(exp(2*x)+1)/(a*exp(
2*x)/(exp(2*x)+1)^2)^(1/2)-5/96/a^2*exp(-2*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp
(2*x)+1)^2)^(1/2)-1/160/a^2*exp(-4*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)
^2)^(1/2)
```

Maxima [A] time = 1.82914, size = 72, normalized size = 1.31

$$\frac{e^{(5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} - \frac{5\,e^{(-3x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) - 5/96*e
^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) + 5/16*e^x/a^(5/2)
```

Fricas [B] time = 2.14912, size = 1885, normalized size = 34.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) + 1)*sinh(x)^10 + 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^9 + 5*(27*cosh(x)^2 + (27*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x
)^8 + 25*cosh(x)^8 + 40*(9*cosh(x)^3 + (9*cosh(x)^3 + 5*cosh(x))*e^(2*x) +
5*cosh(x))*sinh(x)^7 + 10*(63*cosh(x)^4 + 70*cosh(x)^2 + (63*cosh(x)^4 + 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 + 150*cosh(x)^6 + 4*(189*cosh(x)^5
 + 350*cosh(x)^3 + (189*cosh(x)^5 + 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 + 10*(63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 +
 (63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 - 15)*e^(2*x) - 15)*sinh(x)^
4 - 150*cosh(x)^4 + 40*(9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 + (9*cosh
(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sin
h(x)^3 + 5*(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 +
(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 - 25*cosh(x)^2 + (3*cosh(x)^10 + 25*cosh(x)^8 + 150*cosh(x)^
6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 3)*e^(2*x) + 10*(3*cosh(x)^9 + 20*cosh(x
)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 + (3*cosh(x)^9 + 20*cosh(x)^7 + 90*cosh(x
)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [A] time = 15.0781, size = 60, normalized size = 1.09

$$\frac{8\tanh^5(x)}{15a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}-\frac{4\tanh^3(x)}{3a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}+\frac{\tanh(x)}{a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(5/2),x)
```

```
[Out] 8*tanh(x)**5/(15*a**(5/2)*(sech(x)**2)**(5/2)) - 4*tanh(x)**3/(3*a**(5/2)*(
sech(x)**2)**(5/2)) + tanh(x)/(a**(5/2)*(sech(x)**2)**(5/2))
```

Giac [A] time = 1.13454, size = 55, normalized size = 1.

$$-\frac{\left(150\,e^{(4\,x)}+25\,e^{(2\,x)}+3\right)e^{(-5\,x)}-3\,e^{(5\,x)}-25\,e^{(3\,x)}-150\,e^{x}}{480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
150*e^x)/a^(5/2)
```

3.38 $\int \frac{1}{\left(a\,\mathbf{sech}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}+\frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}}+\frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}}+\frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}}$$

```
[Out] Tanh[x]/(7*(a*Sech[x]^2)^(7/2)) + (6*Tanh[x])/(35*a*(a*Sech[x]^2)^(5/2)) +
(8*Tanh[x])/(35*a^2*(a*Sech[x]^2)^(3/2)) + (16*Tanh[x])/(35*a^3*Sqrt[a*Sech
[x]^2])
```

Rubi [A] time = 0.0403825, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}+\frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}}+\frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}}+\frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] Tanh[x]/(7*(a*Sech[x]^2)^(7/2)) + (6*Tanh[x])/(35*a*(a*Sech[x]^2)^(5/2)) +
(8*Tanh[x])/(35*a^2*(a*Sech[x]^2)^(3/2)) + (16*Tanh[x])/(35*a^3*Sqrt[a*Sech
[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{9/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{7/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{24\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)}{35a}$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{16\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{35a^2}$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}$$

Mathematica [A] time = 0.0486604, size = 42, normalized size = 0.57

$$\frac{(1225\sinh(x) + 245\sinh(3x) + 49\sinh(5x) + 5\sinh(7x))\cosh(x)\sqrt{a\operatorname{sech}^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x] + 5
*Sinh[7*x]))/(2240*a^4)
```

Maple [B] time = 0.057, size = 262, normalized size = 3.5

$$\frac{e^{8x}}{896\,a^3\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}} + \frac{7\,e^{6x}}{640\,a^3\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}} + \frac{7\,e^{4x}}{128\,a^3\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}} + \frac{35\,e^{2x}}{128\,a^3\left(e^{2x}+1\right)}\frac{1}{\sqrt{\frac{ae^{2x}}{\left(e^{2x}+1\right)^2}}} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(7/2),x)
```

```
[Out] 1/896/a^3*exp(8*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+7/640/a^3
*exp(6*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+7/128/a^3*exp(4*x)
/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)+35/128/a^3*exp(2*x)/(exp(2*
x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)-35/128/a^3/(exp(2*x)+1)/(a*exp(2*x)
/(exp(2*x)+1)^2)^(1/2)-7/128/a^3*exp(-2*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*
x)+1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)
^(1/2)-1/896/a^3*exp(-6*x)/(exp(2*x)+1)/(a*exp(2*x)/(exp(2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.65516, size = 96, normalized size = 1.3

$$\frac{e^{(7x)}}{896\,a^{\frac{7}{2}}} + \frac{7\,e^{(5x)}}{640\,a^{\frac{7}{2}}} + \frac{7\,e^{(3x)}}{128\,a^{\frac{7}{2}}} - \frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-3x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-5x)}}{640\,a^{\frac{7}{2}}} - \frac{e^{(-7x)}}{896\,a^{\frac{7}{2}}} + \frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) + 7/128*e^(3*x)/a^(7/2) - 35/
128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) - 7/640*e^(-5*x)/a^(7/2) - 1/89
6*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Fricas [B] time = 2.29884, size = 3340, normalized size = 45.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) + 1)*sinh(x)^14 + 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) + c
osh(x))*sinh(x)^13 + 7*(65*cosh(x)^2 + (65*cosh(x)^2 + 7)*e^(2*x) + 7)*sinh
(x)^12 + 49*cosh(x)^12 + 28*(65*cosh(x)^3 + (65*cosh(x)^3 + 21*cosh(x))*e^(
2*x) + 21*cosh(x))*sinh(x)^11 + 7*(715*cosh(x)^4 + 462*cosh(x)^2 + (715*cos
h(x)^4 + 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 + 245*cosh(x)^10 + 70
*(143*cosh(x)^5 + 154*cosh(x)^3 + (143*cosh(x)^5 + 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 + 35*(429*cosh(x)^6 + 693*cosh(x)^4 + 3
15*cosh(x)^2 + (429*cosh(x)^6 + 693*cosh(x)^4 + 315*cosh(x)^2 + 35)*e^(2*x)
 + 35)*sinh(x)^8 + 1225*cosh(x)^8 + 8*(2145*cosh(x)^7 + 4851*cosh(x)^5 + 36
75*cosh(x)^3 + (2145*cosh(x)^7 + 4851*cosh(x)^5 + 3675*cosh(x)^3 + 1225*cos
h(x))*e^(2*x) + 1225*cosh(x))*sinh(x)^7 + 7*(2145*cosh(x)^8 + 6468*cosh(x)^
6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 + (2145*cosh(x)^8 + 6468*cosh(x)^6 + 73
50*cosh(x)^4 + 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 - 1225*cosh(x
)^6 + 14*(715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3
+ (715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 + 35*(143*cosh(x)^10 + 693*cosh(x)
^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 + (143*cosh(x)^10 + 69
3*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 - 7)*e^(2*x)
- 7)*sinh(x)^4 - 245*cosh(x)^4 + 140*(13*cosh(x)^11 + 77*cosh(x)^9 + 210*co
sh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 + (13*cosh(x)^11 + 77*cosh(x)^9 + 2
10*cosh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 - 7*cosh(x))*e^(2*x) - 7*cosh(
x))*sinh(x)^3 + 7*(65*cosh(x)^12 + 462*cosh(x)^10 + 1575*cosh(x)^8 + 4900*c
osh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 + (65*cosh(x)^12 + 462*cosh(x)^10
 + 1575*cosh(x)^8 + 4900*cosh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 - 49*cosh(x)^2 + (5*cosh(x)^14 + 49*cosh(x)^12 + 245*c
osh(x)^10 + 1225*cosh(x)^8 - 1225*cosh(x)^6 - 245*cosh(x)^4 - 49*cosh(x)^2
- 5)*e^(2*x) + 14*(5*cosh(x)^13 + 42*cosh(x)^11 + 175*cosh(x)^9 + 700*cosh(
x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 + (5*cosh(x)^13 + 42*cosh(x)^11 + 175*c
osh(x)^9 + 700*cosh(x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11404, size = 72, normalized size = 0.97

$$-\frac{\left(1225\,e^{(6\,x)}+245\,e^{(4\,x)}+49\,e^{(2\,x)}+5\right)e^{(-7\,x)}-5\,e^{(7\,x)}-49\,e^{(5\,x)}-245\,e^{(3\,x)}-1225\,e^{x}}{4480\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) + 245*e^(4*x) + 49*e^(2*x) + 5)*e^(-7*x) - 5*e^(7*x)
 - 49*e^(5*x) - 245*e^(3*x) - 1225*e^x)/a^(7/2)
```

3.39 $\int \left(a\,\mathbf{sech}^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=121

$$\frac{2}{13}a^2\tanh(x)\mathrm{sech}^4(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{22}{117}a^2\tanh(x)\mathrm{sech}^2(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{154}{585}a^2\tanh(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{154}{195}ia^2\cosh$$

```
[Out] ((154*I)/195)*a^2*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (
154*a^2*Cosh[x]*Sqrt[a*Sech[x]^3]*Sinh[x])/195 + (154*a^2*Sqrt[a*Sech[x]^3]
*Tanh[x])/585 + (22*a^2*Sech[x]^2*Sqrt[a*Sech[x]^3]*Tanh[x])/117 + (2*a^2*S
ech[x]^4*Sqrt[a*Sech[x]^3]*Tanh[x])/13
```

Rubi [A] time = 0.0600378, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$\frac{2}{13}a^2\tanh(x)\mathrm{sech}^4(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{22}{117}a^2\tanh(x)\mathrm{sech}^2(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{154}{585}a^2\tanh(x)\sqrt{a\,\mathrm{sech}^3(x)}+\frac{154}{195}ia^2\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(5/2),x]
```


```
[Out] ((154*I)/195)*a^2*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (
154*a^2*Cosh[x]*Sqrt[a*Sech[x]^3]*Sinh[x])/195 + (154*a^2*Sqrt[a*Sech[x]^3]
*Tanh[x])/585 + (22*a^2*Sech[x]^2*Sqrt[a*Sech[x]^3]*Tanh[x])/117 + (2*a^2*S
ech[x]^4*Sqrt[a*Sech[x]^3]*Tanh[x])/13
```


Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{15}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(11a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{11}{2}}(x)\,dx}{13\text{sech}^{\frac{3}{2}}(x)}$$
$$= \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(77a^2\sqrt{a\text{sech}^3(x)}\right)\int}{117\text{sech}^{\frac{3}{2}}(}$$
$$= \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\,\text{ta}$$
$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\,\text{ta}$$
$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\,\text{ta}$$
$$= \frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x}$$

Mathematica [A] time = 0.0963402, size = 63, normalized size = 0.52

$$\frac{2}{585}a\text{sech}(x)\left(a\text{sech}^3(x)\right)^{3/2}\left(45\tanh(x) + 231\sinh(x)\cosh^5(x) + 77\sinh(x)\cosh^3(x) + 231i\cosh^{\frac{11}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 55\,\text{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] (2*a*Sech[x]*(a*Sech[x]^3)^(3/2)*((231*I)*Cosh[x]^(11/2)*EllipticE[(I/2)*x,
 2] + 55*Cosh[x]*Sinh[x] + 77*Cosh[x]^3*Sinh[x] + 231*Cosh[x]^5*Sinh[x] + 4
5*Tanh[x]))/585
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(a\left(\text{sech}\left(x\right)\right)^3\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(5/2),x)
```

```
[Out] int((a*sech(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{sech}\left(x\right)^3\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\,\mathrm{sech}\,(x)^3}a^2\,\mathrm{sech}\,(x)^6\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)*a^2*sech(x)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\mathrm{sech}\,(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

3.40 $\int \left(a\,\text{sech}^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$-\frac{10}{21} i a \cosh^{\frac{3}{2}}(x)\text{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\,\text{sech}^3(x)}+\frac{10}{21}a\sinh(x)\sqrt{a\,\text{sech}^3(x)}+\frac{2}{7}a\tanh(x)\text{sech}(x)\sqrt{a\,\text{sech}^3(x)}$$

```
[Out] ((-10*I)/21)*a*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (10*
a*Sqrt[a*Sech[x]^3]*Sinh[x])/21 + (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*Tanh[x])/7
```

Rubi [A] time = 0.0398334, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21}a\sinh(x)\sqrt{a\,\text{sech}^3(x)}+\frac{2}{7}a\tanh(x)\text{sech}(x)\sqrt{a\,\text{sech}^3(x)}-\frac{10}{21} i a \cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\,\text{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] ((-10*I)/21)*a*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (10*
a*Sqrt[a*Sech[x]^3]*Sinh[x])/21 + (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*Tanh[x])/7
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{3/2} dx = \frac{\left(a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{9}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{5}{2}}(x)\,dx}{7\text{sech}^{\frac{3}{2}}(x)}$$
$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{\text{sech}(x)}\,dx}{21\text{sech}^{\frac{3}{2}}(x)}$$
$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{1}{21}\left(5a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int$$
$$= -\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tan$$

Mathematica [A] time = 0.0361048, size = 47, normalized size = 0.68

$$\frac{2}{21}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\left(-5i\cosh^{\frac{5}{2}}(x)\text{EllipticF}\left(\frac{ix}{2},2\right) + 3\tanh(x) + 5\sinh(x)\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2]
+ 5*Cosh[x]*Sinh[x] + 3*Tanh[x]))/21
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \left(a\left(\text{sech}\,(x)\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(3/2),x)
```

```
[Out] int((a*sech(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\,\text{sech}\,(x)^3}a\,\text{sech}\,(x)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)*a*sech(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{sech}^3\,(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

3.41 $\int \sqrt{a \operatorname{sech}^3(x)}\,dx$

Optimal. Leaf size=46

$$2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}$$

```
[Out] (2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + 2*Cosh[x]*Sqr
t[a*Sech[x]^3]*Sinh[x]
```

Rubi [A] time = 0.0342023, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^3],x]
```

```
[Out] (2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + 2*Cosh[x]*Sqr
t[a*Sech[x]^3]*Sinh[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \frac{\sqrt{a\text{sech}^3(x)}\int \text{sech}^{\frac{3}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) - \frac{\sqrt{a\text{sech}^3(x)}\int \frac{1}{\sqrt{\text{sech}(x)}}\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) - \left(\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{\cosh(x)}\,dx$$
$$= 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + 2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x)$$

Mathematica [A] time = 0.0187185, size = 36, normalized size = 0.78

$$2\cosh(x)\sqrt{a\text{sech}^3(x)}\left(\sinh(x) + i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^3],x]
```

```
[Out] 2*Cosh[x]*Sqrt[a*Sech[x]^3]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x
])
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \sqrt{a(\text{sech}(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(1/2),x)
```

```
[Out] int((a*sech(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\text{sech}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\,\text{sech}(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{sech}^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \operatorname{sech}(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^3), x)
```

3.42 $\int \frac{1}{\sqrt{a\,\mathbf{sech}^3(x)}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}-\frac{2i\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}$$

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*x, 2])/(Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) + (2*T
anh[x])/(3*Sqrt[a*Sech[x]^3])
```

Rubi [A] time = 0.031463, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$\frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*x, 2])/(Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) + (2*T
anh[x])/(3*Sqrt[a*Sech[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}dx = \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{3}{2}}(x)}dx}{\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}} + \frac{\text{sech}^{\frac{3}{2}}(x)\int \sqrt{\text{sech}(x)}dx}{3\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}} + \frac{\int \frac{1}{\sqrt{\cosh(x)}}dx}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}$$

$$= -\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

Mathematica [A] time = 0.0401381, size = 38, normalized size = 0.79

$$\frac{2\tanh(x) - \frac{2i\text{EllipticF}\left(\frac{ix}{2},2\right)}{\cosh^{\frac{3}{2}}(x)}}{3\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + 2*Tanh[x])/(3*Sqrt[a*Sech[x
]^3])
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\text{sech}\left(x\right)\right)^{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sech(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\text{sech}\left(x\right)^{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\,\text{sech}\,(x)^3}}{a\,\text{sech}\,(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)/(a*sech(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\text{sech}^3\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\text{sech}\,(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

3.43 $\int \frac{1}{\left(a\,\mathbf{sech}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{14\sinh(x)}{45a\sqrt{a\,\text{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\,\text{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\,\text{sech}^3(x)}}$$

```
[Out] (((-14*I)/15)*EllipticE[(I/2)*x, 2])/(a*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) +
(14*Sinh[x])/(45*a*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^2*Sinh[x])/(9*a*Sqrt[a*S
ech[x]^3])
```

Rubi [A] time = 0.0441274, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2639}

$$\frac{14\sinh(x)}{45a\sqrt{a\,\text{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\,\text{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\,\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-14*I)/15)*EllipticE[(I/2)*x, 2])/(a*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) +
(14*Sinh[x])/(45*a*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^2*Sinh[x])/(9*a*Sqrt[a*S
ech[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}}\,dx &= \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int \frac{1}{\operatorname{sech}^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(7\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(7\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\operatorname{sech}(x)}}\,dx}{15a\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{7\int \sqrt{\cosh(x)}\,dx}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}\\
&= -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}} + \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0890085, size = 47, normalized size = 0.61

$$
\frac{33\sinh(x) + 5\sinh(3x) - \frac{84iE\left(\frac{ix}{2}\middle|2\right)}{\cosh^{\frac{3}{2}}(x)}}{90a\sqrt{a\operatorname{sech}^3(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-84*I)*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + 33*Sinh[x] + 5*Sinh[3*x])/
(90*a*Sqrt[a*Sech[x]^3])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$
\int \left(a\left(\operatorname{sech}(x)\right)^3\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sech(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(a\operatorname{sech}(x)^3\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\,\text{sech}\,(x)^3}}{a^2\,\text{sech}\,(x)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)/(a^2*sech(x)^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\text{sech}^3\,(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

3.44 $\int \frac{1}{\left(a\,\mathrm{sech}^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{26i\,\mathrm{EllipticF}\left(\frac{ix}{2},2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\,\mathrm{sech}^3(x)}}$$

```
[Out] (((-26*I)/77)*EllipticF[(I/2)*x, 2])/(a^2*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3])
+ (78*Cosh[x]*Sinh[x])/(385*a^2*Sqrt[a*Sech[x]^3]) + (26*Cosh[x]^3*Sinh[x])
/(165*a^2*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^5*Sinh[x])/(15*a^2*Sqrt[a*Sech[x]
^3]) + (26*Tanh[x])/(77*a^2*Sqrt[a*Sech[x]^3])
```

Rubi [A] time = 0.0642417, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$\frac{26\tanh(x)}{77a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\,\mathrm{sech}^3(x)}}-\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\,\mathrm{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (((-26*I)/77)*EllipticF[(I/2)*x, 2])/(a^2*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3])
+ (78*Cosh[x]*Sinh[x])/(385*a^2*Sqrt[a*Sech[x]^3]) + (26*Cosh[x]^3*Sinh[x])
/(165*a^2*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^5*Sinh[x])/(15*a^2*Sqrt[a*Sech[x]
^3]) + (26*Tanh[x])/(77*a^2*Sqrt[a*Sech[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx &= \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int \frac{1}{\operatorname{sech}^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{\left(13\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\operatorname{sech}^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{\left(39\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\operatorname{sech}^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{\left(39\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)}\,d\cdots}{77a^2\sqrt{a\operatorname{sech}^3(x)}}\\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{\left(13\operatorname{sec}\cdots\right.}{\cdots}\\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{1\cdots}{77a^2\,\mathrm{c}\cdots}\\
&= -\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(\cdots}{15a^2\sqrt{a\operatorname{sech}^3(x\cdots}}
\end{aligned}
$$

Mathematica [A] time = 0.0907472, size = 63, normalized size = 0.52

$$\frac{\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\left(-24960i\sqrt{\cosh(x)}\operatorname{EllipticF}\left(\frac{ix}{2},2\right)+19122\sinh(2x)+4406\sinh(4x)+826\sinh(6x)+77\sinh\cdots\right.}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^3]*((-24960*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
+ 19122*Sinh[2*x] + 4406*Sinh[4*x] + 826*Sinh[6*x] + 77*Sinh[8*x]))/(73920*
a^3)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(a\left(\operatorname{sech}(x)\right)^3\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*sech(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\,\mathrm{sech}\,(x)^3}}{a^3\,\mathrm{sech}\,(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)/(a^3*sech(x)^9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{sech}^3\,(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

3.45 $\int \left(a\,\mathrm{sech}^4(x)\right)^{7/2}\,dx$

Optimal. Leaf size=163

$$a^3\sinh(x)\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\,\mathrm{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{5}{3}a^3\,\mathrm{si}$$

```
[Out] a^3*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - 2*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*T
anh[x] + 3*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3 - (20*a^3*Sqrt[a*Sech[
x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (5*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^7
)/3 - (6*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^9)/11 + (a^3*Sqrt[a*Sech[x
]^4]*Sinh[x]^2*Tanh[x]^11)/13
```

Rubi [A] time = 0.0448859, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a^3\sinh(x)\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\,\mathrm{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{5}{3}a^3\,\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] a^3*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - 2*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*T
anh[x] + 3*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3 - (20*a^3*Sqrt[a*Sech[
x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (5*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^7
)/3 - (6*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^9)/11 + (a^3*Sqrt[a*Sech[x
]^4]*Sinh[x]^2*Tanh[x]^11)/13
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\,\mathrm{sech}^4(x)\right)^{7/2}\,dx &= \left(a^3\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\int \mathrm{sech}^{14}(x)\,dx\\ &= \left(ia^3\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\mathrm{Subst}\left(\int\left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)dx,x,-i\,\mathrm{ta}\right.\\ &= a^3\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}\sinh(x)-2a^3\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh(x)+3a^3\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\end{aligned}$$

Mathematica [A] time = 0.17192, size = 54, normalized size = 0.33

$$\frac{\sinh(x)\cosh(x)(2380\cosh(2x)+1093\cosh(4x)+378\cosh(6x)+92\cosh(8x)+14\cosh(10x)+\cosh(12x)+2048)(}{6006}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] (Cosh[x]*(2048 + 2380*Cosh[2*x] + 1093*Cosh[4*x] + 378*Cosh[6*x] + 92*Cosh[
8*x] + 14*Cosh[10*x] + Cosh[12*x])*(a*Sech[x]^4)^(7/2)*Sinh[x])/6006
```

Maple [A] time = 0.089, size = 72, normalized size = 0.4

$$-\frac{2048\,a^{3}{\mathrm e}^{-2\,x}\left(1716\,{\mathrm e}^{12\,x}+1287\,{\mathrm e}^{10\,x}+715\,{\mathrm e}^{8\,x}+286\,{\mathrm e}^{6\,x}+78\,{\mathrm e}^{4\,x}+13\,{\mathrm e}^{2\,x}+1\right)}{3003\left({\mathrm e}^{2\,x}+1\right)^{11}}\sqrt{\frac{a\,{\mathrm e}^{4\,x}}{\left({\mathrm e}^{2\,x}+1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(7/2),x)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(exp(2*x)+1)^11*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*
(1716*exp(12*x)+1287*exp(10*x)+715*exp(8*x)+286*exp(6*x)+78*exp(4*x)+13*exp
(2*x)+1)
```

Maxima [B] time = 1.65713, size = 837, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] 2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e
^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x)
 + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x)
+ 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) +
 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(
-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-
26*x) + 1) + 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-
6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 12
87*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x)
+ e^(-26*x) + 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 2
86*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*
x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(
-24*x) + e^(-26*x) + 1) + 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*
x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e
^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) +
13*e^(-24*x) + e^(-26*x) + 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) + 78*
e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22
*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) + 78*e
^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1
716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*
x) + 13*e^(-24*x) + e^(-26*x) + 1)
```

Fricas [B] time = 3.05414, size = 9415, normalized size = 57.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) +
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) + 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 + a^3 + (88*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(88*a^3*cosh(x)^2 + a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 + 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 + 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(11
88*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 + 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) + 2*(1188*a^3*cosh(x)^5 + 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3 + (5544*a^3*c
osh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(5544*a^
3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^
3*cosh(x))*e^(4*x) + 2*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
+ 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) + 2*(32670*a^3*cosh(x)^8 + 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(3630*a^3
*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 + 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3
 + (8712*a^3*cosh(x)^10 + 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(8712*a^3*cosh(x)^10 + 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 + a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) + 2*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x) + (792*a^3*cosh(x)^11 + 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a
^3*cosh(x))*e^(4*x) + 2*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 + 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 + 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 +
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 + 483*co
sh(x)^4 + 63*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 + 483*
cosh(x)^5 + 105*cosh(x)^3 + 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 + 12236*cosh(x)^6 + 3990*cosh(x)^4 + 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 + 15732*cosh(x)^7 + 7182*cosh(x)^5 + 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 + 66861*cosh
```

```
(x)^8 + 40698*cosh(x)^6 + 9690*cosh(x)^4 + 765*cosh(x)^2 + 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 + 81719*cosh(x)^9 + 63954*cosh(x)^7 + 21318*c
osh(x)^5 + 2805*cosh(x)^3 + 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 + 490314*cosh(x)^10 + 479655*cosh(x)^8 + 213180*cosh(x)^6 + 42075*c
osh(x)^4 + 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
+ 4056234*cosh(x)^11 + 4849845*cosh(x)^9 + 2771340*cosh(x)^7 + 765765*cosh(
x)^5 + 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 + 676039*cosh(x)^12 + 969969*cosh(x)^10 + 692835*cosh(x)^8 + 255255*
cosh(x)^6 + 45045*cosh(x)^4 + 3003*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 + 156009*cosh(x)^13 + 264537*cosh(x)^11 + 230945*cosh(x)
^9 + 109395*cosh(x)^7 + 27027*cosh(x)^5 + 3003*cosh(x)^3 + 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 + 178296*cosh(x)^14 + 352716*cosh(x)
^12 + 369512*cosh(x)^10 + 218790*cosh(x)^8 + 72072*cosh(x)^6 + 12012*cosh(x
)^4 + 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 + 59432
*cosh(x)^15 + 135660*cosh(x)^13 + 167960*cosh(x)^11 + 121550*cosh(x)^9 + 51
480*cosh(x)^7 + 12012*cosh(x)^5 + 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 + 66861*cosh(x)^16 + 174420*cosh(x)^14 + 2519
40*cosh(x)^12 + 218790*cosh(x)^10 + 115830*cosh(x)^8 + 36036*cosh(x)^6 + 59
40*cosh(x)^4 + 405*cosh(x)^2 + 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
+ 3933*cosh(x)^17 + 11628*cosh(x)^15 + 19380*cosh(x)^13 + 19890*cosh(x)^11
+ 12870*cosh(x)^9 + 5148*cosh(x)^7 + 1188*cosh(x)^5 + 135*cosh(x)^3 + 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 + 6118*cosh(x)^18 + 20349*cos
h(x)^16 + 38760*cosh(x)^14 + 46410*cosh(x)^12 + 36036*cosh(x)^10 + 18018*co
sh(x)^8 + 5544*cosh(x)^6 + 945*cosh(x)^4 + 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 + 966*cosh(x)^19 + 3591*cosh(x)^17 + 7752*cosh(x)
^15 + 10710*cosh(x)^13 + 9828*cosh(x)^11 + 6006*cosh(x)^9 + 2376*cosh(x)^7
+ 567*cosh(x)^5 + 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 + 5313*cosh(x)^20 + 21945*cosh(x)^18 + 53295*cosh(x)^16 + 84150*cos
h(x)^14 + 90090*cosh(x)^12 + 66066*cosh(x)^10 + 32670*cosh(x)^8 + 10395*cos
h(x)^6 + 1925*cosh(x)^4 + 165*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 + 253*cosh(x)^21 + 1155*cosh(x)^19 + 3135*cosh(x)^17 + 5610*cosh(x
)^15 + 6930*cosh(x)^13 + 6006*cosh(x)^11 + 3630*cosh(x)^9 + 1485*cosh(x)^7
+ 385*cosh(x)^5 + 55*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 + 276*cosh(x)^22 + 1386*cosh(x)^20 + 4180*cosh(x)^18 + 8415*cosh(x)^
16 + 11880*cosh(x)^14 + 12012*cosh(x)^12 + 8712*cosh(x)^10 + 4455*cosh(x)^8
 + 1540*cosh(x)^6 + 330*cosh(x)^4 + 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 + 12*cosh(x)^23 + 66*cosh(x)^21 + 220*cosh(x)^19 + 495*cosh(x
)^17 + 792*cosh(x)^15 + 924*cosh(x)^13 + 792*cosh(x)^11 + 495*cosh(x)^9 + 2
20*cosh(x)^7 + 66*cosh(x)^5 + 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 + 13*cosh(x)^24 + 78*cosh(x)^22 + 286*cosh(x)^20 + 715*cosh(x)^18
+ 1287*cosh(x)^16 + 1716*cosh(x)^14 + 1716*cosh(x)^12 + 1287*cosh(x)^10 + 7
15*cosh(x)^8 + 286*cosh(x)^6 + 78*cosh(x)^4 + 13*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20025, size = 69, normalized size = 0.42

$$-\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}+1287\,e^{(10\,x)}+715\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+1\right)}{3003\,\left(e^{(2\,x)}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) + 1287*e^(10*x) + 715*e^(8*x) + 286*e^(6*
x) + 78*e^(4*x) + 13*e^(2*x) + 1)/(e^(2*x) + 1)^13
```

3.46 $\int \left(a\,\mathrm{sech}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=117

$$a^2\sinh(x)\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\,\mathrm{sech}^4(x)}-\frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{6}{5}a^2\sinh^2(x$$

```
[Out] a^2*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (4*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*
Tanh[x])/3 + (6*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5 - (4*a^2*Sqrt[
a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x]^7)/9
```

Rubi [A] time = 0.0351459, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a^2\sinh(x)\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\,\mathrm{sech}^4(x)}-\frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\,\mathrm{sech}^4(x)}+\frac{6}{5}a^2\sinh^2(x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] a^2*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (4*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*
Tanh[x])/3 + (6*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5 - (4*a^2*Sqrt[
a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x]^7)/9
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\,\mathrm{sech}^4(x)\right)^{5/2}\,dx &= \left(a^2\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\int \mathrm{sech}^{10}(x)\,dx\\ &= \left(ia^2\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\mathrm{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-i\tanh(x)\right)\\ &= a^2\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}\sinh(x)-\frac{4}{3}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh(x)+\frac{6}{5}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\,\mathrm{ta}\end{aligned}$$

Mathematica [A] time = 0.0926697, size = 42, normalized size = 0.36

$$\frac{1}{315}\sinh(x)\cosh(x)(130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x)+128)\left(a\,\mathrm{sech}^4(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] (Cosh[x]*(128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*(a
*Sech[x]^4)^(5/2)*Sinh[x])/315
```

Maple [A] time = 0.07, size = 60, normalized size = 0.5

$$-\frac{256\,a^{2}\mathrm{e}^{-2\,x}\left(126\,\mathrm{e}^{8\,x}+84\,\mathrm{e}^{6\,x}+36\,\mathrm{e}^{4\,x}+9\,\mathrm{e}^{2\,x}+1\right)}{315\,\left(\mathrm{e}^{2\,x}+1\right)^{7}}\sqrt{\frac{a\mathrm{e}^{4\,x}}{\left(\mathrm{e}^{2\,x}+1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(5/2),x)
```

```
[Out] -256/315*a^2*exp(-2*x)/(exp(2*x)+1)^7*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*(12
6*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

Maxima [B] time = 1.76867, size = 435, normalized size = 3.72

$$\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\left(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18\,x)}+1\right)}+\frac{}{35\,(9\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) + 126*e^(-8
*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) + e^(-18*x)
 + 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) +
126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) +
e^(-18*x) + 1) + 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^
(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(
-16*x) + e^(-18*x) + 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1) + 256/315*a^(5/2)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1)
```

Fricas [B] time = 2.43031, size = 4632, normalized size = 39.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) + 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 + 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) + 2*a^2*cosh(x)*e^(2*x)
```

```
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 + a^2 + (42*a^2*cosh(x)^2
+ a^2)*e^(4*x) + 2*(42*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 + a^2*cosh(x) + (14*a^2*cosh(x)^3 + a^2*cosh
(x))*e^(4*x) + 2*(14*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 + 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) + 2*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 + 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) + 2*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 +
a^2 + (392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^(4
*x) + 2*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(126*a^2*cosh(x)^8 + 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x) + (56*a^2*cosh(x)
^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(56*a^2*
cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 + 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 + 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 + 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 + 195*cosh(x)^4
+ 39*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 + 273*cosh(x)^5
 + 91*cosh(x)^3 + 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 + 4004
*cosh(x)^6 + 2002*cosh(x)^4 + 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 + 25740*cosh(x)^7 + 18018*cosh(x)^5 + 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 + 6435*cosh(x)^8 + 6006*cosh
(x)^6 + 2310*cosh(x)^4 + 315*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 + 715*cosh(x)^9 + 858*cosh(x)^7 + 462*cosh(x)^5 + 105*cosh(x)^3 +
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 + 858*cosh(x)^10 + 1287*c
osh(x)^8 + 924*cosh(x)^6 + 315*cosh(x)^4 + 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 + 78*cosh(x)^11 + 143*cosh(x)^9 + 132*cosh(x)^7 +
63*cosh(x)^5 + 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 + 455*cosh(x)^12 + 1001*cosh(x)^10 + 1155*cosh(x)^8 + 735*cosh(x)^6 + 245
*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 + 105*
cosh(x)^13 + 273*cosh(x)^11 + 385*cosh(x)^9 + 315*cosh(x)^7 + 147*cosh(x)^5
 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 + 120*cos
h(x)^14 + 364*cosh(x)^12 + 616*cosh(x)^10 + 630*cosh(x)^8 + 392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 + 8*c
osh(x)^15 + 28*cosh(x)^13 + 56*cosh(x)^11 + 70*cosh(x)^9 + 56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 + 9*cosh
(x)^16 + 36*cosh(x)^14 + 84*cosh(x)^12 + 126*cosh(x)^10 + 126*cosh(x)^8 + 8
4*cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1718, size = 53, normalized size = 0.45

$$-\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(e
^(2*x) + 1)^9
```

3.47 $\int \left(a\,\text{sech}^4(x)\right)^{3/2} dx$

Optimal. Leaf size=61

$$a\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

```
[Out] a*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (2*a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x])/3 + (a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5
```

Rubi [A] time = 0.0234475, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] a*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (2*a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x])/3 + (a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a\,\text{sech}^4(x)\right)^{3/2} dx &= \left(a\cosh^2(x)\sqrt{a\,\text{sech}^4(x)}\right)\int \text{sech}^6(x)\,dx\\
&= \left(ia\cosh^2(x)\sqrt{a\,\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right) dx, x, -i\tanh(x)\right)\\
&= a\cosh(x)\sqrt{a\,\text{sech}^4(x)}\sinh(x)-\frac{2}{3}a\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh(x)+\frac{1}{5}a\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh
\end{aligned}$$

Mathematica [A] time = 0.0543789, size = 30, normalized size = 0.49

$$\frac{1}{15}\sinh(x)\cosh(x)(6\cosh(2x)+\cosh(4x)+8)\left(a\,\text{sech}^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*(a*Sech[x]^4)^(3/2)*Sinh[x])/15
```

Maple [A] time = 0.06, size = 46, normalized size = 0.8

$$-\frac{16\,a\mathrm{e}^{-2\,x}\left(10\,\mathrm{e}^{4\,x}+5\,\mathrm{e}^{2\,x}+1\right)}{15\,\left(\mathrm{e}^{2\,x}+1\right)^{3}}\sqrt{\frac{a\mathrm{e}^{4\,x}}{\left(\mathrm{e}^{2\,x}+1\right)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(3/2),x)
```

```
[Out] -16/15*a*exp(-2*x)/(exp(2*x)+1)^3*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*(10*exp
(4*x)+5*exp(2*x)+1)
```

Maxima [B] time = 1.72302, size = 162, normalized size = 2.66

$$\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+\frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(
-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) + 16/15*a^(3/2)/(5*e^(-2*x) + 10*e^(-4*
x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Fricas [B] time = 2.03966, size = 1623, normalized size = 26.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 + a)*e^(4*x) + 2*(12*a*cosh(x)^2 +
 a)*e^(2*x) + a)*sinh(x)^2 + (10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(4*x) +
 2*(10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 + a*cos
h(x) + (4*a*cosh(x)^3 + a*cosh(x))*e^(4*x) + 2*(4*a*cosh(x)^3 + a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 + 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cos
h(x)^4 + 15*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^
6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 + 4*
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
```

```
^10 + 5*cosh(x)^8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(3/2), x)
```

Giac [A] time = 1.17369, size = 36, normalized size = 0.59

$$-\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)} + 5\,e^{(2\,x)} + 1\right)}{15\,\left(e^{(2\,x)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

3.48 $\int \sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=15

$$\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^4(x)}$$

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rubi [A] time = 0.0166341, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 3767, 8}

$$\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\operatorname{sech}^4(x)}\,dx &= \left(\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\int \operatorname{sech}^2(x)\,dx\\ &= \left(i\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\operatorname{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\ &= \cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0051942, size = 15, normalized size = 1.

$$\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Maple [B] time = 0.075, size = 29, normalized size = 1.9

$$-2\sqrt{\frac{a\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}+1\right)^{4}}}\,\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(1/2),x)
```

```
[Out] -2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)
```

Maxima [A] time = 1.5763, size = 18, normalized size = 1.2

$$\frac{2\sqrt{a}}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) + 1)
```

Fricas [B] time = 2.10846, size = 230, normalized size = 15.33

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^{2}+\left(\cosh(x)^{2}+1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 + 1)*e^(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\operatorname{sech}^{4}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**4), x)
```

Giac [A] time = 1.17594, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{a}}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) + 1)
```

3.49 $\int \frac{1}{\sqrt{a\,\mathbf{sech}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] (x*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + Tanh[x]/(2*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.0162538, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + Tanh[x]/(2*Sqrt[a*Sech[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{sech}^4(x)}}\,dx &= \frac{\operatorname{sech}^2(x)\int \cosh^2(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\ &= \frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\operatorname{sech}^2(x)\int 1\,dx}{2\sqrt{a\operatorname{sech}^4(x)}}\\ &= \frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0220314, size = 23, normalized size = 0.64

$$\frac{\tanh(x) + x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2 + Tanh[x])/(2*Sqrt[a*Sech[x]^4])
```

Maple [B] time = 0.085, size = 89, normalized size = 2.5

$$\frac{e^{2x}x}{2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}}+\frac{e^{4x}}{8\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}}-\frac{1}{8\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x+1/8/(a*exp(
4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(4*x)-1/8/(a*exp(4*x)/(exp(2*x
)+1)^4)^(1/2)/(exp(2*x)+1)^2
```

Maxima [A] time = 1.72467, size = 41, normalized size = 1.14

$$-\frac{\left(\sqrt{a}e^{(-4x)} - \sqrt{a}\right)e^{(2x)}}{8a} + \frac{x}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x)/a + 1/2*x/sqrt(a)
```

Fricas [B] time = 2.33109, size = 771, normalized size = 21.42

$$\frac{\left(\left(e^{(4x)} + 2e^{(2x)} + 1\right)\sinh(x)^4 + \cosh(x)^4 + 4\left(\cosh(x)e^{(4x)} + 2\cosh(x)e^{(2x)} + \cosh(x)\right)\sinh(x)^3 + 4x\cosh(x)^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 + 2*x)*e^(4*x) + 2*(3*cosh(x)^2 + 2*x)*e^(2*x) + 2*x)*sinh(x)^
2 + (cosh(x)^4 + 4*x*cosh(x)^2 - 1)*e^(4*x) + 2*(cosh(x)^4 + 4*x*cosh(x)^2
```

```
- 1)*e^(2*x) + 4*(cosh(x)^3 + 2*x*cosh(x) + (cosh(x)^3 + 2*x*cosh(x))*e^(4*
x) + 2*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*
e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**4), x)
```

Giac [A] time = 1.13167, size = 38, normalized size = 1.06

$$-\frac{\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)}-4\,x-e^{(2\,x)}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/sqrt(a)
```

3.50 $\int \frac{1}{\left(a\,\mathbf{sech}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5x\,\text{sech}^2(x)}{16a\sqrt{a\,\text{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\,\text{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\,\text{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\,\text{sech}^4(x)}}$$

```
[Out] (5*x*Sech[x]^2)/(16*a*Sqrt[a*Sech[x]^4]) + (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a
*Sech[x]^4]) + (Cosh[x]^3*Sinh[x])/(6*a*Sqrt[a*Sech[x]^4]) + (5*Tanh[x])/(1
6*a*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.035756, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{5x\,\text{sech}^2(x)}{16a\sqrt{a\,\text{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\,\text{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\,\text{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\,\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sech[x]^2)/(16*a*Sqrt[a*Sech[x]^4]) + (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a
*Sech[x]^4]) + (Cosh[x]^3*Sinh[x])/(6*a*Sqrt[a*Sech[x]^4]) + (5*Tanh[x])/(1
6*a*Sqrt[a*Sech[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = \frac{\text{sech}^2(x)\int \cosh^6(x)\,dx}{a\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}} + \frac{\left(5\text{sech}^2(x)\right)\int \cosh^4(x)\,dx}{6a\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}} + \frac{\left(5\text{sech}^2(x)\right)\int \cosh^2(x)\,dx}{8a\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\text{sech}^4(x)}} + \frac{\left(5\text{sech}^2(x)\right)\int 1\,dx}{16a\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{5x\text{sech}^2(x)}{16a\sqrt{a\text{sech}^4(x)}} + \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\text{sech}^4(x)}}$$

Mathematica [A] time = 0.0360809, size = 38, normalized size = 0.44

$$\frac{(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))\text{sech}^6(x)}{192\left(a\text{sech}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Sech[x]
^4)^(3/2))
```

Maple [B] time = 0.058, size = 230, normalized size = 2.7

$$\frac{5\,e^{2x}x}{16\,a\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{e^{8x}}{384\,a\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{3\,e^{6x}}{128\,a\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{15\,e^{4x}}{128\,a\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} - \frac{}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(3/2),x)
```

```
[Out] 5/16/a*exp(2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*x+1/384/a*
exp(8*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+3/128/a*exp(6*x)/
(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(2*x
)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-15/128/a/(exp(2*x)+1)^2/(a*exp(4*x
)/(exp(2*x)+1)^4)^(1/2)-3/128/a*exp(-2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2
*x)+1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4
)^(1/2)
```

Maxima [A] time = 1.68946, size = 88, normalized size = 1.02

$$\frac{\left(9\,\sqrt{a}e^{(-2\,x)} + 45\,\sqrt{a}e^{(-4\,x)} - 45\,\sqrt{a}e^{(-8\,x)} - 9\,\sqrt{a}e^{(-10\,x)} - \sqrt{a}e^{(-12\,x)} + \sqrt{a}\right)e^{(6\,x)}}{384\,a^2} + \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(9*sqrt(a)*e^(-2*x) + 45*sqrt(a)*e^(-4*x) - 45*sqrt(a)*e^(-8*x) - 9*s
qrt(a)*e^(-10*x) - sqrt(a)*e^(-12*x) + sqrt(a))*e^(6*x)/a^2 + 5/16*x/a^(3/2
)
```

Fricas [B] time = 2.64698, size = 3687, normalized size = 42.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 + 3)*e^(4*x) + 2*(22*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^10 + 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 + 9*cosh(x))*e^(4*x) + 2*(22*cosh(x
)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 + 9*cosh
(x)^2 + (11*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(4*x) + 2*(11*cosh(x)^4 + 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 + 15*co
sh(x)^3 + (11*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) + 2*(11*cosh(x)
^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 + 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 +
315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(4*x) + 2*(154*cosh(x)^6 + 315*cosh
(x)^4 + 210*cosh(x)^2 + 20*x)*e^(2*x) + 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
+ 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x) + (22*cosh(x)^7 + 63*cosh(x)^5
 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(4*x) + 2*(22*cosh(x)^7 + 63*cosh(x)^5 +
70*cosh(x)^3 + 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 + 42*cos
h(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 + (11*cosh(x)^8 + 42*cosh(x)^6 + 70*
cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(4*x) + 2*(11*cosh(x)^8 + 42*cosh(x)^6 +
70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 + (11*cos
h(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) + 2*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 + 135*cosh(x)^8 +
420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 + 135*cosh(
x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 - 3)*e^(4*x) + 2*(22*
cosh(x)^10 + 135*cosh(x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2
 - 3)*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^12 + 9*cosh(x)^10 + 4
5*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4 - 9*cosh(x)^2 - 1)*e^(4*x) + 2
*(cosh(x)^12 + 9*cosh(x)^10 + 45*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4
 - 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(x)^7
 + 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(
x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x))*e^(4*x) + 2*(2*cosh(x)^1
1 + 15*cosh(x)^9 + 60*cosh(x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x
))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x
) + 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\operatorname{sech}^{4}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-3/2), x)
```

Giac [A] time = 1.13889, size = 70, normalized size = 0.81

$$-\frac{\left(110\,e^{(6\,x)} + 45\,e^{(4\,x)} + 9\,e^{(2\,x)} + 1\right)e^{(-6\,x)} - 120\,x - e^{(6\,x)} - 9\,e^{(4\,x)} - 45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))/a^(3/2)
```

3.51 $\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63x\operatorname{sech}^2(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{21\sinh(x}{128a^2\sqrt{a}}$$

```
[Out] (63*x*Sech[x]^2)/(256*a^2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]*Sinh[x])/(128*a^
2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]^3*Sinh[x])/(160*a^2*Sqrt[a*Sech[x]^4]) +
 (9*Cosh[x]^5*Sinh[x])/(80*a^2*Sqrt[a*Sech[x]^4]) + (Cosh[x]^7*Sinh[x])/(10
*a^2*Sqrt[a*Sech[x]^4]) + (63*Tanh[x])/(256*a^2*Sqrt[a*Sech[x]^4])
```

Rubi [A] time = 0.0561511, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{63x\operatorname{sech}^2(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}}+\frac{21\sinh(x}{128a^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-5/2),x]
```


```
[Out] (63*x*Sech[x]^2)/(256*a^2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]*Sinh[x])/(128*a^
2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]^3*Sinh[x])/(160*a^2*Sqrt[a*Sech[x]^4]) +
 (9*Cosh[x]^5*Sinh[x])/(80*a^2*Sqrt[a*Sech[x]^4]) + (Cosh[x]^7*Sinh[x])/(10
*a^2*Sqrt[a*Sech[x]^4]) + (63*Tanh[x])/(256*a^2*Sqrt[a*Sech[x]^4])
```


Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = \frac{\operatorname{sech}^2(x)\int \cosh^{10}(x)\,dx}{a^2\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(9\operatorname{sech}^2(x)\right)\int \cosh^8(x)\,dx}{10a^2\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(63\operatorname{sech}^2(x)\right)\int \cosh^6(x)\,dx}{80a^2\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(21\operatorname{sech}^2(x)\right)\int \cosh^4(x)\,dx}{32a^2\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{(63\operatorname{sech}^2(}{128a}$$
$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{63\tan}{256a^2\sqrt{as}}$$
$$= \frac{63x\operatorname{sech}^2(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x}{10a^2\sqrt{as}}$$

Mathematica [A] time = 0.0762338, size = 55, normalized size = 0.42

$$\frac{(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] + 150
*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] time = 0.069, size = 362, normalized size = 2.7

$$\frac{63\,e^{2x}x}{256\,a^2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{e^{12x}}{10240\,a^2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{5\,e^{10x}}{4096\,a^2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{ae^{4x}}{\left(e^{2x}+1\right)^4}}} + \frac{15\,e^{8x}}{2048\,a^2\left(e^{2x}+1\right)^2}\frac{1}{\sqrt{\frac{a}{(e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(5/2),x)
```

```
[Out] 63/256/a^2*exp(2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*x+1/10
240/a^2*exp(12*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+5/4096/a
^2*exp(10*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/2048/a^2*e
xp(8*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/512/a^2*exp(6*x
)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+105/1024/a^2*exp(4*x)/(e
xp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-105/1024/a^2/(exp(2*x)+1)^2/
(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-15/512/a^2*exp(-2*x)/(exp(2*x)+1)^2/(a*ex
```

```
p(4*x)/(exp(2*x)+1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)+1)^2/(a*exp(4*
x)/(exp(2*x)+1)^4)^(1/2)-5/4096/a^2*exp(-6*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(e
xp(2*x)+1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2
*x)+1)^4)^(1/2)
```

Maxima [A] time = 1.74461, size = 139, normalized size = 1.05

$$\frac{\left(25\,\sqrt{a}e^{(-2\,x)} + 150\,\sqrt{a}e^{(-4\,x)} + 600\,\sqrt{a}e^{(-6\,x)} + 2100\,\sqrt{a}e^{(-8\,x)} - 2100\,\sqrt{a}e^{(-12\,x)} - 600\,\sqrt{a}e^{(-14\,x)} - 150\,\sqrt{a}e^{(-16\,x)} - 25\,\sqrt{\cdot}\right.}{20480\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20480*(25*sqrt(a)*e^(-2*x) + 150*sqrt(a)*e^(-4*x) + 600*sqrt(a)*e^(-6*x)
+ 2100*sqrt(a)*e^(-8*x) - 2100*sqrt(a)*e^(-12*x) - 600*sqrt(a)*e^(-14*x) -
150*sqrt(a)*e^(-16*x) - 25*sqrt(a)*e^(-18*x) - 2*sqrt(a)*e^(-20*x) + 2*sqrt
(a))*e^(10*x)/a^3 + 63/256*x/a^(5/2)
```

Fricas [B] time = 2.761, size = 9443, normalized size = 71.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 + 5)*e^(4*x) + 2*(76*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x)^18 + 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 + 15*cosh(x))*e^(4*x) + 2*(7
6*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 + 255*cosh(x)^2 + (646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(4*x) + 2*(646
*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 + 425*cosh(x)^3 + (646*cosh(x)^5 + 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) + 2*(646*cosh(x)^5 + 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(4*x) + 2*(129
2*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^14
 + 600*cosh(x)^14 + 120*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(4*x) + 2*
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(2*x) + 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4
 + 910*cosh(x)^2 + (4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4 + 910*
cosh(x)^2 + 35)*e^(4*x) + 2*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)
^4 + 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + (4199*co
sh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) + 2*(4199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 + 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x
)^6 + 300300*cosh(x)^4 + 69300*cosh(x)^2 + 2520*x)*e^(4*x) + 2*(184756*cosh
(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^4 + 69300*cos
```

```
h(x)^2 + 2520*x)*e^(2*x) + 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 + 6077
5*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 2520*x*
cosh(x) + (16796*cosh(x)^11 + 60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cos
h(x)^5 + 23100*cosh(x)^3 + 2520*x*cosh(x))*e^(4*x) + 2*(16796*cosh(x)^11 +
60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 + 36465*cosh(x)^10 +
64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 + 36465*cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 346
50*cosh(x)^4 + 7560*x*cosh(x)^2 - 70)*e^(4*x) + 2*(8398*cosh(x)^12 + 36465*
cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*cosh(x)^5 + 2
520*x*cosh(x)^3 + (646*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580
*cosh(x)^7 + 6930*cosh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) + 2*(6
46*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*co
sh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 + 7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8
+ 32340*cosh(x)^6 + 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 +
7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17
640*x*cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(4*x) + 2*(1292*cosh(x)^14 + 7735*c
osh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17640*x*
cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^6 - 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 +
 69300*cosh(x)^7 + 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 +
8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7 + 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(4*x) + 2*(1292*cosh(x)^1
5 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7
+ 52920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(2*x) - 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*
cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cos
h(x)^2 + (323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*cosh(x
)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cosh(x)^2
 - 5)*e^(4*x) + 2*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 200
20*cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
+ 170*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040
*x*cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 + (19*cosh(x)^17 + 170*cosh(x)
^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*cosh(x)^7
- 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(19*cosh(x)^17 + 1
70*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*
cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
*sinh(x)^3 + 5*(76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14 + 10920*co
sh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^6 - 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14
 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^
6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(4*x) + 2*(76*cosh(x)^18 + 765*co
sh(x)^16 + 3600*cosh(x)^14 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x) -
 5)*sinh(x)^2 - 25*cosh(x)^2 + (2*cosh(x)^20 + 25*cosh(x)^18 + 150*cosh(x)^
16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 5040*x*cosh(x)^10 - 2100*cosh(x)^8
- 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 2)*e^(4*x) + 2*(2*cosh(x)^
20 + 25*cosh(x)^18 + 150*cosh(x)^16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 - 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 840
*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 84
0*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*co
sh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(4*cosh(x)^19 + 45*cosh(x)^
17 + 240*cosh(x)^15 + 840*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 -
```

```
1680*cosh(x)^7 - 360*cosh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cos
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\,\mathrm{sech}^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-5/2), x)
```

Giac [A] time = 1.12396, size = 103, normalized size = 0.78

$$-\frac{\left(5754\,e^{(10\,x)}+2100\,e^{(8\,x)}+600\,e^{(6\,x)}+150\,e^{(4\,x)}+25\,e^{(2\,x)}+2\right)e^{(-10\,x)}-5040\,x-2\,e^{(10\,x)}-25\,e^{(8\,x)}-150\,e^{(6\,x)}-600\,e}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e^
(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 600
*e^(4*x) - 2100*e^(2*x))/a^(5/2)
```

3.52 $\int \frac{\sinh^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

```
[Out] -x/(8*a) - (Cosh[x]*Sinh[x])/(8*a) + (Cosh[x]^3*Sinh[x])/(4*a) - Sinh[x]^3/
(3*a)
```

Rubi [A] time = 0.139033, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$-\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] -x/(8*a) - (Cosh[x]*Sinh[x])/(8*a) + (Cosh[x]^3*Sinh[x])/(4*a) - Sinh[x]^3/
(3*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^4(x)}{-a-a\cosh(x)}\,dx\\ &= -\frac{\int \cosh(x)\sinh^2(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh^2(x)\,dx}{a}\\ &= \frac{\cosh^3(x)\sinh(x)}{4a}-\frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a}-\frac{\int \cosh^2(x)\,dx}{4a}\\ &= -\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}-\frac{\int 1\,dx}{8a}\\ &= -\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.101697, size = 28, normalized size = 0.64

$$\frac{24\sinh(x)-8\sinh(3x)+3(\sinh(4x)-4x)}{96a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (24*Sinh[x] - 8*Sinh[3*x] + 3*(-4*x + Sinh[4*x]))/(96*a)
```

Maple [B] time = 0.029, size = 130, normalized size = 3.

$$-\frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\frac{5}{6a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{7}{8a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{8a}\ln\left(\tanh\left(\frac{x}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/4/a/(tanh(1/2*x)+1)^4+5/6/a/(tanh(1/2*x)+1)^3-7/8/a/(tanh(1/2*x)+1)^2+1/
8/a/(tanh(1/2*x)+1)-1/8/a*ln(tanh(1/2*x)+1)+1/4/a/(tanh(1/2*x)-1)^4+5/6/a/(
tanh(1/2*x)-1)^3+7/8/a/(tanh(1/2*x)-1)^2+1/8/a/(tanh(1/2*x)-1)+1/8/a*ln(tan
h(1/2*x)-1)
```

Maxima [A] time = 1.10776, size = 73, normalized size = 1.66

$$-\frac{\left(8\,e^{(-x)}-24\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}-\frac{x}{8\,a}-\frac{24\,e^{(-x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) - 24*e^(-3*x) - 3)*e^(4*x)/a - 1/8*x/a - 1/192*(24*e^(-x)
- 8*e^(-3*x) + 3*e^(-4*x))/a
```

Fricas [A] time = 2.38664, size = 113, normalized size = 2.57

$$\frac{(3\,\cosh(x)-2)\sinh(x)^3+3\left(\cosh(x)^3-2\,\cosh(x)^2+2\right)\sinh(x)-3\,x}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((3*cosh(x) - 2)*sinh(x)^3 + 3*(cosh(x)^3 - 2*cosh(x)^2 + 2)*sinh(x) -
 3*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sinh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(sech(x) + 1), x)/a
```

Giac [A] time = 1.08968, size = 57, normalized size = 1.3

$$-\frac{\left(24\,e^{(3\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+24\,x-3\,e^{(4\,x)}+8\,e^{(3\,x)}-24\,e^{x}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/192*((24*e^(3*x) - 8*e^x + 3)*e^(-4*x) + 24*x - 3*e^(4*x) + 8*e^(3*x) -
24*e^x)/a
```

3.53 $\int \frac{\sinh^3(x)}{a+a\mathbf{sech}(x)} dx$

Optimal. Leaf size=23

$$\frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

```
[Out] Cosh[x]^3/(3*a) - Sinh[x]^2/(2*a)
```

Rubi [A] time = 0.122056, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2835, 2564, 30, 2565}

$$\frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] Cosh[x]^3/(3*a) - Sinh[x]^2/(2*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^3(x)}{-a-a\cosh(x)}\,dx$$
$$= -\frac{\int \cosh(x)\sinh(x)\,dx}{a} + \frac{\int \cosh^2(x)\sinh(x)\,dx}{a}$$
$$= \frac{\text{Subst}(\int x\,dx, x, i\sinh(x))}{a} + \frac{\text{Subst}\left(\int x^2\,dx, x, \cosh(x)\right)}{a}$$
$$= \frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

Mathematica [A] time = 0.0445961, size = 23, normalized size = 1.

$$\frac{3\cosh(x)-3\cosh(2x)+\cosh(3x)-7}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (-7 + 3*Cosh[x] - 3*Cosh[2*x] + Cosh[3*x])/(12*a)
```

Maple [B] time = 0.024, size = 67, normalized size = 2.9

$$8\frac{1}{a}\left(1/24\,(\tanh(x/2)+1)^{-3}-1/8\,(\tanh(x/2)+1)^{-2}+1/8\,(\tanh(x/2)+1)^{-1}-1/24\,(\tanh(x/2)-1)^{-3}-1/8\,(\tanh(x/2)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*sech(x)),x)
```

```
[Out] 8/a*(1/24/(tanh(1/2*x)+1)^3-1/8/(tanh(1/2*x)+1)^2+1/8/(tanh(1/2*x)+1)-1/24/
(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/8/(tanh(1/2*x)-1))
```

Maxima [B] time = 1.16261, size = 62, normalized size = 2.7

$$-\frac{\left(3\,e^{(-x)}-3\,e^{(-2\,x)}-1\right)e^{(3\,x)}}{24\,a}+\frac{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x)/a + 1/24*(3*e^(-x) - 3*e^(-2*x) +
 e^(-3*x))/a
```

Fricas [A] time = 2.30223, size = 100, normalized size = 4.35

$$\frac{\cosh(x)^3+3\,(\cosh(x)-1)\sinh(x)^2-3\,\cosh(x)^2+3\,\cosh(x)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(x)^3 + 3*(cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 3*cosh(x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sinh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(sech(x) + 1), x)/a
```

Giac [A] time = 1.12878, size = 50, normalized size = 2.17

$$\frac{\left(3\,e^{(2\,x)} - 3\,e^{x} + 1\right)e^{(-3\,x)} + e^{(3\,x)} - 3\,e^{(2\,x)} + 3\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*x) - 3*e^x + 1)*e^(-3*x) + e^(3*x) - 3*e^(2*x) + 3*e^x)/a
```

3.54 $\int \frac{\sinh^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] x/(2*a) - Sinh[x]/a + (Cosh[x]*Sinh[x])/(2*a)
```

Rubi [A] time = 0.100531, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2637, 2635, 8}

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/(2*a) - Sinh[x]/a + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^2(x)}{-a - a\cosh(x)}\,dx$$
$$= -\frac{\int \cosh(x)\,dx}{a} + \frac{\int \cosh^2(x)\,dx}{a}$$
$$= -\frac{\sinh(x)}{a} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\sinh(x)}{a} + \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [A] time = 0.059689, size = 16, normalized size = 0.59

$$\frac{x + \sinh(x)(\cosh(x) - 2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (x + (-2 + Cosh[x])*Sinh[x])/(2*a)
```

Maple [B] time = 0.023, size = 78, normalized size = 2.9

$$-\frac{1}{2\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{3}{2\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{1}{2\,a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{2\,a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2} + \frac{3}{2\,a}\left(\tanh\left(\frac{x}{2}\right)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)+1/2/
a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.08288, size = 57, normalized size = 2.11

$$-\frac{\left(4\,e^{(-x)} - 1\right)e^{(2\,x)}}{8\,a} + \frac{x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a + 1/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a
```

Fricas [A] time = 2.48415, size = 47, normalized size = 1.74

$$\frac{(\cosh(x) - 2)\sinh(x) + x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(x) - 2)*sinh(x) + x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sinh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15785, size = 38, normalized size = 1.41

$$\frac{(4\,e^x - 1)e^{(-2\,x)} + 4\,x + e^{(2\,x)} - 4\,e^x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/8*((4*e^x - 1)*e^(-2*x) + 4*x + e^(2*x) - 4*e^x)/a
```

3.55 $\int \frac{\sinh(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cosh(x)}{a} - \frac{\log(\cosh(x)+1)}{a}$$

```
[Out] Cosh[x]/a - Log[1 + Cosh[x]]/a
```

Rubi [A] time = 0.0730798, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\cosh(x)}{a} - \frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + a*Sech[x]),x]

[Out] Cosh[x]/a - Log[1 + Cosh[x]]/a
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cosh(x)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)dx, x, -a\cosh(x)\right)}{a^2} \\
&= \frac{\cosh(x)}{a} - \frac{\log(1+\cosh(x))}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0180007, size = 16, normalized size = 0.94

$$\frac{\cosh(x) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x] - 2*Log[Cosh[x/2]])/a
```

Maple [A] time = 0.017, size = 27, normalized size = 1.6

$$-\frac{\ln(1+\text{sech}(x))}{a} + \frac{1}{a\text{sech}(x)} + \frac{\ln(\text{sech}(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*sech(x)),x)
```

```
[Out] -1/a*ln(1+sech(x))+1/a/sech(x)+1/a*ln(sech(x))
```

Maxima [B] time = 1.12734, size = 47, normalized size = 2.76

$$-\frac{x}{a} + \frac{e^{(-x)}}{2a} + \frac{e^x}{2a} - \frac{2\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^(-x) + 1)/a
```

Fricas [B] time = 2.40772, size = 200, normalized size = 11.76

$$\frac{2x\cosh(x) + \cosh(x)^2 - 4(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1) + 2(x+\cosh(x))\sinh(x) + \sinh(x)^2 + 1}{2(a\cosh(x) + a\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*cosh(x) + cosh(x)^2 - 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(a*cosh(x) + a*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sinh(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15025, size = 43, normalized size = 2.53

$$\frac{x}{a} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{2\,\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^x + 1)/a
```

3.56 $\int \frac{\mathrm{csch}(x)}{a+a\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\mathrm{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\mathrm{csch}(x)}{2a}$$

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rubi [A] time = 0.0979849, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$\frac{\mathrm{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\mathrm{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\operatorname{csch}(x)\,dx}{a} - \frac{\int \coth(x)\operatorname{csch}^2(x)\,dx}{a}\\
&= -\frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\int \operatorname{csch}(x)\,dx}{2a} - \frac{\operatorname{Subst}(\int x\,dx, x, -i\operatorname{csch}(x))}{a}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{2a} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\operatorname{csch}^2(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0485752, size = 44, normalized size = 1.33

$$-\frac{\operatorname{sech}(x)\left(2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1\right)}{2a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -((1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))*Sech[x])/(2*a*(1 +
Sech[x]))
```

Maple [A] time = 0.02, size = 23, normalized size = 0.7

$$\frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*sech(x)),x)
```

```
[Out] 1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

Maxima [A] time = 1.11196, size = 65, normalized size = 1.97

$$-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Fricas [B] time = 2.41551, size = 397, normalized size = 12.03

$$-\frac{\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\left(\cosh\right.\right.}{2\left(a\cosh\left(x\right)^{2}+a\sinh\left(x\right)^{2}+2a\cosh\left(x\right)+2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{csch}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)/(sech(x) + 1), x)/a
```

Giac [A] time = 1.11287, size = 70, normalized size = 2.12

$$-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4a}+\frac{e^{(-x)}+e^{x}-2}{4a\left(e^{(-x)}+e^{x}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

3.57 $\int \frac{\text{csch}^2(x)}{a+a\,\text{sech}(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\text{csch}^3(x)}{3a}-\frac{\coth^3(x)}{3a}$$

```
[Out] -Coth[x]^3/(3*a) + Csch[x]^3/(3*a)
```

Rubi [A] time = 0.138909, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2606, 30, 2607}

$$\frac{\text{csch}^3(x)}{3a}-\frac{\coth^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + a*Sech[x]),x]
```

```
[Out] -Coth[x]^3/(3*a) + Csch[x]^3/(3*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\int \frac{\coth(x)\operatorname{csch}(x)}{-a-a\cosh(x)}\,dx$$
$$= \frac{\int \coth^2(x)\operatorname{csch}^2(x)\,dx}{a} - \frac{\int \coth(x)\operatorname{csch}^3(x)\,dx}{a}$$
$$= -\frac{i\operatorname{Subst}\left(\int x^2\,dx, x, i\coth(x)\right)}{a} - \frac{i\operatorname{Subst}\left(\int x^2\,dx, x, -i\operatorname{csch}(x)\right)}{a}$$
$$= -\frac{\coth^3(x)}{3a} + \frac{\operatorname{csch}^3(x)}{3a}$$

Mathematica [A] time = 0.0381574, size = 25, normalized size = 1.09

$$-\frac{(2\cosh(x)+\cosh(2x)+3)\operatorname{csch}(x)}{6a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Sech[x]),x]

[Out] -((3 + 2*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

Maple [A] time = 0.022, size = 23, normalized size = 1.

$$\frac{1}{4a}\left(-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \left(\tanh\left(\frac{x}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*sech(x)),x)

[Out] 1/4/a*(-1/3*tanh(1/2*x)^3-1/tanh(1/2*x))
```

Maxima [B] time = 1.14342, size = 122, normalized size = 5.3

$$-\frac{4e^{(-x)}}{3\left(2ae^{(-x)}-2ae^{(-3x)}-ae^{(-4x)}+a\right)} - \frac{2e^{(-2x)}}{2ae^{(-x)}-2ae^{(-3x)}-ae^{(-4x)}+a} - \frac{2}{3\left(2ae^{(-x)}-2ae^{(-3x)}-ae^{(-4x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="maxima")

[Out] -4/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) -
a*e^(-4*x) + a)
```

Fricas [B] time = 2.29228, size = 230, normalized size = 10.

$$-\frac{4\,(2\,\cosh(x)+\sinh(x)+1)}{3\left(a\cosh(x)^3 + a\sinh(x)^3 + 2\,a\cosh(x)^2 + (3\,a\cosh(x)+2\,a)\sinh(x)^2 - a\cosh(x) + \left(3\,a\cosh(x)^2 + 4\,a\cosh(x) + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x) + 1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 +
 (3*a*cosh(x) + 2*a)*sinh(x)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) +
 a)*sinh(x) - 2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15226, size = 42, normalized size = 1.83

$$-\frac{1}{2\,a(e^x-1)}+\frac{3\,e^{(2\,x)}+1}{6\,a(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 1)/(a*(e^x + 1)^3)
```

3.58 $\int \frac{\mathbf{csch}^3(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=46

$$\frac{\mathbf{csch}^4(x)}{4a}+\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\mathbf{csch}^3(x)}{4a}-\frac{\coth(x)\mathbf{csch}(x)}{8a}$$

```
[Out] ArcTanh[Cosh[x]]/(8*a) - (Coth[x]*Csch[x])/(8*a) - (Coth[x]*Csch[x]^3)/(4*a
) + Csch[x]^4/(4*a)
```

Rubi [A] time = 0.194662, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\mathbf{csch}^4(x)}{4a}+\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\mathbf{csch}^3(x)}{4a}-\frac{\coth(x)\mathbf{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Sech[x]),x]
```


```
[Out] ArcTanh[Cosh[x]]/(8*a) - (Coth[x]*Csch[x])/(8*a) - (Coth[x]*Csch[x]^3)/(4*a
) + Csch[x]^4/(4*a)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^2(x)}{-a-a\cosh(x)}\,dx\\ &= \frac{\int \coth^2(x)\operatorname{csch}^3(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^4(x)\,dx}{a}\\ &= -\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\int \operatorname{csch}^3(x)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= -\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}-\frac{\int \operatorname{csch}(x)\,dx}{8a}\\ &= \frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.154225, size = 59, normalized size = 1.28

$$\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(-2\operatorname{csch}^2\left(\frac{x}{2}\right)+\operatorname{sech}^4\left(\frac{x}{2}\right)-4\log\left(\sinh\left(\frac{x}{2}\right)\right)+4\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)}{16(a\operatorname{sech}(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*(-2*Csch[x/2]^2 + 4*Log[Cosh[x/2]] - 4*Log[Sinh[x/2]] + Sech[x
/2]^4)*Sech[x])/(16*(a + a*Sech[x]))
```

Maple [A] time = 0.027, size = 45, normalized size = 1.

$$\frac{1}{32\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{1}{16\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{16\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{1}{8\,a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/32/a*tanh(1/2*x)^4-1/16/a*tanh(1/2*x)^2-1/16/a/tanh(1/2*x)^2-1/8/a*ln(tan
h(1/2*x))
```

Maxima [B] time = 1.12578, size = 134, normalized size = 2.91

$$-\frac{e^{(-x)} + 2\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + e^{(-5\,x)}}{4\left(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a\right)} + \frac{\log\left(e^{(-x)} + 1\right)}{8\,a} - \frac{\log\left(e^{(-x)} - 1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(e^(-x) + 2*e^(-2*x) + 10*e^(-3*x) + 2*e^(-4*x) + e^(-5*x))/(2*a*e^(-x
) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) + a)
 + 1/8*log(e^(-x) + 1)/a - 1/8*log(e^(-x) - 1)/a
```

Fricas [B] time = 2.5358, size = 2060, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(x)^5 + 2*(5*cosh(x) + 2)*sinh(x)^4 + 2*sinh(x)^5 + 4*cosh(x)^4
 + 4*(5*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x)^3 + 20*cosh(x)^3 + 4*(5*cosh(x)^
3 + 6*cosh(x)^2 + 15*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 - (cosh(x)^6 + 2*
(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*co
sh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) -
 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 -
12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*co
sh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) +
 sinh(x) + 1) + (cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*co
sh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh
(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4
+ 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3
*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)
 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(5*cosh(x)^4 + 8*cosh(x)^3
 + 30*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 2*cosh(x))/(a*cosh(x)^6 + a*sinh
(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (15*a
*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(x)^3
 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(x)^4
 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(
x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 - a*c
osh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}^{3}(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(sech(x) + 1), x)/a
```

Giac [B] time = 1.1941, size = 122, normalized size = 2.65

$$\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}-\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}+\frac{e^{(-x)}+e^{x}-6}{16\,a\left(e^{(-x)}+e^{x}-2\right)}-\frac{3\left(e^{(-x)}+e^{x}\right)^{2}+12\,e^{(-x)}+12\,e^{x}-4}{32\,a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/16*log(e^(-x) + e^x + 2)/a - 1/16*log(e^(-x) + e^x - 2)/a + 1/16*(e^(-x)
+ e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(3*(e^(-x) + e^x)^2 + 12*e^(-x) +
12*e^x - 4)/(a*(e^(-x) + e^x + 2)^2)
```

3.59 $\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)} dx$

Optimal. Leaf size=34

$$-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^5(x)}{5a}$$

```
[Out] Coth[x]^3/(3*a) - Coth[x]^5/(5*a) + Csch[x]^5/(5*a)
```

Rubi [A] time = 0.146252, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Coth[x]^5/(5*a) + Csch[x]^5/(5*a)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^3(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\operatorname{csch}^4(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^5(x)\,dx}{a}\\
&= \frac{i\operatorname{Subst}\left(\int x^4\,dx,x,-i\operatorname{csch}(x)\right)}{a}+\frac{i\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\coth(x)\right)}{a}\\
&= \frac{\operatorname{csch}^5(x)}{5a}+\frac{i\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\coth(x)\right)}{a}\\
&= \frac{\coth^3(x)}{3a}-\frac{\coth^5(x)}{5a}+\frac{\operatorname{csch}^5(x)}{5a}
\end{aligned}
$$

Mathematica [A] time = 0.0599826, size = 39, normalized size = 1.15

$$\frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x)-15)\operatorname{csch}^3(x)}{60a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] ((-15 - 6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(60*a
*(1 + Cosh[x]))
```

Maple [A] time = 0.026, size = 39, normalized size = 1.2

$$\frac{1}{16\,a}\left(-\frac{1}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5+\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3+2\,(\tanh(x/2))^{-1}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*sech(x)),x)
```

```
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5+2/3*tanh(1/2*x)^3+2/tanh(1/2*x)-1/3/tanh(1/2*x)^
3)
```

Maxima [B] time = 1.17394, size = 394, normalized size = 11.59

$$\frac{8\,e^{(-x)}}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a\right)}-\frac{}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 8/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 8/15*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 8/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a
*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 4*e^(-4*x)/(2*a
*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e
^(-7*x) - a*e^(-8*x) + a) + 4/15/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
+ 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] time = 2.36489, size = 695, normalized size = 20.44

$$-\frac{}{15\left(a\cosh(x)^6 + a\sinh(x)^6 + 2\,a\cosh(x)^5 + 2\,(3\,a\cosh(x) + a)\sinh(x)^5 - 2\,a\cosh(x)^4 + \left(15\,a\cosh(x)^2 + 10\,a\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -8/15*(7*cosh(x)^2 + 4*(4*cosh(x) + 1)*sinh(x) + 7*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - 2*a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - 2*a)*sinh(x)^4 - 6*a*
cosh(x)^3 + 2*(10*a*cosh(x)^3 + 10*a*cosh(x)^2 - 4*a*cosh(x) - 3*a)*sinh(x)
^3 - a*cosh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 12*a*cosh(x)^2 - 18*a
*cosh(x) - a)*sinh(x)^2 + 4*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 -
4*a*cosh(x)^3 - 9*a*cosh(x)^2 + a*cosh(x) + 4*a)*sinh(x) + 2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(sech(x) + 1), x)/a
```

Giac [B] time = 1.182, size = 80, normalized size = 2.35

$$\frac{3\,e^{(2\,x)} - 12\,e^x + 5}{24\,a(e^x - 1)^3} - \frac{15\,e^{(4\,x)} + 60\,e^{(3\,x)} + 10\,e^{(2\,x)} + 20\,e^x + 7}{120\,a(e^x + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) - 12*e^x + 5)/(a*(e^x - 1)^3) - 1/120*(15*e^(4*x) + 60*e^(3
*x) + 10*e^(2*x) + 20*e^x + 7)/(a*(e^x + 1)^5)
```

3.60 $\int \frac{\sinh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=132

$$\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)}{8a^4}-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5}$$

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^5 + ((8*b*(a^2 - b^2) - a*(
3*a^2 - 4*b^2)*Cosh[x])*Sinh[x])/(8*a^4) - ((4*b - 3*a*Cosh[x])*Sinh[x]^3)/
(12*a^2)
```

Rubi [A] time = 0.369727, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2865, 2735, 2659, 205}

$$\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)}{8a^4}-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^5 + ((8*b*(a^2 - b^2) - a*(
3*a^2 - 4*b^2)*Cosh[x])*Sinh[x])/(8*a^4) - ((4*b - 3*a*Cosh[x])*Sinh[x]^3)/
(12*a^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^4(x)}{-b-a\cosh(x)}\,dx$$

$$= -\frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\int \frac{\left(-ab+\left(3a^2-4b^2\right)\cosh(x)\right)\sinh^2(x)}{-b-a\cosh(x)}\,dx}{4a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} - \frac{\int \frac{-ab\left(5a^2-4b^2\right)+\left(3a^4-1\right.}{-b-a\cos}}{8a}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3}{12a^2}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3}{12a^2}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right.\right.}{8a^4}$$

Mathematica [A] time = 0.70483, size = 219, normalized size = 1.66

$$\frac{-144a^2b^2x+24ab\left(5a^2-4b^2\right)\sinh(x)-24a^2\left(a^2-b^2\right)\sinh(2x)+\frac{192b^5\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{384a^2b^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{192}{}}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (36*a^4*x - 144*a^2*b^2*x + 96*b^4*x + (192*a^4*b*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (384*a^2*b^3*ArcTan[((-a + b)*Tanh[x
/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (192*b^5*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*b*(5*a^2 - 4*b^2)*Sinh[x] - 24*
a^2*(a^2 - b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [B] time = 0.036, size = 488, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sech(x)),x)
```

```
[Out] -2/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+4*
b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2
*b^5/a^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-
1/4/a/(tanh(1/2*x)+1)^4+1/2/a/(tanh(1/2*x)+1)^3+1/8/a/(tanh(1/2*x)+1)^2-3/8
/a/(tanh(1/2*x)+1)+3/8/a*ln(tanh(1/2*x)+1)+1/4/a/(tanh(1/2*x)-1)^4+1/2/a/(t
anh(1/2*x)-1)^3-1/8/a/(tanh(1/2*x)-1)^2-3/8/a/(tanh(1/2*x)-1)-3/8/a*ln(tanh
(1/2*x)-1)+1/2/a^3/(tanh(1/2*x)-1)^2*b^2+3/2/a^3*ln(tanh(1/2*x)-1)*b^2-1/a^
5*ln(tanh(1/2*x)-1)*b^4-1/a^2/(tanh(1/2*x)-1)*b+1/2/a^3/(tanh(1/2*x)-1)*b^2
+1/a^4/(tanh(1/2*x)-1)*b^3-1/a^2/(tanh(1/2*x)+1)*b+1/2/a^3/(tanh(1/2*x)+1)*
b^2+1/a^4/(tanh(1/2*x)+1)*b^3+1/3/a^2/(tanh(1/2*x)-1)^3*b+1/2/a^2/(tanh(1/2
*x)-1)^2*b-3/2/a^3*ln(tanh(1/2*x)+1)*b^2+1/a^5*ln(tanh(1/2*x)+1)*b^4+1/3/a^
2/(tanh(1/2*x)+1)^3*b-1/2/a^2/(tanh(1/2*x)+1)^2*b-1/2/a^3/(tanh(1/2*x)+1)^2
*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74173, size = 4609, normalized size = 34.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*co
sh(x) - a^3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)
^2 - 14*a^3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b
^2 + 8*b^4)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh
(x)^3 - 7*a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*
sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 -
180*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a
^3*b - 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)
^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a
^4 - a^2*b^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5
*a^3*b - 4*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*
(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^
4 - 2*a^2*b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b -
 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 - 192*((a^2*
b - b^3)*cosh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*co
sh(x)^2*sinh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(
x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x
) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
 + b)*sinh(x) + a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^
2*b^2)*cosh(x)^5 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*
b - 4*a*b^3)*cosh(x)^4 + a^3*b - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 -
 a^2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^
```

```
5*cosh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4), 1/192*(3*
a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cosh(x) - a^
3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^2 - 14*a^
3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b^2 + 8*b^4
)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(x)^3 - 7*
a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5
+ 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 - 180*(a^4 -
 a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a^3*b - 4*a
*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)^3 + 8*(21
*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a^4 - a^2*b
^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5*a^3*b - 4
*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(7*a^4*cos
h(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^4 - 2*a^2*
b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b - 4*a*b^3)*
cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 + 384*((a^2*b - b^3)*c
osh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*cosh(x)^2*si
nh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(x)^4)*sqrt
(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 8*(3*a^4
*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^2*b^2)*cosh(x)^5 + 12*(3*a^4 -
 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*b - 4*a*b^3)*cosh(x)^4 + a^3*b
 - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x))/(a
^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*cosh(x)^2*sinh(x)^2 + 4*a^5*
cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.1708, size = 266, normalized size = 2.02

$$\frac{3a^3e^{(4x)} - 8a^2be^{(3x)} - 24a^3e^{(2x)} + 24ab^2e^{(2x)} + 120a^2be^x - 96b^3e^x}{192a^4} + \frac{\left(3a^4 - 12a^2b^2 + 8b^4\right)x}{8a^5} + \frac{\left(8a^3be^x - 3a^4 - 24\left(5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) - 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
+ 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x/a^5
+ 1/192*(8*a^3*b*e^x - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*e^(3*x) + 24*(a^4 - a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^x +
 b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5)
```

3.61 $\int \frac{\sinh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\cosh(x)}{a^3}+\frac{b\left(a^2-b^2\right)\log(a\cosh(x)+b)}{a^4}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}$$

```
[Out] -(((a^2 - b^2)*Cosh[x])/a^3) - (b*Cosh[x]^2)/(2*a^2) + Cosh[x]^3/(3*a) + (b
*(a^2 - b^2)*Log[b + a*Cosh[x]])/a^4
```

Rubi [A] time = 0.180163, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$-\frac{(a^2-b^2)\cosh(x)}{a^3}+\frac{b\left(a^2-b^2\right)\log(a\cosh(x)+b)}{a^4}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(((a^2 - b^2)*Cosh[x])/a^3) - (b*Cosh[x]^2)/(2*a^2) + Cosh[x]^3/(3*a) + (b
*(a^2 - b^2)*Log[b + a*Cosh[x]])/a^4
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^3(x)}{-b-a\cosh(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)dx, x, -a\cosh(x)\right)}{a^4}$$
$$= -\frac{(a^2-b^2)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}$$

Mathematica [A] time = 0.126838, size = 66, normalized size = 1.08

$$\frac{(12ab^2-9a^3)\cosh(x)-3a^2b\cosh(2x)+12a^2b\log(a\cosh(x)+b)+a^3\cosh(3x)-12b^3\log(a\cosh(x)+b)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] - 3*a^2*b*Cosh[2*x] + a^3*Cosh[3*x] + 12*a^2*b
*Log[b + a*Cosh[x]] - 12*b^3*Log[b + a*Cosh[x]])/(12*a^4)
```

Maple [B] time = 0.031, size = 361, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2-1/2/a^2/(tanh(1/2*x)+1)^2*b-b/a^2*ln(tanh(1/2*x)+1
)+b^3/a^4*ln(tanh(1/2*x)+1)+1/3/a/(tanh(1/2*x)+1)^3-1/2/a/(tanh(1/2*x)+1)+1
/2/a^2/(tanh(1/2*x)+1)*b+1/a^3/(tanh(1/2*x)+1)*b^2-1/3/a/(tanh(1/2*x)-1)^3-
1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*b-b/a^2*ln(tanh(1/2*x)-1)
+b^3/a^4*ln(tanh(1/2*x)-1)+1/2/a/(tanh(1/2*x)-1)-1/2/a^2/(tanh(1/2*x)-1)*b-
1/a^3/(tanh(1/2*x)-1)*b^2+b/a/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
-b^2/a^2/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-b^3/a^3/(a-b)*ln(a*t
anh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+b^4/a^4/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2
*x)^2*b+a+b)
```

Maxima [B] time = 1.1307, size = 173, normalized size = 2.84

$$-\frac{\left(3\,abe^{(-x)}-a^2+3\left(3\,a^2-4\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}-\frac{3\,abe^{(-2\,x)}-a^2e^{(-3\,x)}+3\left(3\,a^2-4\,b^2\right)e^{(-x)}}{24\,a^3}+\frac{\left(a^2b-b^3\right)x}{a^4}+\frac{\left(a^2b-b^3\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) - a^2*e^(-3*x) + 3*(3*a^2 - 4*b^2)*e^(-x))/a^3 + (a^2*b - b
^3)*x/a^4 + (a^2*b - b^3)*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^4
```

Fricas [B] time = 2.76321, size = 1251, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 - 24*(a^2*b - b^3)*x*cosh(x)^3 - 3*(3*a^3 - 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) - 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 - 12*(a^2*b -
b^3)*x - 6*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 - 3*(3*a^3 - 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 + 4*a*b^2 - 24
*(a^2*b - b^3)*x*cosh(x) - 6*(3*a^3 - 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 24*((
a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b - b^3
)*cosh(x)*sinh(x)^2 + (a^2*b - b^3)*sinh(x)^3)*log(2*(a*cosh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 - 24*(a^2*b - b^3)*
x*cosh(x)^2 - 4*(3*a^3 - 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 - 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*sech(x)), x)
```

Giac [A] time = 1.17214, size = 117, normalized size = 1.92

$$\frac{a^2\left(e^{(-x)} + e^x\right)^3 - 3\,ab\left(e^{(-x)} + e^x\right)^2 - 12\,a^2\left(e^{(-x)} + e^x\right) + 12\,b^2\left(e^{(-x)} + e^x\right)}{24\,a^3} + \frac{\left(a^2b - b^3\right)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(-x) + e^x)^3 - 3*a*b*(e^(-x) + e^x)^2 - 12*a^2*(e^(-x) + e^x)
 + 12*b^2*(e^(-x) + e^x))/a^3 + (a^2*b - b^3)*log(abs(a*(e^(-x) + e^x) + 2*
b))/a^4
```

3.62 $\int \frac{\sinh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=82

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tanh[x/2])/Sqrt[a + b]])/a^3 - ((2*b - a*Cosh[x])*Sinh[x])/(2*a^2)
```

Rubi [A] time = 0.212117, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2865, 2735, 2659, 205}

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tanh[x/2])/Sqrt[a + b]])/a^3 - ((2*b - a*Cosh[x])*Sinh[x])/(2*a^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^2(x)}{-b-a\cosh(x)}\,dx\\
&= -\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}+\frac{\int \frac{-ab+\left(a^2-2b^2\right)\cosh(x)}{-b-a\cosh(x)}\,dx}{2a^2}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{\left(2b\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}+\frac{2\sqrt{a-b}b\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.153419, size = 76, normalized size = 0.93

$$\frac{-8b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)-2a^2x+a^2\sinh(2x)-4ab\sinh(x)+4b^2x}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - 8*b*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[
a^2 - b^2]] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] time = 0.027, size = 213, normalized size = 2.6

$$-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{b}{a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)+1/a^2/(tanh(1/2*x)+1)*b-1/2/
a*ln(tanh(1/2*x)+1)+1/a^3*ln(tanh(1/2*x)+1)*b^2+1/2/a/(tanh(1/2*x)-1)^2+1/2
/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b+1/2/a*ln(tanh(1/2*x)-1)-1/a^3*ln
(tanh(1/2*x)-1)*b^2+2/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+
b)*(a-b))^(1/2))-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a
+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75851, size = 1500, normalized size = 18.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*c
osh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(
x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*
cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*si
nh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2
*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a
*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^2 + 2
*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2), 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 -
 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*cosh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh
(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2
)*x)*sinh(x)^2 - 16*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(
a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - a^2 + 4*(
a^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))
/(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.14827, size = 135, normalized size = 1.65

$$\frac{ae^{(2x)} - 4\,be^{x}}{8\,a^2} - \frac{\left(a^2 - 2\,b^2\right)x}{2\,a^3} + \frac{\left(4\,abe^{x} - a^2\right)e^{(-2x)}}{8\,a^3} + \frac{2\left(a^2 b - b^3\right)\arctan\left(\frac{ae^{x}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x -
a^2)*e^(-2*x)/a^3 + 2*(a^2*b - b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sq
rt(a^2 - b^2)*a^3)
```

3.63 $\int \frac{\sinh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\cosh(x)}{a}-\frac{b\log(a\cosh(x)+b)}{a^2}$$

```
[Out] Cosh[x]/a - (b*Log[b + a*Cosh[x]])/a^2
```

Rubi [A] time = 0.0880297, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\cosh(x)}{a}-\frac{b\log(a\cosh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sech[x]),x]

[Out] Cosh[x]/a - (b*Log[b + a*Cosh[x]])/a^2
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh(x)}{-b-a\cosh(x)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^2}$$
$$= -\frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)dx, x, -a\cosh(x)\right)}{a^2}$$
$$= \frac{\cosh(x)}{a} - \frac{b\log(b+a\cosh(x))}{a^2}$$

Mathematica [A] time = 0.0098156, size = 19, normalized size = 0.95

$$\frac{a\cosh(x)-b\log(a\cosh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sech[x]),x]
```

```
[Out] (a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

Maple [A] time = 0.02, size = 31, normalized size = 1.6

$$\frac{1}{a\text{sech}(x)} + \frac{b\ln(\text{sech}(x))}{a^2} - \frac{b\ln(a+b\text{sech}(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sech(x)),x)
```

```
[Out] 1/a/sech(x)+1/a^2*b*ln(sech(x))-1/a^2*b*ln(a+b*sech(x))
```

Maxima [B] time = 1.13409, size = 62, normalized size = 3.1

$$-\frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a} - \frac{b\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a - b*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^
2
```

Fricas [B] time = 2.57383, size = 246, normalized size = 12.3

$$\frac{2\,bx\cosh(x) + a\cosh(x)^2 + a\sinh(x)^2 - 2\,(b\cosh(x) + b\sinh(x))\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right) + 2\,(bx + a\cosh(x))\sinh(x)}{2\left(a^2\cosh(x) + a^2\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) +
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*sech(x)), x)
```

Giac [A] time = 1.12379, size = 46, normalized size = 2.3

$$\frac{e^{(-x)} + e^{x}}{2\,a} - \frac{b \log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/a - b*log(abs(a*(e^(-x) + e^x) + 2*b))/a^2
```

3.64 $\int \frac{\mathbf{csch}(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[b + a*
Cosh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.118449, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2721, 801}

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[b + a*
Cosh[x]])/(a^2 - b^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)}{-b-a\cosh(x)}\,dx\\
&= \operatorname{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cosh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)}-\frac{b}{(a-b)(a+b)(b-x)}+\frac{1}{2(a+b)(a+x)}\right)dx, x, -a\cosh(x)\right)\\
&= \frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(b+a\cosh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.0727261, size = 37, normalized size = 0.7

$$\frac{b\log(a\cosh(x)+b)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)-b\log(\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*Log[b + a*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

Maple [A] time = 0.022, size = 48, normalized size = 0.9

$$\frac{1}{a+b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b}{(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b+a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sech(x)),x)
```

```
[Out] 1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+
b)
```

Maxima [A] time = 1.10733, size = 80, normalized size = 1.51

$$\frac{b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

Fricas [A] time = 2.47507, size = 181, normalized size = 3.42

$$\frac{b\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sech(x)), x)
```

Giac [A] time = 1.12509, size = 88, normalized size = 1.66

$$\frac{ab\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^3 - ab^2} - \frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{2\,(a - b)} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.65 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=66

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rubi [A] time = 0.132552, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2866, 12, 2659, 205}

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-b-a\cosh(x)}\,dx\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{\int \frac{ab}{-b-a\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{(ab)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{(2ab)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.251885, size = 75, normalized size = 1.14

$$\frac{1}{2}\left(-\frac{4ab\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{\tanh\left(\frac{x}{2}\right)}{b-a}-\frac{\coth\left(\frac{x}{2}\right)}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((-4*a*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
Coth[x/2]/(a + b) + Tanh[x/2]/(-a + b))/2
```

Maple [A] time = 0.023, size = 77, normalized size = 1.2

$$-\frac{1}{2\,a-2\,b}\tanh\left(\frac{x}{2}\right)-\frac{1}{2\,b+2\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+2\,\frac{ab}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)\,(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*a*b/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58839, size = 1156, normalized size = 17.52

$$\left[\frac{2\,a^{3}-2\,ab^{2}-\left(ab\cosh\left(x\right)^{2}+2\,ab\cosh\left(x\right)\sinh\left(x\right)+ab\sinh\left(x\right)^{2}-ab\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2\,ab\cosh(x)-a^{2}}{a\cosh(x)^{2}+a\sinh}\right.}{a^{4}-2\,a^{2}b^{2}+b^{4}-\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{2}-2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
- a*b)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
- a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) + a)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(
a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a
*b^2 + (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - (a^2*b - b
^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{2}(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.13428, size = 86, normalized size = 1.3

$$\frac{2\,ab\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{2\,\left(be^{x}-a\right)}{\left(a^{2}-b^{2}\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*(b*e^x - a)
/((a^2 - b^2)*(e^(2*x) - 1))
```

3.66 $\int \frac{\mathbf{csch}^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{a^2b\log(a\cosh(x)+b)}{\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - (a*Log[1 - Cosh[x]])/(4*(a +
b)^2) + (a*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^2*b*Log[b + a*Cosh[x]])/(a^
2 - b^2)^2
```

Rubi [A] time = 0.238127, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^2b\log(a\cosh(x)+b)}{\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - (a*Log[1 - Cosh[x]])/(4*(a +
b)^2) + (a*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^2*b*Log[b + a*Cosh[x]])/(a^
2 - b^2)^2
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^2(x)}{-b-a\cosh(x)}\,dx\\
&= -\left(a^3\operatorname{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right)\\
&= -\left(a^2\operatorname{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right)\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{\operatorname{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cosh(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{\operatorname{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)}{2(a+b)(a+x)}\right)dx, x, -a\cosh(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{a\log(1-\cosh(x))}{4(a+b)^2} + \frac{a\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^2b\log(b+a\cosh(x))}{\left(a^2-b^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.329667, size = 86, normalized size = 1.01

$$\frac{1}{8}\left(-\frac{4a\left(\left(a^2+b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-2ab\log(\sinh(x))+2ab\log(a\cosh(x)+b)\right)}{(a-b)^2(a+b)^2}-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b}-\frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(Csch[x/2]^2/(a + b)) - (4*a*(2*a*b*Log[b + a*Cosh[x]] - 2*a*b*Log[Sinh[x
]] + (a^2 + b^2)*Log[Tanh[x/2]]))/((a - b)^2*(a + b)^2) - Sech[x/2]^2/(a -
b))/8
```

Maple [A] time = 0.029, size = 82, normalized size = 1.

$$\frac{1}{8a-8b}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{8a+8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{a}{2\,(a+b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{a^2b}{(a+b)^2\,(a-b)^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-(\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sech(x)),x)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2-1/2*a/(a+b)^2*ln(tanh(1/2*x
))-a^2*b/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maximima [A] time = 1.11189, size = 200, normalized size = 2.35

$$-\frac{a^2b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^4-2\,a^2b^2+b^4}+\frac{a\log\left(e^{(-x)}+1\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(e^{(-x)}-1\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\left(a^2-b^2\right)e^{(-2\,x)}+\left(a^2-b^2\right)e^{(-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*log
(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*a*log(e^(-x) - 1)/(a^2 + 2*a*b + b^2
) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2 - b^2)*e^(-2
*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] time = 2.74301, size = 2097, normalized size = 24.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) + 2*(a^2*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^
2*b*sinh(x)^4 - 2*a^2*b*cosh(x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 - a^2*b)*s
inh(x)^2 + 4*(a^2*b*cosh(x)^3 - a^2*b*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b)/(cosh(x) - sinh(x))) - ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 +
 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b +
 a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x)
 + sinh(x) + 1) + ((a^3 - 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 - 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^3 + (a^3 - 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 - 2*a^2*b
 + a*b^2 - 2*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 - 2*a^2*b + a*b^2 -
 3*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 2*a^2*b + a*b^2
)*cosh(x)^3 - (a^3 - 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) - 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh
(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*sech(x)), x)
```

Giac [B] time = 1.14697, size = 235, normalized size = 2.76

$$-\frac{a^{3}b\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^{5}-2\,a^{3}b^{2}+ab^{4}}+\frac{a\log\left(e^{(-x)}+e^{x}+2\right)}{4\left(a^{2}-2\,ab+b^{2}\right)}-\frac{a\log\left(e^{(-x)}+e^{x}-2\right)}{4\left(a^{2}+2\,ab+b^{2}\right)}-\frac{a^{2}b\left(e^{(-x)}+e^{x}\right)^{2}+2\,a^{3}\left(e^{(-x)}+e^{x}\right)-2\,ab^{2}}{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\left(e^{(-x)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*a*l
og(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*a*log(e^(-x) + e^x - 2)/(a^2
 + 2*a*b + b^2) - 1/2*(a^2*b*(e^(-x) + e^x)^2 + 2*a^3*(e^(-x) + e^x) - 2*a*
b^2*(e^(-x) + e^x) - 8*a^2*b + 4*b^3)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e
^x)^2 - 4))
```

3.67 $\int \frac{\mathrm{csch}^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=111

$$\frac{\mathrm{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\mathrm{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}-\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-2*a^3*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a +
b)^(5/2)) - ((3*a^2*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2)
 + ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rubi [A] time = 0.303556, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2866, 12, 2659, 205}

$$\frac{\mathrm{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\mathrm{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}-\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a +
b)^(5/2)) - ((3*a^2*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2)
 + ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\mathrm{csch}^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = -\int \frac{\coth(x)\mathrm{csch}^3(x)}{-b-a\cosh(x)}\,dx$$
$$= \frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)} - \frac{\int \frac{\left(ab-2a^2\cosh(x)\right)\mathrm{csch}^2(x)}{-b-a\cosh(x)}\,dx}{3\left(a^2-b^2\right)}$$
$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\mathrm{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{3a^3b}{-b-a\cosh(x)}\,dx}{3\left(a^2-b^2\right)^2}$$
$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\mathrm{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(a^3b\right)\int \frac{1}{-b-a\cosh(x)}\,dx}{\left(a^2-b^2\right)^2}$$
$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\mathrm{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(2a^3b\right)\mathrm{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\right.}{\left(a^2-b^2\right)^2}$$
$$= -\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\mathrm{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.577201, size = 156, normalized size = 1.41

$$\frac{\mathrm{sech}(x)(a\cosh(x)+b)\left(\frac{48a^3b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{2b\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{2(4a+b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\sinh^4\left(\frac{x}{2}\right)\mathrm{csch}^3(x)}{a-b} - \frac{\sinh(x)\mathrm{csch}^4\left(\frac{x}{2}\right)}{2(a+b)}\right.}{24(a+b\,\mathrm{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((48*a^3*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (2*(4*a + b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3
*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])
/(a - b)^2 - (2*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[x]))
```

Maple [A] time = 0.032, size = 154, normalized size = 1.4

$$-\frac{a}{24\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + \frac{b}{24\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + \frac{3\,a}{8\,(a-b)^2}\tanh\left(\frac{x}{2}\right) - \frac{b}{8\,(a-b)^2}\tanh\left(\frac{x}{2}\right) - \frac{1}{24\,a+24\,b}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sech(x)),x)
```

```
[Out] -1/24/(a-b)^2*a*tanh(1/2*x)^3+1/24/(a-b)^2*b*tanh(1/2*x)^3+3/8/(a-b)^2*a*ta
nh(1/2*x)-1/8/(a-b)^2*tanh(1/2*x)*b-1/24/(a+b)/tanh(1/2*x)^3+3/8/(a+b)^2/ta
nh(1/2*x)*a+1/8/(a+b)^2/tanh(1/2*x)*b-2/(a-b)^2/(a+b)^2*a^3*b/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87179, size = 5416, normalized size = 48.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(6*(a^4*b - a^2*b^3)*cosh(x)^5 + 6*(a^4*b - a^2*b^3)*sinh(x)^5 - 4*a^
5 + 2*a^3*b^2 + 2*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^
4 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^3 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)
^2 + 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2
+ 12*(a^5 - a^3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*c
osh(x)^2 - (5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 + 3*(a^3*b*cosh
(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3
*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a
^3*b*cosh(x)^3 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*
b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a
^3*b*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2
+ 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2
 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(
x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^4*b - a^2*b^3)*cosh(x) + 6*(a^4
*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^
3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*
sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*
sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 -
 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), -2/3*(3
*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 - 2*a^5 + a^3*
```

```
b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^4 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^3
- 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 + 6*(a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5 - a^
3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (5*
a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(x)^6 + 6*a^3*
b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3*a^3*b*cosh(x)
^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a^3*b*cosh(x)^3
 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*b*cosh(x)^2 +
a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a^3*b*cosh(x))*
sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2
)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(a^4*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)
*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*
a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.13639, size = 201, normalized size = 1.81

$$-\frac{2\,a^3b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}-10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x-2\,a^3-ab^2\right)}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(
a^2 - b^2)) - 2/3*(3*a^2*b*e^(5*x) - 3*a*b^2*e^(4*x) - 10*a^2*b*e^(3*x) + 4
*b^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a^2*b*e^x - 2*a^3 - a*b^2)/((a^4 - 2*a^2*b
^2 + b^4)*(e^(2*x) - 1)^3)
```

3.68 $\int \frac{\cosh^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=67

$$\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\mathrm{sech}(x)+a}$$

```
[Out] (15*x)/(8*a) - (4*Sinh[x])/a + (15*Cosh[x]*Sinh[x])/(8*a) + (5*Cosh[x]^3*Si
nh[x])/(4*a) - (Cosh[x]^3*Sinh[x])/(a + a*Sech[x]) - (4*Sinh[x]^3)/(3*a)
```

Rubi [A] time = 0.0961097, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2633}

$$\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\mathrm{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sinh[x])/a + (15*Cosh[x]*Sinh[x])/(8*a) + (5*Cosh[x]^3*Si
nh[x])/(4*a) - (Cosh[x]^3*Sinh[x])/(a + a*Sech[x]) - (4*Sinh[x]^3)/(3*a)
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+a\mathrm{sech}(x)}dx = -\frac{\cosh^3(x)\sinh(x)}{a+a\mathrm{sech}(x)} - \frac{\int \cosh^4(x)(-5a+4a\mathrm{sech}(x))\,dx}{a^2}$$
$$= -\frac{\cosh^3(x)\sinh(x)}{a+a\mathrm{sech}(x)} - \frac{4\int \cosh^3(x)\,dx}{a} + \frac{5\int \cosh^4(x)\,dx}{a}$$
$$= \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\mathrm{sech}(x)} - \frac{(4i)\,\mathrm{Subst}\left(\int \left(1-x^2\right)dx, x, -i\sinh(x)\right)}{a} + \frac{15\int \cosh^2(x}{4a}$$
$$= -\frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\mathrm{sech}(x)} - \frac{4\sinh^3(x)}{3a} + \frac{15\int 1}{8a}$$
$$= \frac{15x}{8a} - \frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\mathrm{sech}(x)} - \frac{4\sinh^3(x)}{3a}$$

Mathematica [A] time = 0.0864632, size = 63, normalized size = 0.94

$$\frac{\mathrm{sech}\left(\frac{x}{2}\right)\left(-360\sinh\left(\frac{x}{2}\right) - 120\sinh\left(\frac{3x}{2}\right) + 40\sinh\left(\frac{5x}{2}\right) - 5\sinh\left(\frac{7x}{2}\right) + 3\sinh\left(\frac{9x}{2}\right) + 360x\cosh\left(\frac{x}{2}\right)\right)}{192a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(360*x*Cosh[x/2] - 360*Sinh[x/2] - 120*Sinh[(3*x)/2] + 40*Sinh[(
5*x)/2] - 5*Sinh[(7*x)/2] + 3*Sinh[(9*x)/2]))/(192*a)
```

Maple [B] time = 0.032, size = 139, normalized size = 2.1

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right) - \frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} + \frac{5}{6a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \frac{15}{8a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{25}{8a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{15}{8a}\ln\left(t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-1/4/a/(tanh(1/2*x)+1)^4+5/6/a/(tanh(1/2*x)+1)^3-15/8/a/(ta
nh(1/2*x)+1)^2+25/8/a/(tanh(1/2*x)+1)+15/8/a*ln(tanh(1/2*x)+1)+1/4/a/(tanh(
1/2*x)-1)^4+5/6/a/(tanh(1/2*x)-1)^3+15/8/a/(tanh(1/2*x)-1)^2+25/8/a/(tanh(1
/2*x)-1)-15/8/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.07872, size = 108, normalized size = 1.61

$$\frac{15\,x}{8\,a} + \frac{168\,e^{(-x)} - 48\,e^{(-2\,x)} + 8\,e^{(-3\,x)} - 3\,e^{(-4\,x)}}{192\,a} - \frac{5\,e^{(-x)} - 40\,e^{(-2\,x)} + 120\,e^{(-3\,x)} + 552\,e^{(-4\,x)} - 3}{192\left(ae^{(-4\,x)} + ae^{(-5\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 15/8*x/a + 1/192*(168*e^(-x) - 48*e^(-2*x) + 8*e^(-3*x) - 3*e^(-4*x))/a - 1
/192*(5*e^(-x) - 40*e^(-2*x) + 120*e^(-3*x) + 552*e^(-4*x) - 3)/(a*e^(-4*x)
```

```
+ a*e^(-5*x))
```

Fricas [B] time = 2.54816, size = 491, normalized size = 7.33

$$\frac{3 \cosh(x)^5 + (15 \cosh(x) - 8)\sinh(x)^4 + 3 \sinh(x)^5 - 8 \cosh(x)^4 + \left(30 \cosh(x)^2 - 8 \cosh(x) + 35\right)\sinh(x)^3 + 4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*cosh(x)^5 + (15*cosh(x) - 8)*sinh(x)^4 + 3*sinh(x)^5 - 8*cosh(x)^4
 + (30*cosh(x)^2 - 8*cosh(x) + 35)*sinh(x)^3 + 45*cosh(x)^3 + (30*cosh(x)^3
 - 48*cosh(x)^2 + 135*cosh(x) - 160)*sinh(x)^2 + 24*(15*x - 2)*cosh(x) - 16
0*cosh(x)^2 + (15*cosh(x)^4 - 8*cosh(x)^3 + 105*cosh(x)^2 + 360*x - 160*cos
h(x) - 288)*sinh(x) + 360*x + 552)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cosh^4(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(sech(x) + 1), x)/a
```

Giac [A] time = 1.18051, size = 116, normalized size = 1.73

$$\frac{15\,x}{8\,a} + \frac{\left(552\,e^{(4\,x)} + 120\,e^{(3\,x)} - 40\,e^{(2\,x)} + 5\,e^{x} - 3\right)e^{(-4\,x)}}{192\,a(e^{x} + 1)} + \frac{3\,a^3e^{(4\,x)} - 8\,a^3e^{(3\,x)} + 48\,a^3e^{(2\,x)} - 168\,a^3e^{x}}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 15/8*x/a + 1/192*(552*e^(4*x) + 120*e^(3*x) - 40*e^(2*x) + 5*e^x - 3)*e^(-4
*x)/(a*(e^x + 1)) + 1/192*(3*a^3*e^(4*x) - 8*a^3*e^(3*x) + 48*a^3*e^(2*x) -
 168*a^3*e^x)/a^4
```

3.69 $\int \frac{\cosh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[
x])/(a + a*Sech[x]) + (4*Sinh[x]^3)/(3*a)
```

Rubi [A] time = 0.0864547, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[
x])/(a + a*Sech[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{\int \cosh^3(x)(-4a+3a\operatorname{sech}(x))\,dx}{a^2} \\
&= -\frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{3\int \cosh^2(x)\,dx}{a} + \frac{4\int \cosh^3(x)\,dx}{a} \\
&= -\frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} + \frac{(4i)\operatorname{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(x)\right)}{a} - \frac{3\int 1\,dx}{2a} \\
&= -\frac{3x}{2a} + \frac{4\sinh(x)}{a} - \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} + \frac{4\sinh^3(x)}{3a}
\end{aligned}
$$

Mathematica [A] time = 0.0728321, size = 53, normalized size = 0.98

$$
\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)-36x\cosh\left(\frac{x}{2}\right)\right)}{24a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

Maple [B] time = 0.029, size = 111, normalized size = 2.1

$$
\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{5}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{3}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{3a}\left(\tan\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1)^2-5/2/a/(tanh(1
/2*x)+1)-3/2/a*ln(tanh(1/2*x)+1)-1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1
)^2-5/2/a/(tanh(1/2*x)-1)+3/2/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.16826, size = 89, normalized size = 1.65

$$
-\frac{3x}{2a}-\frac{21e^{(-x)}-3e^{(-2x)}+e^{(-3x)}}{24a}-\frac{2e^{(-x)}-18e^{(-2x)}-69e^{(-3x)}-1}{24\left(ae^{(-3x)}+ae^{(-4x)}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Fricas [B] time = 2.43033, size = 346, normalized size = 6.41

$$\frac{\cosh(x)^4 + (4\cosh(x) - 1)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + \left(6\cosh(x)^2 - 9\cosh(x) + 20\right)\sinh(x)^2 - 3\,(12\,x - 1)}{24\,(a\cosh(x) + a\sinh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cosh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(sech(x) + 1), x)/a
```

Giac [A] time = 1.14813, size = 95, normalized size = 1.76

$$-\frac{3\,x}{2\,a} - \frac{\left(69\,e^{(3\,x)} + 18\,e^{(2\,x)} - 2\,e^{x} + 1\right)e^{(-3\,x)}}{24\,a(e^{x} + 1)} + \frac{a^2e^{(3\,x)} - 3\,a^2e^{(2\,x)} + 21\,a^2e^{x}}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

3.70 $\int \frac{\cosh^2(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal. Leaf size=41

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]*Sinh[x])
/(a + a*Sech[x])
```

Rubi [A] time = 0.0796549, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2637}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]*Sinh[x])
/(a + a*Sech[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{\int \cosh^2(x)(-3a+2a\operatorname{sech}(x))\,dx}{a^2}$$
$$= -\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{2\int \cosh(x)\,dx}{a} + \frac{3\int \cosh^2(x)\,dx}{a}$$
$$= -\frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)} + \frac{3\int 1\,dx}{2a}$$
$$= \frac{3x}{2a} - \frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}$$

Mathematica [A] time = 0.0480794, size = 45, normalized size = 1.1

$$\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)+12x\cosh\left(\frac{x}{2}\right)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

Maple [B] time = 0.029, size = 87, normalized size = 2.1

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{3}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2}+\frac{3}{2a}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-1/2/a/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+3/2/a*ln(tan
h(1/2*x)+1)+1/2/a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-3/2/a*ln(tanh(1/2
*x)-1)
```

Maxima [A] time = 1.1507, size = 76, normalized size = 1.85

$$\frac{3\,x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}-\frac{3\,e^{(-x)}+20\,e^{(-2\,x)}-1}{8\left(ae^{(-2\,x)}+ae^{(-3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

Fricas [A] time = 2.17091, size = 242, normalized size = 5.9

$$\frac{\cosh(x)^3 + (3\cosh(x) - 4)\sinh(x)^2 + \sinh(x)^3 + (12x - 1)\cosh(x) - 4\cosh(x)^2 + \left(3\cosh(x)^2 + 12x - 4\cosh(\right.}{8\,(a\cosh(x) + a\sinh(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cosh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.12272, size = 69, normalized size = 1.68

$$\frac{3\,x}{2\,a} + \frac{\left(20\,e^{(2\,x)} + 3\,e^{x} - 1\right)e^{(-2\,x)}}{8\,a(e^{x} + 1)} + \frac{ae^{(2\,x)} - 4\,ae^{x}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

3.71 $\int \frac{\cosh(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

```
[Out] -(x/a) + (2*Sinh[x])/a - Sinh[x]/(a + a*Sech[x])
```

Rubi [A] time = 0.0568475, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3819, 3787, 2637, 8}

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] -(x/a) + (2*Sinh[x])/a - Sinh[x]/(a + a*Sech[x])
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\sinh(x)}{a+a\text{sech}(x)}-\frac{\int \cosh(x)(-2a+a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\sinh(x)}{a+a\text{sech}(x)}-\frac{\int 1\,dx}{a}+\frac{2\int \cosh(x)\,dx}{a}\\ &= -\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a+a\text{sech}(x)}\end{aligned}$$

Mathematica [A] time = 0.0602496, size = 32, normalized size = 1.23

$$\frac{-2x+3\tanh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Maple [B] time = 0.027, size = 59, normalized size = 2.3

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/a/(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)+1)-1/a/(tanh(1/2*x)-
1)+1/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.12146, size = 55, normalized size = 2.12

$$-\frac{x}{a}+\frac{5\,e^{(-x)}+1}{2\left(ae^{(-x)}+ae^{(-2\,x)}\right)}-\frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Fricas [A] time = 2.42953, size = 151, normalized size = 5.81

$$-\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cosh\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(sech(x) + 1), x)/a
```

Giac [A] time = 1.1658, size = 47, normalized size = 1.81

$$-\frac{x}{a} - \frac{(5\,e^{x} + 1)e^{(-x)}}{2\,a(e^{x} + 1)} + \frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

3.72 $\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{a\text{sech}(x)+a}$$

```
[Out] Tanh[x]/(a + a*Sech[x])
```

Rubi [A] time = 0.0242042, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3794}

$$\frac{\tanh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x]/(a + a*Sech[x])
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx = \frac{\tanh(x)}{a+a\text{sech}(x)}$$

Mathematica [A] time = 0.0073096, size = 10, normalized size = 0.91

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x/2]/a
```

Maple [A] time = 0.012, size = 9, normalized size = 0.8

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)
```

Maxima [A] time = 1.13703, size = 16, normalized size = 1.45

$$\frac{2}{ae^{(-x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/(a*e^(-x) + a)
```

Fricas [A] time = 2.27118, size = 43, normalized size = 3.91

$$-\frac{2}{a\cosh(x) + a\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)/(sech(x) + 1), x)/a
```

Giac [A] time = 1.14796, size = 15, normalized size = 1.36

$$-\frac{2}{a(e^{x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2/(a*(e^x + 1))
```

3.73 $\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])
```

Rubi [A] time = 0.0658841, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3789, 3770, 3794}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + a*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A] time = 0.028501, size = 22, normalized size = 1.1

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

Maple [A] time = 0.01, size = 21, normalized size = 1.1

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right)+2\,\frac{\arctan\left(\tanh\left(x/2\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)+2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.75343, size = 31, normalized size = 1.55

$$-\frac{2\,\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Fricas [A] time = 2.39204, size = 117, normalized size = 5.85

$$\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{sech}^{2}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.11875, size = 27, normalized size = 1.35

$$\frac{2 \arctan(e^x)}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

3.74 $\int \frac{\mathbf{sech}^3(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a\text{sech}(x)+a}$$

```
[Out] -(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Rubi [A] time = 0.101052, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3790, 3789, 3770, 3794}

$$\frac{\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^3(x)}{a+a\text{sech}(x)}\,dx &= \frac{\tanh(x)}{a}-\int \frac{\text{sech}^2(x)}{a+a\text{sech}(x)}\,dx\\
&= \frac{\tanh(x)}{a}-\frac{\int \text{sech}(x)\,dx}{a}+\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx\\
&= -\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a+a\text{sech}(x)}
\end{aligned}$$

Mathematica [A] time = 0.0851934, size = 45, normalized size = 1.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (2*Cosh[x/2]*Sech[x]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x]
)))/(a*(1 + Sech[x]))
```

Maple [A] time = 0.015, size = 39, normalized size = 1.5

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)+2\,\frac{\tanh\left(x/2\right)}{a\left(\left(\tanh\left(x/2\right)\right)^{2}+1\right)}-2\,\frac{\arctan\left(\tanh\left(x/2\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.67018, size = 61, normalized size = 2.35

$$\frac{2\left(e^{(-x)}+e^{(-2x)}+2\right)}{ae^{(-x)}+ae^{(-2x)}+ae^{(-3x)}+a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

Fricas [B] time = 2.29869, size = 467, normalized size = 17.96

$$-\frac{2\left(\left(\cosh\left(x\right)^{3}+\left(3\cosh\left(x\right)+1\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\cosh\left(x\right)^{2}+\left(3\cosh\left(x\right)^{2}+2\cosh\left(x\right)+1\right)\sinh\left(x\right)+\cosh\left(\right.\right.\right.}{a\cosh\left(x\right)^{3}+a\sinh\left(x\right)^{3}+a\cosh\left(x\right)^{2}+\left(3\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(sech(x) + 1), x)/a
```

Giac [A] time = 1.11443, size = 49, normalized size = 1.88

$$-\frac{2\,\arctan\left(e^{x}\right)}{a}-\frac{2\left(e^{(2\,x)}+e^{x}+2\right)}{a\left(e^{(3\,x)}+e^{(2\,x)}+e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

3.75 $\int \frac{\mathbf{sech}^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\mathrm{sech}^2(x)}{a\mathrm{sech}(x)+a}+\frac{3\tanh(x)\mathrm{sech}(x)}{2a}$$

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]^2*Tanh[x])/(a + a*Sech[x])
```

Rubi [A] time = 0.0849706, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\mathrm{sech}^2(x)}{a\mathrm{sech}(x)+a}+\frac{3\tanh(x)\mathrm{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]^2*Tanh[x])/(a + a*Sech[x])
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{\int \operatorname{sech}^2(x)(2a-3a\operatorname{sech}(x))\,dx}{a^2}\\
&= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{2\int \operatorname{sech}^2(x)\,dx}{a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{a}\\
&= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{(2i)\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a}+\frac{3\int \operatorname{sech}(x)\,dx}{2a}\\
&= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}
\end{aligned}$$

Mathematica [A] time = 0.0839871, size = 51, normalized size = 1.13

$$\frac{\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(\cosh\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)(\operatorname{sech}(x)-2)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]*Sech[x]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + S
ech[x])*Tanh[x])))/(a*(1 + Sech[x]))
```

Maple [A] time = 0.017, size = 61, normalized size = 1.4

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right)-3\,\frac{(\tanh(x/2))^3}{a\left((\tanh(x/2))^2+1\right)^2}-\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+3\,\frac{\arctan(\tanh(x/2))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-1/a/(tanh(1/2*x)^2+1
)^2*tanh(1/2*x)+3/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.69542, size = 99, normalized size = 2.2

$$-\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a}-\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Fricas [B] time = 2.43579, size = 1099, normalized size = 24.42

$$\frac{3\cosh(x)^4 + 3(4\cosh(x)+1)\sinh(x)^3 + 3\sinh(x)^4 + 3\cosh(x)^3 + \left(18\cosh(x)^2 + 9\cosh(x) + 5\right)\sinh(x)^2 + 3}{a\cosh(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15175, size = 65, normalized size = 1.44

$$\frac{3\arctan(e^x)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^x + 2}{a\left(e^{(2x)}+1\right)^2} + \frac{2}{a(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

3.76 $\int \frac{1}{a+a\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\operatorname{sech}(c+dx)+a)}$$

```
[Out] x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))
```

Rubi [A] time = 0.0154826, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\operatorname{sech}(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx &= -\frac{\tanh(c+dx)}{d(a+a\operatorname{sech}(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\operatorname{sech}(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.139259, size = 58, normalized size = 2.

$$\frac{\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(c+\frac{dx}{2}\right)-2\sinh\left(\frac{dx}{2}\right)+dx\cosh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] (Sech[c/2]*Sech[(c + d*x)/2]*(d*x*Cosh[(d*x)/2] + d*x*Cosh[c + (d*x)/2] - 2
*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A] time = 0.036, size = 58, normalized size = 2.

$$-\frac{1}{da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c)),x)
```

```
[Out] -1/d/a*tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/
2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.14442, size = 45, normalized size = 1.55

$$\frac{dx+c}{ad}-\frac{2}{\left(ae^{(-dx-c)}+a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + a)*d)
```

Fricas [A] time = 2.38467, size = 131, normalized size = 4.52

$$\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)+dx+2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) + d*x + 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{\operatorname{sech}\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x)
```

```
[Out] Integral(1/(sech(c + d*x) + 1), x)/a
```

Giac [A] time = 1.14892, size = 42, normalized size = 1.45

$$\frac{dx+c}{ad}+\frac{2}{ad\left(e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)/(a*d) + 2/(a*d*(e^(d*x + c) + 1))
```

3.77 $\int \frac{1}{a-a\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\,\mathrm{sech}(c+dx))}$$

```
[Out] x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Rubi [A] time = 0.0154844, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\,\mathrm{sech}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sech[c + d*x])^(-1),x]

[Out] x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\,\mathrm{sech}(c+dx)}\,dx &= -\frac{\tanh(c+dx)}{d(a-a\,\mathrm{sech}(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\,\mathrm{sech}(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.140193, size = 59, normalized size = 1.97

$$\frac{\mathrm{csch}\left(\frac{c}{2}\right)\mathrm{csch}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(c+\frac{dx}{2}\right)+2\sinh\left(\frac{dx}{2}\right)-dx\cosh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sech[c + d*x])^(-1),x]

[Out] (Csch[c/2]*Csch[(c + d*x)/2]*(-(d*x*Cosh[(d*x)/2]) + d*x*Cosh[c + (d*x)/2]
+ 2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A] time = 0.032, size = 60, normalized size = 2.

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a/tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2
*d*x+1/2*c)-1)
```

Maxima [A] time = 1.11493, size = 47, normalized size = 1.57

$$\frac{dx+c}{ad}+\frac{2}{\left(ae^{(-dx-c)}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 2/((a*e^(-d*x - c) - a)*d)
```

Fricas [A] time = 2.42036, size = 131, normalized size = 4.37

$$\frac{dx\cosh(dx+c)+dx\sinh(dx+c)-dx-2}{ad\cosh(dx+c)+ad\sinh(dx+c)-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) - d*x - 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{\operatorname{sech}(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x)
```

```
[Out] -Integral(1/(sech(c + d*x) - 1), x)/a
```

Giac [A] time = 1.16877, size = 42, normalized size = 1.4

$$\frac{dx+c}{ad}-\frac{2}{ad\left(e^{(dx+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)/(a*d) - 2/(a*d*(e^(d*x + c) - 1))
```

3.78 $\int (a + a\,\text{sech}(c + dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\,\text{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\,\text{sech}(c+dx)+a}}{3d}$$

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(14*a^3*Tanh[c + d*x])/(3*d*Sqrt[a + a*Sech[c + d*x]]) + (2*a^2*Sqrt[a + a*
Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rubi [A] time = 0.120318, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\,\text{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\,\text{sech}(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(14*a^3*Tanh[c + d*x])/(3*d*Sqrt[a + a*Sech[c + d*x]]) + (2*a^2*Sqrt[a + a*
Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d} + \frac{1}{3}(2a)\int \sqrt{a+a\operatorname{sech}(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}a\operatorname{sech}(c\right.$$

$$= \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d} + a^2\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx + \frac{1}{3}\left(7a^2\right)\int \operatorname{sech}$$

$$= \frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}} + \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d} + \frac{\left(2ia^3\right)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\right.}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d} + \frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}} + \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\tanh}{3d}$$

Mathematica [A] time = 0.327859, size = 99, normalized size = 1.01

$$\frac{a^2\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}(c+dx)\sqrt{a(\operatorname{sech}(c+dx)+1)}\left(-6\sinh\left(\frac{1}{2}(c+dx)\right)+8\sinh\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sin\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sech[(c + d*x)/2]*Sech[c + d*x]*Sqrt[a*(1 + Sech[c + d*x])]*(3*Sqrt[2]
*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Cosh[c + d*x]^(3/2) - 6*Sinh[(c + d*x)/
2] + 8*Sinh[(3*(c + d*x))/2]))/(3*d)
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (a+a\operatorname{sech}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(5/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\operatorname{sech}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.77612, size = 2558, normalized size = 26.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(
d*x + c)^2 + a^2)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x
 + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 +
 (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5
 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x
 + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15
*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^
4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*
x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(
d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c)
+ 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(
d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a)*log((a*cosh(d*x
 + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh
(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*co
sh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d
*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(
d*x + c))) + 8*(4*a^2*cosh(d*x + c)^3 + 4*a^2*sinh(d*x + c)^3 - 3*a^2*cosh(
d*x + c)^2 + 3*a^2*cosh(d*x + c) + 3*(4*a^2*cosh(d*x + c) - a^2)*sinh(d*x +
 c)^2 - 4*a^2 + 3*(4*a^2*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) + a^2)*sinh(
d*x + c))*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*si
nh(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36297, size = 216, normalized size = 2.2

$$\frac{\frac{48\,a^{3}\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{4\left(\left(\left(\frac{4\,e^{(dx-3\,c)}}{a^{2}}-\frac{3\,e^{(-4\,c)}}{a^{2}}\right)e^{(dx)}+\frac{3\,e^{(-5\,c)}}{a^{2}}\right)e^{(dx)}-\frac{4\,e^{(-6\,c)}}{a^{2}}\right)}{\left(ae^{(2\,dx+2\,c)}+a\right)^{\frac{3}{2}}}-\frac{3\,e^{(-6\,c)}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)}{a^{\frac{7}{2}}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(48*a^3*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sq
rt(-a))/sqrt(-a) + 4*(((4*e^(d*x - 3*c)/a^2 - 3*e^(-4*c)/a^2)*e^(d*x) + 3*e
^(-5*c)/a^2)*e^(d*x) - 4*e^(-6*c)/a^2)/(a*e^(2*d*x + 2*c) + a)^(3/2) - 3*e^
(-6*c)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) + a)))/a^(7/2)
)/d
```

3.79 $\int (a + a\,\mathbf{sech}(c + dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\operatorname{sech}(c+dx)+a}}$$

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Rubi [A] time = 0.0420447, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\operatorname{sech}(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\operatorname{sech}(c+dx))^{3/2}\,dx &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}} + (2a)\int \frac{\frac{a}{2}+\frac{1}{2}a\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx \\ &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}} + a\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx \\ &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}} + \frac{(2ia^2)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d} \\ &= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d} + \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.19536, size = 75, normalized size = 1.14

$$\frac{a\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\operatorname{sech}(c+dx)+1)}\left(2\sinh\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(3/2),x]
```

```
[Out] (a*Sech[(c + d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])]*(Sqrt[2]*ArcSinh[Sqrt[2]*S
inh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]] + 2*Sinh[(c + d*x)/2]))/d
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (a+a\operatorname{sech}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(3/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.43816, size = 1975, normalized size = 29.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + a^(3/2)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))) + 4*(a*cosh(d*x
 + c) + a*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,\mathrm{sech}\,(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(3/2), x)
```

Giac [B] time = 1.22772, size = 159, normalized size = 2.41

$$\frac{\frac{2\,a^2\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)+\frac{2\left(a^2e^{(dx+c)}-a^2\right)}{\sqrt{ae^{(2\,dx+2\,c)}+a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (2*a^2*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a)
)/sqrt(-a) - a^(3/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c)
+ a))) + 2*(a^2*e^(d*x + c) - a^2)/sqrt(a*e^(2*d*x + 2*c) + a))/d
```

3.80 $\int \sqrt{a+a\,\mathrm{\mathbf{sech}}(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Rubi [A] time = 0.0185252, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\mathrm{sech}(c+dx)}\,dx &= \frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\mathrm{sech}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\mathrm{sech}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0891638, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\mathrm{sech}(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]]*Sech[(c + d
*x)/2]*Sqrt[a*(1 + Sech[c + d*x])])/d
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \sqrt{a + a\mathrm{sech}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\mathrm{sech}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(d*x + c) + a), x)
```

Fricas [B] time = 2.55815, size = 1808, normalized size = 48.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,\mathrm{sech}\,(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(c + d*x) + a), x)
```

Giac [B] time = 1.17821, size = 112, normalized size = 3.03

$$\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))/
sqrt(-a) - sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) +
a))))/d
```

3.81 $\int \frac{1}{\sqrt{a+a\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0736925, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx = \frac{\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx}{a} - \int \frac{\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx$$
$$= \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d} - \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 1.12355, size = 118, normalized size = 1.39

$$\frac{\left(e^{c+dx}+1\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{e^{c+dx}-1}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{\sqrt{2}d\sqrt{e^{2(c+dx)}+1}\sqrt{a(\operatorname{sech}(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] ((1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(-1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] - Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a*(1 + Sech[c + d*x
])])
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+a\operatorname{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+a*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sech(d*x + c) + a), x)
```

Fricas [B] time = 2.8768, size = 2480, normalized size = 29.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) - 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) - 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)) + sqrt(a)*log(-(a*cosh(d*x +
 c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)
*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*
x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*si
nh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 -
 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
 c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh
(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2
- 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a
/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) -
 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh
(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*
sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(
a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \operatorname{sech}(c + dx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(c + d*x) + a), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.82 $\int \frac{1}{(a+a\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(a^(3/2)*d)
- (5*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/2))
```

Rubi [A] time = 0.127655, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(a^(3/2)*d)
- (5*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/2))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\text{sech}(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx}{4a}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{ad} - \frac{(5i)\,\text{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}}$$

Mathematica [A] time = 4.62702, size = 177, normalized size = 1.55

$$\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)\left(4\left(e^{c+dx}+1\right)\sinh^{-1}\left(e^{c+dx}\right)+5\sqrt{2}\left(e^{c+dx}+1\right)\tanh^{-1}\left(\frac{1-e^{c+dx}}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)-4\left(e^{c+dx}+1\right)\tanh\right.}{2d\sqrt{e^{2(c+dx)}+1}(a(\text{sech}(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*Sech[c + d*x]*(4*(1 + E^(c + d*x))*ArcSinh[E^(c + d*x)
] + 5*Sqrt[2]*(1 + E^(c + d*x))*ArcTanh[(1 - E^(c + d*x))/(Sqrt[2]*Sqrt[1 +
 E^(2*(c + d*x))])] - 4*(1 + E^(c + d*x))*ArcTanh[Sqrt[1 + E^(2*(c + d*x))]
] - 2*Sqrt[1 + E^(2*(c + d*x))]*Tanh[(c + d*x)/2]))/(2*d*Sqrt[1 + E^(2*(c +
 d*x))]*(a*(1 + Sech[c + d*x]))^(3/2))
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int (a+a\,\text{sech}\,(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 2.91463, size = 3401, normalized size = 29.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(3*a*cosh(d*x + c)^2 + 3*a
*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cos
h(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*a*cosh(
d*x + c) + 2*(3*a*cosh(d*x + c) - a)*sinh(d*x + c) + 3*a)/(cosh(d*x + c)^2
+ 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) +
 1)) + 4*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x
+ c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 +
5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(
d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sin
h(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) +
 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x
 + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*co
sh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c)
+ 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c)
+ (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*si
nh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 4*(cosh(d*x + c)^2 + 2*(
cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*s
qrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*c
osh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*s
qrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/
(cosh(d*x + c) + sinh(d*x + c))) - 4*(cosh(d*x + c)^3 + (3*cosh(d*x + c) -
1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2
 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(a^2*d*cos
h(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a
^2*d*cosh(d*x + c) + a^2*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \operatorname{sech}\left(c + dx\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.83 $\int \sqrt{a-a\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d
```

Rubi [A] time = 0.0232803, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\operatorname{sech}(c+dx)}\,dx &= -\frac{(2ia)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 2.26553, size = 70, normalized size = 1.84

$$\frac{\sqrt{e^{2(c+dx)}+1}\sqrt{a-a\operatorname{sech}(c+dx)}\left(\sinh^{-1}\left(e^{c+dx}\right)+\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{d\left(e^{c+dx}-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^(2*(c + d*x))]*(ArcSinh[E^(c + d*x)] + ArcTanh[Sqrt[1 + E^(2*(c
 + d*x))]])*Sqrt[a - a*Sech[c + d*x]])/(d*(-1 + E^(c + d*x)))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \sqrt{a - a\operatorname{sech}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a-a*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\operatorname{sech}(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sech(d*x + c) + a), x)
```

Fricas [B] time = 2.5674, size = 1808, normalized size = 47.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log((a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)
^3 + (4*a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a
*cosh(d*x + c)^2 + 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c
)^5 + (5*cosh(d*x + c) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x
+ c)^4 + (10*cosh(d*x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*co
sh(d*x + c)^3 + (10*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c)
 + 7)*sinh(d*x + c)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*
x + c)^3 + 15*cosh(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cos
h(d*x + c) + 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9
*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d
*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x +
c) - a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a \operatorname{sech}\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sech(c + d*x) + a), x)
```

Giac [B] time = 1.18494, size = 136, normalized size = 3.58

$$-\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)\operatorname{sgn}\left(e^{(dx+c)}-1\right)}{\sqrt{-a}}+\sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)\operatorname{sgn}\left(e^{(dx+c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))
*sgn(e^(d*x + c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqr
t(a*e^(2*d*x + 2*c) + a)))*sgn(e^(d*x + c) - 1))/d
```

3.84 $\int \frac{1}{\sqrt{a-a\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.079209, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\operatorname{sech}(c+dx)}}\,dx = \frac{\int \sqrt{a-a\operatorname{sech}(c+dx)}\,dx}{a} + \int \frac{\operatorname{sech}(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\,dx$$
$$= -\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d} + \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 2.16382, size = 118, normalized size = 1.36

$$\frac{\left(e^{c+dx}-1\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{e^{c+dx}+1}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{\sqrt{2}d\sqrt{e^{2(c+dx)}+1}\sqrt{a-a\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] ((-1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] + Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a - a*Sech[c + d*x]
])
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-a\operatorname{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-a*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-a*sech(d*x + c) + a), x)
```

Fricas [B] time = 2.81095, size = 2480, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) + 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 2*cosh(d*x + c) + 1)) + sqrt(a)*log((a*cosh(d*x +
c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + 3*a)*
sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 9*a*cosh(d*x
 + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) + 3)*sin
h(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 +
12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) + 7)*sinh(d*x + c)^2 + 7*cosh(
d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 +
 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) + 4)*sqrt(a)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) +
4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9*a*cosh(d*x + c)^2 + 10*a*cosh(
d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*s
inh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a
)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x + c) - a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\,\mathrm{sech}\,(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sech(c + d*x) + a), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.85 $\int \sqrt{3+3\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=19

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\mathrm{sech}(x)+1}}\right)$$

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Rubi [A] time = 0.0173795, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\mathrm{sech}(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3\,\mathrm{sech}(x)}\,dx &= 6i\,\mathrm{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\tanh(x)}{\sqrt{3+3\,\mathrm{sech}(x)}}\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\mathrm{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.0381295, size = 39, normalized size = 2.05

$$\sqrt{6}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{x}{2}\right)\right)\sqrt{\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{\mathrm{sech}(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] Sqrt[6]*ArcSinh[Sqrt[2]*Sinh[x/2]]*Sqrt[Cosh[x]]*Sech[x/2]*Sqrt[1 + Sech[x]
]
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \sqrt{3+3\,\mathrm{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sech(x))^(1/2),x)
```

```
[Out] int((3+3*sech(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\,\mathrm{sech}\left(x\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*sech(x) + 3), x)
```

Fricas [B] time = 2.4964, size = 833, normalized size = 43.84

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{\cosh\left(x\right)^4+\left(4\,\cosh\left(x\right)-3\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4-3\,\cosh\left(x\right)^3+\left(6\,\cosh\left(x\right)^2-9\,\cosh\left(x\right)+5\right)\sinh\left(x\right)^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(cosh(x)^4 + (4*cosh(x) - 3)*sinh(x)^3 + sinh(x)^4 - 3*cos
h(x)^3 + (6*cosh(x)^2 - 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(
cosh(x) - 1)*sinh(x)^2 + sinh(x)^3 - 3*cosh(x)^2 + (3*cosh(x)^2 - 6*cosh(x)
 + 4)*sinh(x) + 4*cosh(x) - 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x
)^2 + (4*cosh(x)^3 - 9*cosh(x)^2 + 10*cosh(x) - 4)*sinh(x) - 4*cosh(x) + 4)
/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2
*sqrt(3)*log((sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
+ 1) + cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{\mathrm{sech}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(sech(x) + 1), x)
```

Giac [B] time = 1.14755, size = 70, normalized size = 3.68

$$-\sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^{x}+1\right)+\log\left(\sqrt{e^{(2x)}+1}-e^{x}\right)-\log\left(-\sqrt{e^{(2x)}+1}+e^{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1) + log(sqrt(e^(2*x) + 1) - e^x) -
 log(-sqrt(e^(2*x) + 1) + e^x + 1))
```

3.86 $\int \sqrt{3-3\,\textbf{sech}(x)}\,dx$

Optimal. Leaf size=21

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Rubi [A] time = 0.0186269, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-3\,\text{sech}(x)}\,dx &= -\left(6i\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\tanh(x)}{\sqrt{3-3\,\text{sech}(x)}}\right)\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.526214, size = 51, normalized size = 2.43

$$\frac{\sqrt{3}\sqrt{e^{2x}+1}\sqrt{1-\text{sech}(x)}\left(\sinh^{-1}\left(e^x\right)+\tanh^{-1}\left(\sqrt{e^{2x}+1}\right)\right)}{e^x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[1 + E^(2*x)]*(ArcSinh[E^x] + ArcTanh[Sqrt[1 + E^(2*x)]])*Sqrt
[1 - Sech[x]])/(-1 + E^x)
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \sqrt{3-3\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sech(x))^(1/2),x)
```

```
[Out] int((3-3*sech(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3\,\mathrm{sech}\,(x)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*sech(x) + 3), x)
```

Fricas [B] time = 2.47617, size = 833, normalized size = 39.67

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\cosh(x)^4 + (4\,\cosh(x)+3)\sinh(x)^3 + \sinh(x)^4 + 3\,\cosh(x)^3 + \left(6\,\cosh(x)^2 + 9\,\cosh(x)+5\right)\sinh(x)^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((cosh(x)^4 + (4*cosh(x) + 3)*sinh(x)^3 + sinh(x)^4 + 3*cosh
(x)^3 + (6*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(c
osh(x) + 1)*sinh(x)^2 + sinh(x)^3 + 3*cosh(x)^2 + (3*cosh(x)^2 + 6*cosh(x)
+ 4)*sinh(x) + 4*cosh(x) + 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x)
^2 + (4*cosh(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 4)*sinh(x) + 4*cosh(x) + 4)/
(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*
sqrt(3)*log(-(sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
- 1) + cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int \sqrt{1-\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(1 - sech(x)), x)
```

Giac [B] time = 1.1214, size = 93, normalized size = 4.43

$$\sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^{x}+1\right)\operatorname{sgn}\left(e^{x}-1\right)-\log\left(\sqrt{e^{(2x)}+1}-e^{x}\right)\operatorname{sgn}\left(e^{x}-1\right)-\log\left(-\sqrt{e^{(2x)}+1}+e^{x}+1\right)\operatorname{sgn}\left(e^{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1)*sgn(e^x - 1) - log(sqrt(e^(2*x) +
 1) - e^x)*sgn(e^x - 1) - log(-sqrt(e^(2*x) + 1) + e^x + 1)*sgn(e^x - 1))
```

3.87 $\int (a + b\mathbf{sech}(c + dx))^4 \, dx$

Optimal. Leaf size=107

$$\frac{b^2 \left(17a^2 + 2b^2\right) \tanh(c + dx)}{3d} + \frac{2ab \left(2a^2 + b^2\right) \tan^{-1}(\sinh(c + dx))}{d} + a^4 x + \frac{4ab^3 \tanh(c + dx)\mathrm{sech}(c + dx)}{3d} + \frac{b^2 \tan}{}$$

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tanh[c + d*x])/(3*d) + (4*a*b^3*Sech[c + d*x]*Tanh[c + d*x])/(3*d) + (b^
2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d)
```

Rubi [A] time = 0.123799, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$\frac{b^2 \left(17a^2 + 2b^2\right) \tanh(c + dx)}{3d} + \frac{2ab \left(2a^2 + b^2\right) \tan^{-1}(\sinh(c + dx))}{d} + a^4 x + \frac{4ab^3 \tanh(c + dx)\mathrm{sech}(c + dx)}{3d} + \frac{b^2 \tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tanh[c + d*x])/(3*d) + (4*a*b^3*Sech[c + d*x]*Tanh[c + d*x])/(3*d) + (b^
2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\operatorname{sech}(c+dx))^4\,dx = \frac{b^2(a+b\operatorname{sech}(c+dx))^2\tanh(c+dx)}{3d}+\frac{1}{3}\int (a+b\operatorname{sech}(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\operatorname{sech}(c\right.$$

$$= \frac{4ab^3\operatorname{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\operatorname{sech}(c+dx))^2\tanh(c+dx)}{3d}+\frac{1}{6}\int \left(6a^4+12ab\right.$$

$$= a^4x+\frac{4ab^3\operatorname{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\operatorname{sech}(c+dx))^2\tanh(c+dx)}{3d}+\left(2ab\left(2a^2+\right.\right.$$

$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+\frac{4ab^3\operatorname{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\operatorname{sech}}{}$$

$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\tanh(c+dx)}{3d}+\frac{4ab^3\operatorname{sech}(c+}{3}$$

Mathematica [A] time = 0.237227, size = 78, normalized size = 0.73

$$\frac{6ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))+3b^2\tanh(c+dx)\left(6a^2+2ab\operatorname{sech}(c+dx)+b^2\right)+3a^4dx-b^4\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sech[c + d*x])*Tanh[c + d*x] - b^4*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.033, size = 121, normalized size = 1.1

$$a^4x+\frac{a^4c}{d}+8\frac{a^3b\arctan\left(e^{dx+c}\right)}{d}+6\frac{a^2b^2\tanh(dx+c)}{d}+2\frac{ab^3\operatorname{sech}(dx+c)\tanh(dx+c)}{d}+4\frac{ab^3\arctan\left(e^{dx+c}\right)}{d}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+8/d*a^3*b*arctan(exp(d*x+c))+6/d*a^2*b^2*tanh(d*x+c)+2*a*b^
3*sech(d*x+c)*tanh(d*x+c)/d+4/d*a*b^3*arctan(exp(d*x+c))+2/3/d*b^4*tanh(d*x
+c)+1/3/d*b^4*tanh(d*x+c)*sech(d*x+c)^2
```

Maxima [B] time = 1.81674, size = 285, normalized size = 2.66

$$a^4x-4\,ab^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*a*b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))
/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 4/3*b^4*(3*e^(-2*d*x -
2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
```

```
 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) +
4*a^3*b*arctan(sinh(d*x + c))/d + 12*a^2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.48824, size = 2547, normalized size = 23.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 + 12*a*b^3*cosh(
d*x + c)^5 + 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) + 2*a*b^3)*sinh(d*x + c
)^5 - 12*a*b^3*cosh(d*x + c) + 9*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 + 3*a^4*d*x + 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 - 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 +
10*a*b^3*cosh(d*x + c)^2 + 3*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 + 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x - 24*a^2*b^2 - 4*b^
4 + 18*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((2*a^3*
b + a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)*sinh(d*x + c
)^5 + (2*a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)
^4 + 3*(2*a^3*b + a*b^3 + 5*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*(2*a^3*b
 + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c
)^2 + 3*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^4 + 2*a^3*b + a*b^3 + 6*(2*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b + a*b^3)*cosh(d*x
+ c)^5 + 2*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + (2*a^3*b + a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(3*a^4*d*x*co
sh(d*x + c)^5 + 10*a*b^3*cosh(d*x + c)^4 - 2*a*b^3 + 6*(a^4*d*x - 4*a^2*b^2
)*cosh(d*x + c)^3 + (3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x
 + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4
 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d
*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{sech}\,(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**4, x)
```

Giac [A] time = 1.16533, size = 194, normalized size = 1.81

$$\frac{(dx+c)a^4}{d} + \frac{4\left(2\,a^3b + ab^3\right)\arctan\left(e^{(dx+c)}\right)}{d} + \frac{4\left(3\,ab^3e^{(5\,dx+5\,c)} - 9\,a^2b^2e^{(4\,dx+4\,c)} - 18\,a^2b^2e^{(2\,dx+2\,c)} - 3\,b^4e^{(2\,dx+2\,c)} - 3\right.}{3\,d\left(e^{(2\,dx+2\,c)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^4/d + 4*(2*a^3*b + a*b^3)*arctan(e^(d*x + c))/d + 4/3*(3*a*b^3*
e^(5*d*x + 5*c) - 9*a^2*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) -
3*b^4*e^(2*d*x + 2*c) - 3*a*b^3*e^(d*x + c) - 9*a^2*b^2 - b^4)/(d*(e^(2*d*x
 + 2*c) + 1)^3)
```

3.88 $\int (a + b\,\mathbf{sech}(c + dx))^3\, dx$

Optimal. Leaf size=73

$$\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+a^3x+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\operatorname{sech}(c+dx))}{2d}$$

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*d) + (5*a*b^2*Tanh[c + d
*x])/(2*d) + (b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0516835, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+a^3x+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\operatorname{sech}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*d) + (5*a*b^2*Tanh[c + d
*x])/(2*d) + (b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^3\,dx = \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} + \frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\text{sech}(c+dx)+5ab^2\text{sech}^2(c\right.$$
$$= a^3x + \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} + \frac{1}{2}\left(5ab^2\right)\int \text{sech}^2(c+dx)\,dx + \frac{1}{2}\left(b\left(6a^2+b^2\right)\right.$$
$$= a^3x + \frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} + \frac{\left(5iab^2\right)\text{Su}}{}$$
$$= a^3x + \frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{5ab^2\tanh(c+dx)}{2d} + \frac{b^2(a+b\text{sech}(c+dx))\tanh(}{2d}$$

Mathematica [A] time = 0.129793, size = 55, normalized size = 0.75

$$\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))+2a^3dx+b^2\tanh(c+dx)(6a+b\text{sech}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + b^2*(6*a + b*Sech[c +
d*x])*Tanh[c + d*x])/(2*d)
```

Maple [A] time = 0.023, size = 80, normalized size = 1.1

$$a^3x + \frac{a^3c}{d} + 6\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d} + 3\,\frac{ab^2\tanh(dx+c)}{d} + \frac{b^3\text{sech}(dx+c)\tanh(dx+c)}{2d} + \frac{b^3\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+6/d*a^2*b*arctan(exp(d*x+c))+3*a*b^2*tanh(d*x+c)/d+1/2*b^3*
sech(d*x+c)*tanh(d*x+c)/d+1/d*b^3*arctan(exp(d*x+c))
```

Maxima [A] time = 1.8284, size = 154, normalized size = 2.11

$$a^3x - b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right) + \frac{3\,a^2b\arctan(\sinh(dx+c))}{d} + \frac{6\,ab^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*
(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*arctan(sinh(d*x + c
))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.36095, size = 1319, normalized size = 18.07

$$a^3dx\cosh(dx+c)^4 + a^3dx\sinh(dx+c)^4 + b^3\cosh(dx+c)^3 + a^3dx - b^3\cosh(dx+c) + \left(4a^3dx\cosh(dx+c) + b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*x*cosh(d*x + c)^4 + a^3*d*x*sinh(d*x + c)^4 + b^3*cosh(d*x + c)^3 +
a^3*d*x - b^3*cosh(d*x + c) + (4*a^3*d*x*cosh(d*x + c) + b^3)*sinh(d*x + c)
^3 - 6*a*b^2 + 2*(a^3*d*x - 3*a*b^2)*cosh(d*x + c)^2 + (6*a^3*d*x*cosh(d*x
+ c)^2 + 2*a^3*d*x + 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh(d*x + c)^2 + ((6*a
^2*b + b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)
^3 + (6*a^2*b + b^3)*sinh(d*x + c)^4 + 6*a^2*b + b^3 + 2*(6*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(6*a^2*b + b^3 + 3*(6*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 4*((6*a^2*b + b^3)*cosh(d*x + c)^3 + (6*a^2*b + b^3)*cosh(d*x
+ c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (4*a^3*d*x*cos
h(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2 - b^3 + 4*(a^3*d*x - 3*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{sech}(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**3, x)
```

Giac [A] time = 1.13269, size = 131, normalized size = 1.79

$$\frac{(dx+c)a^3}{d} + \frac{\left(6a^2b + b^3\right)\arctan\left(e^{(dx+c)}\right)}{d} + \frac{b^3e^{(3dx+3c)} - 6ab^2e^{(2dx+2c)} - b^3e^{(dx+c)} - 6ab^2}{d\left(e^{(2dx+2c)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^3/d + (6*a^2*b + b^3)*arctan(e^(d*x + c))/d + (b^3*e^(3*d*x + 3
*c) - 6*a*b^2*e^(2*d*x + 2*c) - b^3*e^(d*x + c) - 6*a*b^2)/(d*(e^(2*d*x + 2
*c) + 1)^2)
```

3.89 $\int (a+b\,\mathbf{sech}(c+dx))^2\,dx$

Optimal. Leaf size=33

$$a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0280215, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\text{sech}(c+dx))^2\,dx &= a^2x+(2ab)\int \text{sech}(c+dx)\,dx+b^2\int \text{sech}^2(c+dx)\,dx\\ &= a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{\left(ib^2\right)\text{Subst}\left(\int 1\,dx,x,-i\tanh(c+dx)\right)}{d}\\ &= a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.063079, size = 32, normalized size = 0.97

$$\frac{a\left(adx+2b\tan^{-1}(\sinh(c+dx))\right)+b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] (a*(a*d*x + 2*b*ArcTan[Sinh[c + d*x]]) + b^2*Tanh[c + d*x])/d
```

Maple [A] time = 0.01, size = 42, normalized size = 1.3

$$a^2x + \frac{b^2\tanh(dx+c)}{d} + 4\,\frac{ab\arctan\left(e^{dx+c}\right)}{d} + \frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^2,x)
```

```
[Out] a^2*x+b^2*tanh(d*x+c)/d+4/d*a*b*arctan(exp(d*x+c))+1/d*a^2*c
```

Maxima [A] time = 1.0838, size = 55, normalized size = 1.67

$$a^2x + \frac{2\,ab\arctan(\sinh(dx+c))}{d} + \frac{2\,b^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*arctan(sinh(d*x + c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.43496, size = 428, normalized size = 12.97

$$\frac{a^2dx\cosh(dx+c)^2 + 2\,a^2dx\cosh(dx+c)\sinh(dx+c) + a^2dx\sinh(dx+c)^2 + a^2dx - 2\,b^2 + 4\left(ab\cosh(dx+c)^2 + 2\right.}{d\cosh(dx+c)^2 + 2\,d\cosh(dx+c)\sinh(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 + a^2*d*x - 2*b^2 + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*arctan(cosh(d*x + c) + sin
h(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(
d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{sech}(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**2, x)
```

Giac [A] time = 1.16557, size = 65, normalized size = 1.97

$$\frac{(dx+c)a^2}{d} + \frac{4\,ab\arctan\left(e^{(dx+c)}\right)}{d} - \frac{2\,b^2}{d\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^2/d + 4*a*b*arctan(e^(d*x + c))/d - 2*b^2/(d*(e^(2*d*x + 2*c) +
 1))
```

3.90 $\int (a + b\,\mathbf{sech}(c + dx))\, dx$

Optimal. Leaf size=16

$$ax + \frac{b\tan^{-1}(\sinh(c + dx))}{d}$$

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.009225, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3770}

$$ax + \frac{b\tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\operatorname{sech}(c + dx))\, dx &= ax + b\int \operatorname{sech}(c + dx)\, dx\\ &= ax + \frac{b\tan^{-1}(\sinh(c + dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0017844, size = 16, normalized size = 1.

$$ax + \frac{b\tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Maple [A] time = 0.003, size = 17, normalized size = 1.1

$$ax + \frac{b\arctan(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c),x)
```

```
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Maxima [A] time = 1.21941, size = 22, normalized size = 1.38

$$ax + \frac{b \arctan\left(\sinh\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*arctan(sinh(d*x + c))/d
```

Fricas [A] time = 2.31228, size = 74, normalized size = 4.62

$$\frac{adx + 2\,b \arctan\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + 2*b*arctan(cosh(d*x + c) + sinh(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{sech}\left(c + dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x)
```

```
[Out] Integral(a + b*sech(c + d*x), x)
```

Giac [A] time = 1.13199, size = 23, normalized size = 1.44

$$ax + \frac{2\,b \arctan\left(e^{\left(dx + c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 2*b*arctan(e^(d*x + c))/d
```

3.91 $\int \frac{1}{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0553858, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3783, 2659, 208}

$$\frac{x}{a}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx = \frac{x}{a}-\frac{\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a}$$
$$= \frac{x}{a}+\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$
$$= \frac{x}{a}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.10464, size = 60, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{c}{d}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] (c/d + x + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

Maple [A] time = 0.014, size = 88, normalized size = 1.5

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\,\frac{b}{da\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-2/d/a*b/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51876, size = 648, normalized size = 10.98

$$\left[\frac{\left(a^2-b^2\right)dx-\sqrt{-a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)-a^2+2\,b^2+2\left(a^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{-a^2+b^2}(a\cosh}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx+c)+a}\right.}{\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b
))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d
*x + c) + b)*sinh(d*x + c) + a)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x + 2*s
qrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 -
 b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)), x)
```

Giac [A] time = 1.14122, size = 76, normalized size = 1.29

$$-\frac{2\,b\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*d) + (d
*x + c)/(a*d)
```

3.92 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\,\mathrm{sech}(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Sech[c + d*x]))
```

Rubi [A] time = 0.158986, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\,\mathrm{sech}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-2),x]
```


```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Sech[c + d*x]))
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))} + \frac{\left(2i\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\right.}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))}$$

Mathematica [A] time = 0.402178, size = 203, normalized size = 1.86

$$\frac{b\left(\left(a^2-b^2\right)^{3/2}(c+dx)+ab\sqrt{a^2-b^2}\sinh(c+dx)+\left(4a^2b-2b^3\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)+a\cosh(c+dx)\left(\left(a^2-b\right.\right.}{a^2d(a-b)(a+b)\sqrt{a^2-b^2}(a\cosh(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(
c + d*x)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x] + b*((a^2 - b^2)^(3/2)*(c + d*
x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(b + a*Cosh[c + d*x]))
```

Maple [B] time = 0.046, size = 221, normalized size = 2.

$$\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\,\frac{b^2\tanh(1/2\,dx+c/2)}{da\left(a^2-b^2\right)\left((\tanh(1/2\,dx+c/2))^2a-(\tanh(1/2\,dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a*b
^2/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*
c)^2*b+a+b)-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))+2/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61205, size = 2708, normalized size = 24.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x
 + (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)
*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 +
 (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*
cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x
 + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2*b^3 -
b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^
5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*s
inh(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5
*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(
d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d
*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)), -(2*a^3*b
^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^5 - 2*a^3
*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x - 2*(2*a^
3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)*sinh(d*
x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 + (2*a^3*
b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 - b^2)) + 2*(a^2*b^3 - b^5 - (a^4*b
- 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2
 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*sinh(d*x + c))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^
4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{sech}\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-2), x)
```

Giac [A] time = 1.12446, size = 185, normalized size = 1.7

$$-\frac{2\left(2\,a^2b - b^3\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\left(a^4d - a^2b^2d\right)\sqrt{a^2 - b^2}} - \frac{2\left(b^3e^{(dx+c)} + ab^2\right)}{\left(a^4d - a^2b^2d\right)\left(ae^{(2\,dx+2\,c)} + 2\,be^{(dx+c)} + a\right)} + \frac{dx + c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(2*a^2*b - b^3)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/((a^4*d - a^
2*b^2*d)*sqrt(a^2 - b^2)) - 2*(b^3*e^(d*x + c) + a*b^2)/((a^4*d - a^2*b^2*d
)*(a*e^(2*d*x + 2*c) + 2*b*e^(d*x + c) + a)) + (d*x + c)/(a^2*d)
```

3.93 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\mathrm{sech}(c+dx))}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\mathrm{sech}(c+$$

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tanh[c + d*x])/
(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tanh[c +
 d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sech[c + d*x]))
```

Rubi [A] time = 0.308491, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\mathrm{sech}(c+dx))}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\mathrm{sech}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tanh[c + d*x])/
(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tanh[c +
 d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sech[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)+2ab\operatorname{sech}(c+dx)-b^2\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} + \frac{\int \frac{2\left(a^2-b^2\right)^2-}{a+}}{2a}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} - \frac{\left(b\left(6a\right.\right.}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} - \frac{\left(6a^4-\right.}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} + \frac{\left(i\left(6a\right.\right.}{}$$

$$= \frac{x}{a^3} - \frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx}$$

Mathematica [A] time = 0.726719, size = 205, normalized size = 1.18

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(c+dx)+b)\left(\frac{3ab^2\left(2a^2-b^2\right)\sinh(c+dx)(a\cosh(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{2b\left(-5a^2b^2+6a^4+2b^4\right)(a\cosh(c+dx)+b)^2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{1}{2}(\right.}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{5/2}}\right.}{2a^3d(a+b\operatorname{sech}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cosh[c + d*x])*Sech[c + d*x]^3*(2*(c + d*x)*(b + a*Cosh[c + d*x])^2
 + (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqr
t[a^2 - b^2]]*(b + a*Cosh[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sinh[c +
d*x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cosh[c + d*x])*Sin
h[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sech[c + d*x])^3)
```

Maple [B] time = 0.05, size = 660, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+6/d*b^2
/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/a/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-2/d*b^4/a^2/(t
anh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^3+6/d*b^2/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/d*b^3/a/(tanh(1/2*d
*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)-2/d*b^4/a^2/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-6/d*b*a/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+5
/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.17152, size = 8986, normalized size = 51.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(12*a^6*b^2 - 18*a^4*b^4 + 6*a^2*b^6 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
 - a^2*b^6)*d*x*cosh(d*x + c)^4 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 - 11*a^3*b^
5 + 4*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) -
```

```
 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - 2*(a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^
6 + 6*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x
 + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^
2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4
 + 5*a^2*b^6 - 2*b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a
^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x +
 c)^4 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 -
5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 +
 (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^
6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b
^3 - 8*a^2*b^5 + 4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^
2 + 6*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*
(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4
+ 2*a*b^6 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^
2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5
 + 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*cosh(d*x
+ c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a
*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x
+ c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(17*a^5*b^3 - 25*a^3
*b^5 + 8*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
c) + 2*(17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
- a^2*b^6)*d*x*cosh(d*x + c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6
*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^
4 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 -
3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2
 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
 a^4*b^7)*d)*sinh(d*x + c)^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c) + 2*(3*(a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x
 + c)^2 + 6*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a
^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^
11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4
- a^5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c)^2 + (a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d
*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c
)), -(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^
2*b^6)*d*x*cosh(d*x + c)^4 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*si
nh(d*x + c)^4 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7
 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) - 4*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*d*x + (6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(
a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)^2 + (6*a^
6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*
b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - (6*a^6*b - 5*a^4*b^3
+ 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^4 + (6*a^6*b
- 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b
^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^6*b - 5*a^4
*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^6*b + 7*a^4*b^3 -
 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 +
4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2 + 6*(6*a^5*b^2
```

```
- 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(6*a^5*b^2 - 5*a^
3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^
6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^2 - 5*a^3*b^4 + 2
*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*
x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*x + c) + a*sinh(d*
x + c) + b)/sqrt(a^2 - b^2)) + (17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c) + (17*a^5*b^3 - 25*a^
3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x +
c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x + 3*(7*a^5*b^3 - 11*a^
3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
 c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(a^8 - a^6*b^2 -
3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^4 + (a^11 - 3*a^9*b^2 + 3*
a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*
a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*
cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c)
^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + 2*(3*(a
^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^2 + 6*(a^10*b - 3*a^
8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a^11 - a^9*b^2 - 3*a^7*b^4
+ 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4
 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)
^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^2 + (a^11
 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c) + (a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{sech}\left(c + dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3), x)
```

Giac [A] time = 1.1785, size = 362, normalized size = 2.09

$$-\frac{\left(6\,a^{4}b - 5\,a^{2}b^{3} + 2\,b^{5}\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{7}d - 2\,a^{5}b^{2}d + a^{3}b^{4}d\right)\sqrt{a^{2}-b^{2}}} - \frac{7\,a^{3}b^{3}e^{(3\,dx+3\,c)} - 4\,ab^{5}e^{(3\,dx+3\,c)} + 6\,a^{4}b^{2}e^{(2\,dx+2\,c)} + 9\,a^{2}b^{4}e^{(2\,dx+2\,c)} - 6\,b^{6}e^{(2\,\cdots}}{\left(a^{7}d - 2\,a^{5}b^{2}d + a^{3}b^{4}d\right)\left(ae^{(2\,dx+2\,c)} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(6*a^4*b - 5*a^2*b^3 + 2*b^5)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/
((a^7*d - 2*a^5*b^2*d + a^3*b^4*d)*sqrt(a^2 - b^2)) - (7*a^3*b^3*e^(3*d*x +
 3*c) - 4*a*b^5*e^(3*d*x + 3*c) + 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(
2*d*x + 2*c) - 6*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(
d*x + c) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7*d - 2*a^5*b^2*d + a^3*b^4*d)*(a*e^(
2*d*x + 2*c) + 2*b*e^(d*x + c) + a)^2) + (d*x + c)/(a^3*d)
```

3.94 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.0292714, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad}$$

Mathematica [A] time = 2.34509, size = 168, normalized size = 1.58

$$\frac{2b\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a\cosh(c+dx)+b}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{b-a}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)}{\sqrt{a}d\sqrt{a+b}\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(\operatorname{sech}(c+dx)-1)}{a+b}}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
```

```
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.235, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\mathrm{sech}\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\,\mathrm{sech}\left(dx + c\right) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\,\mathrm{sech}\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

3.95 $\int \frac{\cosh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=146

$$\frac{x\left(4a^2b^2+3a^4+8b^4\right)}{8a^5}-\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4}-\frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}+\frac{\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{8a^3}-\frac{b\sinh(x)\cos}{3a^2}$$

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]) - (b*(2*a^2 + 3*b^2)*Sinh[
x])/(3*a^4) + ((3*a^2 + 4*b^2)*Cosh[x]*Sinh[x])/(8*a^3) - (b*Cosh[x]^2*Sinh
[x])/(3*a^2) + (Cosh[x]^3*Sinh[x])/(4*a)
```

Rubi [A] time = 0.657326, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$\frac{x\left(4a^2b^2+3a^4+8b^4\right)}{8a^5}-\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4}-\frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}+\frac{\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{8a^3}-\frac{b\sinh(x)\cos}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]) - (b*(2*a^2 + 3*b^2)*Sinh[
x])/(3*a^4) + ((3*a^2 + 4*b^2)*Cosh[x]*Sinh[x])/(8*a^3) - (b*Cosh[x]^2*Sinh
[x])/(3*a^2) + (Cosh[x]^3*Sinh[x])/(4*a)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh^3(x)\left(-4b+3a\text{sech}(x)+3b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{4a}$$

$$= -\frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\int \frac{\cosh^2(x)\left(-3\left(3a^2+4b^2\right)-ab\text{sech}(x)+8b^2\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{12a^2}$$

$$= \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh(x)\left(-8b\left(2a^2+3b^2\right)+a\right.}{}}{}$$

$$= -\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x}{4a}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh}{3a^2}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh}{3a^2}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh}{3a^2}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(}{8a^3}$$

Mathematica [A] time = 0.275261, size = 126, normalized size = 0.86

$$\frac{12x\left(4a^2b^2+3a^4+8b^4\right)-24ab\left(3a^2+4b^2\right)\sinh(x)+24a^2\left(a^2+b^2\right)\sinh(2x)+\frac{192b^5\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-8a^3b\sinh(3x}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x + (192*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Sinh[x] + 24*a^2*(
a^2 + b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [B] time = 0.037, size = 406, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sech(x)),x)
```

```
[Out] -2*b^5/a^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-1/4/a/(tanh(1/2*x)+1)^4+1/2/a/(tanh(1/2*x)+1)^3-7/8/a/(tanh(1/2*x)+1)^2+5
/8/a/(tanh(1/2*x)+1)+3/8/a*ln(tanh(1/2*x)+1)+1/4/a/(tanh(1/2*x)-1)^4+1/2/a/
(tanh(1/2*x)-1)^3+7/8/a/(tanh(1/2*x)-1)^2+5/8/a/(tanh(1/2*x)-1)-3/8/a*ln(ta
nh(1/2*x)-1)+1/2/a^3/(tanh(1/2*x)-1)^2*b^2-1/2/a^3*ln(tanh(1/2*x)-1)*b^2-1/
a^5*ln(tanh(1/2*x)-1)*b^4+1/a^2/(tanh(1/2*x)-1)*b+1/2/a^3/(tanh(1/2*x)-1)*b
^2+1/a^4/(tanh(1/2*x)-1)*b^3+1/a^2/(tanh(1/2*x)+1)*b+1/2/a^3/(tanh(1/2*x)+1
)*b^2+1/a^4/(tanh(1/2*x)+1)*b^3+1/3/a^2/(tanh(1/2*x)-1)^3*b+1/2/a^2/(tanh(1
/2*x)-1)^2*b+1/2/a^3*ln(tanh(1/2*x)+1)*b^2+1/a^5*ln(tanh(1/2*x)+1)*b^4+1/3/
a^2/(tanh(1/2*x)+1)^3*b-1/2/a^2/(tanh(1/2*x)+1)^2*b-1/2/a^3/(tanh(1/2*x)+1)
^2*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.81342, size = 5667, normalized size = 38.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 - 8*(a^5*
b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh(x))*si
nh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(a^6 - a
^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6 + 3*a^4
*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*a^5*b +
 a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*(a^6 -
a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh
(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3*b^3)*c
osh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4
- 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 24*(3*a^
```

```
5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b^5 + 21
*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^6 - a^2
*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x) - 30*(
3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^4)*cosh
(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^5*b - a
^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b^2 + 4*
a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 +
 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 - 192*(b^5*cosh(x)^4 +
4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)
^3 + b^5*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2
*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) + a)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6
 - a^4*b^2)*cosh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 1
8*(a^6 - a^2*b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*co
sh(x)^3 - 15*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3
 - 4*a*b^5)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2
)*cosh(x)^4 + 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(
x)^2*sinh(x)^2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh
(x)^4), 1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 -
8*(a^5*b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh
(x))*sinh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(
a^6 - a^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6
+ 3*a^4*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*
a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*
(a^6 - a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^
4)*cosh(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3
*b^3)*cosh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a
^2*b^4 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 2
4*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b
^5 + 21*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^
6 - a^2*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)
 - 30*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^
4)*cosh(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^
5*b - a^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b
^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh
(x)^3 + 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 + 384*(b^5*cosh(
x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*
sinh(x)^3 + b^5*sinh(x)^4)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6 - a^4*b^2)*c
osh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 18*(a^6 - a^2*
b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^3 - 15*
(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3 - 4*a*b^5)*c
osh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2)*cosh(x)^4 +
 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(x)^2*sinh(x)^
2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.14688, size = 246, normalized size = 1.68

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^5}+\frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-72\,a^2be^x-96\,b^3e^x}{192\,a^4}+\frac{\left(3\,a^4+4\,a^2b^2+8\,b^4\right)x}{8\,a^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/192*(3
*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x) - 72*a^2
*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/192*(8
*a^3*b*e^x - 3*a^4 + 24*(3*a^3*b + 4*a*b^3)*e^(3*x) - 24*(a^4 + a^2*b^2)*e^
(2*x))*e^(-4*x)/a^5
```

3.96 $\int \frac{\cosh^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=112

$$-\frac{bx\left(a^2+2b^2\right)}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}+\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Sinh[x])/(3*a^3) -
 (b*Cosh[x]*Sinh[x])/(2*a^2) + (Cosh[x]^2*Sinh[x])/(3*a)
```

Rubi [A] time = 0.423414, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$-\frac{bx\left(a^2+2b^2\right)}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}+\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Sinh[x])/(3*a^3) -
 (b*Cosh[x]*Sinh[x])/(2*a^2) + (Cosh[x]^2*Sinh[x])/(3*a)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\,\mathrm{sech}(x)}\,dx &= \frac{\cosh^2(x)\sinh(x)}{3a}+\frac{\int \frac{\cosh^2(x)\left(-3b+2a\,\mathrm{sech}(x)+2b\,\mathrm{sech}^2(x)\right)}{a+b\,\mathrm{sech}(x)}\,dx}{3a}\\
&= -\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}-\frac{\int \frac{\cosh(x)\left(-2\left(2a^2+3b^2\right)-ab\,\mathrm{sech}(x)+3b^2\mathrm{sech}^2(x)\right)}{a+b\,\mathrm{sech}(x)}\,dx}{6a^2}\\
&= \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}+\frac{\int \frac{-3b\left(a^2+2b^2\right)-3ab^2\mathrm{sech}(x)}{a+b\,\mathrm{sech}(x)}\,dx}{6a^3}\\
&= -\frac{b\left(a^2+2b^2\right)x}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}+\frac{b^4\int \frac{\mathrm{sech}(x)}{a+b\,\mathrm{sech}(x)}\,dx}{a^4}\\
&= -\frac{b\left(a^2+2b^2\right)x}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}+\frac{b^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^4}\\
&= -\frac{b\left(a^2+2b^2\right)x}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}+\frac{\left(2b^3\right)\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{b\left(a^2+2b^2\right)x}{2a^4}+\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.159893, size = 99, normalized size = 0.88

$$\frac{-6bx\left(a^2+2b^2\right)+3a\left(3a^2+4b^2\right)\sinh(x)-\frac{24b^4\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-3a^2b\sinh(2x)+a^3\sinh(3x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*x - (24*b^4*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] - 3*a^2*b*Sinh[2*x] + a^3*
Sinh[3*x])/(12*a^4)
```

Maple [B] time = 0.036, size = 264, normalized size = 2.4

$$-\frac{1}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)+1)^3+1/2/a/(tanh(1/2*x)+1)^2+1/2/a^2/(tanh(1/2*x)+1)^2*
b-1/a/(tanh(1/2*x)+1)-1/2/a^2/(tanh(1/2*x)+1)*b-1/a^3/(tanh(1/2*x)+1)*b^2-1
/2*b/a^2*ln(tanh(1/2*x)+1)-b^3/a^4*ln(tanh(1/2*x)+1)-1/3/a/(tanh(1/2*x)-1)^
3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*b-1/a/(tanh(1/2*x)-1)-1
/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2+1/2*b/a^2*ln(tanh(1/2*x)
-1)+b^3/a^4*ln(tanh(1/2*x)-1)+2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
nh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69503, size = 3675, normalized size = 32.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(x)^6 - 3*(a^4*b - a
^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^2)*cosh(x))*sinh(x)
^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^3 + 3*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4 + 5*(a^5 - a^3*
b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 - a^3*
b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4*b + a^2*b^3 - 2*b^
5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^3*b^
2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 - 5*(a^5 - a^3*b^2)*c
osh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b + a^2*b^3 - 2*b^5)*x*
cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 - 24*(b^4*cosh
(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*
sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 +
 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*
sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*
sinh(x) + a)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^
5 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
 2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 -
a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a
```

```
^4*b^2)*sinh(x)^3), 1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(
x)^6 - 3*(a^4*b - a^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^
2)*cosh(x))*sinh(x)^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh
(x)^3 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*
b^4 + 5*(a^5 - a^3*b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4
+ 2*(10*(a^5 - a^3*b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4
*b + a^2*b^3 - 2*b^5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3
- 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 -
5*(a^5 - a^3*b^2)*cosh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b +
a^2*b^3 - 2*b^5)*x*cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(
x)^2 - 48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^
2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt
(a^2 - b^2)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^5
 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 + a
^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a
^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a^
4*b^2)*sinh(x)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{a + b \operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*sech(x)), x)
```

Giac [A] time = 1.14753, size = 180, normalized size = 1.61

$$\frac{2\,b^4 \arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^4} + \frac{a^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 9\,a^2e^x + 12\,b^2e^x}{24\,a^3} - \frac{\left(a^2b + 2\,b^3\right)x}{2\,a^4} + \frac{\left(3\,a^2be^x - a^3 - 3\left(3\,a^3 + 4\,ab^2\right)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2
*e^(3*x) - 3*a*b*e^(2*x) + 9*a^2*e^x + 12*b^2*e^x)/a^3 - 1/2*(a^2*b + 2*b^3
)*x/a^4 + 1/24*(3*a^2*b*e^x - a^3 - 3*(3*a^3 + 4*a*b^2)*e^(2*x))*e^(-3*x)/a
^4
```

3.97 $\int \frac{\cosh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=85

$$\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Sinh[x])/a^2 + (Cosh[x]*Sinh[x])/(2
*a)
```

Rubi [A] time = 0.264664, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Sinh[x])/a^2 + (Cosh[x]*Sinh[x])/(2
*a)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2a}+\frac{\int \frac{\cosh(x)\left(-2b+a\operatorname{sech}(x)+b\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2a}\\
&= -\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}-\frac{\int \frac{-a^2-2b^2-ab\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{2a^2}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}-\frac{b^3\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}-\frac{b^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}-\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.124125, size = 78, normalized size = 0.92

$$
\frac{\frac{8b^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+2a^2x+a^2\sinh(2x)-4ab\sinh(x)+4b^2x}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + (8*b^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] time = 0.031, size = 174, normalized size = 2.1

$$
-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{b}{a^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)\cdot
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)+1/a^2/(tanh(1/2*x)+1)*b+1/2/
a*ln(tanh(1/2*x)+1)+1/a^3*ln(tanh(1/2*x)+1)*b^2+1/2/a/(tanh(1/2*x)-1)^2+1/2
/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b-1/2/a*ln(tanh(1/2*x)-1)-1/a^3*ln
(tanh(1/2*x)-1)*b^2-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.60045, size = 2067, normalized size = 24.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^2
 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^2)*
cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 - 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(-a^2 +
 b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*
(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b
))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a
)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cosh(x)
^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sin
h(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) + (a^5
 - a^3*b^2)*sinh(x)^2), 1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*si
nh(x)^4 - a^4 + a^2*b^2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3
+ 2*(3*(a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b -
 a*b^3)*cosh(x))*sinh(x)^2 + 16*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^
3*sinh(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 -
 b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cos
h(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)
*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) +
(a^5 - a^3*b^2)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a + b \operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.13296, size = 124, normalized size = 1.46

$$-\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3}+\frac{ae^{(2\,x)}-4\,be^x}{8\,a^2}+\frac{\left(a^2+2\,b^2\right)x}{2\,a^3}+\frac{\left(4\,abe^x-a^2\right)e^{(-2\,x)}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*e
^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x - a^2)*e^(
-2*x)/a^3
```

3.98 $\int \frac{\cosh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqr
t[a - b]*Sqrt[a + b]) + Sinh[x]/a
```

Rubi [A] time = 0.0924905, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3853, 12, 3783, 2659, 205}

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqr
t[a - b]*Sqrt[a + b]) + Sinh[x]/a
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\,\mathrm{sech}(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int \frac{b}{a+b\,\mathrm{sech}(x)}\,dx}{a}\\ &= \frac{\sinh(x)}{a}-\frac{b\int \frac{1}{a+b\,\mathrm{sech}(x)}\,dx}{a}\\ &= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^2}\\ &= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\ &= -\frac{bx}{a^2}+\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.112964, size = 57, normalized size = 0.92

$$\frac{b\left(-\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-x\right)+a\sinh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*(-x - (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
) + a*Sinh[x])/a^2
```

Maple [A] time = 0.031, size = 94, normalized size = 1.5

$$-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{b^2}{a^2\sqrt{(a+b)\,(a-b)}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)+1)-b/a^2*ln(tanh(1/2*x)+1)-1/a/(tanh(1/2*x)-1)+b/a^2*ln(t
anh(1/2*x)-1)+2/a^2*b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6303, size = 1064, normalized size = 17.16

$$\left[-\frac{a^3 - ab^2 + 2\left(a^2b - b^3\right)x\cosh\left(x\right) - \left(a^3 - ab^2\right)\cosh\left(x\right)^2 - \left(a^3 - ab^2\right)\sinh\left(x\right)^2 + 2\left(b^2\cosh\left(x\right) + b^2\sinh\left(x\right)\right)\sqrt{-a^2 -}}{2\left(\left(a^4 - \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) - (a^3 - a*b^2)*cosh(x)^2 -
(a^3 - a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*lo
g((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*co
sh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 2*(
(a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*cosh(x)
+ (a^4 - a^2*b^2)*sinh(x)), -1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) -
 (a^3 - a*b^2)*cosh(x)^2 - (a^3 - a*b^2)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*s
inh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)
) + 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*c
osh(x) + (a^4 - a^2*b^2)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(x\right)}{a + b\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*sech(x)), x)
```

Giac [A] time = 1.16149, size = 84, normalized size = 1.35

$$\frac{2\,b^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^2} - \frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) - b*x/a^2 -
 1/2*e^(-x)/a + 1/2*e^x/a
```

3.99 $\int \frac{\mathbf{sech}(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

[Out] (2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])

Rubi [A] time = 0.0550791, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3831, 2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(a + b*Sech[x]),x]

[Out] (2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.0257107, size = 41, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sech[x]),x]
```

```
[Out] (-2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A] time = 0.013, size = 36, normalized size = 0.9

$$2\frac{1}{\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sech(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33746, size = 446, normalized size = 10.62

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2ab\cosh(x)-a^2+2b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2b\cosh(x)+2(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2-b^2}, -\frac{2\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) +
a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b
)*sinh(x) + a))/(a^2 - b^2), -2*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^
2 - b^2))/sqrt(a^2 - b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sech(x)), x)
```

Giac [A] time = 1.17422, size = 43, normalized size = 1.02

$$\frac{2\arctan\left(\frac{ae^{x}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

3.100 $\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}(\sinh(x))}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b])
```

Rubi [A] time = 0.0995088, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3789, 3770, 3831, 2659, 205}

$$\frac{\tan^{-1}(\sinh(x))}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b])
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{a\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b}$$
$$= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^2}$$
$$= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}$$
$$= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

Mathematica [A] time = 0.0501867, size = 54, normalized size = 1.

$$\frac{2\left(\frac{a\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] + (a*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2]))/b
```

Maple [A] time = 0.014, size = 51, normalized size = 0.9

$$2\,\frac{\arctan(\tanh(x/2))}{b} - 2\,\frac{a}{b\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sech(x)),x)
```

```
[Out] 2/b*arctan(tanh(1/2*x))-2*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60518, size = 587, normalized size = 10.87

$$\left[-\frac{\sqrt{-a^2+b^2}a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)-2\left(a^2-b^2\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) + a)) - 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)
, 2*(sqrt(a^2 - b^2)*a*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2))
 + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.15848, size = 61, normalized size = 1.13

$$-\frac{2\,a\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b}+\frac{2\,\arctan\left(e^x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b) + 2*arctan(e^x
)/b
```

3.101 $\int \frac{\mathbf{sech}^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=64

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

```
[Out] -((a*ArcTan[Sinh[x]])/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]) + Tanh[x]/b
```

Rubi [A] time = 0.14027, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3790, 3789, 3770, 3831, 2659, 205}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -((a*ArcTan[Sinh[x]])/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]) + Tanh[x]/b
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\tanh(x)}{b}-\frac{a\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{\tanh(x)}{b}-\frac{a\int \operatorname{sech}(x)\,dx}{b^2}+\frac{a^2\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^2}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}+\frac{a^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^3}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\tanh(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.105394, size = 63, normalized size = 0.98

$$\frac{-\frac{2a^2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\tanh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-2*a*ArcTan[Tanh[x/2]] - (2*a^2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + b*Tanh[x])/b^2
```

Maple [A] time = 0.024, size = 73, normalized size = 1.1

$$2\frac{\tanh(x/2)}{b\left((\tanh(x/2))^2+1\right)}-2\frac{a\arctan(\tanh(x/2))}{b^2}+2\frac{a^2}{b^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sech(x)),x)
```

```
[Out] 2/b*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/b^2*a*arctan(tanh(1/2*x))+2*a^2/b^2/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70445, size = 1285, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^2*b - 2*b^3 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 + a^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 -
 a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(x) + sinh(x)
))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x)*s
inh(x) + (a^2*b^2 - b^4)*sinh(x)^2), -2*(a^2*b - b^3 + (a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x
) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^
2 + 2*(a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(
x) + sinh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^
4)*cosh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sech(x)), x)
```

Giac [A] time = 1.16892, size = 82, normalized size = 1.28

$$\frac{2\,a^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^2} - \frac{2\,a\arctan\left(e^x\right)}{b^2} - \frac{2}{b\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^2) - 2*a*arcta
n(e^x)/b^2 - 2/(b*(e^(2*x) + 1))
```

3.102 $\int \frac{\mathbf{sech}^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\mathbf{sech}(x)}{2b}$$

```
[Out] ((2*a^2 + b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Tanh[x])/b^2 + (Sech[
x]*Tanh[x])/(2*b)
```

Rubi [A] time = 0.242076, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3851, 4082, 3998, 3770, 3831, 2659, 205}

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\mathbf{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Tanh[x])/b^2 + (Sech[
x]*Tanh[x])/(2*b)
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\operatorname{sech}(x)\tanh(x)}{2b}+\frac{\int \frac{\operatorname{sech}(x)\left(a+b\operatorname{sech}(x)-2a\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2b}\\
&= -\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}+\frac{\int \frac{\operatorname{sech}(x)\left(ab+\left(2a^2+b^2\right)\operatorname{sech}(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2b^2}\\
&= -\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}-\frac{a^3\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^3}+\frac{\left(2a^2+b^2\right)\int \operatorname{sech}(x)\,dx}{2b^3}\\
&= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}-\frac{a^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^4}\\
&= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}-\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan
\end{aligned}
$$

$$
= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}
$$

Mathematica [A] time = 0.200481, size = 82, normalized size = 0.94

$$
\frac{\frac{4a^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+2\left(2a^2+b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\tanh(x)(b\operatorname{sech}(x)-2a)}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*ArcTan[Tanh[x/2]] + (4*a^3*ArcTan[((-a + b)*Tanh[x/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + b*(-2*a + b*Sech[x])*Tanh[x])/(2*b^3)
```

Maple [A] time = 0.021, size = 146, normalized size = 1.7

$$-2\,{\frac{ \left(\tanh \left(x/2 \right) \right) ^{3}a}{{b}^{2} \left(\left(\tanh \left(x/2 \right) \right) ^{2}+1 \right) ^{2}}}-{\frac{1}{b} \left(\tanh \left({\frac{x}{2}} \right) \right) ^{3} \left(\left(\tanh \left({\frac{x}{2}} \right) \right) ^{2}+1 \right) ^{-2}}-2\,{\frac{a\tanh \left(x/2 \right) }{{b}^{2} \left(\left(\tanh \left(x/2 \right) \right) ^{2}+1 \right) ^{2}}}+{\frac{1}{b}\tanh \left({\frac{x}{2}} \right) \left(\left(\tanh \left({\frac{x}{2}} \right. \right. \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sech(x)),x)
```

```
[Out] -2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a-1/b/(tanh(1/2*x)^2+1)^2*tanh(1/2
*x)^3-2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a+1/b/(tanh(1/2*x)^2+1)^2*tanh(
1/2*x)+2/b^3*arctan(tanh(1/2*x))*a^2+1/b*arctan(tanh(1/2*x))-2*a^3/b^3/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.54353, size = 3321, normalized size = 38.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cos
h(x))*sinh(x)^2 - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh
(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sin
h(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sq
rt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*
b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^4 - a^2*b^2 - b^4)*cosh
(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^
4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2
+ 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2
 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b^2 - b^4)*cosh(x))*
sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cosh(x) - (a^2*b^2 - b
^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^2
*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^
3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 -
b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3
 + (a^2*b^3 - b^5)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^
3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 2*(a^3*cosh(x)^4 + 4
```

```
*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*c
osh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + ((2*a^4 -
a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2
*a^4 - a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^
2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b
^2 - b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cos
h(x) - (a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cos
h(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^
5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x
)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b
^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.1367, size = 120, normalized size = 1.38

$$-\frac{2\,a^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^3}+\frac{\left(2\,a^2+b^2\right)\arctan\left(e^x\right)}{b^3}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^x+2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^3) + (2*a^2 +
 b^2)*arctan(e^x)/b^3 + (b*e^(3*x) + 2*a*e^(2*x) - b*e^x + 2*a)/(b^2*(e^(2*
x) + 1)^2)
```

3.103 $\int \frac{\tanh^6(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{a}-\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)(4-3\mathbf{sech}(x))}{12a}-\frac{\tanh(x)(8-3\mathbf{sech}(x))}{8a}$$

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*a) - ((8 - 3*Sech[x])*Tanh[x])/(8*a) - ((4 - 3
*Sech[x])*Tanh[x]^3)/(12*a)
```

Rubi [A] time = 0.0967035, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$\frac{x}{a}-\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)(4-3\mathbf{sech}(x))}{12a}-\frac{\tanh(x)(8-3\mathbf{sech}(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*a) - ((8 - 3*Sech[x])*Tanh[x])/(8*a) - ((4 - 3
*Sech[x])*Tanh[x]^3)/(12*a)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+a\text{sech}(x)}\,dx = -\frac{\int(-a+a\text{sech}(x))\tanh^4(x)\,dx}{a^2}$$
$$= -\frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-4a+3a\text{sech}(x))\tanh^2(x)\,dx}{4a^2}$$
$$= -\frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-8a+3a\text{sech}(x))\,dx}{8a^2}$$
$$= \frac{x}{a} - \frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{3\int \text{sech}(x)\,dx}{8a}$$
$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8a} - \frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a}$$

Mathematica [A] time = 0.117712, size = 60, normalized size = 1.25

$$\frac{\cosh^2\left(\frac{x}{2}\right)\text{sech}(x)\left(6\left(4x-3\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\tanh(x)\left(-6\text{sech}^3(x)+8\text{sech}^2(x)+15\text{sech}(x)-32\right)\right)}{12a(\text{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(6*(4*x - 3*ArcTan[Tanh[x/2]]) + (-32 + 15*Sech[x] + 8
*Sech[x]^2 - 6*Sech[x]^3)*Tanh[x]))/(12*a*(1 + Sech[x]))
```

Maple [B] time = 0.059, size = 117, normalized size = 2.4

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{11}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^7\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-4}-\frac{137}{12a}\left(\tanh\left(\frac{x}{2}\right)\right)^5\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*sech(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-11/4/a/(tanh(1/2*x)^2+1)^4*tanh
(1/2*x)^7-137/12/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5-71/12/a/(tanh(1/2*x)^2
+1)^4*tanh(1/2*x)^3-5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)-3/4/a*arctan(tanh
(1/2*x))
```

Maxima [B] time = 1.70188, size = 126, normalized size = 2.62

$$\frac{x}{a}+\frac{15\,e^{(-x)}-80\,e^{(-2\,x)}-9\,e^{(-3\,x)}-96\,e^{(-4\,x)}+9\,e^{(-5\,x)}-48\,e^{(-6\,x)}-15\,e^{(-7\,x)}-32}{12\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a\right)}+\frac{3\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/12*(15*e^(-x) - 80*e^(-2*x) - 9*e^(-3*x) - 96*e^(-4*x) + 9*e^(-5*x)
 - 48*e^(-6*x) - 15*e^(-7*x) - 32)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6
```

```
*x) + a*e^(-8*x) + a) + 3/4*arctan(e^(-x))/a
```

Fricas [B] time = 2.60885, size = 2288, normalized size = 47.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*x*cosh(x)^8 + 12*x*sinh(x)^8 + 3*(32*x*cosh(x) + 5)*sinh(x)^7 + 48
*(x + 1)*cosh(x)^6 + 15*cosh(x)^7 + 3*(112*x*cosh(x)^2 + 16*x + 35*cosh(x)
+ 16)*sinh(x)^6 + 3*(224*x*cosh(x)^3 + 96*(x + 1)*cosh(x) + 105*cosh(x)^2 -
 3)*sinh(x)^5 + 24*(3*x + 4)*cosh(x)^4 - 9*cosh(x)^5 + 3*(280*x*cosh(x)^4 +
 240*(x + 1)*cosh(x)^2 + 175*cosh(x)^3 + 24*x - 15*cosh(x) + 32)*sinh(x)^4
+ 3*(224*x*cosh(x)^5 + 320*(x + 1)*cosh(x)^3 + 175*cosh(x)^4 + 32*(3*x + 4)
*cosh(x) - 30*cosh(x)^2 + 3)*sinh(x)^3 + 16*(3*x + 5)*cosh(x)^2 + 9*cosh(x)
^3 + (336*x*cosh(x)^6 + 720*(x + 1)*cosh(x)^4 + 315*cosh(x)^5 + 144*(3*x +
4)*cosh(x)^2 - 90*cosh(x)^3 + 48*x + 27*cosh(x) + 80)*sinh(x)^2 - 9*(cosh(x
)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*c
osh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cos
h(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^
2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (96*x*cosh(x)^7 + 288*(x + 1)*cosh(x)^5
 + 105*cosh(x)^6 + 96*(3*x + 4)*cosh(x)^3 - 45*cosh(x)^4 + 32*(3*x + 5)*cos
h(x) + 27*cosh(x)^2 - 15)*sinh(x) + 12*x - 15*cosh(x) + 32)/(a*cosh(x)^8 +
8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(7*a*cosh(x)^2 + a)
*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 + 6*a*cosh(x)^4 + 2*
(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a
*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15
*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^
5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^{6}(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15037, size = 93, normalized size = 1.94

$$\frac{x}{a} - \frac{3 \arctan\left(e^{x}\right)}{4 a} + \frac{15 e^{(7 x)} + 48 e^{(6 x)} - 9 e^{(5 x)} + 96 e^{(4 x)} + 9 e^{(3 x)} + 80 e^{(2 x)} - 15 e^{x} + 32}{12 a \left(e^{(2 x)} + 1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 3/4*arctan(e^x)/a + 1/12*(15*e^(7*x) + 48*e^(6*x) - 9*e^(5*x) + 96*e^
(4*x) + 9*e^(3*x) + 80*e^(2*x) - 15*e^x + 32)/(a*(e^(2*x) + 1)^4)
```

3.104 $\int \frac{\tanh^5(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\operatorname{sech}^3(x)}{3a}+\frac{\operatorname{sech}^2(x)}{2a}+\frac{\operatorname{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

```
[Out] Log[Cosh[x]]/a + Sech[x]/a + Sech[x]^2/(2*a) - Sech[x]^3/(3*a)
```

Rubi [A] time = 0.0591837, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 75}

$$-\frac{\operatorname{sech}^3(x)}{3a}+\frac{\operatorname{sech}^2(x)}{2a}+\frac{\operatorname{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a + Sech[x]^2/(2*a) - Sech[x]^3/(3*a)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx,x,\cosh(x)\right)}{a^4}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)dx,x,\cosh(x)\right)}{a^4}\\ &= \frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0714647, size = 38, normalized size = 1.06

$$\frac{\operatorname{sech}^3(x)(6\cosh(2x)+3\cosh(3x)\log(\cosh(x))+\cosh(x)(9\log(\cosh(x))+6)+2)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] ((2 + 6*Cosh[2*x] + 3*Cosh[3*x]*Log[Cosh[x]] + Cosh[x]*(6 + 9*Log[Cosh[x]])
)*Sech[x]^3)/(12*a)
```

Maple [A] time = 0.037, size = 34, normalized size = 0.9

$$-\frac{(\mathrm{sech}\,(x))^3}{3\,a}+\frac{(\mathrm{sech}\,(x))^2}{2\,a}+\frac{\mathrm{sech}\,(x)}{a}-\frac{\ln(\mathrm{sech}\,(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*sech(x)),x)
```

```
[Out] -1/3*sech(x)^3/a+1/2*sech(x)^2/a+sech(x)/a-1/a*ln(sech(x))
```

Maxima [B] time = 1.50146, size = 100, normalized size = 2.78

$$\frac{x}{a}+\frac{2\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+2\,e^{(-3\,x)}+3\,e^{(-4\,x)}+3\,e^{(-5\,x)}\right)}{3\left(3\,ae^{(-2\,x)}+3\,ae^{(-4\,x)}+ae^{(-6\,x)}+a\right)}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2/3*(3*e^(-x) + 3*e^(-2*x) + 2*e^(-3*x) + 3*e^(-4*x) + 3*e^(-5*x))/(3
*a*e^(-2*x) + 3*a*e^(-4*x) + a*e^(-6*x) + a) + log(e^(-2*x) + 1)/a
```

Fricas [B] time = 2.56271, size = 1403, normalized size = 38.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*cosh(x)^6 + 3*x*sinh(x)^6 + 6*(3*x*cosh(x) - 1)*sinh(x)^5 + 3*(3*
x - 2)*cosh(x)^4 - 6*cosh(x)^5 + 3*(15*x*cosh(x)^2 + 3*x - 10*cosh(x) - 2)*
sinh(x)^4 + 4*(15*x*cosh(x)^3 + 3*(3*x - 2)*cosh(x) - 15*cosh(x)^2 - 1)*sin
h(x)^3 + 3*(3*x - 2)*cosh(x)^2 - 4*cosh(x)^3 + 3*(15*x*cosh(x)^4 + 6*(3*x -
 2)*cosh(x)^2 - 20*cosh(x)^3 + 3*x - 4*cosh(x) - 2)*sinh(x)^2 - 3*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cos
h(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)
^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sin
h(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*x*cosh(x)^5 + 2*(3*x -
2)*cosh(x)^3 - 5*cosh(x)^4 + (3*x - 2)*cosh(x) - 2*cosh(x)^2 - 1)*sinh(x) +
 3*x - 6*cosh(x))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*
cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x
)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^5(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15256, size = 82, normalized size = 2.28

$$\frac{\log\left(e^{(-x)} + e^x\right)}{a} - \frac{11\left(e^{(-x)} + e^x\right)^3 - 12\left(e^{(-x)} + e^x\right)^2 - 12\,e^{(-x)} - 12\,e^x + 16}{6\,a\left(e^{(-x)} + e^x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - 1/6*(11*(e^(-x) + e^x)^3 - 12*(e^(-x) + e^x)^2 - 12*e
^(-x) - 12*e^x + 16)/(a*(e^(-x) + e^x)^3)
```

3.105 $\int \frac{\tanh^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)(2-\operatorname{sech}(x))}{2a}$$

```
[Out] x/a - ArcTan[Sinh[x]]/(2*a) - ((2 - Sech[x])*Tanh[x])/(2*a)
```

Rubi [A] time = 0.0703507, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)(2-\operatorname{sech}(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/(2*a) - ((2 - Sech[x])*Tanh[x])/(2*a)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\int(-a+a\operatorname{sech}(x))\tanh^2(x)\,dx}{a^2}\\
&= -\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int(-2a+a\operatorname{sech}(x))\,dx}{2a^2}\\
&= \frac{x}{a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int\operatorname{sech}(x)\,dx}{2a}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0578742, size = 41, normalized size = 1.32

$$\frac{\cosh^2\left(\frac{x}{2}\right)\text{sech}(x)\left(2\left(x-\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\tanh(x)(\text{sech}(x)-2)\right)}{a(\text{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(2*(x - ArcTan[Tanh[x/2]]) + (-2 + Sech[x])*Tanh[x]))/
(a*(1 + Sech[x]))
```

Maple [B] time = 0.033, size = 75, normalized size = 2.4

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-3\,\frac{(\tanh(x/2))^3}{a\left((\tanh(x/2))^2+1\right)^2}-\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}-\frac{1}{a}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*sech(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^3-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-1/a*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.66518, size = 69, normalized size = 2.23

$$\frac{x}{a}+\frac{e^{(-x)}-2\,e^{(-2\,x)}-e^{(-3\,x)}-2}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}+\frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + (e^(-x) - 2*e^(-2*x) - e^(-3*x) - 2)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
+ arctan(e^(-x))/a
```

Fricas [B] time = 2.51102, size = 711, normalized size = 22.94

$$\frac{x\cosh(x)^4+x\sinh(x)^4+(4\,x\cosh(x)+1)\sinh(x)^3+2\,(x+1)\cosh(x)^2+\cosh(x)^3+\left(6\,x\cosh(x)^2+2\,x+3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + (4*x*cosh(x) + 1)*sinh(x)^3 + 2*(x + 1)*cosh(x
)^2 + cosh(x)^3 + (6*x*cosh(x)^2 + 2*x + 3*cosh(x) + 2)*sinh(x)^2 - (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 (4*x*cosh(x)^3 + 4*(x + 1)*cosh(x) + 3*cosh(x)^2 - 1)*sinh(x) + x - cosh(x
```

```
) + 2)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**4/(sech(x) + 1), x)/a
```

Giac [A] time = 1.14294, size = 57, normalized size = 1.84

$$\frac{x}{a} - \frac{\arctan\left(e^{x}\right)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^{x} + 2}{a\left(e^{(2x)} + 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2)
```

3.106 $\int \frac{\tanh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

```
[Out] Log[Cosh[x]]/a + Sech[x]/a
```

Rubi [A] time = 0.0483187, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 43}

$$\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx,x,\cosh(x)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)dx,x,\cosh(x)\right)}{a^2}\\ &= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0335871, size = 10, normalized size = 0.71

$$\frac{\text{sech}(x)+\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Log[Cosh[x]] + Sech[x])/a
```

Maple [B] time = 0.027, size = 54, normalized size = 3.9

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{a}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+2\frac{1}{a\left((\tanh(x/2))^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)^2+1)+2/a/(t
anh(1/2*x)^2+1)
```

Maxima [B] time = 1.65543, size = 45, normalized size = 3.21

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{ae^{(-2\,x)}+a}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(a*e^(-2*x) + a) + log(e^(-2*x) + 1)/a
```

Fricas [B] time = 2.52471, size = 288, normalized size = 20.57

$$-\frac{x\cosh(x)^2+x\sinh(x)^2-\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+2\,(x\cosh(x)-1)\sinh}{a\cosh(x)^2+2\,a\cosh(x)\sinh(x)+a\sinh(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(x*cosh(x) - 1)*sinh(x) + x - 2*c
osh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tanh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(sech(x) + 1), x)/a
```

Giac [B] time = 1.22351, size = 47, normalized size = 3.36

$$\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{e^{(-x)}+e^{x}-2}{a\left(e^{(-x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - (e^(-x) + e^x - 2)/(a*(e^(-x) + e^x))
```

3.107 $\int \frac{\tanh^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] x/a - ArcTan[Sinh[x]]/a
```

Rubi [A] time = 0.0457572, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3888, 3770}

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/a
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+a\,\text{sech}(x)}\,dx &= -\frac{\int (-a+a\,\text{sech}(x))\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\int \text{sech}(x)\,dx}{a}\\ &= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0295558, size = 15, normalized size = 1.07

$$\frac{x-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (x - 2*ArcTan[Tanh[x/2]])/a
```

Maple [B] time = 0.019, size = 35, normalized size = 2.5

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\,\frac{\arctan\left(\tanh\left(x/2\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*sech(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.65944, size = 22, normalized size = 1.57

$$\frac{x}{a}+\frac{2\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*arctan(e^(-x))/a
```

Fricas [A] time = 2.42832, size = 50, normalized size = 3.57

$$\frac{x-2\,\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (x - 2*arctan(cosh(x) + sinh(x)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\tanh^{2}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15041, size = 19, normalized size = 1.36

$$\frac{x}{a} - \frac{2 \arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/a
```

3.108 $\int \frac{\tanh(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\cosh(x)+1)}{a}$$

```
[Out] Log[1 + Cosh[x]]/a
```

Rubi [A] time = 0.0266844, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + a*Sech[x]),x]
```

```
[Out] Log[1 + Cosh[x]]/a
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+a\operatorname{sech}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cosh(x)\right)\\ &= \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0069035, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Sech[x]),x]
```

```
[Out] (2*Log[Cosh[x/2]])/a
```

Maple [A] time = 0.016, size = 19, normalized size = 2.1

$$\frac{\ln\left(1+\operatorname{sech}\left(x\right)\right)}{a}-\frac{\ln\left(\operatorname{sech}\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*sech(x)),x)
```

```
[Out] 1/a*ln(1+sech(x))-1/a*ln(sech(x))
```

Maxima [A] time = 1.13244, size = 24, normalized size = 2.67

$$\frac{x}{a}+\frac{2\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*log(e^(-x) + 1)/a
```

Fricas [A] time = 2.4193, size = 53, normalized size = 5.89

$$-\frac{x-2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [B] time = 0.20549, size = 19, normalized size = 2.11

$$\frac{x}{a}-\frac{\log\left(\tanh\left(x\right)+1\right)}{a}+\frac{\log\left(\operatorname{sech}\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x)
```

```
[Out] x/a - log(tanh(x) + 1)/a + log(sech(x) + 1)/a
```

Giac [A] time = 1.1518, size = 23, normalized size = 2.56

$$-\frac{x}{a}+\frac{2\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -x/a + 2*log(e^x + 1)/a
```

3.109 $\int \frac{\coth(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=40

$$\frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

```
[Out] 1/(2*a*(1 + Cosh[x])) + Log[1 - Cosh[x]]/(4*a) + (3*Log[1 + Cosh[x]])/(4*a)
```

Rubi [A] time = 0.0581539, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 88}

$$\frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] 1/(2*a*(1 + Cosh[x])) + Log[1 - Cosh[x]]/(4*a) + (3*Log[1 + Cosh[x]])/(4*a)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+a\text{sech}(x)}\,dx &= -\left(a^2\,\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx,x,\cosh(x)\right)\right)\\ &= -\left(a^2\,\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)dx,x,\cosh(x)\right)\right)\\ &= \frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}\end{aligned}$$

Mathematica [A] time = 0.048076, size = 44, normalized size = 1.1

$$\frac{\text{sech}(x)\left(2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\sinh\left(\frac{x}{2}\right)\right)+3\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)+1\right)}{2a(\text{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] ((1 + 2*Cosh[x/2]^2*(3*Log[Cosh[x/2]] + Log[Sinh[x/2]]))*Sech[x])/(2*a*(1 +
 Sech[x]))
```

Maple [A] time = 0.029, size = 47, normalized size = 1.2

$$-\frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*sech(x)),x)
```

```
[Out] -1/4/a*tanh(1/2*x)^2-1/a*ln(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x))-1/a*ln(tan
h(1/2*x)-1)
```

Maxima [A] time = 1.15523, size = 70, normalized size = 1.75

$$\frac{x}{a}+\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}+\frac{3\,\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) + 3/2*log(e^(-x) + 1)/a + 1/2*lo
g(e^(-x) - 1)/a
```

Fricas [B] time = 2.46441, size = 497, normalized size = 12.42

$$-\frac{2\,x\cosh(x)^{2}+2\,x\sinh(x)^{2}+2\,(2\,x-1)\cosh(x)-3\left(\cosh(x)^{2}+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^{2}+2\,\cosh(x)\right.}{2\left(a\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x)^2 + 2*x*sinh(x)^2 + 2*(2*x - 1)*cosh(x) - 3*(cosh(x)^2 +
2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x)
+ 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) - 1) + 2*(2*x*cosh(x) + 2*x - 1)*sinh(x) + 2*x)/(a*cosh
(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)/(sech(x) + 1), x)/a
```

Giac [A] time = 1.16699, size = 76, normalized size = 1.9

$$\frac{3 \log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} - \frac{3\,e^{(-x)} + 3\,e^{x} + 2}{4\,a\left(e^{(-x)} + e^{x} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a - 1/4*(3*e^(-x) +
 3*e^x + 2)/(a*(e^(-x) + e^x + 2))
```

3.110 $\int \frac{\coth^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

```
[Out] x/a - (Coth[x]*(3 - 2*Sech[x]))/(3*a) - (Coth[x]^3*(1 - Sech[x]))/(3*a)
```

Rubi [A] time = 0.0893435, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(3 - 2*Sech[x]))/(3*a) - (Coth[x]^3*(1 - Sech[x]))/(3*a)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\int \coth^4(x)(-a+a\operatorname{sech}(x))\,dx}{a^2}\\
&= -\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}+\frac{\int \coth^2(x)(3a-2a\operatorname{sech}(x))\,dx}{3a^2}\\
&= -\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\int -3a\,dx}{3a^2}\\
&= \frac{x}{a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}
\end{aligned}$$

Mathematica [A] time = 0.0718144, size = 33, normalized size = 0.87

$$\frac{6x - 4\tanh(x) - 4\coth(x) - 2\text{csch}(x) + 6x\text{sech}(x)}{6a\text{sech}(x) + 6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (6*x - 4*Coth[x] - 2*Csch[x] + 6*x*Sech[x] - 4*Tanh[x])/(6*a + 6*a*Sech[x])
```

Maple [A] time = 0.03, size = 56, normalized size = 1.5

$$-\frac{1}{12a}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{1}{a}\tanh\left(\frac{x}{2}\right) + \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/12/a*tanh(1/2*x)^3-1/a*tanh(1/2*x)+1/a*ln(tanh(1/2*x)+1)-1/4/a/tanh(1/2*
x)-1/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.09619, size = 63, normalized size = 1.66

$$\frac{x}{a} - \frac{2\left(5e^{(-x)} - 3e^{(-3x)} + 4\right)}{3\left(2ae^{(-x)} - 2ae^{(-3x)} - ae^{(-4x)} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/3*(5*e^(-x) - 3*e^(-3*x) + 4)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*
x) + a)
```

Fricas [A] time = 2.23765, size = 149, normalized size = 3.92

$$-\frac{2\cosh(x)^2 - ((3x+4)\cosh(x) + 3x + 4)\sinh(x) + 2\sinh(x)^2 + \cosh(x)}{3(a\cosh(x) + a)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cosh(x)^2 - ((3*x + 4)*cosh(x) + 3*x + 4)*sinh(x) + 2*sinh(x)^2 + c
osh(x))/((a*cosh(x) + a)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth^2(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(sech(x) + 1), x)/a
```

Giac [A] time = 1.17326, size = 54, normalized size = 1.42

$$\frac{x}{a} - \frac{1}{2\,a(e^x - 1)} + \frac{15\,e^{(2\,x)} + 24\,e^x + 13}{6\,a(e^x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2/(a*(e^x - 1)) + 1/6*(15*e^(2*x) + 24*e^x + 13)/(a*(e^x + 1)^3)
```

3.111 $\int \frac{\coth^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=68

$$\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

```
[Out] 1/(8*a*(1 - Cosh[x])) - 1/(8*a*(1 + Cosh[x])^2) + 3/(4*a*(1 + Cosh[x])) + (
5*Log[1 - Cosh[x]])/(16*a) + (11*Log[1 + Cosh[x]])/(16*a)
```

Rubi [A] time = 0.0857017, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] 1/(8*a*(1 - Cosh[x])) - 1/(8*a*(1 + Cosh[x])^2) + 3/(4*a*(1 + Cosh[x])) + (
5*Log[1 - Cosh[x]])/(16*a) + (11*Log[1 + Cosh[x]])/(16*a)
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx &= a^4\operatorname{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx,x,\cosh(x)\right)\\ &= a^4\operatorname{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2}+\frac{5}{16a^5(-1+x)}+\frac{1}{4a^5(1+x)^3}-\frac{3}{4a^5(1+x)^2}+\frac{11}{16a^5(1+x)}\right)dx,x,\cos\right.\\ &= \frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1+\cosh(x)}{16a}\end{aligned}$$

Mathematica [A] time = 0.1474, size = 66, normalized size = 0.97

$$\frac{\operatorname{sech}(x)\left(-2\coth^2\left(\frac{x}{2}\right)-\operatorname{sech}^2\left(\frac{x}{2}\right)+4\cosh^2\left(\frac{x}{2}\right)\left(5\log\left(\sinh\left(\frac{x}{2}\right)\right)+11\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)+12\right)}{16a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ((12 - 2*Coth[x/2]^2 + 4*Cosh[x/2]^2*(11*Log[Cosh[x/2]] + 5*Log[Sinh[x/2]])
 - Sech[x/2]^2)*Sech[x])/(16*a*(1 + Sech[x]))
```

Maple [A] time = 0.036, size = 69, normalized size = 1.

$$-\frac{1}{32a}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{5}{16a}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{16a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{5}{8a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{a}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*sech(x)),x)
```

```
[Out] -1/32/a*tanh(1/2*x)^4-5/16/a*tanh(1/2*x)^2-1/a*ln(tanh(1/2*x)+1)-1/16/a/tan
h(1/2*x)^2+5/8/a*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.1447, size = 146, normalized size = 2.15

$$\frac{x}{a}+\frac{5\,e^{(-x)}-6\,e^{(-2\,x)}-14\,e^{(-3\,x)}-6\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{11\,\log\left(e^{(-x)}+1\right)}{8\,a}+\frac{5\,\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/4*(5*e^(-x) - 6*e^(-2*x) - 14*e^(-3*x) - 6*e^(-4*x) + 5*e^(-5*x))/(
2*a*e^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-
6*x) + a) + 11/8*log(e^(-x) + 1)/a + 5/8*log(e^(-x) - 1)/a
```

Fricas [B] time = 2.4752, size = 2475, normalized size = 36.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*x*cosh(x)^6 + 8*x*sinh(x)^6 + 2*(8*x - 5)*cosh(x)^5 + 2*(24*x*cosh(
x) + 8*x - 5)*sinh(x)^5 - 4*(2*x - 3)*cosh(x)^4 + 2*(60*x*cosh(x)^2 + 5*(8*
x - 5)*cosh(x) - 4*x + 6)*sinh(x)^4 - 4*(8*x - 7)*cosh(x)^3 + 4*(40*x*cosh(
x)^3 + 5*(8*x - 5)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) - 8*x + 7)*sinh(x)^3 - 4
*(2*x - 3)*cosh(x)^2 + 4*(30*x*cosh(x)^4 + 5*(8*x - 5)*cosh(x)^3 - 6*(2*x -
 3)*cosh(x)^2 - 3*(8*x - 7)*cosh(x) - 2*x + 3)*sinh(x)^2 + 2*(8*x - 5)*cosh
(x) - 11*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5
 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 +
 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*co
sh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x
)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*co
```

```
sh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^6 + 2*(3*cosh(x) + 1)*si
nh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)
^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*
cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*si
nh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x
)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(24*x*cosh(x)^5 + 5*(8*x - 5)*cosh(x)^4 - 8*(2*x - 3)*cosh(x)^3 - 6*(8*x -
7)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) + 8*x - 5)*sinh(x) + 8*x)/(a*cosh(x)^6 +
 a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
+ (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*co
sh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*co
sh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*
a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^
2 - a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(sech(x) + 1), x)/a
```

Giac [A] time = 1.15991, size = 127, normalized size = 1.87

$$\frac{11\log\left(e^{(-x)}+e^{x}+2\right)}{16a}+\frac{5\log\left(e^{(-x)}+e^{x}-2\right)}{16a}-\frac{5e^{(-x)}+5e^{x}-6}{16a\left(e^{(-x)}+e^{x}-2\right)}-\frac{33\left(e^{(-x)}+e^{x}\right)^{2}+84e^{(-x)}+84e^{x}+52}{32a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 11/16*log(e^(-x) + e^x + 2)/a + 5/16*log(e^(-x) + e^x - 2)/a - 1/16*(5*e^(-
x) + 5*e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(33*(e^(-x) + e^x)^2 + 84*e^(
-x) + 84*e^x + 52)/(a*(e^(-x) + e^x + 2)^2)
```

3.112 $\int \frac{\coth^4(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=55

$$\frac{x}{a}-\frac{\coth^5(x)(1-\mathbf{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\mathbf{sech}(x))}{15a}-\frac{\coth(x)(15-8\mathbf{sech}(x))}{15a}$$

```
[Out] x/a - (Coth[x]*(15 - 8*Sech[x]))/(15*a) - (Coth[x]^3*(5 - 4*Sech[x]))/(15*a
) - (Coth[x]^5*(1 - Sech[x]))/(5*a)
```

Rubi [A] time = 0.120491, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$\frac{x}{a}-\frac{\coth^5(x)(1-\mathbf{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\mathbf{sech}(x))}{15a}-\frac{\coth(x)(15-8\mathbf{sech}(x))}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(15 - 8*Sech[x]))/(15*a) - (Coth[x]^3*(5 - 4*Sech[x]))/(15*a
) - (Coth[x]^5*(1 - Sech[x]))/(5*a)
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\int \coth^6(x)(-a + a\operatorname{sech}(x))\,dx}{a^2}$$
$$= -\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} + \frac{\int \coth^4(x)(5a - 4a\operatorname{sech}(x))\,dx}{5a^2}$$
$$= -\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} - \frac{\int \coth^2(x)(-15a + 8a\operatorname{sech}(x))\,dx}{15a^2}$$
$$= -\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a} - \frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} + \frac{\int 15a\,dx}{15a^2}$$
$$= \frac{x}{a} - \frac{\coth(x)(15-8\operatorname{sech}(x))}{15a} - \frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}$$

Mathematica [A] time = 0.102212, size = 69, normalized size = 1.25

$$\frac{\operatorname{csch}^3(x)\operatorname{sech}(x)(-90x\sinh(x) - 30x\sinh(2x) + 30x\sinh(3x) + 15x\sinh(4x) + 8\cosh(x) + 16\cosh(2x) - 16\cosh(3x) -}{120a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Csch[x]^3*Sech[x]*(-25 + 8*Cosh[x] + 16*Cosh[2*x] - 16*Cosh[3*x] - 23*Cosh
[4*x] - 90*x*Sinh[x] - 30*x*Sinh[2*x] + 30*x*Sinh[3*x] + 15*x*Sinh[4*x]))/(
120*a*(1 + Sech[x]))
```

Maple [A] time = 0.037, size = 78, normalized size = 1.4

$$-\frac{1}{80\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^5 - \frac{1}{8\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{1}{a}\tanh\left(\frac{x}{2}\right) + \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{48\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3} - \frac{3}{8\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/80/a*tanh(1/2*x)^5-1/8/a*tanh(1/2*x)^3-1/a*tanh(1/2*x)+1/a*ln(tanh(1/2*x
)+1)-1/48/a/tanh(1/2*x)^3-3/8/a/tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.19771, size = 142, normalized size = 2.58

$$\frac{x}{a} - \frac{2\left(31\,e^{(-x)} - 31\,e^{(-2\,x)} - 73\,e^{(-3\,x)} + 25\,e^{(-4\,x)} + 65\,e^{(-5\,x)} + 15\,e^{(-6\,x)} - 15\,e^{(-7\,x)} + 23\right)}{15\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/15*(31*e^(-x) - 31*e^(-2*x) - 73*e^(-3*x) + 25*e^(-4*x) + 65*e^(-5*
x) + 15*e^(-6*x) - 15*e^(-7*x) + 23)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3
*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] time = 2.44348, size = 502, normalized size = 9.13

$$-\frac{23\cosh(x)^4-2\,(2\,(15\,x+23)\cosh(x)+15\,x+23)\sinh(x)^3+23\sinh(x)^4+16\cosh(x)^3+2\left(69\cosh(x)^2+24\right.}{30\left((2\,a\cosh(x)+a)\sinh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/30*(23*cosh(x)^4 - 2*(2*(15*x + 23)*cosh(x) + 15*x + 23)*sinh(x)^3 + 23*
sinh(x)^4 + 16*cosh(x)^3 + 2*(69*cosh(x)^2 + 24*cosh(x) - 8)*sinh(x)^2 - 16
*cosh(x)^2 - 2*(2*(15*x + 23)*cosh(x)^3 + 3*(15*x + 23)*cosh(x)^2 - 2*(15*x
 + 23)*cosh(x) - 45*x - 69)*sinh(x) - 8*cosh(x) + 25)/((2*a*cosh(x) + a)*si
nh(x)^3 + (2*a*cosh(x)^3 + 3*a*cosh(x)^2 - 2*a*cosh(x) - 3*a)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(sech(x) + 1), x)/a
```

Giac [A] time = 1.11255, size = 86, normalized size = 1.56

$$\frac{x}{a}-\frac{21\,e^{(2\,x)}-36\,e^{x}+19}{24\,a(e^{x}-1)^3}+\frac{115\,e^{(4\,x)}+380\,e^{(3\,x)}+530\,e^{(2\,x)}+340\,e^{x}+91}{40\,a(e^{x}+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/24*(21*e^(2*x) - 36*e^x + 19)/(a*(e^x - 1)^3) + 1/40*(115*e^(4*x) +
 380*e^(3*x) + 530*e^(2*x) + 340*e^x + 91)/(a*(e^x + 1)^5)
```

3.113 $\int \frac{\tanh^7(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=121

$$\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\,(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(-3a^2b^2+a^4+3b^4)\,\text{sech}(x)}{b^5}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}-\frac{a\text{sech}^4(x)}{4b^2}$$

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rubi [A] time = 0.149396, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\,(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(-3a^2b^2+a^4+3b^4)\,\text{sech}(x)}{b^5}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}-\frac{a\text{sech}^4(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)}\,dx,x,b\text{sech}(x)\right)}{b^6}\\ &= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(-a^2+b^2)}{a^4}\right)+\frac{b^6}{ax}+a\,(a^2-3b^2)\,x-(a^2-3b^2)\,x^2+ax^3-x^4+\frac{(a^2-b^2)^3}{a(a+x)}\right)dx,x,\right.}{b^6}\\ &= \frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5}-\frac{a\,(a^2-3b^2)\,\text{sech}^2(x}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.324951, size = 132, normalized size = 1.09

$$\frac{20b^3\left(a^2-3b^2\right)\operatorname{sech}^3(x)-30ab^2\left(a^2-3b^2\right)\operatorname{sech}^2(x)+60b\left(-3a^2b^2+a^4+3b^4\right)\operatorname{sech}(x)+60a\left(-3a^2b^2+a^4+3b^4\right)\operatorname{lo}}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] (60*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Cosh[x]] - (60*(a^2 - b^2)^3*Log[b + a*
Cosh[x]])/a + 60*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sech[x] - 30*a*b^2*(a^2 - 3*b^
2)*Sech[x]^2 + 20*b^3*(a^2 - 3*b^2)*Sech[x]^3 - 15*a*b^4*Sech[x]^4 + 12*b^5
*Sech[x]^5)/(60*b^6)
```

Maple [B] time = 0.047, size = 415, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^7/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2/b^4/(tanh(1/2*x)^2+1)^2*a^3-
4/b^3/(tanh(1/2*x)^2+1)^2*a^2+4/b/(tanh(1/2*x)^2+1)^2+1/b^6*ln(tanh(1/2*x)^
2+1)*a^5-3/b^4*ln(tanh(1/2*x)^2+1)*a^3+3/b^2*ln(tanh(1/2*x)^2+1)*a-4/b^2/(t
anh(1/2*x)^2+1)^4*a-16/b/(tanh(1/2*x)^2+1)^4+8/3/b^3/(tanh(1/2*x)^2+1)^3*a^
2+8/b^2/(tanh(1/2*x)^2+1)^3*a+8/b/(tanh(1/2*x)^2+1)^3+32/5/b/(tanh(1/2*x)^2
+1)^5+2/b^5/(tanh(1/2*x)^2+1)*a^4+2/b^4/(tanh(1/2*x)^2+1)*a^3-4/b^3/(tanh(1
/2*x)^2+1)*a^2-4/b^2/(tanh(1/2*x)^2+1)*a+2/b/(tanh(1/2*x)^2+1)-a^5/b^6*ln(a
*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+3*a^3/b^4*ln(a*tanh(1/2*x)^2-tanh(1/2*x
)^2*b+a+b)-3*a/b^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/a*ln(a*tanh(1/
2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [B] time = 1.72747, size = 448, normalized size = 3.7

$$\frac{2\left(15\left(a^4-3\,a^2b^2+3\,b^4\right)e^{(-x)}-15\left(a^3b-3\,ab^3\right)e^{(-2\,x)}+20\left(3\,a^4-8\,a^2b^2+6\,b^4\right)e^{(-3\,x)}-15\left(3\,a^3b-7\,ab^3\right)e^{(-4\,x)}+2\left(4\right.}{15\left(5\,b^5e^{(-2\,x)}+10\,b^5e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b - 3*a*b^3)*e^(-2*x) +
 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-3*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 - 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-6
*x) + 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-7*x) - 15*(a^3*b - 3*a*b^3)*e^(-8*
x) + 15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) + 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) + 5*b^5*e^(-8*x) + b^5*e^(-10*x) + b^5) + x/a + (a^5 -
 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^6)
```

Fricas [B] time = 3.27781, size = 9682, normalized size = 80.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 - 30*(a^5*b - 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) - a^5*b + 3*a^3*b^3 - 3*a*b^5)*sin
h(x)^9 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 + 5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4 - 18*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 - 27*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x))*sinh(x)^7
+ 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 - 21*a^2*b^4 - 252*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 + 42*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 - 4*(45*a^5*b - 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 - 45*a^5*b + 115*a^3*b^3 -
99*a*b^5 - 945*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 + 210*(5*b^6*x + 2*a
^4*b^2 - 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^5 + 30*(5*b^6*x
+ 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 + 15*b^6*x + 9
*a^4*b^2 - 21*a^2*b^4 - 378*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 105*(
5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b - 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 2*(45*a^
5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b - 8*a^3*b^3
+ 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 - 63*(a^5*b - 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 + 21*(5*b^6*x + 2*a^4*b^2 -
 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - (45*a^5*b - 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 + 3*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 - 2
16*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x + 84*(5*b^6*x + 2*a^4
*b^2 - 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 - 18*a^2*b^4 - 168*(3*a^5*b - 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4
- 8*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 + 36*(5*b^6*x + 3*a^4*b^2
 - 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 - 30*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x) + 15*((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^10 + 5*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 +
9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 10
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 +
 a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 4*(63*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 15*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh
(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 7*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 5*(9*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6
```

```
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^4 + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*cosh(x)^7 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 4*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - 15*
((a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 + 5*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 9*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 4*(63*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6
 - 3*a^4*b^2 + 3*a^2*b^4 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 + 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 +
 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 - 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 - 27*(a^5*b - 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 + 12*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 - 3*a^5*b + 9*a^3*b^3 - 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^5 - 2*(45*a^5*b - 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 + 12*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 + 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 + 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 + a*b^6*cosh(x))*sinh(x)^7 + a*b^6 + 10*(21*a*b^6*cosh(x)^4 + 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 + 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 + 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 + 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 + 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 + 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 + 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
+ 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^7(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**7/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**7/(a + b*sech(x)), x)
```

Giac [B] time = 1.16096, size = 360, normalized size = 2.98

$$\frac{\left(a^{5}-3\,a^{3}b^{2}+3\,ab^{4}\right)\log\left(e^{(-x)}+e^{x}\right)}{b^{6}}-\frac{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{ab^{6}}-\frac{137\,a^{5}\left(e^{(-x)}+e^{x}\right)^{5}-411\,a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] (a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + e^x)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b^6) - 1/60*(137*a^5*(e^(-
x) + e^x)^5 - 411*a^3*b^2*(e^(-x) + e^x)^5 + 411*a*b^4*(e^(-x) + e^x)^5 - 1
20*a^4*b*(e^(-x) + e^x)^4 + 360*a^2*b^3*(e^(-x) + e^x)^4 - 360*b^5*(e^(-x)
+ e^x)^4 + 120*a^3*b^2*(e^(-x) + e^x)^3 - 360*a*b^4*(e^(-x) + e^x)^3 - 160*
a^2*b^3*(e^(-x) + e^x)^2 + 480*b^5*(e^(-x) + e^x)^2 + 240*a*b^4*(e^(-x) + e
^x) - 384*b^5)/(b^6*(e^(-x) + e^x)^5)
```

3.114 $\int \frac{\tanh^6(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=187

$$\frac{a\left(a^2-3b^2\right)\tanh(x)}{b^4}-\frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{\left(-3a^2b^2+a^4+3b^4\right)\tan^{-1}(\sinh(x))}{b^5}-\frac{\left(a^2-3b^2\right)\tanh(x)\mathbf{sech}(x}{2b^3}$$

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) -
 ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a + b)
^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x])/b
^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2 - 3*
b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]^3)/(
3*b^2)
```

Rubi [A] time = 0.293451, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2897, 2659, 205, 3770, 3767, 8, 3768}

$$\frac{a\left(a^2-3b^2\right)\tanh(x)}{b^4}-\frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{\left(-3a^2b^2+a^4+3b^4\right)\tan^{-1}(\sinh(x))}{b^5}-\frac{\left(a^2-3b^2\right)\tanh(x)\mathbf{sech}(x}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) -
 ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a + b)
^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x])/b
^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2 - 3*
b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]^3)/(
3*b^2)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^5(x)}{b+a\cosh(x)}\,dx$$

$$= -\int \left(-\frac{1}{a} - \frac{\left(a^2-b^2\right)^3}{ab^5(b+a\cosh(x))} + \frac{\left(a^4-3a^2b^2+3b^4\right)\text{sech}(x)}{b^5} + \frac{\left(-a^3+3ab^2\right)\text{sech}^2(x)}{b^4} + \frac{\left(a^2-3b^2\right.}{}\right.$$

$$= \frac{x}{a} + \frac{a\int \text{sech}^4(x)\,dx}{b^2} - \frac{\int \text{sech}^5(x)\,dx}{b} + \frac{\left(a\left(a^2-3b^2\right)\right)\int \text{sech}^2(x)\,dx}{b^4} - \frac{\left(a^2-3b^2\right)\int \text{sech}^3(x)\,dx}{b^3} +$$

$$= \frac{x}{a} - \frac{\left(a^4-3a^2b^2+3b^4\right)\tan^{-1}(\sinh(x))}{b^5} - \frac{\left(a^2-3b^2\right)\text{sech}(x)\tanh(x)}{2b^3} - \frac{\text{sech}^3(x)\tanh(x)}{4b} + \frac{(ia)\,\text{S}}{}$$

$$= \frac{x}{a} - \frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{\left(a^4-3a^2b^2+3b^4\right)\tan^{-1}(\sinh(x))}{b^5} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\tan^{-1}\left(\right.}{ab^5}$$

$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8b} - \frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{\left(a^4-3a^2b^2+3b^4\right)\tan^{-1}(\sinh(x))}{b^5} + \frac{2(a-b}{}$$

Mathematica [A] time = 0.588959, size = 185, normalized size = 0.99

$$\frac{-12\left(-20a^2b^2+8a^4+15b^4\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{48\left(b^5x\sqrt{a^2-b^2}-2\left(a^2-b^2\right)^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{a\sqrt{a^2-b^2}} + b\tanh(x)\text{sech}^3(x)\left(4a\left(9a^2-1\right.\right.}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] (-12*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] + (48*(b^5*Sqrt[a^2 -
b^2]*x - 2*(a^2 - b^2)^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]))/(a*
Sqrt[a^2 - b^2]) + b*(-12*a^2*b + 15*b^3 + 4*a*(9*a^2 - 17*b^2)*Cosh[x] + 3
*b*(-4*a^2 + 9*b^2)*Cosh[2*x] + 12*a^3*Cosh[3*x] - 28*a*b^2*Cosh[3*x])*Sech
[x]^3*Tanh[x])/(48*b^5)
```

Maple [B] time = 0.043, size = 575, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)),x)
```

```
[Out] -15/4/b*arctan(tanh(1/2*x))-2/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2
*x)/((a+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-6*a^3/
b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+6*a/b
/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-44/3/b^2
/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a+1/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^
5*a^2-44/3/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a-1/b^3/(tanh(1/2*x)^2+1)^
4*tanh(1/2*x)^3*a^2+6/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^3+2/b^4/(tanh
(1/2*x)^2+1)^4*tanh(1/2*x)*a^3-4/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a-1/b^
3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^2+2*a^5/b^5/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x
)^7*a^3+1/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^2-4/b^2/(tanh(1/2*x)^2+1)
^4*tanh(1/2*x)^7*a+6/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^3-7/4/b/(tanh(
1/2*x)^2+1)^4*tanh(1/2*x)^7-15/4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5+15/4/b
/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3+7/4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)-2/
b^5*arctan(tanh(1/2*x))*a^4+5/b^3*arctan(tanh(1/2*x))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.43995, size = 12166, normalized size = 65.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2 - 9*a*b^4)*co
sh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^7 + 24*(2*b^5*
x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 16*b^5*x - 8*a^
4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6 + 12*b^5*x -
```

```
3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*a^3*b^2 + a*b^
4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b + 3*a^2*b^3)*c
osh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a^4*b + 7*a^2*b
^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b + 56*a^2*b^3 -
 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b + 3*a^2*b^3)*cos
h(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*b^2 - a*b^4)*c
osh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4*a^3*b^2 - 9*a
*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3 - 10*(4*a^3*b
^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x))*sinh(x)
^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*x*cosh(x)^6 +
48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 152*a^2*b^3 + 36
0*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a*b^4)*cosh(x)^
3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^2 + 12*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^8 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4
+ 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2
 + b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-
a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*co
sh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5
- 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^6 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3
*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 +
15*a*b^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3
*b^2 + 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5
- 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 +
15*a*b^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15
*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5
- 20*a^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b
^2 + 15*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh
(x) + (96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5
*x - a^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 -
 a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3
*b^2 - a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh
(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b
^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2
 + a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(
x)^5 + 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*
(7*a*b^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7
*a*b^5*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^
2 + 8*(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh
(x))*sinh(x)), 1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2
 - 9*a*b^4)*cosh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^
```

```
7 + 24*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 1
6*b^5*x - 8*a^4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6
 + 12*b^5*x - 3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*
a^3*b^2 + a*b^4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b
+ 3*a^2*b^3)*cosh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a
^4*b + 7*a^2*b^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b
+ 56*a^2*b^3 - 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b +
3*a^2*b^3)*cosh(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*
b^2 - a*b^4)*cosh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4
*a^3*b^2 - 9*a*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3
 - 10*(4*a^3*b^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*co
sh(x))*sinh(x)^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*
x*cosh(x)^6 + 48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 15
2*a^2*b^3 + 360*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a
*b^4)*cosh(x)^3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^2 - 24*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8
*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*c
osh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^
2*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a
^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b
^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a
^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x)
+ a*sinh(x) + b)/sqrt(a^2 - b^2)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6
+ 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 + 15*a*b
^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3*b^2 +
 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 - 2
0*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5 - 20*a
^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b^2 + 1
5*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(
x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh(x) +
(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5*x - a
^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 - a*b^4
)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3*b^2 -
 a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh(x))/(
a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b^5*cos
h(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2 + a*b
^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^5 +
 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*(7*a*b
^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7*a*b^5
*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^2 + 8*
(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh(x))*s
inh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^6(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*sech(x)), x)
```

Giac [A] time = 1.13604, size = 338, normalized size = 1.81

$$\frac{x}{a} - \frac{\left(8\,a^4 - 20\,a^2b^2 + 15\,b^4\right)\arctan\left(e^x\right)}{4\,b^5} + \frac{2\left(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6\right)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}ab^5} - \frac{12\,a^2be^{(7\,x)} - 27\,b^3e^{(7\,x)} + 24\,a^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/4*(8*a^4 - 20*a^2*b^2 + 15*b^4)*arctan(e^x)/b^5 + 2*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a
*b^5) - 1/12*(12*a^2*b*e^(7*x) - 27*b^3*e^(7*x) + 24*a^3*e^(6*x) - 72*a*b^2
*e^(6*x) + 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) + 72*a^3*e^(4*x) - 168*a*b^2*e^
(4*x) - 12*a^2*b*e^(3*x) + 3*b^3*e^(3*x) + 72*a^3*e^(2*x) - 152*a*b^2*e^(2*
x) - 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 - 56*a*b^2)/(b^4*(e^(2*x) + 1)^4)
```

3.115 $\int \frac{\tanh^5(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=72

$$-\frac{\left(a^2-2b^2\right)\mathrm{sech}(x)}{b^3}+\frac{\left(a^2-b^2\right)^2\log(a+b\mathrm{sech}(x))}{ab^4}+\frac{a\mathrm{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\mathrm{sech}^3(x)}{3b}$$

```
[Out] Log[Cosh[x]]/a + ((a^2 - b^2)^2*Log[a + b*Sech[x]])/(a*b^4) - ((a^2 - 2*b^2
)*Sech[x])/b^3 + (a*Sech[x]^2)/(2*b^2) - Sech[x]^3/(3*b)
```

Rubi [A] time = 0.0970659, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{\left(a^2-2b^2\right)\mathrm{sech}(x)}{b^3}+\frac{\left(a^2-b^2\right)^2\log(a+b\mathrm{sech}(x))}{ab^4}+\frac{a\mathrm{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\mathrm{sech}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((a^2 - b^2)^2*Log[a + b*Sech[x]])/(a*b^4) - ((a^2 - 2*b^2
)*Sech[x])/b^3 + (a*Sech[x]^2)/(2*b^2) - Sech[x]^3/(3*b)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{a+b\mathrm{sech}(x)}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)}\,dx,x,b\mathrm{sech}(x)\right)}{b^4}\\
&= -\frac{\mathrm{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{\left(a^2-b^2\right)^2}{a(a+x)}\right)dx,x,b\mathrm{sech}(x)\right)}{b^4}\\
&= \frac{\log(\cosh(x))}{a}+\frac{\left(a^2-b^2\right)^2\log(a+b\mathrm{sech}(x))}{ab^4}-\frac{\left(a^2-2b^2\right)\mathrm{sech}(x)}{b^3}+\frac{a\mathrm{sech}^2(x)}{2b^2}-\frac{\mathrm{sech}^3(x)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.170961, size = 85, normalized size = 1.18

$$\frac{3a^2b^2\text{sech}^2(x) - 6ab\left(a^2 - 2b^2\right)\text{sech}(x) - 6a^2\left(a^2 - 2b^2\right)\log(\cosh(x)) + 6\left(a^2 - b^2\right)^2\log(a\cosh(x) + b) - 2ab^3\text{sech}^3(x)}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] (-6*a^2*(a^2 - 2*b^2)*Log[Cosh[x]] + 6*(a^2 - b^2)^2*Log[b + a*Cosh[x]] - 6
*a*b*(a^2 - 2*b^2)*Sech[x] + 3*a^2*b^2*Sech[x]^2 - 2*a*b^3*Sech[x]^3)/(6*a*
b^4)
```

Maple [B] time = 0.04, size = 233, normalized size = 3.2

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2\frac{a}{b^2\left((\tanh(x/2))^2+1\right)^2} + 4\frac{1}{b\left((\tanh(x/2))^2+1\right)^2} - \frac{a^3}{b^4}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+2/b^2/(tanh(1/2*x)^2+1)^2*a+4/
b/(tanh(1/2*x)^2+1)^2-1/b^4*ln(tanh(1/2*x)^2+1)*a^3+2/b^2*ln(tanh(1/2*x)^2+
1)*a-8/3/b/(tanh(1/2*x)^2+1)^3-2/b^3/(tanh(1/2*x)^2+1)*a^2-2/b^2/(tanh(1/2*
x)^2+1)*a+2/b/(tanh(1/2*x)^2+1)+a^3/b^4*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+
a+b)-2*a/b^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/a*ln(a*tanh(1/2*x)^2
-tanh(1/2*x)^2*b+a+b)
```

Maxima [B] time = 1.66744, size = 221, normalized size = 3.07

$$\frac{2\left(3\,abe^{(-2\,x)} + 3\,abe^{(-4\,x)} - 3\left(a^2 - 2\,b^2\right)e^{(-x)} - 2\left(3\,a^2 - 4\,b^2\right)e^{(-3\,x)} - 3\left(a^2 - 2\,b^2\right)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)} + 3\,b^3e^{(-4\,x)} + b^3e^{(-6\,x)} + b^3\right)} + \frac{x}{a} - \frac{\left(a^3 - 2\,ab^2\right)\log\left(e^{(-2\,x)} + 1\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*a*b*e^(-2*x) + 3*a*b*e^(-4*x) - 3*(a^2 - 2*b^2)*e^(-x) - 2*(3*a^2 -
4*b^2)*e^(-3*x) - 3*(a^2 - 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) + 3*b^3*e^(-4*x
) + b^3*e^(-6*x) + b^3) + x/a - (a^3 - 2*a*b^2)*log(e^(-2*x) + 1)/b^4 + (a^
4 - 2*a^2*b^2 + b^4)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^4)
```

Fricas [B] time = 3.19241, size = 3216, normalized size = 44.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b - 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b - 2*a*b^3)*sinh(x)^5 + 3*b^4*x + 3*(3*b^4*x -
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 + 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b - 2*a*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^3*b - 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 + 3*a^3*b - 4*a*b^3 + 15*(a^3*b - 2*a*b^3)*cosh(x)^2 + 3*
(3*b^4*x - 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x - 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x - 2*a^2*b^2 + 20*(a^3*b - 2*a*b^3)*cosh
(x)^3 + 6*(3*b^4*x - 2*a^2*b^2)*cosh(x)^2 + 4*(3*a^3*b - 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b - 2*a*b^3)*cosh(x) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2 + b^4)
*sinh(x)^6 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + b^4
 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 4*(5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + 5*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 + a^4 - 2*a^2*b^2 + 4*(5*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 - 2*a^2*b^2)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + (a^4 - 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b - 2*a*b^3)*cosh(x)^4 + a^3*b - 2*a*b^3 + 2*(3*b^4*x -
2*a^2*b^2)*cosh(x)^3 + 2*(3*a^3*b - 4*a*b^3)*cosh(x)^2 + (3*b^4*x - 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 + 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 + a*b^4 + 3*(5*a*b^4*cosh
(x)^2 + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 + 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 + 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)), x)
```

Giac [B] time = 1.15139, size = 205, normalized size = 2.85

$$-\frac{\left(a^3 - 2\,ab^2\right)\log\left(e^{(-x)} + e^x\right)}{b^4} + \frac{\left(a^4 - 2\,a^2b^2 + b^4\right)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} + e^x\right)^3 - 22\,ab^2\left(e^{(-x)} + e^x\right)^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(e^(-x) + e^x)/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(a*
(e^(-x) + e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) + e^x)^3 - 22*a*b^2*(e
^(-x) + e^x)^3 - 12*a^2*b*(e^(-x) + e^x)^2 + 24*b^3*(e^(-x) + e^x)^2 + 12*a
*b^2*(e^(-x) + e^x) - 16*b^3)/(b^4*(e^(-x) + e^x)^3)
```

3.116 $\int \frac{\tanh^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=94

$$\frac{(2a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}+\frac{x}{a}+\frac{\tanh(x)\mathbf{sech}(x)}{2b}$$

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*(a - b)^(3/2)*(a + b)^
(3/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^3) - (a*Tanh[x])/b^
2 + (Sech[x]*Tanh[x])/(2*b)
```

Rubi [A] time = 0.318866, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3898, 2893, 3057, 2659, 205, 3770}

$$\frac{(2a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}+\frac{x}{a}+\frac{\tanh(x)\mathbf{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*(a - b)^(3/2)*(a + b)^
(3/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^3) - (a*Tanh[x])/b^
2 + (Sech[x]*Tanh[x])/(2*b)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
```

```
Q[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^3(x)}{b+a\cosh(x)}\,dx$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\int \frac{\left(-2a^2+3b^2-ab\cosh(x)-2b^2\cosh^2(x)\right)\text{sech}(x)}{b+a\cosh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\left(a^2-b^2\right)^2\int \frac{1}{b+a\cosh(x)}\,dx}{ab^3} - \frac{\left(-2a^2+3b^2\right)\int \text{sech}(x)\,dx}{2b^3}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}\left(\int \frac{1}{a+b-(-a}\right.}{ab^3}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2} + \text{sech}(x$$

Mathematica [A] time = 0.403349, size = 113, normalized size = 1.2

$$\frac{\text{sech}^2(x)(a\cosh(x)+b)\left(2\cosh(x)\left(a\left(2a^2-3b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)+b^3x\right)+ab(b\,\text{ta}\right.}{2ab^3(a+b\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]^2*(2*(b^3*x + a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]]
+ 2*(a^2 - b^2)^(3/2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])*Cosh[x]
 + a*b*(-2*a*Sinh[x] + b*Tanh[x])))/(2*a*b^3*(a + b*Sech[x]))
```

Maple [B] time = 0.033, size = 248, normalized size = 2.6

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 2\frac{(\tanh(x/2))^3 a}{b^2\left((\tanh(x/2))^2+1\right)^2} - \frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - 2\frac{a}{b^2\left((\text{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2/b^2/(tanh(1/2*x)^2+1)^2*tanh(
1/2*x)^3*a-1/b/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/b^2/(tanh(1/2*x)^2+1)^2*
tanh(1/2*x)*a+1/b/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)+2/b^3*arctan(tanh(1/2*x))
*a^2-3/b*arctan(tanh(1/2*x))-2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
h(1/2*x)/((a+b)*(a-b))^(1/2))+4*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1
/2*x)/((a+b)*(a-b))^(1/2))-2/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.97129, size = 3224, normalized size = 34.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh(
x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^2
 - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*s
inh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2
)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*s
inh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*
cosh(x)*sinh(x)^3 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^
3 - 3*a*b^2)*cosh(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*s
inh(x))*arctan(cosh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2
- a*b^2 + 4*(b^3*x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*co
sh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*
cosh(x)^2 + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))
, (b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh
(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cos
h(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^
2 + 2*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2
)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 -
b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x)
)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b
```

```
^2)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^3
 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^3 - 3*a*b^2)*cosh
(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2 - a*b^2 + 4*(b^3*
x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 +
 a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 + a*b^3
)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.14186, size = 150, normalized size = 1.6

$$\frac{x}{a}+\frac{(2a^2-3b^2)\arctan(e^x)}{b^3}-\frac{2\left(a^4-2a^2b^2+b^4\right)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ab^3}+\frac{be^{(3x)}+2ae^{(2x)}-be^x+2a}{b^2\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a + (2*a^2 - 3*b^2)*arctan(e^x)/b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a
*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*b^3) + (b*e^(3*x) + 2*a*e^(2*
x) - b*e^x + 2*a)/(b^2*(e^(2*x) + 1)^2)
```

3.117 $\int \frac{\tanh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{b}$$

```
[Out] Log[Cosh[x]]/a + ((1 - a^2/b^2)*Log[a + b*Sech[x]])/a + Sech[x]/b
```

Rubi [A] time = 0.0750012, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((1 - a^2/b^2)*Log[a + b*Sech[x]])/a + Sech[x]/b
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx,x,b\text{sech}(x)\right)}{b^2}\\
&= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx,x,b\text{sech}(x)\right)}{b^2}\\
&= \frac{\log(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\text{sech}(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0840565, size = 37, normalized size = 1.06

$$\frac{\left(b^2-a^2\right)\log(a\cosh(x)+b)+a^2\log(\cosh(x))+ab\text{sech}(x)}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (a^2*Log[Cosh[x]] + (-a^2 + b^2)*Log[b + a*Cosh[x]] + a*b*Sech[x])/(a*b^2)
```

Maple [B] time = 0.03, size = 107, normalized size = 3.1

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{a}{b^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+2\frac{1}{b\left((\tanh(x/2))^2+1\right)}-\frac{a}{b^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)^2+1)*a+2/
b/(tanh(1/2*x)^2+1)-a/b^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/a*ln(a*
tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [A] time = 1.56517, size = 90, normalized size = 2.57

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{be^{(-2\,x)}+b}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{b^2}-\frac{\left(a^2-b^2\right)\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) + b) + a*log(e^(-2*x) + 1)/b^2 - (a^2 - b^2)*log
(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^2)
```

Fricas [B] time = 2.5721, size = 543, normalized size = 15.51

$$-\frac{b^2x\cosh(x)^2+b^2x\sinh(x)^2+b^2x-2\,ab\cosh(x)+\left(\left(a^2-b^2\right)\cosh(x)^2+2\left(a^2-b^2\right)\cosh(x)\sinh(x)+\left(a^2-b^2\right)\sinh\right.}{ab^2\cosh(x)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 + b^2*x - 2*a*b*cosh(x) + ((a^2 - b^2)*
cosh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2 + a^2 - b
^2)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) - a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)), x)
```

Giac [B] time = 1.13766, size = 99, normalized size = 2.83

$$\frac{a\log\left(e^{(-x)}+e^{x}\right)}{b^2} - \frac{\left(a^2-b^2\right)\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{ab^2} - \frac{a\left(e^{(-x)}+e^{x}\right)-2\,b}{b^2\left(e^{(-x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*log(e^(-x) + e^x)/b^2 - (a^2 - b^2)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b
^2) - (a*(e^(-x) + e^x) - 2*b)/(b^2*(e^(-x) + e^x))
```

3.118 $\int \frac{\tanh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}+\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}$$

```
[Out] x/a - ArcTan[Sinh[x]]/b + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*b)
```

Rubi [A] time = 0.171266, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3894, 4051, 3770, 3919, 3831, 2659, 205}

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}+\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/b + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*b)
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{-1+\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx\\
&= -\frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{\int \frac{-b-a\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}
\end{aligned}
$$

Mathematica [A] time = 0.0809918, size = 62, normalized size = 1.

$$\frac{-2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right) - 2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + bx}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (b*x - 2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2]
)/Sqrt[a^2 - b^2]])/(a*b)
```

Maple [B] time = 0.026, size = 113, normalized size = 1.8

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 2\frac{\arctan(\tanh(x/2))}{b} + 2\frac{a}{b\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2/b*arctan(tanh(1/2*x))+2*a/b/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2/a*b/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.81215, size = 551, normalized size = 8.89

$$\left[\frac{bx - 2a\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + \sqrt{-a^2 + b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2ab\cosh(x) - a^2 + 2b^2 + 2\left(a^2\cosh(x) + ab\right)\sinh(x) + 2\sqrt{-a^2 +}}{a\cosh(x)^2 + a\sinh(x)^2 + 2b\cosh(x) + 2\left(a\cosh(x) + b\right)\sinh(x)}\right.}{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b*x - 2*a*arctan(cosh(x) + sinh(x)) + sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2
 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)))/(a*b), (b*x - 2*a*ar
ctan(cosh(x) + sinh(x)) - 2*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x)
+ b)/sqrt(a^2 - b^2)))/(a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2\left(x\right)}{a + b\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.17209, size = 70, normalized size = 1.13

$$\frac{x}{a} - \frac{2\arctan\left(e^x\right)}{b} + \frac{2\sqrt{a^2 - b^2}\arctan\left(\frac{ae^x + b}{\sqrt{a^2 - b^2}}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/b + 2*sqrt(a^2 - b^2)*arctan((a*e^x + b)/sqrt(a^2 - b^2
))/(a*b)
```

3.119 $\int \frac{\tanh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+b\mathrm{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}$$

```
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a
```

Rubi [A] time = 0.0316895, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3885, 36, 29, 31}

$$\frac{\log(a+b\mathrm{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+b\mathrm{sech}(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\mathrm{sech}(x)\right)\\ &= -\frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, b\mathrm{sech}(x)\right)}{a}+\frac{\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\mathrm{sech}(x)\right)}{a}\\ &= \frac{\log(\cosh(x))}{a}+\frac{\log(a+b\mathrm{sech}(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0173727, size = 11, normalized size = 0.58

$$\frac{\log(a\cosh(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[b + a*Cosh[x]]/a
```

Maple [A] time = 0.014, size = 21, normalized size = 1.1

$$-\frac{\ln(\operatorname{sech}(x))}{a}+\frac{\ln(a+b\operatorname{sech}(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(sech(x))+ln(a+b*sech(x))/a
```

Maxima [A] time = 1.13219, size = 35, normalized size = 1.84

$$\frac{x}{a}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Fricas [A] time = 2.43681, size = 72, normalized size = 3.79

$$-\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [A] time = 0.585946, size = 41, normalized size = 2.16

$$\begin{cases}\frac{\tilde{\infty}}{\operatorname{sech}(x)} & \text{for } a=0\wedge b=0\\ \frac{x-\log(\tanh(x)+1)}{a} & \text{for } b=0\\ \frac{1}{b\operatorname{sech}(x)} & \text{for } a=0\\ \frac{x}{a}+\frac{\log\left(\frac{a}{b}+\operatorname{sech}(x)\right)}{a}-\frac{\log(\tanh(x)+1)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x)
```

```
[Out] Piecewise((zoo/sech(x), Eq(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1))/a, Eq
(b, 0)), (1/(b*sech(x)), Eq(a, 0)), (x/a + log(a/b + sech(x))/a - log(tanh(
x) + 1)/a, True))
```

Giac [A] time = 1.13428, size = 26, normalized size = 1.37

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

3.120 $\int \frac{\coth(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=66

$$-\frac{b^2\log(a+b\,\mathrm{sech}(x))}{a\left(a^2-b^2\right)}+\frac{\log(1-\mathrm{sech}(x))}{2(a+b)}+\frac{\log(\mathrm{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

```
[Out] Log[Cosh[x]]/a + Log[1 - Sech[x]]/(2*(a + b)) + Log[1 + Sech[x]]/(2*(a - b)
) - (b^2*Log[a + b*Sech[x]])/(a*(a^2 - b^2))
```

Rubi [A] time = 0.106399, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3885, 894}

$$-\frac{b^2\log(a+b\,\mathrm{sech}(x))}{a\left(a^2-b^2\right)}+\frac{\log(1-\mathrm{sech}(x))}{2(a+b)}+\frac{\log(\mathrm{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[1 - Sech[x]]/(2*(a + b)) + Log[1 + Sech[x]]/(2*(a - b)
) - (b^2*Log[a + b*Sech[x]])/(a*(a^2 - b^2))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\,\mathrm{sech}(x)}\,dx &= -\left(b^2\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx,x,b\,\mathrm{sech}(x)\right)\right)\\ &= -\left(b^2\,\mathrm{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)}+\frac{1}{ab^2x}+\frac{1}{a(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)b^2(b+x)}\right)dx,x,\right.\right.\\ &= \frac{\log(\cosh(x))}{a}+\frac{\log(1-\mathrm{sech}(x))}{2(a+b)}+\frac{\log(1+\mathrm{sech}(x))}{2(a-b)}-\frac{b^2\log(a+b\,\mathrm{sech}(x))}{a\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0853563, size = 44, normalized size = 0.67

$$-\frac{a^2(-\log(\sinh(x)))+b^2\log(a\cosh(x)+b)+ab\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b^2*Log[b + a*Cosh[x]] - a^2*Log[Sinh[x]] + a*b*Log[Tanh[x/2]])/(a^3 - a
*b^2))
```

Maple [A] time = 0.029, size = 78, normalized size = 1.2

$$-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{a+b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{b^2}{a\left(a+b\right)\left(a-b\right)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)+1/(a+b)*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)-1/(a+b
)/a*b^2/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [A] time = 1.07207, size = 90, normalized size = 1.36

$$-\frac{b^2\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3-ab^2}+\frac{x}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^2*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^3 - a*b^2) + x/a + log(e^(-x) + 1)
/(a - b) + log(e^(-x) - 1)/(a + b)
```

Fricas [A] time = 2.33004, size = 220, normalized size = 3.33

$$-\frac{b^2\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+\left(a^2-b^2\right)x-\left(a^2+ab\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(a^2-ab\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + (a^2 - b^2)*x - (a^2 + a
*b)*log(cosh(x) + sinh(x) + 1) - (a^2 - a*b)*log(cosh(x) + sinh(x) - 1))/(a
^3 - a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(x\right)}{a+b\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sech(x)), x)
```

Giac [A] time = 1.1175, size = 90, normalized size = 1.36

$$-\frac{b^2 \log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^3 - ab^2} + \frac{\log\left(e^{(-x)} + e^x + 2\right)}{2\,(a - b)} + \frac{\log\left(e^{(-x)} + e^x - 2\right)}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.121 $\int \frac{\coth^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=114

$$-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (a*x)/(a^2 - b^2) - (b^2*x)/(a*(a^2 - b^2)) + (2*b^3*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)) - (a*Coth[x])/(a^2 -
 b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rubi [A] time = 0.20499, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2659, 205}

$$-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b^2*x)/(a*(a^2 - b^2)) + (2*b^3*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)) - (a*Coth[x])/(a^2 -
 b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx &= \int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx \\
&= \frac{a\int \coth^2(x)\,dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)\,dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{a^2-b^2} \\
&= -\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{a\coth(x)}{a^2-b^2}+\frac{a\int 1\,dx}{a^2-b^2}+\frac{(ib)\,\text{Subst}(\int 1\,dx,x,-i\text{csch}(x))}{a^2-b^2}+\frac{b^3\int \frac{1}{b+a\cosh(x)}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{ax}{a^2-b^2}-\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)} \\
&= \frac{ax}{a^2-b^2}-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.334968, size = 81, normalized size = 0.71

$$
\frac{\frac{2b^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2x-a^2\coth(x)+ab\text{csch}(x)-b^2x}{a^3-ab^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a^2*x - b^2*x + (2*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a^2*Coth[x] + a*b*Csch[x])/(a^3 - a*b^2)
```

Maple [A] time = 0.03, size = 104, normalized size = 0.9

$$-\frac{1}{2a-2b}\tanh\left(\frac{x}{2}\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2b+2a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{b^3}{(a-b)\,a\,(a+b)\sqrt{(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+1/a*ln(tanh(1/2*x)+1)-1/2/(a+b)/tanh(1/2*x)-1/a*ln(t
anh(1/2*x)-1)+2/(a-b)/a/(a+b)*b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2
*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35958, size = 1582, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^
2 + b^4)*x*sinh(x)^2 - (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)
^2 - b^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*co
sh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cos
h(x) + b)*sinh(x) + a)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cos
h(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5
- 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*
b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2), (2*a^4
 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^2 + b^4
)*x*sinh(x)^2 + 2*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 -
b^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) +
 (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^3*b - a*b^3 +
 (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^
5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)*sinh
(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)), x)
```

Giac [A] time = 1.15148, size = 111, normalized size = 0.97

$$\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^3-ab^2\right)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^3 - a*b^2)*sqrt(a^2 - b^2)) +
 x/a + 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

3.122 $\int \frac{\coth^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=113

$$\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-\frac{1}{4(a+b)(1-\text{sech}(x))}-\frac{1}{4(a-b)(\text{sech}(x)+1)}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(\text{sech}}{4(a-b)^2}$$

```
[Out] Log[Cosh[x]]/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(4*(a + b)^2) + ((2*a - 3*b
)*Log[1 + Sech[x]])/(4*(a - b)^2) + (b^4*Log[a + b*Sech[x]])/(a*(a^2 - b^2)
^2) - 1/(4*(a + b)*(1 - Sech[x])) - 1/(4*(a - b)*(1 + Sech[x]))
```

Rubi [A] time = 0.190496, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-\frac{1}{4(a+b)(1-\text{sech}(x))}-\frac{1}{4(a-b)(\text{sech}(x)+1)}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(\text{sech}}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(4*(a + b)^2) + ((2*a - 3*b
)*Log[1 + Sech[x]])/(4*(a - b)^2) + (b^4*Log[a + b*Sech[x]])/(a*(a^2 - b^2)
^2) - 1/(4*(a + b)*(1 - Sech[x])) - 1/(4*(a - b)*(1 + Sech[x]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx &= -\left(b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx,x,b\text{sech}(x)\right)\right)\\
&= -\left(b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2}+\frac{2a+3b}{4b^4(a+b)^2(b-x)}+\frac{1}{ab^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)}-\frac{}{4(a}\right.\right.\right.\\
&= \frac{\log(\cosh(x))}{a}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2}+\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-
\end{aligned}$$

Mathematica [A] time = 0.300283, size = 112, normalized size = 0.99

$$\frac{4a\left(2a\left(a^2-2b^2\right)\log(\sinh(x))+b\left(3b^2-a^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+8b^4\log(a\cosh(x)+b)-a(a-b)^2(a+b)\text{csch}^2\left(\frac{x}{2}\right)+a}{8a(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(a*(a - b)^2*(a + b)*Csch[x/2]^2) + 8*b^4*Log[b + a*Cosh[x]] + 4*a*(2*a*(
a^2 - 2*b^2)*Log[Sinh[x]] + b*(-a^2 + 3*b^2)*Log[Tanh[x/2]]) + a*(a - b)*(a
 + b)^2*Sech[x/2]^2)/(8*a*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.043, size = 119, normalized size = 1.1

$$-\frac{1}{8\,a-8\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{8\,a+8\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a}{(a+b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{3\,b}{2\,(a+b)^2}\ln\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-1/a*ln(tanh(1/2*x)+1)-1/8/(a+b)/tanh(1/2*x)^2+a/(a
+b)^2*ln(tanh(1/2*x))+3/2/(a+b)^2*ln(tanh(1/2*x))*b-1/a*ln(tanh(1/2*x)-1)+1
/(a-b)^2*b^4/(a+b)^2/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [A] time = 1.1829, size = 221, normalized size = 1.96

$$\frac{b^4\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+1\right)}{2\left(a^2-2\,ab+b^2\right)}+\frac{(2\,a+3\,b)\log\left(e^{(-x)}-1\right)}{2\left(a^2+2\,ab+b^2\right)}+\frac{be^{(-x)}-2\,ae^{(-2\,x)}+be}{a^2-b^2-2\left(a^2-b^2\right)e^{(-2\,x)}+(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + 1/2*(2*a -
 3*b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + 3*b)*log(e^(-x) - 1)
/(a^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 -
2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x)) + x/a
```

Fricas [B] time = 2.7178, size = 2885, normalized size = 25.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*x*s
inh(x)^4 - 2*(a^3*b - a*b^3)*cosh(x)^3 - 2*(a^3*b - a*b^3 - 4*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4
```

```
)*x)*cosh(x)^2 + 2*(2*a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)
^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*x - 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 2
*(a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(b^4*cosh(x)^4 +
 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4
*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(x))*log(
2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - ((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*
b^3)*cosh(x)^4 + 4*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)*sinh(x)^3
+ (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*sinh(x)^4 + 2*a^4 + a^3*b - 4*a^2*b
^2 - 3*a*b^3 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^2 - 2*(2*a^4
 + a^3*b - 4*a^2*b^2 - 3*a*b^3 - 3*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^3 - (
2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) - ((2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + 4*(2*a^4 - a^3*
b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^3 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3
*a*b^3)*sinh(x)^4 + 2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - 2*(2*a^4 - a^3*b
- 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2 - 2*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 -
 3*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 -
 a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 - (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*
b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*(a^4 - 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 - a^3*b + a*b^3 - 3*(a^3*b - a*b^3)*cosh(x)^2 + 4*(a^4 -
a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a
*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 - 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^4 - 2*(a^5 - 2*a^3*b^2
+ a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^3 - (a^5 - 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sech(x)), x)
```

Giac [A] time = 1.14085, size = 261, normalized size = 2.31

$$\frac{b^4\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2\,ab+b^2\right)}+\frac{(2\,a+3\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2\,ab+b^2\right)}-\frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,ab^2\left(e^{(-}\right.}{2\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*(2*a -
 3*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + 3*b)*log(e^(-x
) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a*b^2*(e^(
-x) + e^x)^2 - 2*a^2*b*(e^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) + 4*a*b^2)/((a
^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))
```

3.123 $\int \frac{\coth^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=207

$$\frac{b^4x}{a\left(a^2-b^2\right)^2}-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\mathrm{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}-$$

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a +
 b)^(5/2)) + (a*b^2*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x])/(a^2 - b^2) - (a*C
oth[x]^3)/(3*(a^2 - b^2)) - (b^3*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2
- b^2) + (b*Csch[x]^3)/(3*(a^2 - b^2))
```

Rubi [A] time = 0.329422, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2659, 205}

$$\frac{b^4x}{a\left(a^2-b^2\right)^2}-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\mathrm{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a +
 b)^(5/2)) + (a*b^2*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x])/(a^2 - b^2) - (a*C
oth[x]^3)/(3*(a^2 - b^2)) - (b^3*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2
- b^2) + (b*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \int \frac{\cosh(x)\coth^4(x)}{b+a\cosh(x)}\,dx$$

$$= \frac{a\int \coth^4(x)\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\mathrm{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a\coth^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ab^2\right)\int \coth^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^3\int \coth(x)\mathrm{csch}(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^4\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\int \coth^2(x)}{a^2-b^2}$$

$$= \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3\left(a^2-b^2\right)} + \frac{b\,\mathrm{csch}(x)}{a^2-b^2} + \frac{b\,\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ab^2\right)\int 1\,dx}{\left(a^2-b^2\right)^2} -$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} + \frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3\left(a^2-b^2\right)} - \frac{b^3\mathrm{csch}(x)}{\left(a^2-b^2\right)^2} + \frac{b\mathrm{cs}}{a^2}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}} + \frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\,\mathrm{co}}{3(a}$$

Mathematica [A] time = 0.762133, size = 166, normalized size = 0.8

$$\frac{\operatorname{sech}(x)(a\cosh(x)+b)\left(\frac{48b^5\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{22b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}-\frac{16a\tanh\left(\frac{x}{2}\right)}{(a-b)^2}-\frac{2(8a+11b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}+\frac{8\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x)}{a-b}-\frac{\sinh(x)\operatorname{cs}}{2(a+}\right.}{24(a+b\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((24*x)/a + (48*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) - (2*(8*a + 11*b)*Coth[x/2])/(a + b)^
2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) -
 (16*a*Tanh[x/2])/(a - b)^2 + (22*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[
x]))
```

Maple [A] time = 0.042, size = 179, normalized size = 0.9

$$-\frac{a}{24\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{b}{24\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3-\frac{5\,a}{8\,(a-b)^2}\tanh\left(\frac{x}{2}\right)+\frac{7\,b}{8\,(a-b)^2}\tanh\left(\frac{x}{2}\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sech(x)),x)
```

```
[Out] -1/24/(a-b)^2*a*tanh(1/2*x)^3+1/24/(a-b)^2*b*tanh(1/2*x)^3-5/8/(a-b)^2*a*ta
nh(1/2*x)+7/8/(a-b)^2*tanh(1/2*x)*b+1/a*ln(tanh(1/2*x)+1)-1/24/(a+b)/tanh(1
/2*x)^3-5/8/(a+b)^2/tanh(1/2*x)*a-7/8/(a+b)^2/tanh(1/2*x)*b-1/a*ln(tanh(1/2
*x)-1)-2/(a-b)^2/(a+b)^2/a*b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)
/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62131, size = 8041, normalized size = 38.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^
5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(
4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*c
osh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3
+ 2*a*b^5)*cosh(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 -
 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^3 - 15*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*
b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)
^3 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x)*cosh(x)^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3
 - 6*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*x)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 - 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(
x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*c
osh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 +
 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 -
 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sin
h(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 +
 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*
b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
 a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x
)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 -
 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)), -1/3*(3*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^
3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos
h(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*
a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15
*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^
4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^6
- 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 - 6*(4*a^6 -
10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5
)*cosh(x))*sinh(x)^2 + 6*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sin
h(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5
```

```
)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh
(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)
^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b -
 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 + 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c
osh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6
 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*s
inh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*co
sh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*sech(x)), x)
```

Giac [A] time = 1.15766, size = 257, normalized size = 1.24

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^5-2\,a^3b^2+ab^4\right)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\left(3\,a^2be^{(5\,x)}-6\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}-2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}\right.}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(
a^2 - b^2)) + x/a + 2/3*(3*a^2*b*e^(5*x) - 6*b^3*e^(5*x) - 6*a^3*e^(4*x) +
9*a*b^2*e^(4*x) - 2*a^2*b*e^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) - 12*a*b^
2*e^(2*x) + 3*a^2*b*e^x - 6*b^3*e^x - 4*a^3 + 7*a*b^2)/((a^4 - 2*a^2*b^2 +
b^4)*(e^(2*x) - 1)^3)
```

3.124 $\int \frac{\coth^5(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=178

$$-\frac{b^6\log(a+b\,\mathrm{sech}(x))}{a\left(a^2-b^2\right)^3}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\mathrm{sech}(x))}{16(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\mathrm{sech}(x)+1)}{16(a-b)^3}-\frac{5a+7}{16(a+b)^2(1-}$$

```
[Out] Log[Cosh[x]]/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(16*(a + b)^3
) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(16*(a - b)^3) - (b^6*Log[
a + b*Sech[x]])/(a*(a^2 - b^2)^3) - 1/(16*(a + b)*(1 - Sech[x])^2) - (5*a +
 7*b)/(16*(a + b)^2*(1 - Sech[x])) - 1/(16*(a - b)*(1 + Sech[x])^2) - (5*a
- 7*b)/(16*(a - b)^2*(1 + Sech[x]))
```

Rubi [A] time = 0.320137, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{b^6\log(a+b\,\mathrm{sech}(x))}{a\left(a^2-b^2\right)^3}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\mathrm{sech}(x))}{16(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\mathrm{sech}(x)+1)}{16(a-b)^3}-\frac{5a+7}{16(a+b)^2(1-}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Sech[x]),x]
```


```
[Out] Log[Cosh[x]]/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(16*(a + b)^3
) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(16*(a - b)^3) - (b^6*Log[
a + b*Sech[x]])/(a*(a^2 - b^2)^3) - 1/(16*(a + b)*(1 - Sech[x])^2) - (5*a +
 7*b)/(16*(a + b)^2*(1 - Sech[x])) - 1/(16*(a - b)*(1 + Sech[x])^2) - (5*a
- 7*b)/(16*(a - b)^2*(1 + Sech[x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = -\left(b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\text{sech}(x)\right)\right)$$

$$= -\left(b^6\,\text{Subst}\left(\int\left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{}{a(a}\right.\right.\right.$$

$$= \frac{\log(\cosh(x))}{a}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\text{sech}(x))}{16(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(1+\text{sech}(x))}{16(a-b)^3}$$

Mathematica [A] time = 0.997587, size = 167, normalized size = 0.94

$$\frac{1}{64}\left(-\frac{8\left(a\left(b\left(-10a^2b^2+3a^4+15b^4\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-8a\left(-3a^2b^2+a^4+3b^4\right)\log(\sinh(x))\right)+8b^6\log(a\cosh(x)+b)\right)}{a(a-b)^3(a+b)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] ((-2*(7*a + 9*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) - (8*(8*b^6*L
og[b + a*Cosh[x]] + a*(-8*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + b*(3*a
^4 - 10*a^2*b^2 + 15*b^4)*Log[Tanh[x/2]])))/(a*(a - b)^3*(a + b)^3) + (2*(7
*a - 9*b)*Sech[x/2]^2)/(a - b)^2 - Sech[x/2]^4/(a - b))/64
```

Maple [A] time = 0.041, size = 215, normalized size = 1.2

$$-\frac{a}{64\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{b}{64\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{3\,a}{16\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{b}{4\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{a}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*sech(x)),x)
```

```
[Out] -1/64/(a-b)^2*tanh(1/2*x)^4*a+1/64/(a-b)^2*tanh(1/2*x)^4*b-3/16/(a-b)^2*tan
h(1/2*x)^2*a+1/4/(a-b)^2*tanh(1/2*x)^2*b-1/a*ln(tanh(1/2*x)+1)-1/64/(a+b)/t
anh(1/2*x)^4-3/16/(a+b)^2/tanh(1/2*x)^2*a-1/4/(a+b)^2/tanh(1/2*x)^2*b+1/(a+
b)^3*ln(tanh(1/2*x))*a^2+21/8/(a+b)^3*ln(tanh(1/2*x))*a*b+15/8/(a+b)^3*ln(t
anh(1/2*x))*b^2-1/a*ln(tanh(1/2*x)-1)-1/(a-b)^3*b^6/(a+b)^3/a*ln(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [B] time = 1.14141, size = 494, normalized size = 2.78

$$-\frac{b^6\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}+\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(e^{(-x)}+1\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(e^{(-x)}-1\right)}{8\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{\left(5\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^6*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/8*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
```

```
b^3) + 1/8*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3) + 1/4*((5*a^2*b - 9*b^3)*e^(-x) - 8*(2*a^3 - 3*a*b^2)*e^(-2*x) +
 (3*a^2*b + b^3)*e^(-3*x) + 16*(a^3 - 2*a*b^2)*e^(-4*x) + (3*a^2*b + b^3)*e
^(-5*x) - 8*(2*a^3 - 3*a*b^2)*e^(-6*x) + (5*a^2*b - 9*b^3)*e^(-7*x))/(a^4 -
 2*a^2*b^2 + b^4 - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2
+ b^4)*e^(-4*x) - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b
^4)*e^(-8*x)) + x/a
```

Fricas [B] time = 3.65215, size = 11934, normalized size = 67.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^8 + 8*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*sinh(x)^8 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(
x)^7 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*x*cosh(x))*sinh(x)^7 + 16*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^6 + 2*(16*a^6 - 40*a^4*b^2 + 24*a^2
*b^4 + 112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 16*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x - 7*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x))*si
nh(x)^6 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^5 - 2*(3*a^5*b - 2*a^3*b^
3 - a*b^5 - 224*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 + 21*(5*a^5
*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^5 - 16*(2*a^6 - 6
*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 -
 2*(16*a^6 - 48*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x*cosh(x)^4 + 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 - 120*(2*a^
6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
)^2 - 24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 5*(3*a^5*b - 2*a^3*b^3 - a
*b^5)*cosh(x))*sinh(x)^4 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 + 2*(2
24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 3*a^5*b + 2*a^3*b^3 +
a*b^5 - 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 160*(2*a^6 - 5*a^4*
b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 10*(
3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 - 6*a^4*b^2 + 4*a^2*b^4
- 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 16*(2*a^6 -
 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
 + 2*(112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 16*a^6 - 40*a^4
*b^2 + 24*a^2*b^4 - 21*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 + 120*(2*
a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh
(x)^4 - 10*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 - 6*a^4*b^2
+ 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2 - 16*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 3*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x))
*sinh(x)^2 + 8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 2*(5*a^5*b - 14*a^3*
b^3 + 9*a*b^5)*cosh(x) + 8*(b^6*cosh(x)^8 + 8*b^6*cosh(x)*sinh(x)^7 + b^6*s
inh(x)^8 - 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 - 4*b^6*cosh(x)^2 + 4*(7*b^6*c
osh(x)^2 - b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cosh(x)^3 - 3*b^6*cosh(x))*sinh(
x)^5 + 2*(35*b^6*cosh(x)^4 - 30*b^6*cosh(x)^2 + 3*b^6)*sinh(x)^4 + 8*(7*b^6
*cosh(x)^5 - 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))*sinh(x)^3 + 4*(7*b^6*cosh(x)
^6 - 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 - b^6)*sinh(x)^2 + 8*(b^6*cosh(x)^7
 - 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x))*log(2*(a*cosh(
x) + b)/(cosh(x) - sinh(x))) - ((8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^8 + 8*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a
^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (8*a^6 + 3*a^5*b - 24*a
^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*sinh(x)^8 - 4*(8*a^6 + 3*a^5*b
```

```
 - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^6 - 4*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 - 7*(8*a^6 + 3*a^
5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^6
 + 8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + 8*(7
*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x
)^3 - 3*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)
*cosh(x))*sinh(x)^5 + 6*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^4 + 2*(24*a^6 + 9*a^5*b - 72*a^4*b^2 - 30*a^3*b^3
+ 72*a^2*b^4 + 45*a*b^5 + 35*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 2
4*a^2*b^4 + 15*a*b^5)*cosh(x)^4 - 30*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3
*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 - 10*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 + 3*(8
*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*
sinh(x)^3 - 4*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*
a*b^5)*cosh(x)^2 + 4*(7*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^6 - 8*a^6 - 3*a^5*b + 24*a^4*b^2 + 10*a^3*b^3 - 24
*a^2*b^4 - 15*a*b^5 - 15*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^
2*b^4 + 15*a*b^5)*cosh(x)^4 + 9*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^6 + 3*a^5*b - 24*a^
4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^7 - 3*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 + 3*(8*a^6 + 3
*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 - (8*a^
6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*sin
h(x))*log(cosh(x) + sinh(x) + 1) - ((8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*
b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^8 + 8*(8*a^6 - 3*a^5*b - 24*a^4*b^2 +
10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)*sinh(x)^7 + (8*a^6 - 3*a^5*b -
24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*sinh(x)^8 - 4*(8*a^6 - 3*a
^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^6 - 4*(8*a^
6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5 - 7*(8*a^6 -
3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(
x)^6 + 8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5 +
8*(7*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*co
sh(x)^3 - 3*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cosh(x))*sinh(x)^5 + 6*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24
*a^2*b^4 - 15*a*b^5)*cosh(x)^4 + 2*(24*a^6 - 9*a^5*b - 72*a^4*b^2 + 30*a^3*
b^3 + 72*a^2*b^4 - 45*a*b^5 + 35*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3
 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^4 - 30*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10
*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^6 - 3*a^
5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^5 - 10*(8*a^
6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^3 +
3*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(
x))*sinh(x)^3 - 4*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
 15*a*b^5)*cosh(x)^2 + 4*(7*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24
*a^2*b^4 - 15*a*b^5)*cosh(x)^6 - 8*a^6 + 3*a^5*b + 24*a^4*b^2 - 10*a^3*b^3
- 24*a^2*b^4 + 15*a*b^5 - 15*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 2
4*a^2*b^4 - 15*a*b^5)*cosh(x)^4 + 9*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*
b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^6 - 3*a^5*b - 2
4*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^7 - 3*(8*a^6 - 3*a^
5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^5 + 3*(8*a^6
 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^3 - (
8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x))
*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*x*cosh(x)^7 - 7*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^6 - 5*a^5*b
+ 14*a^3*b^3 - 9*a*b^5 + 48*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^5 - 5*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh
(x)^4 - 32*(2*a^6 - 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*x)*cosh(x)^3 - 3*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^2 + 16*(2*a^6
 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
```

```
)*sinh(x))/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^8 + 8*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
 - a*b^6)*sinh(x)^8 + a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 4*(a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
 - 7*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*cosh(x))*sinh(x)^5 + 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos
h(x)^4 + 2*(3*a^7 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + 35*(a^7 - 3*a^5*b^2 +
 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 30*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*co
sh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 -
 10*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + 3*(a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*
b^6)*cosh(x)^2 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 7*(a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 + 15*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
*cosh(x)^4 - 9*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 +
 8*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^7 - 3*(a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*cosh(x)^5 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh
(x)^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^5(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*sech(x)), x)
```

Giac [B] time = 1.18878, size = 513, normalized size = 2.88

$$-\frac{b^6 \log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^7 - 3\,a^5b^2 + 3\,a^3b^4 - ab^6} + \frac{\left(8\,a^2 - 21\,ab + 15\,b^2\right)\log\left(e^{(-x)} + e^x + 2\right)}{16\left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)} + \frac{\left(8\,a^2 + 21\,ab + 15\,b^2\right)\log\left(e^{(-x)} + e^x - 2\right)}{16\left(a^3 + 3\,a^2b + 3\,ab^2 + b^3\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^6*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/16*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*
a*b^2 - b^3) + 1/16*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) + e^x - 2)/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 1/4*(3*a^5*(e^(-x) + e^x)^4 - 9*a^3*b^2*(e^(-x)
+ e^x)^4 + 9*a*b^4*(e^(-x) + e^x)^4 - 5*a^4*b*(e^(-x) + e^x)^3 + 14*a^2*b^3
*(e^(-x) + e^x)^3 - 9*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 32*a^
3*b^2*(e^(-x) + e^x)^2 - 48*a*b^4*(e^(-x) + e^x)^2 + 12*a^4*b*(e^(-x) + e^x
) - 40*a^2*b^3*(e^(-x) + e^x) + 28*b^5*(e^(-x) + e^x) - 16*a^3*b^2 + 64*a*b
^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

3.125 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\tanh^5(c+dx)\,dx$

Optimal. Leaf size=169

$$-\frac{2\left(3a^2-2b^2\right)(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a\left(a^2-2b^2\right)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}+\frac{6a(a+b\text{sec}}{7}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d + (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/(3*b^4*d)
 - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) + (6*a*(a + b*
Sech[c + d*x])^(7/2))/(7*b^4*d) - (2*(a + b*Sech[c + d*x])^(9/2))/(9*b^4*d)
```

Rubi [A] time = 0.194028, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$-\frac{2\left(3a^2-2b^2\right)(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a\left(a^2-2b^2\right)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}+\frac{6a(a+b\text{sec}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d + (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/(3*b^4*d)
 - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) + (6*a*(a + b*
Sech[c + d*x])^(7/2))/(7*b^4*d) - (2*(a + b*Sech[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\left(b^2-x^2\right)^2}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^4d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(b^4-a\left(a^2-2b^2\right)x^2+\left(3a^2-2b^2\right)x^4-3ax^6+x^8+\frac{ab^4}{-a+x^2}\right)dx, x\right.}{b^4d}$$
$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a\left(a^2-2b^2\right)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2\left(3a^2-2b^2\right.}{}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a\left(a^2-2b^2\right)(a+b\text{se}}{3b^4d}$$

Mathematica [A] time = 4.83465, size = 160, normalized size = 0.95

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(\left(\frac{6a^2}{b^2}+126\right)\text{sech}^2(c+dx)+\left(\frac{42a}{b}-\frac{8a^3}{b^3}\right)\text{sech}(c+dx)+\frac{16a^4}{b^4}-\frac{84a^2}{b^2}-\frac{5a\text{sech}^3(c+dx)}{b}+\frac{315\sqrt{a\cosh(c+dx)}}{\sqrt{a\,\text{c}}}\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-315 + (16*a^4)/b^4 - (84*a^2)/b^2 + (315*Arc
Tanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]]
)/Sqrt[b + a*Cosh[c + d*x]] + ((-8*a^3)/b^3 + (42*a)/b)*Sech[c + d*x] + (12
6 + (6*a^2)/b^2)*Sech[c + d*x]^2 - (5*a*Sech[c + d*x]^3)/b - 35*Sech[c + d*
x]^4))/(315*d)
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int \sqrt{a+b\text{sech}\,(dx+c)}\,(\tanh\,(dx+c))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Fricas [B] time = 9.43823, size = 10971, normalized size = 64.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^
4*sinh(d*x + c)^8 + 4*b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4
*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*
x + c)^4 + 30*b^4*cosh(d*x + c)^2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4
*cosh(d*x + c)^5 + 10*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*b^4*cosh(d*x + c)^6 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x
+ c)^2 + b^4)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c
)^5 + 3*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log
(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 +
4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^
2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
 c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*c
osh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
 c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (
4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*((16*a^4 - 84*a^2*b^2 - 315*b^4)*c
osh(d*x + c)^8 + (16*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3
*b - 21*a*b^3)*cosh(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2
*b^2 - 315*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 1
89*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))
*sinh(d*x + c)^6 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b -
53*a*b^3 - 14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b
 - 21*a*b^3)*cosh(d*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 +
 2*(35*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b
^2 - 721*b^4 - 70*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^4 + 16*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^3 + 4*(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35
*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 -
78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x +
 c)^2 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*
b^2 - 315*b^4)*cosh(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 +
15*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 -
189*b^4 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^
2 - 721*b^4)*cosh(d*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^2 - 4*(4*a^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b
^2 - 315*b^4)*cosh(d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*
(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*c
```

```
osh(d*x + c)^4 - 4*a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*co
sh(d*x + c)^3 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)))/(b^4*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)
^7 + b^4*d*sinh(d*x + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c
)^4 + 4*b^4*d*cosh(d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*
x + c)^6 + 8*(7*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c
)^5 + b^4*d + 2*(35*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^
4*d)*sinh(d*x + c)^4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^
3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 1
5*b^4*d*cosh(d*x + c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2
 + 8*(b^4*d*cosh(d*x + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c)), -1/315*(315*(b^4*cosh(d*x + c)^8
 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*b^4*cosh(d
*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*
x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*x + c)^4 + 30*b^4*cosh(d*x + c)^
2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*b^4*cosh(d
*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*cosh(d*x + c)^6
 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 +
8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c)^5 + 3*b^4*cosh(d*x + c)^3 + b^
4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh
(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*
sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c))) - 2*((16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^8 + (1
6*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3*b - 21*a*b^3)*cosh
(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cos
h(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84*a^2*b^2 - 315*b^4)
*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 -
4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b - 53*a*b^3 - 14*(16*a
^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b - 21*a*b^3)*cosh(d
*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 + 2*(35*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b^2 - 721*b^4 - 70*(
4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 1
6*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 4*
(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35*(4*a^3*b - 21*a*b^
3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 - 78*a^2*b^2 - 189*b^
4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(48*a^4 -
 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh
(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 + 15*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 - 189*b^4 - 10*(12*a^
3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d
*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(4*a
^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(
d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*(16*a^4 - 78*a^2*b^
2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^4 - 4*
a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^3 - 3*(
12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^4
*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*d*sinh(d*x
 + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(
d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^6 + 8*(7*b^4
*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^5 + b^4*d + 2*(35
*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^4*d)*sinh(d*x + c)^
```

```
4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 15*b^4*d*cosh(d*x +
c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 + 8*(b^4*d*cosh(d*x
 + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*
x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\tanh^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(dx + c) + a}\tanh(dx + c)^5\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

3.126 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\tanh^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2(a+b\mathrm{sech}(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\mathrm{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d - (2*a*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*S
ech[c + d*x])^(5/2))/(5*b^2*d)
```

Rubi [A] time = 0.121907, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2(a+b\mathrm{sech}(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\mathrm{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d - (2*a*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*S
ech[c + d*x])^(5/2))/(5*b^2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}(b^2-x^2)}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\text{sech}(c+dx)}{5b^2d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+}{3b^2d}$$

Mathematica [A] time = 0.962381, size = 108, normalized size = 1.08

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(-\frac{2a^2}{b^2}+\frac{a\text{sech}(c+dx)}{b}+\frac{15\sqrt{a\cosh(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)+b}}+3\text{sech}^2(c+dx)-15\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-15 - (2*a^2)/b^2 + (15*ArcTanh[Sqrt[b + a*Co
sh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]])/Sqrt[b + a*Cosh[
c + d*x]] + (a*Sech[c + d*x])/b + 3*Sech[c + d*x]^2))/(15*d)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \sqrt{a+b\text{sech}\,(dx+c)}\,(\tanh\,(dx+c))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Fricas [B] time = 9.05048, size = 4070, normalized size = 40.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*
cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cos
h(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a
*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c
)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(
d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x
+ c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2*a*b*cosh(d*x
+ c)^3 - (2*a^2 + 15*b^2)*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^
4 + 2*(a*b - 2*(2*a^2 + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh
(d*x + c) - 2*(2*a^2 + 9*b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*
(2*a^2 + 15*b^2)*cosh(d*x + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 -
 15*b^2 + 2*(3*a*b*cosh(d*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a
*b - 2*(2*a^2 + 9*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b
^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 +
b^2*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(15*(b^2*cosh(d*x + c)^4 + 4*b^2
*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^
2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c
)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*
x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*
x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*
x + c) + a*b)*sinh(d*x + c))) - 2*(2*a*b*cosh(d*x + c)^3 - (2*a^2 + 15*b^2)
*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^4 + 2*(a*b - 2*(2*a^2 + 1
5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh(d*x + c) - 2*(2*a^2 + 9*
b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*(2*a^2 + 15*b^2)*cosh(d*x
 + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 - 15*b^2 + 2*(3*a*b*cosh(d
*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a*b - 2*(2*a^2 + 9*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^2
*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x
 + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^
2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{sech}(c + dx)} \tanh^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

3.127 $\int \sqrt{a+b\,\mathrm{sech}(c+dx)}\tanh(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\mathrm{sech}(c+dx)}}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d
```

Rubi [A] time = 0.0535916, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 50, 63, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\mathrm{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

Mathematica [A] time = 0.141392, size = 90, normalized size = 1.76

$$-\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(\sqrt{a\cosh(c+dx)+b} - \sqrt{a\cosh(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)\right)}{d\sqrt{a\cosh(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (-2*(-(ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh
[c + d*x]]) + Sqrt[b + a*Cosh[c + d*x]])*Sqrt[a + b*Sech[c + d*x]])/(d*Sqrt
[b + a*Cosh[c + d*x]])
```

Maple [A] time = 0.03, size = 43, normalized size = 0.8

$$-\frac{1}{d}\left(2\sqrt{a+b\,\text{sech}\,(dx+c)} - 2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{a+b\,\text{sech}\,(dx+c)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x)
```

```
[Out] -1/d*(2*(a+b*sech(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Fricas [B] time = 8.72209, size = 1613, normalized size = 31.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*c
osh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh
(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*
b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)
^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d
*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
+ 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(
d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh
(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x +
 c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*sqrt((a*cosh(d*x
+ c) + b)/cosh(d*x + c)))/d, -(sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(
d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*s
qrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c))) + 2*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\operatorname{sech}{\left(c + dx \right)}}\tanh{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}\left(dx + c\right) + a}\tanh\left(dx + c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

3.128 $\int \coth(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]])/d
```

Rubi [A] time = 0.17479, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1287, 206, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]])/d
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\text{sech}(c+\right.}{d}$$

$$= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x\right.}{d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\tanh^{-}}{}$$

Mathematica [A] time = 2.2199, size = 195, normalized size = 1.84

$$\frac{\sqrt{a\cosh(c+dx)}\sqrt{a+b\text{sech}(c+dx)}\left(2\sqrt{a}\log\left(\sqrt{a\cosh(c+dx)+b}+\sqrt{a\cosh(c+dx)}\right)-\sqrt{-a-b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cosh(c+dx}}{\sqrt{-a-b}\sqrt{a\cosh(c+}}\right.\right.}{\sqrt{a}d\sqrt{a\cosh(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[a*Cosh[c + d*x]]*(-(Sqrt[-a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])]) - Sqrt[a - b]*ArcTanh[(Sqrt[a
]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]])] + 2*Sqrt[
a]*Log[Sqrt[a*Cosh[c + d*x]] + Sqrt[b + a*Cosh[c + d*x]]])*Sqrt[a + b*Sech[
c + d*x]])/(Sqrt[a]*d*Sqrt[b + a*Cosh[c + d*x]])
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \coth(dx+c)\sqrt{a+b\text{sech}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}(dx+c)+a}\,\coth(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Fricas [B] time = 7.56616, size = 22756, normalized size = 214.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*
a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b -
 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 -
8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2
 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x
+ c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x +
 c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*
cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a -
b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*s
inh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x +
c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) +
 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c
)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*
x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(
d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 +
(8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 +
 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh
(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b
)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)
*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x +
c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x
+ c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*
cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^
2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh
(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(c
osh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 +
 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c
) - 4*cosh(d*x + c) + 1)) + 2*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*s
inh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh
(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2
*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*
a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c
)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cos
h(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) +
 a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x +
```

c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d
*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)))/d, -1/4*(4*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*
x + c) + b)*sinh(d*x + c) + a)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*c
osh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*
cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 -
8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x
+ c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a
- b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2
*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^
2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*
b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x
+ c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(
d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*
cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c)
+ 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*
b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(
3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b
+ 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b
)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*
(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2
+ 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x +
c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x +
c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c)
+ 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x +
c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)
)/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^
4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cos
h(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a +
b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b
)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a
- b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(a + b)*log(-((8*a
^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 +
4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)
^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6
*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*(
(2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3
+ 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x
+ c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sin
h(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt
(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(
d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cos
h(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*s

```
inh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2
 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) - 2*sqrt(a)*l
og(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3
+ 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*
a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c)
 + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x
 + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a
*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x
 + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
 (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/4*(4*sqrt(-a)*arctan((cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sq
rt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c
)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*sqr
t(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2
*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x +
c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*
x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b
^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2
 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x
+ c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*co
sh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a +
b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b
)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)
*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^
2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))
*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c)
+ 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x
+ c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/d, 1/4
*(2*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x +
c) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) + sqrt(a - b
)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(
d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2
- 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b
 + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b
 + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cos
h(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a -
 b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) +
2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)
^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2
*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b -
3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b
- 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x
 + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cos
h(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) +
 2*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh
```

```
(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*
x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*c
osh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4
+ a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x
+ c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2
*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x
 + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*
x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)
^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/4*(4*sqrt(-a)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*
sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) +
 a)) - 2*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*
x + c))/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d
*x + c) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - sqrt(
a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*
sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8
*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 -
8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 -
8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*
b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(
2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x +
c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x
 + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c
) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a
*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4
*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cos
h(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*
x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cos
h(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) +
1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) -
 sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/
((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
+ 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - sqrt(a)*log(-
(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*
(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2
+ b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4
*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c
)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cos
h(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c
)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh
(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/2*(2*sqrt(-a)*arctan((cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((
a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2
+ b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + sqrt(-a +
 b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b
)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a
```

```
 - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(-a - b)*arctan(2*
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqr
t(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a + b)*cosh(d*x + c
)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a + b)*cosh(d*x
 + c) + b)*sinh(d*x + c) + 2*a + b)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{sech}(c + dx)} \coth(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx + c) + a} \coth(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

3.129 $\int \coth^3(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=217

$$-\frac{\coth^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{2d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+}}{\sqrt{a-b}}\right.}{d\sqrt{a-b}}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (a*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) + (3*b*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) - (a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Coth[c + d*x]^2*Sqrt[a + b*Sech
[c + d*x]])/(2*d)
```

Rubi [A] time = 0.327092, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3885, 898, 1315, 1178, 12, 1093, 206, 1170, 207}

$$-\frac{\coth^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{2d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+}}{\sqrt{a-b}}\right.}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (a*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) + (3*b*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) - (a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Coth[c + d*x]^2*Sqrt[a + b*Sech
[c + d*x]])/(2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
```

```
p, -1] && GtQ[m, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{b^4\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} - \frac{(2ab^2)\,\text{Subst}\,(\ldots}{}$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} - \frac{(2ab^2)\,\text{Subst}\,(\ldots}{}$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} - \frac{a\,\text{Subst}\left(\int \frac{\ldots}{a+b\ldots}\right.}{}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}\ldots}}{\sqrt{a+\ldots}}\right)}{\sqrt{a+b}d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sec}\ldots}}{\sqrt{a}\ldots}\right)}{4\sqrt{a-b}d}$$

Mathematica [B] time = 20.391, size = 518, normalized size = 2.39

$$\frac{\sqrt{a+b\text{sech}(c+dx)}\left(\frac{8\sqrt{-a\cosh(c+dx)}\tan^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{-a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)+b}} - \frac{2\sqrt{a}\sqrt{-a\cosh(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cosh(c+dx)+b}}{\sqrt{a-b}\sqrt{-a\cosh(c+dx)}}\right)}{\sqrt{a-b}\sqrt{a\cosh(c+dx)+b}} - \frac{2\sqrt{a}\sqrt{-a\cosh(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a+}\ldots}\right)}{\sqrt{a+b}\sqrt{a\cosh(c+d\ldots}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (((8*ArcTan[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]]*Sqrt[-(a*Co
sh[c + d*x])])/Sqrt[b + a*Cosh[c + d*x]] - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[
b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])]*Sqrt[-(a*Cosh
[c + d*x])])/(Sqrt[a - b]*Sqrt[b + a*Cosh[c + d*x]]) - (2*Sqrt[a]*ArcTan[(S
qrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh[c + d*x])])]*S
qrt[-(a*Cosh[c + d*x])])/(Sqrt[a + b]*Sqrt[b + a*Cosh[c + d*x]]) + (3*b*Arc
Tan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]]
)]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a - b]*Sqrt[b + a*Cosh[c + d*x]])
- ((2*a - 3*b)*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqr
t[a*Cosh[c + d*x]])]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a - b]*Sqrt[b + a
*Cosh[c + d*x]]) - (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(
Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqr
t[b + a*Cosh[c + d*x]]) - 2*Coth[c + d*x]^2)*Sqrt[a + b*Sech[c + d*x]])/(4*
d)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^3\sqrt{a+b\text{sech}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx + c) + a} \coth(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx + c) + a} \coth(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

3.130 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\tanh^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2a(a-b)\sqrt{a+b}\cdots}{}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a +
 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b)
)])/d - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rubi [A] time = 0.392432, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4057, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2\tanh(c+dx)\sqrt{a+b\text{se}\cdots}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a +
 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b)
)])/d - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```


Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh^2(c+dx)\,dx = -\int \sqrt{a+b\operatorname{sech}(c+dx)}\left(-1+\operatorname{sech}^2(c+dx)\right)dx$$

$$= -\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} - b\operatorname{sech}(c+dx) + \frac{1}{2}a\operatorname{sec}}{\sqrt{a+b\operatorname{sech}(c+dx)}}$$

$$= -\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2} - b\right)\operatorname{sech}(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+d}{a+b}}}{3b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+d}{a+b}}}{3b^2d}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int \sqrt{a + b\mathrm{sech}\,(dx + c)}\,(\tanh\,(dx + c))^2\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(dx + c) + a}\,\tanh\,(dx + c)^2\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\,\mathrm{sech}\,(dx + c) + a}\,\tanh\,(dx + c)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{sech}\,(c + dx)}\,\tanh^2\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx+c)+a} \tanh(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

3.131 $\int \sqrt{a+b\operatorname{sech}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\operatorname{sech}(c+dx))}{a+b\operatorname{sech}(c+dx)}}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{a+b\operatorname{sech}(c+dx)}}(a+b\operatorname{sech}(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\operatorname{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c +
d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[c +
 d*x]))/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0259978, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\operatorname{sech}(c+dx))}{a+b\operatorname{sech}(c+dx)}}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{a+b\operatorname{sech}(c+dx)}}(a+b\operatorname{sech}(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\operatorname{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c +
d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[c +
 d*x]))/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\,dx = \frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\operatorname{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\operatorname{sech}(c+dx))}{a+b\operatorname{sech}(c+dx)}}\sqrt{\frac{b(1+\operatorname{sech}(c+dx))}{a+b\operatorname{sech}(c+dx)}}(a+b\operatorname{sec}}{\sqrt{a+b}d}$$

Mathematica [F] time = 7.71742, size = 0, normalized size = 0.

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] Integrate[Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \sqrt{a + b\mathrm{sech}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\,\mathrm{sech}\,(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{sech}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

3.132 $\int \coth^2(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{\coth(c+dx)\sqrt{a+b\text{sec}}}{d}$$

```
[Out] (Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]]
)/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*
Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sec
h[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Se
ch[c + d*x]))/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.215404, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3896, 3780, 3875, 3832}

$$-\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```


```
[Out] (Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]]
)/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*
Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sec
h[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Se
ch[c + d*x]))/(Sqrt[a + b]*d)
```


Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
```

```
m}, x]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\int \left(-\sqrt{a+b\text{sech}(c+dx)} - \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\right) dx$$

$$= \int \sqrt{a+b\text{sech}(c+dx)}\,dx + \int \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+\cdots}}\right.\right.}{\cdots}$$

$$= \frac{\sqrt{a+b}\coth(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(\cdots}{a-b}}}{d}$$

Mathematica [B] time = 18.3135, size = 539, normalized size = 2.19

$$\frac{\sqrt{a+b\text{sech}(c+dx)}\left(\frac{2\sqrt{b}\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}\sqrt{\frac{(a+b)(a\cosh(c+dx)+a)}{(a-b)(a-a\cosh(c+dx))}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{a\cosh(c+dx)+b}}{\sqrt{b}\sqrt{a-a\cosh(c+dx)}}\right),-\frac{2b}{a-b}\right)}{a^{3/2}\sqrt{\cosh(c+dx)-1}\sqrt{\cosh(c+dx)+1}\sqrt{\text{sech}(c+dx)}\left(-\frac{a-a\cosh(c+dx)}{a}\right)^{3/2}\sqrt{\frac{a\cosh(c+dx)+a}{a}}\sqrt{-\frac{a(a+b)\cosh(c+dx)}{b(a-a\cosh(c+dx))}}} - \frac{4b\,\text{s}\cdots}{\sqrt{a}\sqrt{a+b}\sqrt{\cosh(c+d\cdots}}\right.}{2d\sqrt{\text{sech}(c+dx)}\sqrt{a\cosh(c+dx)+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] -((Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]])/d) + (Sqrt[a + b*Sech[c + d*x]]
*((2*Sqrt[b]*(a - a*Cosh[c + d*x])^(3/2)*Sqrt[((a + b)*(a + a*Cosh[c + d*x]
))/((a - b)*(a - a*Cosh[c + d*x]))]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[b + a*Co
sh[c + d*x]])/(Sqrt[b]*Sqrt[a - a*Cosh[c + d*x]])], (-2*b)/(a - b)]*Sinh[c
+ d*x])/(a^(3/2)*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[-((a
*(a + b)*Cosh[c + d*x])/(b*(a - a*Cosh[c + d*x])))]*(-((a - a*Cosh[c + d*x]
)/a))^(3/2)*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Sech[c + d*x]]) - (4*b*(a -
a*Cosh[c + d*x])*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]*Sqrt[-((b*(a
+ a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a - b)))]*Sinh[c + d*x])/(Sqrt[a]*Sqr
t[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d
*x]]*Sqrt[-((a - a*Cosh[c + d*x])/a)]*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Se
ch[c + d*x]]*Sqrt[-((b*(a - a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a + b)))]))
)/(2*d*Sqrt[b + a*Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])
```

Maple [F] time = 0.196, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^2\sqrt{a+b\text{sech}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx+c)+a} \coth(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \operatorname{sech}(dx+c)+a} \coth(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b \operatorname{sech}(c+dx)} \coth^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(dx+c)+a} \coth(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

3.133 $\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\left(3a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d}+\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d}-\frac{2(a+b\operatorname{sech}(c+dx))^{7/2}}{7b^4d}+\frac{6a(a+b\operatorname{sech}(c+}{5b^4d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sech[c + d*x]])/(b^4*d) - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c
+ d*x])^(3/2))/(3*b^4*d) + (6*a*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) - (2
*(a + b*Sech[c + d*x])^(7/2))/(7*b^4*d)
```

Rubi [A] time = 0.164909, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$-\frac{2\left(3a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d}+\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d}-\frac{2(a+b\operatorname{sech}(c+dx))^{7/2}}{7b^4d}+\frac{6a(a+b\operatorname{sech}(c+}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sech[c + d*x]])/(b^4*d) - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c
+ d*x])^(3/2))/(3*b^4*d) + (6*a*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) - (2
*(a + b*Sech[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x\sqrt{a+x}}\,dx, x, b\,\mathrm{sech}(c+dx)\right)}{b^4 d}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \left(-a^3+2ab^2+\left(3a^2-2b^2\right)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a+b\,\mathrm{sech}(c+d}\right.}{b^4 d}$$

$$= \frac{2a\left(a^2-2b^2\right)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^4 d} - \frac{2\left(3a^2-2b^2\right)(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^4 d} + \frac{6a(a+b\,\mathrm{sech}}{5b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{2a\left(a^2-2b^2\right)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^4 d} - \frac{2\left(3a^2-2b^2\right)(a+b\,\mathrm{sec}}{3b^4 d}$$

Mathematica [A] time = 4.43192, size = 167, normalized size = 1.13

$$\frac{2\left(\left(70b^4-6a^2b^2\right)\mathrm{sech}^2(c+dx)+\left(24a^3b-70ab^3\right)\mathrm{sech}(c+dx)-140a^2b^2+48a^4+3ab^3\mathrm{sech}^3(c+dx)+\frac{105b^4\sqrt{a\cosh}}{}\right.}{105b^4 d\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(48*a^4 - 140*a^2*b^2 + (105*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[
a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + (24*a^
3*b - 70*a*b^3)*Sech[c + d*x] + (-6*a^2*b^2 + 70*b^4)*Sech[c + d*x]^2 + 3*a
*b^3*Sech[c + d*x]^3 - 15*b^4*Sech[c + d*x]^4))/(105*b^4*d*Sqrt[a + b*Sech[
c + d*x]])
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^5 \frac{1}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^5}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] time = 10.5067, size = 7063, normalized size = 47.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)*sinh(d*x + c)^5 + b^
4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh(d*x + c)^2 + 3*(5*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*b^4*cosh(d*x + c)^3 +
 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(d*x + c)^4 + 6*b^4*co
sh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x + c)^5 + 2*b^4*cosh(
d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x
 + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x
 + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sin
h(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*
x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 +
(6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*
x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b
)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*
(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh
(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)) + 16*((12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^6 + (12*a^4 -
 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c)^5 - (12*
a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 + (36*a^4 - 87*a^2*b^2 + 15*(12*a^
4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^3
 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^3 + 5*(1
2*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 + (15*(12*a^4 -
 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87*a^2*b^2 - 10*(12*a^3*b - 35*a*b^
3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 - 24*(3*a^3*b
 - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (12*a^3*b - 35*a*b^3)*cosh(d*x
 + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^5 - 5*(12*a^3*b - 35*a*b^3)*
cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12*(12*a^4 - 29*a^2*b^2)*cosh(d*x +
 c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^2 + 6*(12*a^4 - 29*a^2*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b
^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*si
nh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4 + 3*a*b^4*d*cosh(d*x + c)^2 + a*b
^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4
*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^4*
d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^2 +
6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cosh(d*x + c)^3 + a*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/105*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)
*sinh(d*x + c)^5 + b^4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh
(d*x + c)^2 + 3*(5*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*
b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(
d*x + c)^4 + 6*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x
```

```
 + c)^5 + 2*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c
) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 8*((12*a^4 - 35*a^2
*b^2)*cosh(d*x + c)^6 + (12*a^4 - 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b -
 35*a*b^3)*cosh(d*x + c)^5 - (12*a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 +
 (36*a^4 - 87*a^2*b^2 + 15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^
3*b - 35*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3
*a^3*b - 5*a*b^3)*cosh(d*x + c)^3 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 3
5*a^2*b^2)*cosh(d*x + c)^3 + 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(1
2*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2
)*cosh(d*x + c)^2 + (15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87
*a^2*b^2 - 10*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b
^2)*cosh(d*x + c)^2 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x
+ c)^5 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12
*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x +
c)^2 + 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d
*x + c) + b)/cosh(d*x + c)))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4
+ 3*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^
4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(5*a*b^4*d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c
)^2 + a*b^4*d)*sinh(d*x + c)^2 + 6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cos
h(d*x + c)^3 + a*b^4*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^5}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

3.134 $\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a +
b*Sech[c + d*x]])/(b^2*d) + (2*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d)
```

Rubi [A] time = 0.108301, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$\frac{2(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a +
b*Sech[c + d*x]])/(b^2*d) + (2*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$
$$= -\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d} + \frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{secl}}\right.}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d} + \frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}$$

Mathematica [A] time = 0.623525, size = 111, normalized size = 1.41

$$\frac{2\left(-2a^2+\frac{3b^2\sqrt{a\cosh(c+dx)+b}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}-ab\text{sech}(c+dx)+b^2\text{sech}^2(c+dx)\right)}{3b^2d\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(-2*a^2 + (3*b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]
]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] - a*b*Sech[c + d*x] + b^
2*Sech[c + d*x]^2))/(3*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^3 \frac{1}{\sqrt{a+b\text{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^3}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] time = 9.94976, size = 2434, normalized size = 30.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c
)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
 c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*
cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c
) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*c
osh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^
3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a
*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 8*(a^2
*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*c
osh(d*x + c) - a*b)*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
))/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2
*d*sinh(d*x + c)^2 + a*b^2*d), -1/3*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*
x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(-a)*arctan((a*cosh(d
*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*s
inh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*c
osh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c))) + 4*(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x
 + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*cosh(d*x + c) - a*b)*sinh(d*x +
c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b^2*d*cosh(d*x + c)^2 + 2
*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^3}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

3.135 $\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0469297, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3885, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]

[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\,\mathrm{sech}(c+dx)\right)}{d}\\ &= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [B] time = 0.129582, size = 73, normalized size = 2.35

$$\frac{2\sqrt{a\cosh(c+dx)+b}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{d\sqrt{a\cosh(c+dx)}\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh
[c + d*x]])/(d*Sqrt[a*Cosh[c + d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

Maple [A] time = 0.034, size = 26, normalized size = 0.8

$$2\,{\frac{1}{d\sqrt{a}}{\rm Artanh} \left({\frac{\sqrt{a+b{\rm sech} \left(dx+c\right)}}{\sqrt{a}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(dx+c\right)}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] time = 9.52898, size = 1492, normalized size = 48.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
 c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
 + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*
x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*si
nh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*si
nh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^
2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
 + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2
```

```
*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))/(sqrt(a)*d), -sqrt(-a)*arctan(
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x +
c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

3.136 $\int \frac{\coth(c+dx)}{\sqrt{a+b\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.147701, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1170, 206, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$
$$= -\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$
$$= -\frac{\left(2b^2\right)\text{Subst}\left(\int \left(-\frac{1}{b^2\left(a-x^2\right)}+\frac{1}{2b^2\left(a+b-x^2\right)}-\frac{1}{2b^2\left(-a+b+x^2\right)}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

Mathematica [B] time = 3.01105, size = 230, normalized size = 2.17

$$\frac{\sqrt{a\cosh(c+dx)+b}\left(-\frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)+b}}\right)}{\sqrt{b-a}}-\frac{\sqrt{-a\cosh(c+dx)-b}\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cosh(c+dx)}}{\sqrt{-a\cosh(c+dx)-b}}\right)}{\sqrt{a+b}\sqrt{a\cosh(c+dx)+b}}+\frac{2\sqrt{b}\sqrt{\frac{a\cosh(c+dx)}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt{\cosh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{a\cosh(c+dx)+b}}\right)}{d\sqrt{\cosh(c+dx)}\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cosh[c + d*x]]*(-(ArcTan[(Sqrt[-a + b]*Sqrt[Cosh[c + d*x]])/Sqr
t[b + a*Cosh[c + d*x]]]/Sqrt[-a + b]) - (ArcTan[(Sqrt[a + b]*Sqrt[Cosh[c +
d*x]])/Sqrt[-b - a*Cosh[c + d*x]]]*Sqrt[-b - a*Cosh[c + d*x]])/(Sqrt[a + b]
*Sqrt[b + a*Cosh[c + d*x]]) + (2*Sqrt[b]*ArcSinh[(Sqrt[a]*Sqrt[Cosh[c + d*x
]])/Sqrt[b]]*Sqrt[1 + (a*Cosh[c + d*x])/b])/(Sqrt[a]*Sqrt[b + a*Cosh[c + d*
x]])))/(d*Sqrt[Cosh[c + d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \coth(dx+c)\frac{1}{\sqrt{a+b\text{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] time = 12.0383, size = 23317, normalized size = 219.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 +
 (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3
+ 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cos
h(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a -
b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b
)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x +
 c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) +
2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x
 + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)
*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cos
h(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x +
c) + 4*cosh(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
 b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b +
3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8
*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*co
sh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a
 + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2
 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt
((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4
*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2
 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)
^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*
x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) + 2*(a^2 - b^2)*sqrt(a)*l
og(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3
```

```
+ 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*
a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c)
 + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x
 + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a
*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x
 + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
 (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*
b)*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
)/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c
) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) + (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b +
 b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^
2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*
a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(
d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*co
sh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d
*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) +
4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3
+ 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^
2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 +
 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x +
 c) + 1)) + 2*(a^2 - b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x
 + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cos
h(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2
+ 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(
d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2
+ 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*
x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*
sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^
3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/
((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqr
t((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b)*cosh(d*x + c)^2 + (2*a -
b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a - b)*cosh(d*x + c) + b)*si
nh(d*x + c) + 2*a - b)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^
2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b
^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^
2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(
d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a +
b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*
(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 +
2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 +
2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a
*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((
8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 +
```

```
4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh
(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*c
osh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2
 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x +
 c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) - 2*(a^2 - b^2)*sqrt(a)*log(
-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4
*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2
 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*co
sh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4
*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), -1/2*((a^2 + a*b)*s
qrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(
(2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) +
 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*b)*sq
rt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2
*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - (a^2 - b^2)*sqrt
(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
+ (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x
 + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sin
h(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
+ 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sin
h(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*
a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh
(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), -1/4*(4*(a^
2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a
*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c)
 + b)*sinh(d*x + c) + a)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3
*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*
a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cos
h(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a
- b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(
3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2
+ 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
+ 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt(
(a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*
((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2
+ 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4
*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^
2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x
 + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - (a^2 - a*b)*sqrt(a + b)*
log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*
x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 +
8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cos
```

h(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b +
 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b +
 b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(
d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b
)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*
a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a
 + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*
b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b +
3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x +
 c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(
d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/((
a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x
 + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d
*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) - 2*(a^2 - a*b)*sqrt(
-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a
+ b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((
2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - (a^2 + a*b)*sqrt(a
- b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*si
nh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a
^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2
)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*
a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*
a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*
cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x +
 c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)
+ 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b
 - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a
*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(
d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*
cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)
))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*c
osh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2 + a*b)*
sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/
((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
+ 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b
^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2
+ (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b +
 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^
2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^
2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4
+ 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 +
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*
x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh
(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x
 + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*
(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 +

```
3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2
*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x +
 c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt(
(a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2
 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + (a^2 + a
*b)*sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c))/((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x +
 c) + 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*
b)*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
)/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c
) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)))/((a^3 - a*b^
2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(coth(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

3.137 $\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a+b)(1-\mathrm{sech}(c+dx))}-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a-b)(\mathrm{sech}(c+dx)+1)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}-$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) + (b*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) - (b*ArcTanh[Sqrt[a + b*Sech
[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d) - Sqrt[a + b*Sech[c + d*x]]/(4*(a + b)*d
*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a - b)*d*(1 + Sech[c
+ d*x]))
```

Rubi [A] time = 0.296669, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3885, 898, 1238, 206, 199, 207}

$$-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a+b)(1-\mathrm{sech}(c+dx))}-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a-b)(\mathrm{sech}(c+dx)+1)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```


```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) + (b*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) - (b*ArcTanh[Sqrt[a + b*Sech
[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d) - Sqrt[a + b*Sech[c + d*x]]/(4*(a + b)*d
*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a - b)*d*(1 + Sech[c
+ d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
&& IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = -\frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\,\mathrm{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^4\right)\mathrm{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^4\right)\mathrm{Subst}\left(\int \left(-\frac{1}{b^4\left(a-x^2\right)}+\frac{1}{4b^3\left(a+b-x^2\right)^2}+\frac{1}{2b^4\left(a+b-x^2\right)}-\frac{1}{4b^3\left(-a+b+x^2\right)^2}-\frac{1}{2b^4\left(-a+b+x^2\right)}\right)dx, \ldots}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d} + \frac{\mathrm{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+\ldots}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{\sqrt{a\ldots}}{4(a+b\ldots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} - \frac{b\tan\ldots}{}$$

Mathematica [B] time = 7.28054, size = 902, normalized size = 3.44

$$\frac{\sqrt{b+a\cosh(c+dx)}\sqrt{\mathrm{sech}(c+dx)}\left(\frac{\left(2a^2-2b^2\right)\left(\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a-b}\sqrt{-a\cosh(c+dx)}}\right)+\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{-a\cosh(c+dx)}}\right)\right)-4\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\ldots\right.}{\sqrt{a-b}\sqrt{a+b}\sqrt{\cosh(c+dx)-1}\sqrt{\cosh(c+dx)+1}\left(a^2-2b^2-\ldots\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cosh[c + d*x]]*((Sqrt[a]*b*(Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b
+ a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[-a - b]*Ar
cTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]
])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x
]))/(Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]
]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((2*a^2 - 3*b^2)*(Sqrt[a +
 b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c
+ d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a
 + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[(-a + a*Cosh[c +
d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Sqrt[Sech[c + d*x]])/(a^
(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*
x]]) + ((2*a^2 - 2*b^2)*(-4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cosh[
c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*
Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt[a
 - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh
[c + d*x])])]))*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a
*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x]
])/(Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]
]*(a^2 - 2*b^2 + 4*b*(b + a*Cosh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))*S
qrt[Sech[c + d*x]])/(4*(a - b)*(a + b)*d*Sqrt[a + b*Sech[c + d*x]]) + ((b +
 a*Cosh[c + d*x])*(-a/(2*(a^2 - b^2)) + ((a - b*Cosh[c + d*x])*Csch[c + d*x
]^2)/(2*(-a^2 + b^2)))*Sech[c + d*x])/(d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int (\coth (dx+c))^3 \frac{1}{\sqrt{a+b\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (dx+c)^3}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(coth(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)^3}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

3.138 $\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=610

$$\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}-\frac{4\sqrt{a+}}{}$$

```
[Out] (-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*
(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b
))])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(15*b
^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a +
b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x]*Sqrt[a + b*Se
ch[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.790606, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3895, 3784, 3837, 3832, 4004, 3860, 4082, 4005}

$$\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-b)\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```


```
[Out] (-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*
(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b
))])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(15*b
^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a +
b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x]*Sqrt[a + b*Se
ch[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```


Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
```

```
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}} + \frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right) dx$$

$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx + \int \frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad} +$$

$$= -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx)}{a-b}}}{b^2d}$$

$$= -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx)}{a-b}}}{b^2d}$$

$$= -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx)}{a-b}}}{b^2d}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^4 \frac{1}{\sqrt{a+b\text{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh\left(dx+c\right)^{4}}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{4}\left(c+dx\right)}{\sqrt{a+b\,\text{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(dx+c\right)^{4}}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

3.139 $\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\coth(c+\cdots}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.258372, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3894, 4059, 3921, 3784, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(\cdots}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx &= -\int \frac{-1+\operatorname{sech}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx\\ &= -\int \frac{-1-\operatorname{sech}(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx-\int \frac{\operatorname{sech}(c+dx)(1+\operatorname{sech}(c+dx))}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx\\ &= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c}{a-b}}}{b^2d}\\ &= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c}{a-b}}}{b^2d}\end{aligned}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]

[Out] $Aborted
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^2\frac{1}{\sqrt{a+b\operatorname{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^2}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^2}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

3.140 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.0214469, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad}$$

Mathematica [A] time = 0.62605, size = 168, normalized size = 1.58

$$\frac{2b\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a\cosh(c+dx)+b}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{b-a}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)}{\sqrt{a}d\sqrt{a+b}\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(\operatorname{sech}(c+dx)-1)}{a+b}}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
```

```
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\mathrm{sech}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\,\mathrm{sech}\,(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\,\mathrm{sech}\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

3.141 $\int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=362

$$-\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\,\text{sech}(c+dx)}}-$$

```
[Out] (Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
 d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (
2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(d*Sqrt[a
 + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c
 + d*x]])
```

Rubi [A] time = 0.436081, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3896, 3784, 3875, 3833, 21, 3829, 3832, 4004}

$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\,\text{sech}(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{a+b\,\text{sech}(c+dx)}}-\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{se}}}{\sqrt{}}\right.\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
 d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (
2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(d*Sqrt[a
 + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c
 + d*x]])
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}dx = -\int \left(-\frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\operatorname{csch}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\right)dx$$
$$= \int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}dx + \int \frac{\operatorname{csch}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}dx$$
$$= \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad} -$$
$$= \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad} -$$
$$= \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad} -$$
$$= \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad} -$$
$$= \frac{\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d} - \frac{\coth(c+dx)}{}$$

Mathematica [F] time = 86.4917, size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^2 \frac{1}{\sqrt{a+b\operatorname{sech}(dx+c)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)^2}{\sqrt{b\operatorname{sech}(dx+c)+a}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c + dx)}{\sqrt{a + b\operatorname{sech}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(coth(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx + c)^2}{\sqrt{b\operatorname{sech}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

3.142 $\int \frac{\tanh^5(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sech[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec
h[c + d*x]])/(b^4*d) + (2*a*(a + b*Sech[c + d*x])^(3/2))/(b^4*d) - (2*(a +
b*Sech[c + d*x])^(5/2))/(5*b^4*d)
```

Rubi [A] time = 0.191165, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sech[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec
h[c + d*x]])/(b^4*d) + (2*a*(a + b*Sech[c + d*x])^(3/2))/(b^4*d) - (2*(a +
b*Sech[c + d*x])^(5/2))/(5*b^4*d)
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)^{3/2}}\,dx, x, b\,\text{sech}(c+dx)\right)}{b^4d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{x^2\left(-a+x^2\right)}\,dx, x, \sqrt{a+b\,\text{sech}(c+dx)}\right)}{b^4d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{\left(a^2-b^2\right)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a\left(a-x^2\right)}\right)dx, x, \sqrt{a+b\,\text{sech}(c+dx)}\right.}{b^4d}$$

$$= -\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\,\text{sech}(c+dx)}}-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\,\text{sech}(c+dx)}}{b^4d}+\frac{2a(a+b\,\text{sech}(c+dx}{b^4d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\,\text{sech}(c+dx)}}-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\,\text{sech}(c+}{b^4d}$$

Mathematica [A] time = 3.07674, size = 155, normalized size = 1.05

$$-\frac{2\left(-2a^2b^2\text{sech}^2(c+dx)+2ab\left(4a^2-5b^2\right)\text{sech}(c+dx)-20a^2b^2+16a^4+ab^3\text{sech}^3(c+dx)-\frac{5b^4\sqrt{a\cosh(c+dx)+b}\tanh^{}}{\sqrt{a\cosh(c+}}\right.}{5ab^4d\sqrt{a+b\,\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*(16*a^4 - 20*a^2*b^2 + 5*b^4 - (5*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]
/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] +
2*a*b*(4*a^2 - 5*b^2)*Sech[c + d*x] - 2*a^2*b^2*Sech[c + d*x]^2 + a*b^3*Sec
h[c + d*x]^3))/(5*a*b^4*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^5 (a+b\,\text{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^5}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 12.5299, size = 9172, normalized size = 61.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cosh(d*x + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x +
c)^5 + 3*a*b^4*cosh(d*x + c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x +
 c)^2 + 2*b^5*cosh(d*x + c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)
^5 + a*b^4 + (15*a*b^4*cosh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*si
nh(d*x + c)^4 + 4*(5*a*b^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^
4*cosh(d*x + c) + b^5)*sinh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5
*cosh(d*x + c)^3 + 18*a*b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^
4)*sinh(d*x + c)^2 + 2*(3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6
*a*b^4*cosh(d*x + c)^3 + 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^
5)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)
^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*c
osh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c)
 + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*co
sh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*
b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*((16*
a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^
4)*sinh(d*x + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b -
 10*a^2*b^3 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c
)^5 + 16*a^5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh
(d*x + c)^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5
*a*b^4)*cosh(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^4 + 32*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*
(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*
cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^
3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*
(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*
cosh(d*x + c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(
4*a^4*b - 5*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*c
osh(d*x + c)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
 c)^4 + 2*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^
2*b^3)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*si
nh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x
 + c)^6 + a^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b
^4*d*cosh(d*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x +
 c)^2 + 2*a^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c
) + a^2*b^5*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5
*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x +
```

```
c)^3 + 5*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d
)*sinh(d*x + c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x +
 c)^3 + 18*a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b
^4*d)*sinh(d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d
*x + c)^4 + 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a
^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x + c)), -1/5*(5*(a*b^4*cosh(d*x
 + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x + c)^5 + 3*a*b^4*cosh(d*x
+ c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x + c)^2 + 2*b^5*cosh(d*x +
 c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)^5 + a*b^4 + (15*a*b^4*c
osh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^4 + 4*(5*a*b
^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*s
inh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5*cosh(d*x + c)^3 + 18*a*
b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^2 + 2*(
3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6*a*b^4*cosh(d*x + c)^3 +
 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c))*sqrt(-
a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*((16*a^5 - 20*a^
3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*sinh(d*x
 + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b - 10*a^2*b^3
 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16*a^
5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^4
 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos
h(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 32
*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*(16*a^5 - 2
0*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (4
8*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*
a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*(4*a^4*b -
5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x +
c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(4*a^4*b - 5
*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c
)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^4 + 2*(
48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^2*b^3)*cosh
(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x + c)^6 + a
^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b^4*d*cosh(d
*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x + c)^2 + 2*a
^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5
*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5*d*cosh(d*x
 + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x + c)^3 + 5*a
^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x
+ c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x + c)^3 + 18*
a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(
d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d*x + c)^4 +
 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*co
sh(d*x + c) + a^2*b^5*d)*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(c+dx)}{(a+b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^5}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

3.143 $\int \frac{\tanh^3(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
)/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*Sqrt[a + b*Sech[c + d*x]])/(b^2*
d)
```

Rubi [A] time = 0.141404, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
)/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*Sqrt[a + b*Sech[c + d*x]])/(b^2*
d)
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$
$$= \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d} + \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

Mathematica [A] time = 0.64969, size = 103, normalized size = 1.17

$$\frac{2\left(2a^2+\frac{b^2\sqrt{a\cosh(c+dx)+b}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}+ab\text{sech}(c+dx)-b^2\right)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^2 - b^2 + (b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*
x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + a*b*Sech[c + d*x]))
/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^3 (a+b\text{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^3}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 11.7931, size = 2801, normalized size = 31.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3*cosh(d*x + c)
+ a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*
cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*
cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*
cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 +
b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x +
 c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b
*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x
+ c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b
^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 4*(2*a^2*b*cosh(d*x + c) + 2*a^3 - a*b^2 + (2
*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)*sinh(d*x + c)^2 + 2*(a^2*b
+ (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^3*b^2*d*sinh(d*x + c)^2 + 2*
a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^2*d*cosh(d*x + c) + a^2*b^3*
d)*sinh(d*x + c)), -((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3
*cosh(d*x + c) + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(
-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
 c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 2*(2*a^2*b*cosh(d
*x + c) + 2*a^3 - a*b^2 + (2*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)
*sinh(d*x + c)^2 + 2*(a^2*b + (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^
3*b^2*d*sinh(d*x + c)^2 + 2*a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^
2*d*cosh(d*x + c) + a^2*b^3*d)*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c + dx)}{(a + b \operatorname{sech}(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

3.144 $\int \frac{\tanh(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - 2/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rubi [A] time = 0.0622013, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 51, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - 2/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{ad}$$

$$= -\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

Mathematica [A] time = 0.242889, size = 79, normalized size = 1.46

$$\frac{2\left(\frac{\sqrt{a\cosh(c+dx)+b}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}-1\right)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(-1 + (ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b +
 a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]]))/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [A] time = 0.018, size = 46, normalized size = 0.9

$$-\frac{1}{d}\left(2\frac{1}{a\sqrt{a+b\text{sech}\,(dx+c)}}-2\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{a+b\text{sech}\,(dx+c)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(2/a/(a+b*sech(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+b*sech(d*x+c))^(1/2)
/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\,(dx+c)}{(b\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 10.0666, size = 2456, normalized size = 45.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cos
h(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*
a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (1
2*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2
 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (
4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*
x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*
a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x +
 c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(
d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2)) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2 + a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x +
 c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*
cosh(d*x + c) + a^2*b*d)*sinh(d*x + c)), -((a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqr
t(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)
/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*(a*cosh(d*x +
 c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt((a*co
sh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x +
c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*cosh(d*x + c) + a^2*b*d)*
sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

3.145 $\int \frac{\coth(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sech[c + d*x]])
```

Rubi [A] time = 0.216624, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3885, 898, 1287, 206}

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sech[c + d*x]])
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2} - \frac{1}{ab^2(a-x^2)} + \frac{1}{2(a-b)b^2(a-b-x^2)} + \frac{1}{2b^2(a+b)(a+b-x^2)}\right) dx, x, \sqrt{a+b\text{se}}\right.}{d}$$

$$= \frac{2b^2}{a(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{}{a(a^2-}$$

Mathematica [B] time = 7.31917, size = 904, normalized size = 6.37

$$\frac{(b+a\cosh(c+dx))^2\left(-\frac{2b^3}{a^2(a^2-b^2)(b+a\cosh(c+dx))} - \frac{2b^2}{a^2(b^2-a^2)}\right)\text{sech}^2(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} - \frac{(b+a\cosh(c+dx))^{3/2}\left(\frac{(a^2-b^2)\left(\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] -((b + a*Cosh[c + d*x])^(3/2)*((-2*Sqrt[a]*b*(Sqrt[a - b]*ArcTan[(Sqrt[a]*S
qrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[-a -
 b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c
+ d*x]])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c
 + d*x]))/(Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c
+ d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((a^2 + b^2)*(Sqrt[a
 + b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[
c + d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt
[a + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[(-a + a*Cosh[c
+ d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Sqrt[Sech[c + d*x]])/(
a^(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c +
d*x]]) + ((a^2 - b^2)*(-4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cosh[c
+ d*x]]/Sqrt[-(a*Cosh[c + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sq
rt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt[a -
 b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh[c
 + d*x])])]))*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*C
osh[c + d*x])]*(a + a*Cosh[c + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x]])
/(Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*
(a^2 - 2*b^2 + 4*b*(b + a*Cosh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))*Sec
h[c + d*x]^(3/2))/(2*a*(-a + b)*(a + b)*d*(a + b*Sech[c + d*x])^(3/2)) + ((
b + a*Cosh[c + d*x])^2*((-2*b^2)/(a^2*(-a^2 + b^2)) - (2*b^3)/(a^2*(a^2 - b
^2)*(b + a*Cosh[c + d*x])))*Sech[c + d*x]^2)/(d*(a + b*Sech[c + d*x])^(3/2)
)
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \coth(dx+c)\,(a+b\operatorname{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

3.146 $\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)^2(\text{sech}(c+$$

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) + (b*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) - (b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - ((2*a + 3*
b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) - (2
*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) - Sqrt[a + b*Sech[c + d
*x]]/(4*(a + b)^2*d*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a
- b)^2*d*(1 + Sech[c + d*x]))
```

Rubi [A] time = 0.426842, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3885, 898, 1335, 206, 199}

$$-\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+}}{4d(a-b)^2(\text{sech}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) + (b*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) - (b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - ((2*a + 3*
b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) - (2
*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) - Sqrt[a + b*Sech[c + d
*x]]/(4*(a + b)^2*d*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a
- b)^2*d*(1 + Sech[c + d*x]))
```


Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = -\frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\,\mathrm{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^4\right)\mathrm{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^4\right)\mathrm{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4\left(a-x^2\right)} - \frac{1}{4(a-b)b^3\left(a-b-x^2\right)^2} + \frac{2a-3b}{4(a-b)^2b^4\left(a-b-x^2\right)} + \frac{\quad}{4b^3(a+b)\ldots}\right.\right.}{d}$$

$$= -\frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{sech}(c+dx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{ad} - \frac{(2a - \ldots}{\ldots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{se}\ldots}}{\sqrt{a}\ldots}\right)}{2(a+b)^{5/2}d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d}$$

Mathematica [B] time = 7.52323, size = 996, normalized size = 3.15

$$\frac{(b+a\cosh(c+dx))^2\left(\frac{2b^5}{a^2\left(a^2-b^2\right)^2(b+a\cosh(c+dx))} + \frac{\left(-a^2+2b\cosh(c+dx)a-b^2\right)\mathrm{csch}^2(c+dx)}{2\left(b^2-a^2\right)^2} - \frac{a^4+b^2a^2+4b^4}{2a^2\left(b^2-a^2\right)^2}\right)\mathrm{sech}^2(c+dx)}{d(a+b\,\mathrm{sech}(c+dx))^{3/2}} + \frac{(b+a\cosh\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cosh[c + d*x])^(3/2)*(((-(a^3*b) + 7*a*b^3)*(Sqrt[a - b]*ArcTan[(Sq
rt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sq
```

```
rt[-a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*
Cosh[c + d*x]])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a
*Cosh[c + d*x]))/(Sqrt[a]*Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]
*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((2*a
^4 - 6*a^2*b^2 - 2*b^4)*(Sqrt[a + b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d
*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*S
qrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh
[c + d*x]]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c
 + d*x])*Sqrt[Sech[c + d*x]])/(a^(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Co
sh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]) + ((2*a^4 - 4*a^2*b^2 + 2*b^4)*(-4*Sq
rt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*
x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sq
rt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b +
 a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh[c + d*x])])]))*Sqrt[-(a*Cosh[
c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c
 + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*Sq
rt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*(a^2 - 2*b^2 + 4*b*(b + a*Co
sh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))*Sech[c + d*x]^(3/2))/(4*a*(a -
b)^2*(a + b)^2*d*(a + b*Sech[c + d*x])^(3/2)) + ((b + a*Cosh[c + d*x])^2*(-
(a^4 + a^2*b^2 + 4*b^4)/(2*a^2*(-a^2 + b^2)^2) + (2*b^5)/(a^2*(a^2 - b^2)^2
*(b + a*Cosh[c + d*x])) + ((-a^2 - b^2 + 2*a*b*Cosh[c + d*x])*Csch[c + d*x]
^2)/(2*(-a^2 + b^2)^2))*Sech[c + d*x]^2)/(d*(a + b*Sech[c + d*x])^(3/2))
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int (\coth (dx+c))^3 (a+b\mathrm{sech}(dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (dx+c)^3}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

3.147 $\int \frac{\tanh^4(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
ch[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*Sqrt[
a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) +
(2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*Sqrt[a + b
]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) + (2*
Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) - (4*a*Tanh[c + d*x])/((a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*Sqrt[a + b*Sech[c + d*x]]) - (2*a^2*Sech[c + d*x]*Tanh[c + d*x])/(b*(a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sech[c
 + d*x]]*Tanh[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.36637, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3895, 3785, 4058, 3921, 3784, 3832, 4004, 3836, 4005, 3845, 4082}

$$-\frac{2\text{sech}(c+dx)\tanh(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}-\frac{4\tanh(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\left(8a^2-5b^2\right)\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+}}{\sqrt{a+b}}\right.\right.}{3b^4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```


```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
ch[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*Sqrt[
a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) +
(2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*Sqrt[a + b
]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
```

h[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]])/(3*b^3*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]])/(a^2*d) - (4*a*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) - (2*a^2*Sech[c + d*x]*Tanh[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*b^2*(a^2 - b^2)*d)

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \left(\frac{1}{(a+b\mathrm{sech}(c+dx))^{3/2}} - \frac{2\mathrm{sech}^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}} + \frac{\mathrm{sech}^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\right) dx$$
$$= -\left(2\int \frac{\mathrm{sech}^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx + \int \frac{\mathrm{sech}^4(c+dx)}{(a+b\mathrm{sech}(c+dx$$

$$= -\frac{4a\tanh(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\mathrm{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\mathrm{sech}(c+dx)}} - \frac{2a^2\mathrm{sech}(c+dx)\,t}{b\left(a^2-b^2\right)d\sqrt{a+}}$$

$$= -\frac{4a\tanh(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\mathrm{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\mathrm{sech}(c+dx)}} - \frac{2a^2\mathrm{sech}(c+dx)\,t}{b\left(a^2-b^2\right)d\sqrt{a+}}$$
$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{4a\coth}{}$$
$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{4a\coth}{}$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^4 (a+b\,\mathrm{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^4}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}\tanh\,(dx+c)^4}{b^2\,\mathrm{sech}\,(dx+c)^2+2\,ab\,\mathrm{sech}\,(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^4/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^4\,(c+dx)}{(a+b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\,(dx+c)^4}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

3.148 $\int \frac{\tanh^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b}{}}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/
(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) -
 (2*Tanh[c + d*x])/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] time = 0.420231, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4061, 4059, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\coth(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/
(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) -
 (2*Tanh[c + d*x])/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx = -\int \frac{-1+\text{sech}^2(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\,\text{sech}(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)+\frac{1}{2}\left(a^2-b^2\right)\text{sech}^2(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\,\text{sech}(c+dx)}} + \frac{\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx}{a} + \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}\left(a^2-b^2\right)\text{sech}(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,d}{a\left(a^2-b^2\right)}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(}{a-b}}}{ab^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(}{a-b}}}{ab^2d}$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int (\tanh(dx+c))^2 (a+b\operatorname{sech}(dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)^2}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\operatorname{sech}(dx+c)+a}\tanh(dx+c)^2}{b^2\operatorname{sech}(dx+c)^2+2\,ab\operatorname{sech}(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (dx + c)^2}{(b\,\mathrm{sech}\,(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

3.149 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}$$

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a +
b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tanh[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] time = 0.337043, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3/2),x]
```


```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a +
b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tanh[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\operatorname{sech}(c+dx)+\frac{1}{2}b^2\operatorname{sech}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\operatorname{sech}(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\operatorname{sech}(c+dx)(1}{\sqrt{a+b\operatorname{sec}}}}{a\left(a^2-\right.}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{}{a\left(a\right.}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\,\mathrm{co}}{}$$

Mathematica [F] time = 81.5779, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] Integrate[(a + b*Sech[c + d*x])^(-3/2), x]
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (a+b\mathrm{sech}\,(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

3.150 $\int \frac{\coth^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=665

$$\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}-\frac{(3a-b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c}{a+b}}}{}$$

```
[Out] (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*
Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (
2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
+ d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3/2)) - (b^2*Ta
nh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (4*a*b^2*Tanh[c
+ d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])
/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] time = 0.98349, antiderivative size = 665, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3896, 3785, 4058, 3921, 3784, 3832, 4004, 3875, 3833, 4003, 4005}

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)(a+b\text{sech}(c+dx))^{3/2}}+\frac{2\sqrt{a+b}\,\text{coth}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```


```
[Out] (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*
Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (
2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
+ d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3/2)) - (b^2*Ta
nh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (4*a*b^2*Tanh[c
+ d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])
/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
```

```
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = -\int \left(-\frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}} - \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\right)dx$$

$$= \int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx + \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{\coth(c+dx)}{d(a+b\operatorname{sech}(c+dx))^{3/2}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{1}{2}(3b)\int \frac{\operatorname{sech}(c+dx}{(a+b\operatorname{sech}(c+}$$

$$= -\frac{\coth(c+dx)}{d(a+b\operatorname{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^{3/2}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(}}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{\text{co}}{d(a+b}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\coth(}{}$$

$$= \frac{4a\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} - \frac{2\coth(}{}$$

Mathematica [F] time = 104.747, size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^2 (a+b\text{sech}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)^2}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{(a+b\,\text{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (dx+c)^2}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

3.151 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(e^{2c(a+bx)}+1\right)^5}$$

```
[Out] (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*
x)))^6) - (192*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(5*b*c*(1 + E^(
2*c*(a + b*x)))^5) + (48*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*
(1 + E^(2*c*(a + b*x)))^4) - (64*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2
])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3)
```

Rubi [A] time = 0.282013, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(e^{2c(a+bx)}+1\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```


```
[Out] (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*
x)))^6) - (192*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(5*b*c*(1 + E^(
2*c*(a + b*x)))^5) + (48*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*
(1 + E^(2*c*(a + b*x)))^4) - (64*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2
])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^7(ac+bcx)\,dx \\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{128x^7}{(1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(128\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^7}{(1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^3}{(1+x)^7}\,dx,x,e^{2c(a+bx)}\right)}{bc} \\
&= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{1}{(1+x)^4}\right)dx\right.}{bc} \\
&= \frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5}+\frac{48\cos}{}
\end{aligned}$$

Mathematica [A] time = 0.087682, size = 84, normalized size = 0.44

$$-\frac{16\left(6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{15bc\left(e^{2c(a+bx)}+1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)]*Sqrt[Sech[c*(a + b*x)]^2])/(15*b*c*(1 + E^(2*c*(a + b*x
)))^6)
```

Maple [A] time = 0.228, size = 91, normalized size = 0.5

$$-\frac{\left(320\,e^{6\,c(bx+a)}+240\,e^{4\,c(bx+a)}+96\,e^{2\,c(bx+a)}+16\right)e^{-c(bx+a)}}{15\left(1+e^{2\,c(bx+a)}\right)^5cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] -16/15/(1+exp(2*c*(b*x+a)))^5*(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(
1/2)*(20*exp(6*c*(b*x+a))+15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))+1)/c/b*exp
(-c*(b*x+a))
```

Maxima [B] time = 1.13235, size = 521, normalized size = 2.73

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{}{bc\left(e^{(12\,bcx+12\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 3.88298, size = 1516, normalized size = 7.94

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}+6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13784, size = 86, normalized size = 0.45

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

3.152 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=141

$$-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

```
[Out] (-4*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^4) + (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3) - (8*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rubi [A] time = 0.166844, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] (-4*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^4) + (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3) - (8*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = \left(\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\text{sech}^5(ac+bcx)\,dx$$

$$= \frac{\left(\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{32x^5}{\left(1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx,x,e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx,x,e^{2c(a+bx)}\right.}{bc}$$

$$= -\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}-\frac{8\cosh(}{}$$

Mathematica [A] time = 0.0657853, size = 72, normalized size = 0.51

$$-\frac{4\left(4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\text{sech}^2(c(a+bx))}}{3bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)]*Sqrt[
Sech[c*(a + b*x)]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^4)
```

Maple [A] time = 0.174, size = 80, normalized size = 0.6

$$-\frac{\left(24\,e^{4\,c(bx+a)}+16\,e^{2\,c(bx+a)}+4\right)e^{-c(bx+a)}}{3\left(1+e^{2\,c(bx+a)}\right)^3cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -4/3/(1+exp(2*c*(b*x+a)))^3*(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/
2)*(6*exp(4*c*(b*x+a))+4*exp(2*c*(b*x+a))+1)/c/b*exp(-c*(b*x+a))
```

Maxima [A] time = 1.1506, size = 282, normalized size = 2.

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 3.12381, size = 797, normalized size = 5.65

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\,\mathrm{c}\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12626, size = 69, normalized size = 0.49

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

3.153 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rubi [A] time = 0.112898, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 264}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx$$
$$= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

Mathematica [A] time = 0.0559396, size = 44, normalized size = 0.79

$$\frac{e^{3c(a+bx)}\sqrt{\operatorname{sech}^2(c(a+bx))}}{bce^{2c(a+bx)}+bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(3*c*(a + b*x))*Sqrt[Sech[c*(a + b*x)]^2])/(b*c + b*c*E^(2*c*(a + b*x)))
```

Maple [A] time = 0.174, size = 69, normalized size = 1.2

$$-2\,\frac{\left(2\,e^{2\,c(bx+a)}+1\right)e^{-c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)cb}\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] -2/(1+exp(2*c*(b*x+a)))*(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*(
2*exp(2*c*(b*x+a))+1)/c/b*exp(-c*(b*x+a))
```

Maxima [A] time = 1.09153, size = 113, normalized size = 2.02

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 3.50969, size = 302, normalized size = 5.39

$$-\frac{2\,(3\,\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right))}{bc\cosh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+bc\sinh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)+\left(3\,bc\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.122, size = 51, normalized size = 0.91

$$-\frac{2\left(2\,e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

3.154 $\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(b
*c)
```

Rubi [A] time = 0.0869514, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(b
*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx$$

$$= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(2\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

Mathematica [A] time = 0.0371545, size = 42, normalized size = 0.95

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[c*(a + b*x)]^2])/(b
*c)
```

Maple [A] time = 0.207, size = 66, normalized size = 1.5

$$\frac{\left(1+e^{2c(bx+a)}\right)\ln\left(e^{2bcx}+e^{-2ac}\right)e^{-c(bx+a)}}{cb}\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] (1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*(1+exp(2*c*(b*x+a)))/c/b*
ln(exp(2*b*c*x)+exp(-2*a*c))*exp(-c*(b*x+a))
```

Maxima [A] time = 1.67855, size = 28, normalized size = 0.64

$$\frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [A] time = 3.22516, size = 97, normalized size = 2.2

$$\frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\sqrt{\operatorname{sech}^2(ac+bcx)}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(sech(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Giac [A] time = 1.13834, size = 27, normalized size = 0.61

$$\frac{\log\left(e^{(2\,bcx)}+e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

3.155 $\int \frac{e^{c(a+bx)}}{\sqrt{\mathbf{sech}^2(ac+bcx)}}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] (E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(4*b*c*Sqrt[Sech[a*c + b*c*x]^2]) + (
x*Sech[a*c + b*c*x])/(2*Sqrt[Sech[a*c + b*c*x]^2])
```

Rubi [A] time = 0.11297, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(4*b*c*Sqrt[Sech[a*c + b*c*x]^2]) + (
x*Sech[a*c + b*c*x])/(2*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Mathematica [A] time = 0.0492781, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}+2bcx\right)\operatorname{sech}(c(a+bx))}{4bc\sqrt{\operatorname{sech}^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sech[c*(a + b*x)])/(4*b*c*Sqrt[Sech[c*(a + b
*x)]^2])
```

Maple [A] time = 0.188, size = 106, normalized size = 1.4

$$\frac{xe^{c(bx+a)}}{2+2\,e^{2\,c(bx+a)}}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}}+\frac{e^{3\,c(bx+a)}}{\left(4+4\,e^{2\,c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/2/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*
x*exp(c*(b*x+a))+1/4/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+e
xp(2*c*(b*x+a)))/c/b*exp(3*c*(b*x+a))
```

Maxima [A] time = 1.1206, size = 39, normalized size = 0.53

$$\frac{1}{2}x+\frac{a}{2\,b}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A] time = 2.95571, size = 163, normalized size = 2.2

$$\frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sech(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.12721, size = 45, normalized size = 0.61

$$\frac{\left(2\,bcxe^{(-ac)}+e^{(2\,bcx+ac)}\right)e^{(ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c*x*e^(-a*c) + e^(2*b*c*x + a*c))*e^(a*c)/(b*c)
```

3.156 $\int \frac{e^{c(a+bx)}}{\mathbf{sech}^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] -Sech[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) + (
3*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (3*x*Sech[a*c + b*c*x])/(8*Sqrt[Sech[a*c + b*c*x]^2])
```

Rubi [A] time = 0.154983, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -Sech[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) + (
3*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (3*x*Sech[a*c + b*c*x])/(8*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3x\operatorname{sech}}{8\sqrt{\operatorname{sech}}}$$

Mathematica [A] time = 0.0642308, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{sech}^3(c(a+bx))}{16bc\operatorname{sech}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*Sech[c*(a + b*x)]^3)/(16*b*c*(Sech[c*(a + b*x)]^2)^(3/2))
```

Maple [A] time = 0.19, size = 216, normalized size = 1.3

$$\frac{3\,x e^{c(bx+a)}}{8+8\,e^{2\,c(bx+a)}}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} + \frac{e^{5\,c(bx+a)}}{\left(32+32\,e^{2\,c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} + \frac{3\,e^{3\,c(bx+a)}}{\left(16+16\,e^{2\,c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} - \frac{e}{(16+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 3/8/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*
x*exp(c*(b*x+a))+1/32/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+
exp(2*c*(b*x+a)))/c/b*exp(5*c*(b*x+a))+3/16/(1/(1+exp(2*c*(b*x+a)))^2*exp(2
*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))/c/b*exp(3*c*(b*x+a))-1/16/(1/(1+exp
(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))/c/b*exp(-c*(b
*x+a))
```

Maxima [A] time = 1.15241, size = 100, normalized size = 0.62

$$\frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Fricas [A] time = 3.32022, size = 323, normalized size = 1.99

$$-\frac{\cosh\left(bcx+ac\right)^{3}+3\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-3\,\sinh\left(bcx+ac\right)^{3}-6\,(2\,bcx+1)\cosh\left(bcx+ac\right)+3\,\big(4\,bcx-3}{32\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\left(\operatorname{sech}^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(sech(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A] time = 1.12733, size = 111, normalized size = 0.69

$$\frac{\left(12\,bcxe^{(-ac)}-2\,\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-3\,ac)}+\left(e^{(4\,bcx+9\,ac)}+6\,e^{(2\,bcx+7\,ac)}\right)e^{(-6\,ac)}\right)e^{(ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 3*a*c
) + (e^(4*b*c*x + 9*a*c) + 6*e^(2*b*c*x + 7*a*c))*e^(-6*a*c))*e^(a*c)/(b*c)
```

3.157 $\int \frac{e^{c(a+bx)}}{\mathbf{sech}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\mathrm{s}}{192bc\sqrt{\mathrm{s}}}$$

```
[Out] -Sech[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) -
(5*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2])
+ (5*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]
) + (5*E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(128*b*c*Sqrt[Sech[a*c + b*c*x]
^2]) + (E^(6*c*(a + b*x))*Sech[a*c + b*c*x])/(192*b*c*Sqrt[Sech[a*c + b*c*x
]^2]) + (5*x*Sech[a*c + b*c*x])/(16*Sqrt[Sech[a*c + b*c*x]^2])
```

Rubi [A] time = 0.196159, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\mathrm{s}}{192bc\sqrt{\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -Sech[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) -
(5*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2])
+ (5*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]
) + (5*E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(128*b*c*Sqrt[Sech[a*c + b*c*x]
^2]) + (E^(6*c*(a + b*x))*Sech[a*c + b*c*x])/(192*b*c*Sqrt[Sech[a*c + b*c*x
]^2]) + (5*x*Sech[a*c + b*c*x])/(16*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right)dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{4c(a}}{128b}
\end{aligned}$$

Mathematica [A] time = 0.103424, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)} - 5e^{-2c(a+bx)} + 10e^{2c(a+bx)} + \frac{5}{2}e^{4c(a+bx)} + \frac{1}{3}e^{6c(a+bx)} + 20bcx\right)\operatorname{sech}^5(c(a+bx))}{64bc\operatorname{sech}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*Sech[c*(a + b*x)]^
5)/(64*b*c*(Sech[c*(a + b*x)]^2)^(5/2))
```

Maple [A] time = 0.204, size = 326, normalized size = 1.3

$$\frac{5\,x e^{c(bx+a)}}{16+16\,e^{2\,c(bx+a)}}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} + \frac{e^{7\,c(bx+a)}}{\left(192+192\,e^{2\,c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} + \frac{5\,e^{5\,c(bx+a)}}{\left(128+128\,e^{2\,c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{e^{2\,c(bx+a)}}{\left(1+e^{2\,c(bx+a)}\right)^2}}} + \frac{}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 5/16/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))
*x*exp(c*(b*x+a))+1/192/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(
1+exp(2*c*(b*x+a)))/c/b*exp(7*c*(b*x+a))+5/128/(1/(1+exp(2*c*(b*x+a)))^2*ex
p(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))/c/b*exp(5*c*(b*x+a))+5/32/(1/(1+
exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))/c/b*exp(3*
c*(b*x+a))-5/64/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*
c*(b*x+a)))/c/b*exp(-c*(b*x+a))-1/128/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*
x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))/c/b*exp(-3*c*(b*x+a))
```

Maxima [A] time = 1.17902, size = 151, normalized size = 0.6

$$\frac{5\,(bcx+ac)}{16\,bc} + \frac{e^{(6\,bcx+6\,ac)}}{192\,bc} + \frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc} + \frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc} - \frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc} - \frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Fricas [A] time = 3.1812, size = 562, normalized size = 2.25

$$-\frac{\cosh(bcx+ac)^5 + 5\,\cosh(bcx+ac)\sinh(bcx+ac)^4 - 5\,\sinh(bcx+ac)^5 - 5\left(10\,\cosh(bcx+ac)^2+9\right)\sinh(bcx+ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12227, size = 149, normalized size = 0.6

$$\frac{\left(120\,bcxe^{(-ac)} - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx-5\,ac)} + \left(2\,e^{(6\,bcx+20\,ac)} + 15\,e^{(4\,bcx+18\,ac)} + 60\,e^{(2\,bcx+16\,ac)}\right)e^{(-1}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x*e^(-a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a
*c) + 1)*e^(-4*b*c*x - 5*a*c) + (2*e^(6*b*c*x + 20*a*c) + 15*e^(4*b*c*x + 1
8*a*c) + 60*e^(2*b*c*x + 16*a*c))*e^(-15*a*c))*e^(a*c)/(b*c)
```

3.158 $\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] (2*x^2)/(21*c^4*Sqrt[Sech[2*Log[c*x]]]) + x^6/(7*Sqrt[Sech[2*Log[c*x]]]) +
(Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*
x], 1/2])/(21*c^5*(c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rubi [A] time = 0.0869211, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 335, 277, 325, 220}

$$\frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```


```
[Out] (2*x^2)/(21*c^4*Sqrt[Sech[2*Log[c*x]]]) + x^6/(7*Sqrt[Sech[2*Log[c*x]]]) +
(Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*
x], 1/2])/(21*c^5*(c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```


Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^6}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^6\,dx, x, cx\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}
$$

Mathematica [C] time = 0.17026, size = 77, normalized size = 0.71

$$
\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\left(\left(c^4x^4+1\right)^{3/2} - {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{7c^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(3/2) - H
ypergeometric2F1[-1/2, 1/4, 5/4, -(c^4*x^4)]))/(7*c^6)
```

Maple [C] time = 0.075, size = 130, normalized size = 1.2

$$\frac{x^{2}\left(3c^{4}x^{4}+2\right)\sqrt{2}}{42c^{4}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}-\frac{\sqrt{2}x}{21c^{4}\left(c^{4}x^{4}+1\right)}\sqrt{1-ic^{2}x^{2}}\sqrt{1+ic^{2}x^{2}}\mathrm{EllipticF}\left(x\sqrt{ic^{2}},i\right)\frac{1}{\sqrt{ic^{2}}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/42*x^2*(3*c^4*x^4+2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-1/21/c^4/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*
(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{5}}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sqrt(sech(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(sech(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

3.159 $\int \frac{x^4}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=28

$$\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\text{sech}(2\log(cx))}}$$

```
[Out] ((c^4 + x^(-4))*x^5)/(6*c^4*Sqrt[Sech[2*Log[c*x]]])
```

Rubi [A] time = 0.0423691, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5551, 5549, 264}

$$\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\text{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] ((c^4 + x^(-4))*x^5)/(6*c^4*Sqrt[Sech[2*Log[c*x]]])
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{\text{sech}(2\log(cx))}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{\sqrt{\text{sech}(2\log(x))}}\,dx,x,cx\right)}{c^5}\\
&= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^5\,dx,x,cx\right)}{c^6\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}\\
&= \frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\text{sech}(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.0472009, size = 44, normalized size = 1.57

$$\frac{\left(c^4x^4+1\right)^2\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] ((1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A] time = 0.035, size = 39, normalized size = 1.4

$$\frac{\sqrt{2}x\left(c^4x^4+1\right)}{12\,c^4}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^4*x^4+1)/c^4
```

Maxima [A] time = 1.81053, size = 41, normalized size = 1.46

$$\frac{\left(\sqrt{2}c^4x^4+\sqrt{2}\right)\sqrt{c^4x^4+1}}{12\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*c^4*x^4 + sqrt(2))*sqrt(c^4*x^4 + 1)/c^5
```

Fricas [B] time = 3.02418, size = 103, normalized size = 3.68

$$\frac{\sqrt{2}\left(c^8x^8+2\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*(c^8*x^8 + 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/(c^6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(sech(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(sech(2*log(c*x))), x)
```

3.160 $\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=203

$$-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot\right.}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log}}$$

```
[Out] 2/(5*c^4*Sqrt[Sech[2*Log[c*x]]]) - 2/(5*c^4*(c^2 + x^(-2))*x^2*Sqrt[Sech[2*
Log[c*x]]]) + x^4/(5*Sqrt[Sech[2*Log[c*x]]]) + (2*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^
(-4))*x*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c
^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^(-4))*x*Sqrt[Se
ch[2*Log[c*x]]])
```

Rubi [A] time = 0.129896, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5551, 5549, 335, 277, 325, 305, 220, 1196}

$$-\frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] 2/(5*c^4*Sqrt[Sech[2*Log[c*x]]]) - 2/(5*c^4*(c^2 + x^(-2))*x^2*Sqrt[Sech[2*
Log[c*x]]]) + x^4/(5*Sqrt[Sech[2*Log[c*x]]]) + (2*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^
(-4))*x*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c
^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^(-4))*x*Sqrt[Se
ch[2*Log[c*x]]])
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$
$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^4\,dx, x, cx\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\operatorname{Subs}}{5c^5\sqrt{1+}}$$
$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\sqrt{\frac{c^4+}{(c^2+}}}{5c^3(}$$

Mathematica [C] time = 0.112481, size = 65, normalized size = 0.32

$$\frac{\left(\frac{c^2x^2}{c^4x^4+1}\right)^{3/2}\left(c^4x^4+1\right)^{3/2}{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{3\sqrt{2}c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-1/2
, 3/4, 7/4, -(c^4*x^4)])/(3*Sqrt[2]*c^4)
```

Maple [C] time = 0.041, size = 134, normalized size = 0.7

$$\frac{x^4\sqrt{2}}{10}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{\frac{i}{5}\sqrt{2}x}{\left(c^4x^4+1\right)c^2}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\left(\operatorname{EllipticF}\left(x\sqrt{ic^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\frac{1}{\sqrt{ic^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/5*I/(I*c^2)^(1/2)*(1-I*c^2*x
^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^2*(EllipticF(x*(I*c^2)^(1/2),I)
-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(sech(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(sech(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

3.161 $\int \frac{x^2}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\mathbf{sech}(2\log(cx))}}+\frac{x^3}{4\sqrt{\mathbf{sech}(2\log(cx))}}$$

```
[Out] x^3/(4*Sqrt[Sech[2*Log[c*x]]]) + ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rubi [A] time = 0.0549709, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 266, 47, 63, 207}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\mathbf{sech}(2\log(cx))}}+\frac{x^3}{4\sqrt{\mathbf{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Sech[2*Log[c*x]]]) + ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^3}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^3\,dx, x, cx\right)}{c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.137535, size = 77, normalized size = 1.15

$$\frac{x\left(c^2x^2\sqrt{c^4x^4+1}+\sinh^{-1}\left(c^2x^2\right)\right)}{4\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 + c^4*x^4] + ArcSinh[c^2*x^2]))/(4*Sqrt[2]*c^2*Sqrt[(c^2
*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] time = 0.053, size = 97, normalized size = 1.5

$$\frac{x^3\sqrt{2}}{8}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{2}x}{8}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}\frac{1}{\sqrt{c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)/(c^4*x^4+1
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Fricas [A] time = 3.01036, size = 194, normalized size = 2.9

$$\frac{2\sqrt{2}\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+\sqrt{2}\log\left(-2\,c^4x^4-2\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + sqrt(2)*log(-
2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(sech(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

3.162 $\int \frac{x}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=87

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] x^2/(3*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^
2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(3*c*(c^4 + x^(-4))*x*Sqrt[Sech[
2*Log[c*x]]])
```

Rubi [A] time = 0.0595054, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5551, 5549, 335, 277, 220}

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^
2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(3*c*(c^4 + x^(-4))*x*Sqrt[Sech[
2*Log[c*x]]])
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^2} \\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^2\,dx, x, cx\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}
$$

Mathematica [C] time = 0.0964022, size = 58, normalized size = 0.67

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-1/2,
1/4, 5/4, -(c^4*x^4)])/c^2
```

Maple [C] time = 0.035, size = 114, normalized size = 1.3

$$\frac{x^2\sqrt{2}}{6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{2}x}{3\,c^4x^4+3}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\operatorname{EllipticF}\left(x\sqrt{ic^2},i\right)\frac{1}{\sqrt{ic^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/3/(I*c^2)^(1/2)*(1-I*c^2*x^2)
^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)
*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(sech(2*log(c*x))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(sech(2*log(c*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(sech(2*log(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(sech(2*log(c*x))), x)
```

3.163 $\int \frac{1}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=59

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] x/(2*Sqrt[Sech[2*Log[c*x]]]) - ArcCsch[c^2*x^2]/(2*c^2*Sqrt[1 + 1/(c^4*x^4)
]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rubi [A] time = 0.0326481, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5545, 5543, 335, 275, 277, 215}

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Sech[2*Log[c*x]]]) - ArcCsch[c^2*x^2]/(2*c^2*Sqrt[1 + 1/(c^4*x^4)
]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5543

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Sech[d*(a
 + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x\,dx, x, cx\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}
$$

Mathematica [A] time = 0.0892578, size = 77, normalized size = 1.31

$$
\frac{x\left(2\sqrt{c^4x^4+1}-2\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (x*(2*Sqrt[1 + c^4*x^4] - 2*ArcTanh[Sqrt[1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(c
^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{\operatorname{sech}(2\ln(cx))}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(1/2),x)
```

[Out] int(1/sech(2*ln(c*x))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(sech(2*log(c*x))), x)

Fricas [B] time = 3.00354, size = 213, normalized size = 3.61

$$\frac{\sqrt{2}cx\log\left(\frac{c^5x^5+2\,cx-2\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{8\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="fricas")

[Out] 1/8*(sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(c^2*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sech(2*ln(c*x))**(1/2),x)

[Out] Integral(1/sqrt(sech(2*log(c*x))), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="giac")

[Out] Timed out

3.164 $\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=36

$$-i\sqrt{\text{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\text{EllipticF}(i\log(cx),2)$$

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Rubi [A] time = 0.0286682, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3771, 2641}

$$-i\sqrt{\text{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}F(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx &= \text{Subst}\left(\int \sqrt{\text{sech}(2x)}\,dx,x,\log(cx)\right)\\
&= \left(\sqrt{\cosh(2\log(cx))}\sqrt{\text{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(2x)}}\,dx,x,\log(cx)\right)\\
&= -i\sqrt{\cosh(2\log(cx))}F(i\log(cx)|2)\sqrt{\text{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.0551606, size = 36, normalized size = 1.

$$-i\sqrt{\text{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\text{EllipticF}(i\log(cx),2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Maple [B] time = 0.417, size = 167, normalized size = 4.6

$$\sqrt{\left(2\left(1/2\,cx+1/2\,\frac{1}{cx}\right)^{2}-1\right)\left(\frac{cx}{2}-\frac{1}{2\,cx}\right)^{2}}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2\,cx}\right)^{2}}\sqrt{-2\left(1/2\,cx+1/2\,\frac{1}{cx}\right)^{2}+1}\mathrm{EllipticF}\left(\frac{cx}{2}+\frac{1}{2\,cx},\sqrt{2}\right)\frac{}{\sqrt{2\left(1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x,x)
```

```
[Out] ((2*(1/2*c*x+1/2/c/x)^2-1)*(1/2*c*x-1/2/c/x)^2)^(1/2)*(-(1/2*c*x-1/2/c/x)^2
)^(1/2)*(-2*(1/2*c*x+1/2/c/x)^2+1)^(1/2)/(2*(1/2*c*x-1/2/c/x)^4+(1/2*c*x-1/
2/c/x)^2)^(1/2)*EllipticF(1/2*c*x+1/2/c/x,2^(1/2))/(1/2*c*x-1/2/c/x)/(2*(1/
2*c*x+1/2/c/x)^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(2 \log(cx))}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x, x)
```

3.165 $\int \frac{\sqrt{\mathbf{sech}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcCsch[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rubi [A] time = 0.0450486, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 275, 215}

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcCsch[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = c\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^2}\,dx, x, cx\right)$$
$$= \left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)$$
$$= -\left(\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\left(\frac{1}{2}\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)$$
$$= -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Mathematica [A] time = 0.118212, size = 55, normalized size = 1.38

$$-\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -((Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*ArcTanh[Sqrt[1 + c^4*x
^4]])/x)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{\operatorname{sech}\left(2\,\ln\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^2, x)
```

Fricas [A] time = 3.04123, size = 124, normalized size = 3.1

$$\frac{1}{4}\,\sqrt{2}c\log\left(\frac{c^5x^5+2\,cx-2\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*c*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c*x^5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.166 $\int \frac{\sqrt{\mathbf{sech}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\mathrm{sech}(2\log(cx))}\mathrm{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}+cx\sqrt{\frac{c^4+}{\left(c^2+\right.}}$$

```
[Out] -(((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/(c^2 + x^(-2))) + c*Sqrt[(c^4 + x
^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticE[2*ArcCot[c*x], 1/2]*Sqr
t[Sech[2*Log[c*x]]] - (c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2
))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rubi [A] time = 0.0992905, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 335, 305, 220, 1196}

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}-\frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\mathrm{sech}(2\log(cx))}F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)+cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/(c^2 + x^(-2))) + c*Sqrt[(c^4 + x
^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticE[2*ArcCot[c*x], 1/2]*Sqr
t[Sech[2*Log[c*x]]] - (c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2
))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx &= c^2\,\mathrm{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^3}\,dx, x, cx\right)\\
&= \left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^4}\,dx, x, cx\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)+\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(\right.\\
&= -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}+c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}
$$

Mathematica [C] time = 0.11815, size = 59, normalized size = 0.43

$$
-\frac{c^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-c^4x^4\right)}{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((c^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^4*x^4)])/(Sqrt[1 + c^4*x^4]*Sq
rt[(c^2*x^2)/(2 + 2*c^4*x^4)]))
```

Maple [C] time = 0.04, size = 134, normalized size = 1.

$$
-\frac{\left(c^4x^4+1\right)\sqrt{2}}{x^2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}+\frac{ic^2\sqrt{2}}{x}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\left(\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}\frac{1}{\sqrt{ic^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^3,x)
```

```
[Out] -(c^4*x^4+1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)+I*c^2/(I*c^2)^(1/2)*(1
-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*(EllipticF(x*(I*c^2)^(1/2),I)-Ellipti
cE(x*(I*c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

3.167 $\int \frac{\sqrt{\mathbf{sech}(2\log(cx))}}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -((c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])/2
```

Rubi [A] time = 0.0403995, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5551, 5549, 261}

$$-\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^4,x]

[Out] -((c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx &= c^3\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^4}\,dx,x,cx\right)\\
&= \left(c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}x^5}\,dx,x,cx\right)\\
&= -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.0380192, size = 33, normalized size = 1.43

$$-\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

```
[Out] -c^2/(2*x*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])
```

Maple [A] time = 0.037, size = 38, normalized size = 1.7

$$-\frac{\sqrt{2}\left(c^4x^4+1\right)}{2\,x^3}\sqrt{\frac{c^2x^2}{c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^4,x)
```

```
[Out] -1/2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x^3*(c^4*x^4+1)
```

Maxima [B] time = 1.60203, size = 57, normalized size = 2.48

$$-\frac{1}{2}\,c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{c^4x^4}+1}}+\frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{c^4x^4}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(sqrt(2)/sqrt(1/(c^4*x^4) + 1) + sqrt(2)/(c^4*x^4*sqrt(1/(c^4*x^4)
 + 1)))
```

Fricas [A] time = 2.96646, size = 81, normalized size = 3.52

$$-\frac{\sqrt{2}\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^4, x)
```

3.168 $\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=80

$$\frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\text{sech}(2\log(cx))}\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}$$

```
[Out] -((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/3 + (c^3*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log
[c*x]]])/6
```

Rubi [A] time = 0.0696465, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 321, 220}

$$\frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\text{sech}(2\log(cx))}F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```


```
[Out] -((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/3 + (c^3*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log
[c*x]]])/6
```


Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx &= c^4\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^5}\,dx, x, cx\right)\\ &= \left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)\\ &= -\left(\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\ &= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{3}\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,\right.\\ &= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xF\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log}\end{aligned}$$

Mathematica [C] time = 0.0982415, size = 65, normalized size = 0.81

$$-\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-c^4x^4\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -(Sqrt[2]*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4]*Hypergeometric2F1
[-3/4, 1/2, 1/4, -(c^4*x^4)])/(3*x^4)
```

Maple [C] time = 0.042, size = 117, normalized size = 1.5

$$-\frac{\left(c^4x^4+1\right)\sqrt{2}}{3\,x^4}\sqrt{\frac{c^2x^2}{c^4x^4+1}}-\frac{c^4\sqrt{2}}{3\,x}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\,\text{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}\frac{1}{\sqrt{ic^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^5,x)
```

```
[Out] -1/3*(c^4*x^4+1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)-1/3*c^4/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*EllipticF(x*(I*c^2)^(1/2),I)*2^
(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

3.169 $\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=122

$$\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] x/(32*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(16*(c^4 + x^(-4))*S
ech[2*Log[c*x]]^(3/2)) + x^9/(12*Sech[2*Log[c*x]]^(3/2)) - ArcTanh[Sqrt[1 +
 1/(c^4*x^4)]]/(32*c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0752549, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5551, 5549, 266, 47, 51, 63, 207}

$$\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(16*(c^4 + x^(-4))*S
ech[2*Log[c*x]]^(3/2)) + x^9/(12*Sech[2*Log[c*x]]^(3/2)) - ArcTanh[Sqrt[1 +
 1/(c^4*x^4)]]/(32*c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{\cdots}{64c^1\cdots}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{S}\cdots}{32c^1\cdots}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\cdots}{32c^1\cdots}$$

Mathematica [A] time = 0.185238, size = 98, normalized size = 0.8

$$\frac{c^3x^3\sqrt{c^4x^4+1}\left(8c^8x^8+14c^4x^4+3\right) - 3cx\sinh^{-1}\left(c^2x^2\right)}{192\sqrt{2}c^9\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(3 + 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSinh[c^2
*x^2])/(192*Sqrt[2]*c^9*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] time = 0.039, size = 121, normalized size = 1.

$$\frac{x^3\left(8c^8x^8+14c^4x^4+3\right)\sqrt{2}}{384c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}} - \frac{\sqrt{2}x}{128c^6}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{c^4x^4+1}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/384*x^3*(8*c^8*x^8+14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 3.25135, size = 242, normalized size = 1.98

$$\frac{2\sqrt{2}\left(8\,c^{13}x^{13}+22\,c^{9}x^{9}+17\,c^{5}x^{5}+3\,cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}+3\sqrt{2}\log\left(-2\,c^{4}x^{4}+2\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)}{768\,c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 + 22*c^9*x^9 + 17*c^5*x^5 + 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 + 2*(c^5*x^5 + c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) - 1))/c^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

3.170 $\int \frac{x^7}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{}{11}$$

```
[Out] 4/(77*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + (6*x^4)/(77*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^8/(11*Sech[2*Log[c*x]]^(3/2)) + (2*Sqrt[(c^4
 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/
(77*c^5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0979203, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 335, 277, 325, 220}

$$\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\text{sech}^{\frac{3}{2}}(2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + (6*x^4)/(77*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^8/(11*Sech[2*Log[c*x]]^(3/2)) + (2*Sqrt[(c^4
 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/
(77*c^5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1+x^4\right)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx$$

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{}{7}$$

2

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + $$

Mathematica [C] time = 0.177372, size = 77, normalized size = 0.55

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\left(\left(c^4x^4+1\right)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{22c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(5/2) - H
ypergeometric2F1[-3/2, 1/4, 5/4, -(c^4*x^4)]))/(22*c^8)
```

Maple [C] time = 0.036, size = 138, normalized size = 1.

$$\frac{x^2 \left(7\,c^8x^8+13\,c^4x^4+4\right)\sqrt{2}}{308\,c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}-\frac{\sqrt{2}x}{77\,c^6\left(c^4x^4+1\right)}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)\frac{1}{\sqrt{ic^2}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/308*x^2*(7*c^8*x^8+13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/77/c^6/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*
EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^7}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sech(2*log(c*x))^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

3.171 $\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=28

$$\frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] ((c^4 + x^(-4))*x^7)/(10*c^4*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0434737, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5551, 5549, 264}

$$\frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 + x^(-4))*x^7)/(10*c^4*Sech[2*Log[c*x]]^(3/2))
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] time = 0.0536219, size = 44, normalized size = 1.57

$$\frac{\left(c^4x^4+1\right)^3\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A] time = 0.033, size = 47, normalized size = 1.7

$$\frac{\sqrt{2}x\left(c^8x^8+2\,c^4x^4+1\right)}{40\,c^6}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^8*x^8+2*c^4*x^4+1)
```

Maxima [A] time = 1.90706, size = 41, normalized size = 1.46

$$\frac{\left(\sqrt{2}c^4x^4+\sqrt{2}\right)\left(c^4x^4+1\right)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 + sqrt(2))*(c^4*x^4 + 1)^(3/2)/c^7
```

Fricas [B] time = 2.92552, size = 122, normalized size = 4.36

$$\frac{\sqrt{2}\left(c^{12}x^{12}+3\,c^8x^8+3\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(2)*(c^12*x^12 + 3*c^8*x^8 + 3*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1))/(c^8*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sech(2*log(c*x))^(3/2), x)
```

3.172 $\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=251

$$-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{15c^3x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx)}$$

```
[Out] -4/(15*c^4*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 4/(1
5*c^4*(c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + (2*x^2)/(15*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^6/(9*Sech[2*Log[c*x]]^(3/2)) + (4*Sqrt[(c^4
+ x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(
15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (2*Sqrt[(c^4 + x^(-4)
)/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(15*c^3*(
c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.14933, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5551, 5549, 335, 277, 325, 305, 220, 1196}

$$\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{4}{15c^4x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\ldots 2$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sech[2*Log[c*x]]^(3/2),x]
```


```
[Out] -4/(15*c^4*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 4/(1
5*c^4*(c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + (2*x^2)/(15*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^6/(9*Sech[2*Log[c*x]]^(3/2)) + (4*Sqrt[(c^4
+ x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(
15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (2*Sqrt[(c^4 + x^(-4)
)/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(15*c^3*(
c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```


Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - $$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - $$

$$= -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{}{15\left(c^4\right.}$$

Mathematica [C] time = 0.118474, size = 65, normalized size = 0.26

$$\frac{\left(\frac{c^2x^2}{c^4x^4+1}\right)^{3/2}\left(c^4x^4+1\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{6\sqrt{2}c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-3/2
, 3/4, 7/4, -(c^4*x^4)])/(6*Sqrt[2]*c^6)
```

Maple [C] time = 0.037, size = 147, normalized size = 0.6

$$\frac{x^4\left(5c^4x^4+11\right)\sqrt{2}}{180\,c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{\frac{i}{15}\sqrt{2}x}{\left(c^4x^4+1\right)c^4}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\frac{1}{\sqrt{ic^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/180*x^4*(5*c^4*x^4+11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/15*I/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^4*(Ellipti
cF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4
*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^5}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sech(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**5/sech(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

3.173 $\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] (3*x)/(16*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(8*Sech[2*Log[c*x]]^
(3/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(16*c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0658133, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 266, 47, 63, 207}

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (3*x)/(16*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(8*Sech[2*Log[c*x]]^
(3/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(16*c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}\\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [A] time = 0.167401, size = 90, normalized size = 0.98

$$
\frac{c^3x^3\sqrt{c^4x^4+1}\left(2c^4x^4+5\right)+3cx\sinh^{-1}\left(c^2x^2\right)}{32\sqrt{2}c^5\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(5 + 2*c^4*x^4) + 3*c*x*ArcSinh[c^2*x^2])/(32*Sq
rt[2]*c^5*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] time = 0.036, size = 113, normalized size = 1.2

$$\frac{x^3\left(2c^4x^4+5\right)\sqrt{2}}{64c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3\sqrt{2}x}{64c^2}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{c^4x^4+1}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/64*x^3*(2*c^4*x^4+5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4+1)^(1
/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\operatorname{sech}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 3.10366, size = 220, normalized size = 2.39

$$\frac{2\sqrt{2}\left(2c^9x^9+7c^5x^5+5cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+3\sqrt{2}\log\left(-2c^4x^4-2\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{128c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 + 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) + 3*sqrt(2)*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) - 1))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/sech(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

3.174 $\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=111

$$-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 2/(7*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^4/(7*Sech[2*Log[c*x]]^(3/2)
) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*Arc
Cot[c*x], 1/2])/(7*c*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0831317, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 277, 220}

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sech[2*Log[c*x]]^(3/2),x]
```


```
[Out] 2/(7*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^4/(7*Sech[2*Log[c*x]]^(3/2)
) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*Arc
Cot[c*x], 1/2])/(7*c*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```


Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4} \\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1+x^4\right)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{7c\left(c^4+\frac{1}{x^4}\right)^2 x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [C] time = 0.106483, size = 61, normalized size = 0.55

$$
\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{2c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
1/4, 5/4, -(c^4*x^4)])/(2*c^4)
```

Maple [C] time = 0.033, size = 129, normalized size = 1.2

$$\frac{x^{2}\left(c^{4}x^{4}+3\right)\sqrt{2}}{28\,c^{2}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{\sqrt{2}x}{\left(7\,c^{4}x^{4}+7\right)c^{2}}\sqrt{1-ic^{2}x^{2}}\sqrt{1+ic^{2}x^{2}}\mathrm{EllipticF}\left(x\sqrt{ic^{2}},i\right)\frac{1}{\sqrt{ic^{2}}}\frac{1}{\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/28*x^2*(c^4*x^4+3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/7/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)
^(1/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{3}}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sech(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/sech(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

3.175 $\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/(2*(c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2)) + x^3/(6*Sech[2*Log[c*x]]^(3/
2)) - ArcCsch[c^2*x^2]/(2*c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^
(3/2))
```

Rubi [A] time = 0.0670216, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 335, 275, 277, 215}

$$\frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 1/(2*(c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2)) + x^3/(6*Sech[2*Log[c*x]]^(3/
2)) - ArcCsch[c^2*x^2]/(2*c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^
(3/2))
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1+x^4\right)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [A] time = 0.168229, size = 88, normalized size = 1.

$$
\frac{x\left(\sqrt{c^4x^4+1}\left(c^4x^4+4\right)-3\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*(Sqrt[1 + c^4*x^4]*(4 + c^4*x^4) - 3*ArcTanh[Sqrt[1 + c^4*x^4]]))/(12*Sq
rt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^{2} \left(\operatorname{sech}\left(2 \ln \left(c x\right)\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/sech(2*ln(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\operatorname{sech}\left(2 \log\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

Fricas [A] time = 3.1002, size = 234, normalized size = 2.66

$$\frac{3 \sqrt{2} c x \log\left(\frac{c^{5} x^{5} + 2 c x - 2 \left(c^{4} x^{4} + 1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4} + 1}}}{c x^{5}}\right) + 2 \sqrt{2} \left(c^{8} x^{8} + 5 c^{4} x^{4} + 4\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4} + 1}}}{48 c^{4} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^
4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^8*x^8 + 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c
^4*x^4 + 1)))/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\operatorname{sech}^{\frac{3}{2}}{\left(2 \log{\left(c x \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/sech(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

3.176 $\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=214

$$-\frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -12/(5*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 6/(5*(c^
4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + x^2/(5*Sech[2*Log[c*x]]^(3/2)) +
(12*c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcC
ot[c*x], 1/2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (6*c*Sqrt[
(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/
2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.116287, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5551, 5549, 335, 277, 305, 220, 1196}

$$\frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[2*Log[c*x]]^(3/2),x]
```


```
[Out] -12/(5*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 6/(5*(c^
4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + x^2/(5*Sech[2*Log[c*x]]^(3/2)) +
(12*c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcC
ot[c*x], 1/2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (6*c*Sqrt[
(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/
2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```


Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + $$

$$= -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(c}$$

Mathematica [C] time = 0.114773, size = 65, normalized size = 0.3

$$-\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-c^4x^4\right)}{2\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -Hypergeometric2F1[-3/2, -1/4, 3/4, -(c^4*x^4)]/(2*Sqrt[2]*c^2*Sqrt[(c^2*x^
2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [C] time = 0.039, size = 159, normalized size = 0.7

$$\frac{\left(c^8x^8-4c^4x^4-5\right)\sqrt{2}}{\left(20c^4x^4+20\right)c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{\frac{3i}{5}\sqrt{2}x}{c^4x^4+1}\sqrt{1-ic^2x^2}\sqrt{1+ic^2x^2}\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\frac{1}{\sqrt{ic^2}}\frac{1}{\sqrt{\frac{c^2}{c^4x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/20*(c^8*x^8-4*c^4*x^4-5)/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/5*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)
*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2
*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{sech}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sech(2*log(c*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\operatorname{sech}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/sech(2*log(c*x))^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x/sech(2*log(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{sech}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/sech(2*log(c*x))^(3/2), x)
```

3.177 $\int \frac{1}{\mathbf{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/(4*(c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2)) + x/(4*Sech[2*Log[c*x]]^(3
/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(4*c^4*(1 + 1/(c^4*x^4))^(3/2)*x^
3*Sech[2*Log[c*x]]^(3/2))
```

Rubi [A] time = 0.0392663, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5545, 5543, 266, 47, 50, 63, 207}

$$-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(-3/2),x]
```


```
[Out] -3/(4*(c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2)) + x/(4*Sech[2*Log[c*x]]^(3
/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(4*c^4*(1 + 1/(c^4*x^4))^(3/2)*x^
3*Sech[2*Log[c*x]]^(3/2))
```


Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5543

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Sech[d*(a
 + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx

Mathematica [C] time = 0.0857202, size = 64, normalized size = 0.7

$$-\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-c^4x^4\right)}{4c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
 -1/2, 1/2, -(c^4*x^4)])/(4*c^4*x^3)
```

Maple [A] time = 0.037, size = 131, normalized size = 1.4

$$\frac{\left(c^8x^8-c^4x^4-2\right)\sqrt{2}}{16\,x\left(c^4x^4+1\right)c^2}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3\,c^2\sqrt{2}x}{16}\ln\left(c^4x^2\frac{1}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\frac{1}{\sqrt{c^4}}\frac{1}{\sqrt{c^4x^4+1}}\frac{1}{\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/16*(c^8*x^8-c^4*x^4-2)/x/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

Fricas [A] time = 3.32069, size = 227, normalized size = 2.47

$$\frac{3\sqrt{2}c^3x^3\log\left(-2\,c^4x^4-2\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)+2\sqrt{2}\left(c^8x^8-c^4x^4-2\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{32\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^
4*x^4 + 1)) - 1) + 2*sqrt(2)*(c^8*x^8 - c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(sech(2*log(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

3.178 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=56

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}+i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)$$

```
[Out] I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Rubi [A] time = 0.035545, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3768, 3771, 2639}

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}+i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx &= \operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(2x)\,dx,x,\log(cx)\right)\\
&= \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))-\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2x)}}\,dx,x,\log(cx)\right)\\
&= \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))-\left(\sqrt{\cosh(2\log(cx))}\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \sqrt{\cosh(2x}\right.\\
&= i\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))}+\sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))
\end{aligned}$$

Mathematica [A] time = 0.100691, size = 45, normalized size = 0.8

$$\frac{\tanh(2\log(cx)) + \frac{iE(i\log(cx)|2)}{\sqrt{\cosh(2\log(cx))}}}{\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] ((I*EllipticE[I*Log[c*x], 2])/Sqrt[Cosh[2*Log[c*x]]] + Tanh[2*Log[c*x]])/Sq
rt[Sech[2*Log[c*x]]]
```

Maple [A] time = 0.314, size = 127, normalized size = 2.3

$$\left(\sqrt{-2\left(1/2\,cx-1/2\,\frac{1}{cx}\right)^{2}-1}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2\,cx}\right)^{2}}\operatorname{EllipticE}\left(\frac{cx}{2}+\frac{1}{2\,cx},\sqrt{2}\right)+2\left(1/2\,cx+1/2\,\frac{1}{cx}\right)\left(1/2\,cx-1/2\,\frac{1}{cx}\right)^{2}\right)\left(\frac{cx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x,x)
```

```
[Out] ((-2*(1/2*c*x-1/2/c/x)^2-1)^(1/2)*(-(1/2*c*x-1/2/c/x)^2)^(1/2)*EllipticE(1/
2*c*x+1/2/c/x,2^(1/2))+2*(1/2*c*x+1/2/c/x)*(1/2*c*x-1/2/c/x)^2)/(1/2*c*x-1/
2/c/x)/(2*(1/2*c*x+1/2/c/x)^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(2*log(c*x))^(3/2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(2\,\log(cx))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x, x)
```

3.179 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] ((c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Rubi [A] time = 0.0395768, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5551, 5549, 261}

$$\frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] ((c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx &= c\,\mathrm{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx,x,cx\right)\\
&= \left(c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^5}\,dx,x,cx\right)\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [A] time = 0.0346873, size = 32, normalized size = 1.28

$$\sqrt{2}c^2x\sqrt{\frac{c^2x^2}{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\operatorname{sech}\left(2\ln\left(cx\right)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

Maxima [A] time = 1.51017, size = 53, normalized size = 2.12

$$c\left(\frac{\sqrt{2}}{\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}}{c^4x^4\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] c*(sqrt(2)/(1/(c^4*x^4) + 1)^(3/2) + sqrt(2)/(c^4*x^4*(1/(c^4*x^4) + 1)^(3/
2)))
```

Fricas [A] time = 3.13494, size = 58, normalized size = 2.32

$$\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^2, x)
```

3.180 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{4c}$$

```
[Out] ((c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2))/2 - ((c^4 + x^(-4))*Sqrt[(c^4 +
 x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x^3*EllipticF[2*ArcCot[c*x], 1/2]
*Sech[2*Log[c*x]]^(3/2))/(4*c)
```

Rubi [A] time = 0.0771525, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 288, 220}

$$\frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] ((c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2))/2 - ((c^4 + x^(-4))*Sqrt[(c^4 +
 x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x^3*EllipticF[2*ArcCot[c*x], 1/2]
*Sech[2*Log[c*x]]^(3/2))/(4*c)
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = c^2\,\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)$$

$$= \left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\ldots}}\right.$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)\operatorname{sech}^{\frac{3}{2}}(\ldots}{4c}$$

Mathematica [C] time = 0.107889, size = 65, normalized size = 0.71

$$\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\left(\sqrt{c^4x^4+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-c^4x^4\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*(1 + Sqrt[1 + c^4*x^4]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(c^4*x^4)])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\operatorname{sech}\left(2\ln(cx)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sech(2*log(c*x))^(3/2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2 \log\left(cx\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

3.181 $\int \frac{\mathbf{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] ((c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2))/2 - (c^6*(1 + 1/(c^4*x^4))^(3/2)*
x^3*ArcCsch[c^2*x^2]*Sech[2*Log[c*x]]^(3/2))/2
```

Rubi [A] time = 0.0574587, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5551, 5549, 335, 275, 288, 215}

$$\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] ((c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2))/2 - (c^6*(1 + 1/(c^4*x^4))^(3/2)*
x^3*ArcCsch[c^2*x^2]*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx &= c^3\,\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)\\
&= \left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2} x^7}\,dx, x, cx\right)\\
&= -\left(\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{\left(1+x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,d\right.\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [C] time = 0.105205, size = 51, normalized size = 0.77

$$\frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};c^4x^4+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] (Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/2,
1 + c^4*x^4])/x
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(\operatorname{sech}\left(2\,\ln(cx)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^4,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^4, x)
```

Fricas [A] time = 3.06589, size = 194, normalized size = 2.94

$$\frac{\sqrt{2}c^3x\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c^3*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4
*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.182 $\int \operatorname{sech}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=63

$$\frac{2e^a x (cx^n)^b \, {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/2
, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rubi [A] time = 0.0586494, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5545, 5547, 263, 364}

$$\frac{2e^a x (cx^n)^b \, {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/2
, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1+e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^a x(cx^n)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

Mathematica [A] time = 0.1466, size = 64, normalized size = 1.02

$$\frac{2e^a x(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, -
(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \operatorname{sech}(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n)),x)
```

```
[Out] int(sech(a+b*ln(c*x^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(b\log(cx^n)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{sech}(b\log(cx^n)+a),x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(sech(a + b*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(b\log\left(cx^n\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

3.183 $\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Rubi [A] time = 0.0698002, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

Mathematica [A] time = 5.41015, size = 126, normalized size = 1.83

$$\frac{x\left(-\frac{e^{2a}(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)}{2bn+1}+{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)+\tanh(a+b\log(cx^n))\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-((E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n), 2 + 1/(2*b
*n), -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)) + Hypergeometric2F1[1, 1/(2*b*
n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Tanh[a + b*Log[c*x^n]]))/(b*
n)
```

Maple [F] time = 1.099, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(a+b\ln(cx^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2x}{bc^{2b}ne^{(2b\log(x^n)+2a)}+bn}+4\int \frac{1}{2\left(bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 4*integrate(1/2/(b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^2, x)
```

3.184 $\int \operatorname{sech}^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=70

$$\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

```
[Out] (8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```

Rubi [A] time = 0.070584, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3,x]
```


```
[Out] (8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```


Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^3}dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^3}dx,x,cx^n\right)}{n}$$
$$= \frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A] time = 0.88645, size = 101, normalized size = 1.44

$$\frac{x\left(2e^a(bn-1)(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)+(bn\tanh(a+b\log(cx^n))+1)\operatorname{sech}(a+b\log(cx^n\right.}{2b^2n^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(2*E^a*(-1 + b*n)*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh
[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Maple [F] time = 1.303, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(a+b\ln(cx^n)))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\left(b^2c^bn^2-c^b\right)\int \frac{e^{(b\log(x^n)+a)}}{8\left(b^2c^{2b}n^2e^{(2b\log(x^n)+2a)}+b^2n^2\right)}dx+\frac{\left(bc^{3b}n+c^{3b}\right)xe^{(3b\log(x^n)+3a)}-\left(bc^bn-c^b\right)xe^{(b\log(x^n)+a)}}{b^2c^{4b}n^2e^{(4b\log(x^n)+4a)}+2b^2c^{2b}n^2e^{(2b\log(x^n)+2a)}+b^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 8*(b^2*c^b*n^2 - c^b)*integrate(1/8*e^(b*log(x^n) + a)/(b^2*c^(2*b)*n^2*e^(
2*b*log(x^n) + 2*a) + b^2*n^2), x) + ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(
x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b
```

```
*log(x^n) + 4*a) + 2*b^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}^{3}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^3, x)
```

3.185 $\int \operatorname{sech}^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Rubi [A] time = 0.0730036, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^4(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^4}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^4}\,dx, x, cx^n\right)}{n}$$
$$= \frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4, \frac{1}{2}\left(4+\frac{1}{bn}\right); \frac{1}{2}\left(6+\frac{1}{bn}\right); -e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

Mathematica [B] time = 13.4016, size = 192, normalized size = 2.78

$$\frac{x\left(\left(8b^2n^2-2\right){}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)+\operatorname{sech}^2(a+b\log(cx^n))\left(\tanh(a+b\log(cx^n))\left(\left(4b^2n^2-1\right)\cosh(2(a+\right.\right.\right.}{12b^3n^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(-2*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n
), 2 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + (-2 + 8*b^2*n^2)*Hypergeometr
ic2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*L
og[c*x^n]]^2*(2*b*n + (-1 + 8*b^2*n^2 + (-1 + 4*b^2*n^2)*Cosh[2*(a + b*Log[
c*x^n])])*Tanh[a + b*Log[c*x^n]])))/(12*b^3*n^3)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(a+b\ln(cx^n)))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\left(4b^2n^2-1\right)\int \frac{1}{48\left(b^3c^{2b}n^3e^{(2b\log(x^n)+2a)}+b^3n^3\right)}\,dx + \frac{\left(2bc^{4b}n+c^{4b}\right)xe^{(4b\log(x^n)+4a)}-2\left(6b^2c^{2b}n^2-bc^{2b}n-c^{2b}\right)xe^{(}}{3\left(b^3c^{6b}n^3e^{(6b\log(x^n)+6a)}+3b^3c^{4b}n^3e^{(4b\log(x^n)+4a)}+3b^3c^{2b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/48/(b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) +
 b^3*n^3), x) + 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) - 2
```

```
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) + 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b^3*n^
3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}^{4}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^4, x)
```

3.186 $\int \left(\left(1-b^2n^2\right)\text{sech}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\text{sech}^3\left(a+b\log\right.\right.$

Optimal. Leaf size=40

$$x\text{sech}\left(a+b\log\left(cx^n\right)\right)+bnx\tanh\left(a+b\log\left(cx^n\right)\right)\text{sech}\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] x*Sech[a + b*Log[c*x^n]] + b*n*x*Sech[a + b*Log[c*x^n]]*Tanh[a + b*Log[c*x^
n]]
```

Rubi [C] time = 0.135373, antiderivative size = 139, normalized size of antiderivative = 3.48, number of steps used = 9, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5545, 5547, 263, 364}

$$\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}+2e^ax(1-bn)\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c*x^n]]
^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*
Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, -(E^(2*a)*(c*x^
n)^(2*b))])/(1 + 3*b*n)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)\right)dx=\left(2b^2n^2\right)\int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)dx+\left(1-\right.$$

$$=\left(2b^2nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a\right.$$

$$=\left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b}}{\left(1+e^{-2a}x\right.}\right.$$

$$=\left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+}}{\left(e^{-2a}+x^2\right.}\right.$$

$$=2e^a(1-bn)x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right.\right.$$

<u>**Mathematica**</u> **[A]** time = 0.286669, size = 29, normalized size = 0.72

$$x\left(bn\tanh\left(a+b\log\left(cx^n\right)\right)+1\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c
*x^n]]^3,x]
```

```
[Out] x*Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh[a + b*Log[c*x^n]])
```

<u>**Maple**</u> **[C]** time = 0.275, size = 509, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*n^2+1)*sech(a+b*ln(c*x^n))+2*b^2*n^2*sech(a+b*ln(c*x^n))^3,x)
```

```
[Out] 2*c^b*(x^n)^b*x/((c^b)^2*((x^n)^b)^2*exp(2*a)*exp(-I*b*Pi*csgn(I*c*x^n)^3)*
exp(I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n
))*exp(-I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))+1)^2*((c^b)^2*((x^n)^b)
^2*b*n*exp(3*a)*exp(-3/2*I*b*Pi*csgn(I*c*x^n)^3)*exp(3/2*I*b*Pi*csgn(I*c*x^
n)^2*csgn(I*c))*exp(3/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(-3/2*I*b*Pi
*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))-exp(a)*exp(-1/2*I*b*Pi*csgn(I*c*x^n)^
3)*exp(1/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(1/2*I*b*Pi*csgn(I*c*x^n)^2
*csgn(I*x^n))*exp(-1/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*b*n+(c^b
)^2*((x^n)^b)^2*exp(3*a)*exp(-3/2*I*b*Pi*csgn(I*c*x^n)^3)*exp(3/2*I*b*Pi*cs
gn(I*c*x^n)^2*csgn(I*c))*exp(3/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(-3
/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))+exp(a)*exp(-1/2*I*b*Pi*csgn(
I*c*x^n)^3)*exp(1/2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(1/2*I*b*Pi*csgn(I
*c*x^n)^2*csgn(I*x^n))*exp(-1/2*I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))
)
```

Maxima [B] time = 2.13237, size = 130, normalized size = 3.25

$$\frac{2\left(\left(bc^{3\,b}n + c^{3\,b}\right)xe^{(3\,b\log(x^n)+3\,a)} - \left(bc^bn - c^b\right)xe^{(b\log(x^n)+a)}\right)}{c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} + 2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="maxima")
```

```
[Out] 2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(
b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) + 2*c^(2*b)*e^(2*b*log(x^n
) + 2*a) + 1)
```

Fricas [B] time = 2.92739, size = 603, normalized size = 15.08

$$\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,(bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)\right.\right.}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="fricas")
```

```
[Out] 2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b*
n*log(x) + b*log(c) + a)^2 - (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)^
3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
inh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a) + 3*cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(2b^{2}n^{2}\operatorname{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)-b^{2}n^{2}+1\right)\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*n**2+1)*sech(a+b*ln(c*x**n))+2*b**2*n**2*sech(a+b*ln(c*x**
n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*sech(a + b*log(c*x**n))**2 - b**2*n**2 + 1)*sech(a +
b*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 2\,b^{2}n^{2}\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{3}-\left(b^{2}n^{2}-1\right)\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="giac")
```

```
[Out] integrate(2*b^2*n^2*sech(b*log(c*x^n) + a)^3 - (b^2*n^2 - 1)*sech(b*log(c*x
^n) + a), x)
```

3.187 $\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=25

$$\frac{2e^{-a}c^6}{\left(\frac{e^{-2a}}{x^2}+c^4\right)^2}$$

```
[Out] (2*c^6)/(E^a*(c^4 + 1/(E^(2*a)*x^2))^2)
```

Rubi [A] time = 0.0389133, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5545, 5547, 261}

$$\frac{2e^{-a}c^6}{\left(\frac{e^{-2a}}{x^2}+c^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*c^6)/(E^a*(c^4 + 1/(E^(2*a)*x^2))^2)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx &= \frac{2\operatorname{Subst}\left(\int x\operatorname{sech}^3(a+2\log(x))\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= \frac{\left(16e^{-3a}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3x^5}\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= \frac{2c^6e^{-a}}{\left(c^4+\frac{e^{-2a}}{x^2}\right)^2}
\end{aligned}$$

Mathematica [B] time = 0.115763, size = 62, normalized size = 2.48

$$-\frac{2(\cosh(a)-\sinh(a))\left(\sinh^2(a)+\cosh^2(a)-2\sinh(a)\cosh(a)+2c^4x^2\right)}{c^2\left(\sinh(a)\left(c^4x^2-1\right)+\cosh(a)\left(c^4x^2+1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (-2*(Cosh[a] - Sinh[a])*(2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((1 + c^4*x^2)*Cosh[a] + (-1 + c^4*x^2)*Sinh[a])^2)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \left(\mathrm{sech}\left(a+2\ln\left(c\sqrt{x}\right)\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

Maxima [B] time = 1.10406, size = 100, normalized size = 4.

$$-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}+\frac{1}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a) + 1/(c^8*
x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Fricas [B] time = 2.87589, size = 104, normalized size = 4.16

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)}{c^{10}x^4e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)/(c^10*x^4*e^(5*a) + 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c*x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c*sqrt(x)))**3, x)
```

Giac [A] time = 1.13385, size = 51, normalized size = 2.04

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}+1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)*e^(-a)/((c^4*x^2*e^(2*a) + 1)^2*c^2)
```

3.188 $\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal. Leaf size=25

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Rubi [A] time = 0.0457748, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5545, 5547, 263, 261}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = -\left((2c^2)\operatorname{Subst}\left(\int \frac{\operatorname{sech}^3(a+2\log(x))}{x^3} dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3 x^9} dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(e^{-2a}+x^4\right)^3} dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= \frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Mathematica [B] time = 0.0960164, size = 64, normalized size = 2.56

$$-\frac{2c^6(\sinh(2a)+\cosh(2a))\left(\sinh(a)\left(c^4-2x^2\right)+\cosh(a)\left(c^4+2x^2\right)\right)}{\left(\sinh(a)\left(c^4-x^2\right)+\cosh(a)\left(c^4+x^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 + 2*x^2)*Cosh[a] + (c^4 - 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((c^4 + x^2)*Cosh[a] + (c^4 - x^2)*Sinh[a])^2
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

```
[Out] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

Maxima [B] time = 1.05959, size = 66, normalized size = 2.64

$$-\frac{2\left(c^{10}e^{(5a)}+2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}+2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Fricas [B] time = 2.85147, size = 107, normalized size = 4.28

$$-\frac{2\left(c^{10}e^{(5a)}+2c^{6}x^{2}e^{(3a)}\right)}{c^{8}e^{(4a)}+2c^{4}x^{2}e^{(2a)}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c/sqrt(x)))**3, x)
```

Giac [A] time = 1.11218, size = 50, normalized size = 2.

$$-\frac{2\left(c^{10}e^{(5a)}+2c^{6}x^{2}e^{(3a)}\right)}{\left(c^{4}e^{(2a)}+x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) + x^2)^2
```

3.189 $\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)dx$

Optimal. Leaf size=89

$$\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] (E^(2*a)*(2 - p)*x*(1 + (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Sech[a - Log[c*x^n
]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rubi [A] time = 0.0894543, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5545, 5549, 261}

$$\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (E^(2*a)*(2 - p)*x*(1 + (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Sech[a - Log[c*x^n
]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(-2+p)}}\right)^p \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}}\right.}{n}$$

$$= \frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 0.715443, size = 57, normalized size = 0.64

$$\frac{(p-2)x\left(e^{2a}\left(cx^n\right)^{\frac{2}{n(p-2)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p))))*Sech[a + Log[c*x^n]/(n*(
-2 + p))]^p)/(2*(-1 + p))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int\left(\operatorname{sech}\left(a+\frac{\ln\left(cx^n\right)}{n\left(-2+p\right)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a+\frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [B] time = 3.20621, size = 1322, normalized size = 14.85

$$\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\right)}{\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2+2\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2+1}\right.\right.}{(p-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*
n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p -
 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 1)))*cosh(
(a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*cosh((a*n*p -
2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh((a*n*p - 2*a*n +
 n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2
*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n
+ n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c)
)/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/
(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(sech(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

3.190 $\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=65

$$\frac{(2-p)x\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] ((2 - p)*x*(1 + 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Sech[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rubi [A] time = 0.0758799, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5545, 5549, 264}

$$\frac{(2-p)x\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Sech[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a-\frac{\log (cx^n)}{n(-2+p)}\right) dx = \frac{\left(x (cx^n)^{-1/n}\right) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \operatorname{sech}^p\left(a-\frac{\log(x)}{n(-2+p)}\right) dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x (cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}} \left(1+e^{-2a} (cx^n)^{\frac{2}{n(-2+p)}}\right)^p \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right) \text{Subst}\left(\int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}} \left(1+\right.\right.}{n}$$
$$= \frac{(2-p)x\left(1+e^{-2a} (cx^n)^{-\frac{2}{n(2-p)}}\right) \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 0.817375, size = 62, normalized size = 0.95

$$\frac{e^{-2a}(p-2)x\left(e^{2a}+(cx^n)^{\frac{2}{n(p-2)}}\right) \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{2n-np}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(E^(2*a) + (c*x^n)^(2/(n*(-2 + p))))*Sech[a + Log[c*x^n]/(2*n -
 n*p)]^p)/(2*E^(2*a)*(-1 + p))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}\left(a-\frac{\ln(cx^n)}{n\left(-2+p\right)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(-a+\frac{\log(cx^n)}{n\left(p-2\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sech(-a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [B] time = 3.12258, size = 1343, normalized size = 20.66

$$(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\,\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\mathrm{lc}}{np-2\,n}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2
*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*
p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*
log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*
p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1)))
*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*cosh(-(
a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh(-(a*n*p
- 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x)
 - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh(-(a*n*p - 2*a*n - n*lo
g(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) - log
(c))/(n*p - 2*n)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}^{p}\left(a-\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(sech(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}\left(a-\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(sech(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

3.191 $\int \frac{\mathrm{sech}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] time = 0.0162663, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3770}

$$\frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{sech}(a+b\log(cx^n))}{x}\,dx &= \frac{\mathrm{Subst}\left(\int \mathrm{sech}(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A] time = 0.0480611, size = 19, normalized size = 1.

$$\frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Maple [A] time = 0.005, size = 20, normalized size = 1.1

$$\frac{\arctan(\sinh(a+b\ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))/x,x)
```

```
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Maxima [A] time = 1.07044, size = 26, normalized size = 1.37

$$\frac{\arctan\left(\sinh\left(b\log\left(cx^{n}\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*log(c*x^n) + a))/(b*n)
```

Fricas [A] time = 3.13184, size = 112, normalized size = 5.89

$$\frac{2\arctan\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))
/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))/x, x)
```

Giac [A] time = 1.12854, size = 36, normalized size = 1.89

$$\frac{2\arctan\left(\frac{c^{2b}x^{bn}e^{a}}{c^{b}}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 2*arctan(c^(2*b)*x^(b*n)*e^a/c^b)/(b*n)
```

3.192 $\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\tanh(a+b\log(cx^n))}{bn}$$

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] time = 0.0276421, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3767, 8}

$$\frac{\tanh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{i\operatorname{Subst}\left(\int 1\,dx, x, -i\tanh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\tanh(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A] time = 0.0585788, size = 18, normalized size = 1.

$$\frac{\tanh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Maple [A] time = 0.013, size = 19, normalized size = 1.1

$$\frac{\tanh\left(a+b\ln\left(cx^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] tanh(a+b*ln(c*x^n))/b/n
```

Maxima [A] time = 1.20167, size = 38, normalized size = 2.11

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Fricas [B] time = 3.03557, size = 219, normalized size = 12.17

$$-\frac{2}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2/x, x)
```

Giac [A] time = 1.15309, size = 38, normalized size = 2.11

$$-\frac{2}{\left(c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}+1\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)*b*n)
```

3.193 $\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Rubi [A] time = 0.0396532, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{2n}\\
&= \frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{2bn}
\end{aligned}$$

Mathematica [A] time = 0.0552964, size = 55, normalized size = 1.

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Maple [A] time = 0.02, size = 51, normalized size = 0.9

$$\frac{\mathrm{sech}\left(a+b\ln\left(cx^{n}\right)\right)\tanh\left(a+b\ln\left(cx^{n}\right)\right)}{2\,bn}+\frac{\arctan\left(\mathrm{e}^{a+b\ln(cx^{n})}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))/b/n+1/b/n*arctan(exp(a+b*ln(c*x
^n)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\,c^{b}\int\frac{e^{(b\log(x^{n})+a)}}{8\left(c^{2\,b}xe^{(2\,b\log(x^{n})+2\,a)}+x\right)}\,dx+\frac{c^{3\,b}e^{(3\,b\log(x^{n})+3\,a)}-c^{b}e^{(b\log(x^{n})+a)}}{bc^{4\,b}ne^{(4\,b\log(x^{n})+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^{n})+2\,a)}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 8*c^b*integrate(1/8*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a) +
x), x) + (c^(3*b)*e^(3*b*log(x^n) + 3*a) - c^b*e^(b*log(x^n) + a))/(b*c^(4*
b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Fricas [B] time = 3.1923, size = 1486, normalized size = 27.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] (cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (cosh(b
*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + (3*cosh(b*n
*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*l
og(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(a + b\log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3/x, x)
```

Giac [B] time = 1.17208, size = 155, normalized size = 2.82

$$c^{3\,b}\left(\frac{\arctan\left(\frac{c^{2\,b}x^{bn}e^{a}}{c^{b}}\right)e^{(-3\,a)}}{bc^{2\,b}c^{b}n} + \frac{\left(c^{2\,b}x^{3\,bn}e^{(2\,a)} - x^{bn}\right)e^{(-2\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1\right)^{2}bc^{2\,b}n}\right)e^{(3\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] c^(3*b)*(arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-3*a)/(b*c^(2*b)*c^b*n) + (c^(2
*b)*x^(3*b*n)*e^(2*a) - x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^
2*b*c^(2*b)*n))*e^(3*a)
```

3.194 $\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh(a+b\log(cx^n))}{bn}-\frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] time = 0.0337831, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3767}

$$\frac{\tanh(a+b\log(cx^n))}{bn}-\frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^4(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{i\operatorname{Subst}\left(\int \left(1+x^2\right)dx,x,-i\tanh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\tanh(a+b\log(cx^n))}{bn}-\frac{\tanh^3(a+b\log(cx^n))}{3bn}\end{aligned}$$

Mathematica [A] time = 0.0475731, size = 42, normalized size = 1.

$$\frac{\tanh(a+b\log(cx^n))}{bn}-\frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A] time = 0.017, size = 36, normalized size = 0.9

$$\frac{\tanh(a+b\ln(cx^n))}{bn}\left(\frac{2}{3}+\frac{(\operatorname{sech}(a+b\ln(cx^n)))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/n/b*(2/3+1/3*sech(a+b*ln(c*x^n))^2)*tanh(a+b*ln(c*x^n))
```

Maxima [B] time = 1.20158, size = 123, normalized size = 2.93

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}+3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) + b*n)
```

Fricas [B] time = 3.05937, size = 869, normalized size = 20.69

$$-\frac{}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+bn\,s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**4/x, x)
```

Giac [A] time = 1.17025, size = 63, normalized size = 1.5

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{3}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^3*b
*n)
```

3.195 $\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$\frac{3\tan^{-1}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{\tanh(a+b\log(cx^n))\operatorname{sech}^3(a+b\log(cx^n))}{4bn}+\frac{3\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\ldots}{8bn}$$

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```

Rubi [A] time = 0.0571088, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{3\tan^{-1}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{\tanh(a+b\log(cx^n))\operatorname{sech}^3(a+b\log(cx^n))}{4bn}+\frac{3\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\ldots}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^5/x,x]
```


```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```


Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^5(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\operatorname{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{4bn}+\frac{3\operatorname{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx,x,\log(c\ldots\right.}{4n}\\ &= \frac{3\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}+\frac{\operatorname{sech}^3(a+b\log(cx^n))\tanh(a+b\log\ldots}{4bn}\\ &= \frac{3\tan^{-1}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{3\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}+\operatorname{sec}\ldots\end{aligned}$$

Mathematica [A] time = 0.0891788, size = 75, normalized size = 0.84

$$\frac{3\tan^{-1}(\sinh(a+b\log(cx^n)))+2\tanh(a+b\log(cx^n))\operatorname{sech}^3(a+b\log(cx^n))+3\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\ldots}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]] + 3*Sech[a + b*Log[c*x^n]]*Tanh[a + b*Log
[c*x^n]] + 2*Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^n]])/(8*b*n)
```

Maple [A] time = 0.022, size = 84, normalized size = 0.9

$$\frac{(\operatorname{sech}(a+b\ln(cx^n)))^3\tanh(a+b\ln(cx^n))}{4bn}+\frac{3\operatorname{sech}(a+b\ln(cx^n))\tanh(a+b\ln(cx^n))}{8bn}+\frac{3\arctan\left(e^{a+b\ln(cx^n)}\right)}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n+3/8*sech(a+b*ln(c*x^n))*t
anh(a+b*ln(c*x^n))/b/n+3/4/b/n*arctan(exp(a+b*ln(c*x^n)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$96\,c^b\int\frac{e^{(b\log(x^n)+a)}}{128\left(c^{2b}xe^{(2b\log(x^n)+2a)}+x\right)}\,dx+\frac{3\,c^{7b}e^{(7b\log(x^n)+7a)}+11\,c^{5b}e^{(5b\log(x^n)+5a)}-11\,c^{3b}e^{(3b\log(x^n)+3a)}-3\,c^{b}}{4\left(bc^{8b}ne^{(8b\log(x^n)+8a)}+4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}+4\,bc^{2b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 96*c^b*integrate(1/128*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a)
 + x), x) + 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) + 11*c^(5*b)*e^(5*b*log(x
^n) + 5*a) - 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) - 3*c^b*e^(b*log(x^n) + a))/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n)
```

Fricas [B] time = 3.31076, size = 4298, normalized size = 48.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*n*log(x) + b*log(c) + a)^7 + 21*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 3*sinh(b*n*log(x) + b*log(c) + a)^7
+ (63*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(c) +
a)^5 + 11*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*(21*cosh(b*n*log(x) + b*log
(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c)
 + a)^4 + (105*cosh(b*n*log(x) + b*log(c) + a)^4 + 110*cosh(b*n*log(x) + b*
log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 11*cosh(b*n*log(x)
+ b*log(c) + a)^3 + (63*cosh(b*n*log(x) + b*log(c) + a)^5 + 110*cosh(b*n*lo
```

```
g(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b
*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x
) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cosh(b*n*l
og(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*cosh(b*n
*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6*cosh(b
*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*c
osh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(
b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)^2 +
 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(b*n*log(x) + b*log(c) + a)^5
 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*s
inh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) + (21*cosh(b*n*log(x) + b*log(c) + a)^6
+ 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(b*n*log(x) + b*log(c) + a)
^2 - 3)*sinh(b*n*log(x) + b*log(c) + a) - 3*cosh(b*n*log(x) + b*log(c) + a)
)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sinh(b*n*log(x) + b*log(c) +
a)^8 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b
*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(
35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*b*n*co
sh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*si
nh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^
6 + 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*lo
g(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*
n*log(x) + b*log(c) + a)^7 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**5/x, x)
```

Giac [A] time = 1.17997, size = 205, normalized size = 2.3

$$\frac{1}{4}\,c^{5\,b}\left(\frac{3\,\arctan\left(\frac{c^{2\,b}x^{bn}e^{a}}{c^{b}}\right)e^{(-5\,a)}}{bc^{4\,b}c^{b}n}+\frac{\left(3\,c^{6\,b}x^{7\,bn}e^{(6\,a)}+11\,c^{4\,b}x^{5\,bn}e^{(4\,a)}-11\,c^{2\,b}x^{3\,bn}e^{(2\,a)}-3\,x^{bn}\right)e^{(-4\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{4}bc^{4\,b}n}\right)e^{(5\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/4*c^(5*b)*(3*arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-5*a)/(b*c^(4*b)*c^b*n) +
 (3*c^(6*b)*x^(7*b*n)*e^(6*a) + 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x
^(3*b*n)*e^(2*a) - 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b
*c^(4*b)*n))*e^(5*a)
```

3.196 $\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}i\left(a+b\right.\right.}{3bn}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sech[a + b*Log[c*x^n]]^(3/2)*
Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rubi [A] time = 0.0614106, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right.\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sech[a + b*Log[c*x^n]]^(3/2)*
Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+b\log\right.}{}$$
$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{3bn} + 2\operatorname{sech}^{\frac{3}{2}}(a$$

Mathematica [A] time = 0.161581, size = 74, normalized size = 0.76

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\left(\sinh(a+b\log(cx^n)) - i\cosh^{\frac{3}{2}}(a+b\log(cx^n))\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Sech[a + b*Log[c*x^n]]^(3/2)*((-I)*Cosh[a + b*Log[c*x^n]]^(3/2)*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(3*b*n)
```

Maple [B] time = 0.47, size = 295, normalized size = 3.

$$\frac{2}{3bn}\left(2\sqrt{-(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sqrt{-2\,(\sinh(a/2+1/2\,b\ln(cx^n)))^2-1}\operatorname{EllipticF}\left(\cosh(a/2+1/2\,b\ln(cx^n)),\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x
^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1
/2*b*ln(c*x^n))^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2
*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*c
osh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)*((2*cosh(1/2*a+1/
2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*
b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(
c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.197 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=93

$$\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}+\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\right|}{bn}$$

```
[Out] ((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]*
Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[
a + b*Log[c*x^n]])/(b*n)
```

Rubi [A] time = 0.0721233, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2639}

$$\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}+\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\right|}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```


```
[Out] ((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]*
Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[
a + b*Log[c*x^n]])/(b*n)
```


Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn} - \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+\right.}{}$$

$$= \frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn} + \frac{2\sqrt{\operatorname{sech}}}{}$$

Mathematica [A] time = 0.08843, size = 72, normalized size = 0.77

$$\frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(\sinh(a+b\log(cx^n)) + i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*Sqrt[Sech[a + b*Log[c*x^n]]]*(I*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(
I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(b*n)
```

Maple [A] time = 0.414, size = 141, normalized size = 1.5

$$2\,\frac{\operatorname{EllipticE}\left(\cosh(a/2+1/2\,b\ln(cx^n)),\sqrt{2}\right)\sqrt{-(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sqrt{-2\,(\sinh(a/2+1/2\,b\ln(cx^n)))^2-1+}}{n\sinh(a/2+1/2\,b\ln(cx^n))\sqrt{2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 2/n*(EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*(-sinh(1/2*a+1/2*b*ln(c
*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)+2*cosh(1/2*a+1/2
*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.198 $\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right),2\right)}{bn}$$

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0692647, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0661949, size = 58, normalized size = 1.

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right),2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Maple [B] time = 0.268, size = 183, normalized size = 3.2

$$2\,\frac{\sqrt{\left(2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1\right)(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sqrt{-(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sqrt{-2\,(\cosh(a/2+1/\cdots}}{n\sqrt{2\,(\sinh(a/2+1/2\,b\ln(cx^n)))^4+(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sinh(a/2+1/2\,b\ln(cx^n))\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.199 $\int \frac{1}{x\sqrt{\mathbf{sech}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0557946, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2639}

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx,x,\log\left(cx\right.\right.}{n}\\ &= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0703197, size = 58, normalized size = 1.

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n*Sqrt[Cosh[a + b*Log[c*
x^n]]]*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Maple [B] time = 0.297, size = 183, normalized size = 3.2

$$-2\frac{\sqrt{\left(2\left(\cosh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}-1\right)\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-2\left(\cosh\left(a/2\right.\right.}}{n\sqrt{2\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{4}+\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sech(a + b*log(c*x**n)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

3.200 $\int \frac{1}{x\,\mathrm{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\mathrm{sech}(a+b\log(cx^n))}}-\frac{2i\sqrt{\mathrm{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,\mathrm{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(3*b*
n*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0728327, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\mathrm{sech}(a+b\log(cx^n))}}-\frac{2i\sqrt{\mathrm{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(3*b*
n*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\text{sech}(a+bx)}dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\right.}{3n}$$
$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\sinh}{3bn\sqrt{\text{sec}}}$$

Mathematica [A] time = 0.111447, size = 76, normalized size = 0.78

$$\frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(\sinh(2(a+b\log(cx^n)))-2i\sqrt{\cosh(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

Maple [A] time = 0.287, size = 237, normalized size = 2.4

$$\frac{2}{3\,bn}\sqrt{\left(2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1\right)\left(\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)^2}\left(4\,(\cosh(a/2+1/2\,b\ln(cx^n)))^5-6\,(\cosh(a/2+1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{sech}(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\,\text{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

3.201 $\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh(a+b\log(cx^n))}{5bn\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{6i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(5*b*
n*Sech[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] time = 0.0702866, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh(a+b\log(cx^n))}{5bn\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{6i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(5*b*
n*Sech[a + b*Log[c*x^n]]^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{5}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(a+bx)}}dx, x, \log(cx^n)\right)}{5n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\left(3\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Sub}}{5n}$$

$$= -\frac{6i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{5bn} + \frac{2\text{si}}{5bns}$$

Mathematica [A] time = 0.128879, size = 87, normalized size = 0.9

$$\frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(\sinh(a+b\log(cx^n))+\sinh(3(a+b\log(cx^n)))-12i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log\right.\right.}{10bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-12*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Ellipti
cE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]] + Sinh[3*(a + b*Lo
g[c*x^n])]))/(10*b*n)
```

Maple [B] time = 0.345, size = 256, normalized size = 2.6

$$\frac{2}{5\,bn}\sqrt{\left(2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1\right)\left(\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)^2}\left(8\,(\cosh(a/2+1/2\,b\ln(cx^n)))^7-16\,(\cosh(a/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*c
osh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*c
osh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n))
,2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2
*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{sech}(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\,\text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\,\text{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
            print("both result and optimal complex");
         fi;
         #both result and optimal complex
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      else #result contains complex but optimal is not
         if debug then
            print("result contains complex but optimal is not");
         fi;
         return "C";
      end if
   else # result do not contain complex
        # this assumes optimal do not as well
        if debug then
            print("result do not contain complex, this assumes optimal do
 not as well");
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
           if debug then
              print("leaf_count_result<=2*leaf_count_optimal");
           fi;
           return "A";
        else
           if debug then
              print("leaf_count_result>2*leaf_count_optimal");
           fi;
           return "B";
        end if
   end if
 else #ExpnType(result) > ExpnType(optimal)
   if debug then
      print("ExpnType(result) > ExpnType(optimal)");
   fi;
   return "C";
 end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```